Tracking Errors and Long-Run Performance of Leveraged ETFs

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Research Objectives

Leveraged exchange-traded funds (ETFs) are relatively new to the world of investments but have become increasingly popular to aggressive investors. While a regular ETF tracks the value of a specific index of stocks, a leveraged ETF attempts to achieve a multiple of the return of the underlying index on a daily basis. This multiple can be positive in the case of bull ETFs or negative in the case of bear (or inverse) ETFs. To accomplish these objectives, leveraged and inverse funds pursue a range of investment strategies through the use of swaps, futures contracts, options and other derivative instruments. Due to the effect of compounding, operating expenses and daily resets, not to mention tracking errors, the performance of leveraged funds over longer periods of time can differ substantially from the performance (or inverse of the performance) of their underlying index or benchmark during the same period of time. Such performance deviations are often quite meaningful and unexpected over the long run.

This paper provides an empirical assessment of how well the leveraged ETFs track their underlying index. The results show that the tracking errors on average are small. However, substantial tracking errors do occur from time to time. Despite the price decay associated with leveraged ETFs, their long-run performance.

Statistical Properties of Leveraged ETFs

Arithmetic vs. Geometric Returns

When evaluating the performance of leveraged and inverse ETFs, we have to distinguish between geometric and arithmetic averages. The latter is the simple average of daily returns over a given period of time. The former, on the other hand, represents an equivalent but constant daily return over the same period.

Consider a time-series of daily returns of the underlying asset for T days: $R_1, R_2, \ldots, R_{T-1}$, and R_T. The arithmetic daily average return, R_A, and the standard deviation of returns, σ, are defined as

$$R_A = \frac{1}{T}\sum_{i=1}^{T} R_i \quad \text{and} \quad \sigma^2 = \frac{1}{T}\sum_{i=1}^{T}(R_i - R_A)^2$$

The geometric mean, R_G, is defined as

$$R_G = -1 + \prod_{i=1}^{T}(1+R_i)^{1/T}$$

It can be shown that the following relationship holds between R_A and R_G:

$$R_G \approx -1 + \prod_{i=1}^{T}\left(1 + \frac{2}{T}R_i + \frac{2-T}{2T^2}R_i^2\right)$$
$$\approx \frac{1}{T}\sum_{i=1}^{T} R_i + \frac{1}{T^2}\sum_{i\neq j}^{T} R_i R_j + \frac{1-T}{2T^2}\sum_{i=1}^{T} R_i^2$$
$$= R_A - \sigma^2/2$$

Price Decay of Leveraged ETFs

Now consider a leveraged bull ETF that aims to change in the same direction by n times the daily change of the underlying index. For simplicity, we assume no tracking error, management fees, or interest charges. Under this assumption, the arithmetic average of this nX ETF will be exactly nR_A, and the standard deviation will be nσ. It can be shown that its geometric average will be approximately

$$nR_G - n(n-1)\,\sigma^2/2$$

The last term, $n(n-1)\,\sigma^2/2$, is often referred to as the price decay that is caused by volatility. The summary for various leveraged ETFs is presented in the top table.

Statistical Properties of Leveraged ETFs and the Underlying Index: A Summary

The underlying index (referred to as 1X) has an arithmetic average of R_A, geometric average of R_G, and standard deviation of σ. The leveraged and inverse ETFs are assumed to achieve exactly n times the return of the index on the daily basis, where n is positive for bull ETFs and negative for bear ETFs.

Leveraged ETF	Description	Arithmetic average	Geometric average	Price Decay
1X	Underlying index	R_A	$R_G \approx R_A - \sigma^2/2$	$\sigma^2/2$
2X	2X Bull ETF	$2R_A$	$2R_G - \sigma^2$	σ^2
3X	3X Bull ETF	$3R_A$	$3R_G - 3\sigma^2$	$3\sigma^2$
-1X	-1X Bear ETF	$-R_A$	$-R_G - \sigma^2$	σ^2
-2X	-2X Bear ETF	$-2R_A$	$-2R_G - 3\,\sigma^2$	$3\sigma^2$
-3X	-3X Bear ETF	$-3R_A$	$-3R_G - 6\sigma^2$	$6\sigma^2$

Data and Sample

This paper examines two groups of leveraged ETFs which are offered from ProShares, the pioneer of leveraged ETFs. The first panel in the following table presents the five ETFs using the S&P 500 index as the underlying index. The second panel lists the five ETFs that are based on the Russell 2000 index. The S&P500 index is essentially a large-cap index while the Russell 200 is a small-cap index. We collect daily returns for these indexes and ETFs from inception to year-end 2013.

Name	Symbol	Objective	Inception	Fee
Panel A: LETFs based on S&P500 Index				
S&P500 EFT (benchmark)	SPY	1X		
Ultra S&P500	SSO	2X	6/19/2006	0.90%
UltraPro S&P500	UPRO	3X	6/23/2009	0.95%
Short S&P500	SH	-1X	6/19/2006	0.90%
UltraShort S&P500	SDS	-2X	7/11/2006	0.90%
UltraProShort S&P500	SPXU	-3X	6/23/2009	0.93%
Panel B: LETFs based on Russell2000 Index				
iShares Russell2000 (benchmark)	IWM	1X		
Ultra Russell2000	UWM	2X	1/23/2007	0.98%
UltraPro Russell2000	URTY	3X	2/9/2010	0.98%
Short Russell2000	RWM	-1X	1/23/2007	0.95%
UltraShort Russell2000	TWM	-2X	1/23/2007	0.95%
UltraPro Short Russell2000	SRTY	-3X	2/9/2010	0.95%

Tracking Errors and Summary Statistics

The main objective of this study is examine how well the leveraged ETF (LETF) tracks its underlying index. For bull ETFs, tracking error is defined as $(R_{LETF} - n\,R_{Index})$; For bear ETFs, tracking error is defined as $(R_{LETF} + n\,R_{Index})$. R_{LETF} is the daily return of the LETF and R_{Index} is the daily return of the index, and n is the stated multiple.

For the S&P500-based LETFs, both the mean and median tracking errors are small. However, the minimum (-0.0667) and the maximum (0.06586) suggest the existence of substantial tracking errors.

Name	Symbol	Daily Objective	Mean	Median	Min	Max	Standard Deviation
S&P500 EFT (benchmark)	SPY	1X	/	/	/	/	/
Ultra S&P500	SSO	2X	-0.00014	-0.00006	-0.06607	0.03095	0.00357
UltraPro S&P500	UPRO	3X	-0.00007	-0.00002	-0.03416	0.03153	0.00272
Short S&P500	SH	-1X	0.00007	-0.00002	-0.02091	0.03691	0.00204
UltraShort S&P500	SDS	-2X	0.00010	0.00005	-0.04039	0.06586	0.00332
UltraProShort S&P500	SPXU	-3X	-0.00006	-0.00007	-0.04095	0.04037	0.00269

The results are similar for the Russell2000-based LETFs. Again, both the mean and median tracking errors are small. However, the minimum (-0.06161) and the maximum (0.05407) suggest the existence of substantial tracking errors.

Name	Symbol	Daily Objective	Mean	Median	Min	Max	Standard Deviation
iShares Russell2000 (benchmark)	IWM	1X	/	/	/	/	/
Ultra Russell2000	UWM	2X	-0.000089	0.000000	-0.049420	0.039220	0.003873
UltraPro Russell2000	URTY	3X	-0.000049	0.000000	-0.018610	0.015380	0.002433
Short Russell2000	RWM	-1X	-0.000011	-0.000070	-0.012470	0.012440	0.002125
UltraShort Russell2000	TWM	-2X	-0.000041	-0.000065	-0.036720	0.054070	0.003593
UltraPro Short Russell2000	SRTY	-3X	-0.000240	-0.000140	-0.061610	0.040540	0.003461

Regression Analyses

Another approach to access the tracking effectiveness is regress the daily return of an EFT on the daily return of its underlying index as follows:

$$R_{LETF} = \alpha + \beta\, R_{Index} + \varepsilon$$

where R_{LETF} is the daily return of the LETF, and R_{Index} is the daily return of the index. The coefficient estimate, β, can be used to test the hypothesis that the leveraged ETF achieves its stated objective, i.e., n times the underlying index's return.

The regression results, presented in the following table, show that the adjusted R^2 is very high and close to 1.0 in all regressions, indicating strong tracking effectiveness. Note that a R^2 value of 1.0 indicates perfect tracking. Additionally, the estimated β is very close to its expected value.

Name	Symbol	Daily Objective	Expected β	Estimated β	Standard error	Adj. R^2
Panel A: LETFs based on S&P500 Index						
Ultra S&P500	SSO	2X	2.0	1.921	0.0055	0.985
UltraPro S&P500	UPRO	3X	3.0	2.977	0.0076	0.996
Short S&P500	SH	-1X	-1.0	-0.9821	0.0033	0.979
UltraShort S&P500	SDS	-2X	-2.0	-1.936	0.0052	0.987
UltraProShort $&P500	SPXU	-3X	-3.0	-2.978	0.0075	0.993
Panel B: LETFs based on Russell2000 Index						
Ultra Russell2000	UWM	2X	2.0	2.002	0.0051	0.989
UltraPro Russell2000	URTY	3X	3.0	2.977	0.0076	0.997
Short Russell2000	RWM	-1X	-1.0	-0.9996	0.0028	0.987
UltraShort Russell2000	TWM	-2X	-2.0	-2.006	0.0047	0.990
UltraPro Short Russell2000	SRTY	-3X	-3.0	-2.955	0.0075	0.994

Price Decay and Long-Run Performance

The statistical properties of LETFs suggests a price decay. Consider the case of the 2X bull ETF, which aims to achieve twice the daily return of the underlying index on a daily basis. Suppose the underlying index (i.e., the S&P 500 Index) has zero one-year total return. Due to the price decay caused by daily resets and volatility, the one-year total return for the bull ETF is -6.1%, even under perfect tracking.

The long-run performance is calculated as the cumulative daily return. The table below presents the year-end 2013 value of one dollar invested at the LETF at inception. For instance, one dollar invested in the UltraPro S&P500 (3X) bull ETF on June 23, 2009 would become $6.697 by the end of 2013, compared to only $2.177 if invested in the benchmark index fund. However, the same dollar would be only $0.041 if invested in the 3X bear ETF.

Name	Symbol	Daily Objective	Inception	Cumulative Return of LETF	Cumulative Return Of Benchmark index
Panel A: LETFs based on S&P500 Index					
Ultra S&P500	SSO	2X	6/19/2006	1.648	1.729
UltraPro S&P500	UPRO	3X	6/23/2009	6.697	2.199
Short S&P500	SH	-1X	6/19/2006	0.452	1.729
UltraShort S&P500	SDS	-2X	7/11/2006	0.129	1.743
UltraProShort S&P500	SPXU	-3X	6/23/2009	0.041	2.199
Panel B: LETFs based on Russell2000 Index					
Ultra Russell2000	UWM	2X	1/23/2007	1.239	1.628
UltraPro Russell2000	URTY	3X	2/9/2010	4.253	2.024
Short Russell2000	RWM	-1X	1/23/2007	0.355	1.628
UltraShort Russell2000	TWM	-2X	1/23/2007	0.060	1.628
UltraPro Short Russell2000	SRTY	-3X	2/9/2010	0.028	2.024

Summary

The study attempts to understand the statistical properties of leveraged and inverse ETFs. It also provides an empirical assessment of how well the leveraged ETFs track their underlying index. The results show that the tracking errors on average are small. However, substantial tracking errors do occur from time to time.

In conclusion, leveraged ETFs may be appropriate for aggressive investors who want to double or triple their short-term returns, but buy-and-hold investors must be warned of the long-run impacts of price decays.