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Three Essays on Skill Heterogeneity in Frictional Labor Markets

Jacklyn R. Buhrmann
Purdue University

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THREE ESSAYS ON SKILL HETEROGENEITY IN FRICTIONAL LABOR
MARKETS

A Dissertation

Submitted to the Faculty

of

Purdue University

by

Jacklyn R. Buhrmann

In Partial Fulfillment of the

Requirements for the Degree

of

Doctor of Philosophy

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West Lafayette, Indiana

**THE PURDUE UNIVERSITY GRADUATE SCHOOL
STATEMENT OF DISSERTATION APPROVAL**

Dr. Cathy Zhang, Chair

Department of Economics, Krannert School of Management

Dr. John Barron

Department of Economics, Krannert School of Management

Dr. Trevor Gallen

Department of Economics, Krannert School of Management

Dr. Victoria Prowse

Department of Economics, Krannert School of Management

Approved by:

Dr. Brian Roberson

Director of Graduate Studies in Economics

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ABSTRACT

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This dissertation is composed of three essays using labor search models to explore the role of skill mismatch in the labor market. The first, “**Skill Mismatch in Frictional Labor Markets**”, provides theory and evidence on pair-specific skill mismatch in the labor market, defined as the gap between an individual’s skills and the requirements of her job. Employment data from the NLSY97 display some degree of positive sorting into occupations on the basis of cognitive skills, but skill mismatch is pervasive and costly. I develop and estimate a labor search model featuring heterogeneity in worker skills and firm skill requirements that demonstrates how search frictions induce voluntary mismatch acceptance. In addition, the model indicates that skill mismatch is countercyclical; as the labor market tightens, mismatch tolerance falls and wages rise for all workers. However, the elasticity of mismatch tolerance with respect to market tightness varies systematically across the skill space, leading to changes in the composition of employment over the business cycle.

While the model generates levels of mismatch broadly consistent with the data, the degree of positive sorting is underestimated for higher-skilled workers. The second chapter, “**Targeted Search in Heterogeneous Labor Markets**”, extends the theory of targeted search by introducing continuous skill heterogeneity among workers and firms in frictional labor markets. Workers are unable to fully direct their search, but instead pay an information cost to reduce the variance of the job offer distribution. A lower variance increases the worker’s expected match quality but decreases the offer arrival rate. Results show higher-skilled workers target their search more intensely,

decreasing the expected level of mismatch among higher-skilled workers and allowing the model to better fit the data on skill mismatch and sorting.

The third chapter, “**Skill Mismatch and the Equilibrium Distribution of Vacancies**”, builds on the model in the first chapter by endogenizing the skill distribution of vacancies. The model generates an equilibrium distribution of vacancies in the skill space that depends on labor market conditions such as bargaining power, matching efficiency, and the distribution of unemployed workers. Job creation depends critically on mismatch tolerance: higher levels of expected mismatch reduce the expected value of a vacancy. The model provides new predictions on the response of job creation to the skill distribution of unemployed workers, which can be tested using data on job postings by occupation.

1. SKILL MISMATCH IN FRICTIONAL LABOR MARKETS

1.1 Introduction

Skill mismatch in the labor market arises when individuals are employed in jobs with skill requirements that don't align with the skills the worker offers. For example, a restaurant server with a bachelor's degree in accounting is likely mismatched, since his education level is higher than the job requires. However, educational requirements are not the only indicator of mismatch. A high school teacher with a bachelor's degree in accounting may also be mismatched, if his job is unable to utilize all of his skills. In this sense, skill mismatch is a continuous, pair-specific measure describing the quality (or lack thereof) of a particular worker-job match.

Recent empirical studies show that skill mismatch is costly to workers in terms of forgone wages and shorter job tenure [Lise and Postel-Vinay, 2015]. Individuals who are poorly matched – either over- or under-skilled relative to the requirements of their occupation – earn lower wages than similarly-skilled workers who are better matched. When multiple dimensions of skill are considered, mismatch in the cognitive skill dimension is shown to be most costly; the wage loss from mismatch is large, and cognitive skills are very slow to accumulate. Additionally, the consequences of skill mismatch are long-lasting; workers who are mismatched have shorter job tenure on average and earn lower future wages even after switching occupations [Guvenen et al., 2015]. Skill mismatch is also costly at the aggregate level. Gautier and Teulings [2015] show that output would be 7.5% to 18.5% higher in the absence of search frictions, and estimate that 60% of this output loss can be attributed to mismatch.

Given the undesirable effects of mismatch on individual as well as aggregate outcomes, it is important to understand workers' mismatch tolerance decisions. How

much skill mismatch do workers accept? What causes the systematic differences in mismatch across different types of workers? How does mismatch change over the business cycle?

This paper presents three stylized facts describing the cognitive skill mismatch existing in the U.S. labor market as a function of workers' skill level, and develops a search and matching model to replicate these facts. Mismatch is measured on the basis of cognitive skills, using data from the 1997 cohort of the National Longitudinal Survey of Youth (NLSY97) and the U.S. Department of Labor's O*NET database. In contrast to Lise and Postel-Vinay [2015] and Guvenen et al. [2015], the mismatch measure in this paper is one-dimensional. It focuses only on cognitive skills, since mismatch in that dimension is most costly and therefore most likely to affect workers' job acceptance decisions. Comparing an individual's cognitive skill with that of their occupation, I quantify how much skill mismatch workers are willing to accept. Because respondents in the NLSY97 are currently young, prime-age workers, it is essential to understand the causes and consequences of skill mismatch among this group. Results indicate mismatch is present among workers of all skill levels, but higher-skilled workers tolerate more skill mismatch than lower-skilled workers since the opportunity cost of unemployment is greater. Workers' average wages decline as mismatch increases; therefore, within-type wage dispersion is higher for higher-skilled workers. In addition, higher-skilled workers experience shorter unemployment spells, and are less likely to be unemployed at any time.

To capture these facts, I construct a labor search model that formalizes the workers' tradeoff between accepting an imperfect match and continuing to search. The model builds off the canonical search-and-matching model of McCall [1970], augmented to allow for heterogeneity in the skills of both workers and firms. Skills are distributed along the unit interval, and workers are vertically differentiated such that a higher value indicates a more skilled worker. Wage offers depend on the worker's type as well as the type of job with which she is matched; for each worker, there is a single job type at which the wage is maximized. The model focuses on the

worker's problem; unemployed workers search randomly for jobs and receive offers according to a Poisson arrival rate. Upon receiving an offer, an unemployed worker chooses whether to begin employment with the current firm or to continue searching. Crucially, it is not an equilibrium strategy to wait for a "perfect" match since the probability of drawing any specific job is measure zero. Instead, workers will accept some range of "good enough" matches.

The level of mismatch accepted is directly linked to the worker's reservation wage, which depends on labor market conditions such as the offer arrival rate, separation rate, and unemployment benefits. When the reservation wage increases workers become more selective, accepting a narrower range of job offers and tolerating less mismatch. The model delivers two key predictions regarding skill mismatch. First, mismatch is present among all types of workers, and the average level of mismatch accepted is (weakly) increasing in the worker's type. Second, mismatch tolerance falls when workers' outside option increases and when the cost of mismatch increases. Therefore, decreases in the job separation rate and increases in unemployment benefits, the arrival rate of job offers, or increases in the curvature of the wage lead to a decrease in mismatch.

When workers are able to search on-the-job, the qualitative results regarding sorting and skill mismatch still hold. Reservation wages fall and expected utility increases for all workers. By gaining the opportunity to move up the job ladder, middle-skill workers see a decrease in expected skill mismatch and an increase in expected wages conditional on employment. High-skill and low-skill workers, on the other hand, experience increased mismatch and lower expected wages. However, there is still a net gain in welfare among these groups of workers due to the increased employment probability.

By making explicit the differences in the equilibrium strategies of workers with different skills, the model also provides a framework to study the effects of changes in labor market conditions on labor market outcomes like wages and unemployment. Models without heterogeneity are generally very tractable and well-suited to address

the response of the average worker. However, by neglecting differences in worker skills, these models are unable to address the compositional changes that can occur. For instance, there are two channels by which aggregate outcomes can change in response to a shift in labor market conditions. First, workers' reservation wage strategies may change; in general, the reservation wages of all workers move in the same direction, but some are more responsive than others. In addition, variation in the elasticity of the reservation wage by skill leads to a change in the composition of employment. Because outcomes such as average wages are systematically different across different types of workers, this compositional effect is relevant for the calculation of aggregate statistics.

To understand whether the theory is consistent with the empirical facts on mismatch, the model is calibrated to the U.S. labor market using NLSY97 data from 2009 to 2013. The calibrated model generates match acceptance behaviors, unemployment rates, and wage differentials that are broadly consistent with the data. Moreover, the model predicts some positive sorting on cognitive skills (the correlation between the worker's skill type and his expected job type is 0.95), but there is also a substantial amount of mismatch. On average, a worker's expected level of skill mismatch is 0.24, or nearly one quartile in the cognitive skill space. While workers in the lower half of the skill space are under-skilled in expectation and workers in the upper half are over-skilled, workers of all skill types are willing to match with jobs both above and below their own skill type.

Solon et al. [1994] and Daly and Hobijn [2016] use PSID and CPS data, respectively, to show that the lack of cyclical in the average wage is attributable to changes in the skill composition of the employed population. The current model highlights the role of mismatch tolerance in explaining this composition effect. In the model, higher-skilled workers are more responsive to changes in labor market conditions than low-skilled workers. When unemployment falls (or when the market tightens), average wages for all workers rise, but the increased employment shares of lower-skilled

workers pushes down the economy-wide average wage. Consequently, growth of the economy-wide average wage is essentially uncorrelated with market tightness.

The remainder of this paper is organized as follows. Section 2 constructs an empirical measure of skill mismatch and describes four facts about skill and skill mismatch in the U.S. labor market. Section 3 presents a model of one-sided search with heterogeneity, and Section 4 illustrates the decentralized equilibrium solution in a calibrated numerical example. Section 5 shows the effect of employment composition on the aggregate wage during the recovery from the Great Recession. Section 6 considers an extension with on-the-job search. Section 7 concludes and offers directions for future work.

1.1.1 Related Literature

Recent empirical work has focuses on describing the effects of skill mismatch. Guvenen et al. [2015] propose a multidimensional measure of skill mismatch and a model in which workers learn about their abilities over time, and use it to show that mismatch is costly in terms of both current and future wages as well as expected job tenure. Lindenlaub [2017] estimates an assignment model with multidimensional skills and finds that changes in technology have contributed to a strong increase in skill complementaries and in the cost of skill mismatch. These studies, along with Lise and Postel-Vinay [2015], show that skill mismatch is a widespread phenomenon in the labor market, with strong negative consequences for workers. The current paper contributes to this literature by quantifying the cognitive skill mismatch tolerated across workers of different skill levels.

A large search and matching literature has been devoted to modeling skill heterogeneity and sorting. Marimon and Zilibotti [1999] was among the first to introduce a continuum of worker and firm types. Their model is made tractable by the fact that skills are situated around a unit circle, so the decision problem and equilibrium

strategy are symmetric across workers. The primary drawback of this model is the inability to address qualitative differences between different types of workers or jobs.

Search models with a hierarchy of types have been studied, primarily in marriage market applications, beginning with Shimer and Smith [2000] and Shimer and Smith [2001]. Studies incorporate two-sided heterogeneity with identical value functions and a fixed pool of agents on each side of the market. Teulings and Gautier [2004] extend the model to a labor market setting comparable to the Diamond-Mortensen-Pissarides framework, and specify an increasing returns to scale matching process in order to approximate the decentralized equilibrium solution and estimate the distortions resulting from search frictions. When equilibrium wages are obtained through Nash bargaining over the total surplus, the worker's incentives are fundamentally unchanged by the inclusion of firms in the model. As a result, the one-sided search problem studied in the current paper allows for structural estimation of the parameters while keeping workers' strategies and outcomes unchanged.

Lise et al. [2016] incorporate productivity shocks and on-the-job search with counter offers into the environment of Shimer and Smith [2000] to estimate the welfare costs of mismatch and explore optimal regulation. They estimate a significant skill complementarity among college-educated workers, and a low complementarity among workers with a high school education or less. The resulting model-predicted match sets for college educated workers align well with the stylized facts described in this paper. However, they predict virtually no sorting among high school educated workers, while the empirical match sets I plot show that lower-skilled workers also exhibit some positive sorting (albeit less strongly) on cognitive skills. Lise and Postel-Vinay [2015] augment the model to allow for multidimensional skills and skill accumulation, and estimate the costs of skill mismatch using the NLSY79 and O*Net datasets. In a related framework, Lise and Robin [2017] attempt to quantify the cyclical dynamics of sorting using a two-sided search model estimated using long-run aggregate moments. However, wage setting and match formation rely on firms engaging in Bertrand com-

petition to poach workers. The resulting implications for mismatch tolerance are not consistent with the evidence presented in the current paper.

Finally, Hagedorn et al. [2017] employ a similar model to prove that sorting can be identified using wages from matched employer-employee data. Extending Shimer and Smith [2000] to allow for on-the-job search, they show using German LIAB data that more productive workers tend to be employed in more productive firms, but that mismatch causes an output loss of 1.83%. In the current paper, I study sorting in the cognitive skill dimension using publicly available U.S. data, providing a complimentary answer to the question of sorting and mismatch in the labor market.

1.2 Skill Mismatch in the U.S. Labor Market

In this section, I construct empirical measures of worker and firm skills, and summarize three main facts regarding skill mismatch in the labor market.

1.2.1 Data

The primary data source in this paper is the NLSY97. Conducted by the Bureau of Labor Statistics, this nationally representative survey samples individuals born between 1980-1984. I restrict the analysis to the 2009-2013 waves of the survey; during this time period, respondents are 25-33 years old. Skill mismatch may be an especially salient feature of the labor market at this time, since workers are likely be less choosy about the jobs they accept during and after recessions. Because survey respondents are young, prime-age workers, it is particularly relevant to study the causes and consequences of mismatch among this group. Of the initial 6,748 individuals in the cross-sectional sample, approximately 6,000 are successfully contacted in 2009. To account for selective attrition, I use custom sample weights calculated over the subsample of individuals who appear in any wave between 2009-2013. Appendix A.3 discusses sample selection criteria and descriptive statistics for the relevant subsample.

I obtain information on occupational skill requirements from O*NET, which provides information on 704 SOC-level occupations. Occupational codes in the NLSY97 are reported as 3- or 4-digit Census codes, a coarser taxonomy than used by the SOC. For occupations that map to multiple SOC codes, I average the skill requirements of the relevant SOC occupations. For each occupation in O*NET, a “level” and an “importance” score are provided for each of 277 descriptors. The level score assigned to a skill indicates the degree of competency in that skill needed to succeed at the occupation; the importance score describes how essential the skill is to the occupation. For example, the skill “Mathematics” is rated as equally important for both Physicists and Post-secondary Mathematics Teachers, but the level of the skill required is substantially higher for Physicists. I rank occupations on the basis of the level requirement for the skills of interest.

1.2.2 Empirical Methodology

Skill mismatch is defined as the difference between a worker’s skill type and the skill type of the occupation in which the worker is employed. For simplicity, both skill types are represented by linear indices, x and y respectively. Skill mismatch is defined as $x - y$; a positive value indicates that the worker is over-skilled relative to his job, and a negative value indicates that the worker is under-skilled. In order to measure and describe skill mismatch in the labor market, measures of worker and job skill types must first be constructed.

To create a measure of skill comparable across individuals, I follow an approach similar to Cawley et al. [2001]. During the first round of the NLSY97, most respondents took the Armed Services Vocational Aptitude Battery (ASVAB) test. The ASVAB consists of 12 component sections, over skills both abstract (eg. mathematics knowledge, paragraph comprehension) and practical (eg. auto information). Scores from four categories, mathematics knowledge, arithmetic reasoning, paragraph comprehension, and word knowledge, are residualized by age and gender, and a principal

components analysis (PCA) is performed on the residuals. The individual’s percentile rank in the first principal component score is referred to as the ASVAB rank.

The ASVAB rank provides some information about a worker’s ability, but because the ASVAB was administered before most educational attainment decisions were made it is likely that rankings have since shifted. To account for this, I combine the ASVAB rank with the respondent’s education level using PCA, where the first component is taken as the individual’s general ability. Finally, I recompute the ranking of individuals using the custom sample weights previously described, and normalize to obtain a skill type $x \in [0, 1]$. This method returns a ranking over individuals such that, conditional on education level, an individual with a higher ASVAB rank is assigned a higher skill type.¹ Figure 1.1 shows the distribution of educational attainment within each percentile bin of worker skills, smoothed using nonparametric local-constant least squares regressions. Workers classified as higher-skilled tend to have higher levels of educational attainment, but the ordering is not perfect. It is possible for a worker with a high school degree and a relatively high ASVAB rank to be classified above a worker with a Bachelor’s degree and a low ASVAB rank. This measure of skill diverges somewhat from the recent literature, which commonly uses ASVAB or AFQT scores as a proxy for ability. Including education level in the construction of worker skills changes the ranking substantially. By including education level, the correlation between worker skill and wage increases, and the aggregate level of mismatch falls.

A job is equivalent to an occupation, or a group of tasks that the worker must perform; data on the skills required for those tasks is included in O*NET. For the purpose of this analysis, the skill type of a job is determined by the levels of the skills “Judgment and Decision Making” (JDM) and “Complex Problem Solving” (CPS) required by that occupation. JDM is the skill most strongly (positively) correlated with the occupation’s median hourly wage. CPS is highly correlated with both JDM

¹See Appendix A.5 for a discussion and robustness checks of other ranking methods for worker and occupation skills.

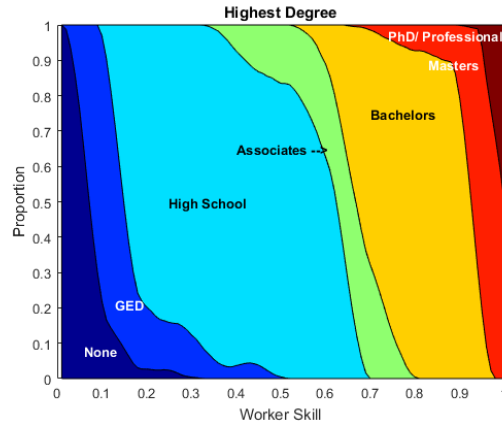


Fig. 1.1.: Empirical probability of educational attainment by individual skill.

and the wage, and its inclusion serves to remove mass points in the occupation skill distribution caused by clustering of skill requirement data around integer values. These two skills are aggregated using PCA, and the first principal component is taken to be the cognitive skill requirement of the occupation. Occupations are weighted according to their employment share in the NLSY97 and a ranking is computed; after normalization, job skill types y also span the interval $[0,1]$. Skill mismatch can then be calculated as the difference between the worker's skill x and the skill y of the occupation in which the worker is currently employed.

1.2.3 Empirical Facts of Skill Mismatch

Using the skill measures constructed in the previous section, I document three stylized facts regarding workers' match acceptance behavior and the presence of skill mismatch in the labor market.

Fact 1 *Higher-skilled workers earn higher incomes, but face increased within-type wage dispersion.*

Figure 1.2a plots the log of average hourly wage, and figure 1.2b shows the log wage differentials between the 90th - 10th percentiles of hourly wage within each worker

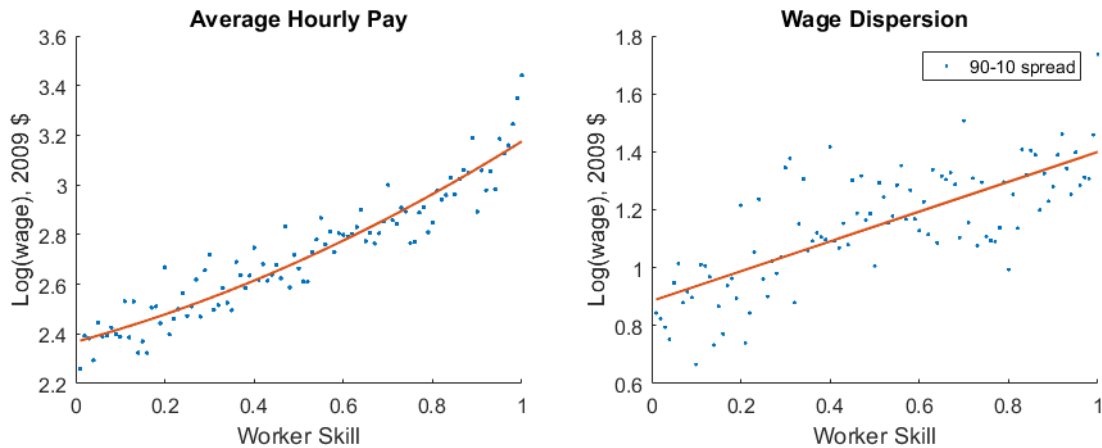


Fig. 1.2.: Empirical wage and wage dispersion by individual skill.

skill type. Workers are grouped into percentile bins such that the plotted points are averages within each bin; the lines in each plot are best fit quadratics.² Average hourly pay is increasing in worker skill; a one-decile increase in skill type increases the hourly wage by \$1.19 on average. However, workers face substantial within-type wage differentials, which are larger among higher-skilled workers. The lowest-skilled workers' wages range from around \$8 per hour at the 10th percentile to about \$20 at the 90th; among the highest-skilled workers, the 10th and 90th percentile individuals earn approximately \$10 and \$40 per hour, respectively. For each decile increase in worker skill, the 90-10 wage differential increases by roughly \$2 on average, and the 90-50 wage differential increases by \$1.14. The increasing wage differential provides preliminary evidence that higher-skilled workers are employed in a wider range of jobs, and that not all of those jobs are able to fully utilize the workers' skills.

The positive correlation between worker skill and hourly wage is consistent with findings in the empirical literature dating back to Mincer [1974]. This well-established result serves as evidence that the measure of worker skill constructed here captures a salient worker characteristic. In addition, the increase in wage dispersion among

²A skill percentile bin i contains workers whose skill type falls between the i and $i + 1$ percentile of the skill space. For example, workers with skill $x \in [.9, .91)$ fall into bin $i = 90$. There are on average 54.3 individuals and 185.5 individual-job observations in each bin.

higher-skilled workers is in line with the empirical literature, as discussed in Mortensen [2005]. In his book, Mortensen characterizes wage dispersion as the amount of variation in wages that is not explained by worker traits, and summarizes empirical results attributing much of the observed wage dispersion to search frictions. In line with this result, the model presented in this paper generates wage dispersion through mismatch, which is caused by search frictions.

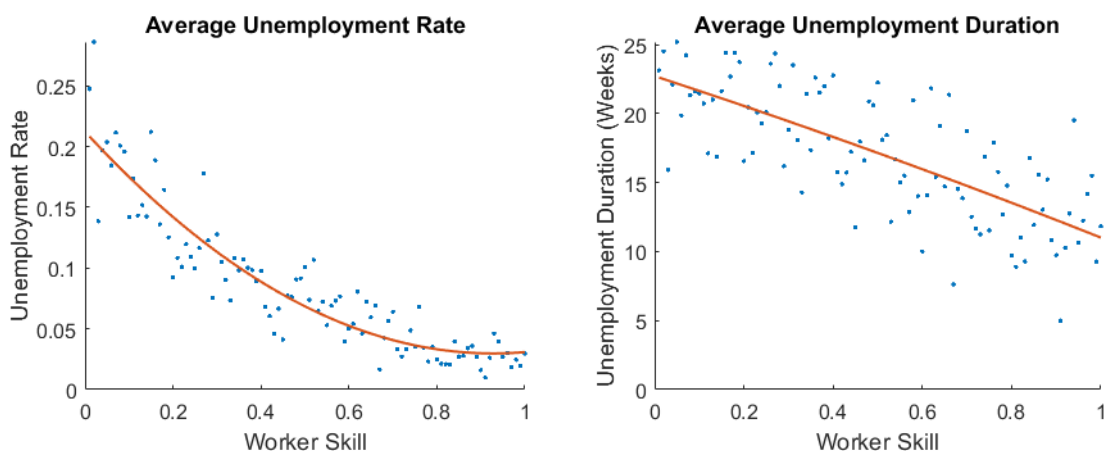


Fig. 1.3.: Empirical unemployment rate and duration by individual skill.

Fact 2 *Higher-skilled workers are less likely to be unemployed and experience shorter unemployment spells.*

Figure 1.3 plots the average unemployment rate and unemployment duration by worker skill type. The unemployment rate and expected duration are negatively correlated with the workers' skill type; a one-decile increase in skill type corresponds to a 1.79 percentage point decrease in the unemployment rate and an unemployment spell that is 1.17 weeks shorter. Fact 2 is in line with previous empirical findings; for example, Becker [1993] show that unemployment rates are negatively correlated with educational attainment, and Heckman et al. [2006] find a negative relationship between unemployment rates and their measure of cognitive ability. Along with Fact

1, this helps to validate that the current measure of worker skill captures meaningful variation across workers in the data.

Fact 3 *There is some degree of positive sorting on the basis of cognitive skills, but matching is not perfectly assortative. On average, there is more skill mismatch among higher-skilled workers.*

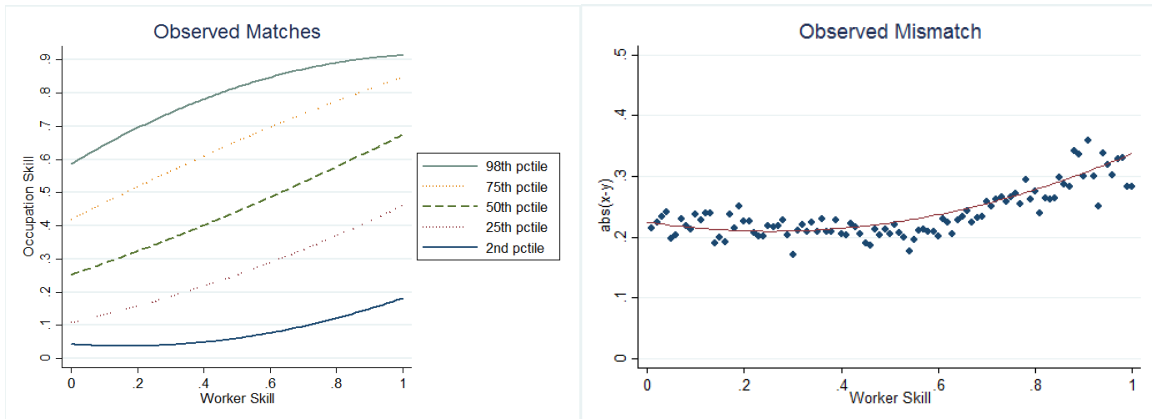


Fig. 1.4.: Empirical match acceptance and average mismatch by individual skill.

Figure 1.4a depicts the range of occupation types each worker type matches with. The figure plots the 95th, 75th, 50th, 25th, and 5th percentiles of observed occupation skills for each percentile bin of worker skill types. It is clear that some level of positive sorting is present. For each decile increase in worker skill, the skill level of the median occupation match increases by 5.6%, indicating that higher-skilled workers occupy higher-skilled sets of occupations on average. However, the range of occupations held by a particular type of worker can be quite large; on average, the difference between a worker's 95th and 5th percentile match is 78.3% of the occupation skill space.

Figure 1.4b plots the average of observed skill mismatch within each percentile of worker type. Workers in the lower half of the skill space experience less skill mismatch than higher-skilled workers. The expected level of mismatch for a worker at the 90th percentile is 28.83% higher than a worker at the 50th percentile, and 26.89% higher than a worker at the 10th percentile.

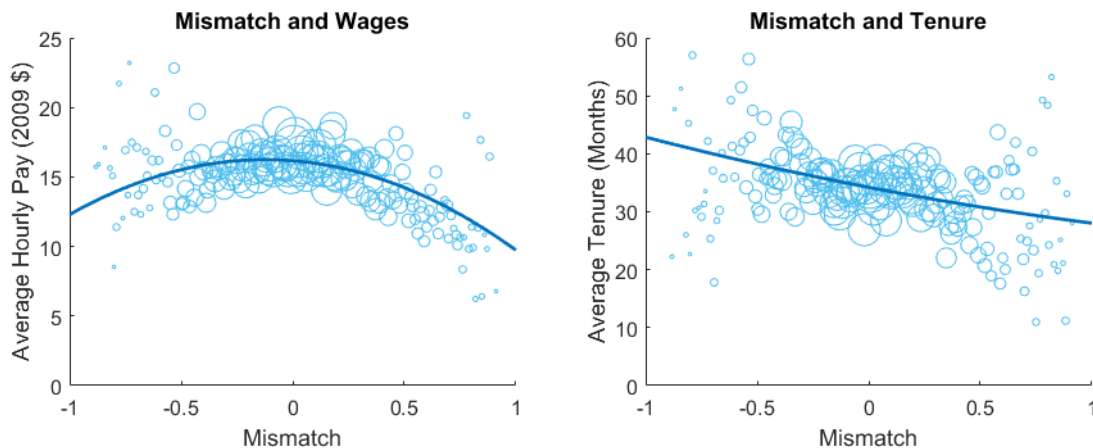


Fig. 1.5.: Empirical relationship of mismatch to wages and job tenure.

Figures 1.5a and 1.5b depict the cost of skill mismatch in terms of forgone wages and reduced job tenure. Since the number of individuals experiencing a particular level of mismatch varies systematically across the space of potential mismatch, larger markers are used to indicate larger groups of workers. Values of mismatch that are less than 0 indicate that the worker's skill index is lower than that of the occupation, or that the worker is under-skilled; values greater than zero indicate that the worker is over-skilled. It is clear that wages are decreasing in mismatch, whether the worker is over- or under-skilled for his occupation. However, the duration of jobs in which the worker is under-skilled is much longer than when he is over-skilled. Over-skilled workers are more likely to be high-skilled; since the average wage is higher for high-skilled workers, the incentive to find a better matched job is larger. Together, these two facts show that mismatch is a salient feature of the labor market.

The relatively high level of mismatch observed among higher-skilled workers may be surprising at first, since higher-skilled workers face a higher opportunity cost of mismatch. However, the opportunity cost of unemployment is also higher for high-skilled workers. Unemployment benefit policies attempt to alleviate this by subsidizing job search, but Fact 3 shows that many high-skilled workers are still willing to accept a poor match in order to exit unemployment more quickly. In the remainder of this

paper, I construct a random search model to explain workers' mismatch acceptance behaviors, and use the model to show how heterogeneity in mismatch acceptance strategies leads to important compositional effects for aggregate labor market outcomes.

1.3 A Model of Job Search and Skill Mismatch

To capture the differences across workers in the expected value of future job offers, I augment the continuous-time, one-sided search environment of McCall [1970] to allow for heterogeneity on both sides of the labor market. Workers are risk-neutral and infinitely-lived, and maximize expected discounted utility which is linear in income. A worker can be either employed or unemployed at any time; all unemployed workers search for jobs, and there is no on-the-job search in the baseline model. Section 6 extends this model to allow for on-the-job search. The extended model provides additional predictions and comparative statics results, but the central results regarding skill mismatch tolerance still hold. In this model, a firm corresponds to one job and may be either vacant (searching for a worker) or filled.

Search frictions in the labor market make job search costly by forcing workers to wait in the low-value state of unemployment until they receive an acceptable job offer. Once a worker receives a job offer, she must decide whether to accept it or to continue searching for a better offer. This tradeoff depends critically on the value of the offers that a worker expects to receive in the future. When that expectation is a function of the worker's ability or skill, workers of different skill levels will make systematically different decisions regarding the range of job offers to accept.

Skill Heterogeneity. Workers are heterogeneous in skills, indexed by type $x \in [0, 1]$, such that higher x indicates a more skilled worker. Firms are also heterogeneous, indexed by skill requirements $y \in [0, 1]$. Worker skill types x and skill requirements of vacant jobs y are distributed according to cdf's L and G , with corresponding pdfs $\ell(x)$ and $g(y)$. The distributions of unemployed and employed workers depend on the

workers' decision problem, and are not necessarily equal to L ; call these distributions F and \tilde{F} , respectively. The firm distribution G includes only vacant firms, and the distribution of vacancies is assumed to be invariant to worker behavior³.

Job Search. All unemployed workers receive job offers at a Poisson rate λ . Search is random; that is, the probability of meeting job y is independent of the worker's type x ⁴. Since workers cannot determine a potential employer's type prior to receiving an offer, offers are randomly drawn from the distribution of vacant jobs $G(y)$. When an offer arrives, the worker must decide whether to accept or reject the job. While unemployed, workers receive a flow of benefits $b(x) \geq 0$. Workers who are employed receive wages $w(x, y)$ that depend on the worker's own skill type as well as the skill type of her job. Utility is obtained only from consumption, so the wage fully summarizes the attractiveness of any particular match. Matches are exogenously separated according to a Poisson process with arrival rate s , which is constant across all worker types. When a match is separated, the job is terminated and the worker becomes unemployed. Workers discount future utility at rate r . See Figure 1.6 for a diagram of the timing of events.

Wage. Let the wage earned by worker x when employed by firm y be given by $w(x, y)$. Define $\mu = |x - y|$ as the skill mismatch of the pair. The wage function must satisfy the following properties:

1. $w(x, y) \geq 0 \forall (x, y)$
2. Given $x = x_0$, $w(x_0, y)$ quasiconcave in y

³See Albrecht and Vroman [2002] for an example of endogenous vacancy creation with two types of workers and firms, where firms respond to the distribution of unemployed workers as well as their mismatch tolerance strategies.

⁴Random search is important to generate skill mismatch in a bilateral matching environment where mismatch is costly. Under competitive search, skill mismatch can exist only when a worker is indifferent across many job types and levels of mismatch. See Shi [2002] for a competitive search model with heterogeneous workers.

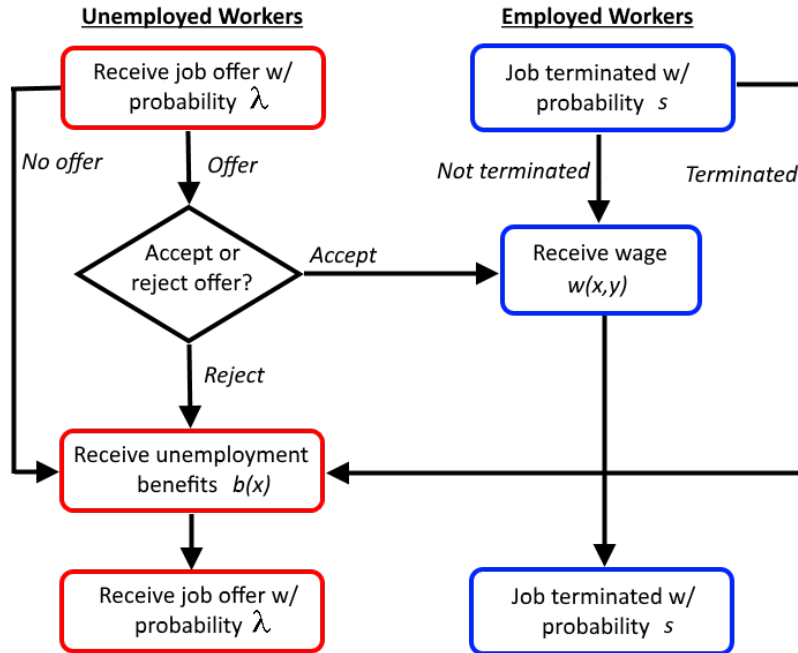


Fig. 1.6.: Timeline of events in model.

Condition (1) simply imposes a minimum wage of 0. Convexity of match sets is ensured by (2); that is, if x is willing to accept jobs y_1 and y_2 , they should also accept all $y \in (y_1, y_2)$.

1.3.1 Equilibrium

The equilibrium strategy for a worker of type x is to choose a reservation wage, $w^*(x)$, such that job offers are accepted if and only if the wage is greater than or equal to $w^*(x)$. The reservation wage depends on the worker's skill type x , and the set of reservation wage strategies $\{w^*(x)\}_{x \in [0,1]}$ is sufficient to characterize the equilibrium in this environment. All proofs can be found in Appendix A.

Value Functions. Define $E(x, y)$ as the value of employment for a worker of type x employed by firm y . The employed worker receives wage $w(x, y)$ from the current job, and has a continuation value of either $E(x, y)$ if the job continues or $U(x)$ if the

job is terminated. In this context, a worker's equilibrium strategy is characterized by a range of firms $[y, \bar{y}]$ with which to accept employment. However, the worker's utility depends only on the wage; given the wage, the employer type is irrelevant. Therefore, it is equivalent to rewrite the worker's value function in terms of the wage offer w rather than the firm type y . Using this transformation, the worker's equilibrium strategy is reduced to a simple reservation wage. The value of employment for worker x in a match that pays w is

$$E(x, w) = \frac{w + s \cdot U(x)}{r + s} \quad (1.1)$$

Next, I construct the value function for an unemployed worker. The unemployed worker receives the flow value of unemployment $b(x)$; the continuation value depends on whether an acceptable job offer arrives. An offer w arrives at rate λ , and is accepted if and only if the value of employment $E(x, w)$ is greater than the value of remaining unemployed. The ex-ante value of an offer is an expectation with respect to the conditional distribution of wages. If no offer arrives, the continuation value is simply the value of unemployment. The flow value of unemployment is

$$rU(x) = b(x) + \lambda \int_{\underline{w}}^{\bar{w}} \max\{E(x, w) - U(x), 0\} d\tilde{G}(w|x) \quad (1.2)$$

where $\tilde{G}(w|x)$ is the distribution of wage offers conditional on the worker's type. This distribution is a transformation of the distribution of jobs, $G(y)$, given the wage offer function $w(x, y)$. If the wage function is invertible, $\tilde{G}(w|x) = G(w^{-1}(w|x))$. However, $w(x, y)$ may be a many-to-one function of y , so $w^{-1}(w|x)$ may be a relation rather than a function. If the second derivative of $w(x, y)$ is constant in y , the relation $w^{-1}(w|x)$ will assign at most two y values for each w . In this case, $\tilde{G}(w|x) = G(\max\{w^{-1}(w|x)\}) - G(\min\{w^{-1}(w|x)\})$.

Reservation Wage. It is in a worker's interest to accept all jobs such that the value of employment in that job is at least as great as the worker's value of unemployment, or $E(x, w) \geq U(x)$. Therefore, the lowest wage a worker will accept is the one that

sets the value of employment exactly equal to the value of unemployment. This wage, denoted $w^*(x)$, is called the reservation wage. The worker's equilibrium strategy is to accept all jobs that offer a wage greater than or equal to the reservation wage. No two types of workers face the same distribution of possible wages $\tilde{G}(w|x)$, so the value of unemployment is a function of the worker's type:

$$\frac{w^*(x) + sU(x)}{r + s} = U(x).$$

This implies that the reservation wage is equal to the flow value of unemployment, or $w^*(x) = rU(x)$. Returning to equation 1.2,

$$w^*(x) = b(x) + \lambda \int_{\underline{w}}^{\bar{w}} \max\{E(x, w) - U(x), 0\} d\tilde{G}(w|x). \quad (1.3)$$

Proposition 1.3.1 *The strategy of a type x worker is to accept a wage offer if and only if it is greater than the reservation wage $w^*(x)$, implicitly defined by*

$$w^*(x) = b(x) + \frac{\lambda}{r + s} \left[\int_{w^*(x)}^{\bar{w}} 1 - \tilde{G}(w|x) dw \right] \quad (1.4)$$

There is a unique solution to (1.4). This is sufficient to characterize match sets, expected wages, and unemployment rates as a function of x , given the labor market parameters $b(x)$, λ , r , and s , the wage function $w(x, y)$, and the distribution of vacant jobs $G(y)$.

Equation (1.4) implicitly pins down the reservation wage $w^*(x)$ as a function of $b(x)$, λ , r , and s , given the conditional distribution of wages \tilde{G} . Define the match acceptance indicator function $\mathbb{1}(x, y)$ to summarize the workers' acceptance strategy.

$$\mathbb{1}(x, y) = \begin{cases} 1 & \text{if } w(x, y) \geq w^*(x) \\ 0 & \text{otherwise} \end{cases} \quad (1.5)$$

The expected wage of an employed worker is

$$\bar{w}(x) = \frac{\int_0^1 w(x, y) \mathbb{1}(x, y) g(y) dy}{\int_0^1 \mathbb{1}(x, y) g(y) dy} \quad (1.6)$$

and the expected accepted mismatch is given by

$$\bar{\mu}(x) = \frac{\int_0^1 |x - y| \mathbb{1}(x, y) g(y) dy}{\int_0^1 \mathbb{1}(x, y) g(y) dy} \quad (1.7)$$

Unemployment. The hazard rate from unemployment to employment is the rate at which an acceptable job offer arrives, which is equal to the offer rate multiplied by the probability that the offer comes from a firm in the acceptable range.

$$\mathcal{H}(x) = \lambda[1 - \tilde{G}(w^*(x)|x)] = \lambda (G(\bar{y}(x)) - G(\underline{y}(x))) \left(\lambda \int_0^1 \mathbb{1}(x, y)g(y) dy \right)$$

Let $u(x)$ denote the unemployment rate for a worker of type x . Equilibrium unemployment rates are given by a steady-state condition on the unemployment rate of each worker type. In steady state where unemployment is constant over time, aggregate flows into and out of unemployment must be equal.

$$\dot{u}(x) = s(1 - u(x)) - u(x)\mathcal{H}(x) = 0$$

Therefore, the equilibrium unemployment rate for worker type x is given by

$$u(x) = \frac{s}{s + \mathcal{H}(x)} \quad (1.8)$$

Define the aggregate unemployment rate \bar{u} as the total measure of unemployed workers in the economy,

$$\bar{u} = \int_0^1 u(x) dx$$

The pdf of unemployed workers is then given by $f(x) = \frac{u(x)}{\bar{u}}$ while the pdf of employed workers is $\tilde{f}(x) = \frac{1-u(x)}{1-\bar{u}}$.

1.4 Quantitative Analysis

Using the reservation wage equation (1.4), I provide a calibrated example of the equilibrium outcome. To align the results with the empirical facts presented in Section 2, workers and firms are uniformly distributed over the unit interval; $L(x) = G(y) = U(0, 1)$.

To deliver predictions consistent with the stylized facts in Section 1.2, two additional assumptions are now imposed on the wage equation:

1. $w_x > 0$

$$2. \forall x_0, \exists y_0 \in [0, 1] \ni w_y(x_0, y_0) = 0$$

where w_x and w_y indicate the partial derivatives of w with respect to x and y . The first assumption is that higher-skilled workers have the ability to earn more. However, there exist some (low-skilled) jobs in which higher-skilled workers may earn less than lower-skilled workers. The second assumption is that there exists a “best” job for each type of worker at which the worker’s wage is maximized. These restrictions ensure that wages in the model reflect those in the data, and are informed by the empirical results in this paper as well as results in Guvenen et al. [2015] and theoretical predictions of Eeckhout and Kircher [2011] among others. These conditions, while not necessary to obtain an equilibrium solution, are key in generating the meaningful differences in the outcomes of workers across different skill levels.

For the numerical exercise in this section, let wages for each worker-firm pair be given by

$$w(x, y) = x - \delta(x - y)^2$$

Here, δ is a scalar representing the substitutability of skills. Increasing δ amplifies the penalty for mismatch, decreasing the range of jobs with which a worker can profitably match. This simple function embodies the empirical results regarding wages and skill mismatch: higher-skilled workers typically earn higher wages, but wages depend also on the type of firm the worker is matched with and the associated match quality. Not only is this consistent with the empirical literature, but it is intuitive as well. Higher-skilled workers possess more human capital and have the potential to be produce more by matching with more productive jobs. In matches with $x > y$, workers do not produce up to their full potential, leading to lower wages. In matches with $x < y$, workers may not produce to the full potential of the jobs they occupy. Therefore, firms may require a larger share of the surplus in this type of match to compensate for the output foregone by hiring an under-skilled worker, again leading to lower wages for mismatched workers. Figure 1.5a shows that the wage penalties for positive and negative mismatch are similar in the data, so for simplicity the wage loss due to mismatch is assumed to be symmetric in this exercise.

The unemployment benefit function is

$$b(x) = b_0 + b_1x$$

By conditioning benefits on x , the unemployment benefit adjusts to better match the workers' expected wages, while keeping the worker's value of unemployment independent of previous employment history.⁵

1.4.1 Calibration

I use the 2009 to 2013 waves of the National Longitudinal Survey of Youth 1997 to calibrate several parameters for this example; at this time, respondents are between the ages of 25-33 years old. Sample selection criteria are as described in Section 2, and sample weights are used to account for selective attrition. The model is calibrated at monthly frequency. Calibrated parameter values used for the numerical example are presented in Table 1.1.

Table 1.1.: Calibrated parameter values in baseline model.

Parameter	Value	Target
b_0	0	(Normalization)
b_1	0.4	Benefits Accuracy Measurement
r	0.001	3-month Treasury bill
s	0.0299	(Job duration) ⁻¹
λ	0.5010	$\bar{u} = .0824, \mathcal{D}_{90,50} = 1.3771$
δ	0.1003	$\bar{u} = .0824, \mathcal{D}_{90,50} = 1.3771$

For simplicity, workers have no base value of leisure; b_0 is set to 0. The monthly interest rate r is chosen to match the average 3-month treasury bill rate from 2009-2013. Other parameters are calibrated using data from the NLSY97. The average reported

⁵Lise et al. [2016] also assume benefits are a function of only the worker's type; Marimon and Zilibotti [1999] and Teulings and Gautier [2004] set equal benefits for all worker types.

job tenure is 33.50 months, leading to a separation rate of $s = 0.0299$. The U.S. Department of Labor’s Office of Unemployment Insurance releases a yearly Benefit Accuracy Measurement report containing each state’s quarterly UI replacement rate. From 2009 to 2013, the weighted average U.S. replacement rate was between 0.405 and 0.470. To provide conservative estimates of the remaining parameters, $b_1 = 0.4$.

I use the Method of Moments to jointly calibrate λ and δ , matching the aggregate unemployment rate $\bar{u} = \int_0^1 u(x) dL(x)$ and the ratio of high-skill to median-skill max-mean wage dispersion $\mathcal{D}_{90,50}$.⁶ Letting w_i^m denote the m^{th} percentile of the hourly wage distribution for a skill percentile bin i , I estimate the max-mean wage dispersion in the data by regressing

$$z_i = \beta_0 + \beta_1(w_i^{90} - w_i^{50})$$

using the 90th percentile wage rather than the 100th to represent the max wage in order to correct for potential misreporting and/or extreme cases. The max-mean wage dispersion ratio is then calculated as

$$\mathcal{D}_{90,50} = \frac{\hat{z}_{90}}{\hat{z}_{50}}$$

Letting the parameter vector be $\theta = \begin{bmatrix} \lambda \\ \delta \end{bmatrix}$ and the moment vector be $M = \begin{bmatrix} \bar{u} \\ \mathcal{D}_{90,50} \end{bmatrix}$,⁶ the loss function $J(\theta) = (M - \hat{M}(\theta))'W(M - \hat{M}(\theta))$ is minimized to obtain the calibrated values for λ and δ . $\hat{M}(\theta)$ represents the model-predicted values for the moments given the parameter vector θ , and the weighting matrix W is set to the identity matrix.

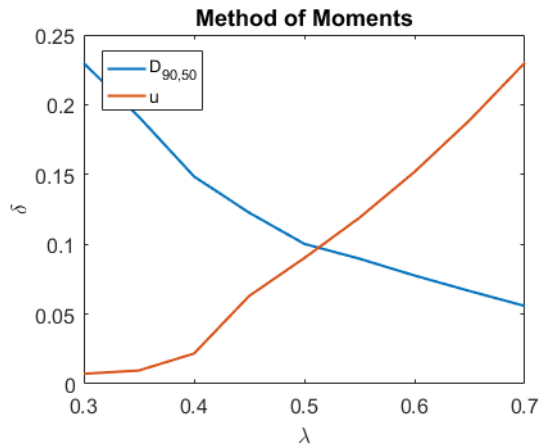
Table 1.2 summarizes the effect of increases in λ and δ on the moments of interest to shed light on the identification method. Holding δ fixed, an increase in λ causes \bar{u} to fall because workers receive offers more quickly; fixing λ , an increase in δ causes \bar{u} to increase since workers become more selective as the wage at each possible job falls. Therefore, the set of (λ, δ) such that $\bar{u} = .0824$ is upward-sloping; higher values for λ

⁶See Appendix A.3 for a discussion of moment and parameter calculation methods, moment identification, and results under alternative moment choices.

Table 1.2.: Effects of λ and δ on \bar{u} and $\mathcal{D}_{90,50}$.

	$\Delta\bar{u}$	$\Delta\mathcal{D}_{90,50}$
$\uparrow \lambda$	-	-
$\uparrow \delta$	+	-

require corresponding increases in δ to offset the decrease in the unemployment rate. To understand the set of (λ, δ) that maintain $\mathcal{D}_{90,50}$, consider the effects of λ and δ on high-skill relative to median-skill workers. Increases in either λ or δ cause workers to become more selective in accepting jobs. However, higher-skilled workers respond more strongly to changes in λ and δ than do median-skill workers. Therefore, match sets for high-skill workers become narrower relative to those of median skill workers, decreasing the difference in wage dispersion between the two skill groups, $\mathcal{D}_{90,50}$. This results in a downward-sloping set of (λ, δ) to match $\mathcal{D}_{90,50} = 1.3771$; an increase in one parameter requires a decrease in the other to obtain the desired moment value. Both the unemployment rate \bar{u} and the ratio of wage dispersion $\mathcal{D}_{90,50}$ can be matched individually by an infinite set of (λ, δ) pairs. However, only one parameterization can match the two moments simultaneously; see Figure 1.7.

Fig. 1.7.: GMM identification of (λ, δ) .

1.4.2 Results

To solve for the reservation wage for each type of worker, I iterate on equation (1.4) until the function converges⁷. The results are shown in the left panel of Figure 1.8. The worker's maximum wage, x , is plotted on the same graph for reference. To better illustrate the tradeoffs each worker faces, the right panel plots the mean wage offer, the reservation wage, and the expected accepted wage relative to each worker's max possible wage. It is clear that the reservation wage is increasing in the worker's type. This result is expected, because better workers are more productive.

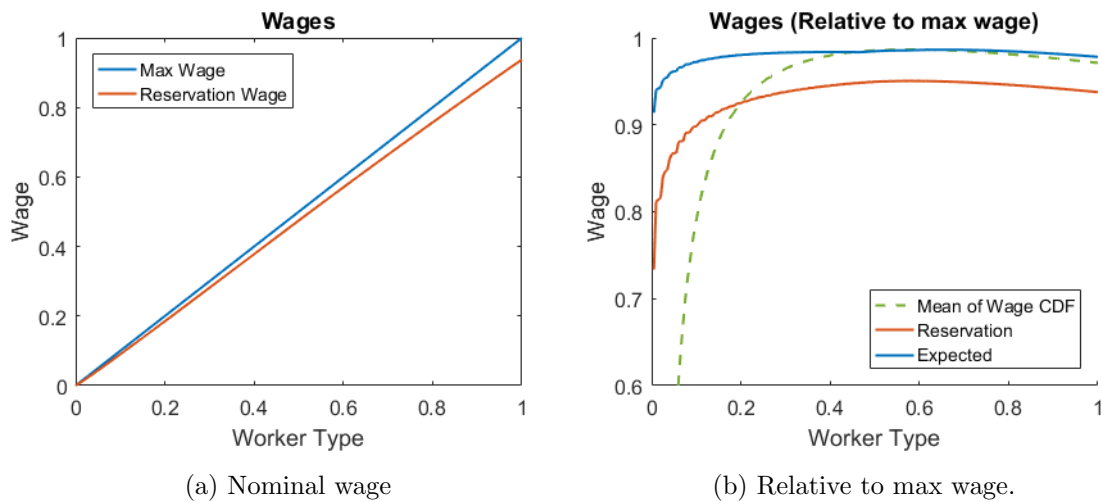


Fig. 1.8.: Model predicted mean wage offer, reservation wage, and expected wage by individual skill.

However, because it is relatively more difficult for workers near the ends of the distribution to find a job with low mismatch, the reservation wage is not linear. Figure 1.8 also shows the expected wage, conditional on being employed, for each type of worker. This is equal to the expected productivity in equation (1.6), following the initial assumption on wage setting. It is useful to note here that this is not equal to the mean of the worker's wage distribution, since the reservation wage truncates

⁷An outline of the computational algorithm is discussed in Appendix A.2.

the lower end of the distribution of wages for employed workers. For most workers, the mean wage and the expected wage are very close. However, workers near the low end of the skill space require a reservation wage higher than the mean of their wage distribution, so the expected wage is above the mean wage as well.

Figure 1.9 shows the within-type wage dispersion generated by the model, represented by the difference between the 90th and 10th percentile of the accepted wage offer distribution for each worker type. Consistent with Fact 1 in the previous section, there is more wage dispersion among higher-skilled workers.



Fig. 1.9.: Model predicted wage dispersion by individual skill.

Conditional on receiving an offer, the probability that a worker of type x receives an acceptable job offer is equal to $1 - \tilde{G}(w_x^*|x)$. Both the wage distribution and the reservation wage vary across x , so the probability of an acceptable offer will also vary. The aggregate acceptance rate is 75.59%. Since the NLSY does not contain data on rejected job offers, it is not possible to directly compare the simulated acceptance rate to the data. However, the Survey of Unemployed Workers in New Jersey contains data on the receipt and acceptance of job offers for a sample of workers during 2009 and 2010; the average acceptance probability in this dataset is 79%.

The variation in acceptance probabilities implies that the expected length of an unemployment spell is not the same across all worker types. The expected unemploy-

ment rate and duration for each type of worker is plotted in Figure 1.10a. Consistent with the stylized facts presented in Section 1.2, the average unemployment rate and duration decrease in the workers' skill type. Because the unemployment rate is not constant for all types of workers, the distributions of employed and unemployed workers differ from the overall worker type distribution $L(x)$; in particular, the distribution of unemployed workers is heavily skewed to the left.

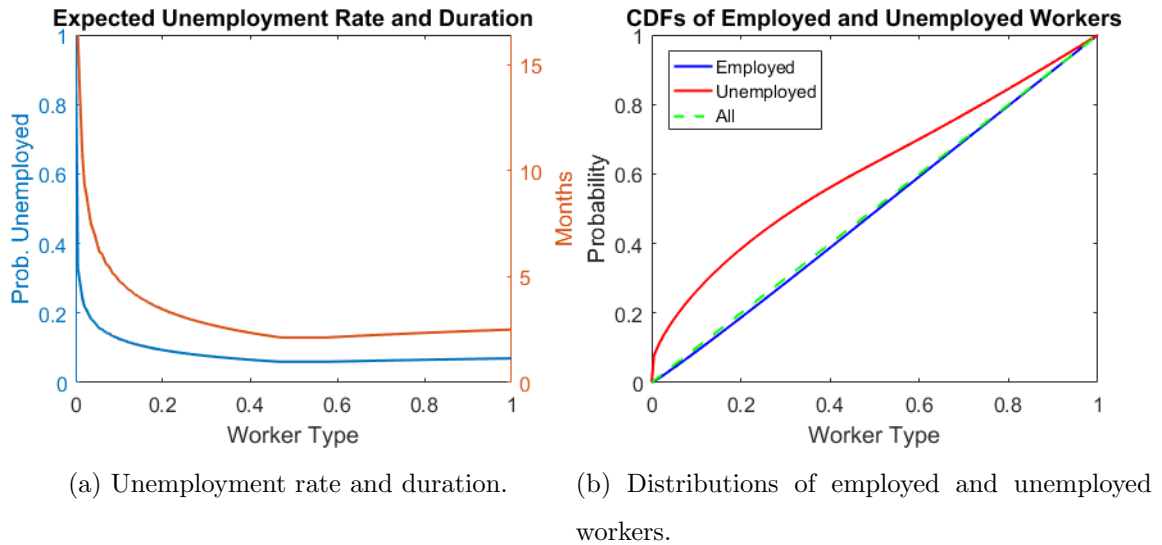


Fig. 1.10.: Model predicted employment outcomes by individual skill.

One of the main questions that the current model is designed to address is whether search frictions can generate the level of mismatch observed in the data and explain the differences in mismatch tolerance across worker skill types. In this example, the level of mismatch associated with a wage w is given by $|\mu| = 1 - \frac{w}{x}$. Substituting in the reservation wage yields the maximum level of mismatch a worker of type x will accept.

The maximum and minimum firm types accepted by each type of worker are plotted in Figure 1.11a; all matches between these two curves are accepted. Figure 1.11b shows that the accepted level of mismatch is monotonically increasing in worker type. For workers near the upper end of the skill space, the difference between the

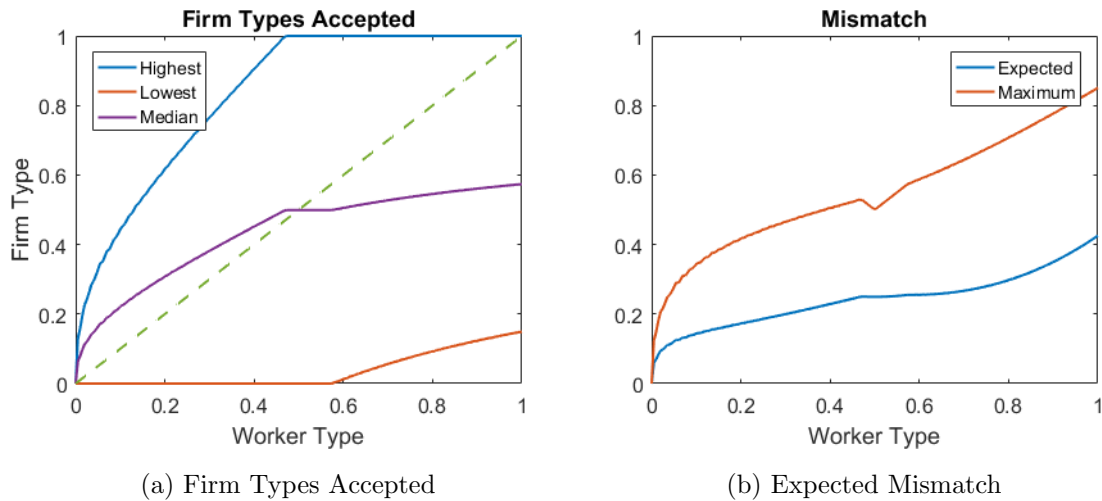


Fig. 1.11.: Model predicted match acceptance and expected mismatch by individual skill.

$E(x, w)$ and $U(x)$ is quite large, even for wages near the low end of the conditional wage offer distribution. On the other hand, this difference is relatively small for low-skilled workers. Because low-skilled workers give up less by remaining unemployed, they are willing to wait longer for a better match. In fact, at the calibrated level of unemployment benefits, low-skilled workers are better off in unemployment than at a job that pays the mean wage offer. Workers accept job offers from all firms with mismatch less than or equal to $|\mu^*| = 1 - \frac{w_x^*}{x}$.

While the workers' equilibrium strategy varies based on the parameters, the model is robust to changes in the calibration. Increasing the replacement rate b_0 increases all workers' reservation wages; increasing the separation rate s decreases the value of employment, causing an increase in reservation wages; increasing the offer arrival rate λ increases the value of unemployment, raising reservation wages; increasing the mismatch penalty δ decreases the value of unemployment (through decreasing the expected value of a job offer), causing a decrease in the reservation wage. The lone parameter that is capable of produce qualitative changes in the results is the base level of unemployment benefits b_0 ; a value greater than 0 would be greater than

the value of employment for some low-skilled workers, causing their reservation wage to be greater than the max possible wage offer and keeping them permanently in unemployment.

Discussion. I assess the model by comparing the results from the calibrated model to the stylized facts in Section 1.2. Figure 1.12a compares the match acceptance sets observed in the data to those predicted by the model. To mitigate the effects of outliers in the data, the 95th and 5th percentiles of observed matches are used in place of the maximum and minimum acceptable matches, and like Section 2 uses nonparametric regressions to smooth the data. The figure plots the boundaries of match acceptance and the median match accepted, with 90% confidence intervals for the fitted values generated by the nonparametric regressions. The model is effective at predicting the lower bound of match acceptance, as well as the median match for all but the highest-skilled workers. However, low-skilled workers in the model are too selective relative to those in the data.

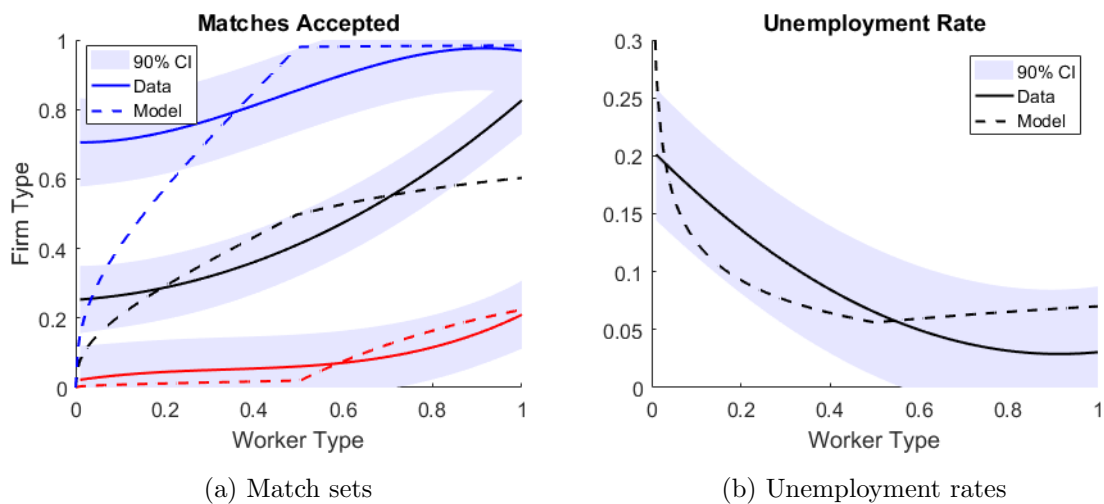


Fig. 1.12.: Comparison of model vs data match sets and unemployment rates.

Consistent with the stylized facts, the model generates an unemployment rate that decreases in the worker's type, as shown in Figure 1.12b. Among workers above

$x = 0.5$, the model-generated unemployment rate is on average 2.45% greater than that in the data. For workers below the median, it is 4.7% too low on average. The model prediction for the expected duration of unemployment is also broadly consistent with the empirical facts; higher-skilled workers are expected to exit unemployment more quickly.

These results highlight several similarities between the stylized facts and the model predictions, as well as two distinct areas of divergence. The model predicts too much selectivity among low-skilled workers, yet it is able to generate a relatively accurate prediction of the median match. In contrast, the range of matches accepted by high-skilled workers is successfully replicated, but the model predicts a median match that is well below that in the data. Together, these two facts suggest that workers are able to sort more effectively than is allowed by the model. While random search provides a simple and tractable way to generate non-singleton match sets, the results here suggest that workers' search is not perfectly random; the implications of a refined search strategy will be explored in subsequent work.

In this paper, workers' search strategy is random. That is, workers have information on their own abilities, but are unable to discern whether a job is a good match prior to meeting the firm or lack information on the location of good matches. A frequently used alternative allows workers perfect information regarding all currently vacant jobs, implying that the market for jobs is segmented by type. Typically, workers choose exactly one market⁸ to search in; Lagos [2000] and Shimer [2005b] are classic examples. The observations in this section suggest a third alternative; perhaps workers have some information about the location and quality of jobs, but do not have sufficient information to be able to successfully choose exactly which jobs to search for.

Suppose that workers are able to *target* their search to jobs in the vicinity of their ideal job. In expectation, job offers will be close to the targeted type, but may also

⁸Decreuse [2008] instead assumes that workers search in every submarket such that the total value of the match is positive.

come from jobs further away with decreasing probability. In this case, low-skilled workers may meet and match with jobs for which they are substantially over-skilled, but on average will sort into jobs near their ideal match. High-skilled workers will also be able to sort into relatively high-type jobs, with the occasional match with low-type firms. Targeted search can therefore explain both of the areas of discrepancy between the stylized facts and the predictions from the model; the implications of this search strategy will be explored in subsequent work.

1.4.3 Wages, Unemployment, and Mismatch During the Recovery

The empirical results in Section 1.2 highlight the fact that during the recovery from the Great Recession many workers were willing to accept substantial skill mismatch, and therefore suffered wage penalties. In standard search models without heterogeneity, as the labor market improves, workers' reservation wages increase. Higher reservation wages imply that average accepted wage also increases. However, the growth of the economy-wide average wage, represented by the median usual weekly earnings reported each quarter by the BLS, was lower than expected during much of the recovery. Despite the increasing vacancy rate and falling unemployment rate, growth of the average wage remained below 2% for several years, well below its long-run average of 3.25%.

In order to explain the lower-than-expected growth of the average wage, it is necessary to break the link between the economy-wide average wage and the average wage of an individual worker. Incorporating heterogeneity into a search model model allows for differential responses to changing labor market conditions across worker type. Changes in the average wage can then be separated into an intensive margin (individual wage changes) and an extensive margin (composition of employment). In this section, I show that the composition effect dominated the individual wage effect during the recovery from the Great Recession, causing the observed change in the average wage to be small.

To decompose average wage growth, I derive the elasticity of the average wage with respect to the contact rate λ , a proxy for market tightness in the one-sided model. Let $\bar{w}(x)$ denote the average wage for a worker of type x ; then $W = \int_0^1 \bar{w}(x) d\tilde{F}(x)$ is the average wage across all workers in the economy. First, differentiate $\bar{w}(x)$ with respect to λ .

$$\frac{d\bar{w}(x)}{d\lambda} = \frac{\tilde{g}(w^*(x)|x) \frac{dw^*(x)}{d\lambda} \int_{w^*(x)}^1 (w - w^*(x)) \tilde{g}(w|x) dw}{\left(\int_{w^*(x)}^1 w \tilde{g}(w|x) dw \right)^2} \quad (1.9)$$

Then differentiate $w^*(x)$.

$$\frac{dw^*(x)}{d\lambda} = \frac{\int_{w^*(x)}^1 (w - w^*(x)) \tilde{g}(w|x) dw}{r + s + \lambda \int_{w^*(x)}^1 \tilde{g}(w|x) dw} \quad (1.10)$$

Figure 1.13a plots (1.10) across the skill space, using the calibrated parameter values from Section 1.4.1. The reservation wage for all workers is increasing in λ . However, higher-skilled workers respond more strongly to increases in the contact rate because their expected gain from employment is larger.

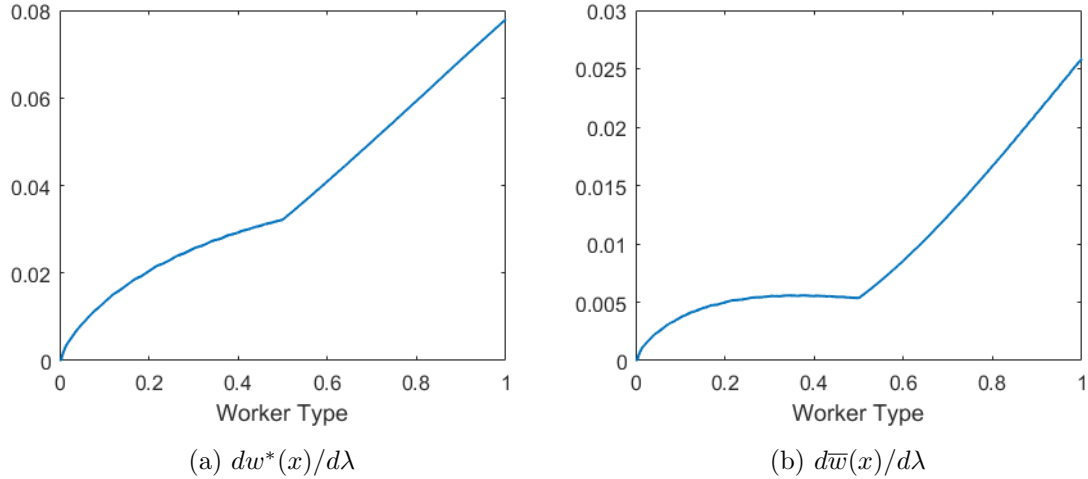


Fig. 1.13.: Derivative of reservation and average wage with respect to λ .

Substituting (1.10) into (1.9),

$$\frac{d\bar{w}(x)}{d\lambda} = \frac{\tilde{g}(w^*(x)|x) \left(\int_{w^*(x)}^1 (w - w^*(x)) \tilde{g}(w|x) dw \right)^2}{(r + s) \left(\int_{w^*(x)}^1 \tilde{g}(w|x) dw \right)^2} \quad (1.11)$$

Figure 1.13b plots the (1.9). Because the change in the reservation wage is positive for all workers, the average wage for each type of worker will increase with λ . However, since the derivative of the reservation wage is increasing in x , the expected wage is also steeper in λ for higher-skilled workers.

Turning now to the average wage across all x ,

$$W = \int_0^1 \bar{w}(x) d\tilde{F}(x) = \frac{1}{1 - \bar{u}} \int_0^1 \bar{w}(x)(1 - u(x)) dx$$

The derivative of the average wage with respect to λ is

$$\frac{dW}{d\lambda} = \int_0^1 \frac{d\bar{w}(x)}{d\lambda} \tilde{f}(x) + \bar{w}(x) \frac{d\tilde{f}(x)}{d\lambda} dx \quad (1.12)$$

This accounts for not only the individual workers' wage changes, but also the change in the distribution of employed workers across the skill space. Differentiating the unemployment rate $u(x)$ with respect to λ ,

$$\frac{du(x)}{d\lambda} = \frac{-s \left(\int_{w^*(x)}^1 \tilde{g}(w|x) dw - \lambda \tilde{g}(w^*(x)|x) \frac{dw^*(x)}{d\lambda} \right)}{\left(s + \lambda \int_{w^*(x)}^1 \tilde{g}(w|x) dw \right)^2} \left(\quad (1.13)$$

There are two competing effects of the contact rate on unemployment. Increasing the contact rate increases the arrival rate of acceptable matches, decreasing the unemployment rate. At the same time, workers become more selective, dampening the effect of the contact rate on unemployment. In the calibrated example, shown in Figure 1.14, the net result is a modest decrease in unemployment that is larger for those workers whose reservation wages are less elastic. Hence, the employment share of higher-skilled workers drops, and that of lower-skilled workers increases. This change in the composition of employment has a salient effect on the average wage. Finally, the change in aggregate unemployment is

$$\frac{d\bar{u}}{d\lambda} = \int_0^1 \frac{du(x)}{d\lambda} dx \quad (1.14)$$

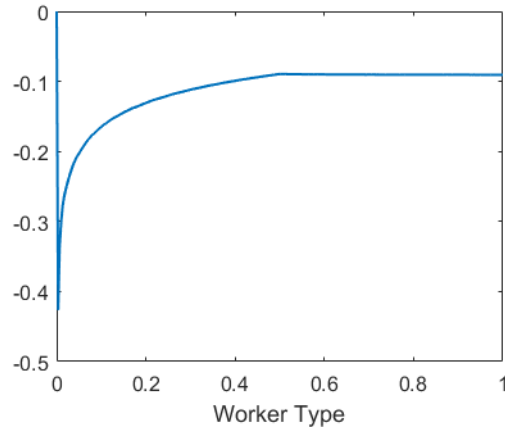


Fig. 1.14.: Derivative of unemployment rate across the skill space.

The elasticity of the economy-wide average wage with respect to the contact rate is

$$\frac{d \ln W}{d \ln \lambda} = \frac{dW}{d\lambda} \cdot \frac{\lambda}{W}$$

In the calibrated model, this elasticity is -0.0006. Although the reservation wages of all workers increase, causing an increase in all workers' average wages, the increase in the economy-wide average wage through this channel is fully offset by the change in the composition of employment. Lower-skilled workers accept jobs at higher rates, increasing their employment shares and decreasing the economy-wide average wage. Imposing a standard Cobb-Douglas matching function with matching elasticity of 0.5, the implied elasticity of the average wage with respect to the unemployment rate, given by equation (1.15) is 0.0009.

$$\frac{d \ln W}{d \ln \bar{u}} = \frac{dW}{d\lambda} \cdot \frac{d\lambda}{d\bar{u}} \cdot \frac{\bar{u}}{W} \quad (1.15)$$

This nearly-zero elasticity helps to explain why the rising vacancies and falling unemployment during the recovery were insufficient to produce much growth in median usual weekly earnings.

In contrast, the average of individual wage elasticity with respect to unemployment, given by

$$\int_0^1 \left(\frac{d\bar{w}(x)}{d\lambda} \frac{d\lambda}{du(x)} \frac{u(x)}{\bar{w}(x)} \right) \tilde{f}(x) dx$$

is -0.015. Nijkamp and Poot [2005] report that the range of estimates in the empirical literature is -0.5 to +0.1, with a mean across 208 empirical estimates of -0.07. Without accounting for compositional changes, the elasticity of the average wage,

$$\left(\frac{1}{1 - \bar{u}} \int_0^1 \frac{d\bar{w}(x)}{d\lambda} (1 - u(x)) dx \right) \left(\frac{d\lambda}{d\bar{u}} \frac{\bar{u}}{\bar{W}} \right)$$

would be -0.016, slightly larger than the average of individual wage elasticity since it doesn't adjust for differences across workers in the elasticity of unemployment with respect to lambda.

Wage Growth in the Data The model calibration is based on U.S. data from the NLSY97, covering 2009-2013. During this time period, the U.S. economy was beginning to recover from the Great Recession. As the unemployment rate fell and the vacancy rate rose, the contact rate for unemployed workers increased. Table 1.3 shows the key labor market variables in 2009 and in 2013. The unemployment rate and average wage are given for the aggregate economy as well as for low-, mid-, and high-skill workers, where each of these groups represents one-third of the worker skill space.⁹

Using the elasticities derived in the beginning of this section, I explore whether the model can generate the appropriate changes in unemployment and wages. For an increase in the contact rate of 0.22, the model-predicted changes in unemployment, mismatch, and wages are shown in Table 1.4.

Because wages in the model are normalized, I equate the standard deviation of log wages in the data to the standard deviation of wages in the model to transform model wages into dollars. Letting σ_n^{data} and σ_n^{model} be the respective standard deviations for

⁹Jobs where the respondent worked less than 35 hours/week are dropped, following the BLS definition of full-time work. The aggregate vacancy rate, \bar{v} is obtained by averaging the monthly vacancy rate in the JOLTS report from the BLS. The aggregate unemployment rate, \bar{u} , and the unemployment rates by skill group are calculated using NLSY97 data on weekly employment status. Market tightness is calculated as $\frac{\bar{v}}{\bar{u}}$, and the contact rate is imputed from the vacancy and unemployment rates using a Cobb-Douglas matching function with matching elasticity 0.5.

Table 1.3.: U.S. labor market in 2009 vs. 2013

	2009	2013	Δ 2009-2013
Vacancy rate	1.83	2.78	0.95
Unemployment rate	9.56	6.07	-3.49
Low-skill	15.28	10.23	-5.05
Mid-skill	7.24	5.33	-1.91
High-skill	3.64	2.06	-1.58
Market tightness	0.19	0.46	0.27
Contact rate	0.41	0.63	0.22
Average wage	16.17	17.76	1.60
Low-skill	13.47	13.75	0.28
Mid-skill	15.56	16.48	0.91
High-skill	19.74	22.87	3.13

Table 1.4.: Empirical vs. model predicted changes, 2009 to 2013

	Unemployment		Mismatch		Wage	
	Data	Model	Data	Model	Data	Model
Average	3.47%	-2.50%	-.0200	-.0283	\$1.60	\$1.57
Low-skill	-5.12	-3.42	-.0128	-.0189	0.28	1.26
Mid-skill	-1.95	-2.07	-.0200	-.0208	0.91	1.52
High-skill	-1.60	-1.98	-.0263	-.0447	3.13	3.10

worker group n (low-, mid-, or high-skill), and w_n^{data} be the average wage among group n , the predicted wage change for group n is

$$\widehat{\Delta w}_n = \exp \left(\log(w_n^{data}) + \Delta w_n^{model} \left(\frac{\sigma_n^{data}}{\sigma_n^{model}} \right) \right) \left(w_n^{data} \right)$$

The above formula predicts the new log wage in the data, exponentiates to get the wage in dollars, and subtracts the old wage to obtain the predicted wage change in dollars. See Appendix A.6 for results for alternative transformations.

Overall, the model is able to match the relative differences across workers, but predicts too large a response in the reservation wages of low-skilled workers. As a result, the model-generated drop in mismatch (and therefore the increase in wage) is too large, and the fall in the unemployment rate is too small relative to the data. Since low-skilled workers make up the largest share of unemployment, the drop in the aggregate unemployment rate is also smaller. This is likely because the contact rate was not the only aspect of the labor market that changed during the recovery. For instance, the additional drop in the unemployment rate could be explained by a change in the separation rate or a change in labor productivity. In either case, the increase in market tightness is able to generate heterogeneous responses across worker types that explain a substantial portion of the changes in outcomes observed during the recovery.

1.5 On-the-job Search

This section extends the baseline model to allow employed workers to search on the job (OTJ). Let λ_u and λ_e be the arrival rates of job offers to unemployed and employed workers, respectively. The continuation value of employment for a type x worker employed by a type y firm now includes the value of on-the-job search, $(1 - s)\lambda_e\phi(x, w)$.

$$\phi(x, w) = \int_{\underline{w}}^{\bar{w}} \max\{E(x, w), E(x, w')\} dG(w'|x)$$

A new job (wage) offer w' will be accepted if and only if it provides a weakly greater value of employment than the current job, ie. $E(x, w') \geq E(x, w)$. Hence, the reservation wage for an employed worker is equal to the current wage, w . The reservation wage for an unemployed worker is given by w_{OTJ}^* such that $E(x, w_{OTJ}^*(x)) = U(x)$.

$$w_{OTJ}^*(x) = rU(x) - (1 - s)\lambda_e (\phi(x, w^*(x)) - U(x))$$

Because unemployed workers do not entirely give up the value of continued search when accepting a job offer, the opportunity cost of accepting a bad match falls relative to the economy without on-the-job search. Increasing the effectiveness (equivalently, the arrival rate) of on-the-job search decreases workers' reservation wages. Integrating by parts, the reservation wage reduces to:

$$w_{OTJ}^*(x) = b(x) + \frac{\lambda_u - (1-s)\lambda_e}{r+s+(1-s)\lambda_e} \int_{w_{OTJ}^*(x)}^{\bar{w}} 1 - \tilde{G}(w|x) dw \quad (1.16)$$

Note that while this has the same general form as the reservation wage in the economy without OTJ search, the new lower limit of integration makes an analytical comparison impossible.

Since workers are able to change jobs, the reservation wage is no longer sufficient to determine the relationship between the offer arrival rate and average match quality. To solve for the expected level of mismatch, the distribution of skill mismatch within each type x must be in steady state. This requires either the distribution of matches or the distribution of wages to be constant over time for each x . It is more general to impose a steady-state assumption on the wage distribution. Following Burdett and Mortensen [1998], the additional equilibrium condition is

$$\forall x : \frac{dH_x^{OTJ}(w)}{dt} = 0 \quad (1.17)$$

where H_x^{OTJ} is the CDF of wages for employed workers of type x . Its time derivative, representing the flow of type x workers into jobs paying less than or equal to w , is given by:

$$\begin{aligned} \frac{dH_x^{OTJ}(w)(1-u(x))}{dt} &= u(x)\lambda_u \int_{\underline{w}}^w \mathbb{1}(x, w) d\tilde{G}(w|x) \\ &\quad - (1-u(x))H_x^{OTJ}(w) \left[s + \lambda_e(1 - \tilde{G}(w|x)) \right] \end{aligned} \quad (1.18)$$

The first term represents the flow of unemployed workers receiving and accepting a wage offer of w or less, and the second term represents the flow of workers currently employed at a wage less than or equal to w who separate to unemployment or receive a

wage offer greater than w . The steady-state distribution of wages earned by employed workers of type x is therefore

$$H_x^{OTJ}(w) = \frac{u(x)\lambda_u \int_{\underline{w}}^w \mathbb{1}(x, w) d\tilde{G}(w|x)}{(1-u(x))(s + \lambda_e(1 - \tilde{G}(w|x)))} \quad (1.19)$$

Given the steady-state distribution of wages, the expected wage for a worker x is given by

$$\bar{w}^{OTJ}(x) = \int_{\underline{w}}^{\bar{w}} w h_x^{OTJ}(w) dw$$

where $h_x^{OTJ}(w) = \frac{dH_x^{OTJ}(w)}{dw}$ is the pdf of the steady state distribution of wages.

$$h_x^{OTJ}(w) = \frac{(1-u(x))u(x)\lambda_u \tilde{g}(w|x) \left[\mathbb{1}(x, w) \cdot (s + \lambda_e(1 - \tilde{G}(w|x))) + \lambda_e \int_{\underline{w}}^w \mathbb{1}(x, w') \tilde{g}(w'|x) dw' \right]}{\left[(1-u(x))(s + \lambda_e(1 - \tilde{G}(w|x))) \right]^2} \quad (1.20)$$

Finally, the expected observed mismatch for a worker of type x can be obtained by inverting the wage function.

$$\bar{\mu}^{OTJ}(x) = w_x^{-1}(\bar{w}^{OTJ}(x)) \quad (1.21)$$

As with the reservation wage, since $\mathbb{1}(x, w)$ changes under OTJ search, it is not possible to compare expected wages or expected mismatch in the two economies analytically.

To determine whether match quality is greater under on-the-job search, we can compare the steady state wage distributions between the two environments. However, the distribution of wages (or mismatch) is not sufficient to determine a worker's expected match. To find the expected match for each worker type, we must make the stronger assumption that the distribution of matches for each worker type x is in steady state. Let the distribution of occupational matches for a worker of type x be given by $M_x^{OTJ}(y)$. As before, when this distribution is in steady state, the time derivative of its cdf will be equal to 0.

$$\frac{dM_x^{OTJ}(x)}{dt} = 0 \quad (1.22)$$

The change in the measure of occupation matches less than or equal to y is the difference between the sum of new matches and job-to-job transitions from existing matches with occupations above y , and the sum of separations to unemployment and to better matches with occupations above y .

$$\begin{aligned} \frac{dM_x^{OTJ}(y)(1-u(x))}{dt} &= u(x)\lambda_u \int_0^y \mathbb{1}(x, y') dG(y') \\ &+ (1-u(x))\lambda_e \int_0^y g(y') \left[\int_y^1 \mathbb{1}[w(x, y') > w(x, y'')] dM_x^{OTJ}(y'') \right] \left(dy' \right. \\ &\left. - (1-u(x))M_x^{OTJ}(y) \left[s + \lambda_e \int_y^x \mathbb{1}[w(x, y') > w(x, y)] dG(y') \right] \right) \end{aligned} \quad (1.23)$$

Setting the above equal to 0, we can write the cdf of matches for worker type x as

$$\begin{aligned} M_x^{OTJ}(y) &= \left[u(x)\lambda_u \int_0^y \mathbb{1}(x, y') dG(y') \right. \\ &\left. + (1-u(x))\lambda_e \int_0^y \int_y^x \mathbb{1}[w(x, y') > w(x, y'')] dM_x^{OTJ}(y'') dG(y') \right] / \left(\right. \\ &\left. \left[(1-u(x)) \left(s + \lambda_e \int_y^x \mathbb{1}[w(x, y') > w(x, y)] dG(y') \right) \right] \right) \end{aligned} \quad (1.24)$$

The pdf $m_x^{OTJ}(y)$ can be used to find the expected occupational match for each worker type.

How much better off are workers when they are able to search on-the-job? Workers' utility is assumed to be linear in income, so the expected flow utility of a type x worker can be expressed as

$$\omega(x) = u(x)b(x) + (1-u(x))\bar{w}(x) \quad (1.25)$$

and aggregate welfare is

$$\mathcal{W} = \int_0^x \omega(x) dL(x) \quad (1.26)$$

1.5.1 Calibration

The extended model contains one additional parameter, λ_e , which must be estimated jointly with λ_u and δ . GMM identification requires a number of moments at

least as great as the number of parameters to be estimated, so an additional moment is required. Let $\mathcal{S}(x)$ denote the rate of job switching among employed type x workers, and $\bar{\mathcal{S}}$ denote the average rate of job switching among all employed workers. That is,

$$\mathcal{S}(x) = \lambda_e(1 - \tilde{G}(\bar{w}(x)|x)) \quad (1.27)$$

and

$$\bar{\mathcal{S}} = \int_{\mathfrak{d}} \mathcal{S}(x) dx \quad (1.28)$$

In the NLSY97 data from 2009 to 2013, 2.85% of individuals in the labor force start a new job in any given month and 47.37% of these new matches are employment-to-employment transitions. Across all employed workers, the average probability of changing jobs in a given month is $\bar{\mathcal{S}} = 0.0147$. The extended model is estimated using GMM, matching the aggregate unemployment rate, the max-min wage differential, and the job switching rate. Table ?? shows the model fit to these moments, and Table 1.5 gives the calibrated parameter values.

Table 1.5.: Calibrated parameter values under on-the-job search.

Parameter	Value	Target
b_0	0	(Normalization)
b_1	0.4	Benefits Accuracy Measurement
r	0.001	3-month Treasury bill
s	0.0299	(Job duration) ⁻¹
λ_u	0.5285	$\bar{u}, \mathcal{D}_{90,50}, \bar{\mathcal{S}}$
λ_e	0.0340	$\bar{u}, \mathcal{D}_{90,50}, \bar{\mathcal{S}}$
δ	0.1407	$\bar{u}, \mathcal{D}_{90,50}, \bar{\mathcal{S}}$

1.5.2 Results

Workers' reservation wages are qualitatively unchanged from the baseline model. The reservation wage increases with the worker's skill type, and workers near the middle of the skills space obtain the highest wages relative to their maximum possible wage offer.

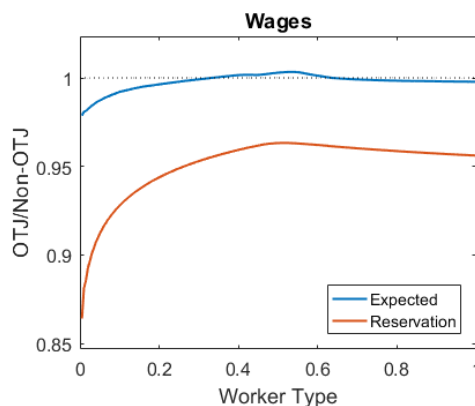


Fig. 1.15.: Reservation and expected wages under OTJ search, relative to those where λ_e is set to zero.

Figure 1.15 shows that while reservation wages fall for all workers under OTJ search, expected wages may increase or decrease. Since workers are able to switch jobs, the continuation value of employment is higher under OTJ search. The increased continuation value decreases the wage necessary to equate the value of employment with that of unemployment, so reservation wages for all workers are lower when they can search on-the-job. In the calibrated model, reservation wages fall by 4.96% on average relative to the counterfactual economy with $\lambda_e = 0$. The decrease in the reservation wage puts downward pressure on workers' average wage, since more low-wage offers are accepted. However, the opportunity to climb the wage ladder pulls average wages up. The net effect is mixed. Medium-skilled workers experience a slight increase in their expected wage because the job-switching effect outweighs that of the

decreased reservation wage, while workers at the ends of the skill space experience a decline.

Unemployment rates and expected unemployment duration across the worker skill space follow a similar pattern to the economy without OTJ search. However, the drop in reservation wages leads to a decrease in unemployment rates and durations for all workers. The average unemployment rate falls from 8.87% in the economy without OTJ search to 7.68% when OTJ search is introduced, and the average unemployment duration falls by just over 2 weeks, from 13.1 to 11.0 weeks.

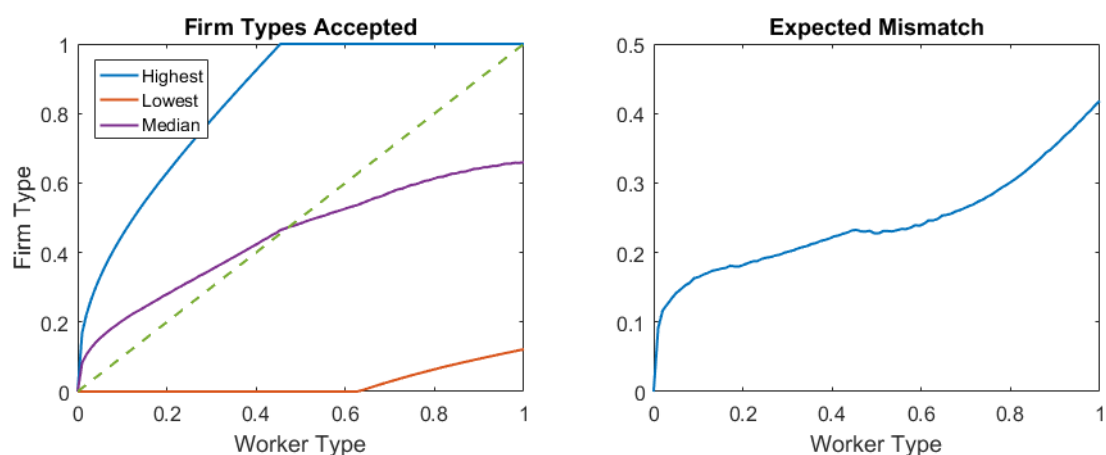


Fig. 1.16.: Model predicted match acceptance and expected mismatch under on-the-job search.

Figure 1.16 plots the match acceptance sets and expected level of mismatch across the worker skill space. The bounds on match acceptance are similar to the baseline case, but sorting (given by the median job type match) is slightly stronger for higher-skilled workers when they are able to search OTJ, as shown in Figure 1.17. However, this is offset by the drop in the reservation wage; expected mismatch rises among this groups of workers. Workers near the middle of the skill space experience a decrease in expected mismatch, while low-skill workers' expected match quality falls.

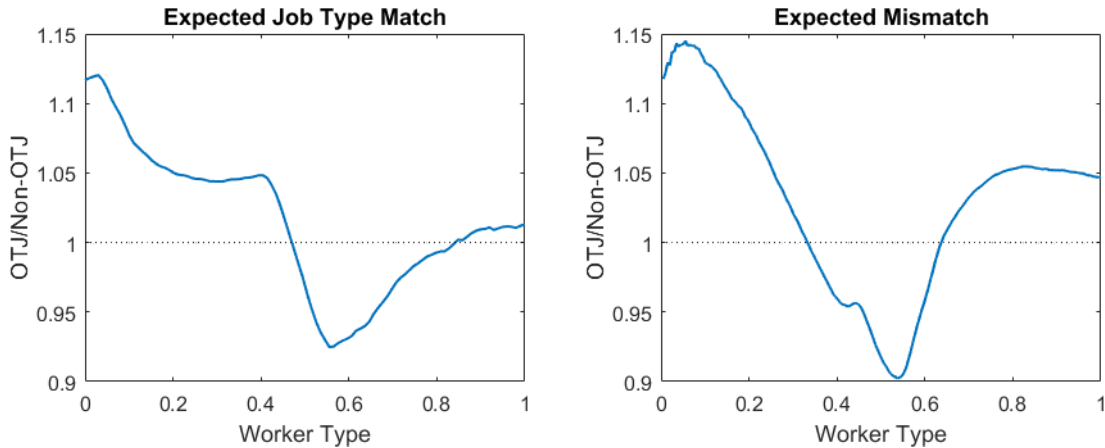


Fig. 1.17.: Model predicted expected match and mismatch under OTJ search, relative to the case where $\lambda_e = 0$.

Finally, I calculate the welfare gain from on the job search using the simple welfare function given in the previous section. Expected income increases for all workers, and aggregate welfare increases by 0.55% relative to the economy without OTJ search.

1.5.3 Mismatch Over the Business Cycle

How does the average level of mismatch in the economy change over the business cycle? Analysis using the baseline model shows that unemployed workers become more selective when the job offer arrival rate increases, so that average mismatch is countercyclical. The counterfactual examples below show the change in reservation wages, expected wages, and expected mismatch resulting from changes in λ_u , λ_e , and from a simultaneous change in both arrival rates. The values are calibrated using the change in unemployment and vacancy rates between 2009 to 2013. Table 1.6 shows the values used for each example. During this expansion, λ_u increased by 54.7%, and λ_e increased by 21.1%.

As in the baseline model, higher values of λ_u increase the value of unemployment and therefore increasing reservation wages. This leads to higher expected wages and

Table 1.6.: Counterfactual job offer arrival rates.

Counterfactual	λ_u	λ_e
2009	.4284	.0299
Increase λ_u	.6627	.0299
Increase λ_e	.4284	.0362
2013	.6627	.0362

lower mismatch for those workers whose reservation wage is above their minimum wage offer.



Fig. 1.18.: Reservation wage change under counterfactual offer arrival rates.

A higher arrival rate of job offers for employed workers increases the continuation value of employment, decreasing workers' reservation wages. The affect on expected wages and expected mismatch is mixed. Those whose reservation wages fall the most experience a decline in their expected wage and an increase in expected mismatch, while the opposite occurs for workers with smaller changes in the reservation wage.

When both arrival rates are changed simultaneously, the λ_u effect dominates that of λ_e . Reservation wages rise across the board, leading to an increase in expected wages and a decrease in mismatch that is nearly identical to the effect of λ_u . How-

ever, workers whose match acceptance probability is equal to 1 in both the 2009 and increased λ_u examples see an additional affect from the change in λ_e .

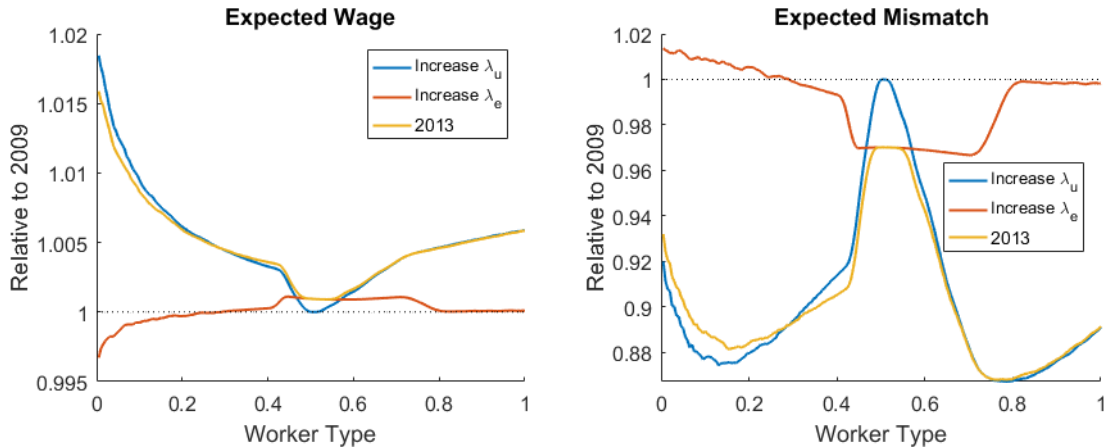


Fig. 1.19.: Changes in expected wages and mismatch under counterfactual offer arrival rates.

1.6 Conclusion

In this paper, I provide new evidence on the presence of cognitive skill mismatch in the labor market, and construct a one-sided search model to explain workers' strategies for the acceptance of skill mismatch. Using data from the NLSY97 and O*NET, I document three stylized facts relating to skill heterogeneity and mismatch. First, I show that higher-skilled workers are less likely to be unemployed, experience shorter unemployment spells, and earn higher incomes. This comes as no surprise, and provides validation that the measure of worker skills used in this paper is capturing a salient worker characteristic. Second, I show that wage dispersion is positively correlated with the worker's skill type. This suggests that each type of worker can be employed in a range of jobs, and that higher-skilled workers may be willing to match with a wider range of occupations. Finally, I construct the match sets in the occupation skill space for each type of worker. I find evidence for some degree of positive

sorting, but matching is not perfect. Higher-skilled workers match with better jobs on average, but also tolerate relatively more mismatch in order to exit unemployment. This novel fact motivates the creation of a search model with heterogeneous workers, in order to better understand the differences in worker strategies.

For the purpose of this paper, I focus only on the worker's problem. Since workers do not internalize the effects of their reservation wage strategies and mismatch tolerance on the decisions of firms, the worker's problem is sufficient to characterize workers' strategies and outcomes. In a follow up paper, I extend the model to a two-sided search framework where firms choose how many and what type of vacancies to post. Under a parameterization comparable to the one in the current paper and with a production function that take the same shape as the wage function in the one-sided model, the results on match sets and mismatch tolerance are qualitatively unchanged.

To understand the relationship between search frictions and skill mismatch tolerance, I augment the McCall [1970] model to allow for heterogeneity among both workers and firms, as well as a match-specific productivity function that is decreasing in the level of mismatch. Workers' decisions are summarized by a reservation wage that depends on the conditional distribution of wage offers as well as the offer arrival rate, separation rate, and discount rate. This model addresses mismatch in a way that models without heterogeneity or models with discrete types are unable to do. Consistent with the empirical facts, the model predicts that higher-skilled workers display in increased tolerance for mismatch. Comparing workers' expected value of employment to the value of unemployment sheds light on this counterintuitive finding; more skilled workers face a larger disparity between the value of employment to that of unemployment. This provides a strong incentive to quickly exit unemployment, leading higher-skilled workers to be relatively less selective in accepting job offers.

Although on-the-job search allows workers to move up the job ladder, it does not alleviate skill mismatch. By increasing the continuation value of employment, on-the-job search reduces the reservation wage. The net result is improved sorting among

some groups of workers, and improved employment rates (but more mismatch) among others.

The model also reveals an important role for skill heterogeneity in the interpretation of aggregate statistics such as the economy-wide average wage. In a labor market with skill heterogeneity, worker strategies vary systematically depending on their skills. Therefore, when the labor market improves after a recession, the increase in market tightness can lead to a decrease in the average wage despite the fact that expected wages increase for all types of workers. This is due to a change in the composition of employment; lower-skilled workers exit unemployment at relatively higher rates, shifting the distribution of employment toward those workers who earn lower wages. This composition effect is consistent with recent explanations for the slow growth of average wages during the recovery from the Great Recession.

While the model in the current paper provides valuable insight into workers' tolerance of skill mismatch and its effect on aggregate labor market outcomes, it also provides a framework in which to address further questions relating to skill heterogeneity and mismatch. A natural follow up is to ask how much mismatch workers should optimally accept and whether there are policies that implement the optimal allocation. Additionally, the empirical relationship between mismatch and job tenure described in Section 2 suggests an important role for endogenous separation and on-the-job search. It may be the case that workers accept high levels of mismatch but search on the job in order to find a better match. However, it may also be that workers' job offer distributions are not uniform. In particular, if each workers' offer distribution is centered around his best match, workers may accept a wide range of jobs while achieving a higher degree of sorting. Each of these modifications could produce a higher degree of sorting, allowing the model to better match the data. Finally, the present model can be extended to allow for endogenous vacancy creation by firms to address the role of worker heterogeneity and mismatch in firms' job creation decisions.

2. TARGETED SEARCH IN LABOR MARKETS WITH SKILL HETEROGENEITY

2.1 Introduction

Who meets whom in a frictional market with heterogeneous agents? While micro-data can tell us who *matches with* whom, meetings that do not result in a match are almost never observable to the researcher. Structural models of markets with frictions must therefore include assumptions about the process by which agents meet. With a few exceptions, search-and-matching models of the labor market typically assume that agents employ either a *random* or a *directed* search strategy when searching for a bilateral meeting. Under random search, the probability of meeting a specific type of firm is independent of the firm's characteristics (wage, productivity, skill requirements, etc.), and meetings are randomly drawn from the population of firms. On the other hand, directed (or competitive) search models assume that workers can observe firm characteristics prior to meeting and choose a firm type to meet without error. However, both categories of models fail to generate some important characteristics present in labor market data, implying that workers' search strategies may be neither perfectly random or directed.¹

Random search models with heterogeneity generate some positive sorting, but fall short in predicting the strength of sorting. Moreover, the distance between the expected match predicted by random search models and the average match observed in the data varies across worker types, suggesting that some types of workers search more randomly than others. On the other hand, competitive search models cannot

¹In a current working paper, Lentz and Moen [2017] propose a model that nests both random and competitive search in an effort to estimate the degree to which search is directed. While the model is promising, the estimation relies on the assumption that workers are identical and requires matched employer-employee data.

generate skill mismatch in an environment where mismatch is costly and wages are positively correlated with productivity.

This paper explores an alternative search strategy known as *targeted* search in the context of a labor market with a continuum of vertically differentiated workers and jobs. When using a targeted search strategy, workers searching for employment cannot perfectly observe a firm's type before meeting, and so are unable to choose a specific firm type to meet. However, they can expend costly search effort to reduce the probability of meeting undesirable jobs. Higher levels of search effort lead to better job offer distributions, so by choosing a level of search effort, workers control how narrowly to target their search. Numerical examples indicate that targeted search can improve the fit of a search-and-matching model to U.S. labor market data on sorting on cognitive skills relative to a random search assumption.

The process of targeted search can be thought of as obtaining some information about a job prior to making contact. For example, an unemployed worker may spend time reading job descriptions in online job ads before applying. In a labor market with an infinite variety of worker skills and job skill requirements, an afternoon spent reading job ads will likely not lead to an error-free identification of the worker's ideal job. However, it may deter him from contacting jobs for which he is obviously not well suited. If he has only a high school degree, he may choose not to spend the time and effort applying to jobs which require a master's degree. As a result, the distribution of jobs that the worker contacts will differ in a predictable way from the global distribution of vacancies posted. The density of applications to relatively well-matched jobs will rise, and the density of applications to very poor matches will fall. In this example, the cost of search effort may be the opportunity cost of time spent on an activity other than leisure, a monetary cost associated with the job search platform, a mental or emotional cost, or any combination of these. In any case, increasing search effort (either effort per unit of time or time spent searching) incurs an increased search cost but yields an improved job offer distribution.

2.1.1 Related Literature

Random search models are based on the assumption that agents are unable to locate a specific partner on the other side of the market. Therefore, contact rates between workers and firms in random search environments are independent of characteristics, and in equilibrium agents accept matches that are sub-optimal in comparison to the frictionless benchmark. Building on the one-sided random search model of McCall [1970], Buhrmann [2018] models the match acceptance decisions of vertically differentiated workers.² I show that a random search model can predict the match acceptance sets of different types of workers, and generates positive sorting on cognitive skills. However, the strength of sorting predicted by the model is less than what is observed in the data. The current paper extends this earlier model by endogenizing the degree of randomness with which workers search for jobs.

In contrast with random search, directed search assumes that agents can identify and meet a chosen partner. Introduced by Moen [1997], competitive search environments feature wage posting (with commitment) by firms with directed search by identical workers. Firms separate into submarkets according to the posted wage, and workers choose one submarket (wage) to search in³. The relative mass of firms and workers in each submarket determines the contact rate, and workers consider both the wage and the expected duration of unemployment when choosing a submarket to search in. In equilibrium, workers are indifferent across all submarkets, leading to wage dispersion among identical workers. However, this model is inconsistent with the existence of skill mismatch in an environment with productive complementarity between worker and firm skills.⁴ Shi [2002] studies a competitive search model with skilled and unskilled workers, but the unique equilibrium does not feature skill mis-

²See Marimon and Zilibotti [1999] for an example without vertical differentiation; Shimer and Smith [2000], Shimer and Smith [2001] for a model with vertical differentiation in marriage markets; Teulings and Gautier [2004] for a model with product differentiation.

³In a goods market model, Yang [2013] introduces error into the choice of submarket, calling it “targeted” search. While the model features buyers with heterogeneous preferences, it cannot address the question of sorting in a labor market.

⁴See Appendix B.1 for discussion of this statement.

match. In an equivalent environment, Shimer [2005c] generalizes the model to include continuous worker and firm heterogeneity.⁵ The model generates skill mismatch in equilibrium, but fails to predict a positive correlation between worker productivity and wages.

To address these and other shortcomings of random and directed search strategies, the literature has proposed a variety of intermediate search strategies.⁶ In a goods market setting, Lester [2011] proposes a model where buyers have heterogeneous information about prices before meeting a seller. Informed buyers direct their search to low-price sellers, while uninformed buyers choose at random, leading to price dispersion in equilibrium. A working paper by Godøy and Moen [2013] presents a related model in a labor market setting, labeling this search strategy “mixed search”. In their model, all workers search using both random and directed strategies simultaneously. The model generates realistic patterns of wage changes at job-to-job transitions, but assumes that workers are identical and therefore cannot be used to address sorting on skills.

Decreuse [2008] describes a “choosy search” strategy in a model based on Marimon and Zilibotti [1999], which allows for worker and firm types distributed on the unit circle with costly mismatch. However, Decreuse assumes that the market is segmented by job type. Rather than receiving a job offer at random from the population of vacancies, workers choose a range of job types to apply to. Workers apply to multiple submarkets at the same time, and solving the model requires that workers apply to all job types with a positive match surplus.⁷ While this model has the advantage of being able to quantify the congestion effects caused by workers searching too widely, the equilibrium match sets are equivalent to those of the random search model in Marimon and Zilibotti [1999].

⁵See Eeckhout and Kircher [2010] for a discussion of conditions required for assortative matching in a related buyer-seller model.

⁶Menzio [2007] constructs a competitive search model with firm productivity heterogeneity and lack of commitment in wage posting that he labels “partially directed” search, but the model cannot address mismatch since workers are assumed to be identical.

⁷Due to the assumption of continuous time, neither workers or firms receive simultaneous contacts.

The model of Gavrel et al. [2012] provides another variation on the horizontal differentiation model of Marimon and Zilibotti [1999]. The authors propose an environment in which search is “oriented” around a worker’s preferred firm type. Workers search randomly in a subset of the firm type space, and each worker meets one firm within a fixed, exogenous distance from his own skill type. Firms may receive multiple applications at once, and always hire the best-matched worker. All matches are assumed to be acceptable, so there is no role for a match acceptance decision by either the worker or the firm. Instead, the authors focus on the decision of firms to provide on-the-job training to reduce mismatch after hiring.

The model most closely related to that of the current paper is Cheremukhin et al. [2016]. The authors propose a search strategy called “targeted search” in which agents choose a probability distribution over potential meetings. The new search strategy is discussed in the context of a marriage market; there are F types of females and M types of males each searching for a match. With some assumptions on preferences and on the surplus created by matches, the model can be used to investigate sorting. Search costs increase as the chosen probability distribution changes away from that of the population (as measured by the Kullback-Leibler divergence), and the model is designed to understand the trade-off between a expected payoff and search costs. As expected, higher probabilities are placed on matches with higher payoffs; this is referred to as the productive motive. However, since the model is designed to address the marriage market, a key feature is the strategic importance of best responding to the agents on the opposite side of the market. Preferences of males and females may not coincide such that one’s preferred type would agree to match. Therefore, the probability distributions chosen by males play a large role in determining the optimal strategies (probability distributions) of females and vice versa. In a labor market environment where skill types are complements in production, workers and firms will always agree on whether a match is acceptable and the so-called strategic motive is not relevant.

2.2 Model

In this section, I construct a search-and-matching model that extends the theory of targeted search to a labor market setting with a continuum of vertically differentiated agents. The model nests both random and directed search as special cases, and allows each type of worker to choose the degree of randomness of his search. I build on the one-sided search framework of Buhrmann (2018) by incorporating an additional decision problem in which workers choose how narrowly to target their search. In the one-sided random search environment, a worker's reservation wage depends on the distribution of wage offers she expects to receive. In typical random search models, this distribution is determined only by the distribution of vacant jobs. However, targeted search gives workers the option to pay an information cost to improve the offer distribution.

The objective of this paper is to construct and estimate a model of targeted search in the labor market, to determine whether the generalized search process can improve the fit of the one-sided search model. In the model, workers are risk-neutral and infinitely-lived, discount the future at rate r , and maximize expected discounted utility which is linear in income. A worker can be either employed or unemployed at any time. All unemployed workers search for jobs, and there is no on-the-job search. An employed worker remains in his job until it is terminated, which occurs at a Poisson rate s . In this environment, a firm corresponds to one vacant job.

Heterogeneity. Workers are heterogeneous in skills, indexed by type $x \in [0, 1]$, such that higher x indicates a more skilled worker. While unemployed, workers receive a flow benefit $b(x)$. Firms are also heterogeneous, indexed by skill type $y \in [0, 1]$. Worker skill types x and skill requirements of vacant jobs y are distributed according to cdf's L and G , with corresponding pdf's $\ell(x)$ and $g(y)$. The distributions of unemployed and employed workers depend on the workers' decision problem, and are not necessarily equal to L ; call these distributions F and \tilde{F} , respectively. The firm

distribution G includes only vacant firms, and the distribution of vacancies is assumed to be invariant to worker behavior.

Skill Mismatch. Skill mismatch, defined as the difference between a worker's skill and the skill requirement of his occupation, is costly. Empirical literature shows that workers who are mismatched earn lower wages than those who are well-matched. Let the wage earned by worker x when employed by firm y be given by $w(x, y)$. Define $\mu = |x - y|$ as the skill mismatch of the pair. The wage function must satisfy the following properties:

1. $w(x, y) \geq 0 \forall (x, y)$
2. Given $x = x_0$, $w(x_0, y)$ quasiconcave in y
3. $w_x > 0$

Condition (1) simply imposes a minimum wage of 0. Convexity of match sets is ensured by (2); that is, if x is willing to accept jobs y_1 and y_2 , they should also accept all $y \in (y_1, y_2)$. Condition (3) ensures that higher-skilled workers have higher earning potential. Because skill mismatch is costly, a lower level of skill mismatch is always preferred. However, because job search is also costly, workers must trade off between lower mismatch (higher wages) and a higher employment rate. Condition (4) is necessary to ensure that the

Job Search. All unemployed workers receive job offers at a Poisson rate. Workers know the distribution of potential job offers (vacancies), but are unable to locate a specified job type with certainty. Offers arrive from a distribution $\Psi_{x,\eta}(y)$ with pdf $\psi_{x,\eta}(y)$, which depends on the worker's type x and his choice of search effort η . The baseline case, $\eta = 0$, corresponds to random search, so that the weighting function is constant across all y . In this case, the offer distribution is equal to the underlying vacancy distribution:

$$\psi_{x,0}(y) = g(y)$$

For search effort $\eta > 0$, increased effort corresponds to a more concentrated job offer distribution. However, as it is not possible to receive offers from job types with no vacancies, the offer distribution must be a re-weighting of the vacancy distribution. Let $\phi_{x,\eta}$ be the weighting function. The realized offer distribution is:

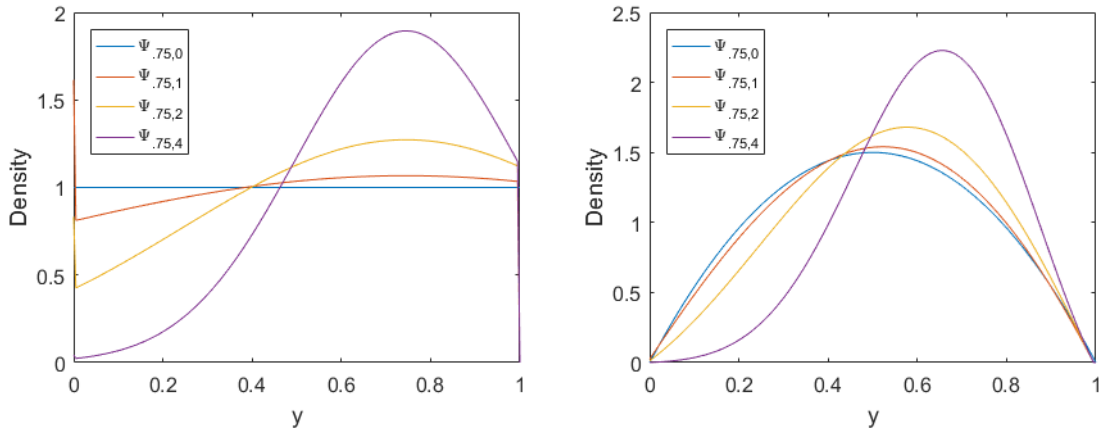
$$\psi_{x,\eta}(y) = \frac{\phi_{x,\eta}(y)g(y)}{\int \phi_{x,\eta}(y)g(y) dy}. \quad (2.1)$$

For any weighting function such that $\phi_{x,\eta}(y)$ is positive and finite for all y , the cdf of the offer distribution, $\Psi_{x,\eta}$, is a valid distribution.

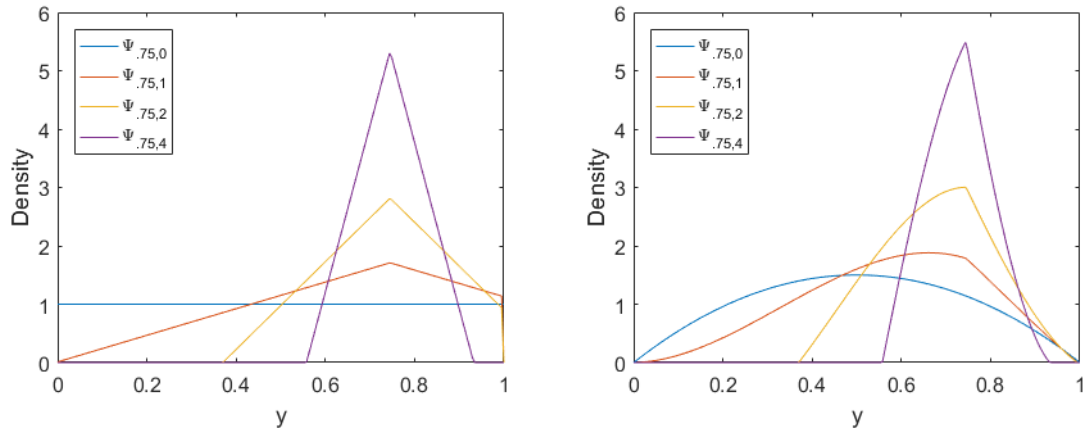
In an environment with costly skill mismatch, workers prefer to match with jobs close to their own skill type, so the weighting function must be (weakly) decreasing in the level of skill mismatch. If search effort is greater than 0, the weighting function should be symmetric and unimodal, peaking at $y = x$. Finally, the weighting function must be able to be expressed in closed-form in terms of two parameters, x and η . For example, the linear weighting function $\phi_{x,\eta}(y) = x - \eta|x - y|$ implies that the probability of contacting job y decreases linearly in skill mismatch, conditional on the underlying distribution of vacancies. On the other hand, if mismatch becomes more costly as the distance between x and y increases, it may be more relevant to use a weighting function with curvature. For example, $\phi_{x,\eta}(y) = \frac{1}{\sigma(\eta)\sqrt{2\pi}}e^{-\frac{(y-x)^2}{2\sigma(\eta)^2}}$ uses the search effort η to control the standard deviation of the weighting function. When $\sigma'(\eta) < 0$, increasing search effort decreases the variance of the offer distribution and increases the probability of meeting a good match. Figure 2.1 shows offer distributions for several example cases, for worker type $x = 0.75$.

Increasing η can be thought of as obtaining information about a job prior to applying. Rather than calling every business with a help wanted ad, a worker may take some time to read the job ad, explore the firm's website, and evaluate the possible match. Let $c(\eta)$ be the cost of choosing search effort η , satisfying:

1. $c(0) = 0$
2. $c'(\eta) \geq 0$



(a) Uniform vacancy distribution with normal distribution weighting function. (b) Beta(2,2) vacancy distribution with normal distribution weighting function.



(c) Uniform vacancy distribution with linear weighting function. (d) Beta(2,2) vacancy distribution with linear weighting function.

Fig. 2.1.: Offer distributions under different weighting functions, for $x = 0.75$.

In addition, decreasing the range of jobs to call may decrease the worker's contact rate. Let λ_0 be the contact rate of a worker choosing to search randomly. Then the contact rate for a worker choosing search effort η is $\lambda(\eta)$, where:

1. $\lambda(\eta) \leq \lambda_0 \forall \eta > 0$
2. $\lambda'(\eta) \leq 0$

The contact rate cannot be greater than that of a randomly searching worker, and must be weakly decreasing in the worker's choice of search effort.

2.2.1 Equilibrium

The equilibrium strategy for a worker of type x is to choose a search effort η and a reservation wage w^* such that a job offering wage w is accepted if and only if $w \geq w^*$, and η maximizes the worker's value of unemployment.

Value Functions. A job is defined by its firm type y , but because utility depends only on income a job can fully summarized by the wage it pays, conditional on the worker's type x . Define $U(x)$ as the value of unemployment for a type x worker and $E(x, w)$ as the value of employment for a type x worker at a job paying wage w . The worker's continuation value is equal to either $U(x)$ if the job is terminated and $E(x, y)$ otherwise.

$$E(x, w) = \frac{w + sU(x)}{r + s} \quad (2.2)$$

An unemployed worker receives flow benefits $b(x)$, and chooses η to maximize his continuation value net of search costs. To target his search, the worker must pay flow cost $c(\eta)$. He then receives wage offers at a Poisson rate $\lambda(\eta)$ from the offer distribution $\tilde{\Psi}_{x,\eta}(w|x)$. The wage offer distribution is a direct transformation of the job offer distribution $\Psi_{x,\eta}(y)$. If the wage function is invertible, $\tilde{\Psi}_{x,\eta}(w|x) = \Psi_{x,\eta}(w^{-1}(w|x))$. However, $w(x, y)$ may be a many-to-one function of y , so $w^{-1}(w|x)$ may be a relation rather than a function. Because the wage function is quasiconcave in y , the relation $w^{-1}(w|x)$ will assign at most two y values for each w . In this case, $\tilde{\Psi}_{x,\eta}(w|x) = \Psi_{x,\eta}(\max\{w^{-1}(w|x)\}) - \Psi_{x,\eta}(\min\{w^{-1}(w|x)\})$. An offer is accepted if

and only if $E(x, w) \geq U(x)$. The model does not allow for borrowing or saving, so a worker may not spend more on searching than is received in unemployment benefits.

$$U(x) = \frac{1}{1+r} \left[b(x) + \max_{\eta \geq 0} \left\{ \left(c(\eta) + (1 - \lambda(\eta))U(x) + \lambda(\eta) \int \left(\max \{E(x, w), U(x)\} \tilde{\psi}_x(w|x) dw \right) \right) \right\} \right] \left(\begin{array}{l} \text{s.t. } b(x) - c(\eta) \geq 0 \end{array} \right)$$

Define the optimal search effort $\eta^*(x)$ as that which maximizes the worker's value of unemployment subject to the budget constraint. Denote the maximized value of unemployment as $U^*(x)$. The maximized flow value of unemployment can then be written as

$$rU^*(x) = b(x) - c(\eta^*(x)) + \frac{\lambda(\eta^*(x))}{r+s} \int \left(\max \{E(x, w) - U(x), 0\} \tilde{\psi}_x^{\eta^*(x)}(w|x) dw \right) \quad (2.3)$$

Reservation Wage. Upon receiving a job offer, a worker will accept the match if and only if the present value, $E(x, w)$, is greater than that of remaining unemployed, $U(x)$. Since utility depends only on income, this induces a reservation wage strategy where a type x worker will accept all jobs offering a wage greater than $w^*(x)$ and reject those with insufficient wage offers. The reservation wage $w^*(x)$ is the wage at which the worker is indifferent between employment and unemployment, or $E(x, w^*(x)) = U^*(x)$.

$$E(x, w^*(x)) - U^*(x) = \frac{w^*(x) - rU^*(x)}{r+s} = 0$$

Hence, the reservation wage is equal to the worker's flow value of unemployment, $rU^*(x)$, and is a function of the worker's optimal choice of search effort $\eta^*(x)$.

$$w^*(x) = b(x) - c(\eta^*(x)) + \frac{\lambda(\eta^*(x))}{r+s} \int \left(\max \{w - w^*(x), 0\} \tilde{\psi}_x^{\eta^*(x)}(w|x) dw \right)$$

Using integration by parts, the reservation wage can be rewritten as:

$$w^*(x) = b(x) - c(\eta^*(x)) + \frac{\lambda(\eta^*(x))}{r+s} \int_{w^*(x)}^{\bar{w}} \left(1 - \tilde{\Psi}_x^{\eta^*(x)}(w|x) \right) dw \quad (2.4)$$

Search Effort. After defining the reservation wage, we can make use of the same integration by parts to define the optimal search effort more concisely. Search effort is chosen simultaneously with the reservation wage, so the optimal choice of search effort $\eta^*(x)$ is given by:

$$\eta^*(x) = \arg \max_{\eta \geq 0} -c(\eta) + \frac{\lambda(\eta)}{r+s} \int_{u^*(x)}^{\bar{w}} 1 - \tilde{\Psi}_{x,\eta}(w|x) dw \quad (2.5)$$

s.t. $b(x) - c(\eta) \geq 0$

When the budget constraint is non-binding, workers target their search more intensely when it is less costly to do so in terms of either $c'(\eta)$ or $\lambda'(\eta)$. Lower $c'(\eta)$ reduces the current utility cost of increasing search effort. Recall that $\lambda'(\eta) \leq 0$, so higher $\lambda'(\eta)$ implies fewer foregone offers relative to the random search benchmark. Additionally, equilibrium search effort is higher under weighting functions that are more efficient at reducing the likelihood of low wage offers. Specifically, workers target more intensely when the average change in the probability of low wage offers, $\int_{u^*(x)}^{\bar{w}} \frac{d\tilde{\Psi}_{x,\eta}(w|x)}{d\eta} dw$, is larger.

2.3 Numerical Example

In this section, I provide a numerical example to illustrate the equilibrium outcome. Workers and vacancies are uniformly distributed across the unit interval; $L(x) = G(y) = U(0, 1)$. When unemployed, workers receive benefits that increase in skill level. In particular, benefits are assumed to be proportional to workers' wage at the best matched occupation.

$$b(x) = \bar{b} \cdot w(x, x)$$

This captures the fact that higher-skilled workers have a higher outside option, while abstracting from the history-dependent UI policies that are in place today. As in Buhrmann (2018), wages depend on the skill type of the worker as well as the occupa-

tion, are increasing in worker skill, and are decreasing in skill mismatch. Specifically, wages take the form

$$w(x, y) = x - \delta(x - y)^2 \quad (2.6)$$

where δ is a skill complementarity parameter describing how important skill mismatch is in determining workers' wages.

In addition to those functions already listed, the current model requires assumptions on the cost of search effort, how search effort affects the offer arrival rate, and on the weighting function. To provide a computational illustration of the equilibrium and comparative statics of this model, I use the following functions. The utility cost of search effort must be weakly increasing, and must be continuous at $\eta = 0$. It is costless to search randomly, expending zero search effort, so I require $\lim_{\eta \rightarrow 0} c(\eta) = 0$. To reflect the idea that the primary cost of search effort is the time associated, I set search costs proportional to the flow value of leisure time. The parameter α_1 is added to allow for some nonlinearity in the cost of search effort.

$$c(\eta) = c_0 \cdot b(x) \cdot \eta^{\alpha_1}$$

The weighting function $\phi_{x,\eta}$ is assumed to be normally distributed with mean x and standard deviation $\sigma(\eta)$.

$$\phi_{x,\eta}(y) = \frac{1}{\sigma(\eta)\sqrt{2\pi}} e^{-\frac{(y-x)^2}{2(1/\sigma(\eta))^2}}$$

The standard deviation of $\phi_{x,\eta}$ should be decreasing in search effort. It must also satisfy $\lim_{\eta \rightarrow 0} \sigma(\eta) = \infty$, so that the search strategy converges to random as search effort approaches 0. This ensures that $\lim_{\eta \rightarrow 0} \psi_{x,\eta}(y) = g(y)$, and the unemployed worker's value function is continuous at $\eta = 0$. I include a scalar multiplier on search effort, α_2 , so that small values of η can produce a substantial improvement in the variance of job offers. This parameter represents the efficiency of search effort in refining the job offer distribution.

$$\sigma(\eta) = \frac{1}{\alpha_2 \eta}$$

The job offer arrival rate may be weakly decreasing in search effort, with a maximum at λ_0 , the offer arrival rate under random search. To maintain continuity of the value function, $\lim_{\eta \rightarrow 0} \lambda(\eta) = \lambda_0$. I allow the arrival rate to be nonlinear in η with a parameter α_3 , so that the marginal cost of search effort in terms of expected job offers may vary.

$$\lambda(\eta) = \min \left\{ \frac{\lambda_0}{(1 + \eta)^{\alpha_3}}, \lambda_0 \right\}$$

2.3.1 Calibration

I use data from the National Longitudinal Survey of Youth (NLSY) 1997 for the time period 2009-2013 to calculate several of the model parameters. At this time, survey respondents are between 25-33 years old. The model is calibrated at monthly frequency.

Table 2.1.: Calibrated parameter values

Parameter	Value	Target
b_1	0.4	Benefits Accuracy Measurement
r	0.001	3-month Treasury bill
s	0.0299	(Job duration) ⁻¹
λ_0	0.5010	Buhrmann (2018)
δ	0.1003	Buhrmann (2018)

The monthly interest rate r is chosen to match the average 3-month treasury bill rate from 2009-2013. The U.S. Department of Labor's Office of Unemployment Insurance releases a yearly Benefit Accuracy Measurement report containing each state's quarterly UI replacement rate. From 2009 to 2013, the weighted average U.S. replacement rate was between 0.405 and 0.470. To provide conservative estimates of the remaining parameters, $b_1 = 0.4$. The other parameters are calibrated using data from the NLSY97. The average reported job tenure is 33.50 months, leading

to a separation rate of $s = 0.0299$. The offer arrival rate λ_0 and the wage loss due to mismatch δ are jointly estimated using GMM in the random-search model of Buhrmann (2018).⁸

Parameters that have not yet been calibrated are the elasticities on search effort, α_1 , α_2 , and α_3 , and the scalar cost parameter c_0 . For the purpose of this example, I set $c_0 = 0.1$, $\alpha_1 = 4$, $\alpha_2 = 2$, and $\alpha_3 = 0$. This parameterization implies that the marginal utility cost of search effort is increasing; at these values, low levels of search effort are very inexpensive, but high search effort becomes prohibitively costly. For simplicity, the offer arrival rate is currently assumed to be constant with respect to search effort.

2.3.2 Results

In this example, all workers choose search effort strictly below the maximum and above zero, and obtain a job offer distribution with a standard deviation at or below 1. Because the cost of search effort is proportional to the worker's unemployment benefit, the maximum level of search effort (determined by the budget constraint) is the same for all workers. Figure 2.2 plots the optimal choice of search effort for workers of each skill level, as well as the resulting job offer distribution. Mid-skill workers, whose expected match under random search is close to their best match, expend the least amount of search effort. For these workers, increased targeting reduces average mismatch but does not significantly change the expected job type match. However, high- and low-skill workers have more to gain by targeting search, and accordingly expend more search effort.

The realized job offer distribution reflects workers' choice of search effort. Low- and high-skill workers obtain narrower offer distributions than workers near the middle of the skill space. Because the underlying job distribution is uniform, the shape of the composite distribution reflects that of the weighting function. Since the weight-

⁸See the appendix for a discussion of constructing worker and occupation skill measures.

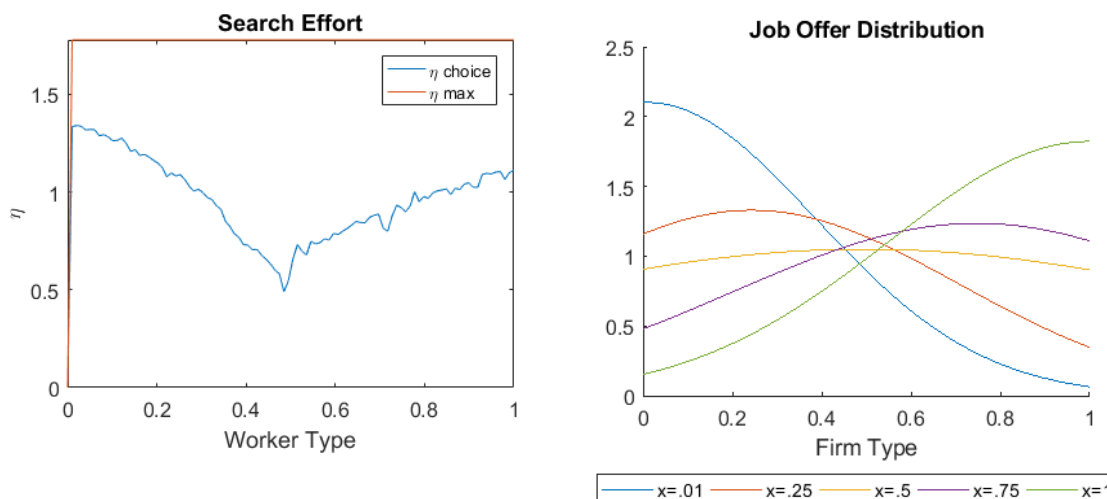


Fig. 2.2.: Optimal choice of search effort, and resulting job offer distribution.

ing function in this example is normal, each worker's offer distribution is a truncated normal distribution that peaks at the occupation corresponding to his own skill type.

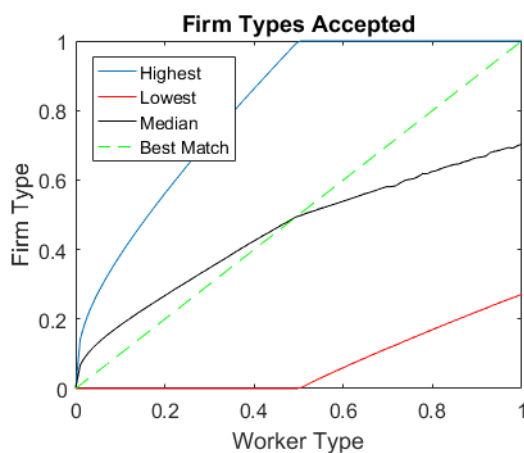


Fig. 2.3.: Optimal match acceptance sets.

Even when search is targeted, workers may meet jobs with skill requirements far from their ideal match. In this example, the highest-skilled workers may still encounter jobs with the lowest skill requirements, though the probability of such a

meeting is low relative to better matches. Workers’ decision strategy is represented by a reservation wage that determines the range of occupations with which they are willing to match. Optimal match acceptance sets are plotted in Figure 2.3. As in the random search case, bounds on match acceptance are relatively wide; the average probability of accepting a match conditional on meeting is 83.6%. As a result, the unemployment rate in this example falls to 6.84%. The random search example discussed in Buhrmann (2018) resulted in an acceptance rate of 75.6%, with a corresponding unemployment rate of 8.24%.

However, workers are slightly more selective under targeted search. Reservation wages are higher than in the random search case, causing the maximum level of mismatch tolerance to fall and match acceptance ranges to narrow for all workers. While mismatch is still widespread, the average level of expected mismatch across all worker skill types is 0.1976, a 12.14% improvement over random search. The correlation between a worker’s skill type and his expected occupation match is increased; in other words, targeted search improves sorting.

2.3.3 Comparison with U.S. labor market data

The primary data source in this paper is the NLSY97. Conducted by the Bureau of Labor Statistics, this nationally representative survey samples individuals born between 1980-1984. I restrict the analysis to the 2009-2013 waves of the survey; during this time period, respondents are 25-33 years old. To account for selective attrition, I use custom sample weights calculated over the subsample of individuals who appear in any wave between 2009-2013. Appendix A.3 discusses sample selection criteria and descriptive statistics for the relevant subsample. I obtain information on occupational skill requirements from O*NET, which provides information on 704 SOC-level occupations, and rank occupations based on the “level” requirement for a subset of cognitive skills⁹.

⁹For each occupation in O*NET, a “level” and an “importance” score are provided for each of 277 descriptors. The level score assigned to a skill indicates the degree of competency in that skill

As in the model, skill mismatch is defined as the difference between a worker’s skill type x and the skill type y of the occupation in which the worker is employed. To measure skill mismatch between a worker-occupation pair, it is necessary to assign skill types to both parties. The construction of individual and occupation skill rankings is described in more detail in Buhrmann [2018]; I summarize the empirical methodology here. To rank individuals in the NLSY97 on the basis of cognitive skills, I use principal components analysis (PCA) to combine information on cognitive test scores from the Armed Services Vocational Aptitude Battery (ASVAB) test and educational attainment.¹⁰ This method returns a ranking over individuals such that, conditional on education level, an individual with a higher ASVAB rank is assigned a higher skill type.¹¹ Occupational skill rankings are computed using PCA to combine information on two cognitive skill descriptors, “Judgment and Decision Making” and “Complex Problem Solving”, and weighting by occupational employment shares.¹²

Figure 2.4 plots the kernel density of occupation skill type matches observed in the NLSY97 data for representative worker skill types. The match densities are clearly not uniform, indicating that workers meet some types of jobs more often than others. This alone strongly suggests that job search is not random. In addition, match densities are roughly single-peaked near the worker’s best match, indicating that the probability of meeting a specific job falls as the level of mismatch increases. Both of these characteristics suggest that targeted search may provide an improvement on the model’s ability to match the data.

To determine whether targeted search can actually improve the fit of the model, I compute the Kullback-Leibler divergence of the predicted match density of each of

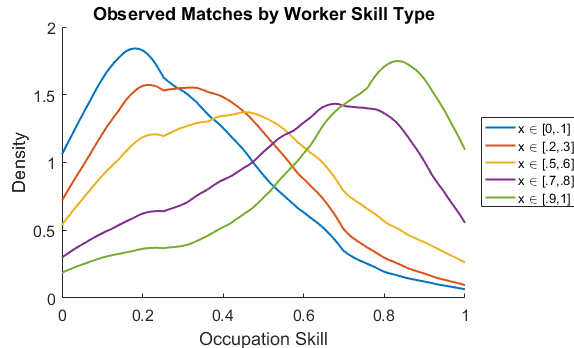
needed to succeed at the occupation; the importance score describes how essential the skill is to the occupation.

¹⁰The ASVAB consists of 12 component sections, over skills both abstract and practical. Four categories, mathematics knowledge, arithmetic reasoning, paragraph comprehension, and word knowledge, are considered “cognitive”.

¹¹Including education level in the construction of worker skills is a divergence from recent empirical literature that increases the correlation between worker skill and wage and decreases average mismatch.

¹²The analysis is robust to other methods of ranking, see the data appendix of Buhrmann [2018] for a full discussion.

Fig. 2.4.: Density of matches by worker skill type.



the models from the empirical density. Because of the size of the dataset, I divide the skill space into 20 bins of size 0.05 and construct an empirical match density for each skill bin.¹³ On average, each bin contains data on 271.4 worker-occupation matches. For each of the 20 bins, I calculate the divergence of each model predicted density from the empirical density as

$$KL_n^j = \sum_y \left(M_n^j(y) \log \left(\frac{M_n^j(y)}{M_n^{data}(y)} \right) \right)$$

where n indicates the skill bin, j indicates the model (targeted or random), $M_n^j(y)$ is the model-generated expected match density over occupations for workers in bin n . Lower values mean that the model-predicted density is less distant from the empirical density. For a skill bin n , the improvement (or lack thereof) of targeted search relative to random search is $KL_n^{targeted}/KL_n^{random}$. To quantify the improvement of the targeted search model relative to random search, I average this ratio of across all skill bins.

$$\frac{1}{20} \sum_n \frac{KL_n^{targeted}}{KL_n^{random}}$$

In the current example, targeted search provides a 7.8% improvement over random search. Since the model parameters were not re-estimated under targeted search, this is only an illustration of the improvement that targeted search can provide.

¹³The result is robust to a bin size of 0.01 as well, but at this level of granularity the empirical density functions are much more noisy.

Estimating or calibrating the new parameters $c_0, \alpha_1, \alpha_2, \alpha_3$ will yield a more accurate estimate of the actual improvement provided by allowing workers to target their search. However, the parameters of the random search model used for comparison here was estimated using GMM to obtain the best possible fit. Though it is an uncalibrated example, the targeted search model is shown to provide a better fit.

2.4 Conclusion

This paper proposes an application of the targeted search strategy introduced by Cheremukhin et al. [2016] to a labor market where agents are continuously vertically differentiated in skills. Targeted search is a generalization of random search where bilateral meetings occur randomly, but the distribution from which meetings are drawn depends on the agent's type. In particular, in this model focusing on the workers' decision problem, workers have the ability to reduce the variance of the job offer distribution by expending search effort. However, targeting one's search incurs a cost; workers choose an optimal level of search effort to balance the gains from improved matching with the utility cost of search effort. The result is a job offer distribution that is systematically different from the underlying distribution of vacancies in the economy.

In a numerical example, targeted search is shown to generate mismatch tolerance strategies that are qualitatively similar to the equilibrium under random search. However, workers have an additional choice variable at their disposal in the current model: search effort. The optimal level of search effort varies with workers' skill type. High- and low-skilled workers, whose expected match under random search is furthest from the wage-maximizing match, choose higher levels of search effort in order to obtain narrower job offer distributions. Workers near the middle of the skill space, whose expected mismatch under random search is lower, choose lower levels of search effort in equilibrium.

Compared to random search, targeted search provides an improvement in the fit of the model to the data on match densities and sorting. The biggest improvement is among higher-skilled workers, where random search clearly failed to produce the strength of positive sorting observed in the data. When averaged across all worker skill groups, the uncalibrated targeted search example discussed here fits the data on match densities 7.8% better than the random search model estimated using GMM, illustrating the potential of targeted search models to help researchers better understand the imperfect but positive sorting on skills that arises in the labor market.

3. SKILL MISMATCH AND THE EQUILIBRIUM DISTRIBUTION OF VACANCIES

3.1 Introduction

When the Great Recession officially ended in June 2009, the U.S. unemployment rate was at 9.5%; after peaking in October 2009, unemployment began to fall before finally reaching pre-recession levels in late 2015. During this time, the job vacancy rate started to increase and the economy moved upward along the Beveridge Curve. However, decreases in the unemployment rate fell behind the increasing vacancy rate, so that a given vacancy rate corresponded to a higher unemployment rate post-recession. This apparent outward shift in the Beveridge Curve suggests that the aggregate vacancy and unemployment rates are not sufficient statistics for understanding the status of a labor market. Job finding rates, expected duration of vacancies, expected match quality, and even wages depend on the distributions of unemployed workers and vacant jobs. How do these distributions arise? In particular, how does the vacancy creation decision respond to the distribution of skills offered by workers?

This paper seeks to understand how skill heterogeneity and mismatch affect occupational vacancy rates and match distributions. Using the Diamond [1982], Mortensen [1982], Pissarides [1985] (DMP) framework of time-consuming search by workers and firms, the paper develops a model with costly skill mismatch and endogenous job creation. Workers and jobs are heterogeneous and vertically differentiated in skills. Pair-specific skill mismatch is defined as the distance between a worker's skill type and that of the job in which he is employed. Unemployed workers and firms with vacant jobs search for a match; upon meeting, the worker and firm jointly decide whether to accept the match or to continue searching. A worker's skill type is fixed when he

enters the labor market, but a firm is able to choose the skill type of any vacancy it opens. Solving the model computationally, I obtain the equilibrium distribution of vacancies across the skill space. The model generates new predictions regarding the response of the vacancy distribution to the distribution of skills offered by unemployed workers, as well as to labor market parameters such as bargaining power and matching elasticity. Skill mismatch also plays an important role in shaping the vacancy distribution. Higher levels of expected mismatch reduce the expected value of a vacancy, suppressing vacancy creation.

The model is calibrated to match U.S. labor market data from 2009 to 2013. The calibrated model yields the equilibrium unemployment rates for each type of worker, vacancy rates for each type of job, and the wage for each worker-job pair. Results show a nonlinear relationship between skill level and mismatch tolerance; like the one-sided model in Buhrmann [2018], highly productive workers and firms accept relatively more mismatch than those near the middle of the skill distribution. The distribution of vacancies generated by the model is non-uniform. Very few jobs are created in low-skilled occupations, despite the supply of low-skill unemployed workers. The vacancy rate is highest among mid-skill occupations near the skill type of the median unemployed worker, because workers of almost all skill types can profitably match with these jobs and the probability of an acceptable match is high. Despite the relatively low numbers of high-skill unemployed workers, jobs in high-skill occupations continue to be created due to the profitability of a good match.

Skill mismatch is shown to be counter-cyclical. Intuitively, workers and firms tolerate less mismatch when the labor market improves. Comparing the equilibrium outcome between 2009 to 2013, this is exactly what occurs. Like in Buhrmann [2018], all workers become more selective in matching, with high-skilled workers responding more strongly to changes in labor market conditions. The corresponding improvement in expected match quality for high-skilled jobs leads to a shift toward higher-skilled occupations in the vacancy distribution.

The remainder of the paper is organized as follows. Section 2 describes the model and defines the equilibrium. Section 3 discusses the calibrated numerical example, illustrates the effect of a change in the underlying distribution of workers, and plots comparative statics. Section 4 concludes.

3.1.1 Related Literature

One of the earliest papers to explore the relationship between skill mismatch and job creation is Acemoglu [1999]. In a model with two types of workers, skilled and unskilled, the paper shows that the composition of jobs is key in determining wages, inequality, and unemployment rates, and that the composition of workers strongly influences the distribution of jobs. In a related model, Albrecht and Vroman [2002] generate skill mismatch by assuming that unskilled workers are only able to perform low-skill jobs, while skilled workers can be employed in either high-skill or low-skill jobs. In this context, mismatch is present when skilled workers are willing to accept low-skill jobs. Blázquez and Jansen [2008] show that the equilibrium is inefficient; too many skilled workers are employed in low-skill jobs, and firms create too many high-skill vacancies. This suggests an important role for skill mismatch in determining the distribution of jobs that will be created. Using a continuous characterization of skill heterogeneity, the model in this paper provides richer predictions for mismatch tolerance and its implications for vacancy creation.

In a marriage market setting, Shimer and Smith [2000] and Shimer and Smith [2001] propose a model with two-sided search and continuous, vertically differentiated agents, as in the current paper. The model is loosely based on the DMP search and matching environment, modified such that there is only one group of agents. The authors impose a log-supermodular production function that does not directly incorporate a cost of mismatch. Under this production function, the vacancy distribution would be degenerate if it were endogenous; all firms would choose to create only the highest skill type of job. However, the authors focus on assortative matching

by assuming a fixed distribution of agents, so the model cannot be used to address the relationship between mismatch tolerance and job creation.

Teulings and Gautier [2004] extend the Shimer-Smith model to a labor market setting with two distinct groups of agents, and specify an increasing returns to scale matching process in order to approximate the decentralized equilibrium solution. To obtain a non-degenerate vacancy distribution, the model assumes that each type of job produces a different output good, with prices set endogenously in commodity markets. However, the productivity function is restricted to be log supermodular, so skill mismatch still has no impact on match productivity; every worker prefers to match with the highest job type, and every job prefers to hire the highest worker type. The empirical literature on skill mismatch has since shown that this is not likely to be the case, since mismatched workers earn lower wages than their equally-skilled but well-matched counterparts. I account for this in the current model by estimating a production function with a direct penalty for skill mismatch, following Lise and Postel-Vinay [2015] and Gautier and Teulings [2015], among others.

In a related environment with worker and job heterogeneity and on-the-job search, Lise and Robin [2017] use U.S. aggregate data from 1951-2012 to estimate a stochastic model. The estimated production function is consistent with the skill complementarity imposed in this paper. The model fits the targeted moments and long-run trends, but predicts that the skill mismatch is procyclical. That is, workers and firms become more selective in accepting matches during recessions. However, a key assumption used to solve the model is that wages are set by firms Bertrand competing to hire workers. As a result, low-type firms cannot afford to retain high-type workers when the aggregate state is bad, leading to a mass of separations. While the data does support an increased separation rate during recessions, Daly and Hobijn [2016] show that these layoffs primarily affect low-wage workers. Under Nash bargaining, the current model finds that mismatch is countercyclical and that the changes in average mismatch are primarily driven by changes in the mismatch tolerance of high-type workers.

3.2 Model

I consider a continuous-time, infinite-horizon model with a labor force of measure 1, augmenting the DMP search and matching environment to allow for heterogeneity on both sides of the labor market in the style of Shimer and Smith [2000].

Heterogeneity. Workers are risk-neutral and infinitely-lived. They discount the future at rate r and maximize expected discounted income. A worker can be either employed or unemployed at any time; all unemployed workers search for jobs, and there is no on-the-job search. Workers are heterogeneous in skills, indexed by type $x \in [0, 1]$, and vertically differentiated such that higher x indicates a more skilled worker. Skills are known, determined at the beginning of life, and fixed for the worker's lifetime. The distribution of worker skills in the economy is given by $L(x)$, with associated density function $\ell(x)$. The distribution of unemployed workers is denoted by $F(x)$ (with density $f(x)$), and is endogenous. The proportion of type x workers who are currently unemployed is given by the unemployment rate $u(x) = \bar{u} \frac{f(x)}{\ell(x)}$, where $\bar{u} = \int_0^1 u(x)\ell(x) dx$ is the aggregate unemployment rate.

A firm corresponds to one job¹, and may be in one of two states: matched or vacant. Matched firms are those currently employing a worker, while vacant firms are those currently searching for a worker. Jobs are heterogeneous and vertically differentiated, and indexed by $y \in [0, 1]$ representing the job's skill requirements. The measure of firms in the economy is endogenous, as is the distribution of jobs in the skill space. Let $G(y)$ denote the distribution of vacant jobs. The vacancy rate for jobs of type y is $v(y) = \bar{v} \frac{g(y)}{\ell(y)}$, where $\bar{v} = \int_0^1 v(y)\ell(y) dy$ is the aggregate vacancy rate. Firms pay a flow cost $c(y)$ to maintain a vacancy, where $c(y)$ is a continuously differentiable function. This cost can be thought of as the cost of advertising the vacancy, interviewing potential workers, and training the new employee.

Match Productivity. Once filled, all jobs produce the same output good. However, the quantity of output produced varies based on the skill types of both the

¹“Firm” and “job” will be used interchangeably.

worker and the job. Let the quantity of output produced by worker x when employed by job y be given by $\rho(x, y)$; this is the match productivity. The production function ρ must satisfy the following properties:

1. $\rho(x, y) \geq 0 \forall (x, y)$
2. Given $x = x_0$, $\rho(x_0, y)$ quasiconcave in y ; similarly, given $y = y_0$, $\rho(x, y_0)$ quasiconcave in x .

Condition (1) simply imposes a minimum match output of 0. Condition (2) ensures convexity of match sets. That is, if match (x_0, y_1) is acceptable and match (x_0, y_2) is acceptable, then (x_0, y) must also be acceptable for all $y \in (y_1, y_2)$. Similarly, if job y_0 is willing to match with workers x_1 and x_2 , y_0 must also be willing to match with all $x \in (x_1, x_2)$.

Job Search. Unemployed workers receive a flow benefit $b(x)$ while searching for a job. Unemployed workers and vacant firms know the distribution of agents on the other side of the market at all times, but are unable to determine the skill type of a particular agent prior to meeting. Firms and workers meet for “interviews” according to a matching function, and must jointly decide whether to accept the match or to continue search.

Let the number of meetings in any period be given by $m(\bar{u}, \bar{v})$, where \bar{u} and \bar{v} are the total measures of unemployed workers and vacant jobs, respectively. Equivalently, \bar{u} and \bar{v} can be thought of as the aggregate unemployment and vacancy rates, since the economy has a unit measure of workers. The matching function exhibits constant returns to scale, and satisfies the standard assumptions:

1. m is increasing in both the measure of unemployed workers and the measure of vacant jobs.
2. m is concave.
3. m satisfies constant returns to scale.

$$4. m(0, \bar{v}) = m(\bar{u}, 0) = 0$$

$$5. m(\bar{u}, \bar{v}) \leq \min\{\bar{u}, \bar{v}\}$$

Let $\theta = \frac{\bar{v}}{\bar{u}}$ be the market tightness. Then the probability that a vacant type y job receives an interview in any period is

$$\frac{m(\bar{u}, \bar{v})}{\bar{v}} = q(\theta),$$

and the probability than an unemployed worker receives an interview is

$$\frac{m(\bar{u}, \bar{v})}{\bar{u}} = \theta q(\theta).$$

Matches are randomly drawn from the distribution of agents on the other side of the market, so the probability of a specific worker matching for an interview with a specific job is constant across all pairs of currently unemployed workers and currently vacant firms. Therefore, workers of all types are equally likely to encounter a job of type y . However, the probability of a worker meeting a job of type y depends on the relative measure of vacancies of type y , and therefore is not equal for all y . The probability of an unemployed worker encountering a job of type y is:

$$P^w(y) = \theta q(\theta) g(y) \tag{3.1}$$

Similarly, the probability of a vacant firm encountering a worker of type x is:

$$P^f(x) = q(\theta) f(x) \tag{3.2}$$

These equations reflect the realization of a random search process; conditional on a worker (firm) receiving an interview, the probability of meeting a type y job (type x worker) must be equal to the proportion of type y vacancies (unemployed workers of type x) in the economy.

Finally, matches are terminated according to a Poisson process with arrival rate s , which is constant across all worker-job type pairs. When a match is terminated, the worker becomes unemployed and must search for a new job.

3.2.1 Equilibrium

The equilibrium outcome is characterized by the set of unemployment rates for each type of worker, vacancy rate for each type of job, and wages for all worker-job pairs. The steady-state equilibrium allocation can be found using the set of worker and firm value functions, together with a steady-state condition on the unemployment rate.

The employed worker's value function depends only on the wage, the value of unemployment, and the parameters r and s .

$$E(x, y) = \frac{w(x, y) + s \cdot U(x)}{r + s}$$

Since the contact rate varies across the job type y , the expected value of continued search must be weighted according to the distribution of vacant jobs.

$$\begin{aligned} rU(x) &= b(x) + \int_0^1 P^w(y) \cdot \max\{E(x, y) - U(x), 0\} dy \\ rU(x) &= b(x) + \frac{\theta q(\theta)}{r + s} \int_0^1 g(y) \cdot \max\{w(x, y) - rU(x), 0\} dy \end{aligned} \quad (3.3)$$

The firm's value functions are similar to those in the Pissarides model, since the structure of the labor market is essentially the same. I first derive the value function for a firm with a type y job employing a worker of type x , denoted by $J(x, y)$. $V(y)$ represents the value function of a firm with a type y job vacancy.

$$J(x, y) = \frac{1}{1 + r} \left[\rho(x, y) - w(x, y) + s \left[\max_y V(y) \right] + (1 - s)J(x, y) \right] \left($$

Rearranging the above equation yields a value function that is identical to that of the Pissarides model.

$$J(x, y) = \frac{\rho(x, y) - w(x, y) + s [\max_y V(y)]}{r + s}$$

The value of opening a vacancy of type y depends on the cost of opening the vacancy, $c(y)$, the expected time until the vacancy is filled, and the expected value of the filled job.

$$V(y) = \frac{1}{1 + r} \left[-c(y) + (1 - q(\theta)) V(y) + \int_0^1 P^f(x) \cdot \max\{V(y), J(x, y)\} dx \right] \left($$

Firms are able to freely enter the market, opening any type of vacancy at any time. Therefore, vacancies in each type of job will continue to open until it is no longer profitable to do so; the value of holding an open vacancy will fall to zero for all types. The vacancy supply condition is then given by

$$\forall y : c(y) = \frac{q(\theta)}{r+s} \int_{\alpha}^{\beta} f(x) \cdot \max\{\rho(x, y) - w(x, y), 0\} dx \quad (3.4)$$

Each job type has a distinct vacancy supply condition. It depends not only on the firm's cost of opening a vacancy, but also on the distribution of matching probabilities and expected profits across available worker types. Firms may create fewer vacancies in regions of the type space where the probability of being matched with an acceptable worker is low, and more vacancies in regions where the probability of hiring a profitable worker is higher. Therefore, the unemployment rates of different types of workers will affect vacancy creation rates differently, depending on how profitable the match would be.

Wage Determination. Wages are determined by generalized Nash bargaining, with the worker's bargaining power equal to β . Hence, the total surplus $S(x, y) = [E(x, y) - U(x)] + [J(x, y) - V(y)]$ is shared between the worker and the firm such that

$$E(x, y) - U(x) = \frac{\beta}{1-\beta} (J(x, y) - V(y))$$

Nash bargaining implies that wages are set such that the interests of the worker and the firm are aligned. Therefore, a worker and firm matched for an interview will always agree on whether or not the match is acceptable. In particular, matches will be accepted if and only if $E(x, y) - U(x) > 0$ (or equivalently, $J(x, y) > 0$). Imposing the free entry condition $V(y) = 0$ and simplifying,

$$\begin{aligned} \frac{w(x, y) - rU(x)}{r+s} &= \frac{\beta}{1-\beta} \cdot \frac{\rho(x, y) - w(x, y)}{r+s} \\ w(x, y) &= \beta\rho(x, y) + (1-\beta)rU(x) \end{aligned} \quad (3.5)$$

Substitution yields the following wage setting condition.

$$w(x, y) = \beta \rho(x, y) + (1 - \beta) \left[b(x) + \frac{\theta q(\theta)}{r + s} \int_{\underline{a}}^x g(y) \cdot \max\{w(x, y) - rU(x), 0\} dy \right] \quad (3.6)$$

It is clear that wages depend on the types of both the worker and the job, since the wage of a pair is increasing in the match-specific productivity.

In steady state, flows into and out of unemployment must be equal for each type of worker; $\forall x$:

$$\dot{u}(x) = s(1 - u(x)) - u(x)\theta q(\theta) \int_{\underline{a}}^x g(y) \cdot \mathbb{1}(x, y) dy = 0 \quad (3.7)$$

where

$$\begin{aligned} \mathbb{1}(x, y) &= 1 \text{ if } E(x, y) - U(x) > 0 \\ &= 0 \text{ otherwise} \end{aligned}$$

and indicates whether the worker would accept a job y if matched for an interview.

The steady-state equilibrium is defined by $(\{w^*(x, y)\}_{x, y \in [0, 1]}, \{u^*(x)\}_{x \in [0, 1]}, \{v^*(y)\}_{y \in [0, 1]})$. While the above equations implicitly define the equilibrium allocation, it is not possible to solve analytically for the equilibrium values. In the Section 3.3, I calibrate the model to provide a numerical example of the equilibrium outcome.

Proposition 1. Equations (3.6), (3.4), and (3.7) define a steady-state equilibrium.

Proposition 2. There is a unique solution for $\{w^*(x, y)\}_{x \in [0, 1]}, \{u^*(x)\}_{x \in [0, 1]}$, \bar{u}^* , and \bar{v}^* . The set of individual vacancy rates $\{v^*(y)\}_{y \in [0, 1]}$ is unique if and only if for all pairs of distinct job types (y_j, y_{-j}) ,

$$((\bar{y}^*)^{-1}(y_j), (\underline{y}^*)^{-1}(y_j)) \neq ((\bar{y}^*)^{-1}(y_{-j}), (\underline{y}^*)^{-1}(y_{-j}))$$

That is, equilibrium vacancy rates are unique if and only if the range of acceptable matches for job type y_j is different from the range of acceptable matches for job type y_{-j} for all pairs of distinct job types (y_j, y_{-j}) . Otherwise, there is a continuum of equilibria, each with a different vacancy distribution $G^*(y)$.

See C.1.1 for a discussion of this proposition.

3.3 Numerical Example

To illustrate the equilibrium defined by equations 3.4, 3.6 and, 3.7 I solve the model numerically. Unemployment benefits are linear in x : $b(x) = b_0 + b_1x$. The meeting function is Cobb-Douglas with elasticity parameter α : $M(\bar{u}, \bar{v}) = A\bar{u}^\alpha\bar{v}^{1-\alpha}$. Match specific productivity is given by $\rho(x, y) = \max\{x - \delta(x - y)^2, 0\}$. Workers are distributed according to $L(x) \sim U(0, 1)$. The distributions of vacant jobs, $G(y)$, and of unemployed workers, $F(x)$, are endogenously determined.

3.3.1 Calibration

I use the 2009 to 2013 waves of the National Longitudinal Survey of Youth 1997 to calibrate several parameters for this example; at this time, respondents are between the ages of 25-33 years old. A time period in this model is taken to be one month. Calibrated parameter values used for the numerical example are presented in Table 3.1.

Table 3.1.: Calibrated parameter values

Parameter	Value	Source
b_0	0	(Normalization)
b_1	0.4	Benefits Accuracy Measurement
r	0.001	3-month Treasury bill
s	0.0299	(Job duration) ⁻¹
A	0.9793	Buhrmann (2018)
δ	0.1003	Buhrmann (2018)
α	0.72	Shimer (2005)
β	0.72	Shimer (2005)

For simplicity, workers have no base value of leisure; b_0 is set to 0. The monthly interest rate r is chosen to match the average 3-month treasury bill rate from 2009-

2013. Other parameters are calibrated using data from the NLSY97 as described in this section. The average reported job tenure is 33.50 months, leading to a separation rate of $s = 0.0299$. The U.S. Department of Labor’s Office of Unemployment Insurance releases a yearly Benefit Accuracy Measurement report containing each state’s quarterly UI replacement rate. From 2009 to 2013, the weighted average U.S. replacement rate was between 0.405 and 0.470. To provide conservative estimates of the remaining parameters, I set $b_1 = 0.4$.

The worker’s bargaining power β and the matching function elasticity α are taken from Shimer [2005a]. The scaling parameter A on the meeting function can be calculated using the job offer arrival rate for unemployed workers. Buhrmann [2018] estimates this at $\lambda = 0.5285$. Using the fact that $\lambda = \frac{M}{u} = A \left(\frac{\bar{y}}{\bar{w}}\right)^{1-\alpha}$, I find $A = 0.9793$. The productivity loss from mismatch, δ , is also taken from the estimation in Buhrmann [2018].

3.3.2 Results

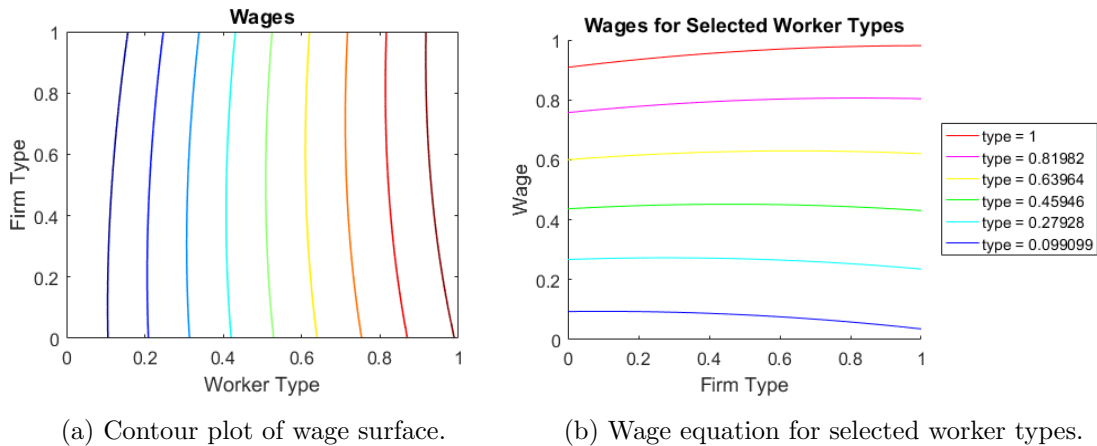


Fig. 3.1.: Equilibrium wage equation.

The equilibrium wage for all worker-job pairs is shown as a contour plot in Figure 3.1a. Wages in this model are consistent with the theoretical results of Eeckhout

and Kircher [2011]; for a given worker type, wages are concave in job type. Figure 3.1b highlights this shape for selected worker types. As in the one-sided model, the upper bound of the wage distribution is increasing in the worker's type, and the lower bound of the wage distribution is non-monotonic in worker type. The intuition for this result carries over; workers at the top of the skill distribution are less productive in lower-skill jobs than workers closer to the average skill type.

Matches are acceptable to both the worker and the firm if and only if the match surplus is positive for both parties. This occurs exactly when the productivity of the match is higher than the bargained wage. Matches such that $\rho(x, y) < w(x, y)$ will be rejected, and both the worker and the firm will continue to search for a new match in the next period. Figure 3.2a plots the worker-job pairs for which wages are equal to productivity; this bounds the set of acceptable matches. In addition, I plot the expected occupation type match for each worker type. This line is (weakly) upward-sloping, indicating that some degree of positive sorting is present. Higher-skilled workers are on average matched in higher-skilled occupations, but mismatch is widespread. In particular, mismatch is highest for workers near the both ends of the skill space; mid-skill workers experience lower mismatch on average despite accepting matches with almost all types of jobs. The reason for this is twofold. First, these workers have more opportunities for relatively low-mismatch jobs simply because they are near the middle of the skill space. Second, in this example more vacancies are created at mid-skill jobs, where the probability of an acceptable match is high. Conditional on being employed, average expected mismatch across all workers is 0.2186. That is, the average worker is employed in an occupation just over two deciles from his ideal match. Aggregate output, calculated as

$$\text{output} = \int_0^1 (1 - u(x)) \left(\int_0^1 \mathbb{1}(x, y) \rho(x, y) g(y) dy \right) dx$$

is equal to 0.4575 in this example.

Skill mismatch affects not only the worker's wage, but the firm's profitability as well. Figure 3.3 shows a contour plot of match profitability for matches with positive surplus. Matches on the diagonal, where $x = y$ are more profitable than off-diagonal

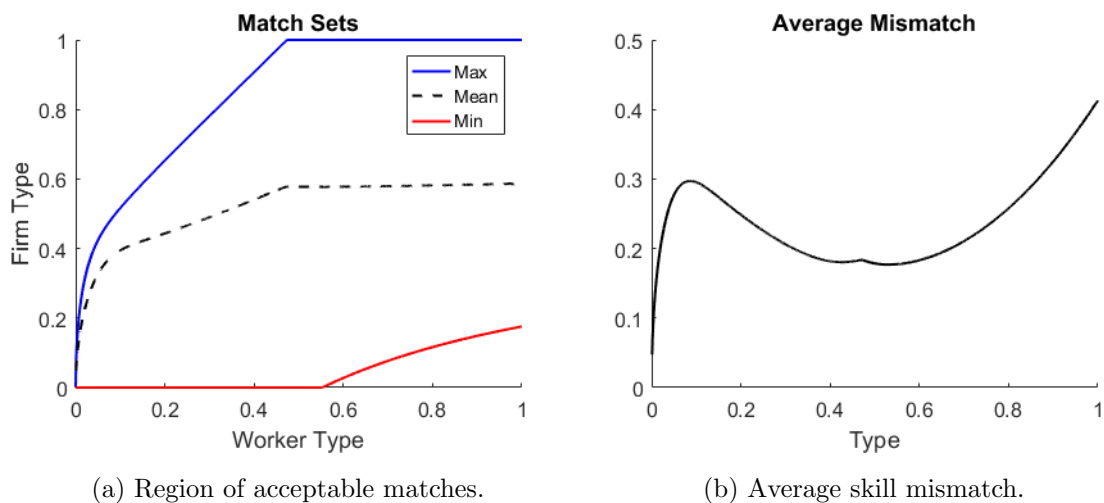


Fig. 3.2.: Matches and mismatch in the calibrated example.

matches. Profit is more dependent on the skill type of the worker than on the firm's type; for a firm, hiring a more skilled worker is usually (but not always) preferred. The most profitable matches are those between high-skilled workers and high-skill occupations.

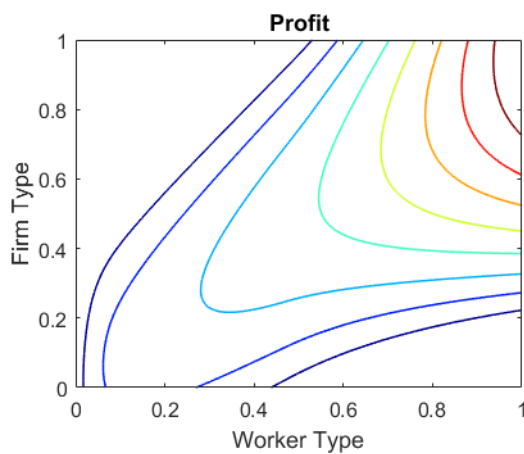


Fig. 3.3.: Match profitability.

The aggregate unemployment rate is 8.71%, slightly above the 8.24% rate in the NLSY97 data. At 2.46%, the equilibrium vacancy rate is also quite close to the



Fig. 3.4.: Unemployment and vacancy rates.

target rate of 2.4% from JOLTS data. Equilibrium unemployment and vacancy rates disaggregated by worker/occupation skill type are shown in Figure 3.4. As with the equilibrium in the one-sided model, workers near the low end of the skill space are more likely to be unemployed, and the unemployment rate is (weakly) decreasing in the worker’s skill level. The primary feature of this model is its ability to generate an equilibrium distribution of vacancies across the skill space. In this example, the vacancy distribution is non-uniform. Although the lowest-skilled workers have the highest unemployment rates, the low profitability of these occupations leads to low job creation for jobs with skill requirements near 0. Instead, the density of vacant jobs is higher near the middle of the skill space. Firms’ job creation decision strikes a balance between the vacancy filling rate and the expected profitability of a match.

3.3.3 Mismatch Over the Business Cycle

Buhrmann [2018] shows that in the absence of any changes in the vacancy distribution, mismatch falls and positive sorting becomes stronger when the labor market improves. Does this result change when firms have a say in matching? To understand the cyclicity of mismatch and sorting, I compare two new parameterizations representing the years 2009 and 2013. In 2009, the Great Recession was coming to an

end, but the labor market was far from recovered. Unemployment peaked at 10% in October 2009, shortly after the vacancy rate bottomed out at 1.7% in August; average unemployment and vacancy rates over the year were 9.5% and 1.83%, respectively. By contrast, the average vacancy rate in 2013 was 2.83%, and unemployment fell to 6.07%. Using these values, I impute two new values for A , representing the change in labor market tightness: $A_{2009} = 0.6514$ and $A_{2013} = 0.8271$. Figure shows the equilibrium outcome in each case, with 2009 represented by dashed lines and 2013 represented by solid lines.

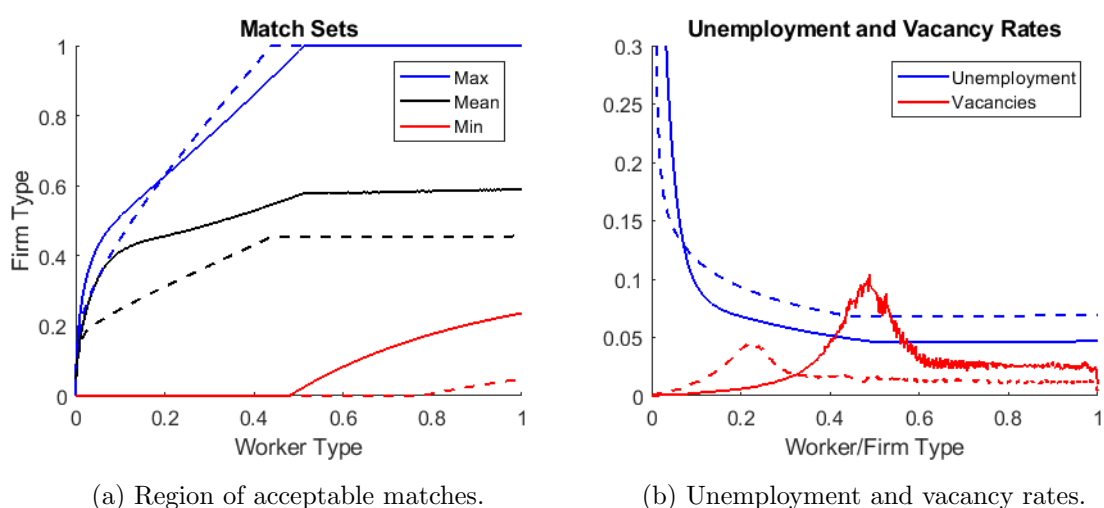


Fig. 3.5.: Equilibrium outcomes in 2009 versus 2013.

Just as in the one-sided search model, workers (and firms) become more selective when the labor market improves. This is especially true among high-skilled workers, whose response to change in labor market conditions is much stronger than that of low-skilled workers. In the 2009 example, average mismatch among employed workers is 0.2447 and aggregate output is 0.4500. In the 2013 case, average mismatch falls to 0.2150, a decrease of almost 13%, while output increases by just over 3% to 0.4636. The vacancy distribution shifts noticeably to the right; although there are fewer high-skilled workers unemployed, the reduction in mismatch tolerance boosts the expected match quality of high-skilled jobs making them more attractive to create.

Relative mismatch tolerance of different types of workers has strong implications for the types of jobs that will be created, illustrating the importance of allowing for vertical differentiation of skills in a model of job creation.

3.3.4 Changes in the Distribution of Worker Skills

What happens when the underlying distribution of workers in the economy changes? We would expect the equilibrium outcome, particularly the distribution of jobs created, to depend on the available pool of worker skills. The skill pool may change over time if, for example, education rates change, previously productive skills become obsolete, or workers systematically enter or leave the labor force. A key feature of this model is the ability to generate a distribution of vacancies that responds to labor market parameters and, importantly, to the distribution of workers. As an example, consider a change in the distribution such that the labor supply of low-skilled workers increases relative to that of high-skilled workers. Specifically, let

$$\begin{aligned} L(x) &= \frac{4}{3}x && \text{for } 0 \leq x \leq 0.5 \\ &= \frac{1}{3} + \frac{2}{3}x && \text{for } 0.5 < x \leq 1 \end{aligned}$$

be the new distribution of worker skills; the distribution is plotted in Figure 3.6.

In this example, aggregate unemployment and vacancy rates remain close to the levels in the calibrated example; equilibrium rates are 8.49% and 2.39%, respectively. However, since there are now twice as many low-skilled workers as there are high-skilled, we should expect that firms will create more vacancies at occupations suited for low-skilled workers. Although low-skilled jobs are less profitable overall than high-skilled jobs, more vacancies should be created in this range due to the increased supply of workers. Figure 3.7 shows exactly that. The vacancy distribution shifts to the left, with lower-skilled occupations opening more vacancies per worker than before. The vacancy rate at higher-skilled occupations remains the same, but plotting job creation in levels emphasizes the difference; there is a sharp drop in the number of jobs created in the range $y > 0.5$.

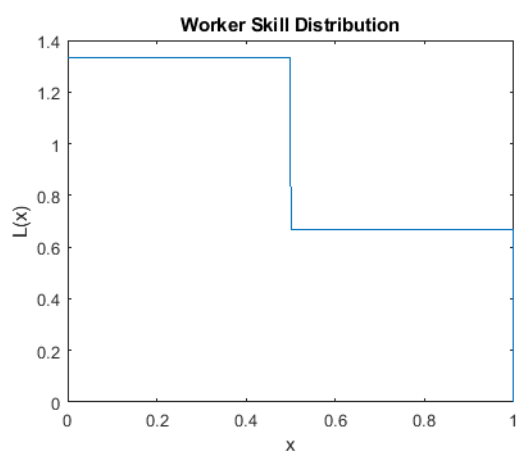
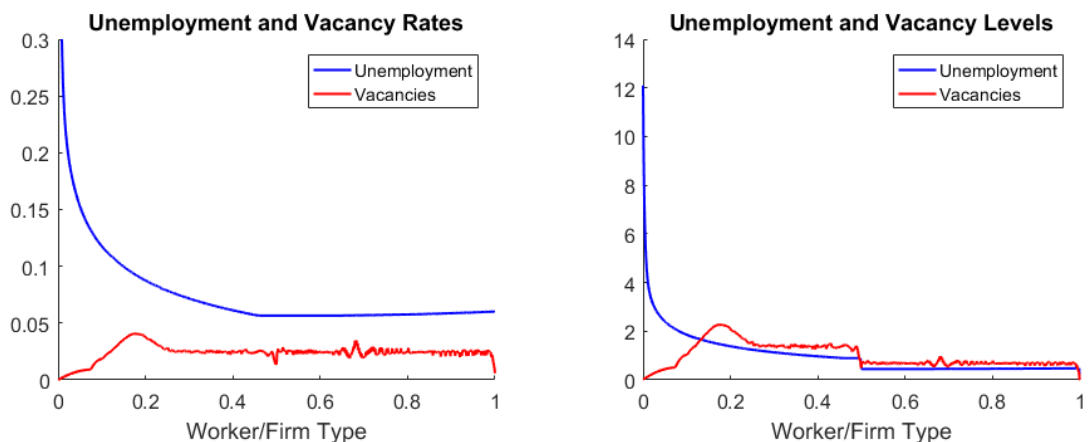
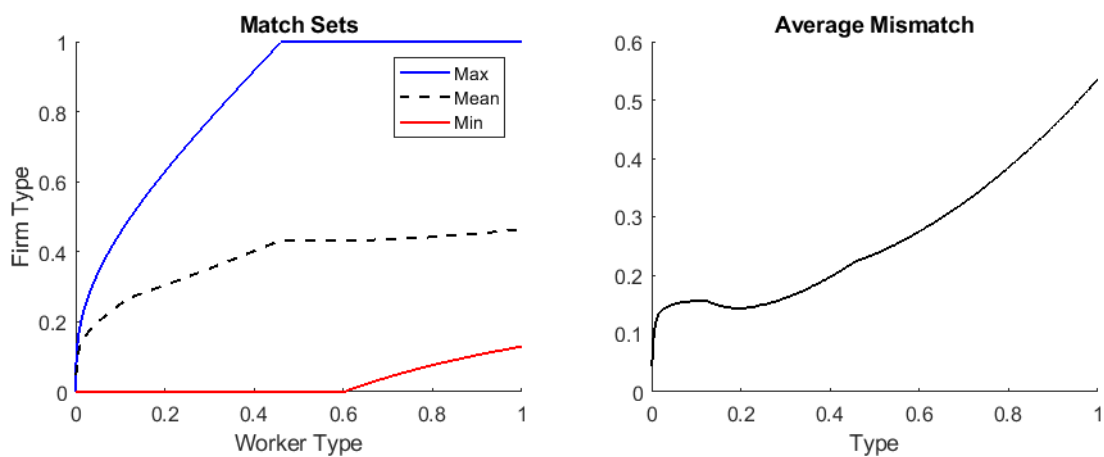


Fig. 3.6.: New distribution of worker skills, with more low-skilled than high-skilled.



(a) Unemployment and vacancy rates under the new skill distribution. (b) Unemployment and vacancy levels under the new skill distribution.

Fig. 3.7.: Equilibrium under a skill distribution with more low-skilled workers.



(a) Region of acceptable matches.

(b) Average skill mismatch.

Fig. 3.8.: Matching under a skill distribution with more low-skilled workers.

The shift in the vacancy distribution changes the mismatch tolerance of workers and firms, as shown in Figure 3.8. Relative to the baseline equilibrium, low-skilled workers become more selective in accepting matches, while high-skilled workers are less selective due to the reduction in the supply of well-matched vacancies. Average mismatch across all employed workers increases to 0.2476, and output falls to 0.4549.

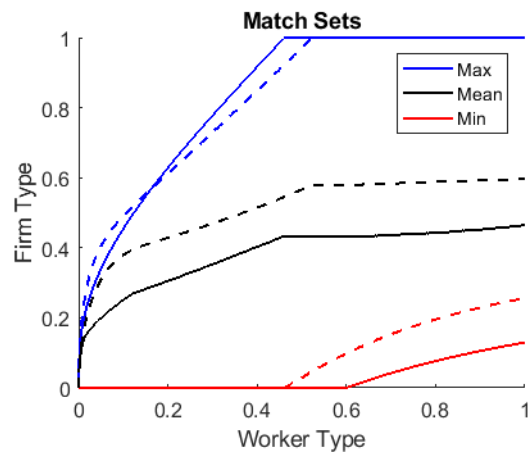


Fig. 3.9.: Match sets with and without endogenous vacancy distribution.

Figure 3.9 highlights the importance of an endogenous vacancy distribution. Match sets from Figure 3.8 are plotted against those from an environment with an exogenous distribution of vacancies equal to that in the baseline example. The equilibrium result allowing for an endogenous vacancy distribution is shown by the solid line, while the dashed line indicates matches under the exogenous distribution. When the skill distribution of vacancies is invariant to the distribution of workers, the implications for workers' mismatch tolerance strategies and ultimately on sorting and aggregate output are very different. In this case, higher-skilled workers are in a better bargaining position due to the excess supply of high-skilled jobs, and experience a reduction in expected mismatch instead of the increase that occurs when the vacancy distribution is endogenous.

3.3.5 Comparative Statics

To understand how the exogenous labor market parameters impact the equilibrium outcome, this section discusses comparative statics for A , α , β , b_1 , and δ .

The replacement rate b_1 is the main policy parameter of interest in this model. Increasing the replacement rate encourages workers to hold out longer for a better match. This raises expected productivity of a match, which leads to a higher match surplus for worker and firm. However, when workers become more choosy they remain in unemployment longer, producing nothing. The effect on expected output per capita depends on which of these channels dominates. In Figure 3.10, I show the equilibrium results of increasing the replacement rate from $b_1 = 0.4$ to $b_1 = 0.6$. Workers become more selective, indicated by narrower match acceptance regions. Average mismatch conditional on being employed falls only slightly, from 0.2186 to 0.2182, while expected output increases from 0.4760 to 0.4818. The unemployment rate increases from 8.71% to 8.83%, and the vacancy rate increases from 2.46% to 2.50%.

The value of the mismatch penalty δ is taken from the calibration of the one-sided model in Buhrmann [2018]. Figures 3.11a and 3.11b show the equilibrium allocation

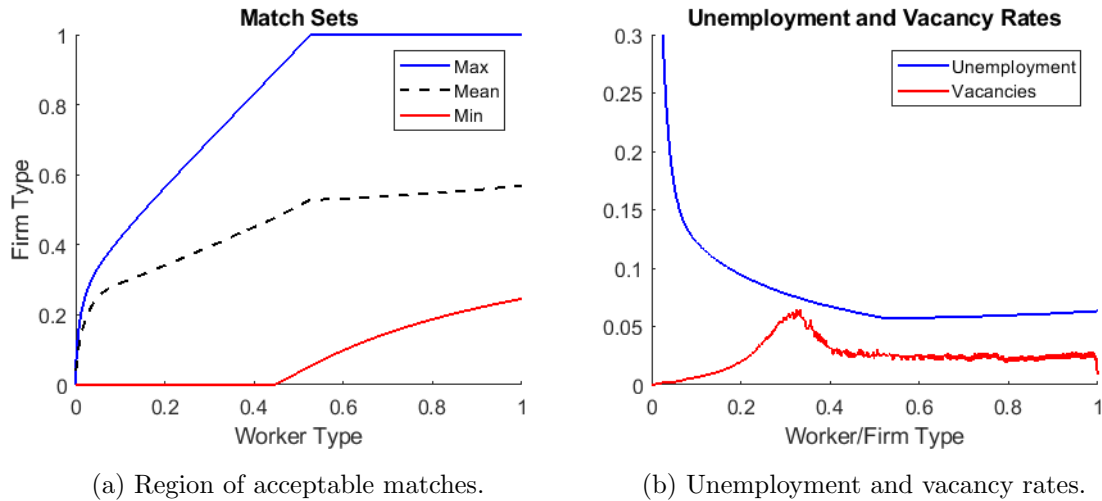


Fig. 3.10.: Equilibrium with $b_1 = 0.6$.

under a doubling of the mismatch penalty to 0.2006, to illustrate the impact of this parameter on the equilibrium outcome. As expected, reducing the productivity of relatively bad matches causes workers and firms to be more selective; average mismatch falls to 0.1838. The set of acceptable matches for any worker or job is reduced, causing average unemployment and vacancy rates to increase to 9.51% and 2.71%, respectively. High-skilled workers respond more strongly to the change, and accordingly experience a greater increase in their unemployment rates. As a result, more vacancies are created at high-skilled jobs. The increase in unemployment dominates the improved match quality in this case; output per capita falls to 0.4700.

Like δ , A is calculated based on the GMM estimation in Buhrmann [2018]. To check the robustness of the model to this calibration, I show the equilibrium results under an increase in A to 0.95. This represents an increase in the number of matches created for every level of unemployment and vacancies. Aggregate unemployment and vacancy rates fall to 6.97% and 1.97%, respectively, but there is no effect on the shape of these distributions. Average mismatch and output per capita are unchanged.

The value for α and β is taken from Shimer [2005a], which he estimates using U.S. labor market data from the BLS between 1951-2003. As he notes, 0.72 is some-

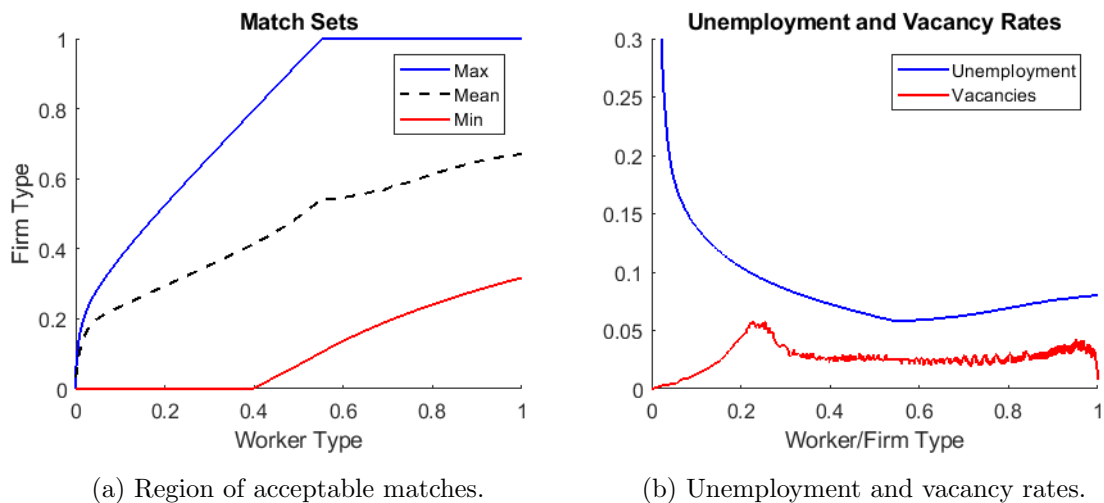


Fig. 3.11.: Equilibrium outcome under $\delta = 0.2006$.

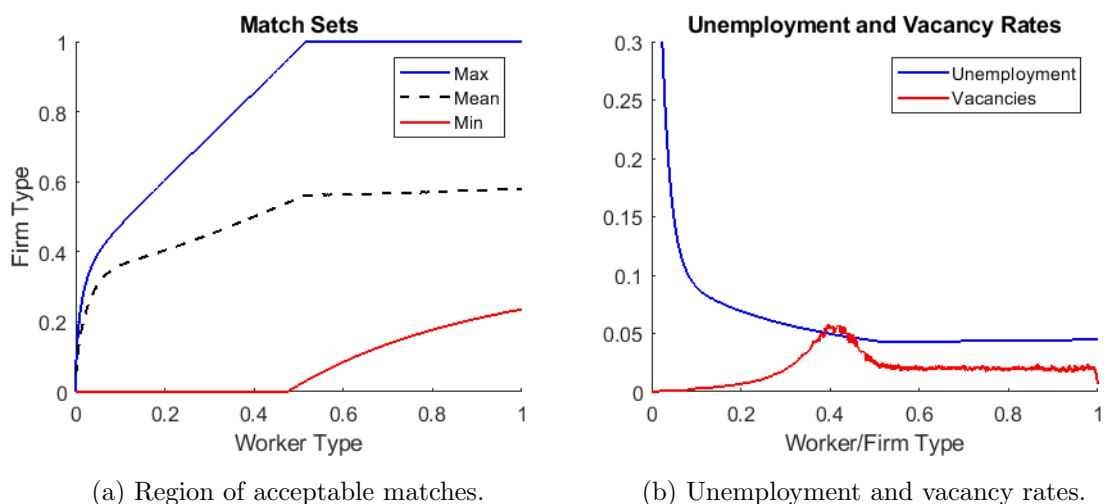


Fig. 3.12.: Equilibrium outcome under $A = 0.95$.

what high; Petrongolo and Pissarides [2001] assert that a “plausible” range for this parameter is between 0.5 to 0.7. To check the robustness of the equilibrium results to changes in these parameters, I plot the equilibrium outcome setting $\alpha = \beta = 0.5$. The reduction in bargaining power causes wages to fall, and decreases the correlation between match quality and wage. As a result, workers are less selective in terms

of the matches accepted. Average mismatch rises to 0.2508, and output per capita falls to 0.4744. Unemployment falls to 7.0%, and the vacancy rate falls to 2.02%. As with other parameter changes, high-skilled workers respond most strongly; in this example, high-skilled workers no longer reject matches with even the lowest-skilled jobs. The highest-skilled jobs are no longer profitable enough to create; the expected match quality is too low to offset the high cost of opening these vacancies.

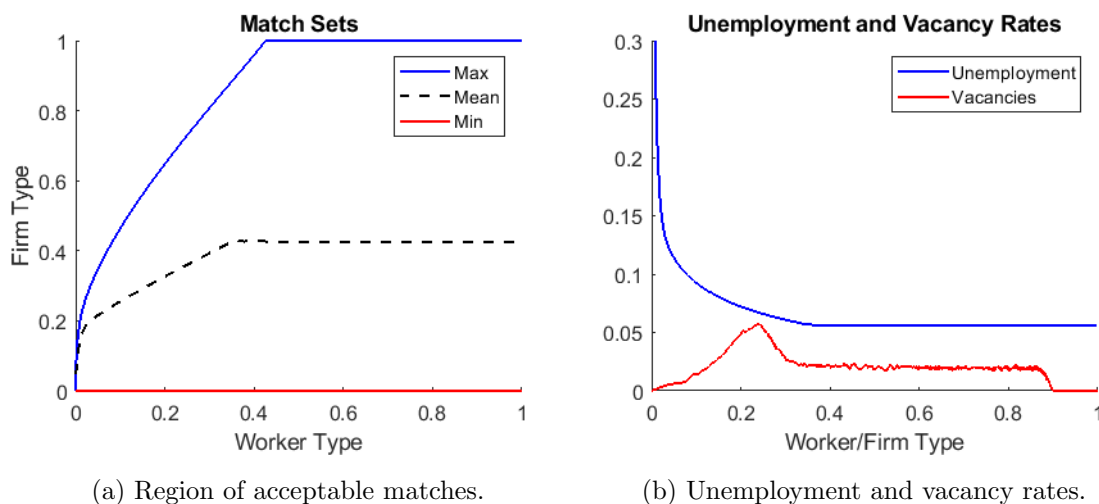


Fig. 3.13.: Equilibrium outcome under $\alpha = \beta = 0.5$.

3.4 Conclusion

This paper proposes a model of labor search with heterogeneous worker and occupations, random search, Nash bargaining over wages, and endogenous vacancy creation. The model is based on Shimer and Smith [2000], but features vertically differentiated agents and an entry decision for firms to take the model from the marriage market into the labor market. Workers are born with an exogenous skill type that determines how productive they can be in various jobs. Occupations are also described by a skill type, which is chosen by the firm at the time of job creation. In general any worker-occupation pair can match and produce output, but the actual

output depends on the match quality of the pair. When the skill types of the worker and occupation are similar, match quality is high and skill mismatch is low; better match quality leads to more output, conditional on the worker's skill type.

The unique feature of this model is its ability to generate an endogenous distribution of vacant jobs using the firms' free entry condition. With the exception of Lise and Robin [2017], most two-sided labor search models assume that the distribution of vacancies is exogenous (and usually uniform). Results from the calibrated numerical example show that the distribution of vacancies is likely not uniform, and depends on the distribution of unemployed workers as well as other labor market parameters. Models that assume an endogenous vacancy distribution will fail to capture the full effects of these changes. This is particularly important for policy analysis, because changes in policies such as unemployment benefits affect welfare in several ways. Consider the effect of an increase in unemployment benefits. The obvious channel is through the worker's flow value of unemployment, which will be captured by models with a fixed vacancy distribution. However, workers' mismatch tolerance strategies inform firms' job creation decisions; when workers tolerate less mismatch, the expected profit of filled jobs increases. Since high-skilled workers respond more strongly to changes in policy than low-skilled workers, it becomes more attractive to create high-skilled vacancies and aggregate output increases despite the rise in unemployment. As a result, the current model will generate different policy recommendations than models with an exogenous vacancy distribution.

In addition to policy exercises, the endogenous distribution of vacancies is key to understanding changes in the labor market over time or drawing comparisons across regions or countries. Education, migration, technical change, and changes in social norms cause the skill distribution of the labor force to vary over time and across locations. Models that assume an exogenous vacancy distribution fail to capture the full effects of these changes in the composition of the labor force. As an example, suppose a region's labor force becomes more educated over time. Firms will respond by creating more high-skilled jobs, and may move lower-skilled jobs to different re-

gions. High-skilled workers will enjoy better match quality, increasing their wages, while low-skilled workers will become less selective in accepting jobs, and as a result will obtain lower wages on average. This simple example underscores the relevance of shifts in the vacancy distribution in understanding changes in key labor market outcomes such as wage dispersion.

This paper explores the interaction between mismatch tolerance and job creation in an environment where skills are vertically differentiated and agents use Nash bargaining to split match surplus. The model presented here provides a framework for that is ideal for applications using new datasets such as the job postings data collected by Burning Glass Technologies. Using data on occupational vacancy rates, the model could be estimated and applied to assessing policies such as unemployment insurance or job creation subsidies, providing a promising avenue for future research.

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APPENDICES

A. APPENDIX: SKILL MISMATCH IN FRICTIONAL LABOR MARKETS

A.1 Proofs

Proposition 1. Setting $w^*(x) = rU(x)$ as given by equation (1.3),

$$\begin{aligned}
 w^*(x) &= b(x) + \lambda \int_{\underline{w}}^{\bar{w}} \max\{E(x, w) - U(x), 0\} d\tilde{G}(w|x) \\
 w^*(x) &= b(x) + \frac{\lambda}{r+s} \int_{\underline{w}}^{\bar{w}} \max\{w(x, y) - rU(x), 0\} d\tilde{G}(w|x) \\
 w^*(x) &= b(x) + \frac{\lambda}{r+s} \int_{\underline{w}}^{\bar{w}} \max\{w(x, y) - w^*(x), 0\} d\tilde{G}(w|x) \\
 w^*(x) &= b(x) + \frac{\lambda}{r+s} \left(\int_{\underline{w}}^{w^*(x)} 0 d\tilde{G}(w|x) + \int_{w^*(x)}^{\bar{w}} (w(x, y) - w^*(x)) d\tilde{G}(w|x) \right) \\
 w^*(x) &= b(x) + \frac{\lambda}{r+s} \int_{w^*(x)}^{\bar{w}} (w(x, y) - w^*(x)) d\tilde{G}(w|x)
 \end{aligned}$$

Using integration by parts,

$$w^*(x) = b(x) + \frac{\lambda}{r+s} \int_{w^*(x)}^{\bar{w}} 1 - \tilde{G}(w|x) dw$$

To see that the solution to the above equation is unique, I differentiate both sides w.r.t. $w^*(x)$. The derivative of the LHS is clearly positive, and Leibniz rule gives

$$\frac{\partial \text{RHS}}{\partial w^*(x)} = \frac{\lambda}{r+s} \left(\tilde{G}(w^*(x)|x) - 1 \right) \leq 0$$

A.2 Computational Algorithm

The baseline model is solved for the equilibrium reservation wage series $\{w_x^*\}$ using an iterative method. Results for expected wages, unemployment rates, etc. can then be computed using the reservation wages. Each worker's decision problem is

independent from that of all other workers, so in practice the problem can be solved for each worker individually as follows.

1. Create a discrete worker type space X .
2. Invert the wage function $w(x, y)$ to obtain $y(x, w)$. This function will return at most two firm types y which will pay worker x a wage equal to w .
3. For each worker type x , transform the firm cdf $G(y)$ into the cdf of wage offers $\tilde{G}(w|x)$ using the function $y(w, x)$.
4. For each $x \in X$, iterate on the reservation wage function until convergence.
 - (a) Guess a reservation wage, for example $w_x^0 = 0$.
 - (b) Using w_x^0 , calculate the flow value of unemployment $r\widehat{U}(x)$ using equation 1.2.
 - (c) Update the reservation wage guess, for example $w_x^1 = w_x^0 + \frac{1}{2} \left(r\widehat{U}(x) - w_x^0 \right)$.
 - (d) Repeat until $|r\widehat{U}(x) - w_x^0|$ is near zero.

A.3 Data

For all calculations using NLSY97 data, a worker-job observation is dropped if:

1. Wage is less than \$2/hour, greater than \$100/hour, or missing.
2. Respondent worked less than 5 hours per week, or hours/week is missing.
3. Respondent held the job while enrolled in school.

A.4 Calibration

Calculation of the aggregate moments uses NLSY97 data from 2009-2013. The moments are constructed as follows, and the resulting values are reported in Table A.2. Custom sample weights are downloaded from www.nlsinfo.org/weights/nlsy97 for individuals who are in any or all of the years: 2009, 2010, 2011, 2013⁴.

⁴The survey shifted to biannual data collection, skipping 2012.

Table A.1.: Descriptive statistics of the cross-sectional sample, NLSY97, 2009-2013

Description	Statistic
Male	51.32%
Highest degree:	
No Degree	7.77%
GED	11.70%
High School	41.06%
Associates	8.27%
Bachelors	22.37%
Masters	6.83%
PhD or Professional	1.85%
Labor Force Participation	81.67%
Avg. Weeks Worked ¹	35.974
Avg. Enrollment ²	10.95%
Any Enrollment ³	24.12%

- Average unemployment rate, using weekly employment status

$$\bar{u} = \frac{1}{260} \sum_{t=1}^{260} \frac{\sum_i \mathbb{1}(e_{i,t} = 0) \cdot \omega_i}{\sum_i \mathbb{1}(e_{i,t} \geq 0) \cdot \omega_i}$$

where $e_{i,t}$ is the unique employer ID (ranging from 9701 to 201313) if individual i is employed in week t , 0 if unemployed, and -1 if out of the labor force, and ω_i is the custom sample weight.

- Average (monthly) hazard rate

$$\mathcal{H} = 1 - \left(1 - \frac{1}{259} \sum_{t=1}^{259} \frac{\sum_i \mathbb{1}(e_{i,t} \in \{-1, 0\} \text{ and } e_{i,t+1} > 0) \cdot \omega_i}{\sum_i \mathbb{1}(e_{i,t} \in \{-1, 0\}) \cdot \omega_i} \right)^{52/12}$$

dropping any instances where the individual is returning to a previous employer.

- Average (monthly) separation rate

$$s = 1 - \left(1 - \frac{1}{259} \sum_{t=1}^{259} \frac{\sum_i \mathbb{1}(e_{i,t} \neq e_{i,t+1}) \cdot \omega_i}{\sum_i \mathbb{1}(e_{i,t} > 0) \cdot \omega_i} \right)^{52/12}$$

dropping instances where the individual appears to separate but returns to the same employer later, since these transitions likely represent vacations, illnesses, maternity leave, etc. rather than true job termination.

- Average job tenure is calculated in two ways

$$\bar{d}_{report}^e = \frac{\sum_t \sum_k \sum_i d_{i,t,k} \cdot \omega_i}{\sum_t \sum_k \sum_i \mathbb{1}(d_{i,t,k} > 0) \cdot \omega_i}$$

where $d_{i,t,k}$ is the reported job tenure (in months) of individual i in year t at job k .

$$\bar{d}_{spell}^e = \frac{\sum_k \sum_i d_{i,k} \cdot \omega_i}{\sum_i K_i^e \cdot \omega_i}$$

where $d_{i,k}$ is the length (in months) of employment spell k for individual i , calculated from the weekly employment status arrays as the number of consecutive weeks where $e_{i,t} > 0$ and $e_{i,t} = e_{i,t-1}$. K_i^e is the number of employment spells observed for i .

- Average unemployment duration

$$\bar{d}_{spell}^u = \frac{\sum_k \sum_i u_{i,k} \cdot \omega_i}{\sum_i K_i^u \cdot \omega_i}$$

where $u_{i,k}$ is the length (in months) of unemployment spell k for individual i , calculated from the weekly employment status arrays as the number of consecutive weeks where $e_{i,t} = 0$. K_i^u is the number of unemployment spells observed for individual i .

- Letting w_i^m denote the m^{th} percentile of the hourly wage distribution for a skill percentile bin i , I estimate the max-mean wage dispersion in the data by regressing

$$z_i = \beta_0 + \beta_1(w_i^{90} - w_i^{50})$$

using the 90th percentile wage rather than the 100th to represent the max wage in order to correct for potential misreporting and/or extreme cases. The max-mean wage dispersion ratio is then calculated as

$$\mathcal{D}_{m1,m2} = \frac{\hat{z}_{m1}}{\hat{z}_{m2}}$$

Specifically, I calculate the ratio of max-mean wage dispersion between high-skill to mid-skill workers, $\mathcal{D}_{90,50}$, and between high-skill to low-skill workers, $\mathcal{D}_{90,10}$. Both measures capture the fact that wage dispersion is increasing in worker skill.

- Average accepted mismatch

$$\bar{\mu} = \frac{\sum_t \sum_k \sum_i (\mu_{i,t,k} \cdot \omega_i)}{\sum_t \sum_k \sum_i (\mathbb{1}(\mu_{i,t,k} > 0) \cdot \omega_i)}$$

where $\mu_{i,t,k} = |x_i - y_{i,t,k}|$ is the mismatch between individual i and the occupation associated with i 's k^{th} job in year t ; x_i and $y_{i,t,k}$ are calculated as detailed in Section 1.2.

- Under the steady state assumption, $\bar{u} = \frac{s}{s+\mathcal{H}}$; this relationship can be used to calculate one of \bar{u} , s , \mathcal{H} given the other two.
- Additionally, s and \mathcal{H} are equal to the inverse of job duration and unemployment duration, respectively.

Table A.2.: Calculated moments and parameters for robustness checks

Moment/Parameter	Value	Method
\bar{u}	8.24%	* Direct
\bar{u}	9.0%	BLS
\mathcal{H}	0.1823	Direct
\mathcal{H}	0.3641	Inferred from $\bar{u} = .0824$, $s = .0327$
\mathcal{H}	0.3329	* Inferred from $\bar{u} = .0824$, $s = .0299$
\mathcal{H}	0.4599	Inferred from $\bar{u} = .0824$, $s = .0413$
\mathcal{H}	0.2273	Inferred from $\bar{d}_{spell}^u = 4.4$
s	0.0327	Direct
s	0.0164	Inferred from $\bar{u} = .0824$, $\mathcal{H} = .1823$
s	0.0204	Inferred from $\bar{u} = .0824$, $\mathcal{H} = .2273$
s	0.0299	* Inferred from $\bar{d}_{report}^e = 33.50$
s	0.0413	Inferred from $\bar{d}_{spell}^e = 24.21$
s	0.0310	JOLTS
\bar{d}^e	33.50 months	* Direct (reported)
\bar{d}^e	24.21 months	Direct (spells)
\bar{d}^e	37.2 months	BLS
\bar{d}^u	4.44 months	* Direct (spells)
\bar{d}^u	4.29 months	BLS
\mathcal{D}_{9050}	1.3772	* Direct
\mathcal{D}_{9010}	2.2107	* Direct
$\bar{\mu}$	0.2256	* Direct

Notes:

* Indicates preferred calibration values used Section 1.4.

BLS statistics are for persons age 25-34, averaged over 2009-2013.

JOLTS statistics are for persons of all ages, averaged over 2009-2013.

For values that are not directly available from NLSY97 data, calibration is as follows:

- $b(x)$: Let $b(x) = b_0 + b_1x$. From 2009 to 2013, the weighted average U.S. replacement rate was between 0.405 and 0.470, according to the U.S. Dept. of Labor's Office of Unemployment Insurance. For consistency with the literature, I use $b_1 = 0.4$ and set the value of leisure time b_0 to 0.
- r : Set to 0.001 to match average 3-month treasury bill rate from 2009-2013.
- λ, δ : jointly calibrated using the method of moments. The available moments are $\bar{u}, \mathcal{H}, \mathcal{D}_{90,50}, \bar{\mu}$; however, \bar{u} and \mathcal{H} cannot be used together since they are directly related in the model.

Table A.3 provides calibrated values of λ, δ from the preferred parameterization, as well as for selected alternative calibration methods. Using the method of moments, the targeted moments will be matched exactly; the simulated value of the untargeted moments are provided as an external check.

Table A.3.: Robustness checks on calibration of λ, δ .

λ	δ	\bar{u}	\mathcal{H}	$\bar{\mu}$	$\mathcal{D}_{90,50}$
.1003	.5010	* .0842	.3789	.2265	* 1.3772
.1227	.4521	.0943	* .3329	.2187	* 1.3771
.0988	.5158	* .0824	.3891	* .2256	1.3653
.1138	.4413	.0942	* .3329	* .2256	1.4247
.2773	.5637	.0991	.3148	* .1568	* 1.1325

Parameterizations that targeting \bar{u} provide the best overall fit, with very similar estimates for λ and δ . The first line in the table gives the preferred calibration.

A.5 Robustness Checks for Stylized Facts

A.5.1 Alternative definitions of skill type

Tables A.4 and A.5 display the R^2 obtained from regressing hourly compensation, H_i , on percentile rank, z_i , for various definitions of worker and occupation skill types.

$$H_i = \beta_0 + \beta_1 z_i + \beta_2 z_i^2$$

This comparison motivates the choice of `x_pcawgt` and `y_cpsjdm` as the preferred methods of ranking individuals and occupations, since these rankings best predict average hourly wages.

A.5.2 “Similar” workers with different jobs

For x to be a good measure of worker skills, it should be the case that characteristics of workers with the same x do not differ systematically across occupations. Figure A.1 shows the average age, average birth year, proportion male, and proportion white for workers of similar skill types (deciles of x) in different occupations (deciles of y). Each connected line represents workers in the same decile who are employed in occupations no more than two deciles away from their own skill type (since the number of observations in a category decreases substantially beyond this point). Flat lines indicate that, conditional on skill type, there is no difference in a specific characteristic across occupations.

Figure A.1a shows that, conditional on x , individuals do not systematically sort into occupations based on their education level. There is positive sorting into occupations on education level, but it is fully accounted for within the skill measure x . Within a skill type, workers in higher-skilled jobs are not more likely to have higher levels of education. The other three panels of Figure A.1 suggest that, while there is clearly variation in the characteristics of workers across different occupations, workers are not sorting in any systematic way on the basis of age, gender, or race after conditioning on skill type.

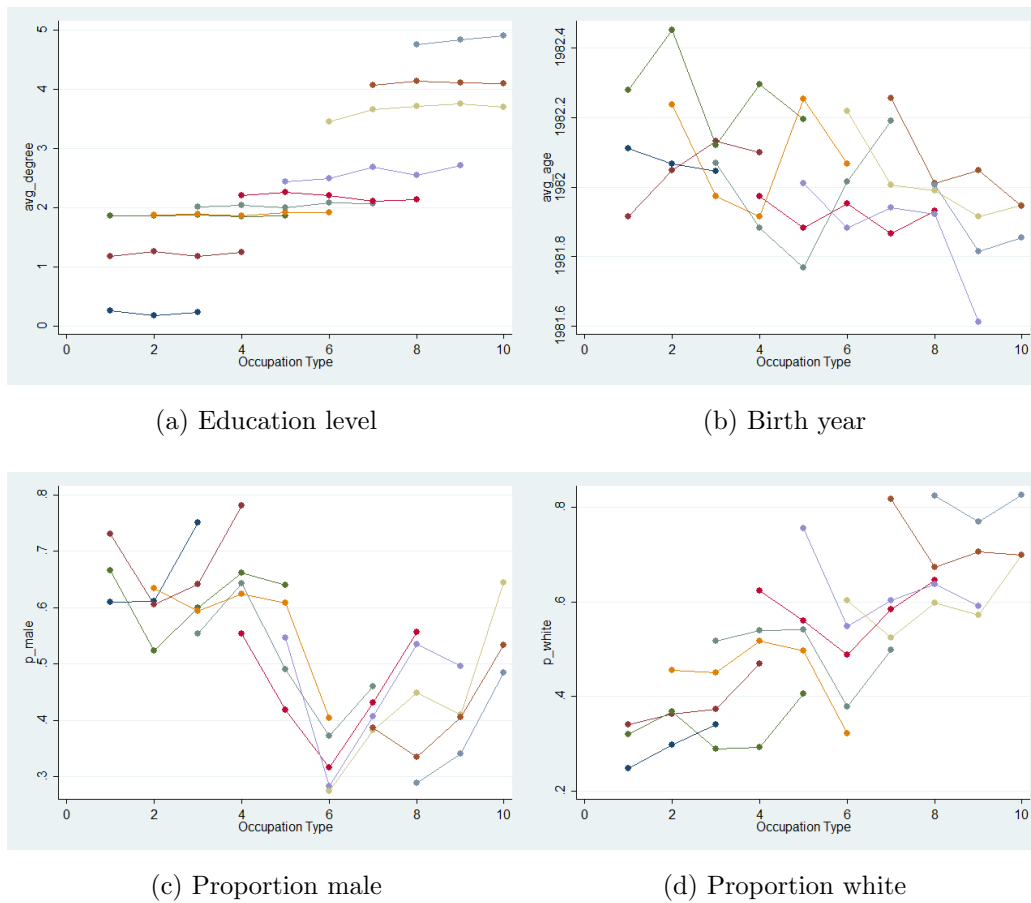


Fig. A.1.: Characteristics of similar workers in different occupations.

Table A.4.: Varying definitions of worker skill type.

x	Ranking definition	R^2
x_pca	PCA of individual's aggregated ASVAB score and highest degree	0.0619
x_pcawgt	PCA of scores in 4 math and verbal ASVAB categories and highest degree; scores residualized by race, gender, and 3-month age cohort (preferred method)	0.0772
x_act	For individuals with high school diploma or less, aggregated ASVAB score. For others, regress $ASVAB_i = \beta_0 + \beta_1 ACT_i + \beta_2 ACT_i^2 + \epsilon_i$ on scores of all individuals with ACT scores, use predicted ASVAB score as x value.	0.0534
x_stack	Rank by education level, and within education level rank by aggregated ASVAB score.	0.0732
x_weight	$x_i = p \cdot ASVAB_i + (1 - p) \cdot \overline{ASVAB}_i$, where $p = 0.8$, $ASVAB_i$ is respondent's aggregated ASVAB score, and \overline{ASVAB}_i is average ASVAB score of respondents with same education level as i .	0.0551
x_optwgt	Same as above, except p is chosen to minimize aggregate mismatch: $p^* = \operatorname{argmin} \left\{ \left(\sum_{t=2009}^{2013} \left(\sum_{k=1}^2 \sum_{i=1}^N (x_i - y_{i,k,t}) \right) \right) \right\}$ where i is an individual, t is the year, k is a job, and $y_{i,k,t} = y_jdm$. $p^* = .143$; there is a small amount of overlap between the top of one education category and the bottom of the next.	0.0743

A.5.3 “Similar” occupations employing different workers

Analogous to the previous section, a good measure of occupation skills would capture all of the characteristics on which workers sort into occupations. Figures A.2a

Table A.5.: Varying definitions of occupation skill type.

y	Ranking definition	R^2
y_ct	Critical Thinking	0.0936
y_cps	Complex Problem Solving	0.1088
y_jdm	Judgment and Decision Making	0.1091
y_cpsjdm	PCA of Complex Problem Solving, Judgment and Decision Making	0.1152
y_jdmtech	PCA of Judgment and Decision Making, Repairing	0.0752
y_cogsk	PCA of all O*NET skill descriptors listed as “cognitive”	0.1129
y_cogab	PCA of all O*NET ability descriptors listed as “cognitive”	0.1109
y_tech	PCA of all O*NET skill descriptors listed as “technical”	0.0072
y_phys	PCA of all O*NET ability descriptors listed as “physical”	0.0419
y_social	PCA of all O*NET skill descriptors listed as “social”	0.0552

and A.2b show that the measure of occupation skills is indeed capturing the sorting of workers according to cognitive skills or abilities. Similarly, conditional on occupation type, higher skilled workers do not sort into occupations requiring systematically different levels of social skills, and in general do not earn higher wages. However, Figures A.2e and A.2f suggest that there remains sorting of workers on other dimensions. After controlling for the occupation skill type (decile of y), workers with higher skill types (by decile of x) sort into occupations that require lower levels of physical abilities and technical skills. This is not entirely unexpected, since the method of ranking skills was chosen to measure only cognitive skills. To attempt to correct for this, I included Repairing, a skill highly correlated with other technical skills and physical abilities, in a principal components analysis and recomputed the rankings of y . However, under this new ranking, jobs of the same type employing different types of workers systematically varied in wages and cognitive skill requirements. Since the

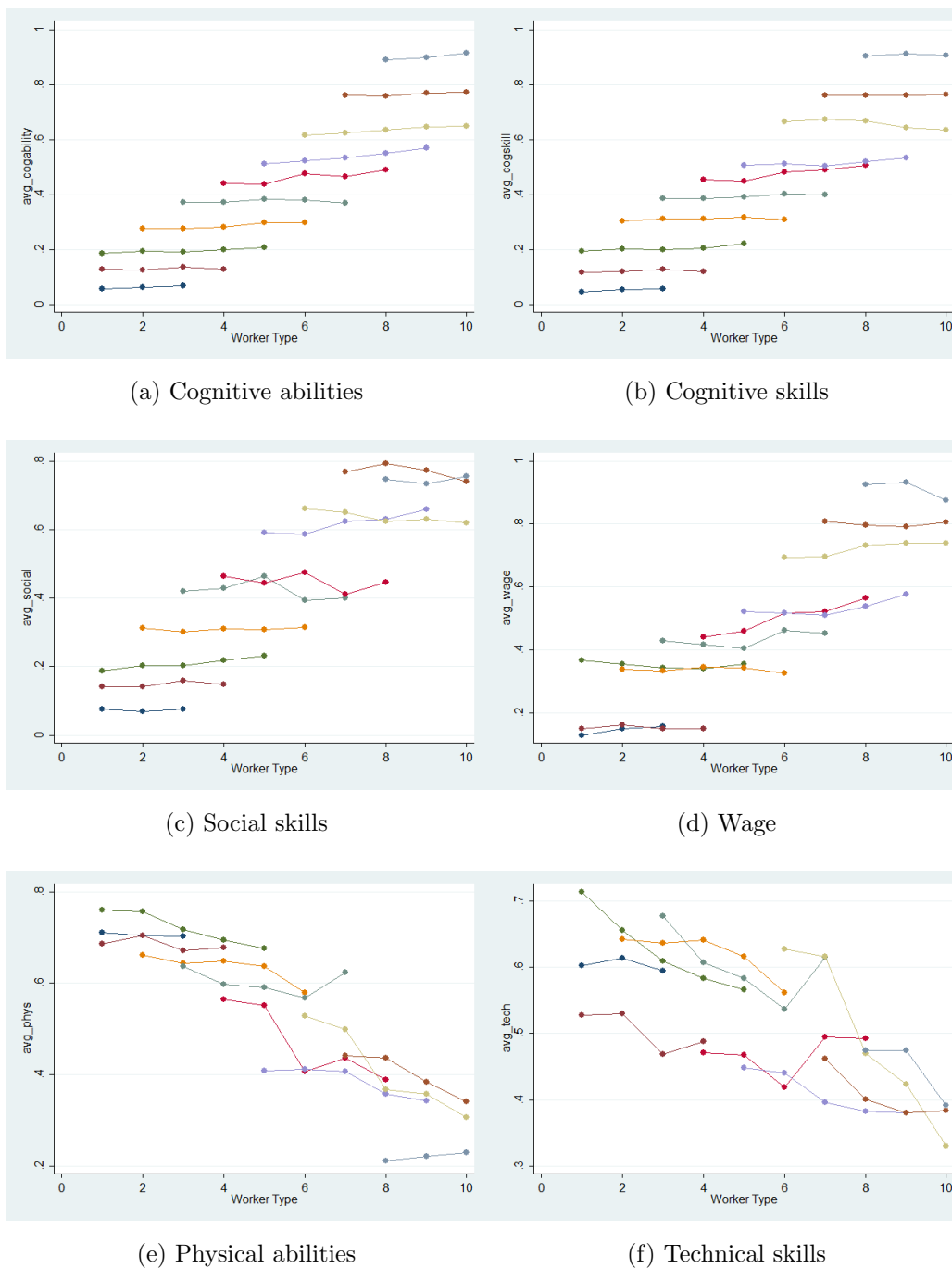


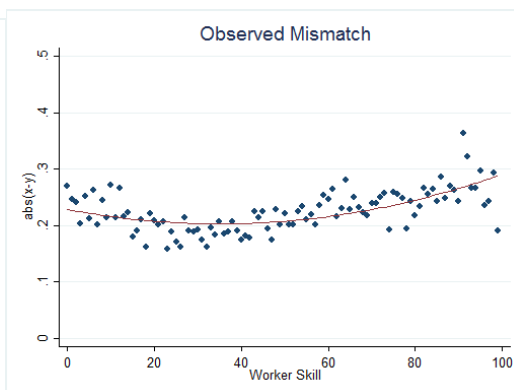
Fig. A.2.: Characteristics of similar occupations employing different workers.

object is to measure rank occupations by cognitive skills, the original measure is more favorable.

A.5.4 Results by Gender



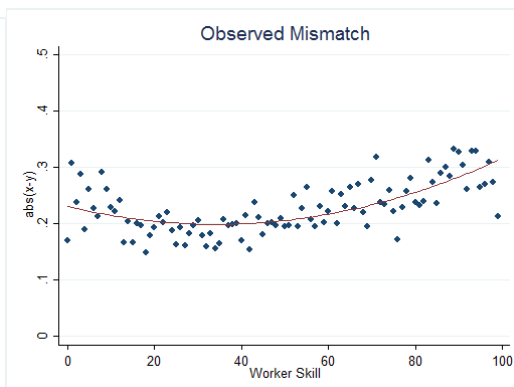
(a) Match sets (males only)



(b) Mismatch (males only)



(c) Match sets (females only)

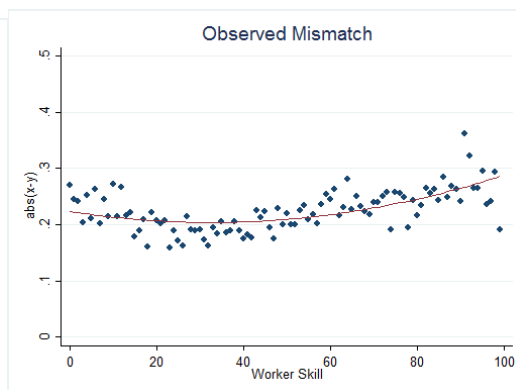


(d) Mismatch (females only)

A.5.5 Results by Race



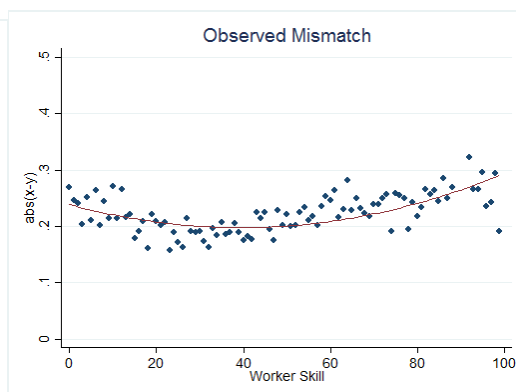
(a) Match sets (white only)



(b) Mismatch (white only)



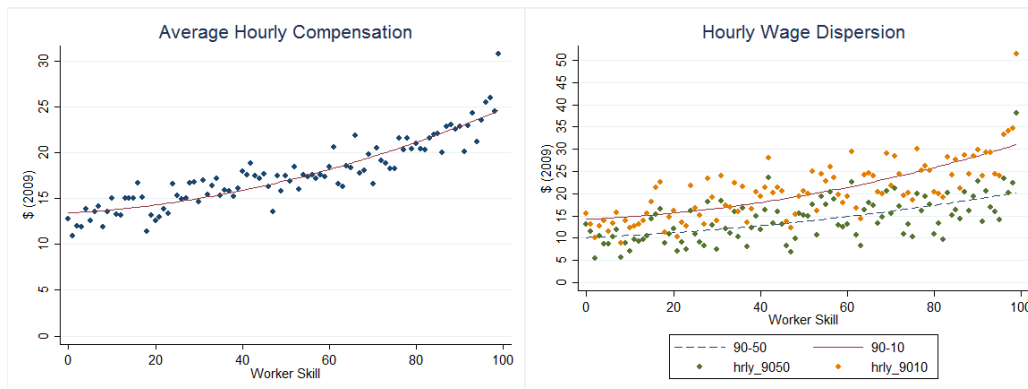
(c) Match sets (black only)



(d) Mismatch (black only)

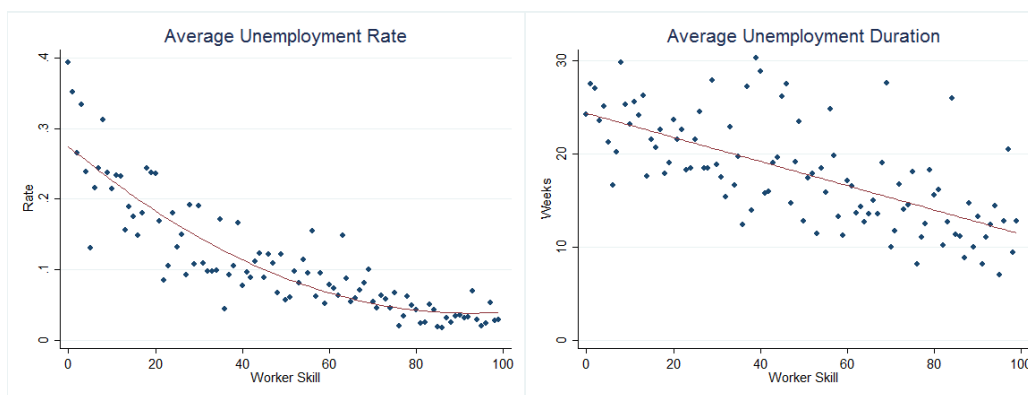
A.5.6 Alternative Skill Ranking for Workers

In this section, `x_optwgt` is used.



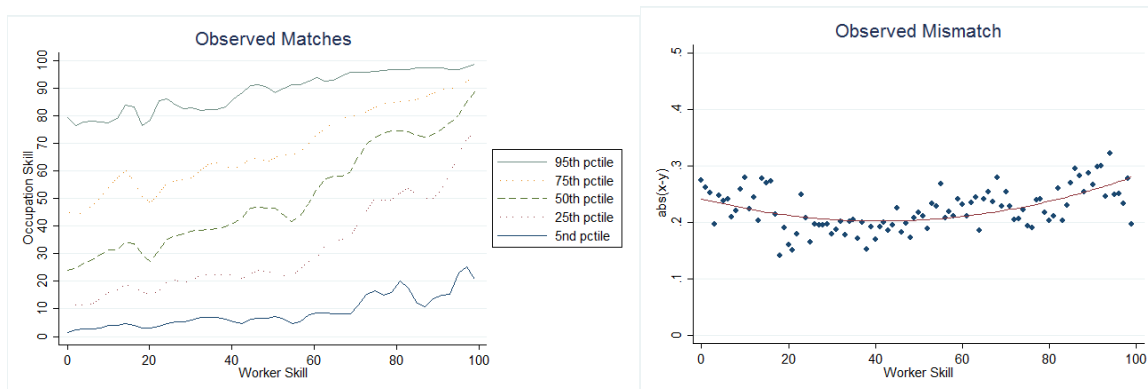
(a) Average wage

(b) Wage dispersion



(c) Unemployment rate

(d) Unemployment duration

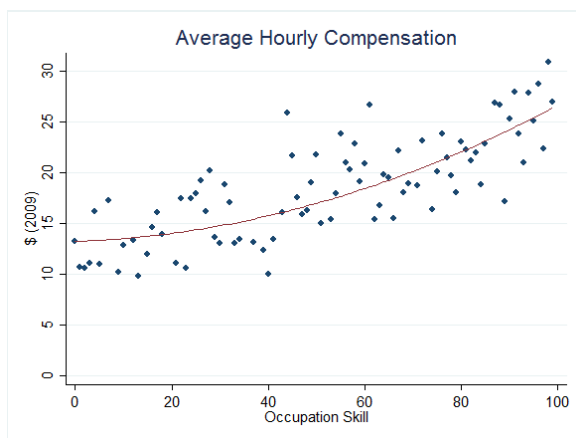


(e) Match sets

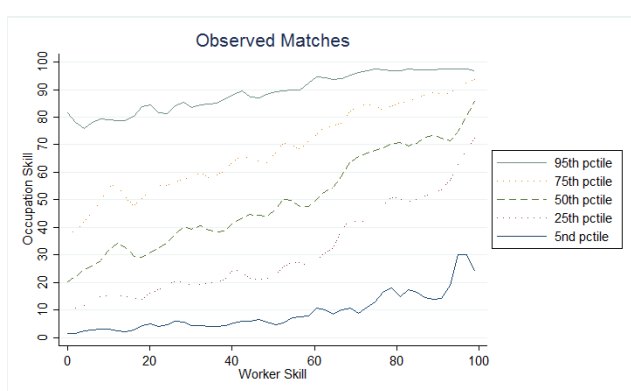
(f) Mismatch

A.5.7 Alternative Skill Ranking for Occupations

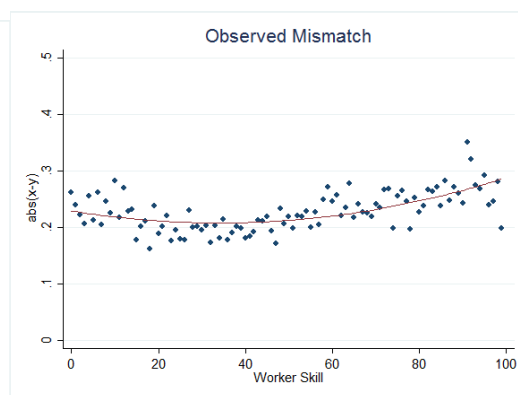
In this section, y_{cogskill} is used.



(a) Average wage by occupation



(b) Match sets



(c) Mismatch

A.6 Wage Growth Transformations

Tables A.6, A.7, and A.8 show the level changes in unemployment rates, mismatch, and wages by skill group for both the data and the model. In addition, the “Relative to average” columns show the ratio of changes within a skill group to the average change, to illustrate the model’s ability to capture differences across worker groups.

Table A.6.: Unemployment: empirical vs. model predicted change

	Level change		Relative to average	
	Data	Model	Data	Model
Average	-3.47%	-2.50%	1	1
Low-skill	-5.12	-3.42	1.48	1.37
Mid-skill	-1.95	-2.07	0.56	0.83
High-skill	-1.60	-1.98	0.46	0.79

Table A.7.: Mismatch: empirical vs. model predicted change

	Level change		Relative to average	
	Data	Model	Data	Model
Average	-.0200	-.0283	1	1
Low-skill	-.0128	-.0189	0.64	0.67
Mid-skill	-.0200	-.0208	1	0.73
High-skill	-.0263	-.0447	1.32	1.58

Comparisons of level changes in unemployment and mismatch are straightforward; these outcomes have the same units in both the data and the model. However, wages in the model are normalized and must be transformed in order to compare the model to the data. Table A.9 shows the level change in wages in the model, as well as two transformed wage changes.

Table A.8.: Average wage: empirical vs. model predicted level change

	Level change		Relative to average	
	Data	Model*	Data	Model
Average	\$1.60	\$1.57	1	1
Low-skill	0.28	1.26	0.17	0.80
Mid-skill	0.91	1.52	0.57	0.97
High-skill	3.13	3.10	1.95	1.98

Table A.9.: Average wage: empirical vs. model predicted % change

	Level change		Transformed	
	Data	Model	Level	Percentage
Average	\$1.60	0.0021	\$2.05	\$1.57
Low-skill	0.28	0.0009	1.37	1.26
Mid-skill	0.91	0.0014	1.57	1.52
High-skill	3.12	0.0040	3.03	3.10

Wage changes in the model can be translated into dollars using one of two assumptions. Both rely on mapping the standard deviation of wages in the model to that in the data, since wage dispersion was a key moment used to calibrate the model.

1. “Level” transformation:

Equate the standard deviation of wages in the model to that in the data.

$$\widehat{\Delta W}_n = \Delta W_n^{model} \left(\frac{\sigma_n^{data}}{\sigma_n^{model}} \right)$$

where σ_n indicates the standard deviation of wages among group n in either the model or the data, ΔW_n indicates the level change in wages, and $\widehat{\Delta W}_n$ is the transformed wage change.

2. “Percentage” transformation:

Equate the standard deviation of wages in the model to the standard deviation of log wages in the data.

$$\widehat{\Delta W}_n = \exp \left(\left(\overline{\log(W_n^{data})} + \Delta W_n^{model} \left(\frac{\sigma_n^{data}}{\sigma_n^{model}} \right) \right) \right) \left(\overline{W}_n^{data} \right)$$

where σ_n^{data} is now the standard deviation of log wages. The above formula predicts the new log wage in the data, exponentiates to obtain the wage in dollars, and subtracts the old wage to obtain the predicted wage change in dollars. Because of the almost-linear wages in the model, this transformation is preferred and is used in the main text.

B. APPENDIX: TARGETED SEARCH IN LABOR MARKETS WITH SKILL HETEROGENEITY

B.1 Discussion of propositions.

Proposition: Competitive search is inconsistent with the existence of skill mismatch in an environment where mismatch is costly to workers.

Under competitive search, identical workers employ identical strategies (either mixed or pure). Workers are assumed to search for the type of job that maximizes expected discounted income. When workers play pure strategies, it is not possible for two identical workers to be employed in different jobs. When workers play mixed strategies, a type x worker must be indifferent between all types of jobs in which type x workers are observed to be employed. Consider a simple example using two types of workers and two types of firms. Let $\rho_{i,j}$ be the productivity of a type i worker at a type j firm, $w_{i,j}$ be the wage paid, and $d_{i,j}$ indicate the expected duration of unemployment for a type i worker searching in submarket j . Suppose that worker and firm types are complements in production; that is, $\rho_{1,1} < \rho_{1,2}$ and $\rho_{2,1} < \rho_{2,2}$. If type 1 workers are employed in both types of jobs, then one of the following must be true: (1) $w_{1,1} > w_{1,2}$ and $d_{1,1} > d_{1,2}$ or (2) $w_{1,1} < w_{1,2}$ and $d_{1,1} < d_{1,2}$. Similarly, if type 2 workers are also employed in both types of jobs, we must have (3) $w_{2,2} > w_{2,1}$ and $d_{2,2} > d_{2,1}$ or (4) $w_{2,2} < w_{2,1}$ and $d_{2,2} < d_{2,1}$. It must be the case that either (1) wages of one worker type are positively correlated with productivity, while wages of the other type are negatively correlated with productivity or (2) the employment probability of a type 1 worker differs from that of a type 2 worker in the same submarket. (1) is clearly not true in the data, and additionally cannot be used to rationalize this issue when there are more than two types. Since meetings in a submarket are generated by a random matching function, (2) implies that a fraction of matches are

rejected. Match surplus is given by $\rho_{i,j} - w_{i,j}$, so there are no grounds for rejecting some workers while employing other identical workers. Data shows that observably identical workers are frequently employed in different jobs and receive different wages, so competitive search can be ruled out.

B.2 Comparative Statics

In this section are comparative statics plots to illustrate the effect of various parameters on workers' optimal search effort. Each plot changes one parameter away from the baseline case of $\alpha_1 = \alpha_2 = (\alpha_3)^{-1} = 4$, $c_0 = 0.1$ shown in the numerical example. Solid lines represent the optimal search effort, while dashed lines show the budget constraint $c(\eta) \leq b(x)$.

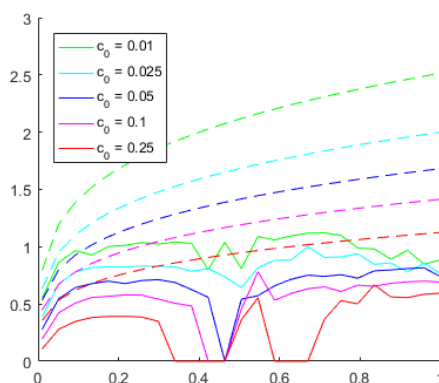


Fig. B.1.: Optimal search effort across the skill space, varying c_0 .

The c_0 parameter in the cost function scales the cost of search effort, so higher levels of c_0 tighten the budget constraint, reducing the maximum level of search effort possible as well as the optimal search effort choice. Figure B.1 shows that as search effort becomes more costly, all workers choose lower levels of search effort. When search costs are particularly high, workers whose expected match under random search is already close to the best match do not target their search at all.

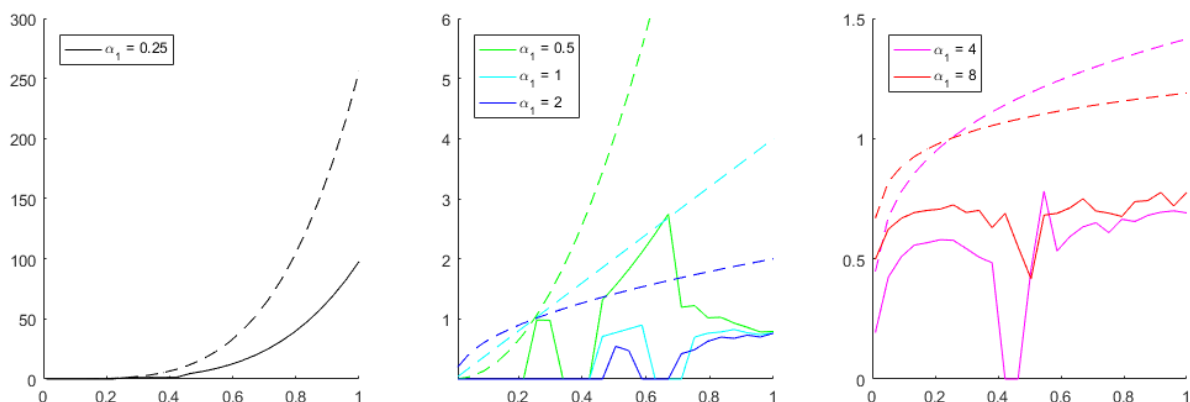


Fig. B.2.: Optimal search effort across the skill space, varying α_1 .

The α_1 parameter in the cost function controls the marginal cost of search effort. $\alpha_1 < 1$ implies a high marginal cost of search effort for η close to zero, and a lower marginal cost as η increases. $\alpha_1 > 1$ implies a low marginal cost for η close to zero, which increases rapidly as η increases. As a result, when α_1 is low search effort tends to be quite high for those workers who choose to target search. As α_1 falls, search effort across the skill type space evens out to a moderate level. Note that the budget constraint also depends on α_1 .

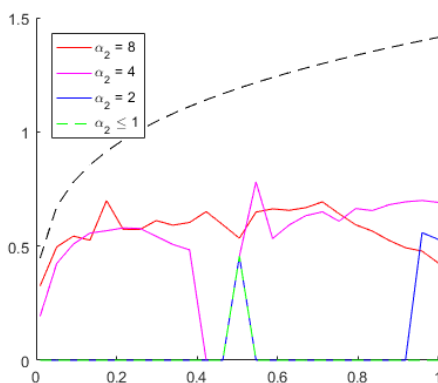


Fig. B.3.: Optimal search effort across the skill space, varying α_2 .

In the $\sigma(\eta)$ function, α_2 controls the efficiency of targeted search, so increases in α_2 incentivise more workers to target their search. When α_2 is relatively low, targeting search is not worth the associated costs, but as α_2 increases targeting becomes more valuable.

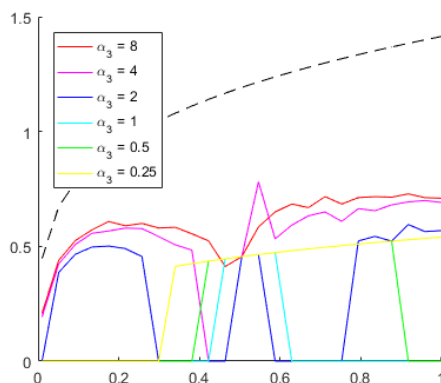


Fig. B.4.: Optimal search effort across the skill space, varying α_3 .

Finally, α_3 represents the marginal cost of targeting search in terms of foregone job offers. When α_3 is low, the offer arrival rate is more sensitive to search effort. As a result, increases in α_3 induce more workers to engage in targeted search.

C. APPENDIX: SKILL MISMATCH AND THE EQUILIBRIUM DISTRIBUTION OF VACANCIES

C.1 Propositions

C.1.1 Discussion of Proposition 2

The steady-state distribution of vacancies $G(y)$ is not necessarily unique; the vacancy supply condition determines only the aggregate vacancy rate \bar{v} . However, since unemployment rates are uniquely determined, the measure of acceptable matches

$$\int_0^1 v(y) \cdot \mathbb{1}(E(x, y) - U(x) > 0) dy = G(\bar{y}(x)) - G(\underline{y}(x))$$

is pinned down by the steady-state condition on unemployment for each worker type x .

Now, suppose $v(y_j)$ is increased. Then $v(y_{-j})$ must be decreased for some $y_{-j} \neq y_j$ in order to maintain \bar{v} . For any worker type x_i such that a match with either y_j or y_{-j} is acceptable while the other is not, the measure of acceptable matches (and therefore the probability of exiting unemployment) for that type x_i will change. Therefore, unless both y_j and y_{-j} are acceptable to exactly the same set of worker types, $G^*(y_j)$ and $G^*(y_{-j})$ are uniquely determined. In order for $G^*(y)$ to be unique for all job types, it must be the case that the range of acceptable matches for y_j is different from the range of acceptable matches for y_{-j} for all pairs of distinct job types (y_j, y_{-j}) .

Let $\bar{y}^*(x)$, and $\underline{y}^*(x)$ be the highest acceptable job and lowest acceptable job for a worker of type x . $G^*(y)$ is unique if and only if there does not exist a job type pair (y_j, y_{-j}) such that the set of x that will accept a match with y_j is exactly the set that accepts a match with y_{-j} . This can be summarized as

$$\forall (y_j, y_{-j}) : ((\bar{y}^*)^{-1}(y_j), (\underline{y}^*)^{-1}(y_j)) \neq ((\bar{y}^*)^{-1}(y_{-j}), (\underline{y}^*)^{-1}(y_{-j})) \quad (\text{C.1})$$

If $\bar{y}^*(x)$ and $\underline{y}^*(x)$ are continuous, the following conditions are sufficient to guarantee uniqueness:

1. $\forall y, (\bar{y}^*)^{-1}(y) \neq \emptyset$
2. (a) If $\bar{y}^*, \underline{y}^*$ are increasing functions: $\bar{y}^*(x_1) \leq \underline{y}^*(x_2)$ for some $x_1 < x_2$
 (b) If $\bar{y}^*, \underline{y}^*$ are decreasing functions: $\underline{y}^*(x_1) \geq \bar{y}^*(x_2)$ for some $x_1 < x_2$

Condition (1) requires that all job types are accepted by at least one worker type. Condition (2) ensures that there cannot exist range of job types accepted by every worker type. If one or both of the match acceptance bounds is not continuous, uniqueness depends on where the discontinuity is, and must be determined by checking that (C.1) is satisfied for all pairs of distinct job types. Under the production function $\rho(x, y) = x - \delta(x - y)^2$, the boundaries of match acceptance are continuous with $\bar{y}(0) = 0$, so there cannot exist a range of job types that is acceptable to all worker types.

Solution Method. Given $\{u^*(x)\}_{x \in [0,1]}$ and $\{w^*(x, y)\}_{x, y \in [0,1]}$, it is possible to back out the equilibrium vacancy distribution and corresponding vacancy rates. Let $\bar{y}^*(x)$, and $\underline{y}^*(x)$ be the highest acceptable job and lowest acceptable job for a worker of type x . Matches are accepted if and only if $\rho(x, y) \geq w(x, y)$, so bounds on match acceptance are fully determined by wages. The steady-state condition on unemployment,

$$s(1 - u^*(x)) = u^*(x)\theta q(\theta) \int_{\underline{y}^*(x)}^{\bar{y}^*(x)} g(y) dy = u^*(x)\theta q(\theta) [G(\bar{y}^*(x)) - G(\underline{y}^*(x))]$$

determines the probability that a type x worker accepts a randomly drawn job, denoted by $\phi(x)$.

$$\phi(x) = [G(\bar{y}^*(x)) - G(\underline{y}^*(x))] = \frac{s(1 - u^*(x))}{\theta q(\theta) u^*(x)}$$

The worker space is subdivided into three parts as depicted in Figure C.1. For a worker in the range X_1 ,

$$accept(x_1) = [G(\bar{y}^*(x_1)) - G(0)]$$

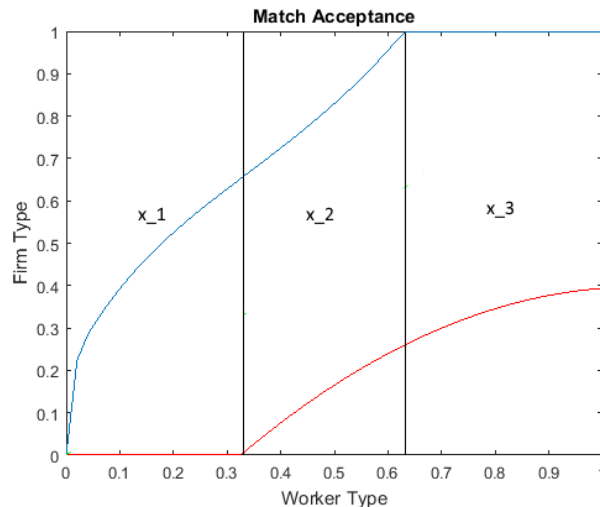


Fig. C.1.: Worker type space subdivided based on match acceptance.

$$G(\bar{y}^*(x_1)) = \frac{s(1 - u^*(x_1))}{\theta q(\theta)u^*(x_1)}$$

For a worker in range X_3 ,

$$accept(x_3) = [1 - G(\underline{y}^*(x_3))]$$

$$G(\underline{y}^*(x_3)) = 1 - \frac{s(1 - u^*(x_3))}{\theta q(\theta)u^*(x_3)}$$

However, this range overlaps with some job types covered by X_1 , so this calculation serves only as a check. Finally, for any worker type in X_2 , there must be a corresponding type in X_1 such that

$$\underline{y}^*(x_2) = \bar{y}^*(x_1)$$

Solving for this job type, I obtain $G(\underline{y}^*(x_2)) = G(\bar{y}^*(x_1))$. Then

$$G(\bar{y}^*(x_2)) = G(\underline{y}^*(x_2)) + \frac{s(1 - u^*(x_2))}{\theta q(\theta)u^*(x_2)}$$

Having calculated $G(y)$ for all job types, it is possible to back out actual vacancy rates. Using a discrete set of job types as in the numerical example, vacancy rates can be computed as

$$v(y_j) = [G(y_j) - G(y_{j-1})]/\ell(y_j)$$

for $y_j \in (0, 1)$.

C.2 Computational Algorithm

1. Guess $w_0(x, y)$, $u_0(x)$, and $v_0(y)$.
2. Calculate $rU_0(x) = b(x) + \frac{\theta_0 q(\theta_0)}{r+s} \int_0^1 \frac{v_0(y)}{v_0} \max\{w_0(x, y) - rU_0(x), 0\} dy$ using *fsolve* to find the fixed point $rU_0(x) \forall x$.
3. Update to $w_1(x, y) = \beta \rho(x, y) + (1 - \beta)rU_0(x)$.
4. Calculate $rU_1(x) = b(x) + \frac{\theta_0 q(\theta_0)}{r+s} \int_0^1 \frac{v_0(y)}{v_0} \max\{w_1(x, y) - rU_1(x), 0\} dy$ using *fsolve* to find the fixed point $rU_1(x) \forall x$.
5. Update to $u_1(x) = \frac{s}{s + \theta_0 q(\theta_0) \int_0^1 \frac{v_0(y)}{v_0} \mathbb{1}(w_1(x, y) \geq rU_1(x)) dy}$
6. Update to $\bar{v}_1 = \bar{u}_1 \left[\frac{\kappa}{r+s} \right]^{1/\alpha}$
7. Update to $v_1(y)$ by reverse engineering $g(y)$ from match sets and acceptance probabilities, as described in Appendix C.1.1.
8. Set $w_0(x, y) = w_1(x, y)$, $u_0(x) = u_1(x)$, and $v_0(y) = v_1(y)$ and repeat (2)-(7) until convergence.