$E^{-\gamma}$
T^{-m}
γ.

A Mathematical Framework for the Energy Spectrum of Primary Cosmic Rays

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Abstract

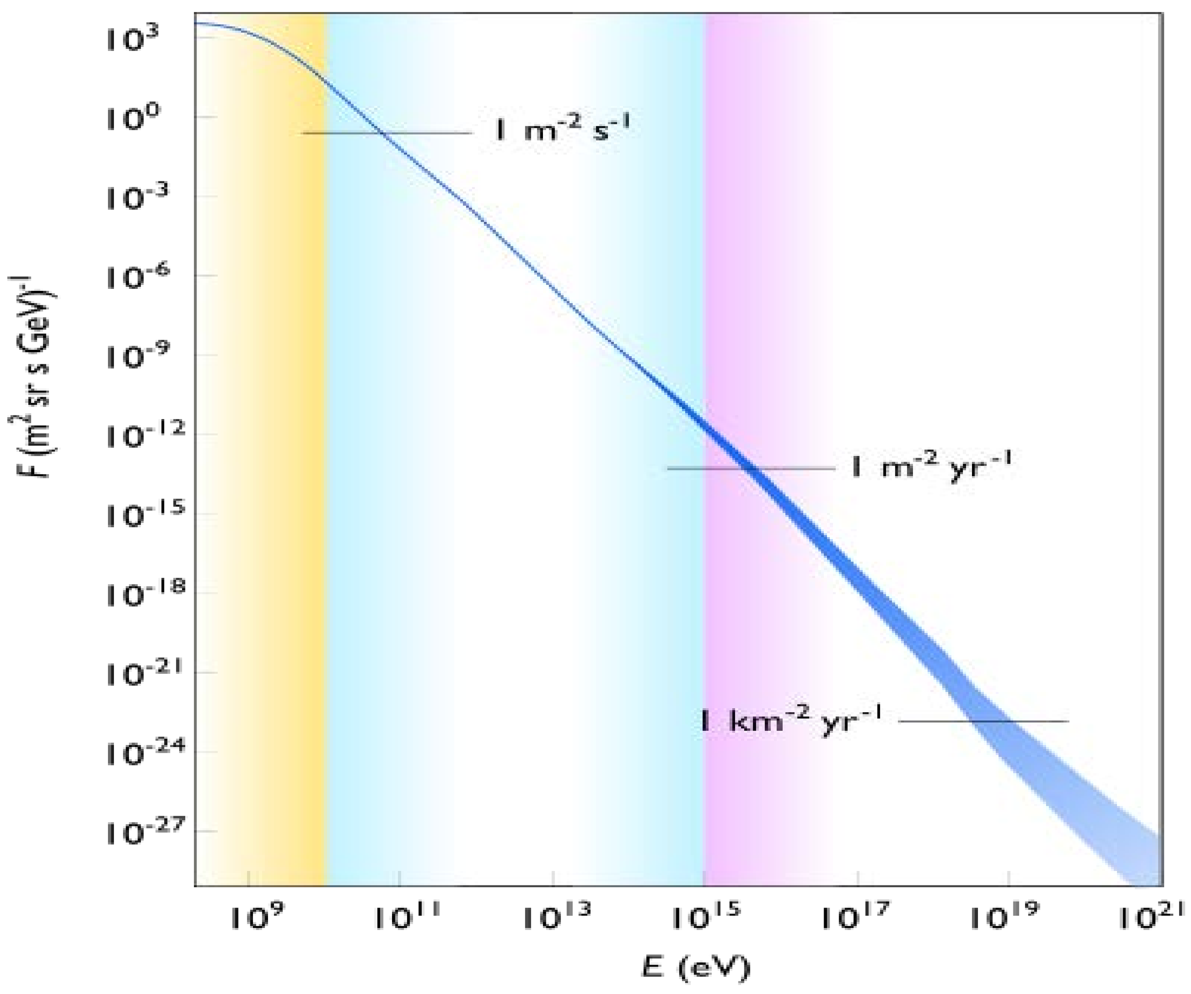
Primary cosmic rays are nucleons from outer space incident upon the Earth's atmosphere. Their flux varies with energy *E* as $E^{-\gamma}$ in which the exponent assumes values between 2.5 and 3.2. We provide herein a framework to account for these values. We consider the particles' kinetic temperature *T* and introduce a damping factor T^{-m} to account for non head-on collisions. It is the presence of the index *m* which can provide values of *γ*.

Introduction

Primary cosmic rays (CRs) are extraterrestrial nuclear particles of high energy which, when incident upon the Earth's atmosphere, interact with atmospheric molecules to create a cascade of numerous secondary particles known collectively as an air shower. Primary CRs consist mostly of protons with lesser amounts of helium nuclei and other elements up to iron. Sources include quasars, neutron stars, supernovae, black holes and the Big Bang. Means to account for the observed high energies include bottom-up mechanisms such as acceleration by shock waves or cosmic magnetic fields and top-down models in which the particles possess an initially high energy at their birth.

The Energy Spectrum

Although CRs have been the subject of intense study since their discovery in 1912, there remain unanswered questions of a fundamental nature. Among them is the origin of the declining power laws which characterize the energy spectrum of primary CRs. Their flux, observed over more than fourteen orders of magnitude of energy, varies with energy as $E^{-\gamma}$ in which the exponent assumes values between 2.5 and 3.2. Such a spectrum is shown below.



It is the slope of this curve which we address here. We adopt a top-down model in which a highly energetic particle strikes a nucleus which we picture as a Bohr-model liquid drop with binding energy E_B and at initial kinetic temperature $T = T_0$. The collision raises *T* and the nucleus is disrupted if the energy transmitted exceeds E_B. The resulting emitted particles comprise the primary CRs.

To study the energy spectrum we assume a Boltzmann distribution for these particles so that the number with temperature *T*, in energy range (*E*, *E* + *dE*) and momentum range [*p*, *p* + *dp*] is given by

$$dN(E) \propto p^2 \exp(-E/kT)dp$$
$$= E^2 \exp(-E/kT)dE \quad \text{for} \quad p \approx E/c.$$

This expression would suffice if all collisions were head-on with maximum transfer of energy and high increase in temperature. Since this is an unlikely scenario we introduce a modification in the form of a factor $T^{-m}, m > 0$, to damp the number of collisions which result in a large increase in *T*. Thus the number in (*E*, *E* + *dE*) produced by collisions resulting in final temperatures between (*T*, *T* + *dT*) is

$$dN(E,T) \propto T^{-m} E^2 \exp(-E/kT)\, dT\, dE.$$

Hence the total in (*E*, *E* + *dE*) that result from all collisions is the integral over *T* (or *kT*). Thus

$$dN(E) = \int_{kT_0}^{kT_M} (kT^{-m})E^2 \exp(-E/kT)d(kT)dE$$

where T_M is the maximum possible temperature to which the struck nucleus can be raised in a single collision. A change of variable to 1/*kT* and *m*-2 integrations by parts yield

$$dN(E) \propto (m-2)!E^{-(m-3)}$$
$$\times [G_m(E)\exp(-E/kT_M) - H_m(E)\exp(-E/kT_0)]dE$$

in which

$$G_m(E) = \sum_{r=0}^{m-2} \frac{1}{r!}\left(\frac{E}{kT_M}\right)^r, \quad H_m(E) = \sum_{r=0}^{m-2} \frac{1}{r!}\left(\frac{E}{kT_0}\right)^r, \quad m \geq 2.$$

Since $T_M \gg T_0$ we retain only the first term; thus

$$dN(E) \propto (m-2)!E^{-(m-3)}G_m(E)\exp(-E/kT_M)dE.$$

Expanding $G_m(E)$ and $\exp(-E/kT_M)$ and forming their product, we find

$$G_m(E)\exp(-E/kT_M) = 1-(E/kT_M)^2 + \ldots \approx 1$$

in which we have neglected higher-orders. The final approximation is justified since few of the struck nuclei attain energies corresponding to T_M. The final form of *dN*(*E*) then becomes

$$dN(E) \propto (m-2)!E^{-(m-3)}dE$$

from which we see that *m* ≈ 5-6 provides the requisite powers of *E*.

References

[1]. L. Motz and D. W. Kraft, *Annals of the NY Academy of Sciences*, **655**, 185 (1992).
[2]. D. W. Kraft, Meeting of American Physical Society, New England Section, Fall 2015.