

8-2021

## Predicting Acoustical Performance of High Surface Area Particle Stacks with a Poro-Elastic Model

Zhuang Mo  
*Purdue University, mo26@purdue.edu*

Guochenhao Song  
*Purdue University, song520@purdue.edu*

J Stuart Bolton  
*Purdue University, bolton@purdue.edu*

Seungkyu Lee  
*3M Company*

Tongyang Shi  
*3M Company, charlesshi720@gmail.com*

*See next page for additional authors*

Follow this and additional works at: <https://docs.lib.purdue.edu/herrick>

---

Mo, Zhuang; Song, Guochenhao; Bolton, J Stuart; Lee, Seungkyu; Shi, Tongyang; and Seo, Yongbeom, "Predicting Acoustical Performance of High Surface Area Particle Stacks with a Poro-Elastic Model" (2021). *Publications of the Ray W. Herrick Laboratories*. Paper 251.  
<https://docs.lib.purdue.edu/herrick/251>

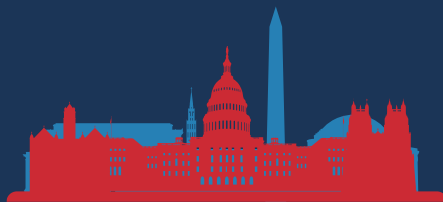
---

**Authors**

Zhuang Mo, Guochenhao Song, J Stuart Bolton, Seungkyu Lee, Tongyang Shi, and Yongbeom Seo

**inter-noise 2021**

**Predicting Acoustical  
Performance of High Surface  
Area Particle  
Stacks with a Poro-Elastic Model  
2437**



**inter-noise**  
**2021** 1 - 4 AUGUST

Zhuang Mo, Guochen hao Song, J. Stuart Bolton  
Ray W. Herrick Laboratories, Purdue University  
Seungkyu Lee, Tongyang Shi, Yongbeom Seo  
3M Company

[internoise2021.org](http://internoise2021.org)

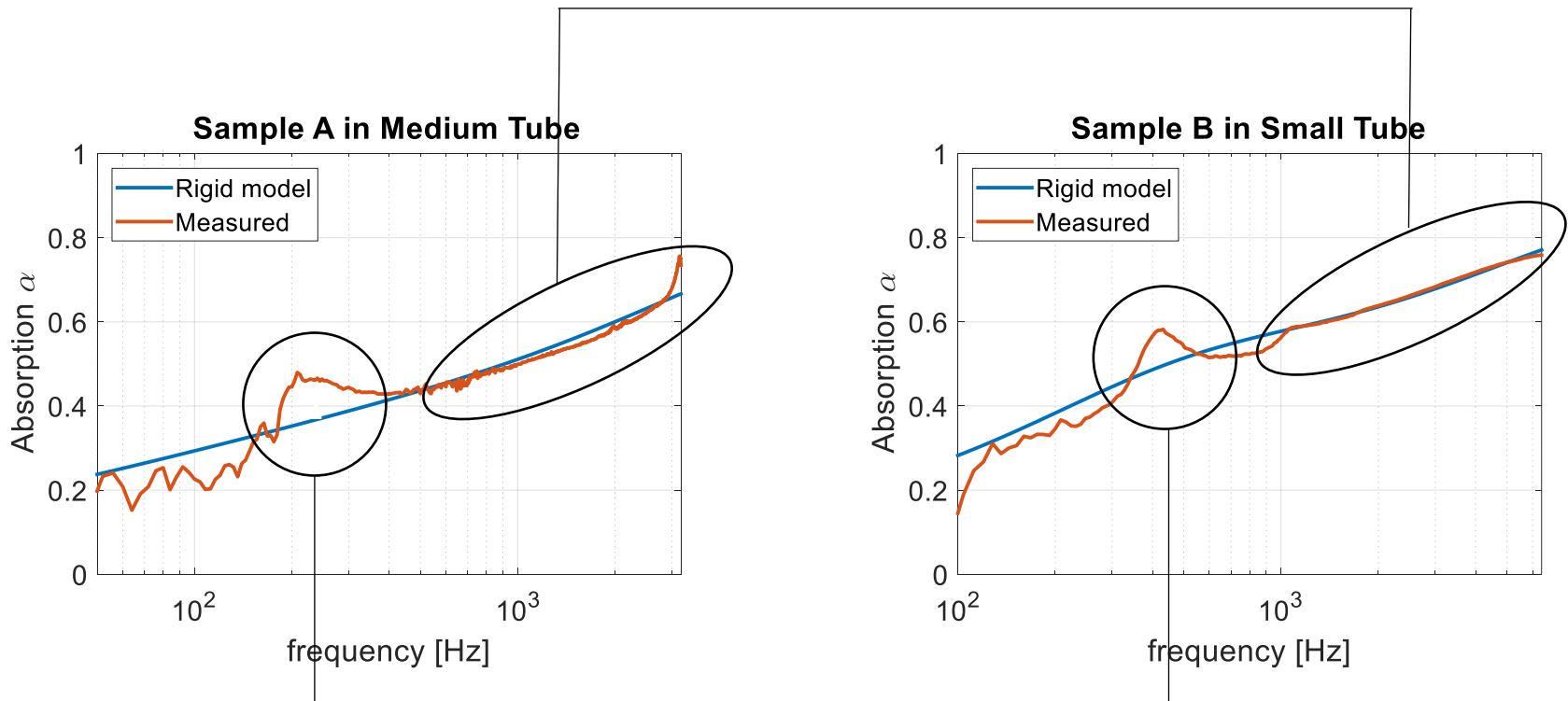


# CONTENT

- Granular Activated Carbon (GAC)  
A review of rigid model for triple porosity particles
- Measurement  
The two-microphone measurement of rigidly backed particle stacks
- Poro-Elastic Model and Its Stable Approach  
The proposed poro-elastic model for GAC
- Particle Swarm Optimization  
Introduce the settings of the fitting procedure
- Fitting Result  
Show fitted results of two types of particles and compare them with corresponding measurement
- Conclusion

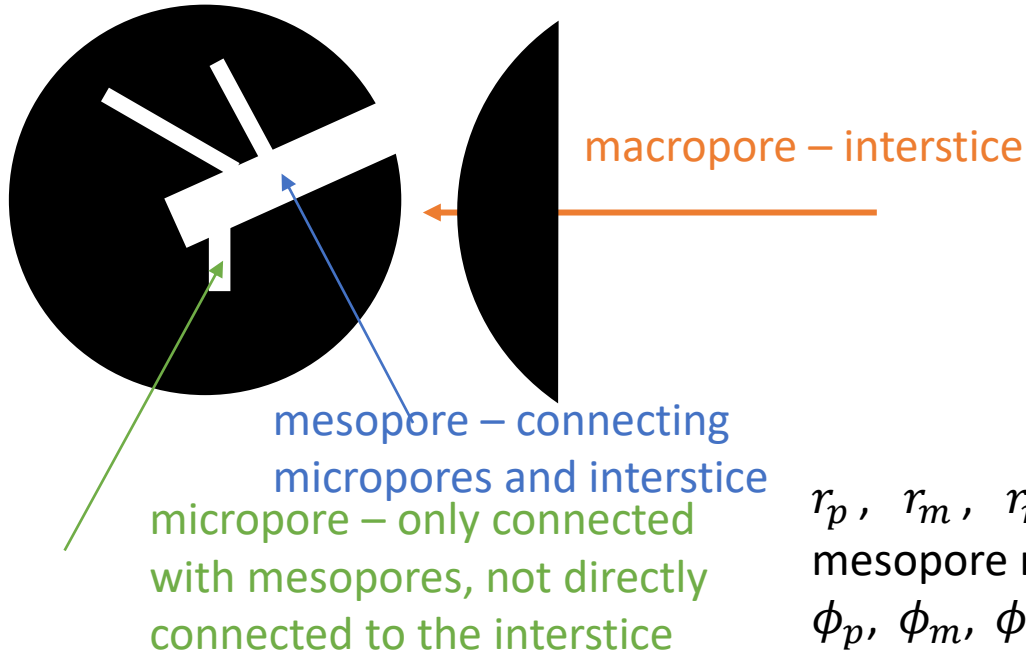
# WHY PORO-ELASTIC MODEL

asymptotic behavior matches



resonance peak not predicted

## TRIPLE POROSITY MODEL



Such material shows excellent low frequency absorption due to its sorption process inside the pores, which brings this material into our interest to further study its properties.

The bulk modulus of different scales are connected in series,

$$B = \left( \frac{1}{B_p} + \frac{1 - \phi_p}{B_u} F_d \right)^{-1}$$

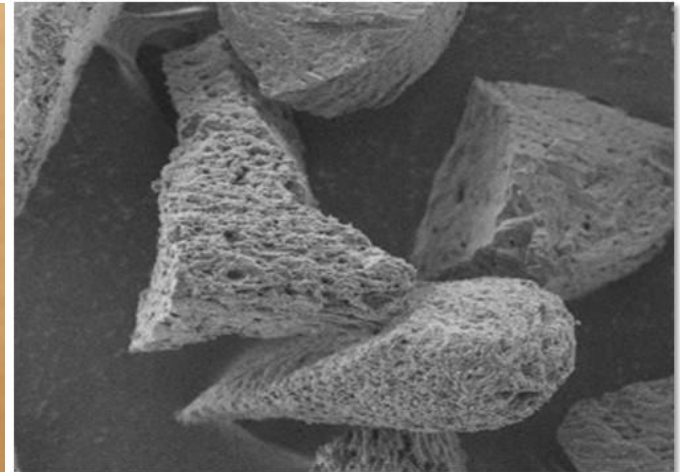
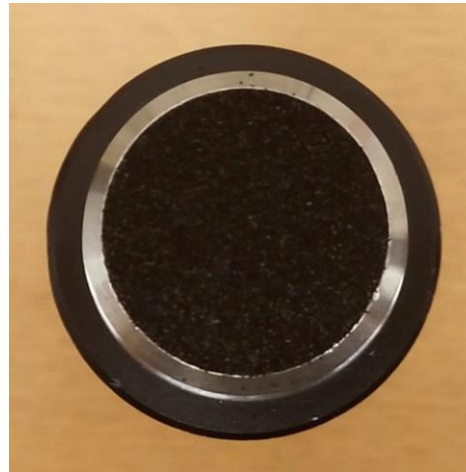
$$B_u = \left( \frac{1}{B_m} + \frac{1 - \phi_m}{B_n} F_{nm} \right)^{-1}$$

$r_p$ ,  $r_m$ ,  $r_n$  denote the particle radius, mesopore radius, and micropore radius.

$\phi_p$ ,  $\phi_m$ ,  $\phi_n$ , and  $\phi_{tb}$  denote the porosity on intergranular scale, mesoscale, micro-scale, and the overall porosity. The relation between the porosities on different scales:

$$\phi_{tb} = \phi_p + (1 - \phi_p)[\phi_m + (1 - \phi_m)\phi_n]$$

# MEASUREMENT



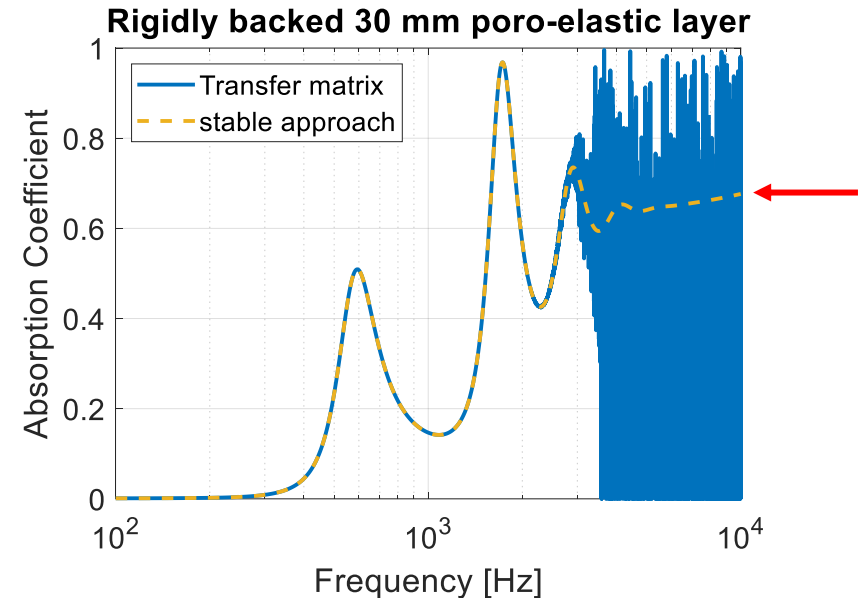
The particle stacks were measured in vertically-positioned standing wave tube with two microphones, following standard E1050.

The stacks were 30mm-thick, and rigidly backed.

The test result in medium tube is valid up to 3200 Hz, while in small tube it is valid up to 6400 Hz.

# PORO-ELASTIC MODEL

- Three waves are propagating in the porous material:
  - Compressional wave in frame
  - Compressional wave in fluid phase
  - Shear wave in frame
- The poroelastic model was built based on the stable approach, proposed by Dazel, Groby, Brouard, and Potel in 2013.
- By comparing the absorption coefficient obtained from the transfer matrix approach and the stabilized approach, one can find perfect matching at low frequencies, before the transfer matrix approach begins to diverge.

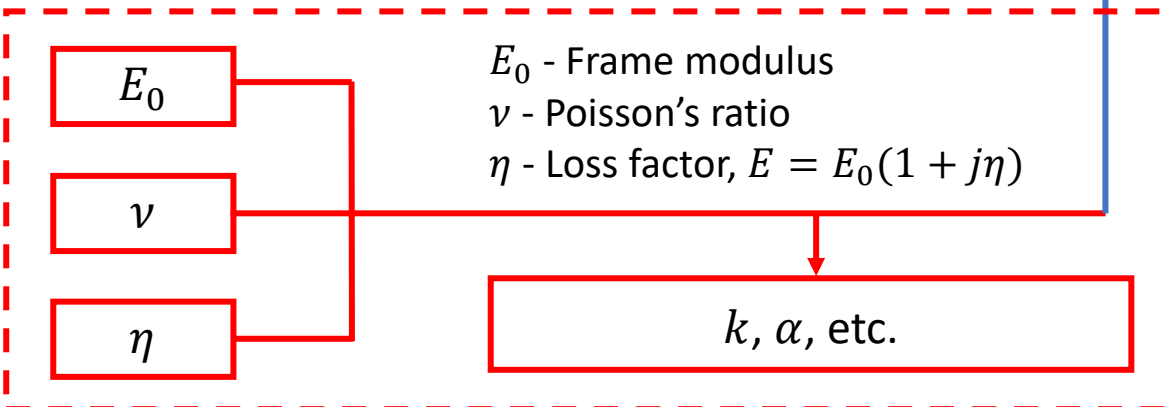
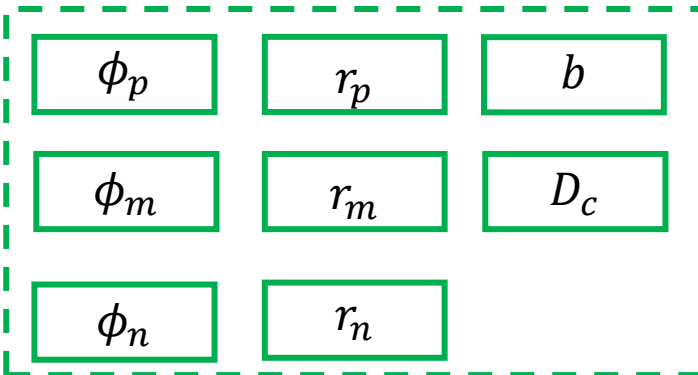


$\sigma$ [rayls/m]	$\phi$	$\alpha_\infty$	$\rho_b$ [kg/m <sup>3</sup> ]
$1.5 \times 10^6$	0.92	1.3	24
$E$ [Pa]	Loss factor	$\nu$	$\theta$
6000	0.004	0.27	0



# PORO-ELASTIC MODEL

## GAC model - rigid



$b$  - Langmuir constant

$D_c$  - Configurational diffusivity

$\phi_p$  - Intergranular porosity

$\phi_m$  - Mesoscale porosity

$\phi_n$  - Microscale porosity

$r_p$  - Particle radius

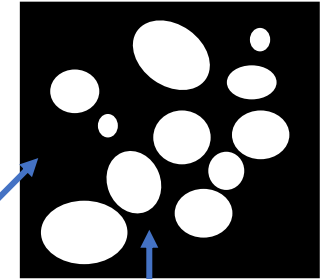
$r_m$  - Mesopore radius

$r_n$  - Micropore radius

$K_f$  - Fluid phase bulk modulus  $K_f, \rho_f$

$\rho_{eq}$  - Fluid equivalent density

$\rho_b$  - Frame bulk density

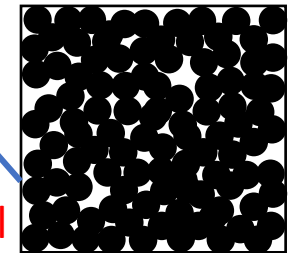


Frame

$K_s, \rho_s$

Cavities/Pores

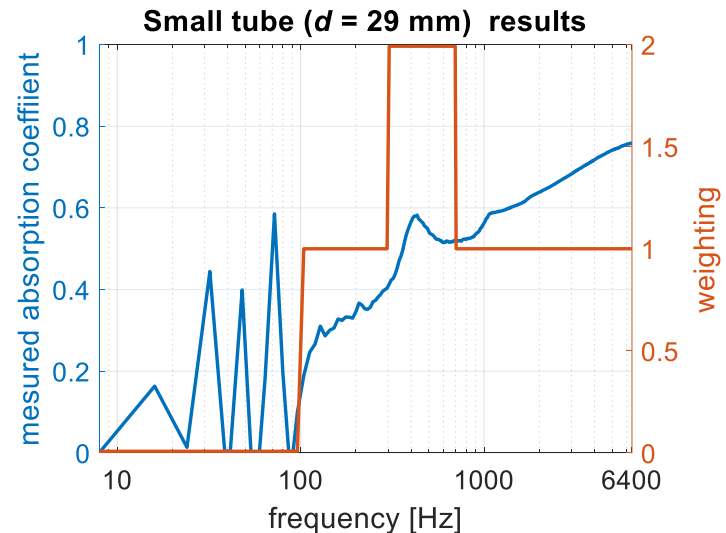
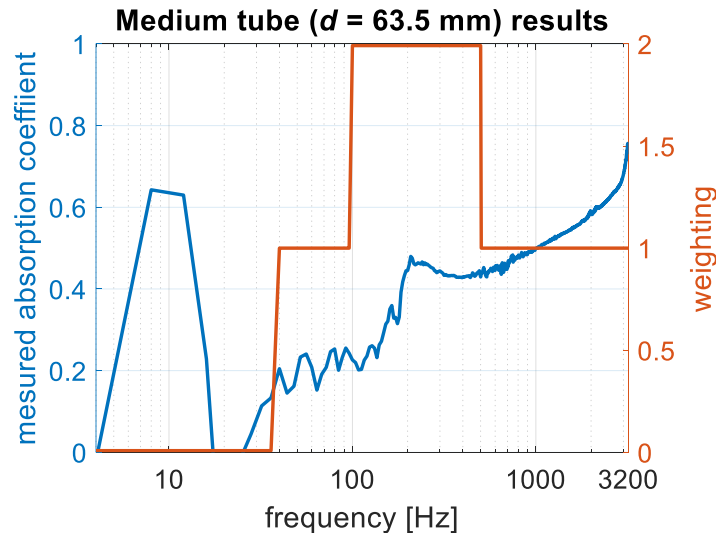
Activated  
carbon  
particles



poroelastic model

# PARTICLE SWARM OPTIMIZATION

All parameters are fitted with constrained particle swarm algorithm, which is realized by a package available at <https://github.com/sdnchen/psomatlab>



$\min_{\mathbf{x}}$   
s. t.

$$f(\mathbf{x}) = \mathbf{w}^T(\mathbf{a} - \mathbf{a}_m)^2/N$$

$$\mathbf{x}_{lb} < \mathbf{x} < \mathbf{x}_{ub}$$

$$\rho_{lb} < \rho_c \left( 1 - \phi_p - (1 - \phi_p)(\phi_m + (1 - \phi_m)\phi_n) \right) < \rho_{ub}$$

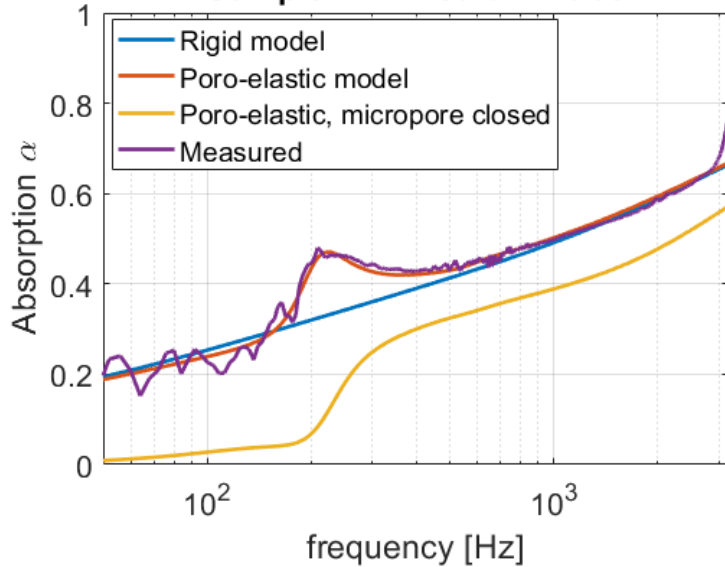
$$0.95\rho_m$$

Where  $\rho_m$  is measured bulk density

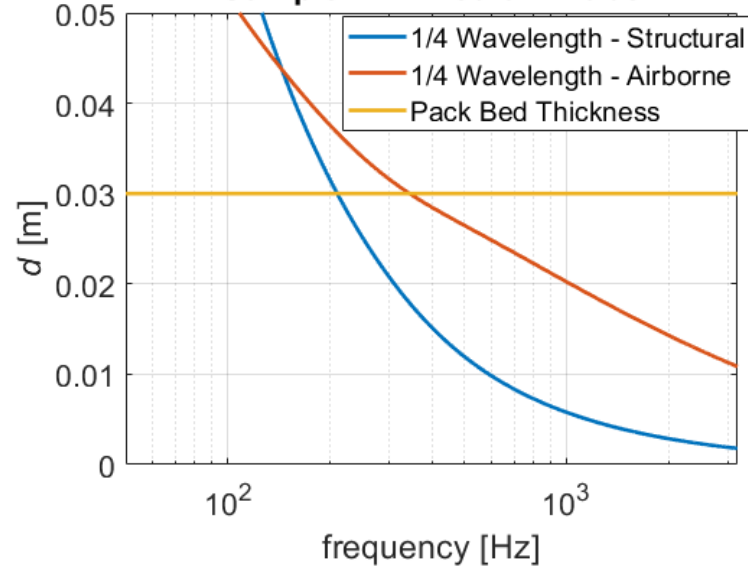
$$1.05\rho_m$$

# FITTING RESULTS

### Sample A in Medium Tube



### Sample A in Medium Tube

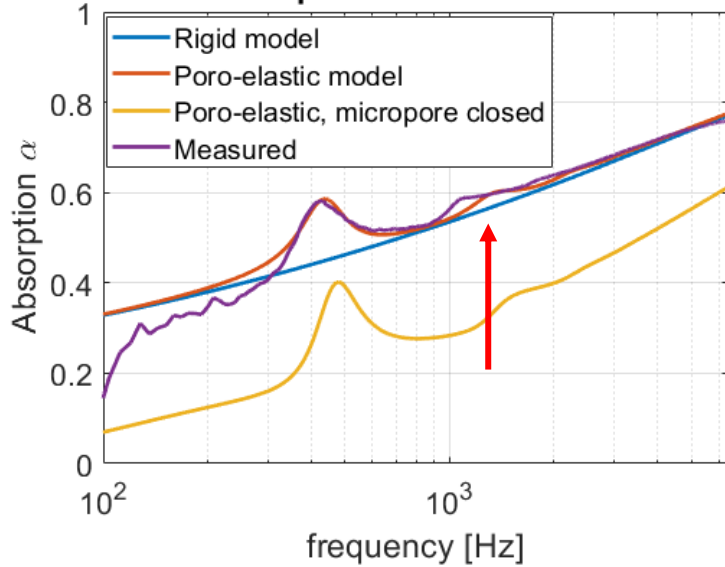


$E_0$ [Pa]	$2.3090 \times 10^5$
$\eta$	0.2963
$\nu$	0.0357

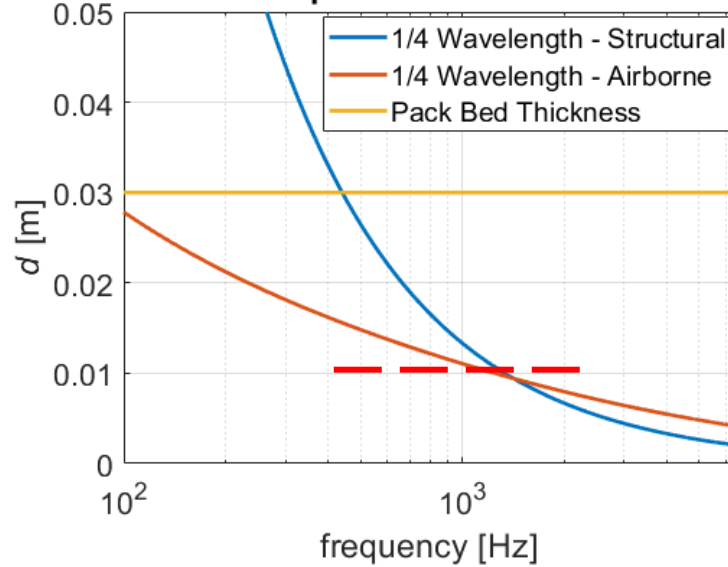
	$r_p$ [mm]	$r_m$ [ $\mu\text{m}$ ]	$r_n$ [nm]	$\phi_p$	$\phi_m$	$\phi_n$	$b$ [ $\text{Pa}^{-1}$ ]	$D_c$ [ $\text{m}^2/\text{s}$ ]
Lower bound	0.105	0.01	0.2	0.260	0.1	0.3	$5 \times 10^{-7}$	$5 \times 10^{-11}$
Fitted value	0.1111	5.2526	0.3637	0.4278	0.1671	0.5518	$5.3967 \times 10^{-6}$	$2.6404 \times 10^{-10}$
Upper bound	0.2125	10	0.5	0.476	0.4	0.8	$1 \times 10^{-5}$	$5 \times 10^{-10}$

# FITTING RESULTS

### Sample B in Small Tube



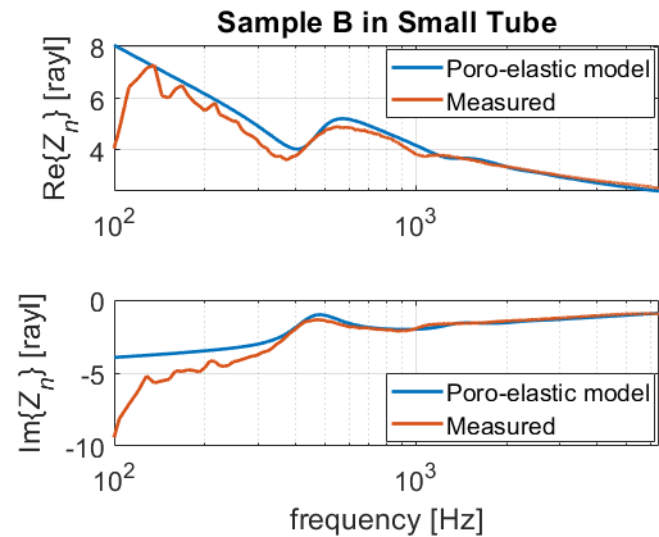
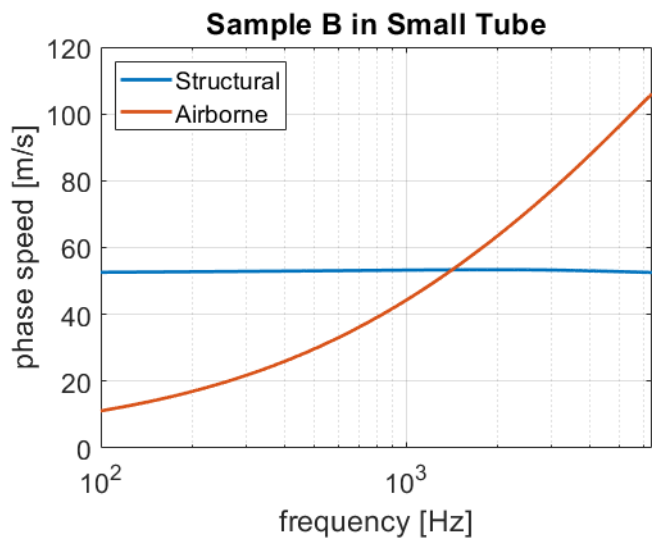
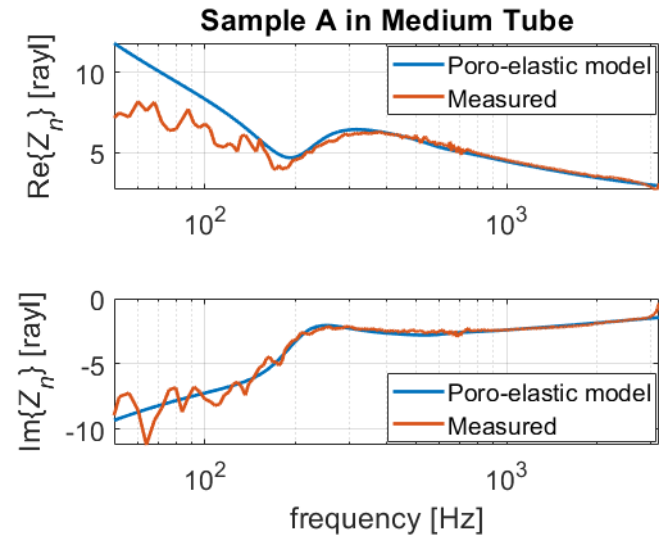
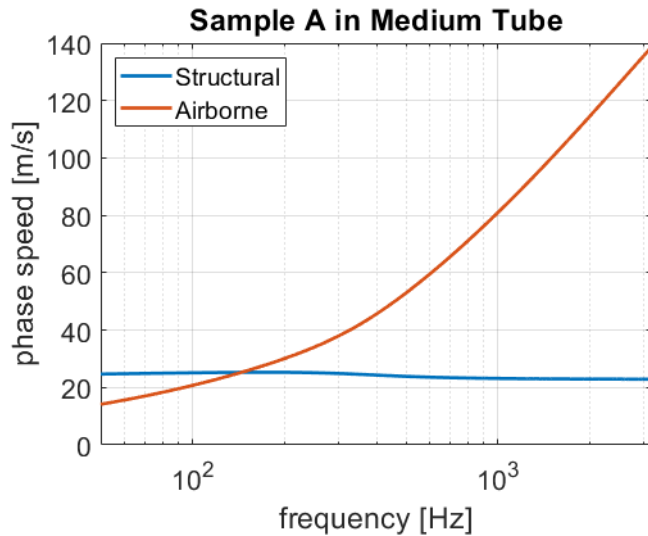
### Sample B in Small Tube



$E_0$ [Pa]	$5.2113 \times 10^5$
$\eta$	0.3552
$\nu$	0.3753

	$r_p$ [mm]	$r_m$ [ $\mu\text{m}$ ]	$r_n$ [nm]	$\phi_p$	$\phi_m$	$\phi_n$	$b$ [ $\text{Pa}^{-1}$ ]	$D_c$ [ $\text{m}^2/\text{s}$ ]
Lower bound	0.075	0.01	0.2	0.260	0.2	0.35	$5 \times 10^{-7}$	$5 \times 10^{-11}$
Fitted value	<b>0.1160</b>	<b>4.5014</b>	<b>0.2000</b>	<b>0.3524</b>	<b>0.4668</b>	<b>0.4989</b>	<b><math>7.2657 \times 10^{-6}</math></b>	<b><math>8.8559 \times 10^{-11}</math></b>
Upper bound	0.161	10	0.5	0.476	0.8	0.55	$1 \times 10^{-5}$	$5 \times 10^{-10}$

# FITTING RESULTS



## CONCLUSIONS

- The poro-elastic model can predict the behavior of the particle stack at high frequencies, where rigid model generates similar results.
- The poro-elastic model can capture the resonance peak, at the frequency where the stack thickness corresponds to a quarter wavelength of structural wave.
- In some cases, a second peak in absorption coefficient is also predicted by the poro-elastic model, at the frequency where the stack thickness corresponds to three quarter wavelengths of structural wave.
- The fitting results from poro-elastic model gives reasonable bulk density prediction, in these two cases, this prediction is constrained in  $\pm 5\%$  range of measured value.
- The absorption coefficient is significantly benefited from the micropores, which is consistent with the conclusion drew from the rigid model.

# ACKNOWLEDGEMENT

We sincerely appreciate the generous help provided by 3M, intellectually, financially and logistically. We also thank Jialin Liu from 3M, for bringing the particle swarm algorithm into our vision.

## REFERENCES

- [1] R. Venegas and O. Umnova. Acoustical properties of double porosity granular materials. *The Journal of the Acoustical Society of America*, 130(5):2765–2776, 2011.
- [2] R. Venegas and O. Umnova. Influence of sorption on sound propagation in granular activated carbon. *The Journal of the Acoustical Society of America*, 140(2):755–766, 2016.
- [3] R. Venegas, C. Boutin, and O. Umnova. Acoustics of multiscale sorptive porous materials. *Physics of Fluids*, 29(8):082006, 2017.
- [4] R. Venegas and C. Boutin. Acoustics of sorptive porous materials. *Wave Motion*, 68:162–181, 2017.
- [5] J. F. Allard and N. Atalla. *Propagation of Sound in Porous Media: Modelling Sound Absorbing Materials*, second edition. John Wiley & Sons, 2009.
- [6] O. Dazel, B. Brouard, C. Depollier, and S. Griffiths. An alternative Biot’s displacement formulation for porous materials. *The Journal of the Acoustical Society of America*, 121(6):3509–3516, 2007.
- [7] Y. Xue. Modeling and design methodologies for sound absorbing porous materials when used as layered vibration dampers. *Purdue University, PhD Thesis*, 2019.
- [8] T. Mellow, O. Umnova, K. Drossos, K. Holland, A. Flewitt, and L. Kärkkäinen. On the adsorption-desorption relaxation time of carbon in very narrow ducts. In *Proceedings of Acoustics 08*, pages 1445–1450, Paris, France, 2008.
- [9] O. Dazel, J.-P. Groby, B. Brouard, and C. Potel. A stable method to model the acoustic response of multilayered structures. *Journal of Applied Physics*, 113(8):083506, 2013.

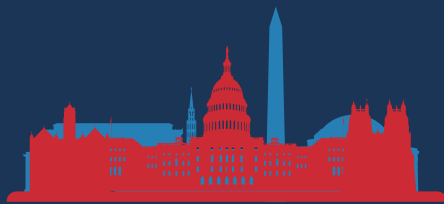


## REFERENCES

- [10] F.-X. Bécot and L. Jaouen. An alternative Biot's formulation for dissipative porous media with skeleton deformation. *The Journal of the Acoustical Society of America*, 134(6):4801–4807, 2013.
- [11] ASTM. Standard test method for impedance and absorption of acoustical materials using a tube, two microphones and a digital frequency analysis system. Standard E1050 - 19, ASTM International, West Conshohocken, PA, 2019, [www.astm.org](http://www.astm.org).
- [12] S. Chen. Constrained particle swarm optimization (2009-2018). MATLAB FILE EXCHANGE. [online] Available: <https://www.mathworks.com/matlabcentral/fileexchange/25986> [Accessed December 2020]

# inter·noise 2021

Thank you!



inter·noise  
2021 1 - 4 AUGUST

[internoise2021.org](http://internoise2021.org)

