Identification of weights in multi-cteria decision problems based on stochastic optimization

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Abstract

Many scientific papers are devoted to solving multi-criteria problems using methods that find discrete solutions. However, the main challenge addressed by our work is the case when new decision-making variants have emerged which have not been assessed. Unfortunately, discrete identification makes it impossible to determine the preferences for new alternatives if we do not know the whole set of parameters, such as criteria weights.

This paper proposes a new approach to identifying a multi-criteria decision model to address this challenge. The novelty of this work is using a discretization in the space of the problem to identify a continuous decisional model. We present a hybrid approach where the new alternative can be assessed based on stochastic optimization and the TOPSIS technique. The stochastic methods are used to find criteria weights used in the TOPSIS method. In that way, we get assessed easily each new alternative based only on the initial set of evaluated alternatives.

Keywords:

TOPSIS; Stochastic algorithms; MCDM; Criteria weights.

Introduction

The growing popularity of Multi-Criteria Decision-Making (MCDM) methods causes a significant increase in interest in this topic. In the world of science, there are more and more ways to use them, and with the growing interest comes the need for new solutions to allow a more accurate analysis of solutions [Kizielewicz et. al 2020]. One of the essential values in decision-making problems is the criteria weights [Paradowski et. al 2021].

Solving decision-making problems using stochastic methods does not guarantee success, but they can solve severe and different problems. Furthermore, stochastic techniques are easy for complex problems to evaluate the ”black box” function [Więckowski et. al 2020]. The mathematical structure of the problem under investigation is more important in understanding the deterministic approach than the stochastic approach [Holman et. Al 2009]. However, deterministic methods are effective in local search. For this particular purpose, algorithms such as Ant Colony Optimization (ACO), Particle Swarm Optimization (PSO), and Differential Evolution (DE) can be used. Each algorithm presents a different approach to solving the problem to choose the best-behaved method finally [Kizielewicz et. al 2021].

Multi-Criteria Decision Methods allow obtaining preference values for a specific alternative. This allows us to use the multi-criteria decision-making method to back-calculate the criteria weights. In this case, the TOPSIS method was used as a fitness function. It is one of the most frequently used methods and, therefore, the most popular one.

This paper presents the use of the methods mentioned above to identify weights from an existing and already solved decision problem. The main novelty of this work is using a discretization in the space of the problem to identify a continuous decisional model. The purpose of the created solution is to analyze the presented decision problem further and compare new options. The main problem addressed in our work is to search for optimal values of weights for assessed sets of decision variants, i.e., those for which the objective function will reach the lowest value. This problem occurs when we calculate decision-making variants' preferences based on a set of assessed alternatives. The objective function in this problem is the absolute difference of the sums of the preferences of the reference decision variants with the calculated ones.

The rest of the paper is organized as follows. In Section 2, the basic assumptions for the methods used are presented. Section 3 describes the study case presenting the obtained results and the performance of the presented solution. The last Section contains conclusions and further directions.

Preliminaries

This section presents the most important aspects of particular methods used in our research. Please refer to the appropriate references for a full description of the methods. The selected stochastic methods and the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) are briefly presented below.

Ant colony optimization (ACO) was initially proposed in 1992 [Koza 1992]. The ACO algorithm was designed to solve optimization problems based on discrete values in their original form. Such an approach sampled discrete values from the search space. The ant colony optimization for continuous domains variant was first presented by Krzysztof Socha and Marco Dorigo in 2008 [Socha and Dorigo 2008]. Their approach to ant colony optimization provided a method capable of tackling both continuous and mixed-variable optimization problems by sampling the Gaussian function. For this variant of the ACO algorithm, the Gaussian kernel probability density function (pdf) needs to be determined, which is defined as $\mu_l^i = s_l^i$. In this study, we set the initial parameters as: Ants – 100, Offspring – 50, Intensification Factor – 0.5, Deviation-Distance Ratio – 1.

The particle swarm optimization (PSO) algorithm was first introduced by Kennedy and Eberhart in 1995 [Kennedy and Eberhart 1995]. Its working principle is that one creates particles with initial parameters that are used to search for the best solution in the search space. This algorithm has many extensions and has been modified to meet certain criteria. Most of the extensions of this algorithm are designed for searching continuous domains, and such a variant was chosen in this research [Wang et. al 2018]. In the PSO algorithm, the fitness function should be used, and for the purpose of further discussion, it will be denoted as f. For the study, we determine the initial parameters presented below: Swarm – 100, Fi Particle – 2.05, Fi Swarm – 2.05, Omega – 0.8, Velocity - 0.5.

Differential evolution (DE) is another metaheuristic algorithm that searchers a large problem space for a solution. Differential evolution was initially presented by Rainer Storn and Kenneth Price in 1997 [Ullensvang et. al 1997]. The principle of this algorithm is to search the problem space using agents that are initialized with a random position in the problem space [Houssein et. al 2021]. This algorithm implements three different principles, namely mutation, crossover and selection. The mutation process is the one that creates new solutions for a given agent. The initial parameters were defined as follows: Population – 100, Cross probability – 0.2, Factor – [0.2, 0.8].

The TOPSIS method was first introduced by Ching-Lai Hwang and Yoon in 1981 [Hwang and Yoon 1981] and then further improved [Lai et. al 1994]. This is one of the most popular methods amongst the researchers, mostly because of its easy execution and being one of the oldest multi-criteria decision-making methods. Numerous researchers extended this method to provide its usage in fields like fuzzy problems, interval problems or group decision making [Behzadian et. al 2012].

Study case

The following research is dedicated to identifying weights in a multi-criteria problem using stochastic methods. The problem assumes that the weights of the criteria used to calculate the preference values are unknown. Meanwhile, the aim was to determine a vector of weights based on the stochastic methods to allow the analysis of already evaluated alternatives and, as accurately as possible, provide the consistent evaluation of new decision variants.

In the problem, we used 10 criteria whose type was determined randomly. To determine the reference preference values, we used entropy to calculate weights, while to evaluate the alternatives TOPSIS method was used. The identification of weights in the problem was carried out on 100 alternatives. The study ran the tests 100 times, where each step lasted 1000 iterations. During this time, we evaluated the identified criteria weights using stochastic optimization methods, namely ACO, PSO, and DE.

The results obtained with the Ant Colony Optimization method are shown in Figure 1. The value of the fitness function decreases with successive iterations, which is the desired effect in the study. The divergence between the upper and lower bounds increased as the algorithm proceeded.

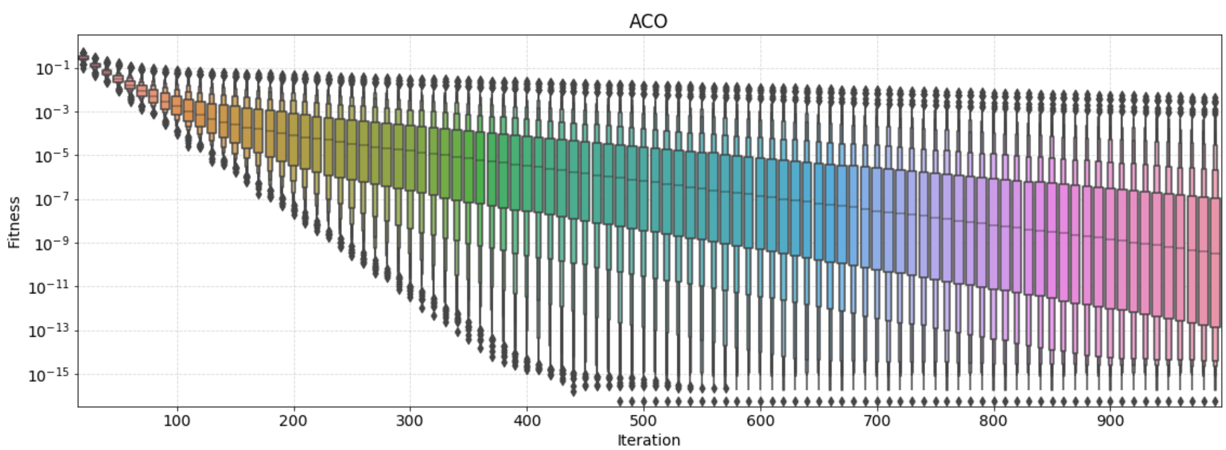

Figure 1. The visualization of the identification process of the ACO method.

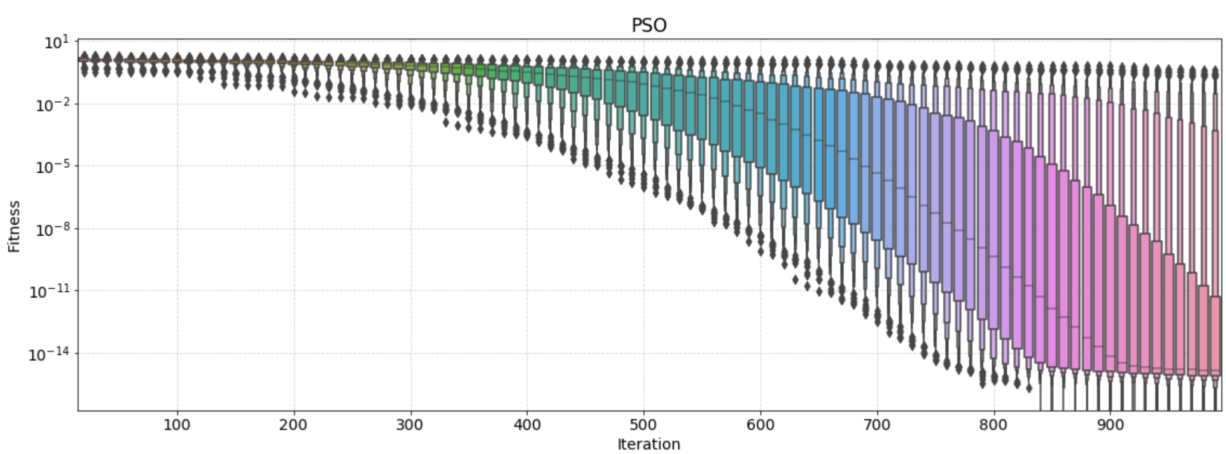

Figure 2. The visualization of the identification process of the PSO method.

The identification process for the Particle Swarm Optimisation method is shown in Figure 2. It is worth noting that, compared to the previously discussed method, the convergence of the values occurred after more iterations. However, the final values achieved were characterized by less divergence.

Figure 3 shows the process of identifying the criteria weights performed by the DE method. It is worth noting that the upper constraint slightly decreases in value with successive iterations, while the main change is seen for the lower constraints on the function values.

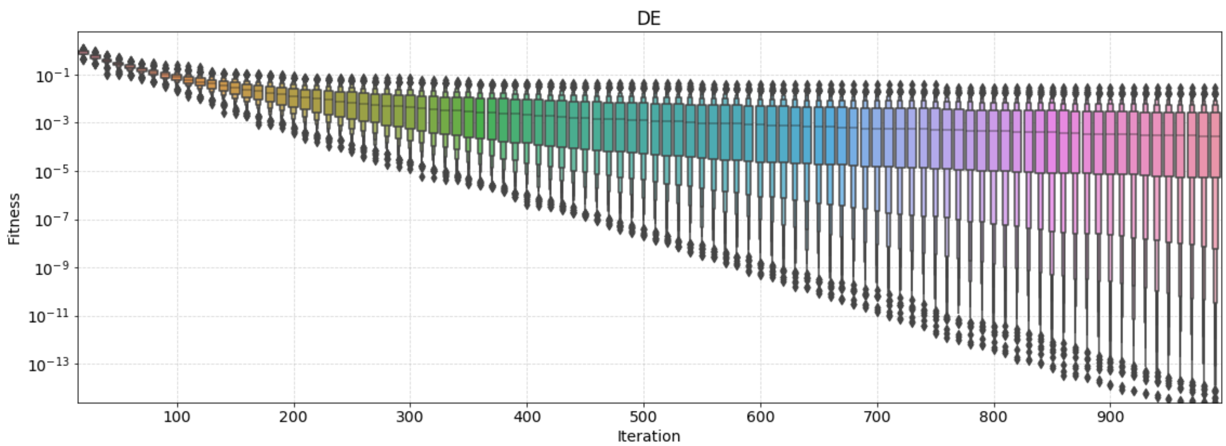

Figure 3. The visualization of the identification process of the DE method.

Based on the research results, it can be noted that the best adjustment of the criteria weights was characterized by the PSO method. Compared to the other methods, it stood out for its greater convergence within the investigated range and the established input parameters of the methods.

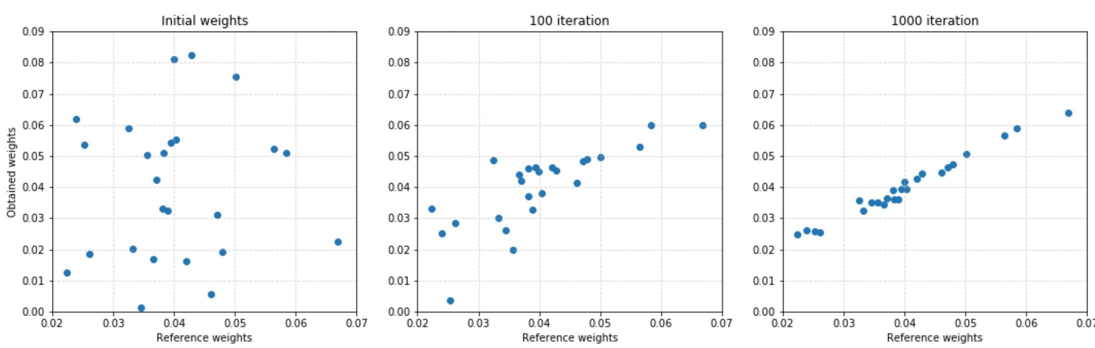

Figure 4. Convergence visualization of weight search process depending on the number of iterations for DE.

Visualization of weights obtained with the DE method is included in Figure 4 to show an example of how values of weights are distributed with successive iterations of the algorithm. It is worth noting that subsequent stages were characterized by much better convergence. In addition, it shows the effectiveness of the algorithm and the rational decisions made to change the values of the criteria weights.

Conclusion

The rapid development of multi-criteria decision-making methods causes new ways for their processing to be sought. For this reason, we have noticed the need to obtain the weights from an existing solution containing preferences for particular criteria. The presented results show that the solution proposed by us fulfills its sentence and allows us to obtain almost identical weights to the input ones. Furthermore, the use of the PSO algorithm allowed for the best convergence, and thus we can say that it turned out to be the most rational method in the case of this study case.

We have presented in this paper a proof of concept that needs to be further investigated. However, it already shows that a hybrid approach between MCDA methods and stochastic methods is reasonable. It would be worthwhile to test the solution for more criteria or use other multi-criteria decision-making methods in future work. Additionally, the behavior in uncertain environments or group decision-making problems could be investigated.

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