Purdue University
Purdue e-Pubs

Open Access Dissertations | Theses and Dissertations

8-2018

Quality Control Along Supply Chain

Didun Peng
Purdue University

Follow this and additional works at: https://docs.lib.purdue.edu/open_access_dissertations

Recommended Citation

Peng, Didun, "Quality Control Along Supply Chain" (2018). *Open Access Dissertations*. 2046.
https://docs.lib.purdue.edu/open_access_dissertations/2046

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries.
Please contact epubs@purdue.edu for additional information.

QUALITY IMPROVEMENT ALONG SUPPLY CHAIN

by

Didun Peng

A Dissertation

Submitted to the Faculty of Purdue University

In Partial Fulfillment of the Requirements for the degree of

Doctor of Philosophy

Krannert School of Management

West Lafayette, Indiana

August 2018

THE PURDUE UNIVERSITY GRADUATE SCHOOL
STATEMENT OF COMMITTEE APPROVAL

Dr. Robert D. Plante, Chair

Krannert Graduate School of Management

Dr. Jen Tang

Krannert Graduate School of Management

Dr. Yanjun Li

Krannert Graduate School of Management

Dr. Mohitosh Kejriwal

Krannert Graduate School of Management

Approved by:

Dr. Yanjun Li

Head of the Graduate Program

"The end justifies the means." I dedicate this thesis.

ACKNOWLEDGMENTS

I express my deepest gratitude to my supervisor, Prof. Robert D. Plante and co-supervisor, Prof. Jen Tang for their expertise, ideas, feedback, time and encouragement.

I would like to thanks my family to support me throughout my life.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

Author: Peng, Didun. PhD
Institution: Purdue University
Degree Received: August 2018
Title: Quality Improvement along Supply Chain.
Committee Chair: Robert Plante

Product quality is the core competitiveness of a brand, prompting brand-owner continuously to pursue. Learning curve is the tool to improve product quality by variance reduction. At the same time, knowledge depreciation with its negative effects attracts attention. Thus, a comprehensive learning curve is introduced in this dissertation. This research focus to explore the quality improvement along the supply chain. There are three contributions in this dissertation: 1) it provides the supply chain's optimal distribution of quality improvement to manufacture and its suppliers; 2) it shows the benefit from coordinated quality improvement among the supply chain; 3) it illustrates the benefit from increased demand and decreased cost from quality improvement.

Three models are presented consecutively in this dissertation. The first model assists the supply chain to coordinate all suppliers for quality investment. Based on the traditional learning curve, the comprehensive learning model is introduced in order to better understand the knowledge accumulation effect. Autonomous learning, induced learning and their respective knowledge depreciation effects are considered in this model. The product quality is measured from several aspects, and each aspect linearly depends on the component quality. Therefore, suppliers' quality improvement contribute to the end product quality. The second model further considers each outsourced components have interaction effects. To better understand knowledge forgetting effect, it adopt Weibull distribution to simulate producing disruption. What's more, it considers the optimal quality investment for the whole producing system and a suboptimal quality investment when there is no coordination in the system (Dyadic Supply Chain). Without coordination, every supplier is trying to save her own quality cost, but the total quality cost is higher than the coordinated system. Thus, incentives are necessary to these suppliers to cooperate in quality improvement. In addition, the second model provides existence proof of the optimal solution. Since the previous models using variance reduction to save cost, it increases demand as well. The

third model starts to consider demand increasing and quality saving simultaneously. Similar to model 2, the third model compares the optimal quality efforts under the coordinated system and the sub-optimal quality efforts under un-coordinated system. Generally, the coordinated system is more efficient. With coordination, 1) marginality is eliminated; 2) cost is lower; 3) demand is higher from higher quality. Since supplier invest more with cooperation, incentives is required to all suppliers.

CHAPTER 1. INTRODUCTION

The research presented in this dissertation is trying to discuss organizational learning, which could lower product producing cost and improving product quality among the supply chain from supplier, brand-owner and retailer. At the same time, higher quality would increase the demand even with higher selling price. Hence, additional profit is possible from lower cost and higher demand. In order to utilize organizational learning achieving more profit, based on learning curve, my research first considers the comprehensive learning curve, which considers autonomous and induced learning along with their respective knowledge depreciation. Second, my research considers how organizational learning reduce quality cost in the supply chain when product is assembling from outsourced components. Further, organization learning improving product quality is discussed in my research as well. Finally, my research shows the whole supply chain could get additional profit as long as suppliers, brand-owner could cooperate. This dissertation progressively discuss a series of problems particularly associated with quality-improvement, demand-increase and cooperation between supplier and brand-owner.

1.1 Comprehensive Learning Curve

Learning curve is a traditional tool to describe producing time cost reducing from repeating work. The more experience, the shorter producing time. This fact was first discovered in 1936 when Wright analysis the aircraft productivity. Hence, knowledge accumulation is autonomous and uncontrollable. Later, people find such experience could save producing cost from other aspects. The best model to depict learning effect is exponential model. At the same time, producer may realize that such experience is so valuable and is willing to pay for some experience. Then, learning curve should extend to this level. This kind of knowledge from managerial activity is induced learning. Later, knowledge accumulates from learning, and get lost from disruption or forgetting. In order to consider all the facts, we proposed a comprehensive learning curve, which consider autonomous learning, induced learning and their respective depreciation. In order to show how accumulated knowledge improve the quality, we introduced Taguchi (1986) squared deviation model, a core part of Six Sigma theorem. We used this model to show how organizational learning reduce the product measurement variation. In other words, the goal of comprehensive learning curve is reduce performance measurement variance through knowledge accumulation. Some knowledge accumulates unconsciously and some accumulates from elaborate plan.

1.2 Comprehensive Learning Curve on Supply Chain

Comprehensive learning model could efficiently reduce product measurement variance. Though there is a cost to get knowledge from induced learning, it worth sometime. The question arise to what is the optimal amount of induced learning. Knowledge brings the quality cost down but additional learning cost incurs. Nowadays, the production process has a complicated system. The process from raw material to end product has a strict structured supply chain. The up-stream supplier components show a compounding effect on down steam end product. In addition, since there are multiple components for the end product, and these components usually come from various suppliers. The optimal induced learning amount along the supply chain for each supplier and for the assembler in order to save the total quality cost is the discussed in my research. Besides the optimal induced learning amount for cooperative system, my research talks the uncooperative system. Since the manufacturer and the suppliers have their own profit, each one are pursuing their own profit. Our research discussed the induced learning amount without coordination as well. In particular, we talk about the un-coordinated system from the marginality

1.3 Dissertation Outline

The rest of this dissertation consists of four parts. Chapter 2 develops comprehensive learning model to consider autonomous learning, induced learning and the associated depreciation effects. An observed data is used to verify this model. With the comprehensive learning, this chapter is able to provide the optimal investments in induced learning while product performances depend on several independent supplier components performance. A total system cost model is then constructed via incorporating the progress function to obtain the minimum cost quality targets allocation. We use a numerical example of internal supplier process to demonstrate this model and to assess the sensitivity of the optimal solutions to different model parameters

Based on Chapter 2, Chapter 3 further consider components interaction effect while disruption occurs accidently or strategically in the cooperative supply chain or un-cooperative supply chain. Planned disruptions is necessary in order to make the whole producing process runs smooth. During the planned disruption or break, employee could get some training and the most important is that the system could get regular recondition in order to maintain uptime until the next break. We used a numerical example to show the benefit from our model and we compare the induced learning amount and quality cost under coordinated and un-coordinated system.

In Chapter 4, we consider the additional profit from increased demand as well. Chapter 2 and 3 showed the comprehensive learning curve could reduce the product quality cost from variance reduction along the supply chain. Since variance is a direct index for product, smaller variance means higher quality, which prompts the demand. We find that managerial quality effort could reduce quality and enhance demand simultaneously. Chapter 4 considers marginality as well. We find that system with cooperation could bring more benefit to brand-owner, suppliers and customers. This is because that under cooperation, there are more quality effort is necessary. We used a numerical example to show this point by comparing quality investment and additional benefits.

Lastly, Chapter 5 summarize the contributions of this dissertation and introduce several topics for further research.

CHAPTER 2. MINIMUM COST ALLOCATION OF QUALITY IMPROVEMENT GOALS: THE EFFECTS OF KNOWLEDGE DEPRECIATIONBEFORE YOU SUBMIT

2.1 Introduction for learning curve

The management of organizational knowledge has drawn attention among various areas of industry and research (Hatch and Mowery 1998, Argote 1999, Lapre et al. 2000). In particular, quality-based organizational learning has received extensive study (Fine 1986, Li and Rajagopalan 1998b, Mukherjee et al. 1998). Recent research efforts have focused on the practice and impact of induced learning resulting from intentional managerial investments in learning activities. Induced learning can be used both to accelerate quality improvement (Ittner et al. 2001, Serel et al. 2003) and to counteract the opportunity loss in quality improvement associated with process disruptions (Howick and Eden 2007, Wang et al. 2013). In this regard, Wang et al. (2013) proposed a model that accounted for the deleterious effect of process disruptions when allocating quality improvement targets to suppliers and obtained the cost minimizing level of induced learning to offset the cost associated with such disruptions.

While organizational learning has been explored in its various components, research on knowledge depreciation is very limited. Knowledge depreciation, or forgetting, is as common and complex a phenomenon as learning, but it is often neglected or simply associated with disruptions within the dimension of autonomous learning. Every year due to the involuntary loss of acquired knowledge, companies suffer immense cost (de Holan et al. 2004). Hence, it is crucial to account for knowledge depreciation and its different forms in managerial decision making. Even though the model proposed by Wang et al. (2013) focused on the effect of disruptions, its formulation did not consider knowledge depreciation and therefore sets the stage for the categorization and inclusion of knowledge depreciation so that a more accurate induced learning investment strategy can be achieved. Particularly, we are interested in assessing to what extent forgetting and knowledge decay affect optimal induced learning investments as well as resulting improvements in quality.

2.2 Forgetting and Knowledge Decay

Differentiating autonomous and induced learning essentially suggests that there are two streams of knowledge creation and accumulation. One is through autonomous learning by doing and the other through the activities and programs that are designed to purposively induce learning. Forces that drive and sustain these two learning dynamics are distinct in nature (Dutton and Thomas 1984, Li and Rajagopalan 1998b). Thus it is reasonable to base our discussion of knowledge depreciation on two separate knowledge accumulating processes. This is also consistent with Yelle's proposition that "the phenomenon of forgetting and relearning needs more attention in the context of the various components of learning," (Yelle 1979) and the categorization of accidental forgetting by de Holan et al. (2004).

As for autonomous learning, forgetting has been observed in various empirical studies. Keachie and Fontana (1966) may have been the first to recognize explicitly the phenomenon of unlearning between production runs. Batchelder et al. (1969) presented an example of a "scallop" learning curve in which manufacturing costs rise abruptly and then decline. According to Batchelder et al. (1969), the "scallop" is generally caused by a major interruption in the production process. Ballof (1970) noted that the total productivity "losses" associated with frequent interruptions of a start-up may greatly outweigh inventory costs and other considerations. While intermittent production is identified as a critical cause for forgetting, scholars have found that other factors, such as worker turnover and product or process changes, also cause forgetting (Darr et al. 1995, Li and Rajagopalan 1998a). For instance, a close examination of the data provided by Badiru (1995) shows that quality is affected by both production down time and production rate. Forgetting occurs not only during production down time, but also when the production rate is decreased. Although various causal factors were claimed to be associated with forgetting during production, most of the proposed forgetting models only account for production disruptions (Badiru 2006). As an attempt to account for other factors, we recognize that these factors often result in decreases in production rates (Ash and Smith-Daniels 2004). And Benkard (2000) found evidence that "experience may depreciate in times of falling production rates." Hence, we will also account for forgetting that is concurrent with the retardation of production.

In the scope of knowledge accumulation through investments in induced learning activities, the literature has not yet addressed forgetting that represents the loss of knowledge accumulated from induced learning activities. We call this forgetting knowledge decay to distinguish it from the forgetting that occurs due to process disruption. The psychology literature suggests three situations where knowledge is likely to decay: (i) lack of similarity between the conditions of encoding and retention of material learned, (ii) old learning interfering with new learning, and (iii) interruption in learning (Badiru 2006). As transparency and interaction among companies increases, knowledge is rapidly created and transferred through additional training programs (Malhotra and Ritzman 1999), which results in frequent occurrence of these scenarios. Therefore, knowledge decay is especially relevant to today's organizations. Empirical studies show that retention of induced knowledge depends on the completeness of induced learning. Negligence of processes, like validation, codification or institutionalization, can impede learning and cause involuntary knowledge decay (Lapre et al. 2000, de Holan et al. 2004). To account for this, we introduce the concept of induced learning capacity. Under-capacity learning implies an insufficient learning period or a lack of follow-up effort to achieve proficiency and retention (Easterby-Smith and Lyles 2006) and will be subject to knowledge decay.

2.3 Development of the Progress Function

Traditionally, quality cost is determined by the expenditure of ensuring conformance to product specifications. However, more contemporary studies have provided theoretical and empirical support for a more general quality cost model for continuous improvement (Ittner 1996, Li and Rajagopalan 1998b).

Within the continuous improvement framework, we adopt Taguchi's quadratic quality loss function, where quality cost depends on the squared deviation of a process design variable from its target value (Taguchi 1986). Hence for a single supplier process with design variable X(t), the quality cost, C(t), is defined as,

$$\begin{aligned} C(t) &= E\left[k\left(X(t)-\tau\right)^2\right] \\ &= k\left[\sigma_{X(t)}^2+\left(\mu_{X(t)}-\tau\right)^2\right] \\ &= k\sigma_{X(t)}^2 \quad \text{(assuming } \mu_{X(t)}=\tau) \end{aligned} \tag{1}$$

where,

$E[\cdot]$	=	expectation operator,
$\mu_{X(t)}$	=	mean of $X(t)$ in period of $(t-1, t]$,
$\sigma^2_{X(t)}$	=	variance of $X(t)$ in period of $(t-1, t]$,
τ	=	the target value for $X(t)$,
k	=	the quadratic cost coefficient, which is the expected cost per unit of squared deviation of $X(t)$ from τ and can be estimated using techniques described by Sullivan (1984).

The last equation in (1) holds because it's assumed that the process target τ for the mean has previously been attained for this process (i.e., $\mu_{X(t)} = \tau$), using methodologies such as those proposed by Plante (2001). This usually involves a one-time process adjustment and is relatively easier to achieve. Based on equation (1), quality depends on the process variance and thereby quality cost can be reduced through decreasing process variance $\sigma^2_{X(t)}$.

2.3.1 Production and Learning Capacity

Quality-based learning (Ittner et al., 2001) suggests that quality cost decreases as knowledge/experience accumulates, which can be described by a progress function (learning curve). The conventional progress function follows the power law with an agglomerate experience term (Argote and Epple 1990). However, Wang et al. (2013) recently introduced an extended learning model, which separates experience accumulated from autonomous and induced learning and also accounts for production disruptions. Their model facilitates the decomposition of knowledge depreciation and provides the basis to separately include forgetting and knowledge decay in our current study. In particular, we introduce production and induced learning capacities (p and q in equation (2) below) where we postulate that, relative to the full capacity scenario, forgetting and knowledge decay occur whenever there exist under-capacity production and/or under-capacity induced learning, respectively. In this way we are able to capture a broader range of forgetting causal factors besides intermittent production, minimizing the underestimation of the adverse effects of knowledge depreciation. The revised progress function is

$$C(t) = k\sigma^2_{X(t)} = C(0)\left[G(t)p\right]^{-\lambda_A}\left[L(t)q\right]^{-\lambda_I}, \tag{2}$$

where

t	=	the end of current planning period $(t-1, t]$,
$C(0)$	=	initial quality cost at the beginning of the time horizon,
$G(t)$	=	cumulative proportions of full-capacity production (for autonomous learning) up till time t $\left(0 \le G(t) \le t\right)$,
$L(t)$	=	cumulative proportions of full-capacity for induced learning up till time t $\left(0 \le L(t) \le t\right)$,
λ_A	=	autonomous learning parameter for the process,
λ_I	=	induced learning parameter $\left(\lambda_I > 0\right)$,
p	=	maximum production capacity for the process under consideration per period (in units),
q	=	maximum induced learning capacity per period (in units).

If we take partial derivatives of (2) with respect to the two sources of knowledge, $G(t)$ and $L(t)$ respectively, we have (t is omitted)

$$\frac{\partial C}{\partial G} = C(0)\left(-\lambda_A p\right)\left(Gp\right)^{-\lambda_A-1}\left(Lq\right)^{-\lambda_I} = \left(-\lambda_A pG^{-1}\right)C, \tag{3}$$

$$\frac{\partial C}{\partial L} = C(0)\left(Gp\right)^{-\lambda_A}\left(-\lambda_I q\right)\left(Lq\right)^{-\lambda_I-1} = \left(-\lambda_I qL^{-1}\right)C. \tag{4}$$

Equations (3) and (4) suggest that the change of quality cost, $C(t)$, is proportional to the current level of quality cost. And cumulative autonomous learning, $G(t)$, and cumulative induced learning $L(t)$, interactively determine the rate of quality cost reduction. These equations are explicit representations of the continuous improvement differential equation introduced by Zangwill and Kantor (1998).

Model (2) suggests that process variance and thereby quality cost depend on the combined effects of both autonomous and induced learning. Adopting the primal technological-based concept of capacity, p and q are the maximum level of production and induced learning activities within a planning period that we could reasonably expect to attain given the capital stock, the state of technology, and the efficient and full utilization of all factors (Morrison 1985). A similar concept, termed performance, was introduced by Carlson and Rowe (1976) in their study of learning and forgetting. Their level of performance ranged from 0% to 100% of production capacity. Therefore the actual amount of cumulative autonomous learning (production) and induced learning are $G(t)p$ and $L(t)q$ (both in units) respectively, such that,

$$G(t) = G(t-1) + g_t, \tag{5}$$

$$L(t) = L(t-1) + l_t, \tag{6}$$

where

g_t = proportion of full-capacity production over $(t-1, t]$ $(0 \leq g_t \leq 1)$,

l_t = proportion of full-capacity induced learning over $(t-1, t]$ $(0 \leq l_t \leq 1)$.

We assume that at the beginning of the current planning period $(t-1, t]$, all information regarding the previous period $(t-2, t-1]$ is available, which in practice becomes increasingly feasible due to continuous advances in information technology and in data analysis/estimation. Thus $G(t-1)$ and $L(t-1)$ are observed. Both g_t and l_t are within [0, 1]. When $g_t = l_t = 1$, production and induced learning are both at full-capacity. And $0 < g_t, l_t < 1$` implies under-capacity production and under-capacity induced learning, resulting in knowledge depreciation. In this Chapter, l_t is the decision variable while g_t is set to pre-specified or planned production levels.

2.3.2 Model of Forgetting and Knowledge Decay

The form of our model is chosen based on a comparative study by Nembhard (2001), in which up to 14 forgetting models are compared, including both statistical and deterministic models. The comparison criteria are (i) efficiency in measuring forgetting, (ii) stability in productivity predictions across the workforce, and (iii) balance of undershoots and overshoots of predictions of productivity following a break. According to the test results, the linear model consistently yielded good predictions among the statistical models investigated by Nembhard (2001). Hence forgetting and knowledge decay will both adopt the linear form in our model.

Further, in the learning and forgetting literature, factors that influence the extent of forgetting have been extensively investigated. The three most common factors are (a) interruption length, the longer of which leads to greater loss of knowledge; (b) amount of accumulated knowledge prior to the interruption, which is often deemed the most important determinant of forgetting in psychology, and (c) nature of the tasks, where forgetting is more prominent for procedural tasks than declarative tasks, and also more for cognitive tasks than motor tasks (Nembhard 2001). In the context of this research, the first two factors are respectively reflected in the unachieved capacity and accumulated knowledge prior to any drop in production or induced learning. Factor (c) is accounted for via the differentiation of two types of learning, autonomous and induced.

We consider the following models for knowledge deterioration,

$$G(t) = \frac{\left[aG(t-1) + g_t\right] + a\left[G(t-1) + g_t\right]}{2}, \quad 0 \le a \le 1, \tag{7}$$

$$L(t) = \frac{\left[bL(t-1) + l_t\right] + b\left[L(t-1) + l_t\right]}{2}, \quad 0 \le b \le 1, \tag{8}$$

where

a = forgetting parameter of knowledge accumulated from autonomous learning. The proportion of accumulated knowledge that is retained.

b = knowledge decay parameter for induced learning. The proportion of accumulated induced learning that is retained.

Then equation (2) becomes

$$C(t) = C(0)\left\{\left[\frac{\left[aG(t-1) + g_t\right] + a\left[G(t-1) + g_t\right]}{2}\right]p\right\}^{-\lambda_A} \times \left\{\left[\frac{\left[bL(t-1) + l_t\right] + b\left[L(t-1) + l_t\right]}{2}\right]q\right\}^{-\lambda_I}. \tag{9}$$

Note that forgetting and knowledge decay are manifested in the discount factors, a and b. Further, forgetting and knowledge decay are embedded within the two progress function components of accumulated production and induced learning, respectively. That is, it provides a direct means of reducing the body of knowledge accumulated through both autonomous and induced learning. When $a = b = 1$, equations (7) and (8) reduce to (5) and (6), respectively. The values of a and b are related to the characteristics of the workforce, the nature of production, and the type of induced learning program implemented (Lapre et al. 2000, Nembhard 2001) and they are assumed to be independent of the learning parameters. Equation (9) is a model with multiple parameters and the estimation of learning and depreciation parameters requires a non-linear estimation procedure, an example of which we will provide in the next section. In a study of aircraft production dynamics, Benkard (2000) used a nonlinear estimator based on the General Method of Moments to estimate the learning and forgetting coefficients for an autonomous learning model. Epple et al. (1991) adopted a maximum likelihood method to estimate the depreciation rate in autonomous learning. Therefore, we assume that the learning parameters, λ_A and λ_I , as well as depreciation parameters, *a* and *b* can be estimated from historical data. In Section 4, using an augmented data set from Badiru (1995), we provide an estimation procedure of these parameters that minimizes the sum of squared deviations of rework unit rate (Darr et al. 1995).

Thus far, we have derived our quality-based learning and forgetting model for a single supplier production process $X(t)$. In Section 3, this model will be applied to a system of suppliers' processes $X_i(t)$, where *i* denotes the *i*-th supplier. We seek an optimal allocation of induced learning by considering the tradeoff between investment in induced learning and the potential positive impact of induced learning, including counteracting forgetting and thereby promoting continuous quality improvement.

2.3.3 Impact of the Learning Model Parameters on Cost

To provide some practical credence to (9), we assess whether this model reflects learning curve patterns that are routinely observed in practice. To illustrate the effect of forgetting and knowledge decay on cost reduction described by (9), four expected cost graphs are drawn over 25 planning periods (Figures 1, 2, 3 and 4). For each illustration, the autonomous and induced learning parameters,   and , are both assumed to be 0.2. Under-capacity production occurs in periods 1 through 3, 6 through 9, 13, 14 and 23, with = 25%, 50%, 75%, 0%, 25%, 50%, 75%, 0%, 90%, 50%, respectively. We assume that induced learning only occurs in periods 1 through 3, 9 through 12 and 18 through 21, with = 75%, 50%, 25%, 100%, 75%, 50%, 25%, 100%, 75%, 50%, 25%, respectively. *p* and *q* are assumed to be 100 and 25 (units), respectively.

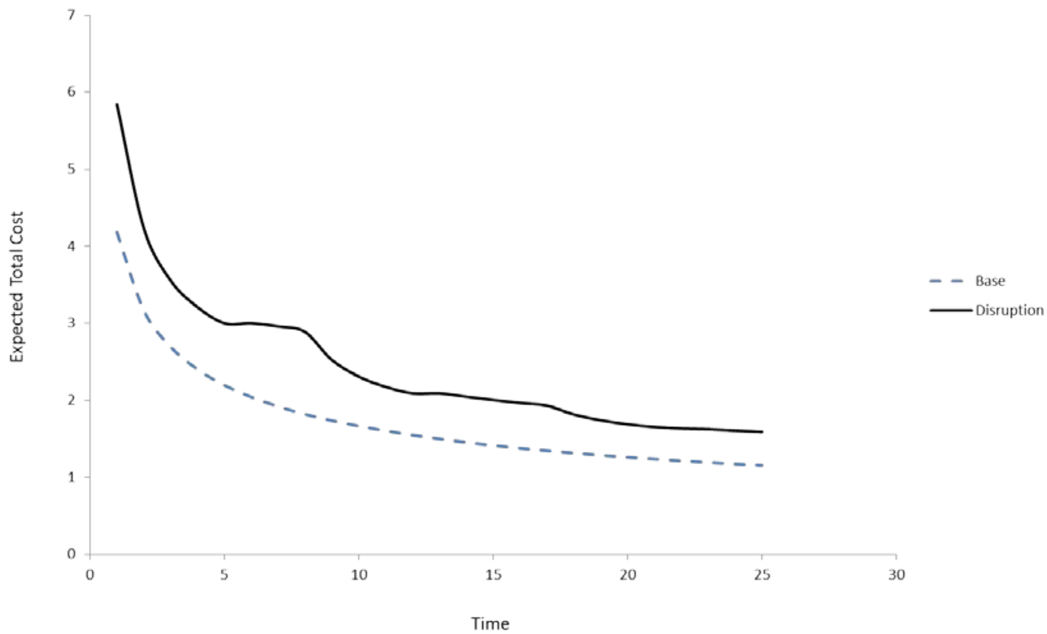

Figure 1. The Effect of Process Disruption on Expected Cost (Wang et al., 2013)

Figure 1 Disruption Effect

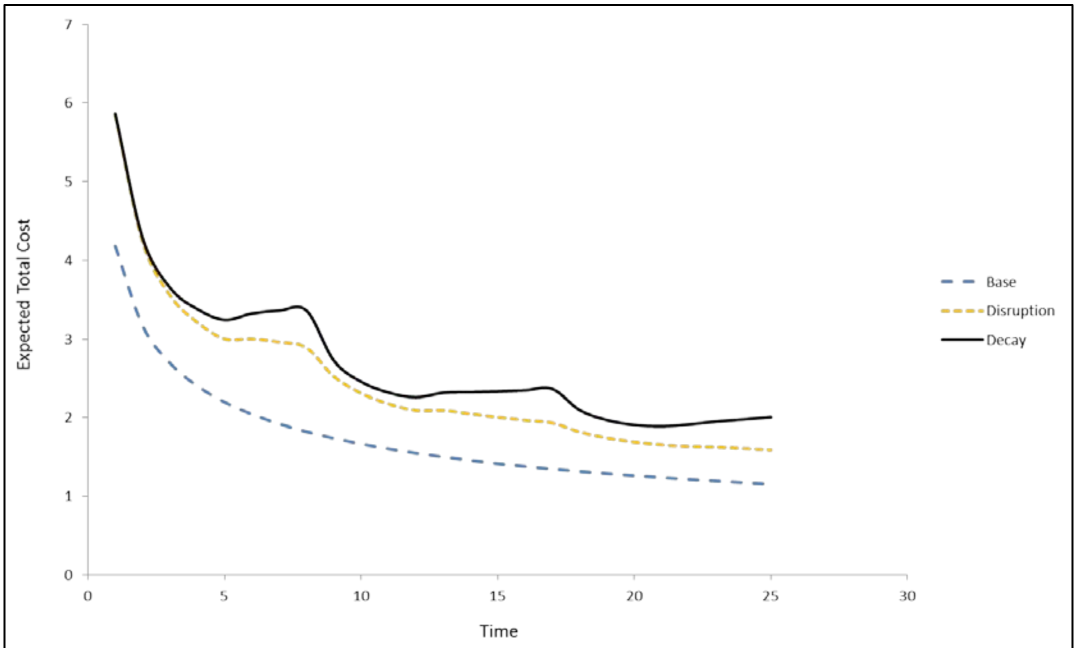

Figure 2. The Effect of Knowledge Decay on Expected Cost

Figure 2 Knowledge Decay Effect

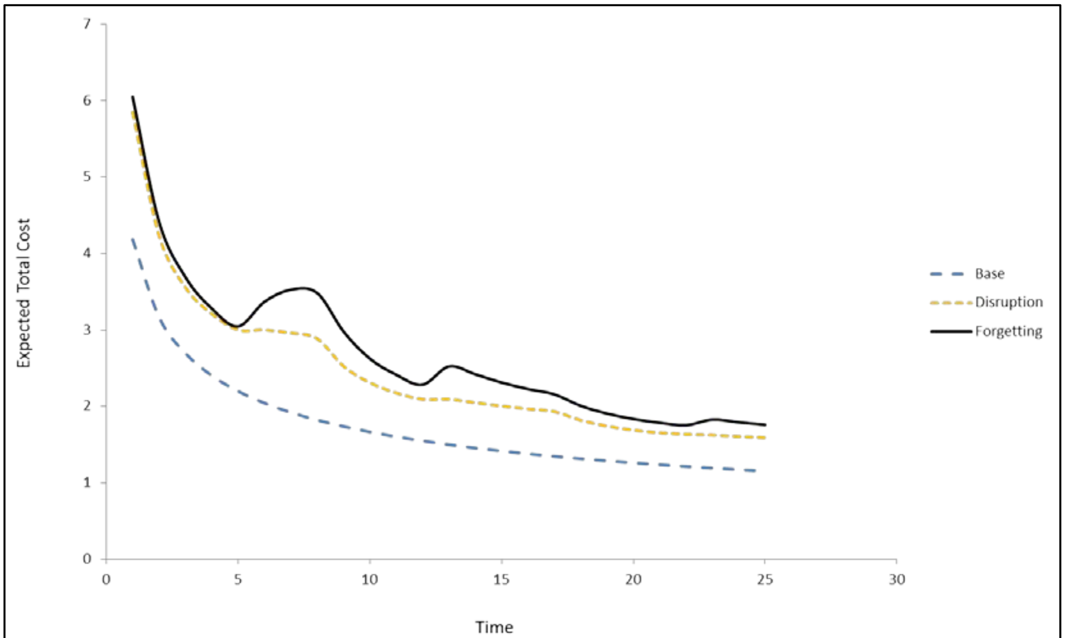

Figure 3. The Effect of Forgetting on Expected Cost

Figure 3 Forgetting Effect

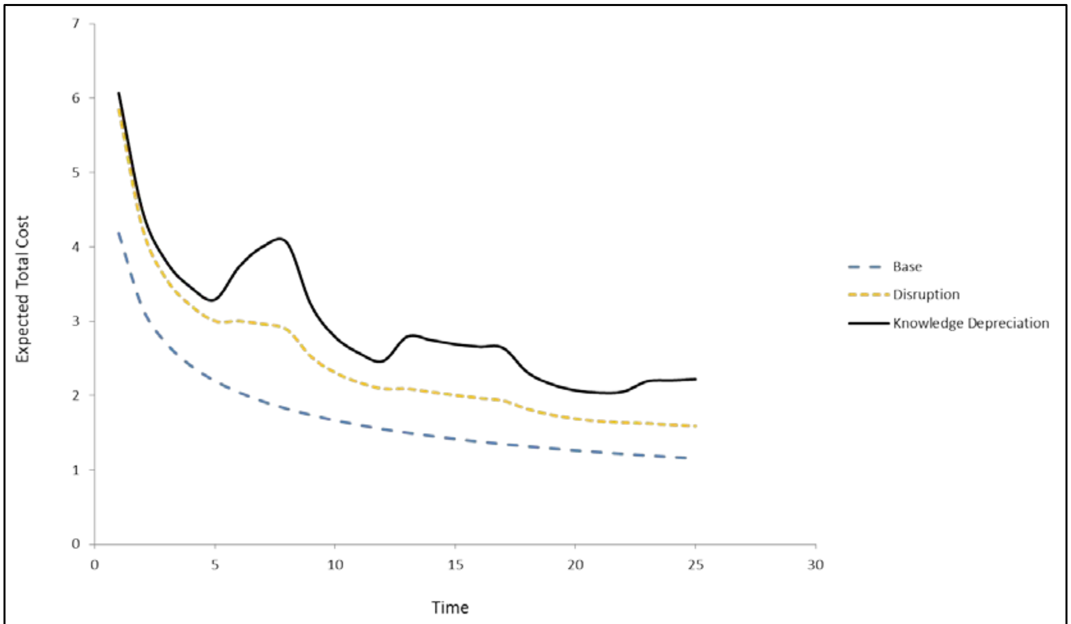

Figure 4. The Effect of Knowledge Depreciation on Expected Cost

Figure 4 Knowledge Depreciaiton Effect

Figure 1 represents the effect of under-capacity production or process disruption which was modeled by Wang et al. (2013). For comparison, each figure also gives a graph that depicts the effect on costs without forgetting (i.e., $a = 1$, $b = 1$) and operating at full capacity (i.e., $g_t = l_t = 1.0$, for all t), which we call the base case. Figure 2 illustrates the case when there is only under-capacity induced learning (i.e., $a = 1$) and therefore only knowledge decay is present with decay parameter $b = 0.95$. It's obvious that new induced learning efforts steepen the learning curve and that discontinuance of induced learning leads to plateaus on the curve. Figure 3 considers the case when there is retardation of production with forgetting parameter $a = 0.005$, but no knowledge decay (i.e., $b = 1$). The effect of forgetting causes regressions on the learning curve, which are the 'scallops.' Figure 4 illustrates the case when both under-capacity production and induced learning are accounted for, where forgetting and knowledge decay are both present with $a = 0.005$ and $b = 0.005$. The resultant progression of cost is consistent with our expectation and what is observed from real data (Badiru 1995, Nembhard 2001), lending some credibility to the model in (9).

2.4 Development of the Progress Function

Assume a supply chain consists of one manufacturer and multiple suppliers. The manufacturer has a set of *M* performance measures, Y_j , j = 1, 2, ..., *M*, each of which is influenced by a common set of *N* design variables (factors), $X_1, X_2, ..., X_N$, associated with *N* supplier processes. The general quality cost model for each design variable is given in Section 2. Our goal is to find the optimal quality improvement targets for each supplier so as to minimize the system's total expected cost. Moskowitz et al. (2001) first introduced a total system cost model in the context of allocating quality improvement targets. In this section, we extend their procedure to develop the total system cost, incorporating both learning and forgetting as shown in (9).

Assuming that there are no interaction effects among the supplier processes, the performance measures relate linearly to design variables at time *t* as follows,

$$Y_j(t) = \sum_{i=1}^{N} c_{ij} X_i(t) + \varepsilon_j, \quad j = 1, 2, ..., M, \tag{10}$$

where,

t	=	the end of current planning period ($t-1$, t],
$Y_j(t)$	=	j-th performance measure (for the manufacturer) over the period ($t-1$, t],
$X_i(t)$	=	design variable by the i-th supplier over the period ($t-1$, t],
c_{ij}	=	marginal effect of the i-th design variable on the j-th performance measure,
ε_j	=	independent noise with mean 0 and constant variance σ_ε^2, and ε_h and ε_k are independent when $h \neq k$.

2.4.1 Impact of the Learning Model Parameters on Cost

Applying Taguchi's model (1) and (9) to each individual supplier process, we have for the *i*-th supplier (i = 1, 2, ..., *N*)

$$\begin{aligned} C_i(t) &= k_i \sigma^2_{X_i(t)} \\ &= C_i(0)\left\{\left[\frac{\left[a_i G_i(t-1)+g_{i,t}\right]+a_i\left[G_i(t-1)+g_{i,t}\right]}{2}\right]p_i\right\}^{-\lambda_{i,A}} \\ &\qquad \times\left\{\left[\frac{\left[b_i L_i(t-1)+l_{i,t}\right]+b_i\left[L_i(t-1)+l_{i,t}\right]}{2}\right]q_i\right\}^{-\lambda_{i,I}}, \end{aligned} \tag{11}$$

where

$\sigma^2_{X_i(t)}$ = variance of the i-th supplier process over period $(t-1, t]$,

k_i = quadratic cost coefficient,

p_i = maximum production capacity (in units),

q_i = maximum induced learning capacity (in units),

$C_i(0)$ = initial quality cost at the beginning of the time horizon,

$G_i(t)$ = cumulative proportions of full-capacity production (autonomous learning) up till time t,

$L_i(t)$ = cumulative proportions of full-capacity induced learning up till time t,

$g_{i,t}$ = realized proportion of full-capacity production over $(t-1, t]$,

$l_{i,t}$ = allocated proportion of full-capacity induced learning over $(t-1, t]$,

$\lambda_{i,A}$ = autonomous learning rate,

$\lambda_{i,I}$ = induced learning rate,

a_i = forgetting parameter, $0 \le a_i \le 1$

b_i = knowledge parameter, $0 \le b_i \le 1$.

From equation (11), where $G_i(t-1) > 0$ and $L_i(t-1) > 0$, the ratio of $C_i(t)$ to $C_i(t-1)$ is:

$$\begin{aligned} \frac{\sigma^2_{X_i(t)}}{\sigma^2_{X_i(t-1)}} &= \left[\frac{\left[a_i G_i(t-1)+g_{i,t}\right]+a_i\left[G_i(t-1)+g_{i,t}\right]}{2G_i(t-1)}\right]^{-\lambda_{i,A}} \\ &\qquad \times\left[\frac{\left[b_i L_i(t-1)+l_{i,t}\right]+b_i\left[L_i(t-1)+l_{i,t}\right]}{2L_i(t-1)}\right]^{-\lambda_{i,I}}, \end{aligned} \tag{12}$$

and hence

$$\sigma^2_{X_i(t)} = \sigma^2_{X_i(t-1)} \left[a_i + \frac{g_{i,t}(1+a_i)}{2G_i(t-1)} \right]^{-\lambda_{i,A}} \times \left[b_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)} \right]^{-\lambda_{i,I}} . \tag{13}$$

Therefore, the total supplier quality cost is:

$$QC_S(t) \equiv \sum_{i=1}^{N} C_i(t) = \sum_{i=1}^{N} k_i \sigma^2_{X_i(t)} \tag{14}$$

$$= \sum_{i=1}^{N} k_i \sigma^2_{X_i(t-1)} \left[a_i + \frac{g_{i,t}(1+a_i)}{2G_i(t-1)} \right]^{-\lambda_{i,A}} \times \left[b_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)} \right]^{-\lambda_{i,I}} . \tag{15}$$

2.4.2 Total Manufacturer Quality Cost

Assuming that the supplier processes are independent, via equation (10), the resultant mean and variance of a manufacturer's j-th performance measure $Y_j(t)$ (j = 1, 2, ..., M) at time t are, respectively,

$$\mu_{Y_j(t)} = \sum_{i=1}^{N} c_{ij} \mu_{X_i(t)} , \tag{16}$$

$$\sigma^2_{Y_j(t)} = \sum_{i=1}^{N} c_{ij}^2 \sigma^2_{X_i(t)} + \sigma^2_\varepsilon . \tag{17}$$

Similar to the model for a supplier's quality cost, the total manufacturer quality cost is also represented by a quadratic loss function as follows,

$$\begin{aligned} QC_M(t) &= E\left[\sum_{j=1}^{M} K_j \left(Y_j(t) - T_j \right)^2 \right] \\ &= \sum_{j=1}^{M} K_j \left[\sigma^2_{Y_j(t)} + \left(\mu_{Y_j(t)} - T_j \right)^2 \right] \\ &= \sum_{j=1}^{M} K_j \left(\sum_{i=1}^{N} c_{ij}^2 \sigma^2_{X_i(t)} + \sigma^2_\varepsilon \right) \quad \left(\text{assuming } \mu_{Y_j(t)} = T_j \right) \end{aligned} \tag{18}$$

$$= \sum_{j=1}^{M} K_j \left\{ \sum_{i=1}^{N} c_{ij}^2 \sigma^2_{X_i(t-1)} \left[a_i + \frac{g_{i,t}(1+a_i)}{2G_i(t-1)} \right]^{-\lambda_{i,A}} \left[b_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)} \right]^{-\lambda_{i,I}} + \sigma^2_\varepsilon \right\}, \tag{19}$$

where

T_j = the target or nominal value for the *j*-th performance measure $Y_j(t)$,

K_j = the quadratic coefficient for $Y_j(t)$.

Consistent with the assumption in (1), equation (18) follows, assuming the target T_j has been achieved so that $\mu_{Y_j(t)} = T_j$. It is clear that the manufacturer can reduce its total expected quality cost by reducing $\sigma^2_{Y_j(t)}$, through reductions in $\sigma^2_{X_i(t)}$, i = 1, 2, ..., N.

2.4.3 Investment in Learning

Investment in induced learning depends on the allocated proportion of induced learning capacity for each supplier, $l_{i,t}$. In a supplier development program, the manufacturer and suppliers collaboratively invest in a supplier's learning process to reduce process variances and the cost is shared proportionately for each party (Moskowitz et al. 2001). Since suppliers may be different in process complexity, cost structure, experience level, etc., the learning cost coefficient varies from supplier to supplier.

If we assume a quadratic relationship between the learning cost and the amount of induced learning, the total investment in learning during the planning period (t − 1, t] for the i-th supplier is then,

$$LC_{X_i(t)} = h_i \left\{ \left[L_i(t) - L_i(t-1) \right] q_i \right\} = h_i (q_i l_{i,t})^2, \tag{20}$$

where

h_i = the quadratic cost coefficient for the *i*th supplier.

Summing over all suppliers, the total induced learning investment is

$$LC(t) = \sum_{i=1}^{N} LC_{X_i(t)}. \tag{21}$$

Note $l_{i,t}$ (i = 1, 2, ..., N) are the decision variables in the model. This quadratic learning cost model was first proposed by Moskowitz et al. (2001). Wang et al. (2013) later adopted this model in their study of induced learning activities used to counteract the negative effect of production disruptions on quality improvement.

2.4.4 Total Expected System Cost of Target Improvement Allocation

Combining the suppliers' quality costs in (15), manufacturer's quality cost in (19) and the learning investment in (21), the total expected system cost at time *t* is

$$\begin{aligned} TEC_t &= LC(t) + QC_S(t) + QC_M(t) \\ &= \sum_{i=1}^{N} \left\{ LC_{X_i(t)} + v_{X_i(t)} \left[a_i + \frac{g_{i,t}(1+a_i)}{2G_i(t-1)} \right]^{-\lambda_{i,A}} \right. \\ &\qquad \left. \times \left[b_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)} \right]^{-\lambda_{i,I}} \right\} + \sigma_\varepsilon^2 \sum_{j=1}^{M} K_j \end{aligned} \tag{22}$$

$$= \sum_{i=1}^{N} TEC_{X_i(t)} + \sigma_\varepsilon^2 \sum_{j=1}^{M} K_j, \tag{23}$$

where

$$v_{X_i(t)} = \sigma^2_{X_i(t-1)} \left[k_i + \sum_{j=1}^{M} K_j c_{ij}^2 \right], \tag{24}$$

and $TEC_{X_i(t)}$ is the total expected cost for supplier *i*,

$$TEC_{X_i(t)} = h_i \left(q_i l_{i,t} \right)^2 + v_{X_i(t)} \left[a_i + \frac{g_{i,t}(1+a_i)}{2G_i(t-1)} \right]^{-\lambda_{i,A}} \left[b_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)} \right]^{-\lambda_{i,I}}. \tag{25}$$

As shown in (23), disregarding the constant term $\sigma_\varepsilon^2 \sum_{j=1}^{M} K_j$, the total expected system cost, TEC_t, may be partitioned by supplier and be minimized by independently minimizing $TEC_{X_i(t)}$, *i* = 1, 2, ..., *N* defined in (25). Minimum cost variance reduction targets $\sigma^2_{X_i(t)}$ for planning period (*t* − 1, *t*] can then be determined from (13). These variance targets are first obtained by determining the optimal learning allocation $0 \le l^*_{i,t} \le 1$ that minimizes (25) for each supplier *i* independently.

2.4.5 Optimal Investment in Learning

Taking the first derivative of (25) with respect to the learning capacity gives,

$$\frac{\partial TEC_{X_i(t)}}{\partial l_{i,t}} = 2h_i q_i^2 l_{i,t} - \lambda_{i,I}\beta_i \left(\frac{1+b_i}{2L_i(t-1)}\right)\left(b_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)}\right)^{-(\lambda_{i,I}+1)}, \tag{26}$$

where

$$\beta_i = v_{X_i(t)}\left[a_i + \frac{g_{i,t}(1+a_i)}{2L_i(t-1)}\right]^{-\lambda_{i,A}}. \tag{27}$$

Taking the second derivative to assure that (25) is strictly convex in $l_{i,t}$:

$$\frac{\partial^2 TEC_{X_i(t)}}{\partial l_{i,t}^2} = 2h_i q_i^2 + \lambda_{i,I}\left(\lambda_{i,I}+1\right)\beta_i \left(\frac{1+b_i}{2L_i(t-1)}\right)^2 \left(b_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)}\right)^{-(\lambda_{i,I}+2)} > 0\,. \tag{28}$$

Since (28) is greater than 0, (25) is strictly convex in $l_{i,t}$ and the optimal learning capacity can be obtained by setting (26) equal to 0.

2.5 Development of the Progress Function

Our learning and forgetting model has a variety of applications in practice. For example, it is applicable to systems with either external supplier processes or internal processes (or multistage manufacturing processes). Furthermore, this model is relevant for processes that suffer significant forgetting and knowledge decay, such as the food franchise industry with high turnover rate, health care services with constant interruptions, and manufacturers with rapid change in technological innovations (Darr et al. 1995, Shafer et al. 2001). In these cases, performance measures can be service delivery time, number of malpractices, and product quality specifics. The design variables, $X_i(t)$'s, represent sources of variation that describe key characteristics of supplier processes. Using this model, the cost minimizing investment on induced learning activities to improve the process and to potentially counteract forgetting can be attained.

In this section, a numerical example is presented to illustrate our methodology and to assess the sensitivity of optimal induced learning allocations to several model parameters. We assess sensitivity in terms of both direction of change in optimal induced learning as well as the magnitude of this change. An algebraic sensitivity analysis is provided in the Appendix (A1 to A4) for direction of change, and is valid for all values of the model parameters. We will refer to these results as we now consider the use a numerical example to assess the magnitude of change.

Since we can optimize separately for each supplier process, and to facilitate our discussion, we will consider only one supplier process. This example considers the manufacture of an electronic product consisting of a single internal sub-process (N = 1). The design variable, X_1, influences a single performance measure, $Y(t)$. We assume that the current planning period is (4, 5], i.e. t = 5 in our implementation. The quadratic cost coefficients are estimated to be K = 500 for the performance measure and k = 1000 for the design variable (Sullivan 1984) provides a procedure for estimating these cost coefficients). And the initial variance at time $t - 1 = 4$ is $\sigma^2_{X(4)} = 22$. The induced learning cost coefficient h is 50; the production and learning capacities are p = 5000 and q = 50, respectively.

We will use an augmented version of the data set provided by Badiru (1995) to estimate the learning and knowledge depreciation parameters. The data represents a 4-year monthly record of a production process for electronics manufacturing. Absent data available to derive the proportion of induced learning capacity directly, we augmented the data, consistent with results from Dorrah et al. (1994), by equating this proportion to the complement of the proportion of production capacity.

This assumption is plausible if one considers a decision scenario where there is a fixed amount of resources, R_i, in period i that can be allocated to production and/or induced learning activities. Hence, the allocation is apportioned as follows,

$$R_i g_i + R_i l_i = R_i, \quad 0 \le g_i, l_i \le 1, \tag{29a}$$

which clearly results in:

$$l_i = 1 - g_i. \tag{29b}$$

We consider the rework/unit as a measure of quality improvement experienced over the 48-month time period. For model estimation, we took the log transformation of equation (9) and used a nonlinear search algorithm to obtain an estimate of the learning and knowledge depreciation parameters that minimize the sum of squared deviations of our model's predictions from the log of the observed data (Darr et al. 1995).

The resulting estimates for the autonomous learning and forgetting parameters are λ_A = 1.322 and a = 0.960, respectively. Continuing, the estimates for the induced learning and knowledge decay parameters are λ_I = 0.545 and b = 0.289, respectively. The planned proportion of production capacity is g_5 = 0.5, and the accumulated production and induced learning experiences are assumed to be $G(4)$ = 3.0 and $L(4)$ = 0.2. The linear function modeling the relationship between the performance measure and the design variable is,

$$Y(t) = 2X_1(t) + \varepsilon \, . \tag{30}$$

Table 1, first row, shows the resulting proportion of induced learning capacity suggested for period 5, the associated total expected cost and the target variance (note: The constraint imposed by equation (29a) in the estimation of the learning and knowledge depreciation parameters is not imposed when determining optimal levels of induced learning activities). Further, Figure 5 depicts the change in Total Expected Cost as the proportion of induced learning capacity in period 5 ranges from 0.0 to 1.0 which results in a clearly convex function as stated in Section 3.5. The cost minimizing (optimal) proportion of induced learning capacity is 0.291 with a resulting expected cost of $61,214 and a resulting expected process variance target of 16.89. Hence, an appropriate quality improvement target for this process would be a variance of 16.89 in period 5, which represents an expected decrease in process variance of about 23.24%. We call this the Base Case and will use it in a comparative assessment of the sensitivity of total expected cost to changes in selected model parameters.

Table 1 Case Results

Case	Deviation from Base Case	Proportion of Induced Learning Capacity	Total Expected Cost	Variance Target	% Reduction in Variance
Base	NA	0.291	$61,214	16.89	23.24%
1	(b = 1)	0.206	$43,780	12.82	41.73%
2	(a = 1, b = 1)	0.201	$41,896	12.29	44.14%
3	(g = 0.75)	0.278	$56,686	15.67	28.79%
4	(g = 1)	0.267	$52,751	14.60	38.62%
5	(σ_4^2 = 30)	0.334	$79,072	21.70	27.68%
6	(σ_4^2 = 15)	0.243	$44,532	12.38	17.47%
7	(λ_I = 1.0)	0.355	$55,247	13.16	40.17%
8	(λ_I = 2.0)	0.392	$42,697	7.85	64.33%
9	(λ_I = 2.0, b = 0.75)	0.272	$24,285	5.02	77.19%

Table 1. Case results for deviations of designated parameters from the Base Case parameters (g = 0.5, λ_A = 1.322, λ_I = 0.545, a = 0.96, b = 0.289, σ_4^2 = 22).

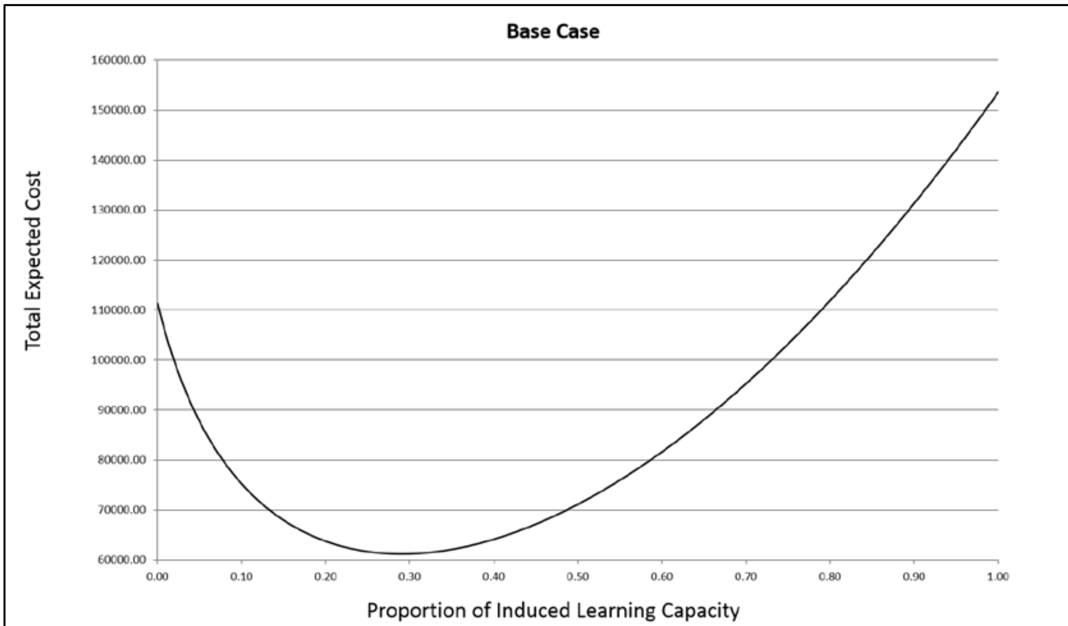

Figure 5. Base Model ($g = 0.5,\ \lambda_A = 1.322,\ \lambda_I = 0.545,\ a = 0.96,\ b = 0.289,\ \sigma_4^2 = 22.0$)

Figure 5 Base Case

2.5.1 Optimal Investment in Learning

In the Appendix-A, we analytically determined the direction of change of the optimal proportion of induced learning capacity for changes in several parameters. In the next few subsections, we will empirically assess for our example problem, the magnitude of these changes on the cost minimizing allocation of the proportion of induced learning capacity to changes in these same parameters. For each case, we will compare the optimal proportion of induced learning capacity and the resulting total expected cost and resulting process variance to those of the base case.

2.5.2 Impact of Knowledge Depreciation, (*a*, *b*)

From the results in the Appendix-A-1, If a_i increases, indicating relatively better retention of the knowledge gained from accumulated experience, the total expected cost $TEC_{X_i(t)}$ is expected to decrease. In this regard, since the system retains comparatively more experiential knowledge, it can be expected to require less induced learning to minimize the cost, i.e., the optimal $l_{i,t}^*$ will be lower. Similarly, if b_i increases, the system suffers less knowledge loss, the expected total cost will decrease and the necessary investment in induced learning can be reduced. The results for cases 1 and 2 in Table 1 are consistent with these directional change results and show the sensitivity of total expected cost to the presence and/or absence of knowledge depreciation, which is achieved in our model through the removal of forgetting (a = 1) and/or the removal of knowledge decay (Case 1: b = 1). Relative to the Base Case, which includes both forgetting and knowledge decay at levels a = 0.96 and b = 0.289, the removal of knowledge decay (a = 0.96, b = 1) has decreased the optimal induced learning rate from 0.291 to 0.206 of induced learning capacity. In comparison, this effect is, understandably quite large relative to the effect of also removing the forgetting parameter given the relatively low retention of accumulated induced learning (28.9%) to that of accumulated autonomous learning (96%). The additional removal of forgetting (Case 2: a = 1, b = 1) has minimally decreased the optimal proportion of induced learning capacity from 0.206 to 0.201, as well as reducing process variance, relative to the Base case, from 16.89 to 12.29, for an expected reduction in process variance of about 44.14%. Also, relative to the Base Case, the total expected cost has been reduced from $61,214 to $41,896, a reduction of about 31.56%. These results clearly depict the effects of not including the knowledge depreciation parameters (Cases 1 and 2) where (*i*) the optimal proportion of induced learning is underestimated (.206 in lieu of .291), (*ii*) the resulting total expected cost is underestimated ($41,896 in lieu of $61,214) and (*iii*) the expected process variance is underestimated (12.29 in lieu of 16.89).

2.5.3 Impact of Planned Production, g_5

In the Appendix-A-2, increasing the level of planned production results in a decrease in the optimal level of allocated induced learning as well as a decrease in total expected cost. Consistent with these results, Cases 3 and 4 in Table 1 show the magnitude of these effects for increasing the planned production in period 5. The three levels of g_5, namely 0.50, 0.75 and 1.00, are chosen to represent the magnitude of lost production, which is $1-g_5$. As g_5 decreases, the planned production level deteriorates, which leads to increases in all three resulting metrics: optimal proportion of induced learning capacity, total expected costs and the expected process variance. On the other hand, as shown in Table 1, when production increases towards full capacity, the process variance reduces from 16.89 to 15.67 to 14.60 for planned production rates of 0.50, 0.75 and 1.00, respectively. And the optimal level of induced learning decreases from 0.291 to 0.278 to 0.267, with resulting decreases in total expected costs from $61,214 to $56,686 to $52,751 having a net decrease of about 13.83% in total expected costs relative to the base case. Here, again, the effect of a low knowledge retention level for induced learning activities is evident. As production, having a relatively higher learning rate as well as a higher retention of knowledge, increases the optimal level of induced learning activities decreases.

2.5.4 Impact of Prior Variance, σ_4^2

The results derived in the Appendix-A-3 indicate that increasing variance in the period prior to the planning period leads to an increase in the optimal level of induced learning as well as an increase in the total expected cost. Consistent with these results, Cases 5 and 6 in Table 1 show that an increase in the previous period process variance from 22 to 30 (Case 5) produces an increase in all three metrics. The optimal induced learning rate increases from 0.291 to 0.392, resulting in an increase for total expected cost from $61,214 to $79,072 and an increase in variance from 16.89 to 21.70. First, these relatively large process variances could imply that a supplier's process may be in the early stages of quality improvement, where it's desirable to invest more in induced learning to reduce quality cost. Complex and/or more volatile processes will need higher investments in training and other induced learning programs, since continual innovations and changes inevitably place the process in the initial stages of a learning curve. Second, when the process variance is higher, it suggests that a relatively larger induced learning level is more economical to achieve a cost effective quality level. Third, for this problem, as process quality improves from a process variance of 22 to 15 (Case 6), the induced learning effort required to achieve the resulting process variance of 12.38 decreases from 0.291 to 0.243. These results suggest that the initial process variance has a substantial effect on the optimal induced learning rate and the resulting total expected cost and the resulting process variance. This suggests that careful assessment of the suppliers' processes is critical for the manufacturer to allocate cost effective quality improvement (i.e., variance reduction) targets.

2.5.5 Impact of Induced Learning Parameter, $\lambda_{i,I}$

Wang et al. (2003) showed how effective an induced learning activity would have to be to counter the impact of the forgetting parameter. In Section 4.1.2 we learned that continuing and/or increasing planned production is particularly effective in reducing the effect of forgetting. Furthermore, because a considerable amount of knowledge can be lost in the presence of process disruptions, it is beneficial to have available effective induced learning activities. This is especially important if process disruptions are expected to continue.

Through managerial intervention, induced learning activities can be designed that could achieve relatively higher induced learning parameters of 1.0 (Case 7) or even 2.0 (Case 8) relative to the .545 of the Base Case. Such gains in the induced learning parameter may be possible depending on the combination of conceptual and operational learning that is used in the design of an induced learning activity (Lapre et al. 2000). Such focused strategies could also lead to increased retention of accumulated induced learning (i.e., decreased knowledge decay). Indeed, during process disruptions the implementation of such effective induced learning activities could go a long way in countering the effect of forgetting due to process disruption.

For most practical cases, the results in the Appendix-A-4 suggest, that as the induced learning parameter increases, the optimal level of induced learning will increase and the total expected cost will decrease. This makes intuitive sense and the results depicted in Cases 7 and 8 in Table 1 are consistent. Here, the impact of increasing the effectiveness of the induced learning activities leads to an increase in the optimal proportion of induced learning capacity from 0.291in the Base Case to 0.355 and 0.392, respectively. Further, as the induced learning parameter increases from 1.0 to 2.0, the expected process variance decreases from 13.16 to 7.85 with a simultaneous decrease in total expected cost from $55,247 to $42,697. Furthermore, by simultaneously designing induced learning activities that also increase the retention of knowledge the impact can be substantial. Case 9 (λ_I = 2.0, b = 0.75) in Table 1 shows the impact of such designs where the Total Expected Cost is greatly reduced to $24,285 and the variance has been reduced to 5.02 for a reduction of about 77.19%. And all of this has been achieved by using relatively low proportions of induced learning capacity. Quite simply, as one would expect, the results of this empirical study suggest that, from a quality improvement and production cost perspective, it is clearly beneficial to design effective induced learning programs.

2.5.6 Brief Discussion

For this problem, the results of the empirical sensitivity analysis show that for relatively more complex production processes, our model suggests that progress along the learning curve is best achieved through continuous production and concurrent effective induced learning activities.

These results also suggest that accounting for forgetting phenomenon in learning models intensifies and underscores the need to counter the impact of under-capacity production and concurrent forgetting through investments in the development and implementation of effective induced learning activities. During planned production interruptions, the development of effective induced learning programs, through a prudent and thoughtful combination of conceptual and operational learning, can introduce analytical procedures and experiential activities, such as the thoughtful collection of relevant process data for the assessment of cause-effect relationships for understanding and implementing process improvements. Thereby, under-capacity production can open up, during such downtimes of process operation, opportunities for new knowledge, which was also suggested by Zellmer-Bruhn (2003).

2.6 Summary of this Chapter

The primary contribution of this Chapter is a quality-based learning-forgetting model, which will enhance manufacturer's effectiveness in its supplier development efforts. The model allows for a continual assessment and update on the parameters in the model, which is especially desirable in a dynamic production environment. And our comprehensive view of supplier processes in light of under-capacity production, including disruptions, and thereby knowledge depreciation provides, for the manufacturer, a more informed allocation of quality improvement targets among suppliers.

This model distinguishes itself from other learning-forgetting models foremost in how we incorporate knowledge depreciation. Past literature treats knowledge depreciation almost as the synonym of forgetting in autonomous learning, which is typically viewed as a result of production disruptions. We associated forgetting with under-capacity production in general, which can be caused by factors other than disruptions. Furthermore, we proposed that a second dimension of knowledge depreciation should also be taken into consideration. This is called knowledge decay due to under-capacity induced learning, which may include the discontinuation or insufficiency of induced learning activities/investments. Thus, we not only partition knowledge accumulation into autonomous and induced learning, but also correspondingly partition knowledge depreciation into forgetting and knowledge decay. Further, we imbed knowledge depreciation directly into the progression model components, such that accumulated knowledge is directly reduced in the presence of forgetting and knowledge decay.

Our results show that knowledge depreciation is a detriment to quality improvement and the oblivion of which leads to overestimation of variance improvement and a suboptimal system quality cost. Therefore, by also accounting for both forgetting and knowledge decay, the optimum investment strategy necessarily requires additional investment in induced learning. Further, since the presence of knowledge decay weakens the counteractive effect of induced learning on process disruption, quality improvement targets should account for this and be adjusted accordingly.

Our model assumes perfect information exchange between manufacturer and supplier. Nevertheless, asymmetric information structures often exist in supply chains and requires consideration for coordination and contracting. Despite the prevalence of research efforts regarding asymmetric information in other contexts (i.e., inventory problems), similar research is scarce in the context of manufacturer and suppliers' coordination regarding quality-based learning (Kim 2000) and provides a topic for future research. Another potential area of future research would be to investigate, following the realization of the achieved variance reductions, how a manufacturer updates the induced learning rates. One potential approach would be via a Bayesian updating model, which may be similar to that used in Tomlin (2009). Additionally, it may be useful to investigate the effect of interactions among supplier processes, which will result in a more complicated quality cost model for the manufacturer (Plante 2000). Lastly, managerial insights could also be drawn when the learning-forgetting model is constructed in a dynamic planning framework. While production is assumed to be predetermined in this Chapter, it will be interesting to explore, in the presence of knowledge depreciation and limited resources, the optimal paths of both production and induced learning investment over a planning horizon (Dorroh et al. 1994).

CHAPTER 3. A COMPREHENSIVE LEARNING MODEL FOR DETERMINING OPTIMAL INVESTMENTS IN A DYADIC SUPPLY CHAIN

We explore the optimal annually contracted investment strategy in quality improvement efforts within a centralized dyadic supply chain, in which a manufacturer outsources product components to several suppliers. The assembled product quality depends not only on the individual main effects of each supplier's planned investments but also on the often ignored, yet ever present collective interactive effects among the suppliers. Taking and advocating a system coordination approach, in lieu of those models having objectives that focus on maximizing the benefits net costs of either the manufacturer or the supplier, we develop, within a dyadic supply chain, a system coordinating model to assess quality improvement targeted investments that are annually contracted throughout the supply chain. Further, it has been well established that there exists a strong and positive causal relationship between continuous learning and continuous process improvement. More succinctly, neglect investments in learning and quality falters. This relationship is consistent with the notion that reducing variance not only improves quality but also reduces costs, increases productivity and, not by accident, these phenomena are also well mirrored and reflected by the learning curve. Management has realized and reaped the rewards of a sustained emphasis, commitment and continued investment in learning activities, many of which have been targeted towards training in variance reduction concepts and methods (i.e., Six Sigma and the like). Relatively recent research has adopted the increasing evidence that during process interruption, forgetting occurs which has been observed in the data and learning curves emerging from this research. Even more recently, this research has suggested that the forgetting phenomena occurring during process interruption is typically under-estimated. In addition, who would have thought, forgetting occurs even during continuous, non-interrupted process operation. In this research, we propose a system model for the annually contracted allocation of quality improvement targets, achieved via variance reduction that (1) accounts for interactive effects among suppliers, (2) uses the learning curve to model the effect of variance reduction achieved through learning by doing, autonomous learning and/or that achieved by managerial investments in induced learning, (3) accounts for forgetting under process interruption, where both the occurrence and the length of an interruption are modeled as exponentially distributed random variables and (4) reflecting relatively recent research that has provided evidence of the occurrence of forgetting even during process operation, where we introduce a novel technique, having the plausible assumption that both production capacity and training capacity are known.

3.1 Background & Theoretical Support

The quality of a product is one of the most important factors affecting its competitiveness, prompting manufacturers continuously to strive to improve and update their quality management processes. Manufacturers can reduce cost and can focus on core competencies such as product design and marketing by outsourcing its components to external/internal suppliers (Bolandifar et al. 2016). As more and more product components are outsourced to various suppliers, there is an increasing recognition and a renewed emphasis on understanding the impact of the quality for supplier sourced component processes on the end product quality (Gray et al. 2009, Chao et al. 2009).

Through the compounding effect of transmitted variation, supplier sourced components influence through the filter of functional relationships, the variance experienced by the manufacture and, ultimately, the customer. Thus, it is important for members of the supply chain to be aware of this compounding effect of transmitted variation up through the coordinated supply chain where reductions in variance result in experiencing variance reductions at all other downstream members of the supply chain even if they have not invested in quality improvement efforts. Hence, it is a very important concept to consider when deciding on the allocation of quality improvement efforts along the supply chain.

The Dyadic supply chain model of the coordinated chain provides for the one-on-one incentive contract negotiations used to align the self-interests of a member of the supply chain with the other members. These negotiations are essentially bilateral games with bargaining where the manufacturer and supplier enter into an incentive contract voluntarily because both benefit from cooperation (Zusman and Etgar 1981). The supplier has an estimate of his expected profit, independent of the supply chain, including plans to invest in quality improvement resulting in a reduction of his quality cost. Hence, the supplier already has in hand a predetermined reservation profit which any incentive contract must exceed to obtain the participation and the trust to share information openly. Supplier quality heavily influences the end product quality, hence the supplier's independent decision may not be enough for manufacture. The manufacture has to present incentives that are designed to sustain the cooperation of the supplier throughout the term of the contract (Tarakci et al., 2006; Alexander, Li and Plante, 2017). Usually, the manufacturer will present an incentive contract to a supplier with a range of tradeoff values for adjusting two of the contract terms. The resulting range has been predetermined by the manufacturer to provide at least the reservation profit of the supplier while the manufacturer reaps all of the additional profit from the coordinated supply chain. Normally, it is feasible to design incentive contracts by considering the supply chain as an integrated manufacturing system that facilitates monitoring and coordinating the improvement of supplier quality (Bernstein and Kok 2009, Ha 2016, Dong et al. 2016). Through the compounding effect of transmitted variation, improving end product quality is essentially the same as improving supplier's quality, since the end product quality heavily depends on both the functional relationships with a single supplier (components with low interdependency) as well as the interaction effects (component with high interdependency) among several suppliers (Agrawal et al. 2017; Baiman et al. 2001; Plante, ???). Under incentives contracts of a coordinated manufacturing system, Texas Instruments and Chrysler both invest in their suppliers' training programs (Trommer 1996); Intel invites suppliers to participate in its quality improvement program (Roos 2001); Techtronic Industries (TTI) forms and sends quality teams to assist its suppliers for training in quality improvement (Zhu et al. 2007).

While the manufacturer and suppliers need to improve the product quality by improving various indices used to describe quality such as process capability measures, including Six Sigma, reducing the variance of a product's quality characteristics is one of the most well-known and useful paths. Its connection to product cost (Taguchi 1986) and process capability (i.e., Six Sigma) have been well established. Further, quality improvement, via variance reduction, can be achieved by learning and amassing what has been learned into a growing body of knowledge. Indeed, continuous quality improvement in production processes occurs via knowledge accumulation in environments conducive to continuous learning, (i.e., Baily and Farrell 2006). Generally, learning contains two types: autonomous learning (Netland and Ferdows 2016, Egelman et al. 2017) and induced learning (Li and Rajagopalan 1998, Plante 2000, Wang et al. 2013). Autonomous learning is a by-product from repeating working and it is called 'Learning-by-doing'. As long as production activities occur, knowledge accumulates along with it. This type of learning normally incurs little additional investment. Induced learning, on the other hand, emerges from targeted managerial interventions and investments, such as training and engineering activities devoted to operational training in the techniques used to reduce variance. Since both learning mechanisms stimulate variance reduction, it is important to account for both of their contributions in any model of quality improvement in a manner that provides for the separate estimation of their effects. In this regard, we adopt the production function of Wang et al. (2013).

While knowledge accumulates with learning, knowledge also deteriorates in the absence of learning. Several studies have reported that during process disruptions, not only has knowledge accumulation, through autonomous learning, is paused, but that the body of knowledge decreases due to a forgetting phenomena (Jaber and Bonney 2003, Froehle and White 2014). Moreover, Agrawal and Muthulingam (2015) found that more than 16% of quality improvement gains from autonomous learning and 13% of gains attributed to induced learning activities depreciate every year. Evidence shows that companies have lost millions of dollars by involuntary loss of knowledge (Holan et al. 2004). Indeed, researchers (i.e., Argote and Epple 1990) believe that forgetting is always present and Wang et al. (2018) have suggested that under-capacity production and underutilized investments in induced learning activities lead to the deterioration of the amassed body of knowledge. Such deterioration incurs increased variance, decreased process capability and increased product cost. Many existing learning curves such as Wang et al. (2013) are limited in this respect. Hence, in order to consider knowledge depreciation as well, a comprehensive learning curve allowing knowledge depreciation (forgetting) is introduced. Besides learning, the comprehensive learning curve captures the knowledge depreciation and the impact due to random production interruptions. Generally, knowledge depreciation can be effectively countered by additional investment in induced learning, especially those forgetting events arising from production interruptions in suppliers' production processes (Wang et al. 2013).

Product architecture affects quality (Anderson et al. 2000) as well. Product architectures can be separable or non-separable. Separable architecture means product failure can be traced to a particular component, and non-separable architecture means it cannot be traced to a particular component (Baiman et al. 2001). In other words, components interaction is also a key factor to control product quality. Further, Novak and Eppinger (2001) used vehicle front-wheel-drive and rear-wheel-drive examples showing how components interact with each other. More importantly, Novak and Eppinger (2001) pointed out, sometimes, any change of one component requires additional coordination with all the interactive components. Thus, the more interconnected are the components in a system, the more complex coordination becomes.

In this Chapter, we consider that components producing processes are independent and uncorrelated with each other but they can interact to affect the performance measures of the end product. According to Plante (2000), it is plausible by using a quadratic form to measure the components interaction. We assume the end product quality totally depends on supplier components. The quality is measured by several performances, so each performance is directly connected to components. Here, the two way interactive effects are expected to efficiently describe other higher order interaction effects. We incorporate the comprehensive learning curve to model suppliers' autonomous and induced learning, respective knowledge depreciation (Darr et al. 1995) and production interruption. Suppliers can invest in induced learning in order to reduce the components quality cost, and thereby the end product quality cost. These investments in induced learning occur in a dyadic supply chain. We develop a model to determine the optimal induced learning for suppliers and the manufacturing system respectively. Generally, optimal induced learning is larger under an integrated manufacturing system. The manufacture can be better off if they incent the supplier to invest more in induced learning.

The Chapter is organized as follows. In Section 2, we bring comprehensive learning curve into our model, and connect end product quality with suppliers components. We then compare the optimal induced learning amount under suppliers' independent decision and under an integrated manufacturing system. Thus, we provide a range for incentive from manufacture to its suppliers. In Section 3, we provide an example to demonstrate the implementation of our model. In addition, we consider the sensitivity of the optimal solution to changes in several factors in the model. Section 4 extends the model to a more general case. Lastly, Section 5 offers concluding remarks.

3.2 Basic Model Formulation

We consider a manufacturer who produce a fixed amount of product, g_t, for the current production period *t*. There are *M* manufacture performance measures that are critical to the quality of the product and each of these are influenced by *N* components outsourced to *N* independent suppliers. Similar to Plante (2000), we assume that each performance measure depends on the components measures of the suppliers. Specifically, albeit the random error, we assume that the functional relationship between the performance measures and components measures can be well approximated by the following quadratic equation:

$$Y_k = c_0^{(k)} + \sum_{i=1}^{N} c_i^{(k)} X_i + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_{ij}^{(k)} X_i X_j \quad , k = 1, \ldots, M ,$$

(1)

where

Y_k	=	The *k*-th manufacturer performance measure,
X_i	=	The *i*-th supplier component measure,
$c_0^{(k)}$	=	The intercept term for the *k*-th performance measure (i.e., fixed effect),
$c_i^{(k)}$	=	The main effect of the *i*-th component on the *k*-th performance measure,
$c_{ij}^{(k)}$	=	The interaction effect of the *i*-th and *j*-th components on the *k*-th performance measure.

Note: For ease of notation, *k* will index the manufacture performance measures, and *i*, *j* will index the suppliers component measures for the rest of this Chapter.

The component measures are designed to the desired levels during the experiments. Central Composite Designs (CCD) are typically used to estimate coefficients of the quadratic equations for each of the performance measures. To facilitate the development and analysis of our model we use the standardized component measures instead of the original component measures. From (1), we have:

$$Y_k = c_0^{(k)} + \sum_{i=1}^{N} c_i^{(k)}(\mu_i + \sigma_i z_i) + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} c_{ij}^{(k)}(\mu_i + \sigma_i z_i)(\mu_j + \sigma_j z_j)$$

$$= c_0^{(k)} + \sum_{i=1}^{N} c_i^{(k)}\mu_i + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} c_{ij}^{(k)}\mu_i\mu_j + \sum_{i=1}^{N} c_i^{(k)}\sigma_i z_i + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} c_{ij}^{(k)}(\mu_i\sigma_j z_j + \mu_j\sigma_i z_i)$$

$$+ \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} c_{ij}^{(k)}\sigma_i\sigma_j z_i z_j .$$

where

z_i = $\dfrac{X_i - \mu_i}{\sigma_i}$, $i = 1, \ldots, N$, is the *i*-th standardized component measures (independently follow standard normal distribution),

μ_i = The mean of the *i*-th component,

σ_i = The standard deviation of the *i*-th component.

Generally, component measures (variances of X_i 's) improve from period to period during production due to autonomous learning or learning-by-doing. We assume all production information from the previous periods (e.g., previous variances) is available from/for both the manufacturer as well as suppliers. Moreover, in the design process, a manufacturer coordinates all suppliers so that the expected value of each performance measure achieves their intended targets (τ_k and τ_i). We start our analysis from previous period (*t*-1) for performance measure *k*, such that,

$$Y_{k,t-1} = \tau_k + \sum_{i=1}^{N} c_i^{(k)}\sigma_{i,t-1} z_i + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} c_{ij}^{(k)}(\mu_i\sigma_{j,t-1} z_j + \mu_j\sigma_{i,t-1} z_i) + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} c_{ij}^{(k)}\sigma_{i,t-1}\sigma_{j,t-1} z_i z_j .$$

where

$\sigma_{i,t-1}$ = The standard deviation of the *i*-th component by the end of production period *t*-1,

We could further rewrite $Y_{k,t-1}$ as:

$$Y_{k,t-1} = \tau_k + \sum_{i=1}^{N} a_{i,t-1}^{(k)} z_i + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} a_{ij,t-1}^{(k)} z_i z_j \quad , k = 1, \ldots, M \tag{2}$$

where

$$\tau_k = c_0^{(k)} + \sum_{i=1}^{N} c_i^{(k)} \tau_i + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_{ij}^{(k)} \tau_i \tau_j \,,$$

$$a_{i,t-1}^{(k)} = c_i^{(k)} \sigma_{i,t-1} + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_{ij}^{(k)} \mu_j \sigma_{i,t-1} + \sum_{j=1}^{i} c_{ji}^{(k)} \mu_j \sigma_{i,t-1}$$

$$a_{ij,t-1}^{(k)} = c_{ij}^{(k)} \sigma_{i,t-1} \sigma_{j,t-1}$$

With the standardized form, we can separate the parameters into two parts, one is uncorrelated with time and the other is related with time. The manufacturer controls the end product quality by controlling the time related parameters in (2), which totally depends on its supplier components. Hence, the manufacturer leans toward having all suppliers improve their quality as much as possible to increase end product quality. At the same time, the suppliers lean towards lowering their quality cost to get the maximum profit. Both can benefit from the reduction in variance.

3.2.1 Quality, Expected Cost and Variance

Since the late 1970's, it has been commonly accepted that the inherent variance in manufacturing largely determines the level of quality and/or capability of a process (e.g., Six Sigma). According to Taguchi (1986), the following function can well approximate expected quality costs by the corresponding process variance:

$$C = k(\sigma^2 + (\mu - \tau)^2) = k\sigma^2 \tag{3}$$

where C is the product quality cost, which is directly linked to product variance (σ^2) and bias $(\mu - \tau)^2$ by a positive parameter k. Without loss of generality, we assume that the expected value of each performance measure equals its corresponding design target. Thus, the expected quality cost only depends on the variance. Clearly, this relationship captures the ubiquitously accepted notion that improving quality via variance reduction, both reduces cost as well as increases customer perceived quality.

3.2.2 Suppliers Learning and Variance

Managerial intervention through intentional investments in operational learning activities that are targeted to reducing variation in processes is called induced learning. Induced learning activities are deliberatively selected, developed and designed to accelerate the accumulation of knowledge via the traditional method of knowledge accumulation that is reflected in the often studied learning curve of a company. Learning curves have become ubiquitous in unambiguously displaying the proclivity of a company to effectively and efficiently absorb and learn how various factors affect the productivity, cost and quality of their processes. This sounds eerily familiar to the always present refrain of the list of production outcomes that are correlated with and derived from every known implementation of those activities that were learned through targeted continuous operational training, those activities devoted to variance reduction. It is our belief that the learning curve is a precise reflection of the cost of variation as described by Taguchi (1986), which, as we know, is the measure of customer perceived quality. It must follow that the progress function used to model a learning curve must also be used to model the changes in cost of the Taguchi (1986) cost function as knowledge accumulates. Thus, quality cost will be modeled via a convex progress function decreasing at a marginally decreasing rate as knowledge accumulates. The implications are enormous, especially when one imagines or brings to bear the large body of research devoted to the learning curve that can now be directed to the unquestioned causal force that begets the learning curve of a company. We now have a metric that goes beyond just estimating the marginal production costs, rather it is now clear that it also represents and reveals improvements in customer perceived quality and reveals that the underlying cause is variance reduction. Of even greater importance, having its own sizable set of implications, is that knowledge accumulation is the undeniable sole contributing force behind variance reduction. This knowledge is rapidly becoming the most important currency of a company, establishing its value in the market place and especially its value within a coordinated supply chain that through the trust of its members thrives on the freely available and open to other members of the supply chain information housed within its knowledge base and communicated in real time to other members. To survive on today's field of competition, especially in today's environment facing the realities of "Big Data" supply chains have to react in real time and accurately through the use of recommender systems to assist customers through the ever-growing plethora of products available to them. The information database used to build the knowledge of the supply chain is, due to social networks, online behavior of customers, and so on are increasing in Volume (the number of rows or observations in the database) and in Variety (he number of columns/variables/products in the database), and arriving at an increasing Velocity and questionable Voracity, the underlying vehicle for mining this data is the removal of noise to separate the true information from the chaff, where inevitably variance reduction will be of utmost importance. Hence, investments in induced learning activities that are targeted to building and improving the capability of a company to reach a greater level of learned knowledge, say through the use of techniques such as the design and analysis of central composite designs, allowing an analyst the ability to build on prior knowledge by augmenting the knowledge base with response surface designs having the analytical ability to estimate quadratic and even cubic functional relationships.

In addition to accelerating a company along the learning curve, induced learning is effective as a means to counteract forgetting due to process disruptions (Wang et. al., 2013). To model the effects of ubiquitous forgetting in supplier operations, Wang, et. al., (2018) used an approach implied by Nembhard (2001), who used the slack in the capacity of a process to estimate the parameters of a "forgetting curve", which is the ever-present counter-point to the "learning curve". Further, as suggested by Wang, et.al. (2018), the company's resulting body of amassed knowledge is vulnerable to loss when the designed capacity for production and/or the designed capacity of induced learning (measured in units of training) are not meant. The implications of this is clear in that forgetting is ever present, not only during random process disruptions (where the occurrence of a process disruption is modeled by a probability distribution having an increasing hazard rate) and planned disruptions of production process disruption, such as regularly scheduled process maintenance. Thus, it is advisable that a model for learning should consider both learning activities that augment a company's knowledge and forgetting events that deteriorate that knowledge. Combining both sources of knowledge accumulation, the effect on the process variance ($\sigma_{i,t}^2$) of supplier component i (i = 1,…,N) at the end of planning period t is represented by the following progress function (Comprehensive Learning Curve):

$$\sigma_{i,t}^2 = \sigma_{i,0}^2 G_{i,t}{}^{-\lambda_{A,i}} L_{i,t}{}^{-\lambda_{I,i}} = \sigma_{i,0}^2 \left(\alpha_i G_{i,t-1} + d_{i,t} g_{i,t}\right)^{-\lambda_{A,i}} \left(\beta_i L_{i,t-1} + l_{i,t}\right)^{-\lambda_{I,i}} \tag{4}$$

where i in subscript stands for supplier i and the definition for each notation having the following definition,

$G_{i,t}$ = The accumulated autonomous learning knowledge at the end of period *t*,

α_i = The proportion of sustainable autonomous learning knowledge from previous,

$d_{i,t}$ = The production interruption for the planned production during period *t*,

$g_{i,t}$ = The accumulated knowledge due to planned production during period *t*,

$L_{i,t}$ = The accumulated knowledge due to induced learning at the end of period *t*,

β_i = The proportion of sustainable induced learning knowledge from previous,

$l_{i,t}$ = The amount of induced learning activities to be determined for period t,

$\lambda_{A,i}$ = The autonomous learning parameter,

$\lambda_{I,i}$ = The induced learning parameter.

Equation (4) fully captures the reduction in variance achieved through both autonomous and induced learning and the forgetting that occurs for under capacity training and production. been lost in one period. The traditional learning curve has α_i =1 (β_i =1).

3.2.3 Production Interruptions

The Wang et. al. (2013) model for disruptions in supplier processes captured the relationship between forgetting that is known to occur during planned suspensions of process operations (i.e., unavoidable planned disruptions of process operations typically are required during regularly scheduled process maintenance, Tarakci (2006)). The strong relationship between knowledge creation and variance reduction dictate that the deterioration of knowledge must be modeled as an increase in process variance with a corresponding increase in cost and customer perceived decrease in quality. Induced learning has been proposed as one means for counteracting the effect of forgetting during process disruption. For example, during planned suspension of production, it would be clearly advisable to take advantage of these planned process downtimes, where all process operators are available to participate in operational training exercises that have been designed and organized to achieve learning objectives which have been previously formed by both managers and process operators based on their current training progress or level understanding of variance reduction methodologies. These training exercises should be explicitly created to provide operational training for specific variance reduction problems where the amount, type and level of training is, in part, determined via the cost minimizing tradeoff between the cost associated with the training exercises/studies and the realized reduction in variance due to process disruption.

We will denote the proportional amount of the knowledge base lost due to process disruption as $d_{i,t}$, $0 \le d_{i,t} \le 1$ and, accounting for the occurrence and length of the disruption, we choose to model a disruption as a power function, such that,

$$d_{i,t} = n^{-\theta}, \quad \theta > 0$$

n = The number of interruptions, and represents,

θ = The interruption effect scale parameter.

Finally, not all process disruptions are planned. Some of the more damaging process disruptions to the knowledge base can occur unpredictably in both length and cost to bring production back on line, such as disruptions due to the power, severity and unsparing whims of Mother Nature (i.e., Tornadoes, Hurricanes, Earthquakes, Volcanic Eruptions, etc.) as well as the less damaging and less costly to repair, yet still unpredictable disruptions that, combined with the uncertainty of demand facing supply chains, inexorably cause major catastrophic upheavals to the best laid plans of supply chain managers. Threatening an undoing of the incentive contracts that have aligned the self-interest of individual supply chain members with the whole. Threatening the continuation of the hard-earned trust throughout the supply chain. Trust forms then very fabric of the coordinated supply chain. Trust provides the nourishment necessary for sustaining and maintaining the willing cooperation of all members to freely share their most privately held knowledge, acquired via the intellectual talent of its staff, resulting in unique competitive advantages. The coordinated supply chain relies on the trust to open this information to the communication architecture of the supply chain, making it a part of the supply chain’s knowledge base that is available in real time to all members of the supply chain. Thereby forming the knowledge base of the supply chain, which in today’s competitive environment is the very currency or storehouse of wealth of the coordinated supply chain.

Hence, it is important to include the unpredictability of process disruptions in our model. Tarakci, et.al., (2006) suggested that the Weibull probability distribution would be appropriate for modeling the occurrence of unplanned process disruptions, as follows,

$$f(t) = kwt^{w-1}$$

where $w \geq 2$ is the shape parameter, and $k > 0$ is the scale parameter. Using this model, the expected number of breakdowns in the time interval $[0, g_{i,t}]$, is:

$$n = kt^{w} = kg_{i,t}{}^{w}.$$

As a result, the expected interruption influence is:

$$d_{i,t} = (kg_{i,t}{}^{w})^{-\theta}$$

Finally, the progress function model of variance can be stated as,

$$\sigma_{i,t}^{2} = \sigma_{i,0}^{2} G_{i,t}{}^{-\lambda_{A,i}} L_{i,t}{}^{-\lambda_{I,i}} = \sigma_{i,0}^{2} \left(\alpha_{i} G_{i,t-1} + (kg_{i,t}{}^{w})^{-\theta} g_{i,t} \right)^{-\lambda_{A,i}} \left(\beta_{i} L_{i,t-1} + l_{i,t} \right)^{-\lambda_{I,i}} \tag{5}$$

Depending on the capacity of induced learning activities available, we can use this model to assist in the planning of additional induced learning activities to counter the knowledge depreciation due to unplanned process disruptions. Another way to assist in handling both planned and unplanned process disruptions is to outsource regular maintenance and repair responsibilities to a contractor who is offered an incentive contract to become part of the supply chain. The parameters that are used to design these incentive contracts, including the expected cost and expected time to perform regular preventive maintenance and the expected cost and expected time to repair unplanned breakdowns in production were suggested by Tarakci et. al., (2006). Further, the extent to which an incentive contract has the ability to sustain supply chain coordination over the term of the contract is important to maintain the trust of the supply chain membership (Alexander, Li and Plante, (2017)).

3.2.4 Improvement in Component and Performance Variances

Using (5), the model for the change in variance from period t-1 and period t, the reduction and/or increase in variance is straightforward and can be stated as,

$$\frac{\sigma_{i,t}^{2}}{\sigma_{i,t-1}^{2}}=\left[\frac{\alpha_{i}G_{i,t-1}+(kg_{i,t}{}^{w})^{-\theta}g_{i,t}}{G_{i,t-1}}\right]^{-\lambda_{A,i}}\left[\frac{\beta_{i}L_{i,t-1}+l_{i,t}}{L_{i,t-1}}\right]^{-\lambda_{I,i}}, \tag{6}$$

where,

$g_{i,t}$ = The forecasted demand and therefore the production planned for period t,

$G_{i,t-1}$ = The accumulated production at the end of period *t-1*, measuring the net contribution of variance reduction achieved via autonomous learning combined with the variance increasing events or forgetting, due to an achieved production level that is less than the design capacity of the production process,

$L_{i,t-1}$ = The accumulated number of training units in period *t-1* of an overall training program that provide to a company a structure to the specially designed programs akin to the degree programs that are designed by qualified faculty who are responsible for deciding on the body of knowledge that must be mastered. A training unit is envisioned to be equivalent to a typical university section where the complexity of the subject matter determines the enrollment cap for each section. And the number of sections is limited by the number of qualified instructors available to guide students.

$l_{i,t}$ is a decision variable to be determined for *i*-th supplier such that $\sigma_{i,t}^{2}/\sigma_{i,t-1}^{2}<1$. Then, the proportionate reduction in the variance and expected cost of the *i*-th component for period *t* is

$$\frac{\left(\sigma_{i,t-1}^{2}-\sigma_{i,t}^{2}\right)}{\sigma_{i,t-1}^{2}}. \tag{7}$$

Note: In order to consider some extreme cases (e.g. 100% forgetting or knowledge decay), we will allow the constraint $\sigma_{i,t}^{2} / \sigma_{i,t-1}^{2} \leq 1$ to be violated in our empirical analysis in Section 4. Further, using (2), the variance of the k-th performance measure for period t-1 $Y_{k,t-1}$ can be stated as follows (see Appendix-A-5 for proof).

$$\sigma_{k,t-1}^{2} = \sum_{i=1}^{N} (a_{i,t-1}^{(k)})^{2} + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (a_{ij,t-1}^{(k)})^{2} . \tag{8}$$

Similarly, we have

$$\sigma_{k,t}^{2} = \sum_{i=1}^{N} (a_{i,t}^{(k)})^{2} + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (a_{ij,t}^{(k)})^{2} .$$

Then, using (6) and (8), the updated variance of the k-th performance measure is,

$$\begin{aligned}
\sigma_{k,t}^{2} &= \sum_{i=1}^{N} (a_{i,t-1}^{(k)})^{2} \left(\frac{\sigma_{i,t}^{2}}{\sigma_{i,t-1}^{2}} \right) + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (a_{ij,t-1}^{(k)})^{2} \left(\frac{\sigma_{i,t}^{2}}{\sigma_{i,t-1}^{2}} \right) \left(\frac{\sigma_{j,t}^{2}}{\sigma_{j,t-1}^{2}} \right) \\
&= \sum_{i=1}^{N} (a_{i,t-1}^{(k)})^{2} \left[\frac{\alpha_i G_{i,t-1} + d_{i,t} g_{i,t}}{G_{i,t-1}} \right]^{-\lambda_{A,i}} \left[\frac{\beta_i L_{i,t-1} + l_{i,t}}{L_{i,t-1}} \right]^{-\lambda_{I,i}} \\
&\quad + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (a_{ij,t-1}^{(k)})^{2} \left[\frac{\alpha_i G_{i,t-1} + d_{i,t} g_{i,t}}{G_{i,t-1}} \right]^{-\lambda_{A,i}} \left[\frac{\beta_i L_{i,t-1} + l_{i,t}}{L_{i,t-1}} \right]^{-\lambda_{I,i}} \times \\
&\quad \times \left[\frac{\alpha_j G_{j,t-1} + d_{j,t} g_{j,t}}{G_{j,t-1}} \right]^{-\lambda_{A,j}} \left[\frac{\beta_j L_{j,t-1} + l_{j,t}}{L_{j,t-1}} \right]^{-\lambda_{I,j}} .
\end{aligned} \tag{9}$$

Using (8) and (9), for a given investment in induced learning, the proportionate reduction in variance and hence the corresponding expected cost for the k-th performance measure is then,

$$\frac{\left(\sigma_{k,t-1}^{2} - \sigma_{k,t}^{2} \right)}{\sigma_{k,t-1}^{2}} . \tag{10}$$

The metrics in (7) and (10) will be used to access our result in the empirical analysis in Section 3.3.

3.2.5 Suppliers Decision

Independent of the manufacturer, the investment in or the induced learning cost of supplier *i* can reasonably be approximated by $h_i(l_{i,t})^2$, where h_i is a quadratic parameter (Moskowitz et al. 2001). Hence, the total quality cost of the Supplier *i* is $C_i = k_i\sigma_{i,t}^2 + h_i(l_{i,t})^2$. $\sigma_{i,t}^2$, a decreasing function with respect to the level of induced learning, and is convex with respect to $l_{i,t}$. Hence, obtaining the derivative of C_i with respect to $l_{i,t}$, setting it equal to zero, provides supplier *i* with the cost minimizing induced learning of $l_{i,t}^s$ and in terms of incentive contracts for coordinated supply chains, the reservation quality cost C_i^s. Any cost higher than C_i^s is not acceptable by supplier *i*. Under this choice, the manufacture has the quality cost $C_M^s = \sum_{k=1}^{M} K_k\sigma_{k,t}^2$. The total system (supply chain) quality cost is $TEC_t^s = \sum_{i=1}^{N} C_i^s + C_M^s$.

3.2.6 Minimum Quality Cost for Coordinated System

Although the supplier has his own optimal induced learning amount, and the reservation cost. It, generally, will not be optimal for the supply chain. The Total Expected Cost (TEC) of the system chain consists of the supplier's quality cost combined with the manufacturer's quality cost. The trade-off between the investments in induced learning activities and the resulting reduction in quality costs realized by these investments can be stated as follows,

$$TEC_t = \sum_{i=1}^{N} C_i + C_M = \sum_{i=1}^{N} (k_i\sigma_{i,t}^2 + h_i l_{i,t}^2) + \sum_{k=1}^{M} K_k\sigma_{k,t}^2. \tag{10}$$

And TEC_t is a function of $l_{i,t}$ for i=1,…,N. In order to find the optimal induced learning amount for the coordinated system, we need the following Lemma.

Lemma: $\sigma_{k,t}^2$ is convex with respect to the induced learning amount $l_{i,t}$. (Proof in Appendix-A-6)

From the Lemma, we know that TEC_t is a convex function with respect to the induced learning amount, since it is sum of several convex functions. Hence, there exists a global optimal induced learning amount ($l_{i,t}^*$ for all *i*) for each supplier. Consequently, although a closed-form solution for induced learning is not available, a univariate numerical search procedure can be used to find the optimal solution. The increase in induced learning will bring the supplier's variance down, then, via transmitted variation, the end product performance variance is thereby reduced, or $\partial\sigma_{k,t}^2 / \partial l_{i,t} < 0$. Then we have $\sum_{i=1}^{N} \partial C_i / \partial l_{i,t}^s + \sum_{k=1}^{M} K_k \, \partial\sigma_{k,t}^2 / \partial l_{i,t}^s < 0$. Thus, the suppliers' independent optimal induced learning is not enough for the coordinated system ($l_{i,t}^* > l_{i,t}^s$). Under the optimal policy, the total cost of the supply chain is less than that achived by thye suppliers' actin g independently, such that $TEC_t^* < TEC_t^s$, but this, of course, places a greater investment by the suppliers, having a higher quality cost $C_i^* > C_i^s$ .. The reduction in costs via variance education for the manufacturer is,

$$\Delta C_M = C_M^s - C_M^* = (TEC_t^s - \sum_{i=1}^{N} C_i^s) - (TEC_t^* - \sum_{i=1}^{N} C_i^*) = TEC_t^s - TEC_t^* + (\sum_{i=1}^{N} C_i^* - \sum_{i=1}^{N} C_i^s)$$

Although we have shown that coordination results in a lower expected quality cost, questions arise regarding how to develop a mechanism that ensure a cooperative setting and how to allocate the extra saving generated from coordination.

3.3 Numerical Analysis

The purposes of this section are (1) to demonstrate an implementation of our model; (2) to compare the suppliers' decisions under dyadic supply chain and coordinated system. To demonstrate an implementation of our model, we use the multivariate data set from Derringer and Suich (1980). In this scenario, we have a manufacturer who assembles a product from three components, where each component is provided by a different and independent supplier. Each product has four performance measures, which are influenced by the components. A central composite experimental design is used to collect data. The data was used to estimate the parameters of the interactive model, representing the relationship between performance and component measures.

These resulting interactive models are:

$$Y_1 = 133.1 - 13.82X_1 - 14.98X_2 - 9.14X_3 + 3.60X_1X_2 + 5.00X_1X_3 + 5.53X_2X_3, \quad R^2 = 94\%$$

$$Y_2 = 1255 + 224.63X_1 + 206.50X_2 + 116.83X_3 + 48.69X_1X_2 + 66.06X_1X_3 + 73.26X_2X_3, R^2 = 53\%$$

$$Y_3 = 417.5 - 83.49X_1 - 26.30X_2 - 61.92X_3 + 6.14X_1X_2 + 4.39X_1X_3 + 0.88X_2X_3, \quad R^2 = 96\%$$

$$Y_4 = 69.78 - 1.18X_1 + 3.62X_2 + 1.37X_3 - 1.14X_1X_2 + 0.09X_1X_3 - 0.18X_2X_3, \quad R^2 = 87\%$$

Note: $R^2 = {SSR}/{SST}$ is the proportion of the variance in the dependent variable that is predictable from the independent variable(s).

Table 2 *Parameter Setting*

$g_{1,t}$	3	$g_{2,t}$	3	$g_{2,t}$	3
α_1	0.9	α_2	0.9	α_3	0.9
$G_1(t-1)$	6	$G_2(t-1)$	6	$G_3(t-1)$	6
$\lambda_{1,A}$	0.1262	$\lambda_{2,A}$	0.1262	$\lambda_{3,A}$	0.1262
σ_1^2	10	σ_2^2	15	σ_3^2	20
k_1	1000	k_2	1000	k_3	1000
β_1	0.95	β_2	0.95	β_3	0.95
$\lambda_{1,I}$	0.33070	$\lambda_{2,I}$	0.33070	$\lambda_{3,I}$	0.33070
$L_1(t-1)$	12	$L_2(t-1)$	12	$L_3(t-1)$	12
h_1	50	h_2	50	h_3	50
$K_i, i = 1,2,3,4$			1		

Table 2 consists of all the parameters settings. The knowledge-depreciating setting are 0.9 for all forgetting and 0.95 for all knowledge decay. Under the normal situation, we believe 5%-10% of the previous knowledge will be lost from previous process. This proportion of lost is unavoidable. There are many reasons leads to the lost, e.g. well-trained worker leaving; new employee come to the production line; production process minor adjustment; material replacement; new technology introduced. In order to compare the difference when system with or without coordination, we have table 3 and table 4 below.

Table 3 *Dyadic System*

	Manufacture Performance Measures			
Optimal Solution	1	2	3	4
% Reduction in variance	16	16	10	13
	Component Performance Measures			
	1	2	3	
Induced Learning	2.23	3.08	3.84	
% Reduction in variance	8	10	11	
Supplier' Quality Cost	9437.6	13984.74	18449	
Total supplier's Quality Cost	41871.34			
	Dyadic Supply Chain			
Total Cost	3780849.36			

Table 4 *Coordinated System*

	Manufacture Performance Measures			
Optimal Solution	1	2	3	4
% Reduction in variance	61	60	45	55
	Component Performance Measures			
	1	2	3	
Induced Learning	44.00	51.61	51.64	
% Reduction in variance	42	45	45	
Supplier' Quality Cost	102600.90	141469.17	144417.63	
Total supplier's Quality Cost	388487.7			
	Coordinated System			
Total Cost	2173149.66			

The optimal induced learning amount exist under both situations. In addition, without any induced learning, the supplier has to pay 43867.00 as quality cost and the whole system has to pay 4307304.81 for quality. From table 3 and 4 we know that right choice of investment in induced learning can save the quality cost a lot. While suppliers make decision on their own behalf, they want to save as much as possible. A little extra induced learning than optima is unacceptable, even this increased induced learning can save a lot for the whole system. On the other hand, each supplier need to pay almost 10 times quality cost than dyadic supply chain under the coordinated system in our model. The total suppliers' quality cost 8 times more. This extra quality cost is directly contributed by the increased induced leaning amount, which helps the whole system especially the manufacturer save a huge quality cost. Actually, the total saved cost is much more than the extra suppliers' quality cost. To be the greatest beneficiary, as long as the manufacture give enough incentives to his suppliers to make sure all suppliers paying less than their dyadic supply chain quality cost, he can make the whole plan go smooth.

In addition, we can see the compound effect for suppliers' variance reduction. In the dyadic supply chain, since the parameters for suppliers are symmetric expect the initial variance, supplier with higher variance (supplier 3) is willing to invest more in induced learning than supplier with lower variance (supplier 1) while they consider their own interest only. In contrast, considering the compound variance reduction from suppliers, the coordinated system make supplier 2 and supplier 3 have the same level of induced learning.

3.4 Summary of this Chapter

In this section, we propose a comprehensive learning model to consider the quality cost of the manufacturer and the suppliers via product variance, where the manufacturer assemble s an end product from suppliers' components. The components interact on the product performance, hence end product variance depend on components variance.

The comprehensive model assumes that both autonomous learning and induced learning could reduce the process variance while interruption could increase it. To account for forgetting during process disruption, we introduce knowledge depreciation parameters. Knowledge depreciation parameter for induced learning could be estimated from historical data and for autonomous learning could be simulated from the number of breakdowns during the producing process. The Weibull distribution is adopted to model the occurrence of process interruptions. There are two supply chain models that we consider, the uncoordinated and the coordinated supply chain. For each model, we obtain the cost minimizing level of induced learning amount. The suppliers only make decisions based on their own self-interest in the uncoordinated system. Whereas, for the coordinated supply chain, their self-interests have been aligned, via incentive contracts, with the interests of the supply chain.

The coordinated supply chain benefits from the compounding effect of components variance reduction. One outcome of this effect is that the level of induced learning for suppliers in the uncoordinated system is substantially less than that of the coordinated system. Hence, there is a greater amount of investment in the coordinated supply chain relative to that of the uncoordinated system. This additional investment could be offset somewhat by the additional income that is realized from the coordinated supply chain.

CHAPTER 4. THE COORDINATED SUPPLY CHAIN, A MICROCOSM OF ECONOMIC COOPERATION: THE COST MINIMIZING ALLOCATION OF VARIANCE REDUCTION TARGETS

4.1 Introduction

Circa 1980, leaders of industry had their long-held belief about quality shattered. Their goal of quality was making 100% of their product within design tolerances. If a customer required better quality (i.e., tighten the tolerance) then they had to be prepared to pay more. However, this perception of quality changed dramatically as evidence mounted from researchers like David Garvin (1984) who reported their fact-based results emerging from the data and analysis of carefully designed studies that exposed this belief in conformity as inferior to a new quality paradigm at work which indeed had the heretofore unknown ability to increase quality (increasing demand) while simultaneously reducing costs. That is, a customer could expect to pay less for better quality, suggesting a substantially new way for managing a business, that if not followed by competitors, would result in a company having a higher quality product at a lower cost to the customer, providing a clear and unambiguous competitive advantage. From an economic perspective, it provided the discovery of a heretofore unknown variable, having the inherent ability to simultaneously shift the demand curve up and the supply curve down. That variable was variation in product performance.

The name of the game had shifted from conformance to specifications to uniformity of all product performance measures. That is, given a design target, each product would be produced consistently on target as an identical clone of others. Indeed, variation was espoused as the root cause of poor quality and that uniformity must replace conformity as the measure of quality. Further, with this definition, the cost of quality could be well approximated for any symmetric, continuous and differentiable loss function, by the scaled squared deviation of an observation about a performance target as follows (Taguchi 1986),

$$\mathrm{Cost} = k\left(Y - T\right)^2 \tag{1}$$

The expected cost is then,

$$Exp\left[k\left(Y-T\right)^2\right]=k\left[\sigma^2+\left(\mu-T\right)^2\right] \tag{2}$$

From this relationship, variance reduction has a direct effect on the bottom line. And given the acceptance of customers that consistent and reliable performance is their measure of quality, we now have an expression that provides the doubling effect of quality. That is as variation decreases, there is a double positive effect where quality and demand increase while cost of production or supply decreases. In addition to perceived higher quality and lower costs, companies carry leaner inventories as requirements for rework and repair are diminished, reduced service call rates as the reliability of their product has increased leading to an increase in customer perceived quality of their product, eliminating the need for the penalizing concessions due to poor quality, and significantly improving the brand image of their product and the supply chain, leading to not only an increase in the product's brand image and market share, but also for other supply chain products sharing the brand name. They experience production levels that routinely achieve the design capacity of their manufacturing facilities as evidenced by the increased first pass productivity of their processes. They further experience a significant drop in working capital over all their operations, lower insurance premiums given the substantial reduction in reported accidents, characterized by a cleaner work environment. The incredible full breadth of the benefits of variance reduction can be well represented by an iceberg where the directly observable or measurable benefits that accrue from reduced variation, such as reduced rework and repair costs and decreased warranty costs, are modeled by the tip or observable portion of the iceberg and the indirect or hidden costs are modeled by the unobservable or below-the-water-level portion of the iceberg. Much like the iceberg, these hidden or indirect costs, depending on process, industry, etc., can be anywhere between 6 to 50 times the tip-of-the-iceberg direct costs, showing the true breadth of variance reduction's influence. And, within the confines of the coordinated supply chain, the influence of variance reduction by any one member has a growing web of influence on the downstream members of the supply chain through the compounding effect of transmitted variation. Some reduction in variation will occur, along with the breadth of benefits realized via the iceberg effects, because of the variance reduction activities undertaken by an upstream supply chain member, the extent of the reduction depends on the filter of the functional relationships that exist (to be discussed in detail later, including an example). Another, often neglected yet important benefit, is the effect of variance reduction on the workforce. Working in a cleaner and safer environment, conducive to continuous learning and training, where workers are treated as respected partners, encouraged, appreciated, recognized and equitably rewarded for their ideas, providing a workforce that is more confident and trusting of management and management in turn empowers them to pursue variance reduction activities that as communication technology increases, their contributions are recorded, becoming a part of the supply chain's knowledge base and immediately available to the supply chain.

4.1.1 Learning

So, what drives variation down? The answer while simply stated, yet often difficult to implement, is learning. The investment in the development of and continuation of training programs that focus on identifying sources of variation and the sustained effort in mitigating their effect on variation must be of overriding importance to managers. Frankly, absent continued investments in learning, understanding what has been learned and the know how to implement what has been learned, quality falters along with the power of its doubling effect. An untended system will strive toward its natural more chaotic, entropic state.

It is well known by economists that when parties are acting independently and not part of a cooperative system, that market results will be sub-optimal as each party, acting in its own self-interest will place a price margin on their product or service. This is called double marginalization that along with the magnified demand uncertainties associated with the bullwhip effect manipulates and distorts the economic demand curve facing the supply chain. These are the detrimental characteristics which emerge from the "prisoner's dilemma" and other non-cooperative games used in negotiating contracts, lacking trust and open communication and lacking the incentive to freely share information.

4.1.2 Demonstration of Marginalization

Starting upstream at the root of the supply chain, we have a supplier who places a marginal price on each unit of product sold to the next downstream member of the supply chain, the manufacturer. For example, suppose the supplier places a 25% marginal profit on the total cost realized per unit of product sold, the resulting unit price would be, Supplier (per unit cost to the manufacturer):

$$Margin = 1.25\ (Marginal\ cost) \quad (8a)$$

Note: In this scenario, an incentive contract proffered by Jeuland and Shugan (1983) that would align the self-interest of the supplier with those of the system would be discount pricing where the buyer receives a discounted price if they place an order that exceeds a predetermined target. The adjustment feature for this incentive contract is the target which the supplier selects to meet his expected reservation profit.

Due to increasing pressure from his customers for more reliable products and ever shorter expected lead times, the manufacturer must find more time to concentrate on the core competencies of his business. Consequently, he sheds the responsibilities and accountabilities of his non-core business functions by the increasingly common practice of outsourcing non-core business functions, such as process maintenance and logistics (Tarakci et. al., 2006). He adopts this strategy with the intention of enhancing, improving, and maintaining the firm's commitment to its core competencies. Of course, each contractor acting independently and in its own self-interest will place a marginal price on their services, which, in turn will be marginalized when it gets to the retailor who is next up in our downstream tour. The marginal price that the retailer faces from the manufacturer is then,

Manufacturer (per unit price to the Retailer):

Margin = 1.25 (Manufacturer's Marginal Cost)

= 1.25 (1.25 Supplier's Marginal Cost + 1.25 Contractor's Marginal Cost + Marginal Cost incurred from Manufacturer's added value)

= ***1.5625*** *(Supplier's Marginal Cost) +* ***1.5625*** *(Contractor's Marginal Cost) +* ***1.25*** *(Manufacturer's Marginal Cost incurred from his added value* (8b)

Note: An incentive contract for this scenario, including knowledge of the expected costs and expected times required for maintenance and repairs, was proffered by Tarakci, Tang, Moskowitz and Plante (2006) where the contractor is given a contract with two adjustable features. The payment per unit of uptime operation and a target for achieved uptime operation, beyond which the contractor would receive a bonus.

The retailer, who is the primary interface with the customer, invests in advertising, brand management, promotions and other activities in support of product sales and uses a markup of 25% over his marginal costs, such that the per unit price to the customer is,

Retailer (per unit price to the customer):

Margin =1.25 (Customer's Marginal Costs)

*= 1.25 [(**1.5625** (Supplier's Marginal Cost) + **1.5625** (Contractor's Marginal Cost) + **1.25** (Manufacturer's Marginal Cost)] + **1.25** (Retailer's Marginal Cost incurred from his added value)*

*= **1.95315** (Supplier's Marginal Cost) + **1.95315** (Contractor's Marginal cost) + + **1.5625** (Manufacturer's Marginal Cost) + **1.25** (Retailer's Marginal Cost incurred from his added value)* (8c)

For a coordinated supply chain, the marginal price to the Customer is:

*Retailer's Marginal Price = **1.25** [(Supplier's Marginal Cost) + (Contractor's Marginal Cost) + (Manufacturer's Marginal Cost) + (Retailer's Marginal Cost)]* (9)

The difference between equations (8c) and (9) establishes the difference in price that the customer experiences as follows,

Delta price = ***0.70315*** *(Supplier's Marginal Cost) +* ***0.70315*** *(Contractor's Marginal Cost) +* ***0.3125*** *(Manufacturer's Marginal Cost)* (10)

Clearly, the actual price to the customer is much greater than that intended by the manufacturer who still only realizes a **25%** markup, yet the customer is asked to pay a markup of **56.25%**, greater than that intended by the Supplier and contractor who still only realize their **25%** markup, yet the customer is asked to pay a markup of **95.135%.** The economists have a name for this. They call it marginalization. We call it the "voodoo" or smoke-and-mirrors curse of a non-coordinated supply chain where the members are all acting in their own self-interest with a clear detriment to the supply chain. The difference in profit is subject to the demand curve facing the supply chain.

4.1.3 Environment for Learning

Given the importance of learning as the fundamental force leading to efficient and effective variance reduction, an informed supply chain must ask itself the following by now familiar and well-practiced litany of questions: What, why and how should the management of their various enterprises be structured where everyone in the company has a vested self-interest in continuous learning? To encourage participation, it is not unusual for a company to establish a credo and an associated slogan that emphasizes its commitment to establishing an environment conducive to continuous learning. Here are a few,

"We are all in this together… All for one and one for all… The whole is worth more than the sum of its parts… What a life we could have if only we would cooperate... all members must win if the group is to win…"

Economists believe, and we have no evidence to the contrary, that individuals will always act in their own self-Interests. Hence, to obtain the cooperation that these slogans profess, we must find a positive means for aligning individuals to the goals of the whole. This can only be achieved using incentives that are designed to cause individuals to perceive the system's goals as surpassing their own self-interests. This requires that incentives should result in leaving the individual no worse off than what he could have expected to accomplish independently. Interestingly, much has been learned from recent supply chain research in designing incentive contracts specific to the type of relationships between the principle agent and the agent involved in the contract negotiation, such as between a manufacturer and retailers, between manufacturers and suppliers, between manufacturers and contractors and between suppliers and contractors. To the extent that all these institutions are aligned within the supply chain, the more likely that cooperation becomes a reality.

These incentive contracts are the glue that both sustains and maintains a coordinated supply chain throughout the term of the contract (Alexander, Li and Plante, 2016; Qin and Tang, 2007; Boyaci, 2005; Cachon and Lariviere, 2005). The key to a coordinated supply chain is to achieve the cooperation of all members where each member benefits. In addition, where knowledge is exchanged freely and openly to produce a product of value to the customer that is greater than that which could have been achieved in a non-integrated system. Clearly, evidence is mounting rapidly as reports from well thought out research combined with reports from well-conducted field studies are validating the expected benefits predicted from research, as well as unexpected yet related benefits such as the incredible size and impact of the benefits realized relative to the uncoordinated and non-cooperative supply chain. We are at a time, similar to the 1980 -paradigm-shifting quality revolution, where it took staring into the abyss of bankruptcy (often called the wake-up call) before companies finally had the incentive to go all-in on the heretofore never-recognized yet exceptionally powerful doubling effect of variance reduction, increasing quality while reducing cost. Companies that had ignored this are no longer with us today or enjoy (tongue-in-cheek) significantly diminished capacity to compete effectively. These companies essentially find themselves in the quicksand of a growing quagmire of confusion and doubt ever-so-slowly sinking, holding onto and being pulled up by the shoestrings of loyal customers. Such loyalty is misplaced and most certainly will evaporate if these companies choose to remain outside of the coordinated supply chain, and persist in the belief that a marginal increase over their costs is the best mechanism for pricing their products. Much like variance reduction, with theoretical support, repeatedly validated from growing practice, the coordinated supply chain operates at a substantially reduced cost and increased quality. This is, in part due to the elimination of the practice of independent marginalization along with the reduction in variance of the Bullwhip effect. However, we suggest and actually claim that the great majority of reduced cost and increased quality experienced by the coordinated supply chain is, again, due to variance reduction. And not just the variance reductions that occur within a member of the supply chain, but of vital importance, through the transmission of this variance reduction throughout the entire constellation of the coordinated supply chain. Much like the stochastic-demand-driven bullwhip effect that begins small and then grows and thrives in a non-cooperative environment having little communication, and much like the compounding effect of increasing the cost to the customer from marginalization, the effect of variance reduction is compounded to all the downstream members of the supply chain.

Hence, variance reduction, through the compounding effect of transmitted variation is the savior to the marginalization's devil. And in this role represents the heaven of Economic cooperation, from which the once-angels of marginalization have been cast down into the hell of Economic non-cooperation. It is high time that we accept the fact that a coordinated supply chain dominates the uncoordinated supply chain. The naïve will depart from the field of competition going quietly into the night.

Much has been written about the environment within which continuous learning thrives. The precepts of the learning organization (Senge, 1990), of the extended enterprise and the integrated supply chain serve as the precursors and motivation for the emergence of the coordinated supply chain. We envision that a truly coordinated supply chain is a microcosm of economic cooperation where the manufacturer is the principle in dyadic cooperative games with each member of the supply chain acting as an agent as envisioned by Zusman, P. and Etgar, M.(1981). Incentive contracts are negotiated and completed that obtain the cooperation of each agent. Thereby aligning the self-interest of each agent with that of the supply chain. The design of the incentive contract is typically initiated by the manufacturer who provides the agent with a contract having an adjustable feature allowing the agent to select and make an adjustment that provides an expected profit that achieves the agent's reservation profit. The reservation profit is essentially the expected profit that the agent would have realized without involvement as a coordinated member of the supply chain.

To illustrate the combined effects of a coordinated supply chain, we will use a supply chain facing a partial demand curve and consisting of one supplier, followed by a manufacturer who has outsourced his non-core business activities to concentrate on improving his core competencies, followed still by a retailer who provides the interface with the customer. Using this scenario, we intend to illustrate what is possible under the configuration of a coordinated supply chain, comparing the coordinated supply chain and the non-coordinated supply chain. For this comparison, we will determine the cost-minimizing allocation of variance reduction targets for both the supplier and the manufacturer.

4.1.4 Compounding effect of Variance Transmitted up through the Supply Chain

The initial variance of our supply chain, assuming probabilistic independence and transmitted variation absent any learning is,

$$\sigma_{SC}^2 = \sigma_{S0}^2 + \sigma_{C0}^2 + \sigma_{M0}^2 + \sigma_{R0}^2 \tag{3}$$

where,

σ_{SC}^2	=	The variance/reliability of the Supply Chain that the customer experiences,
σ_{S0}^2	=	The variance/reliability of the Supplier,
	=	The variance/reliability of the Contractor,
	=	The variance/reliability of the Manufacturer,
	=	The variance/reliability of the Retailer,

The variation transmitted up through the supply chain is often miss-understood as the focus is typically on their own processes. When variation is transmitted, it does so through the filter of a functional relationship as follows,

$$\sigma_{SC}^2 = \sigma_{S0}^2 + \sigma_{C0}^2 + f_{SM}(\sigma_{S0}^2) + f_{CM}(\sigma_{C0}^2) + \sigma_{M1}^2 + f_{SMR}(\sigma_{S0}^2) + f_{CMR}(\sigma_{C0}^2) + f_{MR}(\sigma_{M0}^2) + \sigma_{R1}^2 \tag{4}$$

Example:

The supplier's product has a performance measure that is a random variable having a variance and a mean μ, such that,

$$X = \mu_X + e,$$

where,

e	=	a random error term having variance σ_{S0}^2 and a mean 0.

Following training on the use of process control charts, over adjustment variance was removed from his process and the variance of X has been reduced by σ_{Sadj}^2.

Variance is updated/reduced as follows,

$$\sigma_{S1}^2 = \sigma_{S0}^2 - \sigma_{Sadj}^2$$

Through the proper use of process control charts, the supplier has reduced the noise or variance of his process, such that the effort or costs (i.e., the sample size or the number of replications in a factorial design of experiments), for the identification of a factor or factors that currently are not precisely controlled but could be to reduce the influence of its variation on the variation of the performance measure. the supplier creates a 1/2 fractional replicate of a full factorial design for 5 2-level factors. He uses this design to assess the main effects, under the assumption of no interactive effects of 15 possible factors suspected of influencing the performance measure. Via this design, the supplier finds a significant relationship between X and a factor under his control, Z, such that the mean of X is aZ. such that,

$$X = aZ + e$$

And the variance has been reduced by,

$$\sigma_{S1}^2 = \sigma_{S0}^2 - \sigma_{Sadj}^2 - a^2\sigma_{SZ}^2 \tag{5}$$

The expected cost as a proportion of the initial expected cost for the supplier is,

$$\frac{\text{Expected Cost}_S\,(\text{new})}{\text{Expected Cost}_S\ (\text{new})} = \frac{\sigma_{S0}^2 - \sigma_{Sadj}^2 - a^2\sigma_{SZ}^2}{\sigma_{S0}^2} = \frac{\sigma_{S1}^2}{\sigma_{S0}^2} \tag{6}$$

The Manufacturer has also removed over adjustment from his process, and yet unbeknownst to the manufacturer, the relationship of his performance measure and X is,

$$Y = bX + e$$

The variance of Y is then reduced as,

$$\sigma_{M1}^2 = \sigma_{M0}^2 - \sigma_{Madj}^2 - b^2(\sigma_{S0}^2 - \sigma_{Sadj}^2 - a^2\sigma_{SZ}^2) \tag{7a}$$

$$\frac{\text{Expected Cost}_M\ (\text{new})}{\text{Expected Cost}_M\,(\text{new})} = \frac{\sigma_{M0}^2 - \sigma_{Madj}^2 - b^2(\sigma_{S0}^2 - \sigma_{Sadj}^2 - a^2\sigma_{SZ}^2)}{\sigma_{M0}^2} = \frac{\sigma_{M1}^2}{\sigma_{M0}^2} \tag{7b}$$

In a coordinated supply chain, conducive to continuous learning the reduction in variance that is transmitted and compounded up through the supply chain is the superglue that maintains a coordinated supply chain to the benefit of all members of the supply chain. However, where does the funding for supporting managerial investments in continuous learning activities?

Here is our suggestion: The coordinated supply chain, through the elimination of marginalization and the bullwhip effect, provides more expected profit than the un-coordinated system. We recommend that the additional profit that is realized by a coordinated system be used to determine the allocation of learning activities throughout the supply chain that are focused on training for variance reduction. Hence, in the next few sections, we develop a model for determining the Cost minimizing allocation of variance reduction targets throughout the supply chain.

4.2 Theoretical Support

Higher product quality could efficiently bring more revenue to brand owner. Researchers (Gort and Klepper 1982; Agarwal and Bayus 2002) indicated that product quality improvement results in demand increase. Bagwell and Riordan (1991) stated that consumers are willing to pay higher price for better quality product; Mark (2000) further explained that price is a signal to quality in theoretically. Six Sigma is one of the most fundamental approach to enhance product quality (Hahn et al. 2000). At the same time, Six Sigma is a popular approach to reduce cost by minimizing waste and resource (Harry and Schroeder 2000). The key concept of Six Sigma is about product variation reduction (Walshe et al. 2010). Knowledge Management (KM) and learning curve (Argote and Epple 1990; Wang et al. 2013; Baily and Farrel 2006) built the connection between organizational learning process and variation reduction (Moskowitz et al. 2001). The accumulated knowledge is a primary source of value for the firm (e.g., Grant 1996, Spender 1996), and it benefit customers by significantly increasing product quality (Ofek and Sarvary 2001). Using a structured approach (Choo et al., 2007; Lapre´ et al. 2000), KM help the firm to accumulate greater knowledge (Schroeder et al. 2008). Product performances depend on the components, hence model considering component compounding effect come out (Plante 2000; Agrawal et al. 2017; Wang et al. ?). Since brand owner and outsourcer have different optimal learning amounts for themselves and their partner. They enter into the contract voluntarily because both benefit from cooperation (Zusman and Etgar 1981). Normally, a feasible incentive contracts to satisfy cooperative supply chain decision is available to coordinated variation reduction (Bernstein and Kok 2009, Ha 2016, Dong et al. 2016).

4.3 The Demand Curve

Follow Banker et al. (1998), we use the linear demand curve, linear in product price and quality.

$$D(P,Q) = Q - bP, \qquad b > 0 \tag{8}$$

where,

P = The marginal or per unit price

Q = The customer perceived quality

b = A coefficient representing the rate at which demand decreases price increases

Absent intentional managerial investments to improve product quality through variance reduction, the perceived quality is $Q = Q^0$ and the corresponding demand is $D^0 = Q^0 - bP$. Generally, decreasing product price or increasing product quality can increase the demand. In our model. Without loss of generality and to facilitate model development, we will assume that the retailer, regardless of changes in perceived quality and decreasing costs because of variance reduction activities, plans to maintain the per unit product price constant at the current level. This is just one of several possible competitive strategies where the retailer does not wish to send signals to his competitors.

The manufacturer and supplier have marginal profit functions as follows,

$$\prod_M = \widetilde{P_M} D - C_{MQ} - F_M \tag{9}$$

where,

$\widetilde{P_M}$ = Unit gross profit of the manufacturer

C_{MQ} = Manufacturer's quality costs

F_M = Manufacturer's fixed cost

$$\prod_S = \widetilde{P_S} D - C_{SQ} - F_S \tag{10}$$

where,

$\widetilde{P_S}$ = Unit gross profit of the supplier

C_{SQ} = Supplier's quality costs

F_S = Supplier's fixed cost

4.4 Quality and Quality Cost

We assume that a manufacturer's performance linearly depends on a supplier performance X_S, such that $Y_M = aX_S$. Accounting for transmitted variation, the variance of the product when it reaches the customer is then,

$$\sigma_Y^2 = a^2 \sigma_S^2 + \sigma_M^2 . \tag{11}$$

We also assume that the inverse relationship between quality and variance can be reasonably represented as,

$$Q = 1/\sigma_Y^2 \quad . \tag{12}$$

Then, with scaling parameters, the quality cost as a function of the corresponding variance is $k_S \sigma_S^2$ and $k_M \sigma_M^2$, where k_S and k_M are positive scaling parameters, for the supplier and manufacturer, respectively.

Defining $e_S \geq 0$ and $e_M \geq 0$ as the induced learning efforts (e.g. investments in training) undertaken by management to reduce process variance for the supplier and manufacturer, respectively. We further assume that there are two marginally decreasing convex functions $f_S(e_S)$ and $f_M(e_M)$ that capture the proportion of variance remaining for the supplier and manufacturer, respectively, following a quality improvement exercise, such that, $f_S(0) = 1$, $f_S(e_S) \leq 0$, $f_e{}'(e_S) < 0$ and $f_e{}''(e_S) > 0$ and similarly for the manufacturer's. Following the induced learning activities, the remaining variances are $\sigma_{eS}^2 = f_S(e_S)\sigma_S^2$ and $\sigma_{eM}^2 = f_M(e_M)\sigma_M^2$ for the supplier and manufacturer respectively.

The investment in induced learning as a function of the efforts of the manufacturer and supplier are respectively $h_S e_S^2$ and $h_M e_M^2$, such that the total costs are modeled as follows,

Manufacturer,

$$C_{MQ} = k_M f_M(e_M)\sigma_M^2 + h_M e_M^2 \tag{13}$$

Supplier,

$$C_{SQ} = k_S f_S(e_S)\sigma_S^2 + h_S e_S^2 \tag{14}$$

In the next few sections, we develop the models that determine the profit maximizing quality improvement allocations, $e_S \geq 0$ and $e_M \geq 0$.

4.5 Non-coordinated Supply Chain

In a non-coordinated supply chain, both supplier and manufacturer make decisions based on their own profit function. From the demand curve, we know that

$$\Delta D = \Delta Q - D^0 = \frac{1}{a^2\sigma_{eS}^2 + \sigma_{eM}^2} - D^0 = \frac{1}{a^2 f_S(e_S)\sigma_S^2 + f_M(e_M)\sigma_M^2} - D^0 \tag{15}$$

For manufacturer, he makes decision by maxing the following equation:

$$\Delta\prod_M = \widetilde{P_M}\Delta D - C_{MQ} + C_{MQ}^0 = \widetilde{P_M}\left(\frac{1}{a^2 f_S(e_S)\sigma_S^2 + f_M(e_M)\sigma_M^2} - D^0\right) - k_M f_M(e_M)\sigma_M^2 - h_M e_M^2 + k_M\sigma_M^2 \tag{16}$$

To find the optimal quality effort, set $\partial\Delta\prod_M \big/ \partial e_M = 0$, we have:

$$\frac{\partial\Delta\prod_M}{\partial e_M} = \widetilde{P_M}\frac{-f_M^{'}(e_M)\sigma_M^2}{(a^2 f_S(e_S)\sigma_S^2 + f_M(e_M)\sigma_M^2)^2} - k_M f_M^{'}(e_M)\sigma_M^2 - 2h_M e_M = 0 \tag{17}$$

Thus, the optimal quality effort for manufacturer is a function of quality effort for supplier. From (17) we have $e_M = g(e_S)$.

The supplier makes her decision by the following function

$$\Delta\prod_S = \widetilde{P_S}\Delta D - C_{SQ} + C_{SQ}^0 = \widetilde{P_S}\left(\frac{1}{a^2 f_S(e_S)\sigma_S^2 + f_M(e_M)\sigma_M^2} - D^0\right) - k_S f_S(e_S)\sigma_S^2 - h_S e_S^2 + k_S\sigma_S^2 \tag{18}$$

Replacing e_M by $g(e_S)$, we have

$$\Delta\prod_S = \widetilde{P_S}\left(\frac{1}{a^2 f_S(e_S)\sigma_S^2 + f_M(g(e_s))\sigma_M^2} - D^0\right) - k_S f_S(e_S)\sigma_S^2 - h_S e_S^2 + k_S\sigma_S^2. \tag{19}$$

Hence, the optimal decision for supplier is the solution of $\partial\Delta\prod_S \big/ \partial e_S = 0$.

$$\frac{\partial\Delta\prod_S}{\partial e_S} = -\widetilde{P_S}\frac{a^2 f_S^{'}(e_S)\sigma_S^2 + f_M^{'}(h(e_S))h'(e_S)\sigma_M^2}{(a^2 f_S(e_S)\sigma_S^2 + f_M(g(e_S))\sigma_M^2)^2} - k_S f_S^{'}(e_S)\sigma_S^2 - 2h_S e_S = 0 \tag{20}$$

By solving equation (20) we have the optimal quality effort for supplier, denoted as e_S^{**}, and for the manufacturer as e_M^{**}. The increased demand is ΔD^{**} and the corresponding increased profit are $\Delta\prod_S^{**}$ and $\Delta\prod_M^{**}$ for the supplier and the manufacturer, respectively. The increased profit for the non-coordinated supply chain is,

$$\Delta\prod = \Delta\prod_M^{**} + \Delta\prod_S^{**}. \tag{21}$$

4.6 Coordinated Supply Chain

Combining the profit functions of the supplier and manufacturer, we can represent the change in profit of the coordinated supply chain, avoiding the effect of double marginalization follows,

$$\Delta\prod_{CS}(e_S, e_M) = \Delta\prod_M + \Delta\prod_S + \Delta\prod_R \quad . \tag{22}$$

The optimal solution for the system is e_S^* and e_M^*. The corresponding system profit is $\Delta\prod_{CS}^*$ and we should have $\Delta\prod_{CS}^* > \Delta\prod^*$. Let $\widetilde{P}$ denote the marginal profit for the coordinated supply chain and, due to the elimination of double marginalization, $\widetilde{P} > \widetilde{P_M} + \widetilde{P_S}$. Solving the following two equations, we find the optimal profit maximizing solution e_S^* and e_M^* for the coordinated supply chain as,

$$\frac{\partial\Delta\prod}{\partial e_M} = \widetilde{P}\frac{-f_M^{'}(e_M)\sigma_M^2}{(a^2 f_S(e_S)\sigma_S^2 + f_M(g(e_S))\sigma_M^2)^2} - k_M f_M^{'}(e_M)\sigma_M^2 - 2h_M e_M = 0 \tag{23}$$

$$\frac{\partial\Delta\prod}{\partial e_S} = \widetilde{P}\frac{-a^2 f_S^{'}(e_S)\sigma_S^2}{(a^2 f_S(e_S)\sigma_S^2 + f_M(g(e_S))\sigma_M^2)^2} - k_S f_S^{'}(e_S)\sigma_S^2 - 2h_S e_S = 0 \tag{24}$$

Theorem: The profit maximizing investments in quality improvement for each member of the coordinated supply chain, e_i^* is greater than that of the non-coordinated supply chain, e_i^{**}. (proof provided in Appendix A-7)

Hence, both supplier and manufacturer should invest more on quality effort under the coordinated system. That is $e_S^* \geq e_S^{**}$ and $e_M^* \geq e_M^{**}$.

Lemma1: The achieved improvement in quality of the profit maximizing solution for the coordinated supply chain is greater than that of the uncoordinated supply chain.

Lemma 2: The achieved reduction in costs of the profit maximizing solution for the coordinated supply chain is greater than that of the uncoordinated supply chain.

These lemmas follow directly from the theorem and suggest that the positive doubling effect of variance reduction is greater in the coordinated supply chain. Hence, the demand curve shifts up further under the profit maximizing quality improvement investments of the coordinated supply chain than under the uncoordinated supply chain.

Figure 6 illustrates these effects on the demand curve under Lemma 2. Without any quality effort, the average total cost curve (ATC) is the highest. The equilibrium price and quantity can be found from the intersection between ATC and Demand Curve. The investments in quality improvement drives the ATC curve down while the investment required to achieve the quality improvement pushes against this downward movement until both forces are equivalent at the optimal ATC, after which the level of investments required to achieve the corresponding quality improvement overtake the realized reduction in costs, causing the ATC curve to move upwards as marginal investments become less than the marginal reduction in cost realized by addition efforts in quality improvement.

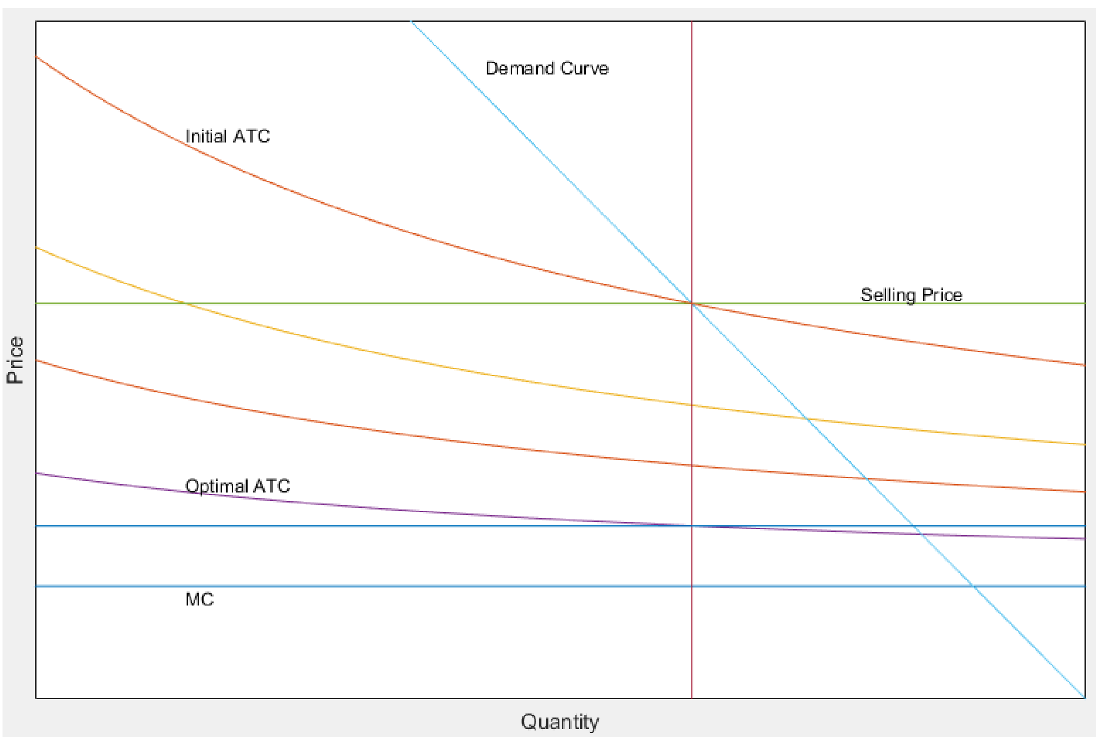

Figure 6 ATC Shitfing

Figure 7 shows the effect of both Lemmas, wherein variance reduction both drives the ATC curve down and shifts the demand curve up. Hence, additional profit enters into the system through an increase in variance reduction efforts. In Figure 7, the initial demand and ATC is represented by a solid line; and the shifted demand and ATC curves are represented by dashed lines. Under this scenario, there are two points in Figure 7 designated by A and B that illustrate several reasonable price strategies for the coordinated supply chain. For example, they could choose to retain the current price avoiding the signaling of a price reduction, resulting in the uncertainty surrounding how competitors might react. Why wake up a sleeping dog? And reaping the rewards of an increase in demand and improved brand image resulting from customer perceived improvements in the quality and reliability of its products. Second, they could cooperate with the retailer to lower the price a bit, calling it a promotional price to move inventory or to introduce new products. If the competitors follow suit, they will be operating at a loss while you are still, unbeknownst to them, making a profit. If the supply chain decides to lower the price to the partial market equilibrium price and competitors follow suit, some of them will not survive. Caution should be used here, since the psychological price effect on quality could cause customers, who have become accustomed with paying higher prices for higher quality products, to react to large price reductions as inconsistent with a higher quality (this is where advertising and promotional messages by the Supply Chain can correct these perceptions and align them with reality).

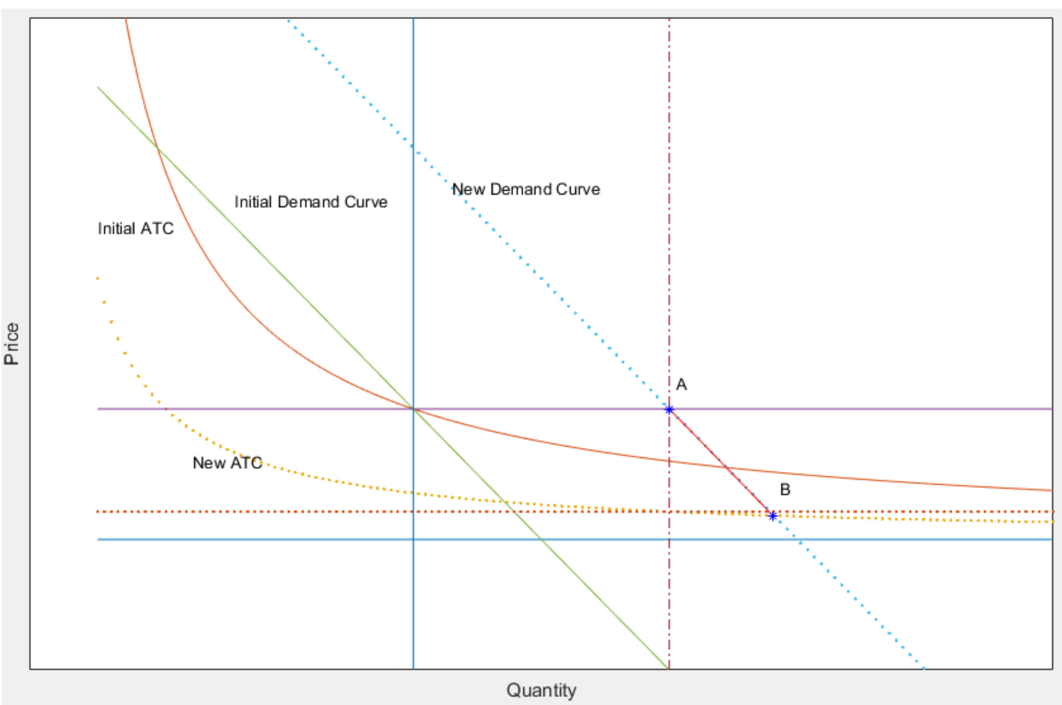

Figure 7 ATC & Demand Curve Shifting

4.7 Numerical Example

In this section, we use a numerical example to demonstrate our analytical results. Namely, (1) quality effort brings profit to the system; (2) product from the coordinated system has higher quality; (3) the coordinated system could benefit more from quality improvement. Table 5 has all the parameters settings. The quality efforts function for manufacturer and supplier areand.

Table 5 *Input parameters for the Example Problem*

Supplier unit Cost	100	Manufacturer unit cost	100
σ_S^2	40	σ_M^2	40
k_S	1,500	k_M	2,500
h_S	80	h_M	50
Retailer unit Cost	100		

Table 6. contains the results for the example problem. To facilitate a comparative assessment of the results for both the uncoordinated and the coordinated supply chains, Table 6 has been formatted such that the 1st row contains the results of the coordinated supply chain. Whereas the 2nd row shows the results achieved by the uncoordinated supply chain. The last row provides the differences between these differing supply chains. These differences can be used to assess the extent of benefits, if any, achieved through the time and effort devoted to develop incentive contracts that have aligned the self-interest of the supplier to those of the supply chain. Thereby earning the mutual trust and cooperation which defines a coordinated supply chain. And the all to often encountered distrust and non-cooperative economic games where each party is expected to make decisions that are in their own self-interests. The Elephant-in-the-room question is: So What? While sizeable differences have been shown in theory and validated in practice in support of the expected benefits realized through the removal of marginalization and the mitigation of the bullwhip effect, can one expect to accrue additional benefits from every day planning activities at a magnitude sufficient for earning the level of trust and cooperation necessary for achieving the economic cooperation of the coordinated supply chain worth the effort. The answer, at least for planning the allocation of quality targets among the members of a supply chain is a resounding yell-from-the-rooftop, YES.

Just a cursory glance of these results is enough for this example. Prior to investing in quality, the results in Table 6 shows that the uncoordinated supply chain earns $516,175 in Total Profit. While the coordinated supply chain earns a more than twice that, having a Total Profit of $1,196,800 for a difference of $680,625. Using our suggestion, we would consider this additional profit of the coordinated supply chain, $680,625, available for allocating investments in induced learning activities. The optimal investment amount for reducing variance is shown in Table 6 as $413,223, leaving $267,402 to be shared or more likely invested. The optimal learning investment for the uncoordinated supply chain is a paltry $374, just .09% of the investment of the coordinated supply chain, which is efforts are greatest for the supplier in both coordinated and uncoordinated supply chains. Table 7 shows the expected reduction in variation achieved through training investments. For both the coordinated and uncoordinated supply chains, the largest proportional reductions in variance occurred at the supplier. This is not surprising given the compounding effect of the supplier efforts through transmitted variance reductions.

Table 6 *Results for the Example Problem (Profit and Investment)*

Profit	Uncoordinated	Coordinated	Difference
W/O Quality Effort	516,175	1,196,800	680,625
W/ Quality Effort	516,278	2,646,374	2,130,096
Difference	103	1,149,574	
Investment in Quality	374	413,223	412,849

Table 7 *Percentage Variance Reduction*

Variance Reduction	Reduction from supplier	Reduction from Manufacturer
Uncoordinated	3%	1%
Coordinated System	34%	6%

Further, the proportional reduction in variance achieved by the supplier in the coordinated supply chain is 34% which is more than ten times that of the uncoordinated supply chain. In addition, the proportional reduction in variance experienced by the manufacturer in the coordinated supply chain is 6 times that of the uncoordinated supply chain.

Finally, the difference between the coordinated system and the uncoordinated system is a resounding $2,646,374 - $516,278 = $2,130,096 in additional expected profits. For this example, the coordinated supply chain had more than 5 times the expected profit of the uncoordinated supply chain. While we had anticipated that the coordinated supply chain would prevail, we did not expect the order of magnitude difference between these two systems that we observe.

4.8 Summary of this Chapter

The not so obvious deleterious effect of marginalization is known to be eliminated by the formation of a coordinated supply chain. A coordinated supply chain is a connected system of suppliers, contractors, manufacturers and retailers working in cooperative dyadic relationships where each member’s self-interest has been aligned with the interest of the supply chain. These aligned interests are obtained by a coordinator of the supply chain, usually a manufacturer, via situation-specific incentive contracts that have been proposed and analyzed in relatively recent research for incentive contracts between a manufacturer and a supplier (price discounts, Jeuland and Shugan (1983)), between a manufacturer and a contractor (Bonus Uptime, Tarakci, Tang, Moskowitz and Plante (2006)) and between a manufacturer and a Retailer (Revenue Sharing. Cachon and Lariviere (2005) and Commission/Compensation, Boyagi (2005)). Generally, an initial incentive contract proposal is offered, wherein the contractor is permitted to make one or two adjustments, within certain ranges, to modify the contract such that the expected profit exceeds the contractor’s reservation profit. The characteristics of a coordinated supply chain include a level of trust and open communication among its members that assures cooperation in the creation of information and the sharing of information in real time, together amassing an accessible data base of knowledge that becomes the supply chains currency establishing its competitive position in the marketplace. Given the real-time communication of the coordinated supply chain, it is salve for the ills wrought on by the bullwhip effect. In combination, the elimination of marginalization and the muting of the bullwhip effect in coordinated supply chains has motivated several ongoing research efforts that should encourage practical implementation. However, much like the resistance experienced at the outset of the quality revolution there remains resistance to this notion of a coordinated supply chain. Managers, who for decades have played the uncooperative economic games of competition have difficulty in believing that a system based on trust and open communication can prevail.

In this Chapter, we suggest that the tipping point greatly in favor of the coordinated supply chain is the concept embedded in targeted learning. That is training programs should be targeted toward variance reduction. Indeed, variance reduction is the only investment decision that has the doubling effect of increasing quality while reducing costs. We further study this concept as it is reflected in the partial economic demand curve where advances in variance reduction simultaneously shifts the demand curve up while shifting the supply curve down. Finally, the nail in the coffin for the demise of the uncoordinated supply chain is transmitted variation. Like variance reduction, transmitted variation thrives on the open communication, sharing, cooperation and trust of the coordinated supply chain. As such the coordinated supply chain provides a microcosm of Economic Cooperation.

Given the importance of learning, we develop a model for determining the optimal investment for a supply chain in training programs. We applied the model to both the uncoordinated and the coordinated supply chains. The results of our research strongly suggest that there is an order of magnitude difference between the uncoordinated and the coordinated systems. The investments in learning of the coordinated system are much higher than those of the uncoordinated system. It follows that there is a much greater positive impact on the demand curve since a greater reduction in variance has been achieved. Further, given the additional profit that is realized by the coordinated supply chain, this provides the source of additional funds that can be used for investments in training throughout the supply chain.

CHAPTER 5. SUMMARY AND FURTHER RESEARCH

This dissertation investigates several important problems within a supply chain as the system continuously to improve product quality. Chapter 2 examines the quality improvement model when the end product has several performances and each performance measurement depends on the quality of components. Different from the traditional learning curve (Fine 1986, Li and Rajagopalan 1998b) or the improved learning curve (Weijia et al 2013), we introduce comprehensive learning curve in order to improve the product quality by variance reduction through the supply chain. The comprehensive learning curve take autonomous learning, induced learning along with their respective knowledge depreciation into account. This learning model can better understand organizational learning effect and provide a better instruction for manufacturer system to invest in employees training (induced learning). In order to reduce the end product measurement variation, Chapter 2 proposes the model improving quality from suppliers components by investing in induced learning. This model optimizes over one-period ahead, which utilize the previous information to continually assess and update on the parameters in the model in a dynamic production environment. Moreover, this Chapter provides a sensitivity analysis to investigate those factors affecting the amount of induced learning investment. There are several contribution in the Chapter 2. On one hand, it shows that faster learning speed could efficiently reduce product measurement and hence improve product quality. On the other hand, it indicates that knowledge depreciation hinder quality improvement. Such detriment will lead to an increase of variance and an expensive system quality cost. There are two kinds of knowledge depreciation—forgetting and knowledge decay. Forgetting is caused by disruption during the producing process and knowledge decay is disruption after training. Therefore, accounting for both forgetting and knowledge decay, the optimum investment strategy necessarily requires additional investment in induced learning. A proper solution concerning the induced learning amount could be obtained by debilitating calculation. In short, accounting for both forgetting and knowledge decay, the optimum investment strategy necessarily requires additional investment in induced learning. Besides the basic solution, the changed of parameters also affects the solution largely. Further, since the presence of knowledge decay weakens the counteractive effect of induced learning, quality improvement targets are adjusted accordingly.

Chapter 3 primarily focus on the quality improvement along the dyadic supply chain. Based on Chapter 2 and Plante (2000), this Chapter introduced interaction effect among components. Hence, a more general model is developed in this Chapter. At the same time, this Chapter consider suppliers, brand-owner's behavior when they facing quality improvement.

Quality improvement in this Chapter is the same as Chapter 2. Product quality is estimated by several performance measurements, and each performance depends on outsourced components. These components have interaction effect, approximated in the quadratic form. Therefore, quality improvement of the end product is transmitted to each supplier. Followed Chapter 2, the quality is estimated by measurement variance, which is rooted from Taguchi (1986). Lower variance indicates better product quality and comprehensive learning curve, introduced in Chapter 2, is used to control the variance. The managerial activity, which adopts comprehensive learning curve, is investment in induced learning. In Chapter 2, we just give an estimation of forgetting. In order to better simulating forgetting effect, this model considers forgetting is caused by machine downtime or process disruption. This Chapter used Weibull distribution to approximate such disruption. It is suggested by Tarakci et al. (2006). This chapter provides existence proofs of the optimal induced learning amount for every suppliers. However, under dyadic supply chain, each supplier want to minimize her own cost, a suboptimal solution exists for each supplier. Since components have compounding effect on the end product, the optimal induced learning amount is greater than the sub-optimal induced learning amount. Under to suboptimal solution strategy, a reservation cost is obtained for each supplier. More investment is necessary for each components supplier under the coordinated system. In order to coordinate the system and guarantee each supplier invest in the optimal amount, an incentive reasonable from the manufacturer to each supplier. Since the coordinated system has less quality cost than the dyadic supply chain, the manufacturer still get additional benefits after the incentives. Last, a numerical example is used to illustrate our model. The solution of the numerical example perfect matches our theoretical conclusion.

In Chapter 4, a more general model is extended from Chapter 2 and Chapter 3. From previous Chapters we know that proper investment in induced learning is actually bringing the quality cost down. This is because such quality effort bringing the product variation down. With lower variation, product perform high quality. This Chapter considers these two aspects. In general, quality investment not only saves quality cost, but also creates additional demand. To better simulate the coordinated system, this Chapter considers the assembling variance as well. Taguchi (1986) connected quality cost to variance. Comprehensive learning curve further connect variance into quality investment. All these model is limited to quality cost area. This Chapter makes connection between variance and product quality. What's more, in order to understand component effects through the whole supply chain, this model illustrates the compounding effect; in order to understand benefits from the coordinated system, this model illustrate the marginality. The contribution of this Chapter is obvious, under to coordination, the system requires more quality efforts, and there is more profits with coordination. Since coordinated system could create more profit than dyadic supply chain, the additional profit could be used to quality improvement. There are two accompanying conclusion based on it: under the coordinated system, product perform better in the sense of quality and the quality cost is less. This Chapter also discuss the price strategy after quality improving. Basically, there are three strategies. First, they could maintain the same price to keep a stable market. Second, just lower the price to the profit maximization point to get the highest profit. Alternatively, lower the price to the average total cost, this strategy will occupy the whole market and become a monopoly in future. By the end of this Chapter, a numerical example is used to show the contributions. The example used the price strategy one. It shows that marginality reduced the profit indeed. In addition, the additional profit create by the coordination can be used to reinvest into quality improvement, which will bring more profit in further.

This research provides substantial insights into the impact of comprehensive learning curve and collaborative quality improvement efforts as well as manufacturer and suppliers' quality investment strategy. There are still several unanswered interesting questions. First, coordinates system create additional profit, how to allocate the additional profit among the suppliers and the brand-owner in order to realize the planned strategy? In our research, we just assumes that everyone gets at least the reservation profit. A quantitative model could extend this and a feasible solution with the changing of parameters need further discuss. Second, our models assumes perfect information exchange among the supply chain. Nevertheless, asymmetric information structure often exists within supply chains and requires consideration of coordination and contracting. Despite the prevalence of research efforts regarding asymmetric information in other contexts (i.e., inventory problems), similar research is scarce in the context of manufacturer and suppliers' coordination regarding quality-based learning (Kim 2000) and provides a topic for future research. A third potential research is extended the supplier into a more general form. The suppliers in our model only provide the components to one manufacturer. In the reality, the suppliers provide components to multiple manufacturer, and they have competitor as well. A model considering component quality and obtained additional profit is desired from supplier’s view.

APPENDIX

For sensitivity analysis, we first note that the optimal $l_{i,t}^*$ can never be negative because the second term of (25) is a decreasing function of $l_{i,t}$, so $l_{i,t} = 0$ is always a better solution than any negative value.

To solve the optimal investment in induced learning, we set (26) to zero to obtain the following necessary condition for the optimal :

$$l_{i,t}\left(b_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)}\right)^{-(\lambda_{i,I}+1)} = \frac{\lambda_{i,I}\left(\frac{l_{i,t}(1+b_i)}{2L_i(t-1)}\right)}{2h_i q_i^2} v_{X_i(t)}\left[a_i + \frac{g_{i,t}(1+a_i)}{2L_i(t-1)}\right]^{-\lambda_{i,A}} . \qquad \text{(A.1)}$$

Let $f(l_{i,t}) = l_{i,t}\left(b_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)}\right)^{\lambda_{i,I}+1}$, for $l_{i,t} \geq 0$. Obviously, $f(l_{i,t})$ is a strictly increasing function of $l_{i,t}$.

A-1. Directional Effect of Knowledge Depreciation, (a_i, b_i).

Mathematically, if a_i increases, then β_i in (27) decreases and, consequently, the right side of (A.1) will decrease. This in turn implies a smaller $f(l_{i,t})$ on the left side of (A.1). But because $f(l_{i,t})$ is a strictly increasing function of $l_{i,t}$, the optimal $l_{i,t}^*$ must be smaller as a result of an increasing a_i. With respect to the expected total cost, assume a_i increases to $\overline{a_i}$. Let $l_{i,t}^*(a_i)$ and $l_{i,t}^*(\overline{a_i})$ denote the corresponding optimal investments in induced learning. From (25), it is obvious that $TEC_{X_i(t)}(\overline{a_i}, l_{i,t}^*(a_i)) \leq TEC_{X_i(t)}(a_i, l_{i,t}^*(a_i))$. Furthermore, since $l_{i,t}^*(\overline{a_i})$ is the optimal investment for induced learning corresponding to $\overline{a_i}$, we must have $TEC_{X_i(t)}(\overline{a_i}, l_{i,t}^*(\overline{a_i})) \leq TEC_{X_i(t)}(a_i, l_{i,t}^*(a_i))$. Combining the last two inequalities, we show the minimum $TEC_{X_i(t)}$ decreases when a_i increases.

Next, we consider the influence of b_i, which describes the proportion of knowledge a supplier can retain from previous training/induced learning. Mathematically, we see from (A.1) that there is no explicit solution for the optimal $l_{i,t}^*$. Actually, the optimal $l_{i,t}^*$ is an implicit function of b_i. In order to find the directional effects, we will use the Implicit Function Theorem (IFT) as follows. For all $l_{i,t} \geq 0$ and $0 \leq b_i \leq 1$, define $F(b_i, l_{i,t}) = \dfrac{\partial TEC_{X_i(t)}}{\partial l_{i,t}}$, then we have the following partial derivatives:

$$F_{b_i} = \frac{\lambda_{i,I}\beta_i}{2L_i(t-1)}\left(b_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)}\right)^{-(\lambda_{i,I}+2)} \frac{\lambda_{i,I}(1+b_i)(2L_i(t-1)+l_{i,t}}{2L_i(t-1)} + 1 \geq 0 \ ,$$

and

$$F_{l_{i,t}} = 2h_i q_i + \lambda_{i,I}(\lambda_{i,I}+1)\frac{(1+b_i)}{2L_i(t-1)}\beta_i \frac{1+a_i}{2L_i(t-1)}\left(\gamma_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)}\right)^{-(\lambda_{i,I}+1)} > 0 \, .$$

By the IFT, any solution(b_i, $l_{i,t}$)to $F = 0$ must satisfy $\dfrac{\partial l_{i,t}}{\partial b_i} = -\dfrac{F_{b_i}}{F_{l_{i,t}}} \leq 0$. This result implies that a larger b_i gives a smaller optimal $l_{i,t}^*$. Finally, we can follow the same argument as for a_i to show that $TEC_{X_i(t)}$ in (25) decreases as b_i increases.

In summary, we conclude that, if the knowledge detention parameters for the autonomous and induced learning increase, both investment in induced learning and total expected cost will decrease.

A-2. Directional Effect of Planned Production $g_{i,t}$.

According to Dorroh et al. (1994), planned production, minimizing total expected cost, should increase over the planning horizon. Therefore, having a relatively greater $g_{i,t}$ for the current period, the amount of accumulative knowledge will be comparatively larger and the expected total cost is expected to be lower. From (27), a larger $g_{i,t}$ will lead to a smaller β_i, and hence a smaller $l_{i,t}^*$.

To study the effect of $g_{i,t}$ on the expected total cost, we re-write the necessary condition for optimality as (c.f. (26))

$$2h_i q_i^2 l_i = \lambda_{i,I} \beta_i \frac{1+b_i}{2L_i(t-1)} \left(b_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)} \right)^{-(\lambda_{i,I}+1)} . \tag{A.2}$$

When $l_{i,t}^*$ is smaller (as a result of a smaller $g_{i,t}$), the left side of (A.2) is smaller. So, maintaining the equality, the right side has to be smaller as well. Since the optimal $l_{i,t}^*$ is always non-negative, a smaller $l_{i,t}^*$ results in a smaller first term of the expected total cost in (25). The second term can be rewritten as

$$v_{X_i(t)} \left(a_i + \frac{g_{i,t}(1+a_i)}{2G_i(t-1)} \right)^{-\lambda_{i,A}} \left(b_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)} \right)^{-\lambda_{i,I}} = \lambda_{i,I} \beta_i \frac{1+b_i}{2L_i(t-1)} \left(b_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)} \right)^{-\lambda_{i,I}-1} \frac{b_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)}}{\lambda_{i,I} \frac{1+b_i}{2L_i(t-1)}}$$

where the first term is exactly equal to the right side of (A.2). The last ratio above also becomes smaller as $l_{i,t}^*$ decreases. Hence, the total expected cost in (25) is smaller when $g_{i,t}$ increases.

To summarize, a higher full-capacity production proportion $g_{i,t}$ leads to a smaller induced learning investment $l_{i,t}$ and a smaller total expected cost $TEC_{X_i(t)}$.

A-3. Directional Effect of Prior Variance $\sigma^2_{X_i(t-1)}$

If the prior variance $\sigma^2_{X_i(t-1)}$ increases, the quality cost should increase, and the investment in learning should increase to offset the effect increasing variance. Intuitively, since the system is relatively less stable resulting in a greater cost of quality, the company should invest more in training. In this scenario, the quality cost is comparatively higher and additional investment in induced learning is recommended (Dorrah et al. (1994)).

Mathematically, a larger $\sigma^2_{X_i(t-1)}$ leads to a larger $v_{X_i(t)}$ in (24). Hence, the left side of (A.1) is strictly increasing in $l_{i,t}$. Thus, as $v_{X_i(t)}$ increases, the right side of (A.1) increases and, for the equality to hold, the optimal $l_{i,t}^*$ has to increase as well. To estimate the effect of $\sigma_{X_i(t-1)}^2$ on $TEC_{X_i(t)}$, we will use the same argument that we used for assessing the effect of increasing b_i. Both the first term and second term of (25) increase due to a larger $\sigma_{X_i(t-1)}^2$. Thus, a larger variance $\sigma_{X_i(t-1)}^2$ leads to a greater investment in induced learning and a relatively higher expected total cost $TEC_{X_i(t)}$.

A-4. Directional Effect of the Induced Learning Parameter, $\lambda_{i,I}$

$\lambda_{i,I}$ is the induced learning parameter and, if increased, the system has a better use of the existing knowledge, the expected total cost $TEC_{X_i(t)}$ will decrease. Similar to the analysis for a, a larger $\lambda_{i,I}$ will lead to a smaller $TEC_{X_i(t)}$.

Since $l_{i,t}$ is an implicit function of $\lambda_{i,I}$, we have

$$
\begin{aligned}
F_{\lambda_i,I} &= -\beta_i \frac{1+a_i}{2L_i(t-1)}\left(\gamma_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)}\right)^{-(\lambda_{i,I}+1)} - \lambda_{i,I}\beta_i \frac{1+a_i}{2L_i(t-1)}\left(\gamma_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)}\right)^{-(\lambda_{i,I}+1)} \ln\left(\frac{1}{\gamma_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)}}\right) \\
&= -\beta_i \frac{1+a_i}{2L_i(t-1)}\left(\gamma_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)}\right)^{-(\lambda_{i,I}+1)} - \lambda_{i,I}\beta_i \frac{1+a_i}{2L_i(t-1)}\left(\gamma_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)}\right)^{-(\lambda_{i,I}+1)} \ln\left(\frac{L_i(t-1)}{L_i(t)}\right) \\
&= -\beta_i \frac{1+a_i}{2L_i(t-1)}\left(\gamma_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)}\right)^{-(\lambda_{i,I}+1)} \left(1+\lambda_{i,I}\ln\left(\frac{L_i(t-1)}{L_i(t)}\right)\right)
\end{aligned}
$$

According to the IFT, we obtain

$$
\frac{\partial l_{i,t}}{\partial \lambda_{i,t}} = -\frac{F_{\lambda_{i,I}}}{F_{l_{i,t}}} = \frac{\beta_i \frac{1+a_i}{2L_i(t-1)}\left(\gamma_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)}\right)^{-(\lambda_{i,I}+1)} 1+\lambda_{i,I}\ln\left(\frac{L_i(t-1)}{L_i(t)}\right)}{2h_i q_i + \lambda_{i,I}(\lambda_{i,I}+1)\frac{(1+b_i)}{2L_i(t-1)}\beta_i \frac{1+a_i}{2L_i(t-1)}\left(\gamma_i + \frac{l_{i,t}(1+b_i)}{2L_i(t-1)}\right)^{-(\lambda_{i,I}+1)}},
$$

so the sign of $\partial l_{i,t} / \partial \lambda_{i,I}$ depends on the sign of $1+\lambda_{i,I}\ln(L_i(t-1)/L_i(t))$ since other factors on the right side are all positive. For some manufacturers, the accumulated induced learning knowledge increases over the entire production horizon, we have $L_i(t-1)/L_i(t) \le 1$ and thus $\ln(L_i(t-1)/L_i(t)) \le 0$. As we can see, there exist a $\tilde{\lambda}_{i,I}$ such that $1+\lambda_{i,I}\ln(L_i(t-1)/L_i(t)) = 0$. Then, $\partial l_{i,t}/\partial\lambda_{i,I} \ge 0$ for $\lambda_{i,I} \le \tilde{\lambda}_{i,I}$, and $\partial l_{i,t}/\partial\lambda_{i,I} \le 0$ for $\lambda_{i,I} \ge \tilde{\lambda}_{i,I}$. So, we can conclude that as $\lambda_{i,I}$ increases, $l_{i,t}^*$ first increases and then decreases. On the other hand, if $L_i(t-1)/L_i(t) > 1$ at time *t*, we have $\partial l_{i,t}/\partial\lambda_{i,I} \ge 0$, and we'll always have an increasing $l_{i,t}^*$ when $\lambda_{i,I}$ increases. The reason for it is very simple. $L_i(t-1)/L_i(t) > 1$ means accumulated induced knowledge decreases for the current period, or, the accumulated induced knowledge is not as sufficient as before. While $\lambda_{i,I}$ increase, it indicates that a better training program is introduced, at this time, it is very natural to invest more into the new efficient training program in order to get extraordinary induced knowledge.

A-5. Derivations of Performance Variances

From (2) and (7) we can get the variance of $Y_{j,t-1}$ as:

$$\begin{aligned}
\sigma_{k,t-1}^2 &= E(y_{k,t-1}-m_k)^2 \\
&= E\Big(\sum_{i=1}^{N} a_{i,t-1}^{(k)} z_i + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} a_{ij,t-1}^{(k)} z_i z_j\Big)^2 \\
&= E\Big(\Big(\sum_{i=1}^{N} a_{i,t-1}^{(k)} z_i\Big)^2 + 2\sum_{i=1}^{N} a_{i,t-1}^{(k)} z_i \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} a_{ij,t-1}^{(k)} z_i z_j + \Big(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} a_{ij,t-1}^{(k)} z_i z_j\Big)^2\Big) \\
&= \sum_{i=1}^{N} (a_{i,t-1}^{(k)})^2 \operatorname{var}(z_i) + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} (a_{ij,t-1}^{(k)})^2 \operatorname{var}(z_i)\operatorname{var}(z_j) \\
&= \sum_{i=1}^{N} (a_{i,t-1}^{(k)})^2 + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} (a_{ij,t-1}^{(k)})^2 .
\end{aligned}$$

Similarly, we have:

$$\sigma_{k,t}^2 = \sum_{i=1}^{N} (a_{i,t}^{(k)})^2 + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} (a_{ij,t}^{(k)})^2$$

Furthermore, from the transformation process, we know that $a_{i,t}^{(k)} \propto \sigma_{i,t}$ and $a_{ij,t}^{(k)} \propto \sigma_{i,t}\sigma_{j,t}$. So, we have:

$$a_{i,t}^{(k)} = a_{i,t}^{(k)} \frac{\sigma_{i,t}}{\sigma_{i,t-1}} \text{ and } a_{ij,t}^{(k)} = a_{ij,t-1}^{(k)} \frac{\sigma_{i,t}\sigma_{j,t}}{\sigma_{i,t-1}\sigma_{j,t-1}}.$$

Hence, we have the first equality in equation (8).

A-6. Convexity of Performance Variance

From (8),

$$\begin{aligned}
\sigma_{k,t}^2 &= \sum_{i=1}^{N} (a_{i,t-1}^{(k)})^2 \left(\frac{\sigma_{i,t}^2}{\sigma_{i,t-1}^2}\right) + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} (a_{ij,t-1}^{(k)})^2 \left(\frac{\sigma_{i,t}^2}{\sigma_{i,t-1}^2}\right)\left(\frac{\sigma_{j,t}^2}{\sigma_{j,t-1}^2}\right) \\
&= \sum_{i=1}^{N} ((a_{i,t-1}^{(k)})^2 + \sum_{j=i+1}^{N} (a_{ij,t-1}^{(k)})^2 \frac{\sigma_{j,t}^2}{\sigma_{j,t-1}^2}) \frac{\sigma_{i,t}^2}{\sigma_{i,t-1}^2} \\
&= \sum_{i=1}^{N} \left[(a_{i,t-1}^{(k)})^2 + \sum_{j=i+1}^{N} (a_{ij,t-1}^{(k)})^2 (\frac{\alpha_j G_{j,t-1} + g_{j,t}}{G_{j,t-1}})^{-\lambda_{A,j}} (\frac{\beta_j L_{j,t-1} + l_{j,t}}{L_{j,t-1}})^{-\lambda_{I,j}} \right] \times \\
&\quad \times \left(\frac{\alpha_i G_{i,t-1} + g_{i,t}}{G_{i,t-1}}\right)^{-\lambda_{A,i}} \left(\frac{\beta_i L_{i,t-1} + l_{i,t}}{L_{i,t-1}}\right)^{-\lambda_{I,i}} \\
&= \sum_{i=1}^{N} s_{i,t} \left\{\frac{\beta_i L_{i,t-1} + l_{i,t}}{L_{i,t-1}}\right\}^{-\lambda_{I,i}} \left[(a_{i,t-1}^{(k)})^2 + \sum_{j=i+1}^{N} (a_{ij,t-1}^{(k)})^2 s_{j,t} \left\{\frac{\beta_j L_{j,t-1} + l_{j,t}}{L_{j,t-1}}\right\}^{-\lambda_{I,j}} \right]
\end{aligned}$$

where, $s_{j,t} = (\frac{\alpha_j G_{j,t-1} + g_{j,t}}{G_{j,t-1}})^{-\lambda_{A,j}}$ is positive, and when $i = N$, = 0.

Since the variance of performance k above is the sum of terms in, if each terms is convex, then is convex. Now, for the i-th term,

$$\begin{aligned}
& s_{i,t} \left\{\frac{\beta_i L_{i,t-1} + l_{i,t}}{L_{i,t-1}}\right\}^{-\lambda_{I,i}} \left\{ (a_{i,t-1}^{(k)})^2 + \sum_{j=i+1}^{N} (a_{ij,t-1}^{(k)})^2 s_{j,t} \left[\frac{\beta_j L_{j,t-1} + l_{j,t}}{L_{j,t-1}}\right]^{-\lambda_{I,j}} \right\} \\
&= s_{i,t} (a_{i,t-1}^{(k)})^2 \left[\frac{\beta_i L_{i,t-1} + l_{i,t}}{L_{i,t-1}}\right]^{-\lambda_{I,i}} + \sum_{j=i+1}^{N} (a_{ij,t-1}^{(k)})^2 s_{i,t} s_{j,t} \left[\frac{\beta_i L_{i,t-1} + l_{i,t}}{L_{i,t-1}}\right]^{-\lambda_{I,i}} \left[\frac{\beta_j L_{j,t-1} + l_{j,t}}{L_{j,t-1}}\right]^{-\lambda_{I,j}} \\
&= s_{i,t} (a_{i,t-1}^{(k)})^2 \beta_i^{-\lambda_{I,i}} \left[\frac{\beta_i L_{i,t-1} + l_{i,t}}{L_{i,t-1}}\right]^{-\lambda_{I,i}} + \sum_{j=i+1}^{N} (a_{ij,t-1}^{(k)})^2 s_{i,t} s_{j,t} \beta_i^{-\lambda_{I,i}} \beta_j^{-\lambda_{I,j}} \left[1 + \frac{l_{i,t}}{\beta_i L_{i,t-1}}\right]^{-\lambda_{I,i}} \left[1 + \frac{l_{j,t}}{\beta_j L_{j,t-1}}\right]^{-\lambda_{I,j}}.
\end{aligned}$$

For the first term, the second order derivative of $\left[1+{l_{i,t}}\Big/{\beta_i L_{i,t-1}}\right]^{-\lambda_{I,i}}$ with respect to $l_{i,t}$ is positive. And the coefficient of $\left[1+{l_{i,t}}\Big/{\beta_i L_{i,t-1}}\right]^{-\lambda_{I,i}}$ is clearly positive, since $s_{i,t}$, $(a_{i,t-1}^{(k)})^2$ and $\beta_i^{-\lambda_{I,i}}$ are all positive constants. Thus, the first part of the last equation is convex.

The second part is a sum of terms, and, if each of these functions is convex, the sum is convex too. Similarly to part 1, each of the coefficients (i.e, $(a_{ij,t-1}^{(k)})^2$, $s_{i,t}$, $s_{j,t}$, $\beta_i^{-\lambda_{I,i}}$ and $\beta_j^{-\lambda_{I,j}}$) is positive. To complete the proof of convexity for our case, we need to show that the product is convex. Since the denominators are positive constants, we can use the general expression of $h(x,y)=(1+x)^{-a}(1+y)^{-b}$ with $x,y\geq 0$, to prove the convexity of the products. We can show the convexity by its Hessian:

$$\begin{aligned}
\det(\nabla^2 h(x,y)) &= \begin{vmatrix} h_{xx}(x,y) & h_{xy}(x,y) \\ h_{yx}(x,y) & h_{yy}(x,y) \end{vmatrix} \\
&= \begin{vmatrix} a(a+1)(1+x)^{-a-2}(1+y)^{-b} & ab(1+x)^{-a-1}(1+y)^{-b-1} \\ ab(1+x)^{-a-1}(1+y)^{-b-1} & b(b+1)(1+x)^{-a}(1+y)^{-b-2} \end{vmatrix} \\
&= (a(a+1)b(b+1)-a^2b^2)(1+x)^{-2a-2}(1+y)^{-2b-2} \\
&= ab(a+b+1)(1+x)^{-2a-2}(1+y)^{-2b-2} \geq 0
\end{aligned}$$

Also, since $h_{xx}(x,y)=a(a+1)(1+x)^{-a-2}(1+y)^{-b}\geq 0$ and $h_{yy}(x,y)=b(b+1)(1+x)^{-a}(1+y)^{-b-2}\geq 0$, the Hessian of $h(x,y)$ is positive definite.

A-7. Dyadic Supply Chain vs. Coordinated system

From (3), we have

$$\widetilde{P_M}\,\frac{-f_M^{'}(e_M^{**})\underline{\sigma_M^2}}{(a^2 f_S(e_S^{**})\underline{\sigma_S^2}+f_M(e_M^{**})\underline{\sigma_M^2})^2}-k_M\,f_M^{'}(e_M^{**})\underline{\sigma_M^2}=2h_M e_M^{**}$$

The first term on the left side is the marginal profit from selling extra product and the second term on the left side is the marginal quality cost saving by quality effort. In total, the left side is the marginal revenue by manufacturer quality effort, which is always positive. Since the property of , we know that the marginal revenue is decreasing. The right side means the marginal quality effort cost, a linear function. If the left side and right side get the same value, which means the marginal revenue equal the marginal cost, the manufacturer stop to invest in quality efforts. From (7) we know that the marginal revenue under the coordinated system is always great than the manufacturer's marginal revenue, though the marginal is decreasing as well. Hence, an increased quality effort from manufacturer is required. There is an intuition behind it. Marginal profit from selling is increased, the current marginal quality effort cost is less than the marginal total profit. To maximize the total profit, the manufacturer must keep investing in quality effort. This relation can be shown by the graph (Figure 8). As we can see from the graph, higher marginal revenue will lead the manufacturer invest more on quality effort. Based on this, we can easily to find that supplier must invest more on quality effort as well.

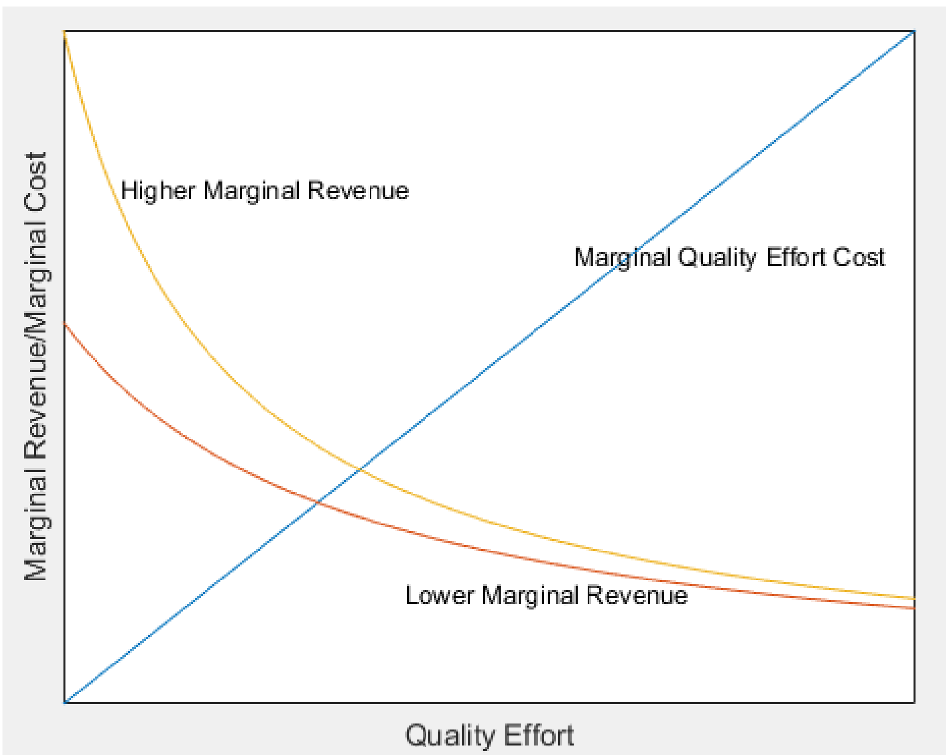

Figure 8 Marginal Revenue Effect

REFERENCES

Anderson, S. W., D. Gleen, K. L. Sedatole. 2000. Sourcing parts of complex products: evidence on transactions costs, high-powered incentives and ex-post opportunism. Accounting, *Organ. Soc.* 25(8): 723–749.

Agrawal, A., S. Muthulingam. 2015. Does organizational forgetting affect vendor quality performance?—An empirical investigation. *Manuf. Serv. Oper. Manage.* 17(3): 350–367.

Agrawal, A., S. Muthulingam, T. Rajapakshe. 2017. How sourcing of interdependent components affects quality in automotive supply chains. *Prod. Oper. Manage.* 26(8): 1512-1533.

Agarwal, R., B. L. Bayus. 2002. The Market Evolution and Sales Takeoff of Product Innovations. *Manage. Sci.* 48(8):1024-1041

Alexander, A., Li, Y. and Plante, R.D., 2017. Sustaining System Coordination in Outsourcing the Maintenance Function of a Process Having a Linear Failure Rate. *IISE Trans.*, 49(5): 544-552

Argote L, Epple D. 1990. Learning curves in manufacturing. *Sci.* 247: 920–924.

Argote, L. 1999. *Organizational Learning: Creating, Retaining and Transferring Knowledge*. Kluwer Academic Publishers, Boston, MA.

Ash, R., Smith-Daniels D. 2004. Managing the impact of customer support disruptions in new product development projects. *Project Manage. J.* 35: 3–10.

Badiru, A. 1995. Multivariate analysis of the effect of learning and forgetting on product quality. *Int. J. of Prod. Research* 33: 777–794.

Badiru, A. 2006. *Handbook of Industrial and Systems Engineering*. CRC / Taylor and Francis Group, Boca Raton, FL.

Baiman, S., P. E. Fischer, M. V. Rajan. 2001. Performance measurement and design in supply chains. *Manage. Sci.* 47(1): 173-188.

Baily, M., D. Farrell. 2006. Increasing global competition and labor productivity: Lessons from the U.S. auto industry. Presentation, The supplier industry in transition: The new geography of auto production, April 18, Federal Reserve Bank of Chicago, Detroit.

Bagwell, K., M. Riordan. 1991. High and Declining Prices Signal Product Quality. *The American Econ. Review*, 81(1): 224-239.

Ballof, N. 1970. Startup management. *IEEE Trans. on Eng. Manage.* EM-17: 132–141.

Banker, R.D, I. Khosla, K. K. Sinha. 1998. Quality and Competition. *Manage. Sci.* 44(9):1179-1192.

Batchelder, C. A., H. E. Boren, H. G. Campbell, J. A. Dei Rossi, J.P. Large. 1969. *An Introduction to Equipment Cost Estimating*. Rand Memorandum, Santa Monica, CA.

Benkard, C.L. 2000 Learning and forgetting: The dynamics of aircraft production. *The American Eco. Review* 90(4): 1034–1054.

Bernstein, F., A. G. Kok. 2009. Dynamic cost reduction through process improvement in assembly networks. *Manage. Sci.* 55(4): 552–567.

Bolandifar, E., P. Kouvelis, F. Zhang. 2016. Delegation vs. control in supply chain procurement under competition. *Prod. Oper. Manage.* 25(9): 1528-1541.

Boyaci, T. 2005. Competitive stocking and coordination in a multiplechannel distribution system, *IIE Trans.*, 37(5): 407-427.

Cachon, G. P., M.A. Lariviere. 2005. Supply Chain Coordination with Revenue-Sharing Contracts: Strengths and Limitations. *Manage. Sci.* 51(1): 30-44.

Chao, G. H., S. M. Iravani, R. C. Savaskan. 2009. Quality improvement incentives and product recall cost sharing contracts. *Manage. Sci.* 55(7): 1122-1138.

Choo, A.S., Linderman, K.W., Schroeder, R.G., 2007. Method and psychological effects on learning behaviors and knowledge creation in quality improvement projects. *Manage. Sci.* 53 (3): 437–450.

Darr, E.D., L. Argote, D. Epple. 1995. The acquisition, transfer, and depreciation of knowledge in service organizations: Productivity in franchises. *Manage. Sci.* 41(11): 1750–1762.

de Holan PM, Phillips N, Lawrence TB. 2004. Managing organizational forgetting. *MIT Sloan Manage. Review* 45(2): 45–51.

Derringger, G., R. Suich. 1980. Simultaneous optimization of several response variables, *J. Quality Tech.* 12(4): 214-219.

Dong, Y., K. Xu, Y. Xu, X. Wan. 2016. Quality management in multi-level supply chains with oursourced manufacturing. *Prod. Oper. Manage.* 25(2): 290-305.

Dorroh, J.R., T.R. Gulledge, N.K. Womer. 1994 Investment in knowledge: A generalization of learning by experience. *Manage. Sci.* 40(8): 947–958.

Dutton JM, A. Thomas. 1984. Treating progress functions as a managerial opportunity. *The Academy of Manage. Review* 9(2): 235–247.

Easterby-Smith, M. M.A. Lyles. 2006. *The Blackwell Handbook of Organizational Learning and Knowledge Management*. Blackwell Publishing Ltd., Malden, MA.

Egelman, C. D., D. Epple, L. Argote, E. R. H. Fuchs. 2017. Learning by doing in multiproduct manufacturing: variety, customizations, and overlapping product generations. *Manage. Sci.* 63(2): 405-423.

Epple D, L. Argote, L, R. Devadas. 1991. Organizational learning curves: A method for investigating intra-plant transfer of knowledge acquired through learning by doing. *Organ. Sci.* 2(1): 58–70.

Fine, C. 1986. Quality improvement and learning in productive systems. *Manage. Sci.* 32(10): 1301–1315.

Froehle, C. M., D. L. White. 2014. Interruption and forgetting in knowledge-intensive service environments. *Prod. Oper. Manage.* 23(4): 702-722.

Garvin, D. 1984. Product Quality: An Important Strategic Weapon," *Business Horizons*, 27: 40-43.

Gort, M., S. Klepper. 1982. Time paths in the diffusion of product innovations. *Econom. J.* 92(3): 630–653.

Grant, R. M. 1996. Prospering in dynamically-competitive environments: Organizational capability as knowledge integration. *Organ. Sci.* 7: 375–387.

Gray, J. V., T. Brian, A. V. Roth. 2009. Outsourcing to a powerful contract manufacturer: the effect of learning-by-doing. *Prod. Oper. Manage.* 18(5): 487-505

Ha, A., X. Long, J. Nasiry. 2016. Quality in supply chain encroachment. Manuf. Serv. *Oper. Manage.* 18(2): 280-298.

Hahn, G.J., N. Doganaksoy, R. Hoerl. 2000. The evolution of Six Sigma. *Quality Engineering* 12 (3): 317–326

Harry, M.J., R. Schroeder. 2000. Six Sigma: The Breakthrough Management Strategy Revolutionizing the World's Top Corporations. Doubleday, New York.

Hatch, N., D. Mowery. 1998. Process innovation and learning by doing in semiconductor manufacturing. *Manage. Sci.* 44(11): 1461–1477.

Howick, S., C. Eden. 2007. Learning in disrupted projects: On the nature of corporate and personal learning. *Int. J. of Pro. Research* 45(12): 2775–2797.

Ittner, C. D. 1996 Exploratory evidence on the behavior of quality costs. *Oper. Research* 44(1): 114–130.

Ittner, C.D, V. Nagar, M. V. Rajan. 2001. An empirical examination of dynamic quality-based learning models. *Manage. Sci.* 47(4): 563–578.

Jaber, M. Y., M. Bonney. 2003. Lot sizing with learning and forget-ting in set-ups and in product quality. *Int. J. Prod. Econ.* 83(1): 95–111.

Jeuland, A., S. M. Shugan. 1983. Managing Channel Profits. *Marketing Sci.* 2: 239-72.

Keachie, E. C., R. J. Fontana. 1966. Effects of learning on optimal lot size. *Manage. Sci.* 13(2): 102–108.

Kim, B. 2000. Coordinating an innovation in supply chain management. *Eur. J. of Oper. Research* 123(3): 568–584.

Lapre, M. A., A. S. Mukherjee, L. V. Wassenhove. 2000. Behind the learning curve: Linking learning activities to waste reduction. *Manage. Sci.* 46(5): 597–611.

Li, G., S. Rajagopalan. 1998. A learning curve model with knowledge depreciation. *Eur. J. of Oper. Research* 105: 143–154.

Li, G., S. Rajagopalan. 1998. Process improvement, quality, and learning effects. *Manage. Sci.* 44(11): 1517–1532.

Malhotra, M. K., L. P. Ritzman. 1999. Resource flexibility issues in multistage manufacturing. *Decision Sci.* 21(4): 673–690.

Stiving, M. 2000. Price-Endings When Prices Signal Quality. Manage. Sci. 46(12):1617-1629.

Montgomery, D. C. 1996. *Design and Analysis of Experiments*. John Wiley and Sons, New York.

Morrison, C. 1985. Primal and dual capacity utilization: An application to productivity measurement in the US automobile industry. *J. of Business and Econ. Statistics* 3: 312–324.

Moskowitz, H., R. D. Plante, J. Tang. 2001. Allocation of quality improvement targets based on investments in learning. *Naval Research Logistics* 48: 684–709.

Mukherjee, A. S., M. A. Lapre, L. V. Wassenhove. 1998. Knowledge driven quality improvement. *Manage. Sci.* 44(11): S35–S49.

Nembhard, D.A. 2001. An empirical comparison of forgetting models. *IEEE Trans. on Engineering Manage.* 48: 283–291.

Netland, T. H., K. Ferdows. 2016. The S-curve effect of lean implementation. *Prod. Oper. Manage.* 25(6): 1106-1120.

Novak, S., S. D. Eppinger. 2001. Sourcing by design: Product complexity and the supply chain. *Manage. Sci.* 47(1): 189-204.

Ofek, E., M. Sarvary. 2001. Leveraging the Customer Base: Creating Competitive Advantage Through Knowledge Management. *Manage. Sci.* 47(11):1441-1456.

Plante, R. D. 2000. Allocation of variance reduction targets under the influence of supplier interaction. *Int. J. of Prod. Research* 38(2): 2815–2827.

Plante, R. D. 2001. Process capability: A criterion for optimizing multiple response product and process design. *IIE Trans.* 33(6): 497–509.

Qin, Z., J. Yang. 2007. Analysis of a revenue-sharing contract in supply chain management. *Int. J. of Logistics: Research and Applications* 11(1): 17-29

Roos, G. 2001. It takes quality to be preferred by world's biggest chipmaker. *Purchasing* 130(22): 21-23.

Schroeder, R., K. Linderman, C. Liedtke, A. Choo. 2008. Six Sigma: Definition and underlying theory. *J. of Oper. Manage.* 26(4): 536–554.

Senge, P. 1990. The Fifth Discipline. London: Century Business.

Serel, D. A., M. Dada, H. Moskowitz, R. D. Plante. 2003. Investing in quality under autonomous and induced learning. *IIE Transactions* 35(6): 545–555.

Shafer, S.M., D. A. Nembhard, M. V. Uzumeri. 2001. The effects of worker learning, forgetting, and heterogeneity on assembly line productivity. *Manage. Sci.* 47(12): 1639–1653.

Snee, R. D., R. W. Hoerl. 2003. Leading Six Sigma: A Step-by-Step Guide Based on Experience with GE and Other Six Sigma Companies. Financial Times Prentice Hall, Upper Saddle River, NJ.

Spender, J. C. 1996. Making knowledge the basis of a dynamic theory of the firm. *Strategic Manage. J.* 17: 27–43.

Sullivan, L. P. 1984. Reducing variability: A new approach to quality. *Quality Progress* 17(7): 15–21.

Tarakci, H., K. Tang, H. Moskowitz, R. Plante. 2006. Incentive maintenance outsourcing contracts for channel coordination and improvement. *IIE Trans.* 38(8):671-684.

Tarakci, H., K. Tang, S. Teyarachakul. 2013. Learning and forgetting effects on maintenance outsourcing. *IIE Trans.* 45(4): 449-463.

Taguchi, G. 1986. *Introduction to Quality Engineering: Designing Quality Into Product and Processes*. Asian Productivity Organization, Tokyo.

Tomlin, B. 2009. Impact of supply learning when suppliers are unreliable. Manuf. Serv. *Oper. Manage.* 11(2): 192–209.

Trommer, D. 1996. Training is key to quality-Group provides venue for supplier education. Electronic Engrg. Times (Novermber 11): 76.

Walshe, K., G. Harvey, P. Jas. 2010. Connecting Knowledge and Performance in Public Services: From Knowing to Doing. Cambridge University Press. p. 175

Wang, W, R. D. Plante, J. Tang. 2013. Allocation of quality improvement targets under supplier process disruption. *Eur. J. of Oper. Research* 228(2): 388-396.

Xiao, W., C. Gaimon. 2013. The effect of learning and integration investment on manufacturing outsourcing decisions: a game theoretic approach. *Prod. Oper. Manage.* 22(6): 1579-1592.

Yelle, L. E. 1979. The learning curve: Historical review and comprehensive survey. *Decision Sci.* 10(2): 302–328.

Zangwill, W. I., P. B. Kantor. 1998. Toward a theory of continuous improvement and the learning curve. *Manage. Sci.* 44(7): 910–920.

Zellmer-Bruhn, M.E. 2003. Interruptive events and team knowledge acquisition. *Manage. Sci.* 49(4): 514–528.

Zhu, K., R. Q. Zhang, F. Tsung. 2007. Pushing quality improvement along supply chains. *Manage. Sci.* 53(3): 421-436

Zusman, P., M. Etgar. 1981. The Marketing Channel As an Equilibrium Set of Contracts. *Manage. Sci.* 27(3): 284–302.