Statistical Properties and Empirical Assessments of Leveraged ETFs

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Research Objective

Leveraged exchange-traded funds (ETFs) are relatively new to the world of investments but have become increasingly popular to aggressive investors. While a regular ETF tracks the value of a specific index of stocks, a leveraged ETF attempts to achieve a multiple of the return of the underlying index on a daily basis. This multiple can be positive in the case of bull ETFs or negative in the case of bear (or inverse) ETFs. To accomplish these objectives, leveraged and inverse funds pursue a range of investment strategies through the use of swaps, futures contracts, options and other derivative instruments.

Due to the effect of compounding, operating expenses and daily resets, not to mention tracking errors, the performance of leveraged funds over longer periods of time can differ substantially from the performance (or inverse of the performance) of their underlying index or benchmark during the same period of time. Such performance deviations are often quite meaningful and unexpected over the long run.

This paper evaluates the nature and statistical properties of leveraged and inverse ETFs and in particular the long-run impacts as compared to their underlying indexes. It also provides an empirical assessment a sample of such ETFs.

Statistical Properties of Leveraged ETFs

Arithmetic vs. Geometric Returns

When evaluating the performance of leveraged and inverse ETFs, we have to distinguish between geometric and arithmetic averages. The latter is the simple average of daily returns over a given period of time. The former, on the other hand, represents an equivalent but constant daily return over the same period.

Consider a time-series of daily returns of the underlying asset for T days: $R_1, R_2, \ldots, R_{T-1}$, and R_T. The arithmetic daily average return, R_A, and the standard deviation of returns, σ, are defined as

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$$R_A = \frac{1}{T}\sum_{i=1}^{T} R_i \quad \text{and} \quad \sigma^2 = \frac{1}{T}\sum_{i=1}^{T}(R_i - R_A)^2$$

The geometric mean, R_G, is defined as

$$R_G = -1 + \prod_{i=1}^{T}(1+R_i)^{1/T}$$

It can be shown that the following relationship holds between R_A and R_G:

$$R_G \approx -1 + \prod_{i=1}^{T}\left(1 + \frac{2}{T}R_i + \frac{2-T}{2T^2}R_i^2\right)$$
$$\approx \frac{1}{T}\sum_{i=1}^{T} R_i + \frac{1}{T^2}\sum_{i\neq j}^{T} R_i R_j + \frac{1-T}{2T^2}\sum_{i=1}^{T} R_i^2$$
$$= R_A - \sigma^2/2$$

Price Decay of Leveraged ETFs

Now consider a leveraged bull ETF that aims to change in the same direction by n times the daily change of the underlying index. For simplicity, we assume no tracking error, management fees, or interest charges. Under this assumption, the arithmetic average of this nX ETF will be exactly nR_A, and the standard deviation will be nσ. It can be shown that its geometric average will be approximately

$$nR_G - n(n-1)\,\sigma^2/2$$

The last term, $n(n-1)\,\sigma^2/2$, is often referred to as the price decay that is caused by volatility. The summary for various leveraged ETFs is presented in the top table.

Table Statistical Properties of Leveraged ETFs and the Underlying Index

The underlying index (referred to as 1X) has an arithmetic average of R_A, geometric average of R_G, and standard deviation of σ. The leveraged and inverse ETFs are assumed to achieve exactly n times the return of the index on the daily basis, where n is positive for bull ETFs and negative for bear ETFs.

Leveraged ETF	Description	Arithmetic average	Geometric average	Price Decay
1X	Underlying index	R_A	$R_G \approx R_A - \sigma^2/2$	$\sigma^2/2$
2X	2X Bull ETF	$2R_A$	$2R_G - \sigma^2$	σ^2
3X	3X Bull ETF	$3R_A$	$3R_G - 3\sigma^2$	$3\sigma^2$
-1X	-1X Bear ETF	$-R_A$	$-R_G - \sigma^2$	σ^2
-2X	-2X Bear ETF	$-2R_A$	$-2R_G - 3\,\sigma^2$	$3\sigma^2$
-3X	-3X Bear ETF	$-3R_A$	$-3R_G - 6\sigma^2$	$6\sigma^2$

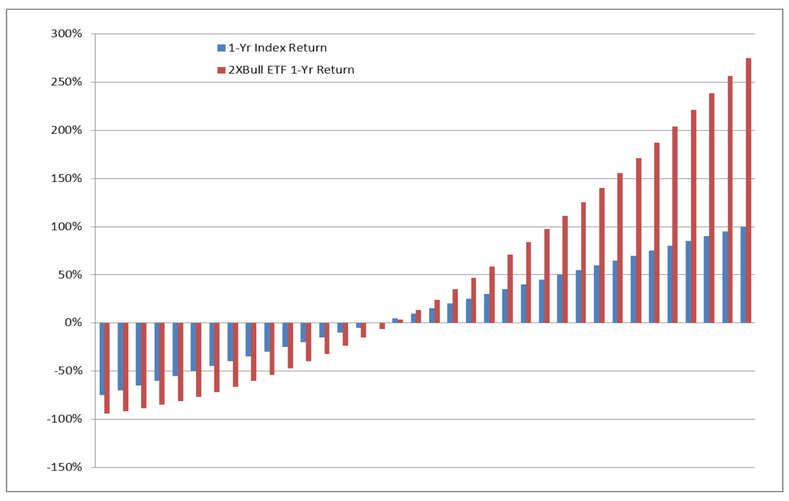
Data and Sample

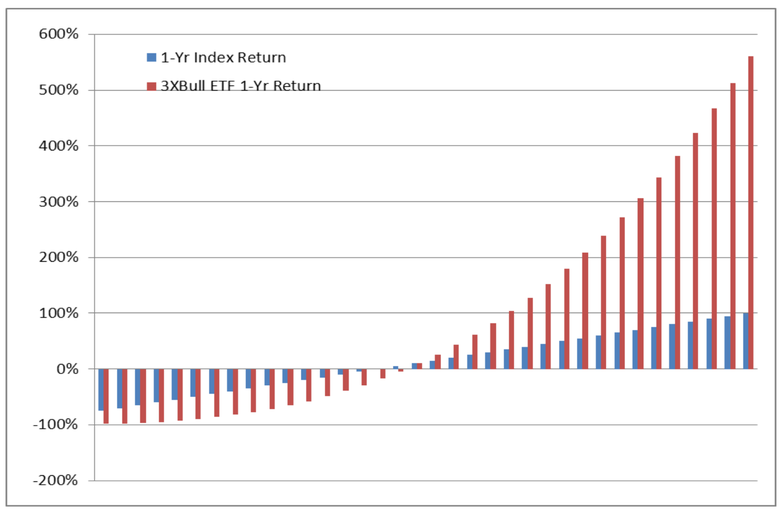
This paper examines two groups of leveraged ETFs which are offered from ProShares, the pioneer of leveraged ETFs. The first panel in the following table includes the five ETFs that use the S&P 500 index as the underlying index. The second panel lists the five ETFs that are based on the Russell 2000 index.

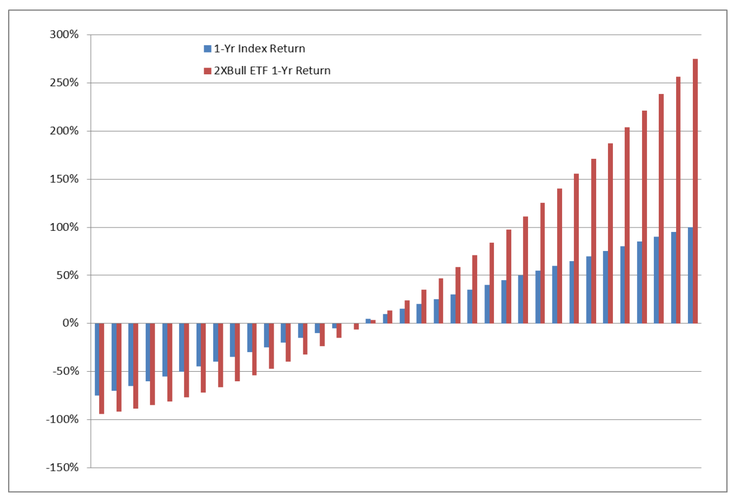
Name	Symbol	Objective	Inception	Fee
Panel 1				
Spider (benchmark)	SPY	1X		
Ultra S&P500	SSO	2X	6/19/2006	0.90%
UltraPro S&P500	UPRO	3X	6/23/2009	0.95%
Short S&P500	SH	-1X	6/19/2006	0.90%
UltraShort S&P500	SDS	-2X	7/11/2006	0.90%
UltraProShort S&P500	SPXU	-3X	6/23/2009	0.93%
Panel 2				
iShares Russell2000 (benchmark)	IWM	1X		
Ultra Russell2000	UWM	2X	1/23/2007	0.98%
UltraPro Russell2000	URTY	3X	2/9/2010	0.98%
Short Russell2000	RWM	-1X	1/23/2007	0.95%
UltraShort Russell2000	TWM	-2X	1/23/2007	0.95%
UltraPro Short Russell2000	SRTY	-3X	2/9/2010	0.95%

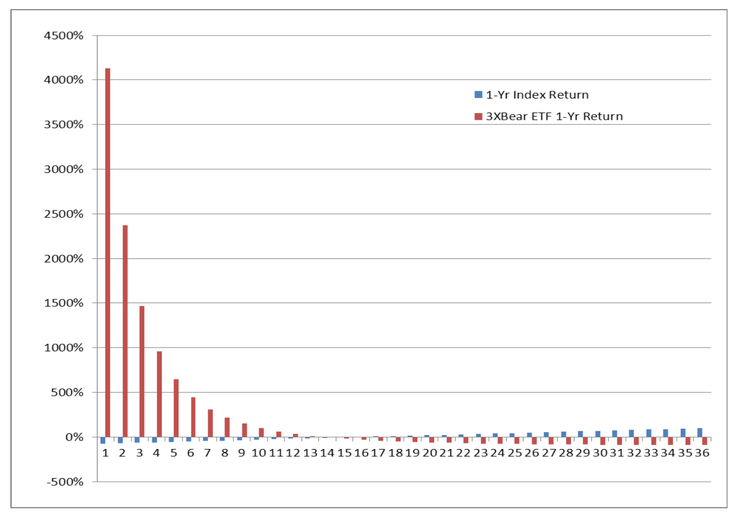
Buy-and-Hold Returns: Simulations

Buy-and-hold investors must be aware that daily volatility will have a significant effect on performance over longer periods of time. Such performance deviations are often quite meaningful and unexpected. Here we offer simulations of one-year return for the 2X bull ETF and -2X bear ETF, as compared to the underlying index. We assume that the standard deviation is 25%.









Implications to Investors

Consider the case of the 2X bull ETF (first figure), which aims to achieve twice the daily return of the underlying index on a daily basis. Suppose the underlying index (i.e., the S&P 500 Index) has zero one-year total return. Due to the price decay caused by daily resets and volatility, the one-year total return for the bull ETF is -6.1%. If the one-year index return is 10%, the return of the bull ETF is only 13.7% during the same period.

The price decay is more severe for the 3X bull ETF (figure 2). When the underlying index has zero one-year return, the 3X bull ETF will have a one-year return of -17.1%, due to price decay.

For bear ETFs, the impact of price decay over longer periods of time is even more severe. Suppose the underlying index has a zero return over one year. The returns over the same period will be -6.1%, -17.1% and -31.3%, respectively, for 1X, 2X, and 3X bear ETFs.

That's why in 2009, the Financial Industry Regulatory Authority (FINRA) and the Securities and Exchange Commission (SEC) jointly issued an Investor Alert entitled "Leveraged and Inverse ETFs: Specialized Products with Extra Risks for Buy-and-Hold Investors."

Summary

The purpose of the current study is to understand the statistical properties of leveraged and inverse ETFs and to examine the impact of price decay on buy-and –hold returns over longer periods of time.

The simulation results indicate that the performance of leveraged ETFs over longer periods of time can differ substantially from the performance of their underlying benchmark during the same period of time. Such performance deviations are often quite meaningful and unexpected over the long run.

In conclusion, leveraged ETFs may be appropriate for aggressive investors who want to double or triple their short-term returns, but buy-and-hold investors must be warned of the long-run impacts of price decays.