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Essays on Supply Chain Economic Networks for Disaster Management Inspired by the COVID-19 Pandemic

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ESSAYS ON SUPPLY CHAIN ECONOMIC NETWORKS FOR DISASTER MANAGEMENT INSPIRED BY THE COVID-19 PANDEMIC

A Dissertation Presented

by

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DEDICATION

To my beloved family.

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ABSTRACT

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The COVID-19 pandemic, which was declared by the World Health Organization on March 11, 2020, negatively impacted virtually all economic and social activities across the globe. As of March 14, 2022, more than 6 million deaths have been associated with COVID-19 disease. This health disaster, unlike many other disasters, is not limited to time or location. It has resulted in intense global competition for many essential products, from Personal Protective Equipment (PPE) to ventilators and vaccines and food products. In this dissertation, I construct, analyze, and quantitatively

solve a spectrum of supply chain economic network models inspired by realities in the COVID-19 pandemic in four essays.

In this dissertation, I first develop a game theory network model for integrating financial and logistical challenges that humanitarian organizations involved in disaster management are faced with. This part of the dissertation illustrates how game theory can be utilized in the modeling and analysis of the behavior of multiple decision-makers that interact with each other in disaster supply chain economic networks under different constraints.

I, subsequently, construct the first Generalized Nash Equilibrium (GNE) model with stochastic demands to model competition among organizations at demand points for medical supplies inspired by the COVID-19 pandemic. The theoretical constructs are provided, and a Variational Equilibrium is utilized to enable alternative variational inequality formulations. Then, I delve more deeply into an important characteristic of disasters, that of uncertainty, by developing a two-stage stochastic game theory network model. Specifically, the first multistage stochastic GNE model is constructed for the study of competition among multiple countries for limited supplies of medical items in the disaster preparedness and response phases in the COVID-19 pandemic. Illustrative examples and algorithmically solved numerical examples, inspired by the need for N95 masks and ventilators, are presented.

Finally, I turn to a key aspect of pandemic disaster management, which is the evaluation of trade instruments that governments have been applying during the pandemic to protect their citizens. Specifically, a unified variational inequality framework in the context of spatial price network equilibrium problems is constructed that focuses on a plethora of essential products, that are in high demand in the pandemic, but short in supply globally. The model allows one to seamlessly introduce various trade measures, including tariffs, quotas, as well as price floors and ceilings.

TABLE OF CONTENTS

LIST OF TABLES

Table **Page**

LIST OF FIGURES

Figure **Page**

CHAPTER 1

INTRODUCTION AND RESEARCH MOTIVATION

Every year, many countries face a variety of disasters and related crises that threaten people's lives and properties, with some disasters being unpredictable, and with others predictable, but with differing degrees of certainty (Nagurney and Qiang (2009), FEMA (2020)). With advances in science and technology, preparation for and response to many disasters have improved, especially in the developed world. Furthermore, although devastating, major natural disasters such as Hurricane Katrina in 2005, the earthquake in Haiti in 2010, and Hurricanes Harvey, Irma, and Maria in 2017, never became a global challenge because of such disasters' limitations in both time and space. In contrast, since 2020, the world has been faced with a health disaster in the form of a global pandemic that has affected all countries. The COVID-19 pandemic was declared on March 11, 2020, by the World Health Organization (WHO) (WHO (2022)). Raker, Zacher, and Lowe (2020) have called the pandemic "a disaster of unprecedented scale and scope."

The International Federation of Red Cross and Red Crescent Societies (IFRC) defines a disaster as "a sudden, calamitous event that seriously disrupts the functioning of a community or society and causes human, material, and economic or environmental losses that exceed the community's or society's ability to cope using its own resources. Though often caused by nature, disasters can have human origins" (IFRC (2021a)). The number of disasters, both sudden-onset and slow-onset ones, and their impacts on the economy and people's lives are increasing. Although 2020 will always be remembered as the year in which one of the most catastrophic crises in history began, the COVID-19 pandemic was not the only disaster that afflicted the globe that year. The United States, in 2020, broke the record in the number of billion-dollar weather and climate disasters in a single year with 22 such disasters (NOAA (2021)).

The growing trend of natural disasters and the COVID-19 pandemic demonstrate the increasing importance of research in the area of disaster management. Disaster management has many different aspects that are generally divided into the four phases: mitigation, preparedness, response, and recovery. Numerous decision-makers, such as governments, relief organizations, transportation companies, nonprofit organizations, manufacturers, and many others, with different roles, are involved in the various phases of disaster management. There are also many factors that affect the outcomes of their efforts, including costs, budgets, disaster characteristics, uncertainties, decisions and strategies, trade policies, and competition among those involved. Such decision-makers and factors play roles in supply chain networks and humanitarian logistics and, thus, deserve attention when we investigate disaster management.

In the subsequent sections of this chapter, I provide details on the characteristics of recent disasters and the challenges that they pose with a focus on the COVID-19 pandemic. Also, the background and motivation for research on humanitarian supply chain networks are provided.

1.1. Recent Disasters and Their Consequences

Disasters can be classified in different ways. The Emergency Events Database (EM-DAT) divides them into two general categories of natural disasters and technological ones (EM-DAT (2022)). The Indian Ocean Tsunami in 2004, Hurricane

Katrina in 2005, and the Haitian Earthquake in 2010 are among the worst natural disasters of the 21st century. Until 2020, Hurricane Katrina in 2005, with $161 billion in damage, was the costliest natural disaster in U.S. history. Year 2017, when Hurricanes Harvey, Irma, and Maria made history and left tens of millions of Americans without power, was the costliest year with the cumulative costs of the 16 separate billion-dollar weather events estimated at more than $306.2 billion (COAST NOAA (2022)). 22 billion-dollar weather and climate disasters, a record number in one year, caused more than $95 billion in damage in 2020, more than double the 41-year average, with Hurricane Laura causing $19 billion in damages, the Western wildfires $16.5 billion, and the August derecho $11 billion in damage (Smith (2021)).

However, in 2020, a health disaster occurred whose damage was far bigger than any disaster we have previously encountered, and the figures for the damage caused by it are not comparable to any of the disasters mentioned above. On December 31, 2019, cases of viral pneumonia with unknown causes were detected in Wuhan, China, and reported to the World Health Organization's (WHO's) Country Office in China (WHO (2022)). This was the beginning of the pandemic. COVID-19 is the disease caused by the novel coronavirus that has symptoms such as: cough, fever, shortness of breath, muscle aches, sore throat, unexplained loss of taste or smell, diarrhea, and headache. COVID-19 cases can be severe, with some resulting in death (Johns Hopkins Medicine (2021)). On January 11, 2020, the first death caused by the coronavirus was announced by the Wuhan Municipal Health Commission. On January 21, the first case on US soil was confirmed by officials (CNN (2022)), and nine days later, WHO declared a Public Health Emergency of International Concern (WHO (2022)). The first death due to the illness in the United States was announced on February 29. Finally, on March 11, WHO declared the novel coronavirus outbreak a pandemic, and, two days later, a national emergency was declared in the United States (Neilson and Woodward (2020), CNN (2022)).

As of March 14, 2022, globally, more than 459,000,000 individuals have been infected by the coronavirus SARS-CoV-2, with the number of documented deaths reaching 6,045,000. In the United States, the country with the most cases and deaths due to COVID-19, authorities have reported more than 79,500,000 cases and 965,000 deaths (Johns Hopkins Coronavirus Resource Center (2022)). The COVID-19 pandemic made 2020 a completely different year. The borders of some countries are still restricted to travelers as of 2022. Many major events including the 2020 Tokyo Olympics were canceled or postponed. Schools and universities had to adapt and to continue to work remotely, and even gatherings with friends and loved ones became a dream for many people. Although, with the development of effective vaccines and the vaccination of a large number of people (CDC (2022)), the situation is improving after two years, still many are being infected with new variants of the virus every day, and it cannot be said that the pandemic is over yet.

In this pandemic, the mitigation phase of disaster management was especially challenging due to many flights between countries, and other transportation connections, as well as many unknowns about the novel virus. With the onset of the outbreak in China, some countries, such as New Zealand, Australia, Iceland, and Denmark responded more quickly and more effectively than others (Kamal et al. (2020), Lafortune (2020), Gilbert et al. (2020)). This unprecedented disaster has not only disrupted the lives of billions of people around the world but has also created secondary crises. The deadly blow to the economy continues and governments are trying to control the consequences. The World Trade Organization (WTO) had estimated that trade in 2020 would fall by 13% to 32%, a historically large plunge (WTO (2020a)). The International Monetary Fund (IMF) warned about the severe impacts of the COVID-19 pandemic on low-income families and the fight against poverty. They also forecasted that, in 2020, the global growth rate would be as low as -4.9%

(IMF (2020)), although recent reports suggest that the world experienced the lowest growth rate in 40 years at -3.1% (IMF (2022)).

In the COVID-19 pandemic, as in other disasters, the main goal of governments has been to reduce losses, especially human losses. Although the management of this unprecedented global health disaster has many aspects, the main priority of governments has been to provide the necessary medical supplies. Therefore, it is important to study how countries behave in the current pandemic to achieve their goals. This motivation creates a foundation for the mathematical models that I construct in Chapters 2, 3, 4, and 5. Chapter 2 illustrates the challenges, such as resource scarcity, that humanitarian organizations face in helping those affected by a disaster, and the model presented in this chapter examines the effect of these challenges on the humanitarian organizations' decisions and strategies. In Chapter 3, I construct a model that investigates the behavior of decision-makers at demand points who try to minimize shortages of medical supplies to reduce the damage from the pandemic. The model in Chapter 4 considers governments that plan and work to procure the various types of medical supplies that are needed in the two stages before and after the pandemic declaration. And, in the model that I construct in Chapter 5, governments use trade measures in order to secure the essential products for their nation to deal with the pandemic.

1.2. Importance of Humanitarian Logistics

Governments and humanitarian organizations do not have an easy job in responding to disasters and face many challenges. Humanitarian organizations may be faced with sudden surges in demand post a disaster with the timing, location, and scale being not easily predictable (Murray (2005)). The uncertainty around many important parameters makes the decision-making process associated with disaster management complex. Another issue that often challenges relief operations is the compromised infrastructure, including that of transportation networks, which can result in extended delays in delivering aid (Nagurney, Yu, and Qiang (2011)).

To overcome such obstacles, it is imperative that governments and humanitarian organizations focus on logistics. One of the notable aspects of the disaster relief efforts after the Indian Ocean tsunami in 2004, for example, was that logistics was publicly acknowledged to play an extremely important role in relief (Thomas and Kopczak (2005)). The Council of Logistics Management defines logistics as "the process of planning, implementing, and controlling the efficient, effective flow and storage of goods, services, and related information from point of origin to point of consumption for the purpose of conforming to customer requirements" (Cooper, Lambert, and Pagh (1997)). The International Federation of Red Cross and Red Crescent Societies notes that the primary task of humanitarian logistics is to provide and deliver the requested items and services at the needed time and place and at the lowest cost. In times of disaster, these items include medicine, food, water, shelter, and other essential items (IFRC (2021b)).

In the COVID-19 pandemic, similar to other disasters, shortages of essential goods and uncertainty around many parameters were among the logistics challenges, but on a different scale. The spread of the virus and the efforts of governments and the public to combat it have led to a sharp increase in demand for medical supplies worldwide (Kamdar (2020)). The Centers for Disease Control and Prevention (CDC) emphasizes that a physical distance of at least 6 feet between persons is necessary to prevent the transmission of the coronavirus, which causes the COVID-19 disease (CDC (2020a), Johns Hopkins Medicine (2021), Greenstone and Nigam (2020)). It has been scientifically established that one of the most effective ways to mitigate contagion, coupled with social distancing, is the use of Personal Protective Equipment

(PPE) (CDC (2020b), Herron et al. (2020), Ferguson et al. (2020)). These recommendations have been the basis for many policies of governments and decision-makers since the beginning of this pandemic.

The measures mentioned above led to a significant increase in demand for medical products, especially PPEs, and a sudden shortage of them. In the early days of the pandemic, the WHO estimated that 89 million masks and 76 million examination gloves would be needed monthly to deal with the pandemic, and that production of these items would have to increase by 40 percent to meet the demand (WHO (2020)). However, the pandemic has not only caused an increase in demand for medical items but it has also disrupted supply chains for various goods. It has led to major changes in people's lifestyles, altering patterns of supply and demand, and even grocery shopping (Severson (2020)), and is threatening the economic stability and well-being of both people and governments (Suau-Sanchez, Voltes-Dorta, and Cuguer-Escofet (2020), Sheth (2020)). Observing the pandemic trend, everyone soon realized that supply chains were facing unprecedented challenges and that global logistics efforts were needed to address the crisis.

Logistics and supply chain networks play a significant role in disaster management. I take into account this issue in the models that I construct in Chapters 2, 3, 4, and 5. Therein, I examine the effects of such challenges as: a sudden increase in demand, limited transportation capacity, increasing purchasing and shipping costs, limited supply capacity, and uncertainty in pre- and post-disaster operations.

1.3. Competition for Limited Resources

One of the most crucial issues in disaster management is resource scarcity. Resources can be budgets, the supply capacity, the capacity of the shipping fleet as well as storage facilities on which relief operations depend. The resource limitations and shortages, in turn, can create competition among those who are involved in pre- and post-disaster operations.

One of the most important restrictions in humanitarian operations is the cost, with logistics estimated to account for 80% of the total cost related to disaster relief (cf. Van Wassenhove (2006)). Hence, competition for financial resources among humanitarian organizations is a common feature. The pandemic has revealed unprecedented competition for essential/vital products at various levels, from people in stores to healthcare institutions, states, as well as nations. Generally, essential commodities are those whose shortages endanger the health and safety of people. However, they are not only a collection of fixed products, depending on the country's needs at different times, varying items may be considered essential. India, for example, enacted the Essential Commodities Act in 1955 to protect the production and distribution of certain goods and to prevent shortages and sudden price increases. This act covers items such as food grains, fertilizers, edible oil, drugs, fuels, and petroleum. Recently, masks and hand sanitizers have been added to the list (Gupta (2020)). In another example, the state of Florida defines an essential commodity as a commodity that "its consumption or use is critical to the maintenance of the public health, safety or welfare during the declared emergency" and puts commodities such as protective masks, sanitizing and disinfecting supplies, and all PPE on the list while mentioning that they continue to monitor the situation and revise the list as needed (Florida Attorney General (2020)).

In the COVID-19 pandemic, competition for medical items from PPEs to ventilators has been prevalent, due to the sudden increase in demand and the disruptions in supply chains. Also, although several vaccines have been approved (Zimmer, Corum, and Wee (2022)), there is, nevertheless, global competition for them. The same holds for medicinal treatments for patients suffering from COVID-19 (Perrone (2022)).

In terms of disaster preparedness of the United States and the world for pandemics, the lack of PPEs has always been an important issue (Lopez (2020a)). Different estimates of the required number of ventilators during this pandemic, in turn, were, early on, from hundreds of thousands to one million. The number of these devices available in the United States was estimated at between 60,000 and 160,000, and the national strategic reserves were insignificant compared to the needs (Ranney, Griffeth, and Jha (2020)). The United States Strategic National Stockpile had approximately 12 million N95 masks and 30 million surgical masks while the Department of Health and Human Services had projected that the country would need 3.5 billion face masks in the event of a year-long pandemic (Jacobs, Richtel, and Baker (2020)).

In today's world, every product has a path from production to demand points, and, in some cases, it goes through several countries. PPEs are no exception, and their production process depends not only on the capacity of the factories, but also on raw materials (UNICEF (2020)). For example, paper needed to produce protective gowns became so scarce that a company had to approach five different countries to find it to keep its supply chain open (Diaz, Sands, and Alesci (2020))

China has historically produced half of the world's face masks, but with the coronavirus originating in Wuhan, China, the country dedicated the majority of the supply for their own citizens, whereas other countries, such as Germany, even banned the export of PPEs (Lopez (2020b)). The intense competition for PPEs led to a dramatic increase in the price, with some prices rising by more than 1,000%, according to the report by The Society for Healthcare Organization Procurement Professionals (2020). For example (cf. Diaz, Sands, and Alesci (2020)), the price of N95 masks grew from \$0.38 to \$5.75 each (a 1,413% increase), isolation protective gowns experienced a price increase from \$0.25 to \$5.00 (a 1900% increase), and the price of reusable face shields went from \$0.50 to \$4.00 (a 700% increase). In addition, because the coronavirus SARS-CoV-2 that causes COVID-19 may result in severe respiratory problems in certain individuals, various healthcare organizations, including hospitals, were clamoring for ventilators for their patients (Gelles and Petras (2020), Namendys-Silva (2020)). This is an example of, yet another medical item for which there was and continues to be intense competition globally, and with limited supply availability (see Goudie et al. (2020), Kamdar (2020), Pifer (2020), SCCM (2020), Schlanger (2020)).

Such challenges have led many governments to institute different trade policies and measures during the COVID-19 pandemic in order to reduce the risk of essential/vital product shortages. When this global health disaster hit one country after another, some countries, which were among the few exporters of PPEs, faced a very high demand within their own national boundaries and, therefore, prioritized meeting their needs first. Hence, they banned the export of medical products (Boykoff, Sebastian, and Di Donato (2020)). According to Global Trade Alert (GTA), as of 25 April 2020, 122 new export bans in more than 75 countries including the US, China, and the European Union (EU) were issued on medical supplies such as antibiotics, face masks, and ventilators. There is also a large number of countries that has reduced the tariffs on essential goods to accelerate the import of such products (Evenett (2020a), Pelc (2020), Evenett (2020b)). For example, China temporarily decreased import tariffs on several types of products such as medical supplies, raw materials, agricultural products, and meat (ITC MACMAP (2022)). The United States temporarily excluded certain products from the additional duty of 25% on a list of 19 products from China and put restrictions on exports of five types of PPEs which need explicit approval from FEMA before export (WTO (2021)). Belarus imposed temporary restrictions on exports of food products such as onion and garlic due to the pandemic (WTO (2021)).

Governments have also taken steps to address concerns about the prices of essential goods during the pandemic. China warned that the price of essential goods should not increase, and also announced how to enforce the law against the increase in the price of face masks. The European Competition Network issued a joint statement by the European Commission and the European Union's national competition agencies highlighting that it is very important that the prices of the goods that are necessary for the health of the people, such as face masks and sanitizing gel, remain within the competitive range (OECD (2020)). The United States Senate expressed concern that the prices of vaccines being developed with the help of the federal government be affordable (Owermohle (2020)). The US Department of Justice, on the other hand, warned that criminal prosecution awaits those that fix prices or rig bids for PPEs such as sterile gloves and face masks (Department of Justice (2020), OECD (2020)).

Although such measures are necessary to manage critical situations, governments must be careful in formulating trade measures and in implementing them to make sure that they are useful and that there are no adverse effects. For example, Dr. Anthony Fauci, the Director of the National Institute of Allergy and Infectious Diseases, pointed out that high medicine prices created problems in countries plagued by the COVID-19 pandemic. But he also mentioned that, if you put a lot of pressure on a company and restrict it, the company may no longer work with you (Dearment (2020)). In India, the government tried to control the price of lifesaving oxygen gas, but this policy was accompanied by shortcomings that led to the emergence of the oxygen black market (Biswas (2020)).

Chapter 2 of this dissertation focuses on the behavior of humanitarian organizations in disaster supply chain economic networks as they compete for limited resources, such as product supply and transportation capacity. In Chapter 3, the competition among organizations at demand points for medical supplies in the COVID-19 pandemic is discussed. In Chapter 4, I look at the competition among governments for the limited resources of essential medical supplies before and after the pandemic declaration. In Chapter 5, the use of trade measures and policies by different countries that seek to facilitate the supply of essential goods to their people is investigate. All the constructed models and accompanying theoretical results and algorithms are based on the theory of variational inequalities, which provides for an elegant, unifying mathematical framework (cf. the books by Nagurney (1999, 2006)).

1.4. Literature Review

This section discusses the relevant literature on several topics covered in this dissertation. A review of the literature is provided in various subsections to better identify the work done and the gaps filled in this dissertation.

1.4.1 Game Theory and Disaster Management

Since various governmental and non-governmental agencies, with their limitations and goals, are involved in the humanitarian supply chain, game theory is a powerful tool to address the interactions among them. However, Muggy and Heier Stamm (2014), in their survey of game theory in humanitarian operations noted that the applications of game theory to this important domain have been limited. Seaberg, Devine, and Zhuang (2017), more recently, provided an excellent, panoramic review of 57 papers over a ten-year horizon from 2006 to 2016 and noted that the response phase of disaster management has been the phase researched the most extensively. Coles, Zhang, and Zhuang (2018) in their novel work investigate how noncooperative game theory can be utilized to assist different agencies, such as businesses, government agencies, militaries, and nonprofit organizations, in better decision-making as to partnership selection during the disaster response and recovery phases.

Coles and Zhuang (2011) discussed the potential of cooperative game theory in disaster recovery operations, along with a critique and references, which is a promising direction for future research. Muggy and Heier Stamm (2014) also emphasized that cooperative models may assist in the identification of methods for partnering agencies to achieve greater impact than what is possible individually and independently. Nagurney and Qiang (2020) identified potential synergies associated with the possible teaming of humanitarian organizations from a supply chain network perspective. The excellent survey article of Gutjahr and Nolz (2016) on multicriteria optimization in humanitarian aid includes references to both deterministic and stochastic models. The authors, in their future research directions section, emphasize the need for papers that consider the diverging interests of multiple and sometimes competing stakeholders.

To date, there have been very few Generalized Nash Equilibrium (GNE) models in the setting of disaster relief. In the GNE model, feasible sets consisting of the strategies of the players, defined by the underlying constraints, depend also on the strategies of their rivals. A frequently encountered class of Generalized Nash games deals with a common coupling constraint that the players' strategies are required to satisfy (Kulkarni and Shanbhag (2012)). These games are also known as Generalized Nash games with shared constraints (Facchinei and Kanzow (2010), Rosen (1965)). Removal of the common/shared constraints from the problem, in turn, yields a classical Nash Equilibrium (Nash (1950, 1951)).

Nagurney, Alvarez Flores, and Soylu (2016) developed the first GNE model for disaster relief to address some of the challenges that humanitarian organizations face during relief operations. Muggy and Heier Stamm (2014) recognized that relief organizations compete for media publicity as well as for financial resources (see also, Zhuang, Saxton, and Wu (2014) and Toyasaki and Wakolbinger (2014)). The model of Nagurney, Alvarez Flores, and Soylu (2016) integrated the financial side, in terms of the securing of donations, based on the visibility of humanitarian organizations, and the logistical side of disaster response operations, in terms of the cost-effectiveness of the delivery of relief items. In the case of game theory, each humanitarian organization as a player aims to maximize its utility function by determining its optimal strategies, while competing for financial donations. It should be mentioned that each player's strategy has an effect on the others' utilities, and, if they do not share the constraints, then we are dealing with a Nash Equilibrium (Nash (1950, 1951)) in the case of noncooperation. On the other hand, if some of the constraints are common/shared, then we have a Generalized Nash Equilibrium (Debreu (1952), Rosen (1965)).

The Nagurney, Alvarez Flores, and Soylu (2016) model, because of its special structure, enabled an optimization reformulation of the GNE conditions. A case study focusing on Hurricane Katrina demonstrated that humanitarian organizations may end up delivering supplies where it is easier/cheaper to do so, unless the demand bounds are imposed by a higher-level authority, with the consequence that demands may not be met and there may be an oversupply at demand points that are easier to reach. Nagurney et al. (2018) extended the previous game theory model by incorporating more general logistical cost, altruism benefit, and financial donation functions. The new model, in contrast, did not allow for an optimization reformulation. Hence, the authors utilized a Variational Equilibrium concept to reformulate the governing equilibrium conditions of their model as a variational inequality. Bensoussan (1974) formulated the GNE problem as a quasivariational inequality. However, GNE problems are challenging to solve when formulated as quasivariational inequality problems for which the state-of-the-art in terms of algorithmic procedures is not as advanced as that for variational inequality problems. In Kulkarni and Shanbhag (2012), the authors provide sufficient conditions to establish the theory of Variational Equilibrium as a refinement of the GNE.

It should be emphasized that game theory is quite relevant to many healthcare issues in the COVID-19 pandemic. For example, recently, Nagurney and Dutta (2021) introduced an equilibrium donor model for convalescent plasma. The model captures the competition among nonprofit and for-profit organizations seeking convalescent plasma donations, which is a characteristic of this new market. Convalescent plasma is now being investigated as a treatment for COVID-19 and there have been documented instances of enhanced patient survival after transfusions. Using a game theory model, Mamani, Chick, and Simchi-Levi (2013) showed that lack of coordination can lead to unbalanced distribution and shortage or excess of influenza vaccine in different regions. In this study, the use of cost-sharing contracts that increased coordination between different players improved global vaccine allocation.

1.4.2 Optimization Frameworks in Humanitarian Logistics

It is important to mention that there have been numerous studies focusing on optimization frameworks in the context of disaster relief as well as humanitarian logistics. In these settings it is imperative to construct the appropriate objective functions and constraints with the former being distinctly different from, for example, profit maximization used widely in commercial supply chains (cf. Nagurney (2006) and the references therein). For example, Tzeng, Cheng, and Huang (2007) proposed a dynamic selection of the amount of relief items to be transported from depots to demand points such that three objectives are achieved: minimum total cost, minimum travel time, and maximum demand satisfaction. Haghani and Oh (1996) considered commodity carry-over, routing, and mode transfer. Ozdamar, Ekinci, and Kkyazici (2004), Yi and Kumar (2007), and Yi and Ozdamar (2007) included split delivery and the sum of unmet demands in their models. Vitoriano et al. (2011) constructed a multicriteria model for humanitarian relief distribution with criteria of response time, equity, and security. Huang, Smilowitz, and Balcik (2012), in their model, incorporated multiple relevant disaster relief objectives of efficiency, efficacy, and equity with the consideration of both vehicle routing and resource distribution.

Multicriteria optimization in humanitarian aid, in particular, is very relevant, since a decision-maker may be faced with multiple objectives in decision-making associated with disaster relief, such as optimizing in terms of efficiency, optimizing effectiveness, etc., and these may even be in conflict. For example, Qiang and Nagurney (2012) presented a supply chain network model for critical needs in the case of disruptions. The objective of the model was to minimize the total network costs, which are generalized costs that may include the monetary, risk, time, and social costs. The model assumed that disruptions may have an impact on both the network link capacities as well as on the product demands. Earlier, Nagurney, Yu, and Qiang (2011) had constructed an integrated framework for the supply chain networks design for vital products such as vaccines, medicines, food, etc., which may be used in the preparation (and response) to pandemics, disasters, attacks, etc. The supply chain network model focused on cost minimization as the primary objective and captured rigorously the uncertainty associated with the demand for critical products at the several demand points.

I refer the reader to the survey of optimization models in emergency logistics by Caunhye, Nie, and Pokharel (2012) and for additional references on models in humanitarian logistics, see Duran et al. (2013) and the survey by Ortuño et al. (2013).

1.4.3 Optimization Under Uncertainty and Disaster Relief

As it was discussed in the previous sections, uncertainty is one of the most important characteristics of disasters and it is necessary to consider this factor in disaster preparedness and response planning. The need to include uncertainty in disaster management models has intensified the research in this area.

Nagurney and Nagurney (2016) developed a supply chain network model for disaster relief under cost and demand uncertainty, but there was a single decision-maker and, hence, game theory was not needed. In another paper, Nagurney and Qiang (2020) considered multiple humanitarian organizations engaged in disaster relief and constructed multiproduct supply chain network models that include uncertainty associated with costs of supply chain activities, including procurement, storage, and distribution, under multiple disaster scenarios, along with uncertainty associated with the demand for the disaster relief products at the demand points. For a survey of uncertainty in humanitarian logistics in disaster management, I refer the interested reader to Liberatore et al. (2013). For an even more recent survey, see Hoyos, Morales, and Akhavan-Tabatabaei (2015). The latter survey reviews the literature on Operations Research (OR) models with some stochastic component applied to Disaster Operations Management (DOM), along with an analysis of these stochastic components and the techniques used by different authors.

Regarding uncertainty in disaster relief, there have been multiple two-stage stochastic optimization models proposed for different phases of disaster management (see Rawls and Turnquist (2010), Mete and Zabinsky (2010), Falasca and Zobel (2011), Grass and Fischer (2016)). In a disaster management two-stage stochastic programming framework, pre-disaster decisions such as pre-positioning of supplies are referred to as first-stage decisions, and decisions regarding shipment of disaster relief items to the demand nodes, such as flows on network links, are second-stage decisions. First-stage decisions should be made prior to realizing the uncertain factors, taking into account all possible scenarios of the incident. When the event happens, and unknown factors are revealed, second-stage decisions are modified and carried out (Grass and Fischer (2016)).

Indeed, since 2010 and, specifically, following the work by Rawls and Turnquist (2010), the number of publications on two-stage stochastic programming in disaster management has increased significantly (see Grass and Fischer (2016)). Rawls and Turnquist (2010) presented a model that assists decision-makers in determining the relief item amounts and the location of facilities for pre-positioning in terms of preparedness for a possible hurricane. The objective of their model is to minimize the total costs. Falasca and Zobel (2011) constructed a two-stage stochastic model for procurement in humanitarian relief supply chains in which purchasing decisions are made after a disaster strikes. Tofighi, Torabi, and Mansouri (2016) focused on a two-echelon humanitarian logistics network design problem, for handling pre-disaster and post-disaster challenges, considering uncertainty and probabilistic characteristics of a potential earthquake. Torabi et al. (2018), afterward, introduced a novel two-stage scenario-based mixed fuzzy-stochastic programming model for integrated relief pre-positioning and procurement planning based on a quantity flexibility contract under a mixture of uncertain data, accompanied by an effective multi-step solution method.

He and Zhuang (2016) constructed a two-stage, dynamic model to assess the trade-off between pre-disaster preparedness and post-disaster relief with the goal of minimizing the total expected damage and the costs of preparedness and aid. The authors modeled the disaster magnitude as a random parameter following either a discrete or continuous probability distribution, while the damage was a function of pre-disaster preparedness, disaster magnitude, and post-disaster relief. Mete and Zabinsky (2010) developed a stochastic programming model that, in the first stage, determines the location of warehouses and the inventory level for medical supplies. In the second stage, based on the disaster scenario, the model chooses the quantities of the medical supplies to be shipped to the hospitals. Nagurney et al. (2020) addressed the issue of uncertainty but with the consideration of multiple, competing decision-makers in the first Stochastic Generalized Nash Equilibrium model for disaster relief. In that model, relief organizations compete for existing resources before and after the disaster, and adjust their decisions based on the probability of different scenarios. Each organization is faced with a two-stage stochastic optimization problem where it can buy and store relief items before a disaster occurrence, and may purchase additional items based on the circumstances after the disaster, or utilize their warehouse reserves to meet the demand.

For background on the two-stage scenario-based stochastic programming problem, I refer the interested reader to Dupacova (1996) and Barbarosoglu and Arda (2004). The books by Birge and Louveaux (1997) and Shapiro, Dentcheva, and Ruszczynski (2009) have excellent coverage of the theory and applications of stochastic programming, with the book by Derman, Gleser, and Olkin (1973) serving as a fundamental resource for probability theory and applications.

In Chapter 3 of this dissertation, I discuss the competition among organizations at different points of demand for medical supplies in the COVID-19 pandemic. In that model, demands are stochastic. Chapter 4 also pays special attention to uncertainty in the pandemic disaster management. Unlike the model in Chapter 3 which is a single-stage stochastic model, I present a two-stage stochastic game theory network model in Chapter 4 which captures the effects of uncertainty associated with different factors in the planning and the performance of decision-makers in pre- and post-disaster management.

1.4.4 COVID-19 Pandemic and Supply Chains

With the onset of the COVID-19 pandemic and the emergence of serious disruptions in supply chains, researchers played a critical role in informing decision-makers. Since the pandemic was declared in 2020 and is unprecedented in terms of both scale and scope and quite different as compared to other disasters, which are limited in both geography and time, researchers have turned to investigating various aspects of the pandemic, including the impacts on product flows. A research agenda was mapped out by Queiroz et al. (2020) through the construction of a structured literature review of recent studies on the COVID-19 pandemic and the impacts of previous epidemic outbreaks on supply chains.

Ivanov (2020a) conducted simulation-based research focused on the impacts of the COVID-19 pandemic on global supply chains. He found that the extent of these effects depends on the timing of the closing and opening of the facilities at different supply chain tiers, and the lead-time, the pandemic transmission speed, and the upstream and downstream disruption periods. The impacts that the COVID-19 pandemic and the supply chains have on one other are pervasive; for example, Choi (2020) discussed the effects of this disaster on changing the way businesses operate in order to survive. He argued that the innovative "bring-service-near-your-home" mobile service operation (MSO) is one way for businesses to continue during the time that people no longer visit stores because of their safety concerns. Nagurney (2020) developed a supply chain network optimization model for perishable food in the COVID-19 pandemic, which included the critical labor resource. The model can be used to investigate the impacts of labor disruptions, due to illnesses, death, etc., on prices and product flows. Focusing on demand management, Govindan, Mina, and Alavi (2020) proposed a decision support system that would help better manage the healthcare supply chain situation by categorizing community members during an outbreak such as the COVID-19 pandemic. In another study, Ivanov (2020b) discussed the viability of the supply chain, an issue that has become increasingly important since the onset of the COVID-19 pandemic.

Earlier, Chick, Mamani, and Simchi-Levi (2008) constructed the first integrated supply chain / health economics model for the distribution of the influenza vaccine. In this model, two players have key roles, the government, and the manufacturer, and coordination between them has a significant impact on the outcome of the operation for both players. Liu and Zhang (2016) constructed a dynamic logistics model to study the medical resource allocation in response to epidemic diffusion. Byktahtakn, des-Border, and Kibis (2018) presented an epidemic-logistics model to be used in the control of an Ebola epidemic. The objective of their mixed-integer programming model was to minimize the total number of infections and fatalities by determining the optimal amount, timing, and location of resources.

Anparasan and Lejeune (2019) developed a resource allocation model based on the 2010 cholera outbreak in Haiti to support emergency response to an epidemic outbreak in resource-limited countries. In a two-stage model, Long, Nohdurft, and Spinler (2018) investigated the allocation of resources in the event of an epidemic. In the first stage, the model predicts how the epidemic moves between the neighboring areas, and, in the second stage, the necessary resources are allocated to intervene in the epidemic. Enayati and Ozaltin (2020) presented a vaccine distribution model for influenza transmission that minimizes the number of vaccines necessary to control an emerging pandemic in its initial stages. Their model also incorporates transmission dynamics and isolation.

1.4.5 COVID-19 Pandemic, Trade Measures, and Relevant Spatial Price Equilibrium Models

The coronavirus, which causes COVID-19, spread rapidly worldwide and pushed governments and policy-makers to adopt new trade policies to pave the way to provide essential supplies including medical ones for their nations. Researchers and experts have varying views on the methods and effects of these trade policies and are publishing research results while we are still in the midst of the pandemic. Baldwin and Evenett (2020) suggested that the governments should not turn inward in response to the COVID-19 pandemic because national trade obstacles would make the production of medical supplies harder for countries. But the protectionist trade policies that we are observing during the COVID-19 pandemic are mostly anti-export. Smith and Glauber (2020) reported that although protectionist policies in food international trade have not worked well in past experiences such as the 2007-08 crisis, policy-makers are still keen to use these strategies. Indeed, in the current global health disaster, certain governments have imposed restrictions such as export bans and export quotas on the food trade, including on wheat and rice.

Stellinger, Berglund, and Isakson (2020) mentioned that we are dealing with a dual crisis, a health crisis, and an economic one. They argued that some trade measures, such as import bans and buy-national, are unnecessary and, while they may be useful in protecting domestic manufacturing, they do not have a role in protecting patients. Bown (2020), pointing out that the European Union imports 90% of its PPEs, suggested that even temporary export bans might cause mistrust and retaliation that would harm such countries. Also, the European Union export restrictions would put Eastern Europe, northern Africa, and sub-Saharan Africa countries at risk.

Shingal (2020) stated that, although the pandemic is a health crisis, with proper trade measures, secondary financial crises can be prevented, and new trade barriers can replace traditional trade instruments and even new jobs and skills may be created. Pauwelyn (2020) and Lawrence (2020) argued that establishing trade measures that restrict the export of essential medical supplies and food is not legal under normal circumstances under WTO and EU law, but is not subject to the law in critical situations such as the current pandemic in which people's lives are in danger. Leibovici and Santacreu (2020) investigated the role of international trade of essential supplies in reducing or exacerbating the effects of a pandemic. They observed that a country's imbalance in the trade of essential goods has a significant role in the effects on the country, so that net importers would be worse off than net exporters. By simulating the effects of trade barriers under different scenarios, Grassia et al. (2020) showed that, although a country would benefit from implementing the trade restrictions in isolation, the generalized use of these measures makes most countries' situations worse than theirs in a no-ban scenario. They also estimated that there would be price increases in many countries that impose trade restrictions.

Fiorini, Hoekman, and Yildirim (2020), examining recent examples, argued that, instead of restricting international trade, governments should work with industry actors to find and improve essential products' supply chain bottlenecks. Nagurney, Besik, and Li (2019) constructed an oligopolistic supply chain network equilibrium with trade policy where firms compete for product quantities and product quality and they are subjected to lower bounds and upper bounds on quality standards. They found that the governments might threaten the consumer welfare of their own nation by imposing trade restrictions. Evenett (2020a) argued that, if governments restrict the export of goods, producers will no longer have an incentive to increase the production level to meet the foreign demand. He also pointed out that manufacturers consider tariffs and non-tariff barriers when deciding whether to sell to foreign buyers.

Spatial price equilibrium (SPE) models are very useful in research on the role and impacts of commodity trade instruments, such as tariffs and quotas, in markets where there are multiple supply markets and multiple demand markets. SPE models originated in the works of Samuelson (1952) and Takayama and Judge (1964, 1971). Since then, such models have been expanded and widely applied in practice, with variational inequality theory, in particular, utilized to formulate increasingly more general SPE models (cf. Nagurney (1999, 2006), Daniele (2004), Li, Nagurney, and Yu (2018), Nagurney, Besik, and Dong (2019), and the references therein). Nagurney, Besik, and Dong (2019) constructed a spatial price equilibrium model with multiple countries and supply and demand markets in each country. In that model, a tariff-rate quota, which is a two-tiered tariff, was applied. They provided a unified variational inequality framework for qualitative analysis and algorithmic solution. They also solved and discussed numerical examples inspired by the dairy industry where a tariff-rate quota was imposed by the US on cheese from France. Nagurney, Besik, and Nagurney (2019), inspired by the ongoing trade wars, provided a modeling and computational framework for competitive global supply chain networks, operating as an oligopoly, where trade instruments are imposed in the form of tariff-rate quotas.

1.5. Dissertation Overview

This dissertation consists of six chapters with the first chapter dealing with the research motivation and literature review. In the following subsections, I present the contributions in Chapters 2 through 5 of the dissertation. Chapter 6 provides conclusions and suggestions for future research.

1.5.1 Contributions in Chapter 2

In this chapter, a game theory model is constructed to capture the competition among humanitarian organizations with the goal of bringing greater realism in terms of the constraints that they face, such as budget ones, and in expanding the scope, while continuing to integrate logistical and financial aspects. In particular, I extend the model of Nagurney et al. (2018), in substantive and significant ways, in order to incorporate and quantify the following issues:

Purchasing of the products: An important part of the humanitarian organizations' costs is the purchasing of the relief items that are then transported to the demand areas. There are different suppliers which humanitarian organizations (HOs) may be able to purchase products from and these may be local to the disaster or non-local with distinct associated prices. There may, hence, be trade-offs, as noted, for example, by Balcik and Beamon (2008): although local supplies may not be available in the quantity and quality needed, local procurement is, typically, characterized by shorter lead times and has lower logistics costs. The Nagurney et al. (2018) model assumed that the products were prepositioned. The model in this chapter, in contrast, allows for the purchasing of the relief items post-disaster.

Multiple freight service providers: As the focus is on logistics, it is important to consider different freight service providers (FSPs) that can ship relief items from the purchase locations of the HOs to the demand points. Each freight service provider may have specific advantages and disadvantages and may charge the HOs accordingly (see also Nagurney (2018)). Procuring globally may have the advantage of lower prices, higher quality and capacity, but brings the disadvantages of longer response time and higher transportation costs due to long distances (Duran et al. (2013)). In this chapter, unlike in the model in Nagurney et al. (2018), there are multiple possible freight service providers the HOs can select services from.

Capacity constraints: The humanitarian organizations, in disaster situations, may be constrained by the freight service providers' shipment capacities due to their available facilities and the impacted regions' infrastructures. No such capacity constraints were considered in Nagurney et al. (2018).

Budget constraints: The budget constraint that an HO faces is the most critical constraint in any humanitarian relief operation. HOs do not have an unlimited budget; hence, they must determine the optimal way in which to allocate their resources without exceeding the limitation. This also demonstrates to donors and stakeholders that they are financially responsible. Since relief agencies have limited funding, procurement procedures in the humanitarian aid sector are principally accomplished through price-based competitive bidding (Balcik et al. (2010) and Gossler et al. (2019)). No budget constraints were included in the model in Nagurney et al. (2018).

One of the novel features of the model introduced in this chapter is that humanitarian organizations have the flexibility of purchasing the relief items from different locations at associated prices and then having them shipped by freight service providers from the purchase locations to the points of demand, also at associated prices. This allows for making optimal resource allocations, given a budget constraint, based on local/nonlocal purchase prices and freight service provision costs. In addition, in the new model in this chapter, in contrast to the models of Nagurney, Alvarez Flores, and Soylu (2016) and Nagurney et al. (2018), it is not assumed that the relief items are prepositioned, but, rather, that they must be purchased. Moreover, this model has explicit budget constraints whereas in the earlier studies the financial aspect was captured through functions that quantified donations based on the visibility of the HOs in terms of relief item deliveries and concomitant media attention. Also, the budget constraints can handle nonlinear freight service costs.

A Lagrange analysis of the marginal utilities of the humanitarian organizations is provided for the model in this chapter. To-date, the only work, other than this dissertation, that includes Lagrange analysis for a humanitarian logistics model in the context of game theory, is the paper by Nagurney et al. (2018) and therein all the constraints were linear and there were no purchasing costs nor budget constraints.

The comprehensive framework presented in this chapter lays the foundation for investigating the competition among decision-makers involved in supply chain economic networks in various types of disasters.

This chapter is based on the paper by Nagurney, Salarpour, and Daniele (2019).

1.5.2 Contributions in Chapter 3

In this chapter, I develop a competitive game theory network model for medical supplies inspired by the COVID-19 pandemic. It features salient characteristics of the realities of this pandemic in terms of competition among organizations/institutions for supplies under limited capacities globally as well as uncertain demands due to the fact that so much about this novel coronavirus remains unknown and has yet to be discovered. Since the organizations, notably, healthcare ones such as hospitals and nursing homes but also medical practices, etc., compete with one another for the limited supplies, given the prices and their associated logistical costs as well as the expected loss due to possible shortages or surpluses, the model is a Generalized Nash Equilibrium (GNE) model (cf. Arrow and Debreu (1954)) rather than a Nash equilibrium one. To date, there have been very few GNE models in the setting of disaster relief. The COVID-19 pandemic is a global health disaster on a monumental scale, which, unlike other disasters (cf. Nagurney and Qiang (2009), Kotsireas, Nagurney, and Pardalos (2016, 2018)), is not limited in space and time. Furthermore, the model in this chapter has stochastic elements which are elaborated below.

In this model, and as is vividly occurring in the COVID-19 pandemic, the supplies of the items, which in the model are medical items, are constrained. Also, the demand for the medical items is uncertain with associated penalties for shortages or surpluses, with the former expected to be much higher due to potential loss of life, increased pain and suffering, etc. The constructs that are utilized for handling the uncertain demands for medical items are based on results of Dong, Zhang, and Nagurney (2004), who introduced a supply chain equilibrium model with random demands, and on the results of Nagurney, Yu, and Qiang (2011) and Nagurney, Masoumi, and Yu (2012, 2015), who focused on optimization models in disaster relief and healthcare.

In the model, there are multiple independent demand points and they compete for the medical item supplies with one another. This model also includes general transportation costs and each demand point is subject to uncertain demand for the medical supplies. This chapter is based on the paper by Nagurney et al. (2021).

1.5.3 Contributions in Chapter 4

In this chapter, I construct a two-stage stochastic game theory model with consideration of the unique characteristics of the COVID-19 pandemic in order to examine the behavior of national governments in this global health disaster and their competition for essential medical supplies in both the preparation and response phases.

The research questions that are addressed in this chapter include:

1. What are the characteristics of competition among countries for essential goods; specifically, medical supplies, in preparation and response to the pandemic?

2. What factors affect the competition?

3. What are the optimal strategies of governments in achieving their objectives, and can these be determined quantitatively through rigorous modeling and computations?

The specific contributions in this chapter are delineated below.

In this chapter, the vital issue of multicountry competition for multicommodity medical items in the pandemic is addressed. Governmental decisions and strategies made both before and after the declaration of a pandemic are considered. Hence, this chapter builds on the work of Nagurney et al. (2020) but with the following significant extensions/modifications:

Competition among multiple countries under uncertainty: This chapter examines the competition of several national governments for medical supplies. The competition arises as each government faces a two-stage stochastic optimization problem for pre- and post-pandemic management in its country.

Multiple medical items: It is considered that, in times of a global health disaster, demand is not limited to a single medical item. Governments must identify their optimal strategies in procuring the different medical products, such as PPEs and ventilators. It is also important to note that, unlike in previously noted disasters, the essential goods that have suffered severe shortages in the COVID-19 pandemic are, in fact, medical supplies.

Limited supplies of medical items: The countries compete with one another for the limited medical supplies, leading to a Generalized Nash Equilibrium. This is only the second study that constructs a two-stage stochastic Generalized Nash Equilibrium model for disaster relief and is actually the first one focused on healthcare in terms of medical supplies.

Penalties on the shortages of medical items: A distinct penalty is assessed by each country on the unmet demand for each medical item.

This chapter is based on the paper by Salarpour and Nagurney (2021).

1.5.4 Contributions in Chapter 5

In this chapter, I develop a multiproduct, multicountry spatial price equilibrium model that integrates a plethora of trade measures that different countries can impose (and have been imposing) on essential products such as medical supplies, certain raw materials, and agricultural goods, as they continue to deal with the pandemic and work for the interests of their nations. In the model, there are multiple supply markets and multiple demand markets in each country.

This chapter builds on the work of Nagurney, Besik, and Dong (2019) but with the following significant extensions/modifications:

Multiple essential products: Unlike the earlier model, this model can handle multiple products and that is especially important in the COVID-19 pandemic due to the importance of such products as medical supplies, a variety of foods, as well as certain raw materials.

Prices and product flows as variables: In the new model, not only are product flows variables but prices at the supply markets and the demand markets in different countries are variables as well. This feature allows to more seamlessly introduce trade measures in the form of price floors and ceilings, which have been incorporated in this pandemic, and which is elaborated upon in the next contribution below.

Multiplicity of trade measures including price floors and ceilings: The model in this chapter integrates a plethora of trade measures for multiple products, including tariffs, quotas, as well as price floors and ceilings. The model allows for price ceilings on both supply market prices and on demand market prices in the countries. It also allows for positive price floors on supply market prices in the countries with price floors on the demand market prices being set to zero.

Special relevance to the COVID-19 pandemic: By incorporating multiple distinct trade measures in a unifying framework, the multiproduct, multicountry model is of specific relevance in the pandemic, since governments are using multiple trade measures.

This chapter is based on the paper by Nagurney, Salarpour, and Dong (2022).

CHAPTER 2

AN INTEGRATED FINANCIAL AND LOGISTICAL GAME THEORY MODEL FOR HUMANITARIAN ORGANIZATIONS WITH PURCHASING COSTS, MULTIPLE FREIGHT SERVICE PROVIDERS, AND BUDGET, CAPACITY, AND DEMAND CONSTRAINTS

In this chapter, a game theory model for disaster relief is constructed that incorporates both financial and logistical aspects of humanitarian organizations involved in the purchasing and delivery of relief items, post-disaster, using freight services. The model allows for the purchasing of the relief items, both locally and nonlocally, includes a budget constraint for each humanitarian organization, along with imposed lower and upper bound demand constraints at each point of demand by a higher-level organization. The governing concept is that of Generalized Nash Equilibrium, since not only does the utility function of a given humanitarian organization depend on its own strategies and the strategies of the other humanitarian organizations, but the constraints do as well. The concept of a variational equilibrium is utilized to derive the variational inequality formulation of the governing equilibrium conditions and the model is analyzed qualitatively. Lagrange analysis of the marginal utilities is conducted to gain insights on the impact of the constraints and an alternative variational inequality constructed, with nice features for computations. An algorithm is proposed and explicit formulae provided for the logistical flows and Lagrange multipliers at each iteration. Numerical examples, inspired by Hurricane Harvey hitting Houston, Texas, as occurred in August 2017, illustrate the framework.

This work adds to the still nascent literature on game theory and disaster relief and also to the literature on variational inequality models with nonlinear constraints, which is limited.

This chapter is organized as follows. In Section 2.1, I describe how the humanitarian organizations compete noncooperatively to maximize their respective utilities from the disaster relief operation, and present the components of the utility functions and the constraints. The Generalized Nash Equilibrium model is constructed and the Variational Equilibrium is presented, followed by the derivation of the variational inequality formulation. In Section 2.2, Lagrange theory is utilized to investigate the role of each constraint and to gain insights and also alternative variational inequality formulations are provided, one of which then utilized for computational purposes. In Section 2.3, an algorithm is presented, which yields closed form expressions for each of the relief item flows and Lagrange multipliers at a given iteration, along with numerical examples. The numerical examples are inspired by Hurricane Harvey hitting Houston, Texas, as occurred in August 2017. In Section 2.4, I summarize the results and present the conclusions.

2.1. The Integrated Financial and Logistical Game Theory Model for Humanitarian Organizations with Purchasing Costs, Multiple Freight Service Providers, and Budget, Capacity, and Demand Constraints

There are m humanitarian organizations, with a typical one denoted by i, involved in delivering relief supplies to n locations, with a typical location denoted by j. The humanitarian organizations may avail themselves of t freight service providers (FSPs) with a typical freight service provider denoted by l. Each humanitarian organization

(HO) may purchase the relief item at o possible locations, with a typical such location denoted by k. The network structure of the problem is given in Figure 2.1.

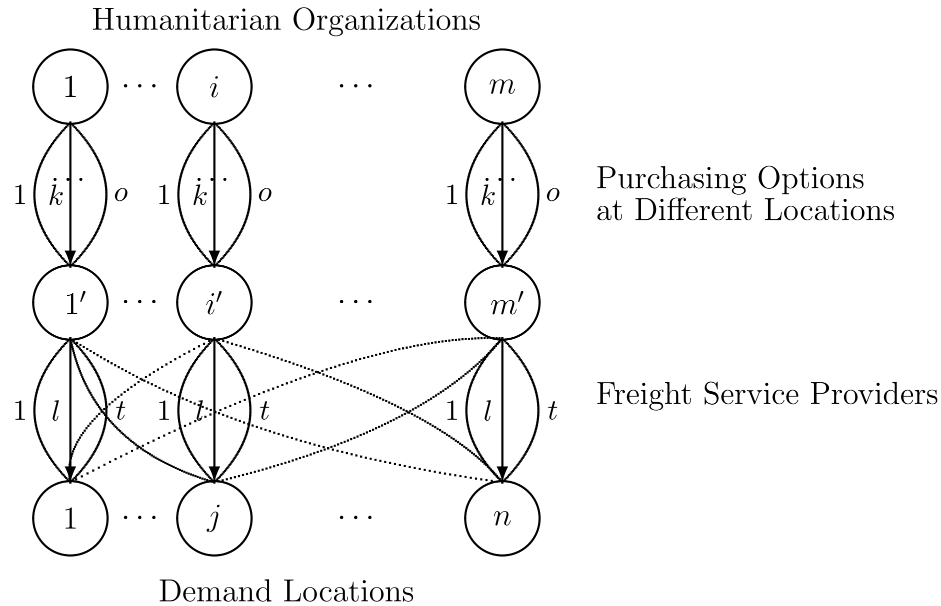

Figure 2.1. The Network Structure of the Game Theory Model with Multiple Purchasing Options and Multiple Freight Service Providers

The volume of relief items purchased by HO i at location k and shipped to demand location j by FSP l is denoted by $q_{ijk,l}$. The relief item shipments of HO i are grouped into the vector $q_i \in R_+^{not}$. This is the strategy vector of HO i. Then the strategy vectors of all the HOs are grouped into the vector $q \in R_+^{mnot}$.

The price associated with purchasing the relief item at location k is denoted by ρ_k. Hence, the total financial outlay for purchasing the relief items at the various locations for HO i; $i = 1, \ldots, m$, is

$$\sum_{k=1}^{o} \rho_k \sum_{j=1}^{n} \sum_{l=1}^{t} q_{ijk,l}. \tag{2.1}$$

In addition to the purchasing outlay, each HO i; $i = 1, \ldots, m$, is responsible for having the relief items transported to the various points of demand following the disaster. Let $c_{ijk,l}$ denote the transportation cost that HO i pays to get its relief items delivered to the demand point j by freight service provider l from purchase location k. The total outlay associated with the logistical costs, hence, can be expressed for HO i; $i = 1, \ldots, m$, as:

$$\sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q). \tag{2.2}$$

Note that, according to expression (2.2), for the sake of generality, and also to further capture the competitive aspects for freight service provision, in general, the cost can depend on the vector of relief item shipments. Hence, the logistical costs of shipment depend not only on the procurement/shipment volumes of a specific HO, but also on those of the other HOs. These cost functions are assumed to be convex and continuously differentiable.

Since HOs are nonprofit organizations, they cover their costs by attracting donations. However, in doing so they compete with each other. The more effectively they provide relief at the demand points, the more attention they receive from potential and existing donors. Hence, benefit functions that measure the effectiveness of the HOs' activities are utilized. According to Nagurney, Alvarez Flores, and Soylu (2016), HOs may benefit not only from their own efforts but also from other HOs' visibility at the demand points. A benefit function associated with HO i; $i = 1, \ldots, m$, is denoted by $B_i(q)$, and with it a nonnegative monetization weight ω_i is associated as follows:

$$\omega_i B_i(q). \tag{2.3}$$

The benefit functions, which also may be interpreted as altruism functions, are assumed to be concave and continuously differentiable. Note that the benefit functions may, in general, depend on the vector of all the procurement/shipment strategies of all the HOs.

The utility function $U_i(q)$ for HO i; $i = 1, \ldots, m$, given the above, can be expressed as:

$$U_i(q) = \omega_i B_i(q) - \sum_{k=1}^{o} \rho_k \sum_{j=1}^{n} \sum_{l=1}^{t} q_{ijk,l} - \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q). \tag{2.4}$$

Now let's describe the constraints faced by the HOs. Some of these are specific to each HO, whereas others are common, that is, shared. It is the latter type of constraint that makes the competitive game theory model one that is governed by a Generalized Nash Equilibrium and not simply by a Nash Equilibrium.

The first set of constraints is that the volume of the relief item of each HO i; $i = 1, \ldots, m$, to any demand point $j; j = 1, \ldots, n$, purchased at location k; $k = 1, \ldots, o$, and transported by FSP $l; l = 1, \ldots, t$, consists of the nonnegativity constraints:

$$q_{ijk,l} \geq 0, \quad \forall j, k, l. \tag{2.5}$$

The next constraint faced by each HO i; $i = 1, \ldots, m$, is its budget constraint:

$$\sum_{k=1}^{o} \rho_k \sum_{j=1}^{n} \sum_{l=1}^{t} q_{ijk,l} + \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q) \leq b_i, \tag{2.6}$$

where b_i is HO i's budget allocated to both procurement of the disaster relief items and transportation to points of demand.

The feasible set K_i corresponding to HO i is defined as:

$$K_i \equiv \{q_i | (2.5) \text{ holds}\} \tag{2.7}$$

and let $K \equiv \prod_{i=1}^{m} K_i$.

The HOs compete among themselves for freight capacity and, hence, are faced with the following common set of constraints. Indeed, while the HOs may be willing to send as much of the relief item as they can, FSPs have limited capacity due to their facilities, vehicle portfolio and availability, and also the disaster regions' infrastructures, which may have been severely compromised, if not destroyed. Therefore, the HOs cannot send more than a certain volume of relief items through a specific freight service provider from a particular purchase location. With $u_{k,l}$ denoting the shipment capacity from purchase location k of FSP l, the common capacity constraints are:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} q_{ijk,l} \leq u_{k,l}, \quad k = 1,\dots,o; l = 1,\dots,t. \tag{2.8}$$

Furthermore, to ensure that the minimum demands are satisfied at the demand points, while not exceeding the maximum demands, in order to reduce possible congestion as well as materiel convergence, as in Nagurney, Alvarez Flores, and Soylu (2016), the following constraints exist, which are imposed by a higher authority. By applying lower bound and upper bounds on the demands, it is guaranteed that at each demand point j; $j = 1,\dots,n$, the volume of relief items will not be less than $\underline{d}_j$ and, at the same time, it will not exceed $\bar{d}_j$, that is,

$$\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t} q_{ijk,l} \geq \underline{d}_j, \quad j = 1,\dots,n, \tag{2.9}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t} q_{ijk,l} \leq \bar{d}_j \quad j = 1,\dots,n. \tag{2.10}$$

In Nagurney, Alvarez Flores, and Soylu (2016) and in Nagurney et al. (2018), the importance of such constraints was demonstrated in case studies, in which their removal resulted in an oversupply in certain regions and an undersupply in other (typically, in those regions where it was more difficult and costly to deliver relief items). Hence, these important constraints are retained in the model.

The feasible set $\mathcal{S}$ of shared constraints is defined as:

$$\mathcal{S} \equiv \{q | (2.6) \text{ holding for all } i, \text{ and } (2.8), (2.9), (2.10) \text{ hold}\}. \tag{2.11}$$

Here it is assumed that the sum of the budgets of all the HOs, i.e., $\sum_{i=1}^{m} b_i$, is sufficient to meet the sum of all the minimum demands, that is, $\sum_{j=1}^{n} \underline{d}_j$ so that the set $\mathcal{K} \equiv K \cap \mathcal{S}$ will be nonempty.

Definition 2.1: Generalized Nash Equilibrium for the Humanitarian Organizations

A relief item flow vector $q^* \in K, q^* \in \mathcal{S}$ is a Generalized Nash Equilibrium if for each HO $i; i = 1, \ldots, m$:

$$U_i(q_i^*, \hat{q_i^*}) \geq U_i(q_i, \hat{q_i^*}), \quad \forall q_i \in K_i, \forall q \in \mathcal{S}, \tag{2.12}$$

where $\hat{q_i^*} \equiv (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_m^*)$.

The above definition means that not one of the HOs is willing to deviate from his current relief item flow pattern, given the relief flow item patterns of the other HOs. Observe that each HO's utility depends not only on his own strategies but also on those of the others' strategies, and so do their feasible sets, since their feasible sets are intertwined. The latter condition makes the problem a Generalized Nash Equilibrium model (Debreu (1952)). The feasible sets K_i are convex for each i, as is the set $\mathcal{S}$, under the imposed assumptions.

It is well-known that Generalized Nash Equilibria can be formulated as quasivariational inequality problems (cf. Fischer, Herrich, and Schonefeld (2014)); however, the state of the art of algorithms for the solution of such problems is not as advanced as for variational inequality problems. As noted in Nagurney, Yu, and Besik (2017), one may take advantage of a refinement of the GNE known as a Variational Equilibrium, which is a specific type of GNE (cf. Kulkarni and Shabhang (2012)), and enables a variational inequality formulation.

Definition 2.2: Variational Equilibrium

A relief item flow vector q^* is a Variational Equilibrium of the above Generalized Nash Equilibrium problem if $q^* \in K, q^* \in \mathcal{S}$ is a solution to the following variational inequality:

$$-\sum_{i=1}^{m} \langle \nabla_{q_i} U_i(q^*), q_i - q_i^* \rangle \geq 0, \quad \forall q \in K, \forall q \in \mathcal{S}. \tag{2.13}$$

In particular, it is worth mentioning that the variational equilibrium corresponds to Lagrange multipliers associated with the common constraints being the same for all the HOs. This has a nice fairness and equity interpretation and is very reasonable for humanitarian organizations involved in disaster relief.

Expanding variational inequality (2.13), we obtain:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left[\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k - \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}}\right] \times \left[q_{ijk,l} - q^*_{ijk,l}\right] \geq 0,$$

$$\forall q \in K, \forall q \in \mathcal{S}. \tag{2.14}$$

Now let's put variational inequality (2.14) into standard form (cf. Nagurney (1999)).

Definition 2.3: Standard Finite-Dimensional Variational Inequality Problem

The finite-dimensional variational inequality problem, VI$(F, \mathcal{K})$, is to determine a vector $X^* \in \mathcal{K} \subset R^{\mathcal{N}}$, such that:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{2.15}$$

where F is a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, $\mathcal{K}$ is a given closed, convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space.

For the model in this chapter, $N = mnot$. Let's define $X \equiv q$ and $F(X)$ as having components:

$$F_{ijk,l}(X) \equiv \sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{irp,s}(q)}{\partial q_{ijk,l}} + \rho_k - \omega_i \frac{\partial B_i(q)}{\partial q_{ijk,l}}, \quad \forall i, j, k, l, \tag{2.16}$$

with $\mathcal{K} \equiv K \cap \mathcal{S}$, and the standard form follows.

Remark 2.1: Existence and Uniqueness

Since the function $F(X)$ that enters the variational inequality problem (2.15) with components as in (2.16) is, under the imposed conditions, continuous and, clearly, the feasible set $\mathcal{K}$ is not only convex, but compact because of the demand and budget constraints, we know that a solution X^* exists from the standard theory of variational inequalities (Kinderlehrer and Stampacchia (1980)).

2.2. Lagrange Theory and Analysis of the Marginal Utilities

In this section, the Lagrange theory associated with the variational inequality (2.14) is investigated. Then, through the use of the Lagrange multipliers, the marginal utilities and the role of each constraint in the model are analyzed. Also alternative variational inequalities to the one in (2.14) are derived, one of which is then utilized for computational purposes.

By setting:

$$C(q) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left[\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k - \omega_i\frac{\partial B_i(q^*)}{\partial q_{ijk,l}}\right] \times \left[q_{ijk,l} - q^*_{ijk,l}\right], \tag{2.17}$$

variational inequality (2.14) can be rewritten as the following minimization problem:

$$\min_{\mathcal{K}} C(q) = C(q^*) = 0. \tag{2.18}$$

Based on the previous assumptions, all the involved functions in (2.18) are convex and continuously differentiable.

In order to construct the Lagrange function, the constraints are reformulated as below, with the associated Lagrange multiplier next to the corresponding constraint:

$$\begin{aligned}
e_{k,l} &= \sum_{i=1}^{m}\sum_{j=1}^{n} q_{ijk,l} - u_{k,l} \le 0, \quad \epsilon_{k,l}, \forall k, \forall l,\\
f_i &= \sum_{k=1}^{o}\rho_k \sum_{j=1}^{n}\sum_{l=1}^{t} q_{ijk,l} + \sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t} c_{ijk,l}(q) - b_i \le 0, \quad \gamma_i, \forall i,\\
g_{ijk,l} &= -q_{ijk,l} \le 0, \quad \lambda_{ijk,l}, \forall i, \forall j, \forall k, \forall l,\\
a_j &= \underline{d}_j - \sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t} q_{ijk,l} \le 0, \quad \alpha_j, \forall j,\\
b_j &= \sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t} q_{ijk,l} - \bar{d}_j \le 0, \quad \beta_j, \forall j,
\end{aligned} \tag{2.19}$$

and

$$\Gamma(q) = (e_{k,l}, f_i, g_{ijk,l}, a_j, b_j)_{i=1,\ldots,m; j=1,\ldots,n; k=1,\ldots,0; l=1,\ldots,t}. \tag{2.20}$$

Now let's construct the Lagrange function:

$$\mathcal{L}(q, \epsilon, \gamma, \lambda, \alpha, \beta) =$$
$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left[\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k - \omega_i\frac{\partial B_i(q^*)}{\partial q_{ijk,l}}\right] \times \left[q_{ijk,l} - q^*_{ijk,l}\right]$$
$$+\sum_{k=1}^{o}\sum_{l=1}^{t} e_{k,l}\epsilon_{k,l} + \sum_{i=1}^{m} f_i\gamma_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t} g_{ijk,l}\lambda_{ijk,l} + \sum_{j=1}^{n} a_j\alpha_j + \sum_{j=1}^{n} b_j\beta_j,$$
$$\forall q \in R_+^{mnot},\ \forall\alpha \in R_+^n,\ \forall\beta \in R_+^n,\ \forall\epsilon \in R_+^{ot},\ \forall\gamma \in R_+^m,\ \forall\lambda \in R_+^{mnot}, \qquad (2.21)$$

where α is the vector of all α_js, β is the vector of all β_js, ϵ is the vector of all $\epsilon_{k,l}$s, γ is the vector of all γ_is, and λ is the vector of all $\lambda_{ijk,l}$s.

Since the feasible set $\mathcal{K}$ is convex and the Slater condition is satisfied, if q^* is a minimal solution to problem (2.18) there exist $\epsilon^* \in R_+^{ot}$, $\gamma^* \in R_+^m$, $\lambda^* \in R_+^{mnot}$, $\alpha^* \in R_+^n$, $\beta^* \in R_+^n$, such that the vector $(q^*, \epsilon^*, \gamma^*, \lambda^*, \alpha^*, \beta^*)$ is a saddle point of the Lagrange function (2.21):

$$\mathcal{L}(q^*, \epsilon, \gamma, \lambda, \alpha, \beta) \le \mathcal{L}(q^*, \epsilon^*, \gamma^*, \lambda^*, \alpha^*, \beta^*) \le \mathcal{L}(q, \epsilon^*, \gamma^*, \lambda^*, \alpha^*, \beta^*) \qquad (2.22)$$

and

$$e^*_{k,l}\epsilon^*_{k,l} = 0, \qquad \forall k, \forall l,$$
$$f^*_i\gamma^*_i = 0, \qquad \forall i,$$
$$g^*_{ijk,l}\lambda^*_{ijk,l} = 0, \qquad \forall i, \forall j, \forall k, \forall l,$$
$$a^*_j\alpha^*_j = 0, \qquad \forall j,$$
$$b^*_j\beta^*_j = 0, \qquad \forall j. \qquad (2.23)$$

From the right-hand side of (2.22), it follows that $q^* \in R_+^{mnot}$ is a minimal point of the function $\mathcal{L}(q, \epsilon^*, \gamma^*, \lambda^*, \alpha^*, \beta^*)$ in the whole space R^{mnot}, and, hence, for all

$i = 1, \ldots, m$, for all $j = 1, \ldots, n$, for all $k = 1, \ldots, o$, and for all $l = 1, \ldots, t$, we have that:

$$\frac{\partial \mathcal{L}(q^*, \epsilon^*, \gamma^*, \lambda^*, \alpha^*, \beta^*)}{\partial q_{ijk,l}} = \left[\sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k - \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}} \right]$$

$$+\epsilon^*_{k,l} + \gamma^*_i \left(\rho_k + \sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} \right) - \lambda^*_{ijk,l} - \alpha^*_j + \beta^*_j = 0, \tag{2.24}$$

together with conditions (2.23).

Theorem 2.1: Alternative Variational Inequality Formulations

Conditions (2.23) and (2.24) represent an equivalent formulation of variational inequality (2.14) given by: determine $(q^*, \epsilon^*, \gamma^*, \lambda^*, \alpha^*, \beta^*) \in R_+^{2mnot+ot+m+2n}$ *such that*

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} \left[(\sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k)(1 + \gamma^*_i) - \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}} + \epsilon^*_{k,l} - \lambda^*_{ijk,l} - \alpha^*_j + \beta^*_j \right]$$

$$\times \left[q_{ijk,l} - q^*_{ijk,l} \right] + \sum_{k=1}^{o} \sum_{l=1}^{t} \left[u_{k,l} - \sum_{i=1}^{m} \sum_{j=1}^{n} q^*_{ijk,l} \right] \times \left[\epsilon^*_{k,l} - \epsilon^*_{k,l} \right]$$

$$+ \sum_{i=1}^{m} \left[b_i - \sum_{k=1}^{o} \rho_k \sum_{j=1}^{n} \sum_{l=1}^{t} q^*_{ijk,l} - \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q^*) \right] \times \left[\gamma_i - \gamma^*_i \right]$$

$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} q^*_{ijk,l} \times (\lambda_{ijk,l} - \lambda^*_{ijk,l})$$

$$+ \sum_{j=1}^{n} \left[\sum_{i=1}^{m} \sum_{k=1}^{o} \sum_{l=1}^{t} q^*_{ijk,l} - \underline{d}_j \right] \times \left[\alpha_j - \alpha^*_j \right] + \sum_{j=1}^{n} \left[\sum_{i=1}^{m} \sum_{k=1}^{o} \sum_{l=1}^{t} \bar{d}_j - q^*_{ijk,l} \right] \times \left[\beta_j - \beta^*_j \right] \geq 0,$$

$$\forall (q, \epsilon, \gamma, \lambda, \alpha, \beta) \in R_+^{2mnot+ot+m+2n}, \tag{2.25a}$$

or, more simply: determine $(q^*, \epsilon^*, \gamma^*, \alpha^*, \beta^*) \in R_+^{mnot+m+2n}$ *such that*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left[(\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}+\rho_k)(1+\gamma_i^*)-\omega_i\frac{\partial B_i(q^*)}{\partial q_{ijk,l}}+\epsilon_{k,l}^*-\alpha_j^*+\beta_j^*\right]$$

$$\times\left[q_{ijk,l}-q_{ijk,l}^*\right]+\sum_{k=1}^{o}\sum_{l=1}^{t}\left[u_{k,l}-\sum_{i=1}^{m}\sum_{j=1}^{n}q_{ijk,l}^*\right]\times\left[\epsilon_{k,l}^*-\epsilon_{k,l}^*\right]$$

$$+\sum_{i=1}^{m}\left[b_i-\sum_{k=1}^{o}\rho_k\sum_{j=1}^{n}\sum_{l=1}^{t}q_{ijk,l}^*-\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}c_{ijk,l}(q^*)\right]\times\left[\gamma_i-\gamma_i^*\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}q_{ijk,l}^*-\underline{d}_j\right]\times\left[\alpha_j-\alpha_j^*\right]+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}\bar{d}_j-q_{ijk,l}^*\right]\times\left[\beta_j-\beta_j^*\right]\geq 0,$$

$$\forall(q,\epsilon,\gamma,\alpha,\beta)\in R_+^{mnot+ot+m+2n}. \qquad (2.25b)$$

Proof: If we multiply (2.24) by $(q_{ijk,l}-q_{ijk,l}^*)$ and sum up with respect to i, j, k, and l we get:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left[\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}+\rho_k-\omega_i\frac{\partial B_i(q^*)}{\partial q_{ijk,l}}\right](q_{ijk,l}-q_{ijk,l}^*)$$

$$=-\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\epsilon_{k,l}^*q_{ijk,l}+\underbrace{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\epsilon_{k,l}^*q_{ijk,l}^*}_{=\epsilon_{k,l}^*u_{k,l}}$$

$$-\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(\rho_k+\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}\right)q_{ijk,l}$$

$$+\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(\rho_k+\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}\right)q_{ijk,l}^*$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\lambda_{jk,l}^*q_{ijk,l}-\underbrace{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\lambda_{jk,l}^*q_{ijk,l}^*}_{=0}$$

$$+\sum_{j=1}^{n}\alpha_j^*\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}q_{ijk,l}-\underbrace{\sum_{j=1}^{n}\alpha_j^*\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}q^*_{ijk,l}}_{=\alpha_j^*\underline{d}_j}$$

$$-\sum_{j=1}^{n}\beta_j^*\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}q_{ijk,l}+\underbrace{\sum_{j=1}^{n}\beta_j^*\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}q^*_{ijk,l})}_{=\beta_j^*\overline{d}_j}$$

$$=-\sum_{k=1}^{o}\sum_{l=1}^{t}\epsilon^*_{k,l}\underbrace{\sum_{i=1}^{m}\sum_{j=1}^{n}(q_{ijk,l}-u_{k,l})}_{\leq 0}+\underbrace{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\lambda^*_{ijk,l}q_{ijk,l}}_{\geq 0}$$

$$+\sum_{j=1}^{n}\alpha_j^*\underbrace{\left(\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}q_{ijk,l}-\underline{d}_j\right)}_{\geq 0}-\sum_{j=1}^{n}\beta_j^*\underbrace{\left(\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}q_{ijk,l}-\overline{d}_j\right)}_{\leq 0}$$

$$-\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(\rho_k+\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}\right)q_{ijk,l}$$
$$+\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(\rho_k+\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}\right)q^*_{ijk,l}. \tag{2.26}$$

Let's consider the last term in (2.26):

$$-\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(\rho_k+\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}\right)q_{ijk,l}$$

$$+\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(\rho_k+\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}\right)q^*_{ijk,l}. \tag{2.27}$$

From the second line of (2.19), we obtain:

$$\sum_{k=1}^{o}\rho_k\sum_{j=1}^{n}\sum_{l=1}^{t}q_{ijk,l}+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}c_{ijk,l}(q)-b_i\leq 0,\quad \forall i;$$

hence,

$$\sum_{i=1}^{m}\gamma_i^*\left(\sum_{k=1}^{o}\rho_k\sum_{j=1}^{n}\sum_{l=1}^{t}q_{ijk,l}+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}c_{ijk,l}(q)-b_i\right)\leq 0. \tag{2.28}$$

From the second line of (2.23), we get:

$$\gamma_i^* \left(\sum_{k=1}^{o} \rho_k \sum_{j=1}^{n} \sum_{l=1}^{t} q^*_{ijk,l} + \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q^*) - b_i \right) = 0, \quad \forall i,$$

and, therefore, we have that:

$$\sum_{i=1}^{m} \gamma_i^* \left(\sum_{k=1}^{o} \rho_k \sum_{j=1}^{n} \sum_{l=1}^{t} q^*_{ijk,l} + \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q^*) - b_i \right) = 0. \tag{2.29}$$

As a consequence,

$$- \left[\sum_{i=1}^{m} \gamma_i^* \left(\sum_{k=1}^{o} \rho_k \sum_{j=1}^{n} \sum_{l=1}^{t} \left(q_{ijk,l} - q^*_{ijk,l} \right) + \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} \left(c_{ijk,l}(q) - c_{ijk,l}(q^*) \right) \right) \right] \geq 0. \tag{2.30}$$

Since it is assumed that the logistical cost functions $c_{ijk,l}(q)$ are convex, $\forall i, j, k, l$, then, applying the convexity properties, for all i, j, k, and l, we have:

$$c_{ijk,l}(q) - c_{ijk,l}(q^*) \geq \left(\sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{ijk,l}(q)}{\partial q_{irp,s}} - \sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{ijk,l}(q^*)}{\partial q_{irp,s}} \right) (q_{irp,s} - q^*_{irp,s}). \tag{2.31}$$

Summing (2.31) with respect to i, j, k, and l, and multiplying by γ_i^*, we get:

$$\sum_{i=1}^{m} \gamma_i^* \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q) - \sum_{i=1}^{m} \gamma_i^* \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q^*)$$

$$\geq \sum_{i=1}^{m} \gamma_i^* \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} \left(\sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{ijk,l}(q)}{\partial q_{irp,s}} - \sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{ijk,l}(q^*)}{\partial q_{irp,s}} \right) \left(q_{irp,s} - q^*_{irp,s} \right). \tag{2.32}$$

Therefore,

$$-\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(c_{ijk,l}(q)-c_{ijk,l}(q^*)\right)$$

$$\leq -\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{ijk,l}(q)}{\partial q_{irp,s}}-\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{ijk,l}(q^*)}{\partial q_{irp,s}}\right)\left(q_{irp,s}-q^*_{irp,s}\right). \tag{2.33}$$

Now, adding $-\sum_{i=1}^{m}\gamma_i^*\sum_{k=1}^{o}\rho_k\sum_{j=1}^{n}\sum_{l=1}^{t}\left(q_{ijk,l}-q^*_{ijk,l}\right)$ to each side of (2.33), we obtain:

$$-\sum_{i=1}^{m}\gamma_i^*\sum_{k=1}^{o}\rho_k\sum_{j=1}^{n}\sum_{l=1}^{t}\left(q_{ijk,l}-q^*_{ijk,l}\right)$$

$$-\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{ijk,l}(q)}{\partial q_{irp,s}}-\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{ijk,l}(q^*)}{\partial q_{irp,s}}\right)\left(q_{irp,s}-q^*_{irp,s}\right)$$

$$\geq -\sum_{i=1}^{m}\gamma_i^*\sum_{k=1}^{o}\rho_k\sum_{j=1}^{n}\sum_{l=1}^{t}\left(q_{ijk,l}-q^*_{ijk,l}\right)-\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(c_{ijk,l}(q)-c_{ijk,l}(q^*)\right)\geq 0. \tag{2.34}$$

So, also the last term in (2.26) is greater than or equal to zero. The conclusion follows that variational inequality (2.14) is equivalent to the variational inequality (2.25a). Variational inequality (2.25b) then follows from (2.25a) since the nonnegativity of q is guaranteed by the feasible set in (2.25b).

I now provide interpretations of the Lagrange multipliers. Let's focus on the case where $q^*_{ijk,l}>0$; namely, the volume of relief items purchased by HO i at location k and shipped to demand location j by FSP l is positive; otherwise, if $q^*_{ijk,l}=0$, the problem is not interesting. Then, from the third line in (2.23), we have that $\lambda^*_{ijk,l}=0$.

Let us consider the situation when the constraints are not active, that is, $\sum_{i=1}^{m}\sum_{j=1}^{n}q^*_{ijk,l}<u_{k,l}$, $\forall k$, $\forall l$, $\sum_{k=1}^{o}\rho_k\sum_{j=1}^{n}\sum_{l=1}^{t}q^*_{ijk,l}+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}c_{ijk,l}(q^*)<b_i$, $\forall i$, and $\underline{d}_j<\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}q^*_{ijk,l}<\overline{d}_j$, $\forall j$.

In this case, all the associated Lagrange multipliers are equal to zero, specifically:

$$\epsilon^*_{k,l} = 0,\, \forall k,\, \forall l;\;\; \gamma^*_i = 0,\, \forall i;\;\; \alpha^*_j = 0,\, \forall j;\;\; \beta^*_j = 0,\, \forall j;$$

hence, (2.24) yields:

$$\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k = \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}}. \tag{2.35}$$

This means that the weighted marginal altruism is equal to the sum of the marginal logistical cost and the relief item purchase price (for the respective i, j, k, l).

If, now, constraint (2.8) is active for k, l; namely, $\sum_{i=1}^{m}\sum_{j=1}^{n} q^*_{ijk,l} = u_{k,l}$, then, from the first line of (2.23), the associated Lagrange multiplier $\epsilon^*_{k,l}$ is greater than zero and (2.24) becomes:

$$\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k + \epsilon_{k,l} = \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}}, \tag{2.36}$$

which means that the weighted marginal altruism exceeds the sum of the marginal logistical cost and the relief item purchase price (for the respective i, j, k, l) and this is a desirable situation since we are dealing with a humanitarian organization.

If constraint (2.6) is active for i; namely, $\sum_{k=1}^{o} \rho_k \sum_{j=1}^{n}\sum_{l=1}^{t} q^*_{ijk,l} + \sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t} c_{ijk,l}(q^*) = b_i$, then from the second line of (2.23), the associated Lagrange multiplier γ^*_i is greater than zero and (2.24) becomes:

$$(1+\gamma^*_i)\left[\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k\right] = \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}}, \tag{2.37}$$

which implies also a desirable situation.

If constraint (2.9) is active for j; namely, $\sum_{i=1}^{m}\sum_{k=1}^{n}\sum_{l=1}^{t} q^*_{ijk,l} = \underline{d}_j$, then, from the fourth line of (2.23), the associated Lagrange multiplier α^*_j is greater than zero and (2.24) becomes:

$$\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k - \alpha_j = \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}}, \tag{2.38}$$

which is an undesirable situation. Indeed, one can see that, in this case, the volume of disaster relief items to the demand point is only at its lower bound and, hence, the marginal logistical cost and the relief item purchase cost exceed the weighted marginal altruism.

If, on the other hand, constraint (2.9) is active for j; namely, $\sum_{i=1}^{m}\sum_{k=1}^{n}\sum_{l=1}^{t} q^*_{ijk,l} = \overline{d}_j$, then, from the fifth line of (2.23), the associated Lagrange multiplier β^*_j is greater than zero and (2.24) becomes:

$$\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k + \beta_j = \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}}, \tag{2.39}$$

which means that the weighted marginal altruism exceeds the sum of the marginal logistical cost and the relief item purchase price (for the respective i, j, k, l) and this is again a desirable situation. Observe that, in this case, the victims of the disaster at that demand point receive a volume of relief items at the upper bound.

From the above analysis of the Lagrange multipliers and marginal utilities at the equilibrium solution, it can be concluded that the most desirable case, in terms of weighted altruism, is the one when $\sum_{i=1}^{m}\sum_{j=1}^{n} q^*_{ijk,l} = u_{k,l}$, $\sum_{i=1}^{m}\sum_{k=1}^{n}\sum_{l=1}^{t} q^*_{ijk,l} = \overline{d}_j$, and $(1+\gamma^*_i)\left[\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k\right] = \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}}$. Indeed, in this case, the full resources of the freight service provider for shipping to the demand point are utilized and the victims of the disasters at that demand point acquire the volume of relief items at the maximum amount demanded, that is, at the imposed upper bound.

2.3. Algorithm and Numerical Examples

In this subsection, before discussing the numerical examples, the algorithm that is used for the computations is described. The algorithm is the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). As Dupuis and Nagurney (1993) establish, for convergence of the general iterative scheme, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$. Convergence conditions for different types of network-based problems are presented in Nagurney and Zhang (1996) and Nagurney (2006).

Specifically, variational inequality (2.25b) is utilized for the computations.

The Euler method yields a specific closed form expression for each relief item flow and Lagrange multiplier at iteration $\tau + 1$ as follows.

2.3.1 Explicit Formulae for the Euler Method Applied to the Game Theory Model

Specifically, at an iteration $\tau + 1$, the following is the closed form expression for the relief item flow that each HO $i = 1, \ldots, m$, purchases at location $k = 1, \ldots, o$, and has then transported to the demand point $j = 1, \ldots, n$, by FSP $l = 1, \ldots, t$:

$$q_{ijk,l}^{\tau+1} = \max\{0, q_{ijk,l}^{\tau} + a_\tau(\omega_i \frac{\partial B_i(q^\tau)}{\partial q_{ijk,l}} - (\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^\tau)}{\partial q_{ijk,l}} + \rho_k)(1+\gamma_i^\tau) + \alpha_j^\tau - \beta_j^\tau - \epsilon_{k,l}^\tau)\}. \tag{2.40}$$

The explicit formula for the Lagrange multipliers associated with the budget constraint (2.6), respectively, for $i = 1, \ldots, m$, at iteration $\tau + 1$, is:

$$\gamma_i^{\tau+1} = \max\{0, \gamma_i^{\tau} + a_{\tau}(-b_i + \sum_{k=1}^{o} \rho_k \sum_{j=1}^{n} \sum_{l=1}^{t} q_{ijk,l}^{\tau} + \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q^{\tau}))\}. \quad (2.41)$$

The closed form expression for the Lagrange multiplier for each capacity constraint (2.8) for $k = 1, \ldots, o$; $l = 1, \ldots, t$, in turn, at iteration $\tau + 1$, is:

$$\epsilon_{k,l}^{\tau+1} = \max\{0, \epsilon_{k,l}^{\tau} + a_{\tau}(-u_{k,l} + \sum_{i=1}^{m} \sum_{j=1}^{n} q_{ijk,l}^{\tau})\}. \quad (2.42)$$

The Lagrange multiplier for the demand lower bound constraint (2.9) at demand points $j = 1, \ldots, n$, at iteration $\tau + 1$, is computed according to:

$$\alpha_j^{\tau+1} = \max\{0, \alpha_j^{\tau} + a_{\tau}(-\sum_{i=1}^{m} \sum_{k=1}^{o} \sum_{l=1}^{t} q_{ijk,l}^{\tau} + \underline{d}_j)\}. \quad (2.43)$$

The Lagrange multiplier for the demand upper bound constraint (2.10) at demand points $j = 1, \ldots, n$, at iteration $\tau + 1$, on the other hand, is computed as follows:

$$\beta_j^{\tau+1} = \max\{0, \beta_j^{\tau} + a_{\tau}(-\bar{d}_j + \sum_{i=1}^{m} \sum_{k=1}^{o} \sum_{l=1}^{t} q_{ijk,l}^{\tau})\}. \quad (2.44)$$

2.3.2 Numerical Examples

The examples in this subsection are inspired by Hurricane Harvey. Hurricane Harvey was a Category 4 storm that hit Texas on August 25, 2017 with epic flooding. Hurricane Harvey caused \$125 billion in damage, according to the National Hurricane Center, and affected almost 13 million people. That is more than any other natural disaster in U.S history except for Hurricane Katrina (Amadeo (2018)).

The Episcopal Health Foundation analyzed the FEMA assistance applications. There were over 880,000 applications across 41 Texas counties (Keyser (2017)). In the analysis, it was clear that some regions of Texas had needed more help, such as Port Arthur, where over 13,654 applications were submitted, whereas other locations had fewer recorded cases, such as Bay City, with 6500, and Silsbee, with 3,232 registered applications.

One of the active organizations in disaster relief in Houston was the American Red Cross (ARC) (cf. FEMA (2017)). The Salvation Army was another humanitarian organization that had a role in disaster relief post Hurricane Harvey (The Salvation Army (2017)). Hence, these two humanitarian organizations are utilized in the numerical examples.

Example 2.1

In Example 2.1, the basic disaster relief supply chain network depicted in Figure 2.2 is utilized. It consists of two humanitarian organizations, The Salvation Army and the American Red Cross, respectively. The Salvation Army is a smaller relief organization as compared to the American Red Cross, which has a larger budget. There are three demand points: Port Arthur, Bay City, and Silsbee, respectively. The major devastation occurred in the Port Arthur region and, therefore, the need for relief items at this demand point was much greater than at the two other areas.

The HOs are involved in the purchasing and delivery of relief item kits. Here "items" and "kits" are used interchangeably. The HOs have two location options at which to purchase the relief items: Purchasing Location 1 (PL 1) and Purchasing Location 2 (PL 2). PL 1, unlike PL 2, offers the relief items at reasonable prices because the market is far from the affected area and the disaster has not had an effect on its prices. However, PL 2, which is a local market, offers a similar product at a higher price due to the increased demand after the disaster. The relief item kits are sold at the two purchasing locations at the following prices:

$$\rho_1 = 50, \quad \rho_2 = 70.$$

The above prices are reasonable, based on data from FEMA (2012) and the American Red Cross (2018).

It is assumed that there are two Freight Service Providers, FSP 1 and FSP 2, involved in the shipment of the relief items from the purchase locations to the demand points. In contrast to FSP 2, FSP 1 has less equipment and capability, so it provides service at a lower capacity as compared to FSP 1.

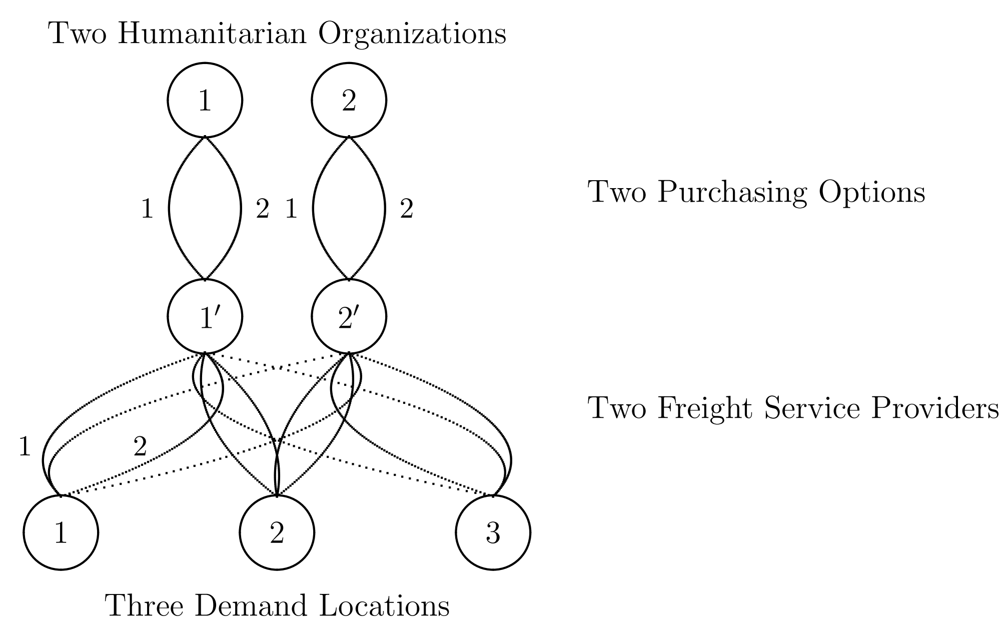

Figure 2.2. Example 2.1: Two Humanitarian Organizations, Three Demand Locations, Two Purchasing Options, Two Freight Service Providers

The data for Example 2.1 are as follows.

The humanitarian organizations' budgets, in dollars, are:

$$b_1 = 3 \times 10^6, \quad b_2 = 6 \times 10^6.$$

According to FEMA (2017), the American Red Cross provided \$45 million in disaster relief immediately after Hurricane Harvey hit Houston. Given that the three demand points in the numerical example are representative, although a fraction of the sites impacted, and The Salvation Army's relative size to the ARC, the budgets that are utilized for the two humanitarian organizations are reasonable.

The HOs' altruism functions are:

$$B_1(q) = \sum_{k=1}^{2} \sum_{l=1}^{2} (300 q_{11k,l} + 200 q_{12k,l} + 100 q_{13k,l}),$$
$$B_2(q) = \sum_{k=1}^{2} \sum_{l=1}^{2} (400 q_{21k,l} + 300 q_{22k,l} + 200 q_{23k,l}),$$

and the monetization weights associated with these benefit functions are:

$$\omega_1 = 1, \quad \omega_2 = 1.$$

Similar altruism functions were used in Nagurney et al. (2018) and are quite reasonable. Indeed, Demand Point 1 has had more devastation and, as a result, attracts the most media attention, so the coefficient of $q_{i1k,l}$ is the highest one. Demand Points 2 and 3 have the next highest such coefficients, accordingly. And, generally, HO 2 has higher coefficients than HO 1 because it has a more recognized brand.

The lower and upper bounds for the relief items at the demand points are:

$$\underline{d}_1 = 10000, \quad \bar{d}_1 = 20000,$$

$$\underline{d}_2 = 1000, \quad \bar{d}_2 = 10000,$$

$$\underline{d}_3 = 1000, \quad \bar{d}_3 = 10000.$$

These lower and upper bound demands are in concert with the number of FEMA assistance applications for these locations, as noted at the beginning of Subsection 2.3.2.

The FSPs' capacities are as follows:

$$u_{1,1} = 3000, \quad u_{1,2} = 6000,$$

$$u_{2,1} = 5000, \quad u_{2,2} = 8000.$$

Each FSP, according to its facilities and location of origin of purchase and destination of delivery, encumbers different logistical costs, with these cost functions being:

$$c_{i11,1}(q) = 0.2q_{i11,1}^2 + 2q_{i11,1} + q_{j11,1}, \quad c_{i21,1}(q) = 0.2q_{i21,1}^2 + 5q_{i21,1} + 2.5q_{j21,1},$$

$$c_{i31,1}(q) = 0.2q_{i31,1}^2 + 7q_{i31,1} + 3.5q_{j31,1},$$

$$c_{i12,1}(q) = 0.15q_{i12,1}^2 + 2q_{i12,1} + q_{j12,1}, \quad c_{i22,1}(q) = 0.15q_{i22,1}^2 + 5q_{i22,1} + 2.5q_{j22,1},$$

$$c_{i32,1}(q) = 0.15q_{i32,1}^2 + 7q_{i32,1} + 3.5q_{j32,1},$$

$$c_{i11,2}(q) = 0.15q_{i11,2}^2 + 2q_{i11,2} + q_{j11,2}, \quad c_{i21,2}(q) = 0.15q_{i21,2}^2 + 5q_{i21,2} + 2.5q_{j21,2},$$

$$c_{i31,2}(q) = 0.15q_{i31,2}^2 + 7q_{i31,2} + 3.5q_{j31,2},$$

$$c_{i12,2}(q) = 0.1q_{i12,2}^2 + 2q_{i12,2} + q_{j12,2}, \quad c_{i22,2}(q) = 0.1q_{i22,2}^2 + 5q_{i22,2} + 2.5q_{j22,2},$$

$$c_{i32,2}(q) = 0.1q_{i32,2}^2 + 7q_{i32,2} + 3.5q_{j32,2},$$

$$\forall i = 1, 2; j \neq i.$$

Observe that FSP 2 has lower costs than FSP 1 and higher shipment capacities.

The Euler method was implemented in FORTRAN and a Linux system at the University of Massachusetts Amherst used for the computations. The algorithm was initialized so that the flows on the links leading to each demand point are equal, and their summation is equal to the lower bound on the demand at that point. All the Lagrange multipliers are set to 0.00. The algorithm is considered to have converged when the absolute value of all the computed variables at two successive iterates are less than or equal to 10^{-5}. The sequence $\{a_\tau\} = .1(1, 1/2, 1/2, 1/3, 1/3, 1/3, \ldots)$.

The results for Example 2.1 are given below.

The computed equilibrium relief item flows for $i = 1, 2; j = 1, 3$, are:

$$q^*_{ij1,1} = \begin{bmatrix} 874.22 & 362.42 & 107.52 \\ 134.20 & 0.00 & 357.42 \end{bmatrix}, \quad q^*_{ij2,1} = \begin{bmatrix} 1098.80 & 416.54 & 76.70 \\ 1432.14 & 749.74 & 409.87 \end{bmatrix},$$

$$q^*_{ij1,2} = \begin{bmatrix} 1165.47 & 483.20 & 143.28 \\ 1498.80 & 816.10 & 476.53 \end{bmatrix}, \quad q^*_{ij2,2} = \begin{bmatrix} 1648.22 & 624.48 & 115.05 \\ 2148.17 & 1123.9 & 614.48 \end{bmatrix}.$$

The amount of the relief item kits delivered to each demand point is:

$$\sum_{i=1}^{2}\sum_{k=1}^{2}\sum_{l=1}^{2} q^*_{i1k,l} = 10000.00, \quad \sum_{i=1}^{2}\sum_{k=1}^{2}\sum_{l=1}^{2} q^*_{i2k,l} = 4576.76,$$

$$\sum_{i=1}^{2}\sum_{k=1}^{2}\sum_{l=1}^{2} q^*_{i3k,l} = 2300.86.$$

Since the demand is at the lower bound at the first demand point, α_1^* is positive, while the other Lagrange multipliers associated with the lower bound demand constraints are equal to zero:

$$\alpha_1^* = 101.72, \quad \alpha_2^* = \alpha_3^* = 0.$$

Since all the demand points receive volumes of relief items less than their corresponding upper bound, all the Lagrange multipliers associated with the upper demand bound constraints are equal to zero, that is,

$$\beta_1^* = \beta_2^* = \beta_3^* = 0.$$

The volumes of relief items carried by each FSP $l; l = 1, 2$, from each purchasing location $k; k = 1, 2$, are:

$$\sum_{i=1}^{2}\sum_{j=1}^{3} q_{ijk,l}^* = \begin{bmatrix} 1835.79 & 4583.70 \\ 4183.79 & 6274.38 \end{bmatrix}.$$

None of the FSPs have consumed their full capacities and, therefore:

$$\epsilon_{1,1}^* = \epsilon_{1,2}^* = \epsilon_{2,1}^* = \epsilon_{2,2}^* = 0.$$

The logistical costs for the humanitarian organizations are, respectively,

$$\sum_{j=1}^{3}\sum_{k=1}^{2}\sum_{l=1}^{2} c_{1jk,l}(q^*) = 976,436.92$$

and

$$\sum_{j=1}^{3}\sum_{k=1}^{2}\sum_{l=1}^{2} c_{2jk,l}(q^*) = 1,598,108.48.$$

The total expenditure of each humanitarian organization in this disaster relief operation, which includes the cost of purchasing and shipping the relief items, is, respectively:

$$\sum_{k=1}^{2} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q^*_{1jk,l} + \sum_{j=1}^{3} \sum_{k=1}^{2} \sum_{l=1}^{2} c_{1jk,l}(q^*) = 1,419,224.00,$$

$$\sum_{k=1}^{2} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q^*_{2jk,l} + \sum_{j=1}^{3} \sum_{k=1}^{2} \sum_{l=1}^{2} c_{2jk,l}(q^*) = 2,208,465.25.$$

Observe that HO 1's logistical costs are 69% of its total costs, whereas the logistical costs of HO 2 are 72% of its total costs. This is reasonable since, as noted in Section 1.3, logistics is estimated to account for 80% of the total cost associated with disaster relief.

Since both organizations spend less than their budgets, the Lagrange multipliers associated with their budget constraints are equal to zero:

$$\gamma_1^* = \gamma_2^* = 0.$$

The benefit/altruism that each humanitarian organization gains from helping the disaster victims is:

$$B_1(q^*) = 1,857,600.50, \quad B_2(q^*) = 3,264,018.25.$$

Putting all the terms in the respective objective functions together, the utility of each HO, after the disaster relief operation, is:

$$U_1(q^*) = 438,376.50, \quad U_2(q^*) = 1,055,553.00.$$

Observe that the American Red Cross, which is the larger organization, tends to be more active than The Salvation Army. The American Red Cross delivers 9,761.73 relief item kits, which is more than the 7,115.92 that The Salvation Army delivers. The high volume of relief item kit shipments encumbers a high cost to the ARC, but, at the same time, it brings more benefits to it and, ultimately, the American Red Cross has a higher utility than The Salvation Army.

FSP 2 achieves a large share of the transportation market by benefiting from its lower costs and larger shipment capacities. FSP 2 carries a volume of 10,858.08 disaster relief item kits, while FSP 1 carries just 6,019.58.

In the relief item kit sales market, 10.458.16 relief item kits are purchased from PL 2 and, despite having a higher price, PL2 is preferred by the HOs due to the lower shipping costs. PL 1 also, because of its lower item price, still enjoys a good share of market with 6,419.48 relief item kits delivered to the demand points from PL 1.

Example 2.2

In this example, the HOs have a new location option for purchasing the relief items. The new purchasing location, denoted by PL 3, is a local one. It charges a lower price than the existing local purchasing location, PL 2, in order to compete and gain a good market share but its price is still higher than PL 1's price. The supply chain network topology for Example 2.2 is depicted in Figure 2.3.

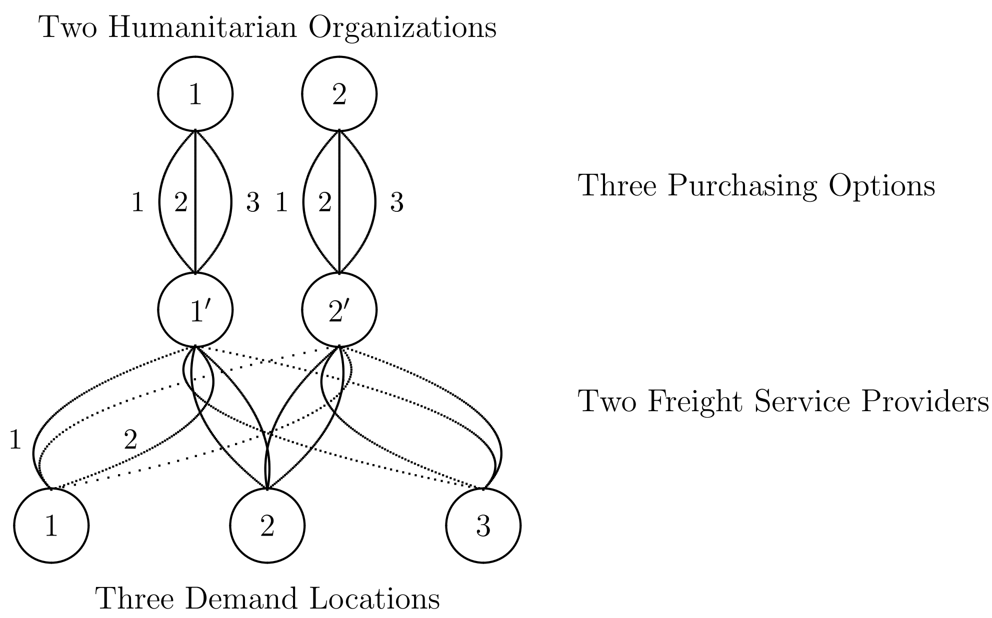

Figure 2.3. Example 2.2: Two Humanitarian Organizations, Three Demand Locations, Three Purchasing Options, Two Freight Service Providers

The data on the new purchasing location, added to Example 2.1, are presented below.

The relief items are sold at PL 3 at the price:

$$\rho_3 = 60.$$

The FSPs' capacities for shipping products from the purchasing locations to the affected area have been updated as below:

$$u_{1,1} = 3000, \quad u_{1,2} = 6000,$$

$$u_{2,1} = 4000, \quad u_{2,2} = 7000.$$

$$u_{3,1} = 4000, \quad u_{3,2} = 7000.$$

The HOs' altruism functions are:

$$B_1(q) = \sum_{k=1}^{3}\sum_{l=1}^{2}(300q_{11k,l} + 200q_{12k,l} + 100q_{13k,l}),$$

$$B_2(q) = \sum_{k=1}^{3}\sum_{l=1}^{2}(400q_{21k,l} + 300q_{22k,l} + 200q_{23k,l}).$$

The logistical cost functions from the new purchasing location PL 3 to the affected area are as follows:

$$c_{i13,1}(q) = 0.15q_{i13,1}^2 + 2q_{i13,1} + q_{j13,1}, \quad c_{i23,1}(q) = 0.15q_{i23,1}^2 + 5q_{i23,1} + 2.5q_{j23,1},$$

$$c_{i33,1}(q) = 0.15q_{i33,1}^2 + 7q_{i33,1} + 3.5q_{j33,1},$$

$$c_{i13,2}(q) = 0.1q_{i13,2}^2 + 2q_{i13,2} + q_{j13,2}, \quad c_{i23,2}(q) = 0.1q_{i23,2}^2 + 5q_{i23,2} + 2.5q_{j23,2},$$

$$c_{i33,2}(q) = 0.1q_{i33,2}^2 + 7q_{i33,2} + 3.5q_{j33,2}.$$

$$\forall i = 1, 2; j \neq i.$$

The computed equilibrium relief item flows for $i = 1, 2; j = 1, 3$ are now:

$$q^*_{ij1,1} = \begin{bmatrix} 620.14 & 362.42 & 107.48 \\ 0.00 & 0.00 & 357.42 \end{bmatrix}, \quad q^*_{ij2,1} = \begin{bmatrix} 760.20 & 416.54 & 76.69 \\ 1092.91 & 749.74 & 409.87 \end{bmatrix},$$

$$q^*_{ij1,2} = \begin{bmatrix} 826.80 & 483.20 & 143.28 \\ 1159.56 & 816.40 & 476.53 \end{bmatrix}, \quad q_{ij2,2} = \begin{bmatrix} 1139.12 & 624.48 & 114.94 \\ 1639.00 & 1123.96 & 614.48 \end{bmatrix},$$

$$q^*_{ij3,1} = \begin{bmatrix} 793.51 & 449.87 & 109.97 \\ 1126.623 & 783.07 & 443.20 \end{bmatrix},$$

$$q^*_{ij3,2} = \begin{bmatrix} 1189.10 & 674.47 & 164.88 \\ 1688.99 & 1173.96 & 664.47 \end{bmatrix}.$$

The volume of relief item kits received at each demand point is:

$$\sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i1k,l} = 12,035.55, \quad \sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i2k,l} = 7658.10,$$
$$\sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i3k,l} = 3,683.22.$$

Observe that all the demand points receive a volume of relief item kits greater than their respective demand lower bound and, therefore, all the Lagrange multipliers for the lower bound demand constraints are equal to zero:

$$\alpha^*_1 = \alpha^*_2 = \alpha^*_3 = 0.$$

Also, the demand points receive an amount of relief items less than their demand upper bound. Hence, all the Lagrange multipliers associated with the upper bound demand constraints are also equal to zero:

$$\beta^*_1 = \beta^*_2 = \beta^*_3 = 0.$$

The relief item kit volumes carried from each purchasing location $k; k = 1, 2, 3$, by each FSP $l; l = 1, 2$ are:

$$\sum_{i=1}^{2}\sum_{j=1}^{3} q^*_{ijk,l} = \begin{bmatrix} 1447.47 & 3905.76 \\ 3505.94 & 5255.98 \\ 3705.85 & 5555.87 \end{bmatrix}.$$

Not one of the FSPs has reached its capacity and, hence, we have that:

$$\epsilon^*_{1,1} = \epsilon^*_{1,2} = \epsilon^*_{2,1} = \epsilon^*_{2,2} = \epsilon^*_{3,1} = \epsilon^*_{3,2} = 0.$$

The total cost of each organization in this operation, which includes the cost of purchasing and shipping the relief items, is:

$$\sum_{k=1}^{3} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q^*_{1jk,l} + \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{2} c_{1jk,l}(q^*) = 1,454,783.75,$$
$$\sum_{k=1}^{3} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q^*_{2jk,l} + \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{2} c_{2jk,l}(q^*) = 2,821,781.50.$$

Since both organizations spend less than their budgets, the Lagrange multipliers associated with the budget constraints are equal to zero:

$$\gamma_1^* = \gamma_2^* = 0.$$

The benefit/altruism of each organization is:

$$B_1(q^*) = 2,272,585.50, \quad B_2(q^*) = 4,670,005.50.$$

The utility of each HO, after the disaster relief operation, is:

$$U_1(q^*) = 817,796.75, \quad U_2(q^*) = 1,848,224.00.$$

In Example 2.2, with the addition of a new purchasing location, both organizations take advantage of this opportunity and purchase more relief item kits for delivery to the affected areas. The American Red Cross, the larger organization, manages to increase the amount of items shipped to 14,319.79 - an increase of almost 5,000, as compared to that in Example 2.1. The Salvation Army provides 9,057.09 relief item kits, while its contribution was only slightly above 7,000 kits in Example 2.1. Both humanitarian organizations pay more, after the addition of a new PL, but this higher cost has led to a significant increase in their utilities.

In the relief item kit sales market, as expected, the new PL is able to take a great market share due to its lower price than the other local PL and with lower associated logistical costs than the nonlocal PL. Both of the previous purchasing locations drop sales with the arrival of the new PL. PL 1 and PL 2 sell 5,353.23 and 8,761.92 relief item kits, respectively, while PL 3 is very successful at selling 9,261.72 items.

The increase in the purchasing power of the HOs has also boosted the logistical / transportation market. Both FSPs ship higher volumes of relief items as compared to Example 2.1. FSP 1 and FSP 2 ship 14,717.61 and 8,659.26 relief items, respectively, with the major increase being in the shipments of the relief items from the newly added PL to the affected region. All of the purchasing locations experience a drop in sales for the disaster with the total volume purchased at PL 1 being: 4,157.66; that at PL 2: 6,996.47, and at PL 3: 7,413.07.

Example 2.3

Example 2.3 has the same data as that in Example 2.2 except that now additional disruptions in transportation are considered so that all the logistical costs are as in Example 2.2 except that the nonlinear component is multiplied by a factor of 10.

The new computed equilibrium relief item flows $i = 1, 2; j = 1, 3$ are:

$$q^*_{ij1,1} = \begin{bmatrix} 575.14 & 249.96 & 0.00 \\ 0.00 & 0.00 & 266.55 \end{bmatrix}, \quad q^*_{ij2,1} = \begin{bmatrix} 700.21 & 266.54 & 0.00 \\ 1032.92 & 599.75 & 199.94 \end{bmatrix},$$

$$q^*_{ij1,2} = \begin{bmatrix} 766.86 & 333.21 & 0.00 \\ 1099.57 & 666.41 & 266.55 \end{bmatrix}, \quad q_{ij2,2} = \begin{bmatrix} 1049.17 & 399.73 & 0.00 \\ 1549.01 & 899.45 & 299.75 \end{bmatrix},$$

$$q^*_{ij3,1} = \begin{bmatrix} 0.00 & 299.88 & 0.00 \\ 1066.25 & 633.08 & 233.27 \end{bmatrix},$$

$$q^*_{ij3,2} = \begin{bmatrix} 0.00 & 499.73 & 0.00 \\ 1599.00 & 949.45 & 349.74 \end{bmatrix}.$$

Due to the increase in the logistical/transportation costs, an increasing number of the computed equilibrium flow variables are now equal to 0. In Example 2.2, there were two flows at level 0.00 and now there are ten such flows, including the same two as in Example 2.2. Some purchasing/delivery combinations are no longer attractive under the higher logistical costs.

The volume of relief item kits received at each demand point is now:

$$\sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i1k,l} = 11,270.81, \quad \sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i2k,l} = 5,747.18,$$

$$\sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i3k,l} = 1,549.20.$$

All the demand points receive a volume of relief item greater than their demand lower bound and, therefore, all the Lagrange multipliers for the lower bound demand constraints are equal to zero:

$$\alpha^*_1 = \alpha^*_2 = \alpha^*_3 = 0.$$

Also, the demand points receive an amount of relief items less than their demand upper bound. As a result, all the Lagrange multipliers associated with the upper bound demand constraints are also equal to zero:

$$\beta^*_1 = \beta^*_2 = \beta^*_3 = 0.$$

The relief item kit volumes delivered from each purchasing location $k; k = 1, 2, 3$ by each FSP $l; l = 1, 2$ are now:

$$\sum_{i=1}^{2} \sum_{j=1}^{3} q^*_{ijk,l} = \begin{bmatrix} 1025.06 & 3132.59 \\ 2799.36 & 4197.11 \\ 2966.01 & 4447.06 \end{bmatrix}.$$

FSP 2 remains the dominant freight service provider in this disaster operation.

The FSPs have not exhausted their capacities and, therefore:

$$\epsilon^*_{1,1} = \epsilon^*_{1,2} = \epsilon^*_{2,1} = \epsilon^*_{2,2} = \epsilon^*_{3,1} = \epsilon^*_{3,2} = 0.$$

The total cost of each humanitarian organization is now:

$$\sum_{k=1}^{3} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q^*_{1jk,l} + \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{2} c_{1jk,l}(q^*) = 1,459,130.25,$$

$$\sum_{k=1}^{3} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q^*_{2jk,l} + \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{2} c_{2jk,l}(q^*) = 2,657,446.75.$$

Again, both organizations spend less than their full budgets, and, therefore, the Lagrange multipliers associated with the budget constraints are equal to zero:

$$\gamma^*_1 = \gamma^*_2 = 0.$$

HO 1 purchases, and has delivered, 6,923.11 relief item kits, whereas HO 2 purchases 11,644.08 relief item kits and has that volume shipped to the points of demand.

The benefit/altruism of each organization is:

$$B_1(q^*) = 1,877,028.75, \quad B_2(q^*) = 3,972,979.50.$$

The utility of each HO, after the disaster relief operation, is:

$$U_1(q^*) = 417,898.50, \quad U_2(q^*) = 1,315,532.75.$$

Both HOs have a reduced incurred altruism, as compared to those in Example 2.2, and also lower utilities.

Example 2.4

Example 2.4 is constructed from Example 2.3 and considers even a greater disruption in the transportation network, with even larger associated costs. The data remain as in Example 2.3, but now each nonlinear term in each logistical cost function is multiplied by a factor of 3.

The new computed equilibrium relief item flows for $i = 1, 2; j = 1, 3$ are:

$$q^*_{ij1,1} = \begin{bmatrix} 541.48 & 83.33 & 23.15 \\ 294.84 & 0.00 & 106.48 \end{bmatrix}, \quad q^*_{ij2,1} = \begin{bmatrix} 699.75 & 88.89 & 8.64 \\ 810.86 & 199.99 & 119.75 \end{bmatrix},$$

$$q^*_{ij1,2} = \begin{bmatrix} 721.97 & 111.11 & 30.86 \\ 833.08 & 222.21 & 141.98 \end{bmatrix}, \quad q_{ij2,2} = \begin{bmatrix} 1049.63 & 133.32 & 12.96 \\ 1216.30 & 299.96 & 179.63 \end{bmatrix},$$

$$q^*_{ij3,1} = \begin{bmatrix} 710.86 & 100.00 & 19.75 \\ 821.97 & 211.10 & 130.87 \end{bmatrix},$$

$$q^*_{ij3,2} = \begin{bmatrix} 1066.30 & 149.98 & 29.63 \\ 1232.97 & 316.63 & 196.30 \end{bmatrix}.$$

The volume of relief item kits received at each demand point is now:

$$\sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i1k,l} = 10,000.00, \quad \sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i2k,l} = 1,916.52,$$

$$\sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i3k,l} = 1,000.00.$$

Observe that now, due to the increase in logistical costs, the demands at the first and third demand points are at their respective lower bounds and, hence, the corresponding Lagrange multipliers are now positive; that is:

$$\alpha^*_1 = 419.84, \quad \alpha^*_2 = 0.00, \quad \alpha^*_3 = 47.78.$$

The Lagrange multipliers associated with the upper bound demand constraints are, as in Example 2.3, again, all equal to zero:

$$\beta^*_1 = \beta^*_2 = \beta^*_3 = 0.$$

The relief item kit volumes delivered from each purchasing location $k; k = 1, 2, 3$, by each FSP $l; l = 1, 2$ are:

$$\sum_{i=1}^{2}\sum_{j=1}^{3} q^*_{ijk,l} = \begin{bmatrix} 1049.28 & 2061.21 \\ 1927.88 & 2891.81 \\ 1994.55 & 2994.55 \end{bmatrix}.$$

The FSPs have not reached their capacities and, hence, we have that:

$$\epsilon^*_{1,1} = \epsilon^*_{1,2} = \epsilon^*_{2,1} = \epsilon^*_{2,2} = \epsilon^*_{3,1} = \epsilon^*_{3,2} = 0.$$

The total cost of each humanitarian organization is now:

$$\sum_{k=1}^{3} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q^*_{1jk,l} + \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{2} c_{1jk,l}(q^*) = 2,152,526.75,$$

$$\sum_{k=1}^{3} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q^*_{2jk,l} + \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{2} c_{2jk,l}(q^*) = 2,782,132.00.$$

The total costs increase for both HOs, as compared to Example 2.3, with HO 1 encumbering the greater increase in its costs. HO 1 delivers a total of 5,581.62 relief item kits, whereas HO 2 delivers 7,334.92 relief item kits. These values are significantly lower than those obtained in Example 2.3. Moreover, HO 2 experiences the greater decrease in volume purchased and shipped than HO 1.

The volume of purchases at PL 1 is now: 3,110.49; that at PL 2 is: 4,819.69, whereas the amount purchased at Pl 3 is now: 4,986.36.

Again, both humanitarian organizations spend less than their full budgets, and, consequently, the Lagrange multipliers associated with the budget constraints are equal to zero:

$$\gamma_1^* = \gamma_2^* = 0.$$

The benefit/altruism of each organization is now:

$$B_1(q^*) = 1,582,822.00, \quad B_2(q^*) = 2,633,978.00.$$

The utility of each HO, after the disaster relief operation, is now:

$$U_1(q^*) = -569,704.75, \quad U_2(q^*) = -148,154.00.$$

Observe that, due to the very high logistical costs associated with this disaster relief operation, the humanitarian organizations now encounter negative utilities. Nevertheless, as humanitarian organizations, and with the imposed minimum demands, they are required to meet the demands of the disaster victims.

Also, observe that, interestingly, in Example 2.3, eight of the flows were equal to 0.00, whereas now only one flow is equal to 0.00. With the increased nonlinear term factor, the HOs spread their logistical flows in order to reduce the costs, which is reasonable.

Example 2.5

In the fifth, and final numerical example, the impacts of the removal of the common/shared demand constraints, that is, the upper and lower bounds on the demands at the demand points are explored. Example 2.5, hence, has the same data as Example 2.4, except that the constraints (2.9) and (2.10), which are expected to be imposed by a higher level authority, are removed. It is easy to adapt the Euler method to handle the removal of such constraints and that is done.

The new computed equilibrium relief item kit flows are for $i = 1, 2; j = 1, 3$ now:

$$q^*_{ij1,1} = \begin{bmatrix} 191.67 & 83.33 & 0.00 \\ 0.00 & 0.00 & 66.67 \end{bmatrix}, \quad q^*_{ij2,1} = \begin{bmatrix} 233.34 & 88.89 & 0.00 \\ 344.46 & 199.99 & 66.67 \end{bmatrix},$$

$$q^*_{ij1,2} = \begin{bmatrix} 255.57 & 111.11 & 0.00 \\ 366.69 & 0.00 & 88.89 \end{bmatrix}, \quad q_{ij2,2} = \begin{bmatrix} 350.04 & 133.32 & 0.00 \\ 516.74 & 299.96 & 99.99 \end{bmatrix},$$

$$q^*_{ij3,1} = \begin{bmatrix} 244.45 & 100.00 & 0.00 \\ 355.57 & 211.10 & 77.78 \end{bmatrix},$$

$$q^*_{ij3,2} = \begin{bmatrix} 366.71 & 149.98 & 0.00 \\ 533.40 & 316.63 & 166.66 \end{bmatrix}.$$

The volume of relief item kits received at each demand point is now:

$$\sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i1k,l} = 3,758.65, \quad \sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i2k,l} = 1,916.52,$$

$$\sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i3k,l} = 516.66.$$

Note that, without the imposition of the lower bounds on the demands, in order to guarantee sufficient relief for the disaster victims, both at Demand Point 1 and at Demand Point 3 the volumes of relief item kits received suffer a significant shortfall, since there was a lower bound of 10,000 at Demand Point 1 and a lower bound of 1,000 at Demand Point 3. Humanitarian organizations, hence, may not deliver the necessary supplies if such important constraints are not added.

The relief item kit volumes delivered from each purchasing location $k; k = 1, 2, 3$, by each FSP $l; l = 1, 2$ are:

$$\sum_{i=1}^{2}\sum_{j=1}^{3} q^*_{ijk,l} = \begin{bmatrix} 341.68 & 1044.46 \\ 933.35 & 1400.05 \\ 988.91 & 1483.38 \end{bmatrix}.$$

Since the FSPs, with lower volumes of shipments, do not reach their capacities, we have that:

$$\epsilon^*_{1,1} = \epsilon^*_{1,2} = \epsilon^*_{2,1} = \epsilon^*_{2,2} = \epsilon^*_{3,1} = \epsilon^*_{3,2} = 0.$$

The total cost of each humanitarian organization is now:

$$\sum_{k=1}^{3} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q^*_{1jk,l} + \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{2} c_{1jk,l}(q^*) = 486,594.13,$$

$$\sum_{k=1}^{3} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q^*_{2jk,l} + \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{2} c_{2jk,l}(q^*) = 886,547.00.$$

HO 1 delivers a total of 2,308.40 relief item kits, whereas HO 2 delivers 3,883.42 relief item kits. These values are approximately half of the respective amounts delivered by the HOs in Example 2.4, demonstrating, as well the importance of the demand lower bound constraints.

The volume of purchases at PL 1 is now: 3,110.49; that at PL 2 is: 4,819.69, whereas the amount purchased at Pl 3 is now: 4,986.36.

Again, both humanitarian organizations spend less than their full budgets, and, consequently, the Lagrange multipliers associated with the budget constraints are equal to zero:

$$\gamma_1^* = \gamma_2^* = 0.$$

The benefit/altruism of each organization is now:

$$B_1(q^*) = 625,858, \quad B_2(q^*) = 1,325,047.00.$$

The utility of each HO, after the disaster relief operation, is now:

$$U_1(q^*) = 139,264.75, \quad U_2(q^*) = 438,500.00.$$

The altruism is decreased for each HO, as compared to the altruism enjoyed in Example 2.4. However, the total costs are as well, yielding a positive utility for each humanitarian organization.

Note that the above examples, although stylized, illustrate the types of problems that can be addressed and analyzed using this modeling and computational game theory framework. It is expected that larger-scale examples would also be effectively solvable using the Euler method given that it has performed well on large-scale network equilibrium problems, with problems with hundreds of variables solved in less than 1 CPU second (cf. Nagurney and Zhang (1996)).

2.4. Summary and Conclusions

The number of disasters is increasing as well as the number of people affected by them, posing great challenges to governments, the population, as well as humanitarian organizations that provide disaster relief. In this chapter, an integrated financial and logistical game theory model for humanitarian organizations with several notable features is developed. In particular, the new model extends existing models in the literature in several ways:

1. The model includes both relief item purchasing costs and freight service shipping costs, with the former being possible both locally and nonlocally, if feasible, and with the latter including competition, under capacity constraints, among the humanitarian organizations.

2. The governing equilibrium conditions, given common/shared constraints associated with the demands for relief items at the demand points, plus the freight capacity constraints, yield a Generalized Nash Equilibrium, which can be challenging to solve. Nevertheless, a variational inequality formulation is constructed through the concept of a variational equilibrium. This study is one of only a handful, outside of the work of Nagurney, Alvarez Flores, and Soylu (2016), Nagurney et al. (2018) and Gossler et al. (2019) in which the GNE concept is applied to disaster relief.

3. The model is qualitatively analyzed and a Lagrange analysis provided - the latter is especially valuable since it yields insights on the impacts of the constraints - and is one of the very few such analyses conducted for variational inequality problems with nonlinear constraints. Moreover, an alternative variational inequality formulation is provided. The interpretation of the Lagrange analysis reveals which situations are beneficial to both humanitarian organizations and victims of disasters.

4. The proposed algorithm, when applied to the alternative variational inequality formulation, which is over the nonnegative orthant, yields closed form expressions, at each iteration, which enables ease of computer implementation.

The numerical examples, inspired by Hurricane Harvey, one of the most expensive natural disasters to ever hit the United States, illustrate the flexibility of the modeling and computational framework. The examples explore the impacts of the addition of a freight service provider; the further deterioration of the logistical infrastructure with the associated increasing costs, as well as the removal of demand constraints on the incurred costs, benefits, and utilities. The final example further reinforces the need for the imposition of lower and upper bounds on the volume of disaster relief supplies at demand points, since otherwise, there may be insufficient relief supplies delivered to deserving victims.

The framework adds to the literature on game theory and disaster relief as well as to the literature on variational inequalities with nonlinear constraints. This chapter presents a comprehensive model for studying the behavior of different decision-makers in supply chain economic networks in the event of a disaster. A distinctive feature of this model is the competition for limited resources, while a range of constraints and challenges of disaster management and their effects on the outcome of operations are also considered. The approach and methods used in this chapter can be utilized to examine competition among different stakeholders for a variety of limited resources in various disasters, such as earthquakes, hurricanes, and pandemics.

CHAPTER 3

COMPETITION FOR MEDICAL SUPPLIES UNDER STOCHASTIC DEMAND IN THE COVID-19 PANDEMIC: A GENERALIZED NASH EQUILIBRIUM FRAMEWORK

As mentioned in Chapter 1, with the rapid onset of the COVID-19 pandemic, fierce competition took place for medical supplies, especially PPEs and ventilators. The importance of medical products in combating the COVID-19 pandemic highlights the necessity of studying the various aspects of competition for medical supplies.

The previous chapter illustrated how a game theory framework can be used to model and analyze the behavior of multiple decision-makers involved in complex disaster management operations while being faced with different logistical and financial challenges. In this chapter, utilizing the methods and concepts discussed in the previous chapter, I construct the first Generalized Nash Equilibrium model with stochastic demands to model competition among organizations at demand points for medical supplies. The model includes multiple supply points and multiple demand points, along with prices of the medical items and generalized costs associated with transportation. The theoretical constructs are provided and a Variational Equilibrium utilized to enable alternative variational inequality formulations. A qualitative analysis is presented and an algorithm proposed, along with convergence results. Illustrative examples are detailed as well as numerical examples that are solved with the implemented algorithm.

Chapter 3 is organized as follows. In Section 3.1, the Generalized Nash Equilibrium network model for medical supplies is presented and alternative variational inequality formulations of the governing equilibrium conditions are provided. Section 3.2 discusses some qualitative properties of the model as well as the function that enters the variational inequality that is then utilized to solve the numerical examples in Section 3.3. Section 3.4 summarizes the results and presents the conclusions.

3.1. The Generalized Nash Equilibrium Network Model for Medical Supplies Under Stochastic Demand

In this section, I construct the Generalized Nash Equilibrium network model for medical supplies in the COVID-19 pandemic, in which decision-makers from different demand points compete with each other to procure medical items such as PPEs from a variety of supply points.

There are m locations that are supply locations for the medical supplies, with a typical supply point denoted by i, and n locations that are demand points, with a typical demand point denoted by j. Note that supply points can be locations in different regions, states, or even countries. Demand points are locations where the medical supplies are needed such as hospitals, nursing homes, medical clinics, prisons, etc. The bipartite structure of the game theory problem is depicted in Figure 3.1. The notation for the model is given in Table 3.1. All vectors are column vectors.

The demand for the medical item at the demand points is uncertain due to the unpredictability of the actual demand at the demand points. The literature contains examples of supply chain network models with uncertain demand and associated shortage and surplus penalties (see, e.g., Dong, Zhang, and Nagurney (2004), Nagurney, Yu, and Qiang (2011), Nagurney, Masoumi, and Yu (2015)). Nagurney and

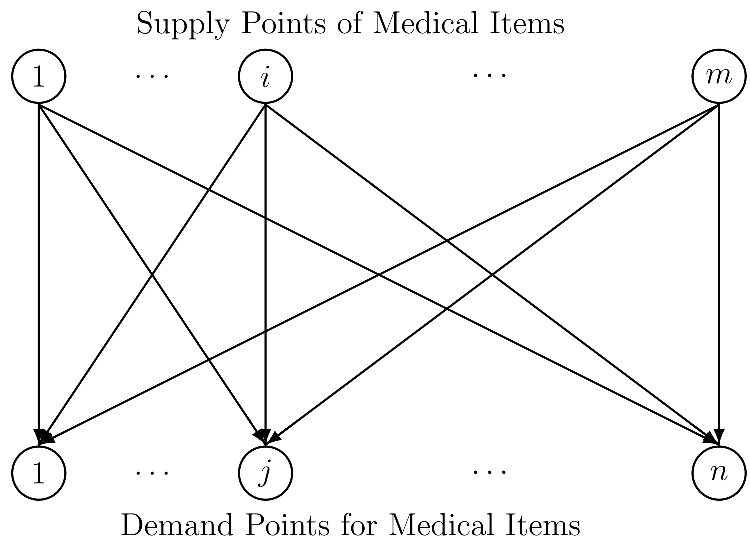

Figure 3.1. The Network Structure of the Competitive Game Theory Model for Medical Supplies

Nagurney (2016) developed a model for disaster relief under cost and demand uncertainty. The probability distribution of demand for medical items can be obtained using census data and/or information gathered during the pandemic preparedness phase.

Before constructing the objective function, some needed preliminaries are presented.

Since d_j denotes the actual (uncertain) demand at destination point j, we have:

$$P_j(D_j) = P_j(d_j \leq D_j) = \int_0^{D_j} \mathcal{F}_j(t)dt, \qquad j = 1, \ldots, n, \tag{3.1}$$

where P_j and $\mathcal{F}_j$ denote the probability distribution function, and the probability density function of demand at point j, respectively.

Recall from Table 3.1 that v_j is the "projected demand" for the medical item at demand point $j; j = 1, \ldots, n$. The amounts of shortage and surplus at demand point j are calculated, respectively, according to:

Table 3.1. Notation for the Medical Supply Generalized Nash Equilibrium Network Model

Notation	Definition
q_{ij}	the amount of the medical item purchased from supply location i by j. First all the i elements $\{q_{ij}\}$ are grouped into the vector q_j and then such vectors for all j are grouped into the vector $q \in R^{mn}_+$.
v_j	the projected demand at demand point j; $j = 1, \ldots, n$.
d_j	the actual (uncertain) demand for the medical item at demand location j; $j = 1, \ldots, n$.
Δ_j^-	the amount of shortage of the medical item at demand point j; $j = 1, \ldots, n$.
Δ_j^+	the amount of surplus of the medical item at demand point j; $j = 1, \ldots, n$.
λ_j^-	the unit penalty associated with a shortage of the the medical item at demand point j; $j = 1, \ldots, n$.
λ_j^+	the unit penalty associated with a surplus of the medical item at demand point j; $j = 1, \ldots, n$.
ρ_i	the price of the medical item at supply location i; $i = 1, \ldots, m$.
$c_{ij}(q)$	the generalized cost of transportation associated with transporting the the medical item from supply location i to demand location j, which includes the financial cost, any tariffs/taxes, time, and risk. All the generalized costs are grouped into the vector $c(q) \in R^{mn}$.
S_i	the nonnegative amount of the medical item available for purchase at supply location i; $i = 1, \ldots, m$.
μ_i	the nonnegative Lagrange multiplier associated with the supply constraint at supply location i. The Lagrange multipliers are grouped into the vector $\mu \in R^m_+$.

$$\Delta_j^- \equiv \max\{0, d_j - v_j\}, \qquad j = 1, \ldots, n, \tag{3.2a}$$

$$\Delta_j^+ \equiv \max\{0, v_j - d_j\}, \qquad j = 1, \ldots, n. \tag{3.2b}$$

The expected values of shortage and surplus at each demand point are, hence:

$$E(\Delta_j^-) = \int_{v_j}^{\infty} (t - v_j)\mathcal{F}_j(t)dt, \qquad j = 1, \ldots, n, \tag{3.3a}$$

$$E(\Delta_j^+) = \int_0^{v_j} (v_j - t)\mathcal{F}_j(t)dt, \qquad j = 1, \ldots, n. \tag{3.3b}$$

The expected penalty incurred by demand point j due to the shortage and surplus of the medical item is equal to:

$$E(\lambda_j^- \Delta_j^- \ + \ \lambda_j^+ \Delta_j^+) = \lambda_j^- E(\Delta_j^-) + \lambda_j^+ E(\Delta_j^+), \qquad j = 1, \ldots, n. \tag{3.4}$$

It is assumed that $\lambda_j^+ + \lambda_j^-$ is greater than zero, for each demand point j.

The projected demand at demand point j, v_j, is equal to the sum of flows of the medical item to j, that is:

$$v_j \equiv \sum_{i=1}^{m} q_{ij}, \qquad j = 1, \ldots, n. \tag{3.5}$$

Each demand location j seeks to minimize the total costs associated with the purchasing of the medical item plus the total cost of transportation plus the expected cost due to a shortage or surplus at j.

The objective function of each demand point j is, hence, given by:

$$\text{Minimize} \quad \sum_{i=1}^{m} \rho_i q_{ij} + \sum_{i=1}^{m} c_{ij}(q) + \lambda_j^- E(\Delta_j^-) + \lambda_j^+ E(\Delta_j^+) \tag{3.6}$$

subject to the following constraints:

$$\sum_{j=1}^{n} q_{ij} \leq S_i, \quad i = 1, \ldots, m, \tag{3.7}$$

$$q_{ij} \geq 0, \quad i = 1, \ldots, m. \tag{3.8}$$

The first term in the objective function (3.6) represents the purchasing costs, whereas the second term represents the generalized total transportation costs. The third term in (3.6) captures the expected cost due to shortage or surplus of the medical items at the demand point of the organization. It is expected that the weight λ_j^- would be significantly higher than the weight λ_j^+ for each j since a shortage of the medical items can result in greater suffering and loss of life.

The constraints (3.7) represent common, that is, a shared constraints in that the demand locations compete for the medical items that are available for purchase at the supply locations at a maximum available supply. The constraints in (3.8) are the nonnegativity assumption on the medical item purchase volumes.

It is assumed that the total generalized transportation cost functions are continuously differentiable and convex. Note that, in this model, the transportation costs can, in general, depend upon the vector of medical item flows since there is competition for freight service provision in the pandemic.

Now some preliminaries are presented that would allow the expression of the partial derivatives of the expected total shortage and discarding costs of the medical items at the demand points only in terms of the medical item flow variables. I then prove that the third term in the Objective Function (3.6) is also convex.

Note that, for each demand point j:

$$\frac{\partial E(\Delta_j^-)}{\partial q_{ij}} = \frac{\partial E(\Delta_j^-)}{\partial v_j} \times \frac{\partial v_j}{\partial q_{ij}}, \qquad \forall i. \tag{3.9}$$

By Leibniz's integral rule (Flanders (1973)), we have:

$$\frac{\partial E(\Delta_j^-)}{\partial v_j} = \frac{\partial}{\partial v_j}\left(\int_{v_j}^{\infty}(t-v_j)\mathcal{F}_j(t)d(t)\right) = \int_{v_j}^{\infty}\frac{\partial}{\partial v_j}(t-v_j)\mathcal{F}_j(t)d(t)$$

$$= P_j(v_j) - 1, \qquad j = 1, \ldots, n. \tag{3.10a}$$

Therefore,

$$\frac{\partial E(\Delta_j^-)}{\partial v_j} = P_j\left(\sum_{i=1}^{m} q_{ij}\right) - 1, \qquad j = 1, \ldots, n. \tag{3.10b}$$

On the other hand, we have:

$$\frac{\partial v_j}{\partial q_{ij}} = \frac{\partial}{\partial q_{ij}} \sum_{l=1}^{m} q_{lj} = 1, \qquad \forall i; j = 1, \ldots, n. \tag{3.11}$$

Therefore, from (3.10b) and (3.11), we conclude that

$$\frac{\partial E(\Delta_j^-)}{\partial q_{ij}} = \left[P_j \left(\sum_{i=1}^{m} q_{ij} \right) - 1 \right], \qquad \forall i; j = 1, \ldots, n. \tag{3.12}$$

Analogously, for the surplus, we have:

$$\frac{\partial E(\Delta_j^+)}{\partial q_{ij}} = \frac{\partial E(\Delta_j^+)}{\partial v_j} \times \frac{\partial v_j}{\partial q_{ij}}, \qquad \forall i; j = 1, \ldots, n, \tag{3.13}$$

$$\frac{\partial E(\Delta_j^+)}{\partial v_j} = \frac{\partial}{\partial v_j} \left(\int_0^{v_j} (v_j - t)\mathcal{F}_j(t)d(t) \right) = \int_0^{v_j} \frac{\partial}{\partial v_j} (v_j - t)\mathcal{F}_j(t)d(t) = P_j(v_j),$$
$$j = 1, \ldots, n. \tag{3.14a}$$

Thus,

$$\frac{\partial E(\Delta_j^+)}{\partial v_j} = P_j \left(\sum_{i=1}^{m} q_{ij} \right), \qquad j = 1, \ldots, n. \tag{3.14b}$$

From (3.14b) and (3.11) we have:

$$\frac{\partial E(\Delta_j^+)}{\partial q_{ij}} = P_j \left(\sum_{i=1}^{m} q_{ij} \right), \qquad \forall i; j = 1, \ldots, n. \tag{3.15}$$

Lemma 3.1

The expected shortage and surplus cost function $\lambda_j^- E(\Delta_j^-) + \lambda_j^+ E(\Delta_j^+)$ *is convex.*

Proof: We have:

$$\frac{\partial^2}{\partial {q_{ij}}^2}\left[\lambda_j^- E(\Delta_j^-)+\lambda_j^+ E(\Delta_j^+)\right]=\lambda_j^- \frac{\partial^2 E(\Delta_j^-)}{\partial {q_{ij}}^2}+\lambda_j^+ \frac{\partial^2 E(\Delta_j^+)}{\partial {q_{ij}}^2}, \qquad \forall i; j=1,\ldots,n. \tag{3.16a}$$

Substituting the first order derivatives from (3.12) and (3.15) into (3.16a) yields:

$$\frac{\partial^2}{\partial {q_{ij}}^2}\left[\lambda_j^- E(\Delta_j^-)+\lambda_j^+ E(\Delta_j^+)\right]=\lambda_j^- \frac{\partial}{\partial q_{ij}}\left[P_j\left(\sum_{i=1}^m q_{ij}\right)-1\right]+\lambda_j^+ \frac{\partial}{\partial q_{ij}} P_j\left(\sum_{i=1}^m q_{ij}\right)$$

$$=(\lambda_j^-+\lambda_j^+)\mathcal{F}_j\left(\sum_{i=1}^m q_{ij}\right)\geq 0, \qquad \forall i; j=1,\ldots,n. \tag{3.16b}$$

The above inequality holds provided that $(\lambda_j^-+\lambda_j^+)$, i.e., the sum of shortage and surplus penalties, is positive. Hence, $\lambda_j^- E(\Delta_j^-)+\lambda_j^+ E(\Delta_j^+)$, and, as a consequence, the objective function in (3.6) is also convex. $\square$

The objective function (3.6) for j is referred to as the disutility of j and is denoted by $DU_j(q)$; $j=1,\ldots,n$.

Let's define the feasible sets $K_j \equiv \{q_j \geq 0\}$; $j=1,\ldots,n$, and $K \equiv \prod_{i=1}^I K_i$. Also define the feasible set $\mathcal{S} \equiv \{q | q \,\text{satisfying}\, (3.7))\}$, which consists of the shared constraints.

Definition 3.1: Generalized Nash Equilibrium for Medical Items

A vector of medical items $q^* \in K \cap \mathcal{S}$ is a Generalized Nash Equilibrium if for each demand point j; $j=1,\ldots,n$:

$$DU_j(q_j^*, \hat{q}_j^*) \leq DU_j(q_j, \hat{q}_j^*), \quad \forall q_j \in K_j \cap \mathcal{S}, \tag{3.17}$$

where $\hat{q}_j^* \equiv (q_1^*,\ldots,q_{j-1}^*,q_{j+1}^*,\ldots,q_n^*)$.

According to (3.17), an equilibrium is established if no demand point has any incentive to unilaterally change its vector of medical item purchases/shipments. Observe that in this model not only does the objective function of a demand point depend not only on the vector of strategies of its own strategies and on those of the other demand points, but the feasible set does as well. Hence, this model is not a Nash (1950, 1951) model, but, rather, it is a Generalized Nash Equilibrium model. This model captures the reality of the intense competitive landscape in the COVID-19 pandemic.

Here, the concept of a *Variational Equilibrium* is utilized to formulate the above GNE conditions as the solution to a finite-dimensional variational inequality problem. Hence, rather than making use of quasi-variational inequalities, for which the algorithms are not as advanced, variational inequality algorithms can be applied to numerically solve the model. Indeed, as emphasized in Nagurney, Yu, and Besik (2017) and in Chapter 2, a Variational Equilibrium can be defined which is a refinement and a specific type of GNE (cf. Kulkarni and Shabhang (2012)) that enables a variational inequality formulation.

I define the feasible set $\mathcal{K} \equiv K \cap \mathcal{S}$.

Definition 3.2: Variational Equilibrium

A vector of medical items $q^* \in \mathcal{K}$ is a Variational Equilibrium of the above Generalized Nash Equilibrium problem if it is a solution to the following variational inequality:

$$\sum_{j=1}^{n} \sum_{i=1}^{m} \frac{\partial DU_j(q^*)}{q_{ij}} \times (q_{ij} - q_{ij}^*) \geq 0, \quad \forall q \in \mathcal{K}, \tag{3.18}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in mn-dimensional Euclidean space.

In expanded form, the variational inequality in (3.18) is: determine $q^* \in \mathcal{K}$ such that

$$\sum_{j=1}^{n}\sum_{i=1}^{m}\left[\rho_i+\sum_{l=1}^{m}\frac{\partial c_{lj}(q^*)}{\partial q_{ij}}+\lambda_j^+P_j(\sum_{l=1}^{m}q_{lj}^*)-\lambda_j^-(1-P_j(\sum_{l=1}^{m}q_{lj}^*))\right]\times\left[q_{ij}-q_{ij}^*\right]\geq 0,$$
$$\forall q \in \mathcal{K}. \qquad (3.19)$$

Note that the variational equilibrium guarantees that the Lagrange multipliers associated with the common constraints are the same for all the demand points. This feature yields an elegant fairness and equity interpretation, which is very relevant during this pandemic.

Now, using arguments as in Chapter 2, I put variational inequality (3.19) into the standard form (2.15). Let $X \equiv q$ and $F(X)$ be the vector with elements: $\{\frac{\partial DU_j(q^*)}{q_{ij}}\}$, $\forall j, i$ with $\mathcal{K}$ as originally defined and $N = mn$.

Also it is worth noting that existence of a solution q^* to variational inequality (3.19) is guaranteed under the classical theory (see Kinderlehrer and Stampacchia (1980)) since the function that enters the variational inequality is continuous and the feasible set $\mathcal{K}$ is not only convex but also compact because the supplies of the medical items are bounded. Hence, the following theorem is immediate.

Theorem 3.1: Existence

A solution to variational inequality (3.19) exists.

Now let's provide an alternative variational inequality to (3.18) (and (3.19)). A nonnegative Lagrange multiplier μ_i is associated with constraint (3.7), for each supply location $i = 1, \ldots, m$. All the Lagrange multipliers are grouped into the vector $\mu \in R^m_+$. And I define the feasible set $\mathcal{K}^2 \equiv \{(q, \mu) | q \geq 0, \mu \geq 0\}$.

Then, using arguments as in Chapter 2, an alternative variational inequality for (3.19) is: determine $(q^*, \mu^*) \in \mathcal{K}^2$ such that

$$\sum_{j=1}^{n} \sum_{i=1}^{m} \left[\rho_i + \sum_{l=1}^{m} \frac{\partial c_{lj}(q^*)}{\partial q_{ij}} + \lambda_j^+ P_j(\sum_{l=1}^{m} q^*_{lj}) - \lambda_j^- (1 - P_j(\sum_{l=1}^{m} q^*_{lj}) + \mu^*_i \right] \times \left[q_{ij} - q^*_{ij}\right]$$

$$+ \sum_{i=1}^{m} \left[S_i - \sum_{j=1}^{n} q^*_{ij} \right] \times [\mu_i - \mu^*_i] \geq 0, \quad \forall (q, \mu) \in \mathcal{K}^2. \tag{3.20}$$

Variational inequality (3.20) can also be put into standard form (2.15) if we define $X \equiv (q, \mu)$ and $F(X) \equiv (F^1(X), F^2(X))$ where $F^1(X)$ has as its (i, j)-th component: $\rho_i + \sum_{l=1}^{m} \frac{\partial c_{lj}(q)}{\partial q_{ij}} + \lambda_j^+ P_j(\sum_{l=1}^{m} q_{lj}) - \lambda_j^- (1 - P_j(\sum_{l=1}^{m} q_{lj}) + \mu_i$; $i = 1, \ldots, m$; $j = 1, \ldots, n$, and the i-th component of $F^2(X)$ is $S_i - \sum_{j=1}^{n} q_{ij}$, for $i = 1, \ldots, m$. Furthermore, $\mathcal{K} \equiv \mathcal{K}^2$ and $N = mn + m$.

3.1.1 Illustrative Examples

In this subsection, three small numerical examples are presented for illustrative purposes. These examples are inspired by the COVID-19 pandemic and associated challenges in procuring N95 face masks, which are among the most needed medical products in dealing with this health disaster. Note that the equilibrium Lagrange multipliers provide valuable information since they represent the shadow prices of the supply constraints. In particular, if an equilibrium Lagrange multiplier is positive then this is the amount of the cost (or the loss) that could be saved with an extra unit of the supply of the medical item.

Illustrative Example 3.1: One Supply Point and One Demand Point

In this example there is a single supply point and a single demand point, as depicted in Figure 3.2.

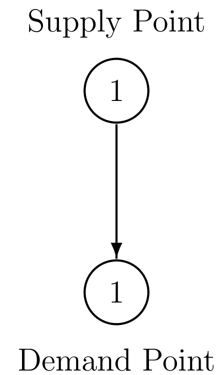

Figure 3.2. Network Topology for Illustrative Example 3.1

The supply point sells 20-pack N95 masks in the form of large bulks of 1000 packs each; therefore, one unit of item flow from the supply point to a demand point, q_{ij}, represents 1000 of 20-pack N95 masks. The demand at the demand point is uniformly distributed between 100 and 1,000 of large bulks. To determine the price of a unit item flow, ρ_i at supply point i, it is assumed that the price of each 20-pack N95 mask during the pandemic is \$25, so that the purchase price of each large bulk is $\rho_1 = 25,000$. Although a face mask is not, under normal conditions, an expensive product, it has been proved to essential in reducing the spread of the virus. Based on this, here it is assumed that, for every 2,000 people who do not use the face mask, one person would die due to the disease. Although it is not easy to value people's lives, it is assumed a \$200,000 equivalent for each loss. As a result, the penalty, λ_1^-, on the shortage of one item flow, which is equivalent to 20,000 N95 masks, is set at \$2,000,000. Also, since the supply chain has been severely disrupted at the time of the declaration of the pandemic, overloading can cause many problems in transportation and processing at entry points for countries. To prevent this, a penalty of $\lambda_1^+ = 100,000$ on surplus item flows is considered. The data for this example is as follows:

$$\rho_1 = 25,000, \quad S_1 = 1,000, \quad c_{11}(q) = q_{11}^2 + 3q_{11}, \quad \lambda_1^- = 2,000,000, \quad \lambda_1^+ = 100,000.$$

One can rewrite variational inequality (3.20) for this example as: determine $(q^*, \mu^*) \in \mathcal{K}^2$ such that:

$$\left[25000 + 2q_{11}^* + 3 + 100000(\frac{q_{11}^* - 100}{900}) - 2000000(\frac{1000 - q_{11}^*}{900}) + \mu_1^*\right] \times [q_{11} - q_{11}^*]$$

$$+ [1000 - q_{11}^*] \times [\mu_1 - \mu_1^*] \geq 0, \quad \forall (q, \mu) \in \mathcal{K}^2$$

The solution to the above variational inequality, which obtained analytically, is:

$$q_{11}^* = 945.62, \quad \mu_1^* = 0.00.$$

Observe that the organization at the demand point procures a huge number of masks because of the great importance of PPEs in preventing the further spread of the virus and the potential damage that could be caused by an insufficient number of N95 face masks. The projected demand value $v_1 = 945.62$ lies between the lower and the upper bounds of the uniform distribution range. Note that the projected demand is very close to the upper bound. The decision-makers at the organization at the demand point are aware of the importance of the masks and have assigned a much larger penalty on a shortage as compared to the surplus penalty. The disutility of the organization in this logistical operation is equal to 67,543,534.04.

Illustrative Example 3.2: Two Supply Points and One Demand Point

In the second illustrative example, a new supply point has been added to the supply chain network, as depicted in Figure 3.3.

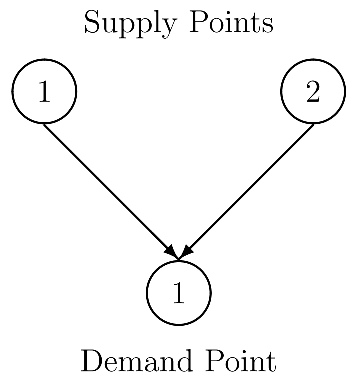

Figure 3.3. Network Topology for Illustrative Example 3.2

Hence, now, the decision-makers at the demand point have two options for procuring the face masks. The new supply point offers masks for less than half the price of the other supply point, but its supply capacity is half that of the previous one. Also, the generalized transportation cost rate from the origin of the N95 masks of the new supply point to the demand point is higher than the rate of the other supply point. The data on the new supply point are as follows.

$$\rho_2 = 10,000, \quad S_2 = 500, \quad c_{21}(q) = 2q_{21}^2 + 4q_{21}.$$

Variational inequality (3.20) can be rewritten as follows for this example: determine $(q^*, \mu^*) \in \mathcal{K}^2$ such that

$$\left[25000 + 2q_{11}^* + 3 + 100000(\frac{q_{11}^* + q_{21}^* - 100}{900}) - 2000000(\frac{1000 - q_{11}^* - q_{21}^*}{900}) + \mu_1^*\right]$$
$$\times [q_{11} - q_{11}^*]$$
$$+ \left[10000 + 4q_{21}^* + 4 + 100000(\frac{q_{11}^* + q_{21}^* - 100}{900}) - 2000000(\frac{1000 - q_{11}^* - q_{21}^*}{900}) + \mu_2^*\right]$$
$$\times [q_{21} - q_{21}^*]$$
$$+ [1000 - q_{11}^*] \times [\mu_1 - \mu_1^*] + [500 - q_{21}^*] \times [\mu_2 - \mu_2^*] \geq 0, \quad \forall (q, \mu) \in \mathcal{K}^2.$$

The solution to the above variational inequality, obtained analytically, is:

$$q_{11}^* = 446.05, \quad q_{21}^* = 500.00, \quad \mu_1^* = 0.00, \quad \mu_2^* = 13,891.80.$$

Observe that, with the addition of a new supply point, the decision-makers' strategy has changed. Since the price of the product offered by the new supply point is much lower than that at the first supply point, the decision-makers purchase more items from Supply Point 2, despite the fact that the generalized transportation cost to the demand point from Supply Point 2 is higher than that from Supply Point 1. However, the supply capacity of the new supply point is half that of the first supply point, and we see that all its capacity has been used. Therefore, the associated equilibrium Lagrange multiplier is positive. Again, the projected demand falls between the lower and the upper bounds of the uniform distribution and is closer to the upper bound for the same reason as in the previous example. But, now, with greater flexibility in the supply chain due to the addition of a new supply point, the disutility of the organization at the demand point has declined, dropping to 59,860,548.75.

Illustrative Example 3.3: Two Supply Points and Two Demand Points

This example is constructed from the previous examples, with the difference that now there are two demand points trying to procure N95 masks and competing for limited supplies; see Figure 3.4.

The demand for the new demand point is uniformly distributed between 100 and 500. The generalized transportation cost functions and the penalty coefficients associated with the second demand point are:

$$c_{12}(q) = 2q_{12}^2 + 3q_{12}, \quad c_{22}(q) = 3q_{22}^2 + 4q_{22}, \quad \lambda_2^- = 2,000,000, \quad \lambda_2^+ = 100,000.$$

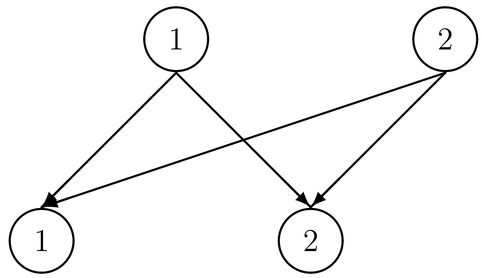

Figure 3.4. Network Topology for Illustrative Example 3.3

Variational inequality (3.20) for this example is as below: determine $(q^*, \mu^*) \in \mathcal{K}^2$ such that

$$\left[25000 + 2q_{11}^* + 3 + 100000(\frac{q_{11}^* + q_{21}^* - 100}{900}) - 2000000(\frac{1000 - q_{11}^* - q_{21}^*}{900}) + \mu_1^*\right]$$

$$\times [q_{11} - q_{11}^*]$$

$$+ \left[10000 + 4q_{21}^* + 4 + 100000(\frac{q_{11}^* + q_{21}^* - 100}{900}) - 2000000(\frac{1000 - q_{11}^* - q_{21}^*}{900}) + \mu_2^*\right]$$

$$\times [q_{21} - q_{21}^*]$$

$$+ \left[25000 + 4q_{12}^* + 3 + 100000(\frac{q_{12}^* + q_{22}^* - 100}{400}) - 2000000(\frac{500 - q_{12}^* - q_{22}^*}{400}) + \mu_1^*\right]$$

$$\times [q_{12} - q_{12}^*]$$

$$+ \left[10000 + 6q_{22}^* + 4 + 100000(\frac{q_{12}^* + q_{22}^* - 100}{400}) - 2000000(\frac{500 - q_{12}^* - q_{22}^*}{400}) + \mu_2^*\right]$$

$$\times [q_{22} - q_{22}^*]$$

$$+ [1000 - q_{11}^*] \times [\mu_1 - \mu_1^*] + [500 - q_{21}^*] \times [\mu_2 - \mu_2^*] \geq 0, \quad \forall (q, \mu) \in \mathcal{K}^2.$$

The solution to this variational inequality, again, obtained analytically, is:

$$q_{11}^* = 634.14, \quad q_{21}^* = 311.74, \quad q_{12}^* = 287.71, \quad q_{22}^* = 188.26,$$

$$\mu_1^* = 0.00, \quad \mu_2^* = 15,020.30.$$

With the addition of another demand point, there is increased competition for the valuable N95 masks. The strategies of the organization at Demand Point 1 have changed as compared to the previous example. It can be seen that the full capacity of Supply Point 2 has not been assigned to Demand Point 1, since the organization at Demand Point 1 now competed with the organization at Demand Point 2. As a result, the major part of the Demand Point 1's procurement of the N95 masks is from Supply Point 1 that has a larger capacity as compared to Supply Point 2. And, similar to the previous example, the equilibrium Lagrange multiplier associated with the supply capacity of Supply Point 2 is positive since it has sold all its available supply of N95 masks, while the other supply point has not exhausted its capacity. Both demand points receive a large amount of face masks and their projected demands lie in their respective uniform probability distribution range. Both projected demands are closer to the upper bound since the penalty on shortage is much higher than the penalty on surplus. The addition of a new demand point to the competition has changed the strategies of the organization at Demand Point 1, and we can see the impact on its disutility. Its disutility has now increased to 62,580,546.57. The disutility of the second demand point is 28,457,845.74.

3.2. Qualitative Properties and the Algorithm

I now discuss some properties of the model, specifically, those that guarantee that the conditions for convergence of the modified projection method (cf. Korpelevich (1977) and Nagurney (1999)) that is used to compute solutions to numerical examples in this next section are met. The algorithm is guaranteed to converge to a solution of variational inequality (3.20) if the function $F(X)$ that enters the variational inequality is monotone and Lipschitz continuous, and that a solution exists.

Recall that the function $F(X)$ is said to be monotone, if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{3.21}$$

Theorem 3.2: Monotonicity

The function $F(X)$ is monotone, for all $X \in \mathcal{K}$, if all the generalized transportation cost functions c_{ij}, $i = 1, \ldots, m$; $j = 1, \ldots, n$, are convex.

Proof: $\forall X^1, X^2 \in \mathcal{K}$, let $v_j^1 = \sum_{i=1}^m q_{ij}^1$ and $v_j^2 = \sum_{i=1}^m q_{ij}^2$.

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle$$

$$= \sum_{j=1}^n \sum_{i=1}^m \left[\sum_{l=1}^m \frac{\partial c_{lj}(q^1)}{\partial q_{ij}} - \sum_{l=1}^m \frac{\partial c_{lj}(q^2)}{\partial q_{ij}} \right] \times (q_{ij}^1 - q_{ij}^2) \tag{3.22}$$

$$+ \sum_{j=1}^n (\lambda_j^+ + \lambda_j^-) \times (P_j(v_j^1) - P_j(v_j^2)) \times (v_j^1 - v_j^2). \tag{3.23}$$

Given the convexity of the generalized transportation cost functions, equation (3.22) is greater or equal to zero. Since a probability function P_j, $\forall j$, is an increasing function, the expression in equation (3.23) is greater or equal to zero. Hence, $F(X)$ is monotone. □

If the conditions in Theorem 3.1 are slightly strengthened so that the the vector function that enters into the variational inequality problem (3.20) is strictly monotone, then its solution is unique (see, e.g., Nagurney (1999)).

Theorem 3.3: Uniqueness

The function $F(X)$ is strictly monotone for all $X \in \mathcal{K}$, if all the generalized transportation cost functions c_{ij}; $i = 1, \ldots, m$; $j = 1, \ldots, n$, are strictly convex. Then the variational inequality (3.20) has a unique solution in $\mathcal{K}$

Theorem 3.4: Lipschitz Continuity

If the generalized transportation cost functions c_{ij}, for all i and j, have bounded second order partial derivatives, then the function $F(X)$ that enters the variational inequality problem (3.20) is Lipschitz continuous; that is, there exists a constant $L > 0$, known as the Lipschitz constant, such that

$$\|F(X^1) - F(X^2)\| \leq L\|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{3.24}$$

Proof: Since each probability function P_j; $j = 1, \ldots, n$, is always less than or equal to 1, the result is direct by applying a mid-value theorem from calculus to the vector function $F(X)$ that enters the variational inequality problem (3.20). See also Nagurney and Zhang (1996) and Nagurney (1999). $\square$

The iterative steps of the modified projection method, with τ denoting an iteration counter, are as follows:

3.2.1 The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $\tau := 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{L}$, where L is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^\tau$ by solving the variational inequality subproblem:

$$\langle \bar{X}^\tau + \beta F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{3.25}$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + \beta F(\bar{X}^\tau) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{3.26}$$

Step 3: Convergence Verification

If $|X^\tau - X^{\tau-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau := \tau + 1$ and go to Step 1.

The modified projection method for the model governed by variational inequality (3.20) yields closed form expressions for the medical item flows and for the Lagrange multipliers in both Steps (3.25) and (3.26). This is a nice feature for computer implementation.

Theorem 3.5: Convergence

Assume that the function that enters the variational inequality (3.20) (or (3.19)) has at least one solution and all the generalized transportation cost functions are convex, then the modified projection method described above converges to the solution of the variational inequality (3.20) (or (3.19)).

Proof: According to Korpelevich (1977), the modified projection method converges to the solution of the variational inequality problem of the form (2.15), provided that the function F that enters the variational inequality is monotone and Lipschitz continuous and that a solution exists. Existence of a solution follows from Theorem 3.1. Monotonicity follows Theorem 3.2. Lipschitz continuity, in turn, follows from Theorem 3.4. □

I now provide the explicit formula for the medical item flows and the Lagrange multipliers at iteration τ for Step 1. The analogues for Step 2 can be easily derived accordingly.

Specifically, we have:

Explicit Formula for the Medical Item Flow for Each i, j at Iteration τ of Step 1

Determine $\bar{q}_{ij}^{\tau}$ for each i, j at Step 1 iteration τ according to:

$$\bar{q}_{ij}^{\tau} = \max\{0, q_{ij}^{\tau-1} + \beta(-\rho_i - \sum_{l=1}^{m} \frac{\partial c_{lj}(q^{\tau-1})}{\partial q_{ij}} - \lambda_j^{+} P_j(\sum_{l=1}^{m} q_{lj}^{\tau-1})$$
$$+\lambda_j^{-}(1 - P_j(\sum_{l=1}^{m} q_{lj}^{\tau-1})) - \mu_i^{\tau-1})\}. \quad (3.27)$$

Explicit Formula for the Lagrange Multiplier for Each i at Iteration τ of Step 1

Determine $\bar{\mu}_i^{\tau}$ for each i at Step 1 iteration τ according to:

$$\bar{\mu}_i^{\tau} = \max\{0, \mu_i^{\tau-1} + \beta(-S_i + \sum_{j=1}^{n} q_{ij}^{\tau-1})\}. \quad (3.28)$$

3.3. Numerical Examples

In this section, the modified projection method is applied to compute solutions to numerical examples. It is an alternative algorithm compared to the one used in the previous chapter to show the breadth of the computational techniques. The algorithm was implemented in FORTRAN and the computer system used was a Linux system at the University of Massachusetts Amherst. The algorithm was initialized by setting all the medical item flows and the Lagrange multipliers to 0.00. The convergence condition for all the examples was that the absolute value of two successive variable iterates was less than or equal to 10^{-8}. The β parameter in the modified projection method was set to: .1.

The examples are of increasing complexity. All the input and the output data are reported for transparency purposes and reproducibility.

In this section, the focus is on the procurement of N95 masks but in the scenario of increasing demand among smaller healthcare organizations in the form of medical practices.

Numerical Example 3.1: One Supply Point and One Demand Point

In the first numerical example, for which the solution is computed using the code implemented, there is a single supply point and a single demand point as in the network in Figure 3.2. The q_{ij}s are in units since these medical practices are small relative to hospitals, etc. A uniform probability distribution in the range $[100, 1000]$ is assumed at the demand point. The additional data for this example are:

$$\rho_1 = 2, \quad S_1 = 1,000, \quad c_{11}(q) = .005q_{11}^2 + .01q_{11}, \quad \lambda_1^- = 1,000, \quad \lambda_1^+ = 10.$$

The computed equilibrium solution is:

$$q_{11}^* = 980.56, \quad \mu_1^* = 0.00.$$

The projected demand of 980.56 is close to the upper bound of the probability distribution at the demand point.

Numerical Example 3.2: One Supply Point and Two Demand Points

This example has the same data as that in Numerical Example 3.1 except for added data for the second demand point. The network topology is as in Figure 3.5.

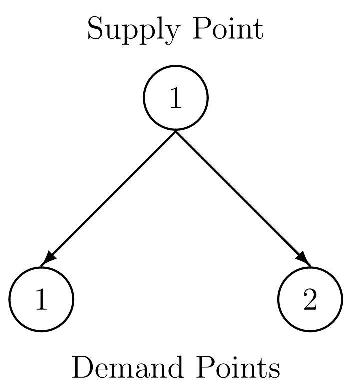

Figure 3.5. Network Topology for Numerical Example 3.2

The probability distribution at the second demand point had the same lower and upper bounds as in the first demand point.

This example has the same data as Numerical Example 3.1 except for the following additional data for the new demand point:

$$c_{12}(q) = .01q_{12}^2 + .02, \quad \lambda_2^- = 1000, \quad \lambda_2^+ = 10.$$

The modified projection method converged to the following equilibrium solution:

$$q_{11}^* = 502.20, \quad q_{12}^* = 497.80, \quad \mu_1^* = 541.61.$$

With increased competition for N95 mask supplies from the second demand point, the first demand point has a large reduction in procured supplies, as compared to the volume received in Numerical Example 3.1. The available supply of 1,000 N95 masks is exhausted between the two demand points, and, hence, the associated Lagrange multiplier μ_1^* is positive. The equilibrium conditions hold with excellent accuracy.

Numerical Example 3.3: Two Supply Points and Two Demand Points

In Numerical Example 3.3, the impacts of the addition of a second supply point to Numerical Example 3.2 is considered. The topology is as in Figure 3.4. Hence, the data are as above with the following additions:

$$S_2 = 500, \quad \rho_2 = 3, \quad c_{21}(q) = .015q_{21}^2 + .03, \quad c_{22}(q) = .02q_{22}^2 + .04q_{22}.$$

The modified projection method yielded the following equilibrium solution:

$$q_{11}^* = 526.31, \quad q_{12}^* = 473.69, \quad q_{21}^* = 225.57, \quad q_{22}^* = 274.43,$$

$$\mu_1^* = 261.17, \quad \mu_2^* = 258.65.$$

With the addition of a new supply point, both demand points gain significantly in terms of the volume of N95 that each procures and the supplies at each supply point are fully sold out. As a result, both equilibrium Lagrange multipliers are positive.

Numerical Example 3.4: Two Supply Points and Three Demand Points

Numerical Example 3.4 is constructed from Numerical Example 3.3 with Demand Point 3 added, as in Figure 3.6.

Numerical Example 3.4 has the same data as Numerical Example 3.3 but with the addition of data for Demand Point 3 as follows:

$$c_{13}(q) = .01q_{13}^2 + .02q_{13}, \quad c_{23}(q) = .015q_{23}^2 + .03q_{23}, \quad \lambda_3^- = 1000, \quad \lambda_3^+ = 10.$$

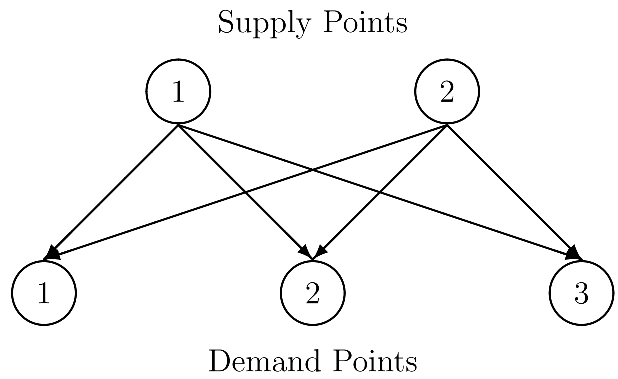

Figure 3.6. Network Topology for Numerical Example 3.4

The probability distribution for the N95 masks associated with Demand Point 3 is uniform with a lower bound of 200 and an upper bound of 1000.

The modified projection method yielded the following equilibrium solution:

$$q_{11}^* = 360.11, \quad q_{12}^* = 318.83, \quad q_{13}^* = 321.06,$$

$$q_{21}^* = 122.29, \quad q_{22}^* = 161.10, \quad q_{23}^* = 216.62, \quad \mu_1^* = 565.25, \quad \mu_2^* = 564.16.$$

Observe that with increasing competition for the N95 masks with another demand point, both Demand Points 1 and 2 experience decreases in procurement of supplies. The two supply points again fully sell out of their N95 masks and the associated equilibrium Lagrange multipliers are both positive.

Numerical Example 3.5: Two Supply Points and Four Demand Points

In the final example, Numerical Example 3.5, I consider yet another demand point addition to the demand points in Numerical Example 3.4. Please refer to Figure 3.7. Smaller medical practices are increasingly concerned about being able to secure the much needed PPEs to protect the health of their employees and the viability of their practices.

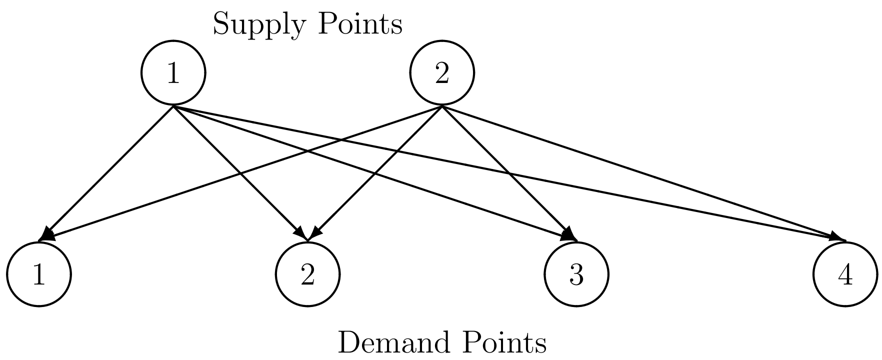

Figure 3.7. Network Topology for Numerical Example 3.5

The data for this example is as the data for Numerical Example 3.4, and the probability distribution structure for the demand at demand point is the same, with the following additional data for the new Demand Point 4:

$$c_{14}(q) = .015q_{14}^2 + .03q_{14}, \quad c_{24}(q) = .025q_{24}^2 + .05q_{24}, \quad \lambda_4^- = 1000, \quad \lambda_4^+ = 10.$$

The modified projection method now yielded the following equilibrium solution:

$$q_{11}^* = 260.73, \quad q_{12}^* = 229.36, \quad q_{13}^* = 251.22, \quad q_{14}^* = 258.69,$$

$$q_{21}^* = 79.57, \quad q_{22}^* = 109.17, \quad q_{23}^* = 160.46, \quad q_{24}^* = 150.81,$$

$$\mu_1^* = 725.71, \quad \mu_2^* = 724.91.$$

Again, the equilibrium conditions hold with excellent accuracy for this example, as was the case for all the other numerical example computed solutions. The suppliers of the N95 sell out their supplies. However, the demand points lose in term of supply procurement for their organizations with the increased demand and competition from yet another demand point.

Note that although the above numerical examples are stylized, this mathematical, computational framework enables the investigation of numerous scenarios and sensitivity analyses. For example, one can consider the impacts of the removal of supply points and/or demand points; the addition of supply and/or demand points; changes in the prices of the medical item under study, as well as changes to the generalized transportation costs. Furthermore, one can investigate the impacts of alternative probability distribution functions.

The above numerical results are consistent with what one can expect to observe in reality in terms of how organizations would procure critical medical supplies such as N95 masks under demand unpredictability and competition. The findings confirm that more supply points with sufficient supplies are needed to ensure that organizations are not deprived of critical supplies due to competition. As a result of this competition and limited local availability; in particular in the case of supplies such as masks and even coronavirus test kits, we are seeing several countries now setting up local production sites (Bradsher (2020)).

3.4. Summary and Conclusions

Medical supplies are essential in the battle against the coronavirus that causes COVID-19. The demand for medical supplies globally from PPEs to ventilators has created an intense competition. PPEs are essential in protecting healthcare workers and it has been recognized that masks can reduce the transmission of the novel coronavirus. Ventilators, on the other hand, can be life-saving for patients with severe cases of COVID-19 and convalescent plasma has become a possible interim treatment. With the pandemic, supply chains, including those for medical items, have been disrupted adding to the intense competition for such supplies.

The COVID-19 pandemic is not limited to space or time and, therefore, there have been many shortages of medical items. In order to elucidate the competition for such supplies in this pandemic, a Generalized Nash Equilibrium model is developed in this chapter. The model consists of multiple supply points for the medical items and multiple demand points with the demand at the latter being stochastic. Using some recently introduced machinery, alternative variational inequality formulations of the equilibrium conditions were provided. I then utilized the variational inequality with not only medical item product flows as variables but also the Lagrange multipliers associated with the supply capacities of the medical items at the supply point. The model was studied both qualitatively and quantitatively - the latter through illustrative examples that were solved analytically as well as via numerical examples for which an algorithm was proposed. The algorithm, for which convergence results were also provided, resolved the variational inequality problem into a series of subproblems for which closed form expressions in the variables were identified.

CHAPTER 4

A MULTICOUNTRY, MULTICOMMODITY STOCHASTIC GAME THEORY NETWORK MODEL OF COMPETITION FOR MEDICAL SUPPLIES INSPIRED BY THE COVID-19 PANDEMIC

Chapter 3 addressed competition among decision-makers at different demand points for a single type of medical product, when the amount of demand was not certain.The model in that chapter, similar to the one in Chapter 2, examined the behavior of decision-makers at one stage of disaster management. In this chapter, a more comprehensive model is presented that investigates the competition among countries for several types of medical supplies, and that handles disaster planning and management in the pre- and post-disaster stages. In this two-stage stochastic game theory network model, not only the amount of demand, but also other important factors such as the price of products, supply capacities, and the state of the transportation network are uncertain and change in different possible scenarios.

This chapter is organized as follows. In Section 4.1, the pandemic stochastic game theory network model for medical supplies is developed. This section describes how the national governments compete for the medical supplies to minimize their respective expected loss/disutility from the pandemic in terms of their individual two-stage stochastic optimization problems. I construct the Stochastic Generalized Nash Equilibrium, define the Variational Equilibrium, and derive alternative variational inequality formulations. Also illustrative examples are presented. In Section

4.2, I outline an algorithm, which resolves the variational inequality problem into subproblems yielding closed form expressions for each of the medical supply flows and the Lagrange multipliers associated with the supply capacities at a given iteration. In Section 4.3, solutions to numerical examples inspired by the COVID-19 pandemic are computed, and, in Section 4.4, a summary of the chapter along with the conclusions are provided.

4.1. The Pandemic Stochastic Game Theory Network Model for Medical Supplies

In this section, the multicountry, multicommodity stochastic game theory network model for medical supplies in the COVID-19 pandemic is constructed. The notation is presented in Table 4.1. The vectors are all column vectors.

The two stages are displayed in Figure 4.1. There are I national governments of countries with a typical one denoted by i; $i = 1, \ldots, I$. Each of these countries requires the acquisition of medical items as they face the pandemic. There are I possible countries where the items are purchased from with a typical one denoted by j; $j = 1, \ldots, I$. There are K different medical items with a typical one denoted by k. In the first stage, before the pandemic occurs, the government of country i can purchase medical supplies from its own country and/or from the other countries. After the declaration of the pandemic, and better information as to the demand, the governments procure needed medical supplies to meet the demand as closely as possible; again, either domestically or from other countries, but now under different supply characteristics.

Table 4.1. Notation for the Pandemic Stochastic Game Theory Network Model

Notation	Parameters
$\omega \in \Omega$	the disaster scenarios.
p_ω	the probability of disaster scenario ω in stage 2; $\forall \omega \in \Omega$.
$S^1_{j,k}$	the supply of medical item k in country j in stage 1; $j = 1, \ldots, I$; $k = 1, \ldots, K$.
$S^{2,\omega}_{j,k}$	the supply of medical item k in country j when scenario ω occurs in stage 2; $j = 1, \ldots, I$; $k = 1, \ldots, K$; $\omega \in \Omega$.
$d^{2,\omega}_{i,k}$	the demand for medical item k in country i when scenario ω occurs in stage 2; $i = 1, \ldots, I$; $k = 1, \ldots, K$; $\omega \in \Omega$.
$\beta_{i,k}$	the unit penalty encumbered by country i on the unmet demand of medical item k; $i = 1, \ldots, I$; $k = 1, \ldots, K$.
$\rho_{j,k}$	the unit price of medical item k at country j before the pandemic; $j = 1, \ldots, I$; $k = 1, \ldots, K$.
$\rho^{\omega}_{j,k}$	the unit price of medical item k at country j when the scenario ω occurs in stage 2; $j = 1, \ldots, I$; $k = 1, \ldots, K$; $\omega \in \Omega$.
Notation	**Variables**
$q^1_{ij,k}$	the amount of medical item k purchased by country i from country j in stage 1. All the j and k elements are grouped into the vector q^1_i and then such vectors for all i are grouped into the vector q^1.
$q^{2,\omega}_{ij,k}$	the amount of medical item k purchased by country i from country j when the scenario ω occurs in stage 2. All the j and k elements are grouped into the vector $q^{2,\omega}_i$ and then such vectors for all i are grouped into the vector $q^{2,\omega}$. Finally, these vectors for all ω are grouped into the vector q^2. The q^1 and q^2 vectors are grouped into the vector $q \in R^{IIK+\|\Omega\|(IIK)}_+$.
Notation	**Cost Functions**
$c^1_{ji,k}(q^1)$	the total transportation cost that country i pays to have the medical items k delivered from country j where the items are purchased before the pandemic hits the country.
$c^{2,\omega}_{ji,k}(q^{2,\omega})$	the total transportation cost that country i pays to have medical items k delivered from country j when the scenario ω occurs in stage 2.

4.1.1 The Countries' Two-stage Stochastic Optimization Problems

Faced with a pandemic, the primary goal of each of the national governments is to save lives by having the demands for medical supplies be met as closely as possible. However, it can be challenging as well as expensive to procure such items since a pandemic is global in nature. The objective function that each government seeks to

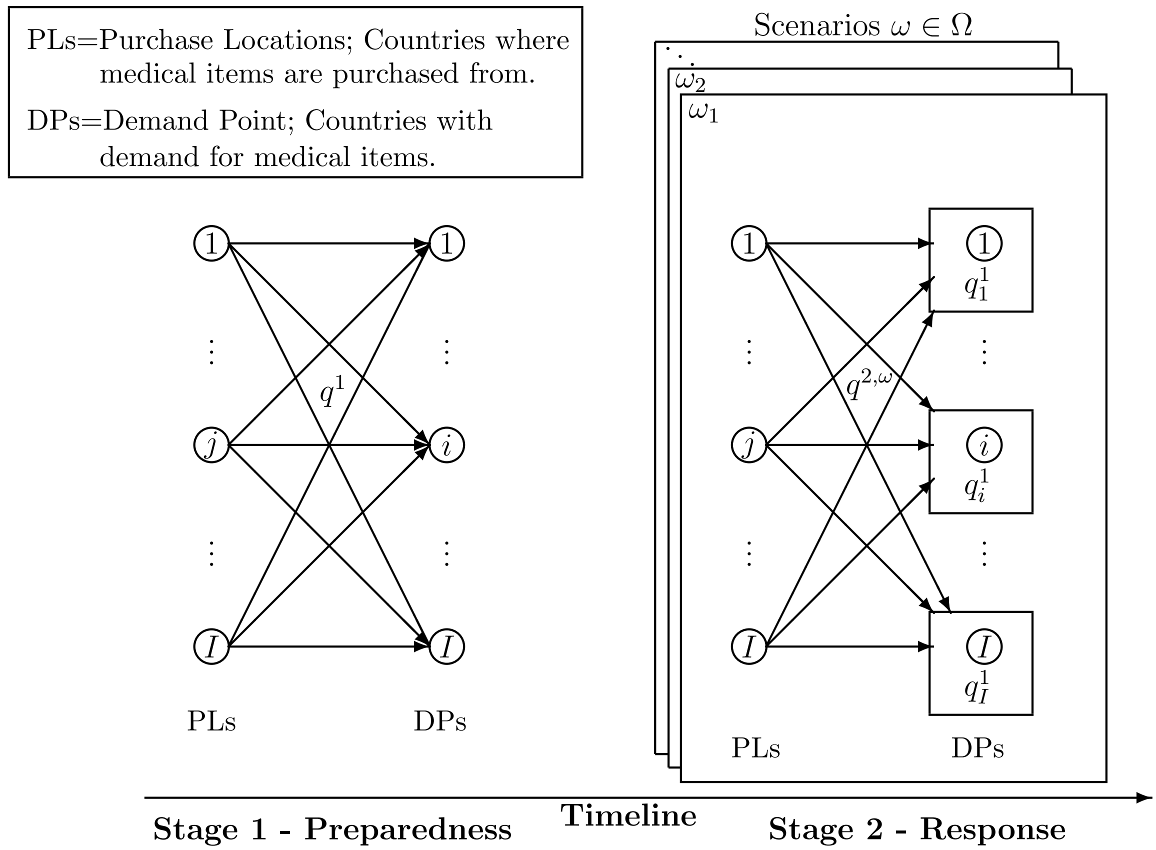

Figure 4.1. The Timeline of the Pandemic Disaster Preparedness and Response

minimize in this model consists of the expected diutility caused by the shortages of medical items and the associated purchasing and logistical costs. The actions that the government takes in the first stage, before the pandemic, and the cost of these actions, are deterministic and, hence, are included in the model deterministically. However, a national government's recourse actions in stage 2, once the pandemic has been declared, depend on the possible scenarios and the realization of the uncertain parameters.

Therefore, each national government is faced with the following two-stage stochastic optimization model in which they minimize the expected disutility:

$$\text{Minimize} \quad \sum_{j=1}^{I}\sum_{k=1}^{K}\rho_{j,k}q^1_{ij,k} + \sum_{j=1}^{I}\sum_{k=1}^{K}c^1_{ji,k}(q^1) + E_\Omega\left[Q^2_i(q^2,\omega)\right] \tag{4.1}$$

subject to:

$$\sum_{i=1}^{I} q^1_{ij,k} \leq S^1_{j,k}, \quad j=1,\ldots,I; \quad k=1,\ldots,K, \tag{4.2}$$

$$q^1_{ij,k} \geq 0, \quad j=1,\ldots,I; \quad k=1,\ldots,K. \tag{4.3}$$

The first two terms in the objective function (4.1) indicate the deterministic costs that government i incurs before the pandemic is declared to meet part of the country's needs of medical items in the pandemic and to prepare for it. The first term in the objective function (4.1) indicates the cost of purchasing the medical items, and the second term represents the total transportation cost that country i pays to have the purchased items delivered. In stage 1, before the pandemic is declared, governments face important constraints, those of supply availability in each country of each medical item. The constraints in (4.2) capture the supply availability of medical item k in country j for each k and each j. The constraints in (4.3) guarantee that the medical item purchases/shipments in the first stage are nonnegative.

The last term in the objective function (4.1) is the expected value of the loss to country i in stage 2, including the procurement costs and the consequences of unmet demand: $E_\Omega\left[Q^2_i(q^2,\omega)\right] = \sum_{\omega\in\Omega} p_\omega \left[Q^2_i(q^2,\omega)\right]$, where the loss in scenario ω, for each country i, is obtained by solving the following second stage optimization problem:

$$\text{Minimize} \quad Q^2_i(q^2,\omega) \equiv \sum_{j=1}^{I}\sum_{k=1}^{K}\rho^\omega_{j,k}q^{2,\omega}_{ij,k} + \sum_{j=1}^{I}\sum_{k=1}^{K}c^{2,\omega}_{ji,k}(q^{2,\omega}) + \sum_{k=1}^{K}\beta_{i,k}[d^{2,\omega}_{i,k} - \sum_{j=1}^{I}(q^1_{ij,k}+q^{2,\omega}_{ij,k})] \tag{4.4}$$

subject to:

$$\sum_{i=1}^{I} q^{2,\omega}_{ij,k} \leq S^{2,\omega}_{j,k}, \quad j=1,\ldots,I; \quad k=1,\ldots,K, \tag{4.5}$$

$$q^{2,\omega}_{ij,k} \geq 0, \quad j=1,\ldots,I; \quad k=1,\ldots,K. \tag{4.6}$$

In stage 2, when the information about the severity of the pandemic and the level of demand for medical items are revealed, national government i completes its stage 1's efforts by taking the recourse actions. It does this by minimizing the total logistical costs and the damage caused by the unmet demand. The first two terms in objective/recourse function (4.4) are the costs associated with, respectively, the purchase of the medical items, and the cost of transportation of the items to the country when the disaster scenario ω occurs. The last term in objective function (4.4) represents the consequences of shortages of the medical items and the losses that the country will suffer from not satisfying the demand for the various medical items.

After the declaration of the pandemic, which may result in disruptions to the medical supply chains, the supply availabilities for the medical items also can be expected to change. In some cases, due to a shortage of raw materials, worker illnesses, and/or factory closures, production capability may be severely reduced. On the other hand, in some cases, governments may increase the production capacity of these items and, thus, the supply of medical items, by invoking different laws and, with the cooperation of large companies, may have greater capacity. Constraints in (4.5) indicate the supply availability of each medical item k at each country j in stage 2 when scenario ω occurs. Constraints in (4.6) are the nonnegativity assumptions on the shipments in stage 2.

The model can assist the national governments in making the best decisions about procuring medical supplies before the onset of the pandemic in their countries. It also gives the optimal response strategies for possible scenarios in the second stage, when the pandemic is declared, so that the governments can act quickly to respond to the health disaster as soon as the uncertain parameters are realized.

Based on standard stochastic programming theory (see the excellent books by Birge and Louveaux (1997) and Shapiro, Dentcheva, and Ruszczynski (2009)), the first- and second-stage problems together form the following minimization problem for each country i:

$$\text{Minimize } \sum_{j=1}^{I}\sum_{k=1}^{K}\rho_{j,k}q^1_{ij,k} + \sum_{j=1}^{I}\sum_{k=1}^{K}c^1_{ji,k}(q^1)$$
$$+\sum_{\omega\in\Omega}p_\omega\left[\sum_{j=1}^{I}\sum_{k=1}^{K}\rho^{\omega}_{j,k}q^{2,\omega}_{ij,k} + \sum_{j=1}^{I}\sum_{k=1}^{K}c^{2,\omega}_{ji,k}(q^{2,\omega}) + \sum_{k=1}^{K}\beta_{i,k}[d^{2,\omega}_{i,k} - \sum_{j=1}^{I}(q^1_{ij,k}+q^{2,\omega}_{ij,k})]\right] \tag{4.7}$$

subject to:

$$\sum_{i=1}^{I}q^1_{ij,k} \leq S^1_{j,k}, \quad j=1,\ldots,I; \quad k=1,\ldots,K, \tag{4.8}$$

$$\sum_{i=1}^{I}q^{2,\omega}_{ij,k} \leq S^{2,\omega}_{j,k}, \quad j=1,\ldots,I; \quad k=1,\ldots,K; \forall\omega\in\Omega, \tag{4.9}$$

$$q^1_{ij,k} \geq 0, \quad j=1,\ldots,I; \quad k=1,\ldots,K, \tag{4.10}$$

$$q^{2,\omega}_{ij,k} \geq 0, \quad j=1,\ldots,I; \quad k=1,\ldots,K; \forall\omega\in\Omega. \tag{4.11}$$

Let feasible set $\mathcal{K}_i$ correspond to country i. It depends only on the strategy vector of country i, where $\mathcal{K}_i \equiv \{q_i \text{ such that (4.10) and (4.11) hold}\}$, where recall that q_i is the vector of country i's medical item flows. Define $\mathcal{K}^1 \equiv \prod_{i=1}^{I}\mathcal{K}_i$. Also, let $\mathcal{S}$ denote the feasible set of shared constraints: $\mathcal{S} \equiv \{q|\text{(4.8) and (4.9) hold}\}$, where q is the vector of all countries' medical item flows, and the feasible set $\mathcal{K}^2 \equiv \mathcal{K}^1 \cap \mathcal{S}$.

Objective function (4.7) is referred as the Expected Disutility $E(DU_i)$ for $i = 1,\ldots,I$. Assuming that the cost functions for each country are convex and continuously differentiable, then:

Definition 4.1: Stochastic Generalized Nash Equilibrium for the Countries

A strategy vector $q^* \in \mathcal{K}^2$ is a Stochastic Generalized Nash Equilibrium if for each country i; $i = 1, \ldots, I$:

$$E(DU_i(q_i^*, \hat{q_i^*})) \leq E(DU_i(q_i, \hat{q_i^*})), \quad \forall q_i \in \mathcal{K}_i \cap \mathcal{S}, \tag{4.12}$$

where $\hat{q_i^*} \equiv (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_I^*)$.

Each government wishes to minimize its expected disutility. The above definition states that no government, given the circumstances and the strategies of the other national governments, at equilibrium, is willing unilaterally to change its vector of strategies, because it may end up with a higher expected disutility. Observe that the expected disutility of each country depends not only on the decisions of its government, but also on the strategies of other countries. Also, their feasible sets are interconnected because of the shared constraints. The latter condition makes the problem a Generalized Nash Equilibrium model (Debreu (1952)).

As noted in Nagurney, Yu, and Besik (2017) and in Chapter 2, a Variational Equilibrium can be defined, which is a refinement and a specific type of GNE (cf. Kulkarni and Shanbhag (2012)), and enables a variational inequality formulation. It was used for the first time in Nagurney et al. (2020) in a stochastic setting for disaster relief.

Definition 4.2: Variational Equilibrium

A medical item flow vector q^* is a Variational Equilibrium of the above Stochastic Generalized Nash Equilibrium problem if $q^* \in \mathcal{K}^2$ is a solution to the following variational inequality:

$$\sum_{i=1}^{I} \langle \nabla_{q_i} E[DU_i(q^*)], q_i - q_i^* \rangle \geq 0, \quad \forall q \in \mathcal{K}^2, \tag{4.13}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in $IIK+|\Omega|(IIK)$-dimensional Euclidean space.

Note that the Variational Equilibrium guarantees that the Lagrange multipliers associated with the common constraints are the same for all the countries. This feature provides a helpful fairness and equity interpretation, and it makes perfect sense for countries involved in disaster management.

Hence, the well-developed theory of variational inequalities (see Nagurney (1999) and the references therein) is utilized. Expanding variational inequality (4.13), we have:

$$\sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K}\left[\rho_{j,k} + \sum_{r=1}^{I}\sum_{s=1}^{K}\frac{\partial c^1_{ri,s}(q^{1*})}{\partial q^1_{ij,k}} - \beta_{i,k}\right] \times \left[q^1_{ij,k} - q^{1*}_{ij,k}\right]$$
$$+\sum_{\omega\in\Omega} p_\omega \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K}\left[\rho^\omega_{j,k} + \sum_{r=1}^{I}\sum_{s=1}^{K}\frac{\partial c^{2,\omega}_{ri,s}(q^{2,\omega*})}{\partial q^{2,\omega}_{ij,k}} - \beta_{i,k}\right]\times\left[q^{2,\omega}_{ij,k} - q^{2,\omega*}_{ij,k}\right] \geq 0, \quad \forall q \in \mathcal{K}^2. \tag{4.14}$$

Now, using arguments as in Chapter 2, I put variational inequality (4.13) into the standard form (2.15). Let $X \equiv q$, $F(X) \equiv (F^1(X), F^2(X))$, and $\mathcal{K} \equiv \mathcal{K}^2$ where:

$$F^1_{ij,k}(X) \equiv \left[\rho_{j,k} + \sum_{r=1}^{I}\sum_{s=1}^{K}\frac{\partial c^1_{ri,s}(q^1)}{\partial q^1_{ij,k}} - \beta_{i,k}\right], \quad \forall i, j, k,$$

$$F^{2,\omega}_{ij,k}(X) \equiv p_\omega\left[\rho^\omega_{j,k} + \sum_{r=1}^{I}\sum_{s=1}^{K}\frac{\partial c^{2,\omega}_{ri,s}(q^{2,\omega})}{\partial q^{2,\omega}_{ij,k}} - \beta_{i,k}\right], \quad \forall i, j, k, \omega. \tag{4.15}$$

Under the imposed conditions, the function $F(X)$ that enters variational inequality problem (4.13) is continuous, and the feasible set $\mathcal{K}^2$ is compact; therefore, the existence of a solution to variational inequality (4.13) is guaranteed from the classical theory of variational inequalities (cf. Kinderlehrer and Stampacchia (1980)).

4.1.2 Illustrative Examples

In this subsection, three illustrative examples are provided that help to illuminate the model. The corresponding network figure depictions are given, respectively, in Figures 4.2, 4.3, and 4.4. Henceforth, in this chapter, in the notation for the superscripts, 1 is utilized for ω_1, 2 is utilized for ω_2, and so forth.

Illustrative Example 4.1: One National Government and One Scenario

In the first example, there is a single national government that is facing a pandemic with demand for N95 masks, a form of PPE. Also, the country will be faced with scenario $\omega_1 = 1$ with $p_{\omega_1} = 1$. The data for this example are: $\rho_{1,1} = 2$, $\rho^1_{1,1} = 25$, $\beta_{1,1} = 3000$, $c^1_{11,1} = (q^1_{11,1})^2$, $c^{2,1}_{11,1} = 2(q^{2,1}_{11,1})^2$. The supply and the demand data are: $S^1_{1,1} = 2000$, $S^{2,1}_{1,1} = 500$, and $d^{2,1}_{1,1} = 3000$.

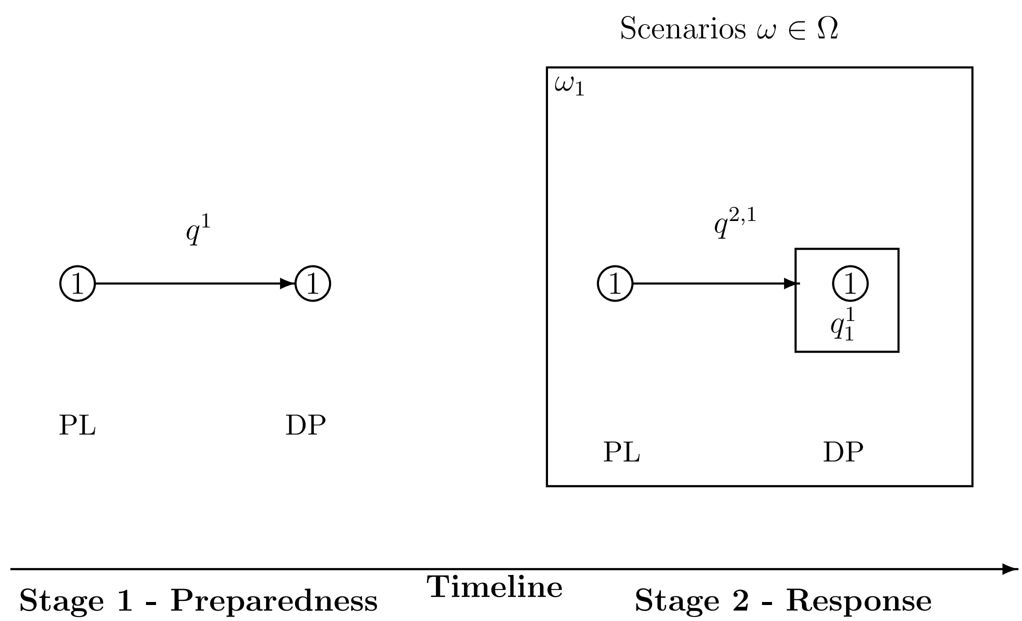

Figure 4.2. The Timeline of Illustrative Example 4.1

One can rewrite variational inequality (4.13); equivalently, variational inequality (4.14), for this example as: determine $q^{1*}_{11,1}$ and $q^{2,1*}_{11,1}$, where $q^{1*}_{11,1} \leq 2000$ and $q^{2,1*}_{11,1} \leq 500$, such that:

$$\left[2 + 2(q^{1*}_{11,1}) - 3000\right] \times \left[q^{1}_{11,1} - q^{1*}_{11,1}\right] + (1)\left[25 + 4(q^{2,1*}_{11,1}) - 3000\right] \times \left[q^{2,1}_{11,1} - q^{2,1*}_{11,1}\right] \geq 0, \tag{4.16}$$

for all $q^{1}_{11,1}$ and $q^{2,1}_{11,1}$ such that

$$q^{1}_{11,1} \leq 2000, \quad q^{2,1}_{11,1} \leq 500.$$

Let's denote the expression in (4.16) preceding the greater than or equal to sign by A. A can be rewritten as:

$$A = B \times B' + C \times C'.$$

Using A, observe that

$$\text{If } q^{1*}_{11,1} = 0 \Longrightarrow \text{always } B \times B' \leq 0;$$

$$\text{If } q^{1*}_{11,1} = 2000 \Longrightarrow \text{always } B \times B' \leq 0;$$

$$\text{If } 0 < q^{1*}_{11,1} < 2000 \Longrightarrow B' \text{ is unrestricted } \Longrightarrow \text{it is desired that } B = 0 \Longrightarrow q^{1*}_{11,1} = 1499;$$

$$\text{Since } q^{2,1}_{11,1} \leq 500 \Longrightarrow \text{always } C < 0 \Longrightarrow \text{it is desired that } C' \leq 0 \Longrightarrow q^{2,1*}_{11,1} = 500.$$

Therefore, it is concluded that the solution to the associated variational inequality for this example is:

$$q^{1*}_{11,1} = 1499, \quad q^{2,1*}_{11,1} = 500.$$

I now demonstrate that variational inequality (4.14) is, indeed, satisfied. Note that this solution lies in the feasible set $\mathcal{K}^2$. Also, substitution of the values into A yields:

$$[2+2(1499)-3000]\times\left[q^{1}_{11,1}-1499\right]+(1)\left[25+4(500)-3000\right]\times\left[q^{2,1}_{11,1}-500\right]$$
$$=[0]\times\left[q^{1}_{11,1}-1499\right]+[-975]\times\left[q^{2,1}_{11,1}-500\right]$$
$$=487,500-975\times q^{2,1}_{11,1},$$

which is clearly greater than or equal to zero since $q^{2,1}_{11,1}\leq 500$.

Observe that the government purchases a total of 1,999.00 items in two stages, with 1499.00 units purchased in the first stage, which can be placed in the national stockpile, and 500 items in the second stage. However, it still faces a shortage of 1,001.00 N95 masks and that is because of the high logistical costs in Stage 1 and the supply capacity in Stage 2. The government's expected disutility under this solution is: $E(DU_1(q^*)) = 5,765,499.00$. The national government may wish to invest in enhancing production capacity in Stage 2.

Illustrative Example 4.2: One National Government and Two Scenarios

In this example, in addition to the scenario $\omega_1 = 1$ from Example 4.1, the government predicts another scenario with a greater severity in Stage 2. The probability of occurrence of these two scenarios according to government estimates are: $p_{\omega_1} = 0.3$, $p_{\omega_2} = 0.7$. In scenario $\omega_2 = 2$, due to the severity of the pandemic, demand increases sharply to $d^{2,2}_{1,1} = 5,000$ and the price of the medical item is raised to $\rho^{2}_{1,1} = 80$. The domestic PPE supply $S^{2,2}_{1,1} = 1,000$ in Stage 2 in scenario ω_2 and the transportation cost functions are similar in the two scenarios, $c^{1}_{11,1} = (q^{1}_{11,1})^2$, $c^{2,1}_{11,1} = 2(q^{2,1}_{11,1})^2$, $c^{2,2}_{11,1} = 2(q^{2,2}_{11,1})^2$.

One can rewrite variational inequality (4.13); equivalently, variational inequality (4.14), for this example as: determine $q^{1*}_{11,1}$, $q^{2,1*}_{11,1}$, and $q^{2,2*}_{11,1}$ such that

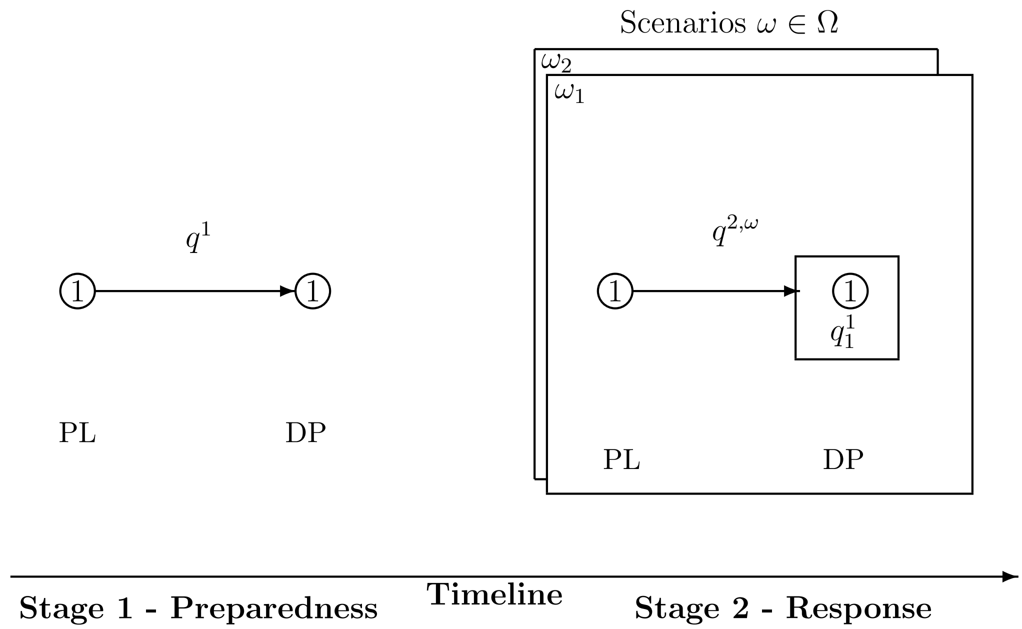

Figure 4.3. The Timeline of the Illustrative Example 4.2

$$q^{1*}_{11,1} \leq 2000, \quad q^{2,1*}_{11,1} \leq 500, \quad q^{2,2*}_{11,1} \leq 1000$$

and

$$\left[2 + 2(q^{1*}_{11,1}) - 3000\right] \times \left[q^1_{11,1} - q^{1*}_{11,1}\right] + (0.3)\left[25 + 4(q^{2,1*}_{11,1}) - 3000\right] \times \left[q^{2,1}_{11,1} - q^{2,1*}_{11,1}\right]$$
$$+(0.7)\left[80 + 4(q^{2,2*}_{11,1}) - 3000\right] \times \left[q^{2,2}_{11,1} - q^{2,2*}_{11,1}\right] \geq 0,$$

for all $q^1_{11,1}$, $q^{2,1}_{11,1}$, and $q^{2,2}_{11,1}$ satisfying:

$$q^1_{11,1} \leq 2000, \quad q^{2,1}_{11,1} \leq 500, \quad q^{2,2}_{11,1} \leq 1000.$$

Proceeding in a similar manner as that in Example 4.1, yields:

$$q^{1*}_{11,1} = 1,499.00, \quad q^{2,1*}_{11,1} = 500.00, \quad q^{2,2*}_{11,1} = 730.00,$$

Indeed, substitution of the above medical item pattern into the variational inequality yields:

$$[2+2(1499)-3000] \times \left[q_{11,1}^{1}-1499\right] + (0.3)\,[25+4(500)-3000] \times \left[q_{11,1}^{2,1}-500\right]$$

$$+(0.7)\,[80+4(730)-3000] \times \left[q_{11,1}^{2,2}-730\right]$$

$$= [0] \times \left[q_{11,1}^{1}-1499\right] + [-292.5] \times \left[q_{11,1}^{2,1}-500\right] + [0] \times \left[q_{11,1}^{2,2}-730\right]$$

$$= 146250 - 292.5 q_{11,1}^{2,1},$$

which is greater than or equal to zero since $q_{11,1}^{2,1} \leq 500$.

The expected disutility increases to $E(DU_1(q^*)) = 9{,}910{,}689.00$ in Example 4.2. Observe that with the addition of a severe disaster scenario, the government ends up in a much worse situation than in Example 4.1. Despite the higher supply availability, the country continues to face severe shortages: 1,001 items under scenario ω_1 and 2,771 items under scenario ω_2.

Illustrative Example 4.3: Two National Governments and One Scenario

This example is constructed from the first example, with the difference that now there are two governments trying to procure N95 masks for their countries and competing for limited supplies. The data for the second country are: $\rho_{2,1} = 2$, $\rho_{2,1}^{1} = 120$, $\beta_{2,1} = 3000$, $S_{2,1}^{1} = 2000$, $S_{2,1}^{2,1} = 500$, $d_{2,1}^{2,1} = 5000$, $c_{22,1}^{1} = (q_{22,1}^{1})^2$, $c_{22,1}^{2,1} = 2(q_{22,1}^{2,1})^2$. Also, the transportation cost functions between the two countries are as follows:

$$c_{21,1}^{1}(q^1) = 2(q_{12,1}^{1})^2 + 5q_{12,1}^{1}, \quad c_{21,1}^{2,1}(q^{2,1}) = 12(q_{12,1}^{2,1})^2 + 5q_{12,1}^{2,1},$$

$$c_{12,1}^{1}(q^1) = 2(q_{21,1}^{1})^2 + 5q_{21,1}^{1}, \quad c_{12,1}^{2,1}(q^{2,1}) = 6(q_{21,1}^{2,1})^2 + 5q_{21,1}^{2,1}.$$

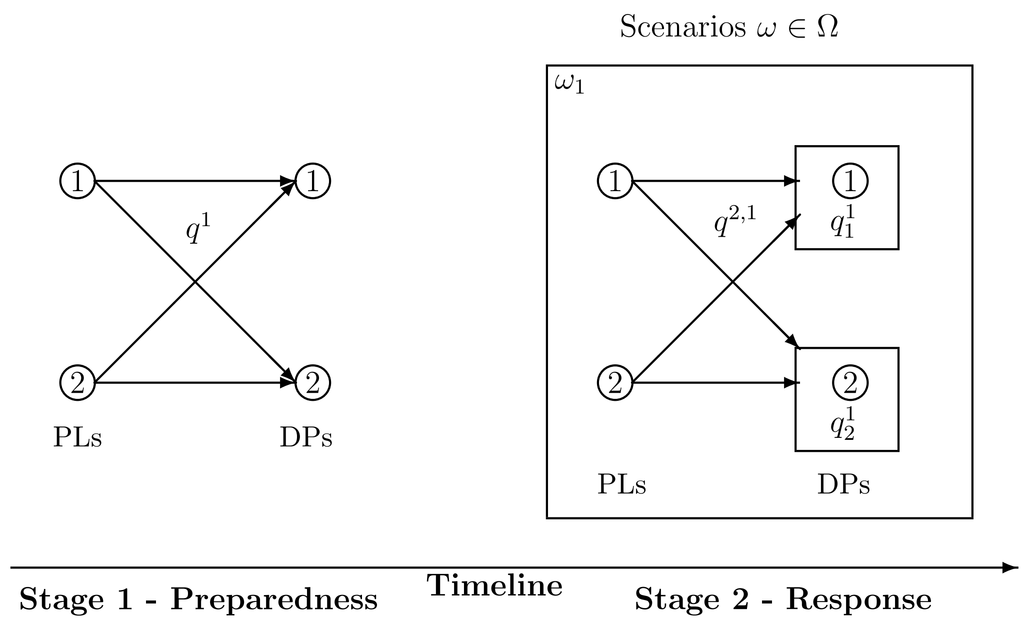

Figure 4.4. The Timeline of the Illustrative Example 4.3

Making use of the variational inequality (4.14), it is straightforward to determine that the equilibrium solution for this example is:

$$q^{1*}_{11,1} = 1,334.17, \quad q^{1*}_{12,1} = 665.83, \quad q^{1*}_{21,1} = 665.83, \quad q^{1*}_{22,1} = 1,334.17,$$

$$q^{2,1*}_{11,1} = 375.31, \quad q^{2,1*}_{12,1} = 71.25, \quad q^{2,1*}_{21,1} = 124.69, \quad q^{2,1*}_{22,1} = 428.75.$$

The above solution is feasible and satisfies variational inequality (4.14). Indeed, observe that:

$$[2 + 2(1334.17) - 3000] \times \left[q^1_{11,1} - 1334.17\right] + [2 + 4(665.83) + 5 - 3000]$$

$$\times \left[q^1_{12,1} - 665.83\right]$$

$$+ [2 + 4(665.83) + 5 - 3000] \times \left[q^1_{21,1} - 665.83\right] + [2 + 2(1334.17) - 3000]$$

$$\times \left[q^1_{22,1} - 1334.17\right]$$

$$+ [25 + 4(375.31) - 3000] \times \left[q^{2,1}_{11,1} - 375.31\right] + [120 + 24(71.25) + 5 - 3000]$$

$$\times \left[q^{2,1}_{12,1} - 71.25\right]$$

$$+ [25 + 12(124.69) + 5 - 3000] \times \left[q^{2,1}_{21,1} - 124.69\right] + [120 + 4(428.75) - 3000]$$

$$\times \left[q^{2,1}_{22,1} - 428.75\right]$$

$$= [-329.66] \times \left[q^{1}_{11,1} - 1334.17\right] + [-329.66] \times \left[q^{1}_{12,1} - 665.83\right]$$

$$+ [-329.66] \times \left[q^{1}_{21,1} - 665.83\right] + [-329.66] \times \left[q^{1}_{22,1} - 1334.17\right]$$

$$+ [-1473.72] \times \left[q^{2,1}_{11,1} - 375.31\right] + [-1165] \times \left[q^{2,1}_{12,1} - 71.25\right]$$

$$+ [-1473.72] \times \left[q^{2,1}_{21,1} - 124.69\right] + [-1165] \times \left[q^{2,1}_{22,1} - 428.75\right]$$

$$= 2638000 - 329.66 \times (q^{1}_{11,1} + q^{1}_{12,1} + q^{1}_{21,1} + q^{1}_{22,1})$$

$$-1473.72 \times (q^{2,1}_{11,1} + q^{2,1}_{21,1}) - 1165 \times (q^{2,1}_{12,1} + q^{2,1}_{22,1}),$$

which is greater than or equal to zero for any feasible medical item shipments.

In the first stage, both national governments manage to procure 2,000 items and utilize all their domestic supplies. Observe that each government makes more purchases domestically, which is due to lower logistical costs than buying from the other country. However, in Stage 2, the prices of the N95 masks in both countries have risen sharply. Country 2, which has a higher demand than Country 1, has a higher price than Country 1. Also, the cost of transporting the masks from Country 2 to Country 1 after the pandemic declaration is much higher than the cost on the opposite route. As a result, in Stage 2, Country 1 adds 446.56 medical items, while Country 2 purchases 553.44 items. The effect of a sharp increase in the transportation cost from Country 2 can be seen, which has led to most of its mask supply being purchased by Country 2.

Country 1 faces a shortage of 553.44 medical items and its expected disutility is equal to $E(DU_1(q^*)) = 4,695,240.86$. Country 2 faces a severe shortage of

2446.56 medical items while the expected disutility of Country 2 is $E(DU_2(q^*)) = 10,529,807.32$. An interesting and significant point in Example 4.3 is that country 1 has a lower shortage than what it was dealing with in Example 4.1. The reason is the addition of another country with supplies gives governments more options. In times of disaster, having more suppliers can help and, unlike in Examples 4.1 and 4.2, there is no unsold capacity in Example 4.3. The national governments purchase all the available medical supplies to reduce the shortages of N95 masks in their countries.

4.2. The Algorithm and Alternative Variational Inequality Formulation

Before presenting the closed form expressions of the algorithm, I define the necessary Lagrange multipliers related to the constraints and provide an alternative variational inequality to variational inequality (4.14), which is utilized here to derive the closed form expressions for the variables at each iteration of the proposed algorithmic scheme. Lagrange multiplier $\alpha^1_{j,k}$ is associated with the supply constraint (4.8) on the availability of medical item k in country j, for each j and each k. Also let $\gamma^{2,\omega}_{j,k}$ be the Lagrange multiplier associated with supply constraint (4.9) on medical item k in country j when scenario ω occurs, for each j and k. These Lagrange multipliers are gathered into the respective vectors: $\alpha^1 \in R^{IK}_+$ and $\gamma^2 \in R^{|\Omega|(IK)}_+$.

Therefore, variational inequality (4.14) can be reformulated as the following variational inequality (cf. arguments in Chapter 2):

Determine $(q^*, \alpha^{1*}, \gamma^{2*}) \in R_+^{IIK+|\Omega|(IIK)+IK+|\Omega|(IK)}$ such that

$$\sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K}\left[\rho_{j,k}+\sum_{r=1}^{I}\sum_{s=1}^{K}\frac{\partial c_{ri,s}^{1}(q^{1*})}{\partial q_{ij,k}^{1}}-\beta_{i,k}+\alpha_{j,k}^{1*}\right]\times\left[q_{ij,k}^{1}-q_{ij,k}^{1*}\right]$$
$$+\sum_{\omega\in\Omega}p_{\omega}\sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K}\left[\rho_{j,k}^{\omega}+\sum_{r=1}^{I}\sum_{s=1}^{K}\frac{\partial c_{ri,s}^{2,\omega}(q^{2,\omega*})}{\partial q_{ij,k}^{2,\omega}}-\beta_{i,k}+\gamma_{j,k}^{2,\omega*}\right]\times\left[q_{ij,k}^{2,\omega}-q_{ij,k}^{2,\omega*}\right]$$
$$+\sum_{j=1}^{I}\sum_{k=1}^{K}\left[S_{j,k}^{1}-\sum_{i=1}^{I}q_{ij,k}^{1*}\right]\times\left[\alpha_{j,k}^{1}-\alpha_{j,k}^{1*}\right]$$
$$+\sum_{\omega\in\Omega}\sum_{j=1}^{I}\sum_{k=1}^{K}\left[S_{j,k}^{2,\omega}-\sum_{i=1}^{I}q_{ij,k}^{2,\omega*}\right]\times\left[\gamma_{j,k}^{2,\omega}-\gamma_{j,k}^{2,\omega*}\right]\geq 0,$$
$$\forall(q,\alpha^{1},\gamma^{2})\in R_+^{IIK+|\Omega|(IIK)+IK+|\Omega|(IK)}. \tag{4.17}$$

4.2.1 Revisiting Examples 4.1 Through 4.3

Accordingly, functions in VI (4.17) can be constructed using data in Example 4.1 as below, where $\alpha_{1,1}^{1*}=0$ and $\gamma_{1,1}^{2,1*}=975$:

$$[2+2(1499)-3000+0]\times\left[q_{11,1}^{1}-1499\right]+(1)\,[25+4(500)-3000+975]\times\left[q_{11,1}^{2,1}-500\right]$$
$$+[2000-1499]\times\left[\alpha_{1,1}^{1}-0\right]+[500-500]\times\left[\gamma_{1,1}^{2,1}-975\right].$$

Noting that the first, second, and fourth terms in the above expression are equal to zero and the third term is greater than or equal to zero for nonnegative $\alpha_{1,1}^{1}$, VI (4.17) holds for Example 4.1.

In Example 4.2, the full supply capacity is not used in Stage 1 and in Scenario 2 of Stage 2. Hence, the associated Lagrange multipliers, $\alpha_{1,1}^{1*}$, and $\gamma_{1,1}^{2,2*}$, are equal to zero. However, the country government utilizes all the supply in Scenario 1 of Stage 2, with $\gamma_{1,1}^{2,1*}=975$. Using the above and the equilibrium medical item flow solution of Example 4.2 in VI (4.17), we obtain:

$$[2+2(1499)-300+0]\times\left[q^1_{11,1}-1499\right]+(0.3)\,[25+4(500)-3000+975]\times\left[q^{2,1}_{11,1}-500\right]$$
$$+(0.7)\,[80+4(730)-3000+0]\times\left[q^{2,2}_{11,1}-730\right]$$
$$+[2000-1499]\times\left[\alpha^1_{1,1}-0\right]+[500-500]\times\left[\gamma^{2,1}_{1,1}-975\right]+[5000-730]\times\left[\gamma^{2,2}_{1,1}-0\right].$$

All terms in the expression above are equal to zero except for the fourth and final ones, which are both greater than or equal to zero, respectively, for any nonnegative $\alpha^1_{1,1}$ and $\gamma^{2,2}_{1,1}$. Hence, clearly, VI (4.17) holds.

In Example 4.3, all the available supplies of the medical item are purchased and the supply constraints are, therefore, binding. The associated Lagrange multipliers are all positive with values at equilibrium as below:

$$\alpha^{1*}_{1,1}=329.66,\quad \alpha^{1*}_{2,1}=329.66,\quad \gamma^{2,1*}_{1,1}=1473.72,\quad \gamma^{2,1*}_{2,1}=1165.00,$$

Indeed, using the data from Example 4.3, the function F(X) that enters VI (4.17) can be constructed for Example 4.3 as:

$$\left[2+2q^{1*}_{11,1}-3000+\alpha^{1*}_{1,1}\right]\times\left[q^1_{11,1}-q^{1*}_{11,1}\right]+\left[2+4q^{1*}_{12,1}+5-3000+\alpha^{1*}_{2,1}\right]\times\left[q^1_{12,1}-q^{1*}_{12,1}\right]$$
$$+\left[2+4q^{1*}_{21,1}+5-3000+\alpha^{1*}_{1,1}\right]\times\left[q^1_{21,1}-q^{1*}_{21,1}\right]+\left[2+2q^{1*}_{22,1}-3000+\alpha^{1*}_{2,1}\right]$$
$$\times\left[q^1_{22,1}-q^{1*}_{22,1}\right]$$
$$+\left[25+4q^{2,1*}_{11,1}-3000+\gamma^{2,1*}_{1,1}\right]\times\left[q^{2,1}_{11,1}-q^{2,1*}_{11,1}\right]+\left[120+24q^{2,1*}_{12,1}+5-3000+\gamma^{2,1*}_{2,1}\right]$$
$$\times\left[q^{2,1}_{12,1}-q^{2,1*}_{12,1}\right]$$
$$+\left[25+12q^{2,1*}_{21,1}+5-3000+\gamma^{2,1*}_{1,1}\right]\times\left[q^{2,1}_{21,1}-q^{2,1*}_{21,1}\right]+\left[120+4q^{2,1*}_{22,1}-3000+\gamma^{2,1*}_{2,1}\right]$$
$$\times\left[q^{2,1}_{22,1}-q^{2,1*}_{22,1}\right]$$
$$+\left[2000-q^{1*}_{11,1}-q^{1*}_{21,1}\right]\times\left[\alpha^1_{1,1}-\alpha^{1*}_{1,1}\right]+\left[2000-q^{1*}_{12,1}-q^{1*}_{22,1}\right]\times\left[\alpha^1_{2,1}-\alpha^{1*}_{2,1}\right]$$

$$+\left[500-q_{11,1}^{2,1*}-q_{21,1}^{2,1*}\right]\times\left[\gamma_{1,1}^{2,1}-\gamma_{1,1}^{2,1*}\right]+\left[500-q_{12,1}^{2,1*}-q_{22,1}^{2,1*}\right]\times\left[\gamma_{2,1}^{2,1}-\gamma_{2,1}^{2,1*}\right].$$

Substituting the above values for the Lagrange multipliers as well as the product flows reported for Example 4.3 in Subsection 4.1.2, we obtain:

$$(2+2(1334.17)-3000+329.66)\times(q_{11,1}^{1}-1334.17)+(2+4(665.83)+5-3000+329.66)$$
$$\times(q_{12,1}^{1}-665.83)$$
$$+(2+4(665.83)+5-3000+329.66)\times(q_{21,1}^{1}-665.83)+(2+2(1334.17)-3000+329.66)$$
$$\times(q_{22,1}^{1}-1334.17)$$
$$+(25+4(375.31)-3000+1473.72)\times(q_{11,1}^{2,1}-375.31)+(120+24(71.25)+5-3000+1165)$$
$$\times(q_{12,1}^{2,1}-71.25)$$
$$+(25+12(124.69)+5-3000+1473.72)\times(q_{21,1}^{2,1}-124.69)+(120+4(428.75)-3000+1165)$$
$$\times(q_{22,1}^{2,1}-428.75).$$

But the above expression can be rewritten as:

$$\epsilon^1\times(q_{11,1}^{1}-1334.17)+\tau^1\times(q_{12,1}^{1}-665.83)+\epsilon^1\times(q_{21,1}^{1}-665.83)+\tau^1\times(q_{22,1}^{1}-1334.17)$$
$$+\epsilon^2\times(q_{11,1}^{2,1}-375.31)+\tau^2\times(q_{12,1}^{2,1}-71.25)+\epsilon^2\times(q_{21,1}^{2,1}-124.69)+\tau^2\times(q_{22,1}^{2,1}-428.75),$$

or

$$\epsilon^1\times(q_{11,1}^{1}+q_{21,1}^{1}-2000)+\tau^1\times(q_{12,1}^{1}+q_{22,1}^{1}-2000)$$
$$+\epsilon^2\times(q_{11,1}^{2,1}+q_{21,1}^{2,1}-500)+\tau^2\times(q_{12,1}^{2,1}+q_{22,1}^{2,1}-500).$$

Since $\epsilon^1=\epsilon^2=\tau^1=\tau^2=0$, VI (4.17) holds and the above is, indeed, the Generalized Nash Equilibrium solution.

4.2.2 The Algorithm

In this subsection, I provide the closed form expressions of the modified projection method to solve the variational inequality (4.17).

Steps 1 and 2 of the modified projection method (cf. (3.25) and (3.26) in Chapter 3) result in explicit formulae for the computation of the medical supply flows and the Lagrange multipliers at each iteration of the algorithm. In particular, at each iteration of the algorithm, we have the following explicit formulae for Step 1. Similar explicit formulae can be determined accordingly for Step 2.

Explicit Formulae for Step 1 for the Medical Supply Flows in Stage 1

For each i, j, k, compute

$$(\bar{q}^1_{ij,k})^\tau = \max\{0, (q^1_{ij,k})^{\tau-1} - \beta(\rho_{j,k} + \sum_{r=1}^{I}\sum_{s=1}^{K} \frac{\partial c^1_{ri,s}((q^1)^{\tau-1})}{\partial q^1_{ij,k}} - \beta_{i,k} + (\alpha^1_{j,k})^{\tau-1})\}; \quad (4.18)$$

Explicit Formulae for Step 1 for the Medical Supply Flows in Stage 2

For each ω, i, j, k, compute

$$(\bar{q}^{2,\omega}_{ij,k})^\tau = \max\{0, (q^{2,\omega}_{ij,k})^{\tau-1} - \beta p_\omega(\rho^\omega_{j,k} + \sum_{r=1}^{I}\sum_{s=1}^{K} \frac{\partial c^{2,\omega}_{ri,s}((q^{2,\omega})^{\tau-1})}{\partial q^{2,\omega}_{ij,k}} - \beta_{i,k} + (\gamma^{2,\omega}_{j,k})^{\tau-1})\}; \quad (4.19)$$

Explicit Formulae for Step 1 the Lagrange Multipliers Associated with the Supply Constraints in Stage 1

For each j, k, compute

$$(\bar{\alpha}^1_{j,k})^\tau = \max\{0, (\alpha^1_{j,k})^{\tau-1} - \beta(S^1_{j,k} - \sum_{i=1}^{I}(q^1_{ij,k})^{\tau-1})\}; \quad (4.20)$$

Explicit Formulae for Step 1 for the Lagrange Multipliers Associated with the Supply Constraints in Stage 2

For each ω, j, k, compute

$$(\bar{\gamma}_{j,k}^{2,\omega})^{\tau} = \max\{0, (\gamma_{j,k}^{2,\omega})^{\tau-1} - \beta(S_{j,k}^{2,\omega} - \sum_{i=1}^{I}(q_{ij,k}^{2,\omega})^{\tau-1})\}. \quad (4.21)$$

4.3. Numerical Examples

The examples in this section are inspired by the COVID-19 pandemic. When the global outbreak occurred, the demand for PPEs was much higher than the inventory and production capacities; for example, the United States, before the pandemic, needed about 50 million N95 masks annually, but during the COVID-19 pandemic, the demand increased significantly to about 140 million face masks during a 90-day peak-use period (Lopez (2020b)). Disruptions in the supply chain also added to the shortage crisis of medical supplies. Over 90% of international PPE shipments have been disrupted in some way. For example, the air freight delivery time from China to the United States, which used to be about 4-6 days, has now reached 8-14 days. Also, the cost of shipping PPEs from China has gone up to 3-4 times of the pre-pandemic rates; additionally, an increase in the price of masks plus transportation disruptions have resulted in the growth in cargo insurance rates as well (C.H. Robinson (2020)).

In this section, as in previous chapters, examples are provided to examine decision-makers' competition for limited resources in disaster management. These examples are inspired by the COVID-19 pandemic and are solved by making use of the model and methods presented in this chapter to explore new aspects of pandemic disaster management, including pre- and post-disaster competition.

4.3.1 Numerical Examples Detailed

The information about the current pandemic is utilized in the following examples to investigate the competition for medical supplies under different scenarios. The modified projection method was implemented in MATLAB and the computer system used was a Microsoft Windows 10 system. The algorithm was initialized so that all the medical item flows and the Lagrange multipliers were set to 0.00. The convergence tolerance for all the examples was that the absolute value of each of the computed variable values at two successive iterations was less than or equal to 0.0001. The β parameter in the modified projection method was set to 0.1.

Numerical Example 4.1: Two Countries, One Type of Medical Supply, Two Scenarios

In the first numerical example, there are two countries and two scenarios. The medical item is the N95 mask. In the first scenario, $\omega_1 = 1$, the pandemic is severe and the demand for such PPEs in both countries is very high. The second scenario, $\omega_2 = 2$, is one in which the consequences of the pandemic are less and the countries have a lower demand. As a result, the national governments in both countries take action in two stages, before and after the pandemic declaration, to purchase the PPEs. They purchase the N95 masks in large bulks of 1000 masks each; therefore, $q^1_{ij,1}$ represents 1000 of N95 masks purchased by country i from country j in Stage 1. Both countries have supplies of N95 masks. The example's data are as follows.

$$p_{\omega_1} = 0.7, \quad p_{\omega_2} = 0.3,$$

$$\rho_{1,1} = 1,000, \quad S^1_{1,1} = 20,000 \qquad \rho^1_{1,1} = 5,000, \quad S^{2,1}_{1,1} = 7,000,$$

$$\rho^2_{1,1} = 2,000, \quad S^{2,2}_{1,1} = 20,000,$$

$$\rho_{2,1} = 1,000, \quad S^1_{2,1} = 25,000 \qquad \rho^1_{2,1} = 2,500, \quad S^{2,1}_{2,1} = 25,000,$$

$$\rho^2_{2,1} = 2,000, \quad S^{2,2}_{2,1} = 40,000,$$

$$\beta_{1,1} = \beta_{2,1} = 100,000, \quad d^{2,1}_{1,1} = d^{2,1}_{2,1} = 80,000, \quad d^{2,2}_{1,1} = d^{2,2}_{2,1} = 55,000,$$

$$c^1_{11,1} = 2(q^1_{11,1})^2, \quad c^1_{22,1} = 2(q^1_{22,1})^2,$$

$$c^1_{21,1}(q^1) = 5(q^1_{12,1})^2 + 3q^1_{12,1}, \quad c^1_{12,1}(q^1) = 5(q^1_{21,1})^2 + 3q^1_{21,1},$$

$$c^{2,1}_{11,1} = 5(q^{2,1}_{11,1})^2, \quad c^{2,1}_{22,1} = 3(q^{2,1}_{22,1})^2,$$

$$c^{2,1}_{21,1}(q^{2,1}) = 9(q^{2,1}_{12,1})^2 + 6q^{2,1}_{12,1}, \quad c^{2,1}_{12,1}(q^{2,1}) = 9(q^{2,1}_{21,1})^2 + 6q^{2,1}_{21,1},$$

$$c^{2,2}_{11,1} = 2(q^{2,2}_{11,1})^2, \quad c^{2,2}_{22,1} = 2(q^{2,2}_{22,1})^2,$$

$$c^{2,2}_{21,1}(q^{2,2}) = 5(q^{2,2}_{12,1})^2 + 3q^{2,2}_{12,1}, \quad c^{2,2}_{12,1}(q^{2,2}) = 5(q^{2,2}_{21,1})^2 + 3q^{2,2}_{21,1}.$$

The computed equilibrium solution via the modified projection method for this example is:

$$q^{1*}_{11,1} = 14,285.92, \quad q^{1*}_{12,1} = 7,142.64, \quad q^{1*}_{21,1} = 5,714.07, \quad q^{1*}_{22,1} = 17,857.35,$$

$$q^{2,1*}_{11,1} = 4,500.22, \quad q^{2,1*}_{12,1} = 5,416.33, \quad q^{2,1*}_{21,1} = 2,499.78, \quad q^{2,1*}_{22,1} = 16,250.00,$$

$$q^{2,2*}_{11,1} = 14,285.92, \quad q^{2,2*}_{12,1} = 9,799.70, \quad q^{2,2*}_{21,1} = 5,714.07, \quad q^{2,2*}_{22,1} = 24,500.00,$$

$$\alpha^{1*}_{1,1} = 41,856.28, \quad \alpha^{1*}_{2,1} = 27,570.57,$$

$$\gamma^{2,1*}_{1,1} = 49,997.79, \quad \gamma^{2,1*}_{2,1} = 0.00, \quad \gamma^{2,2*}_{1,1} = 40,856.28, \quad \gamma^{2,2*}_{2,1} = 0.00.$$

Observe that Country 1 is very vulnerable in the face of Scenario 1. The available supply of the masks in the country have been drastically reduced, while the transportation costs have also increased due to the pandemic. As a result, Country 1 faces a shortage of 48,654.87 units, which is equal to $48,654,870$ N95 masks. On the other hand, the government of Country 2 has a higher supply of masks available in the country in Scenario 1 and is in a better position than Country 1. However, due to insufficient total available supply, Country 2 also faces a large shortage of 37,678.78

units. In Scenario 2, where the consequences of the pandemic are less severe, both countries have better conditions than in Scenario 1, but Country 2, with a significantly greater supply in its country, faces a much smaller shortage than Country 1.

Both countries are not prepared sufficiently to respond to the pandemic, but Country 2, which has greater flexibility and resilience in terms of its country's supply, performs better in reducing the shortage of face masks post the disaster. Another noteworthy point is that Country 2 expends a lot of money to purchase and transport the PPEs, which leads to almost equal expected disutilities for the two countries, $E(DU_1(q^*)) = 1,318,107,953.24$, $E(DU_2(q^*)) = 1,337,948,865.88$.

Numerical Example 4.2: Two Countries, Two Types of Medical Supplies, Two Scenarios

In Numerical Example 4.2, in addition to the face masks, the countries are also trying to meet their demand for ventilators. The need for these devices is less than the demand for the face masks, but the value of a ventilator in saving lives in this pandemic is very high. Also, these devices are expensive, especially once the pandemic occurs. The additional data needed for this example are:

$$\rho_{1,2} = 10,000, \quad S^1_{1,2} = 2,000 \qquad \rho^1_{1,2} = 45,000, \quad S^{2,1}_{1,2} = 2,000,$$

$$\rho^2_{1,2} = 20,000, \quad S^{2,2}_{1,2} = 2,000,$$

$$\rho_{2,2} = 10,000, \quad S^1_{2,2} = 10,000 \qquad \rho^1_{2,2} = 20,000, \quad S^{2,1}_{2,2} = 10,000,$$

$$\rho^2_{2,2} = 15,000, \quad S^{2,2}_{2,2} = 20,000,$$

$$\beta_{1,2} = \beta_{2,2} = 1,000,000, \quad d^{2,1}_{1,2} = d^{2,1}_{2,2} = 50,000, \quad d^{2,2}_{1,2} = d^{2,2}_{2,2} = 25,000,$$

$$c^1_{11,2} = 2(q^1_{11,2})^2, \quad c^1_{22,2} = 2(q^1_{22,2})^2,$$

$$c^1_{21,2}(q^1) = 5(q^1_{12,2})^2 + 3q^1_{12,2}, \quad c^1_{12,2}(q^1) = 5(q^1_{21,2})^2 + 3q^1_{21,2},$$

$$c^{2,1}_{11,2} = 5(q^{2,1}_{11,2})^2, \quad c^{2,1}_{22,2} = 3(q^{2,1}_{22,2})^2,$$

$$c^{2,1}_{21,2}(q^{2,1}) = 9(q^{2,1}_{12,2})^2 + 6q^{2,1}_{12,2}, \quad c^{2,1}_{12,2}(q^{2,1}) = 9(q^{2,1}_{21,2})^2 + 6q^{2,1}_{21,2},$$

$$c^{2,2}_{11,2} = 2(q^{2,2}_{11,2})^2, \quad c^{2,2}_{22,2} = 2(q^{2,2}_{22,2})^2,$$

$$c^{2,2}_{21,2}(q^{2,2}) = 5(q^{2,2}_{12,2})^2 + 3q^{2,2}_{12,2}, \quad c^{2,2}_{12,2}(q^{2,2}) = 5(q^{2,2}_{21,2})^2 + 3q^{2,2}_{21,2}.$$

The computed equilibrium solution for this example is:

$$q^{1*}_{11,1} = 14,285.92, \quad q^{1*}_{12,1} = 7,142.64, \quad q^{1*}_{21,1} = 5,714.07, \quad q^{1*}_{22,1} = 17,857.35,$$

$$q^{2,1*}_{11,1} = 4,500.22, \quad q^{2,1*}_{12,1} = 5,416.33, \quad q^{2,1*}_{21,1} = 2,499.78, \quad q^{2,1*}_{22,1} = 16,250.00,$$

$$q^{2,2*}_{11,1} = 14,285.92, \quad q^{2,2*}_{12,1} = 9,799.70, \quad q^{2,2*}_{21,1} = 5,714.07, \quad q^{2,2*}_{22,1} = 24,500.00,$$

$$q^{1*}_{11,2} = 1,428.78, \quad q^{1*}_{12,2} = 2,856.92, \quad q^{1*}_{21,2} = 571.21, \quad q^{1*}_{22,2} = 7,143.07,$$

$$q^{2,1*}_{11,2} = 1,285.93, \quad q^{2,1*}_{12,2} = 2,499.75, \quad q^{2,1*}_{21,2} = 714.07, \quad q^{2,1*}_{22,2} = 7,500.25,$$

$$q^{2,2*}_{11,2} = 1,428.78, \quad q^{2,2*}_{12,2} = 5,714.07, \quad q^{2,2*}_{21,2} = 571.21, \quad q^{2,2*}_{22,2} = 14,285.92,$$

$$\alpha^{1*}_{1,1} = 41,856.28, \quad \alpha^{1*}_{2,1} = 27,570.57, \quad \alpha^{1*}_{1,2} = 984,284.85, \quad \alpha^{1*}_{2,2} = 961,427.71,$$

$$\gamma^{2,1*}_{1,1} = 49,997.79, \quad \gamma^{2,1*}_{2,1} = 0.00, \quad \gamma^{2,2*}_{1,1} = 40,856.28, \quad \gamma^{2,2*}_{2,1} = 0.00,$$

$$\gamma^{2,1*}_{1,2} = 942,140.65, \quad \gamma^{2,1*}_{2,2} = 934,998.49, \quad \gamma^{2,2*}_{1,2} = 974,284.85, \quad \gamma^{2,2*}_{2,2} = 927,856.28.$$

Country 2 has a much higher supply of the ventilators than Country 1 even before the pandemic declaration, and this helps them a lot in doing a much better than Country 1 in both scenarios. On the other hand, Country 1, which is not able to increase the supply of these relatively complex and vital devices, is faced with a severe shortage. This important and significant advantage of Country 2 in accessing the supply of a life-saving device leads to a much lower expected disutility for this country than that for Country 1, $E(DU_1(q^*) = 5,640,364,039.56$, $E(DU_2(q^*)) = 2,529,423,007.30$.

Numerical Example 4.3: Two Countries, Two Types of Medical Supplies, Two Scenarios, Conservative Strategies

In Numerical Example 4.3, the key issue of restrictions on the export of vital medical supplies in times of a pandemic is addressed. It has been seen that, in some cases, the import of essential medical items from foreign countries has become very difficult or expensive for various reasons, such as the severe disruptions in international transportation and/or the enactment of laws by governments. In this example, the effects of such restriction on countries' strategies is examined by increasing the international transportation rates as compared to the previous example. The data for this example are:

$$c^{2,1}_{21,1}(q^{2,1}) = 25(q^{2,1}_{12,1})^2 + 10q^{2,1}_{12,1}, \quad c^{2,1}_{12,1}(q^{2,1}) = 25(q^{2,1}_{21,1})^2 + 10q^{2,1}_{21,1},$$

$$c^{2,2}_{21,1}(q^{2,2}) = 25(q^{2,2}_{12,1})^2 + 10q^{2,2}_{12,1}, \quad c^{2,2}_{12,1}(q^{2,2}) = 25(q^{2,2}_{21,1})^2 + 10q^{2,2}_{21,1},$$

$$c^{2,1}_{21,2}(q^{2,1}) = 25(q^{2,1}_{12,2})^2 + 10q^{2,1}_{12,2}, \quad c^{2,1}_{12,2}(q^{2,1}) = 25(q^{2,1}_{21,2})^2 + 10q^{2,1}_{21,2},$$

$$c^{2,2}_{21,2}(q^{2,2}) = 25(q^{2,2}_{12,2})^2 + 10q^{2,2}_{12,2}, \quad c^{2,2}_{12,2}(q^{2,2}) = 25(q^{2,2}_{21,2})^2 + 10q^{2,2}_{21,2}.$$

The computed equilibrium solution for this example is:

$$q^{1*}_{11,1} = 14,285.92, \quad q^{1*}_{12,1} = 7,142.64, \quad q^{1*}_{21,1} = 5,714.07, \quad q^{1*}_{22,1} = 17,857.35,$$

$$q^{2,1*}_{11,1} = 5,833.50, \quad q^{2,1*}_{12,1} = 1,949.80, \quad q^{2,1*}_{21,1} = 1,166.50, \quad q^{2,1*}_{22,1} = 16,250.00,$$

$$q^{2,2*}_{11,1} = 18,518.70, \quad q^{2,2*}_{12,1} = 1,959.80, \quad q^{2,2*}_{21,1} = 1,481.298, \quad q^{2,2*}_{22,1} = 24,500.00,$$

$$q^{1*}_{11,2} = 1,428.78, \quad q^{1*}_{12,2} = 2,856.92, \quad q^{1*}_{21,2} = 571.21, \quad q^{1*}_{22,2} = 7,143.07,$$

$$q^{2,1*}_{11,2} = 1,666.84, \quad q^{2,1*}_{12,2} = 1,071.25, \quad q^{2,1*}_{21,2} = 333.16, \quad q^{2,1*}_{22,2} = 8,928.75,$$

$$q_{11,2}^{2,2*} = 1,852.03, \quad q_{12,2}^{2,2*} = 1,481.29, \quad q_{21,2}^{2,2*} = 147.96, \quad q_{22,2}^{2,2*} = 18,518.70,$$

$$\alpha_{1,1}^{1*} = 41,856.28, \quad \alpha_{2,1}^{1*} = 27,570.57, \quad \alpha_{1,2}^{1*} = 984,284.85, \quad \alpha_{2,2}^{1*} = 961,427.71,$$

$$\gamma_{1,1}^{2,1*} = 36,664.99, \quad \gamma_{2,1}^{2,1*} = 0.00, \quad \gamma_{1,1}^{2,2*} = 23,925.18, \quad \gamma_{2,1}^{2,2*} = 0.00,$$

$$\gamma_{1,2}^{2,1*} = 938,331.50, \quad \gamma_{2,2}^{2,1*} = 926,427.49, \quad \gamma_{1,2}^{2,2*} = 972,591.85, \quad \gamma_{2,2}^{2,2*} = 910,925.18.$$

Comparing the results with the previous example's results, we see that the difficulties in imports have increased the relative shortage of face masks in both countries, but still Country 2 is less affected than Country 1. However, the situation is quite different in the case of ventilators. Country 2, which dominates the supply of these vital devices, has benefited greatly from the disruption in exchanges between the two countries. Most of the country's supply is allocated to its own demand and the country's competitor has received a very small share. On the other hand, Country 1, which has access to very limited resources of these devices domestically, is faced with severe shortages and cannot find another source to meet its needs. The impact of the disruption on the trade exchanges between the two countries is also easily visible in their expected disutility values. Country 2, a country that is a leader in the supply of a strategic medical supply, has drastically reduced its expected disutility to 1,577,985,084.63, while Country 1 is faced with a serious crisis and its expected disutility increases to 6,809,140,792.95.

4.3.2 Additional Discussion of the Results

As stated before and, based on what have been seen in past research, it is important to pay attention to both disaster preparedness and to response when we are planning for a pandemic. This is something that was observed in the solution of the model in the above examples. Since governments cannot easily predict what level of disaster they will face, they must be prepared for different scenarios. In the examples, it was shown that, in the event of a low-severity pandemic scenario, countries may not face serious shortages and prices at home and abroad will not change much. However, in the event of a severe disaster in countries, harsh global supply shortages and rising prices significantly affect countries with low domestic production capacity. This finding is consistent with what we have seen in reality in the efforts of various governments to address the COVID-19 pandemic. Countries that were able to increase their domestic production earlier faced fewer problems than countries that were dependent on imports of essential commodities and had to pay huge prices.

Maintaining readiness for a sudden increase in supply is one of the most important strategies that managers and policy-makers should pay special attention to. Firstly, in many cases, it is not possible to predict the occurrence of a disaster in the long run, and, secondly, it is not easy to store and maintain many goods in large volumes; hence, as revealed in the examples, the ability to increase supply in the short term is the most important strength of successful countries in this competition. On the other hand, unlike many previous studies that considered only one product, this model can capture competition for several types of products. It was also shown in the examples that a government must maintain its readiness to supply all kinds of medical supplies. Some products, such as face masks, are less expensive and needed in large numbers, whereas others, such as ventilators, are needed in smaller numbers but are more expensive and they have different effects on controlling the disaster. Decision-makers should consider and analyze the effects of different types of medical products on the amount of damage that might be incurred by the society and take the necessary actions to procure these products before and after the occurrence of a possible disaster, such as the pandemic.

4.4. Summary and Conclusions

Taking into account the specific features of the current pandemic, this chapter examines countries' competition for the purchase of medical supplies under limited availability in a stochastic Generalized Nash Equilibrium model. Specific features of the model include: the uncertainty of the scenarios, the supply capacities of the medical items, and the fluctuating prices before and after the pandemic declaration, as well as disruptions to the global supply chains. The model is formulated as a variational inequality problem applying the concept of a Variational Equilibrium. Also, I utilize an alternative variational inequality formulation with Lagrange multipliers associated with the medical item supply capacities in each country. The model is studied both qualitatively and quantitatively. In this regard, illustrative examples and numerical examples are provided, with the former solved analytically, and the latter, algorithmically.

This study is the first to address not only the stages of preparedness and response, along with the uncertainty, but also the critical issue of resource competition among countries, which was not investigated in previous studies on epidemic crises. Also, most previous research has considered the occurrence of an epidemic to be limited to a single region or, ultimately, to one country, and, as a result, the issue of global supply shortages in a pandemic has not been addressed through game theory and variational inequality theory before.

The results reveal that countries that have more flexibility and resilience in increasing their domestic supply after the pandemic declaration are better at dealing with the pandemic and meeting the need for medical supplies. In times of disaster, uncertainty in many cases, including the supply chain status, plays a key role in a country's success or failure in disaster management. Hence, countries must be ready to supply strategic medical supplies domestically. This study adds to the literature on game theory and two-stage stochastic models in disaster management with the focus on specific features of the COVID-19 pandemic.

CHAPTER 5

MODELING OF COVID-19 TRADE MEASURES ON ESSENTIAL PRODUCTS: A MULTIPRODUCT, MULTICOUNTRY SPATIAL PRICE EQUILIBRIUM FRAMEWORK

The previous two chapters examined the competition among decision-makers, including governments, for vital medical supplies in response to the pandemic. Various aspects of this complicated issue were addressed, including uncertainty in several parameters, planning and decision-making at various stages, and their impacts on the outcome of disaster management operations. Due to the intense competition for essential commodities at the international level in the recent global health disaster, one of the most important tools that governments utilized to ensure the supply of the products for their nations was trade policy measures, such as tariffs, quotas, and price floors and ceilings.

In this chapter, I develop a unified variational inequality framework in the context of spatial price network equilibrium problems that handles multiple products with multiple demand and supply markets in multiple countries as well as multiple transportation routes. The model incorporates a plethora of distinct trade measures, which is particularly important in the pandemic, since PPEs and other essential products are in high demand, but short in supply globally. Qualitative properties are also analyzed. Numerical examples are provided to illustrate the impacts of the trade measures on equilibrium product path and link flows, and on prices, and demand and supply quantities.

This chapter is organized as follows. The multiproduct, multicountry spatial price equilibrium model is constructed in Section 5.1. The governing equilibrium conditions are stated and the variational inequality formulation presented therein. In Section 5.2, the closed form expressions of the algorithm and qualitative properties are presented. In Section 5.3, the modified projection method is applied to compute the equilibrium product flow, the price and the Lagrange multiplier patterns in a series of numerical examples that demonstrate the generality and applicability of this model. The final section, Section 5.4, summarizes the results and presents the conclusions.

5.1. The Multiproduct, Multicountry Spatial Price Equilibrium Model with Trade Measures

The multiproduct, multicountry spatial price equilibrium model with trade measures for essential products is constructed in this section. Each country has multiple possible supply markets as well as multiple possible demand markets. An essential product is denoted by h, with $h = 1, \ldots, H$. As noted in Chapter 1, an essential product can be a medical product, food item, etc. There are n countries in the network economy, with country i; $i = 1, \ldots, n$, having k_i supply markets where the essential products can be produced and l_j; $j = 1, \ldots, n$ demand markets where these product supplies can be delivered to. The total number of supply markets is $n_k = \sum_{i=1}^{n} k_i$ and the total number of demand markets is $n_l = \sum_{j=1}^{n} l_j$. Underlying the supply markets and demand markets is a network consisting of a graph $G = [N, L]$, where N is the set of nodes (supply market origin nodes, demand market destination nodes, intermediate nodes for transshipment, etc.) and L is the set of directed links. An illustrative network figure is given in Figure 5.1. The network topology in Figure 5.1 can be adapted according to the specific application.

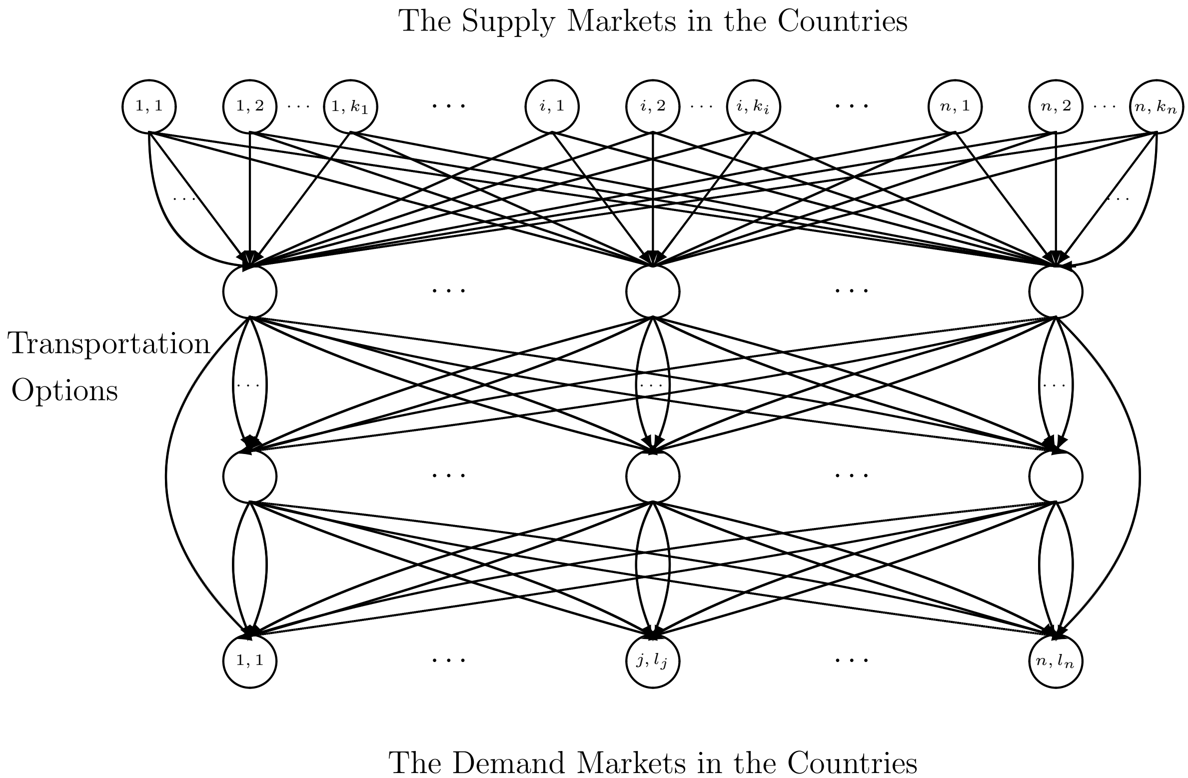

Figure 5.1. An Example of a Multicountry, Multiple Supply Market and Multiple Demand Market Spatial Price Network

The set of supply markets of country i is denoted by O_i and the set of demand markets of country j is denoted by D_j. Here, $P^{(i,k)}_{(j,l)}$ denotes the set of paths joining a country supply market origin node (i,k) with a country demand market node (j,l). $P^{(i,k)}$ denotes the set of paths, such that each path originates at origin node (i,k) and terminates at one of the demand market nodes. $P_{(j,l)}$, in turn, denotes the set of paths, such that each path terminates at the destination node (j,l) and originates at one of the supply market nodes. The set P then denotes the set of all paths joining the country supply market origin nodes with the country demand market destination nodes. There are n_P paths in the network and n_L links.

The basic notation for the model is given in Table 5.1. All vectors are column vectors. The product link unit transportation cost functions, supply functions, and demand functions are all assumed to be continuous.

Table 5.1. Notation for the Multiproduct, Multicountry Spatial Price Equilibrium Model with Trade Measures

Notation	**Definition**
x_p^h	the flow of essential product h on path p. The path flows are grouped into the vector $x \in R_+^{Hn_P}$.
f_a^h	the flow of essential product h on link a. The product link flows are grouped into the vector $f \in R_+^{Hn_L}$.
$\pi_{(i,k)}^h$	the supply price of product h at supply market k in country i. All the supply market prices are grouped into the vector $\pi \in R^{Hn_k}$.
$\rho_{(j,l)}^h$	the demand price of product h at demand market l in country j. All the demand market prices are grouped into the vector $\rho \in R^{Hn_l}$.
$c_a^h(f)$	the unit cost of transportation of product h on link a.
$C_p^h(x)$	the unit cost of transportation of product h on path p. All the path costs are grouped into the vector $C \in R^{Hn_P}$.
$s_{(i,k)}^h(\pi)$	the supply of product h at supply market k of country i. All the supply functions are grouped into the vector $s(\pi) \in R^{Hn_k}$.
$d_{(j,l)}^h(\rho)$	the demand for product h at demand market l of country j. All the demand functions are grouped into the vector $d(\rho) \in R^{Hn_l}$.
τ_{ij}^h	the unit tariff imposed by country j on product h from country i.
$\underline{\pi}_{(i,k)}^h$	the supply price lower bound on product h imposed on supply market k of country i by country i.
$\bar{\pi}_{(i,k)}^h$	the supply price upper bound on product h imposed on supply market k of country i by country i.
$\bar{\rho}_{(j,l)}^h$	the demand price upper bound on product h imposed on demand market l of country j by country j.
G_g^h	the group consisting of all the countries i and j comprising the group and the supply markets of i and demand markets of j for which a strict quota of $\bar{Q}_g^h$ on product h is imposed. There is a total of n_G quotas imposed, with n_{G^h} denoting the number of quotas imposed on product h.

First the conservation of flow equations are presented. Then the relationship between essential product path costs and costs on the links is identified, and an amplified discussion of the trade measures, followed by the governing equilibrium conditions are provided. The product unit transportation costs, supply and demand market prices, plus the price floors and ceilings, and the unit tariffs are all in a common currency.

The essential product path flows must be nonnegative, that is:

$$x_p^h \geq 0, \quad \forall p \in P, \forall h. \tag{5.1}$$

The product link flows, in turn, are related to the product path flows thus:

$$f_a^h = \sum_{p \in P} x_p^h \delta_{ap}, \quad \forall a \in L, \forall h. \tag{5.2}$$

According to (5.2), the product flow on a link is equal to the sum of flows of that product on paths that contain that link.

In addition, the unit transportation cost on a path associated with the transportation of an essential product is equal to the sum of the unit transportation costs on the links that make up the path; in other words:

$$C_p^h(x) = \sum_{a \in L} c_a^h(f) \delta_{ap}, \quad \forall p \in P, \tag{5.3}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise.

The goal is to construct an integrated model in which the impacts of a multiplicity of different trade measures that have been imposed on such essential products as medical supplies and food in the COVID-19 pandemic can be assessed. Specifically, here the following trade measures are considered. First, it is allowed for a unit tariff τ_{ij}^h imposed by a country j on country i's essential product h. If there is no such tariff τ_{ij}^h

between a pair of countries i and j and product h then the associated value can be set to 0. Also, a country i may wish to support producers of a specific essential product h through a price support in the form of a lower bound on the supply market prices in the form $\underline{\pi}^h_{(i,k)}$, for supply market k in its country. On the other hand, a country j may wish to impose an upper bound on the demand market prices of an essential product h in its country of $\bar{\rho}^h_{(j,l)}$, for its demand market l. It is also allowed for an upper bound on the supply market price of each product h at each supply market k in each country i of $\bar{\pi}^h_{(i,k)}$. Of course, when such trade measures are not instituted, then the lower supply market price bound can be just set to 0 and the demand market and supply market price ceilings can be set to a high number. In addition, and, as noted in Table 5.1, a country may wish to impose a quota and it is important to emphasize that this can be either an export quota of an essential product or an import quota of such a product. Such trade measures are handled through the use of groups G^h_g, for relevant gs and hs, and associated quotas $\bar{Q}^h_g$ (please refer to Table 5.1). Associated with each quota there is a corresponding nonnegative Lagrange multiplier λ^h_g. These Lagrange multipliers are grouped into the vector $\lambda \in R^{n_G}_+$. Also, $P_{G^h_g}$ is defined as the set of paths corresponding to group G^h_g.

The feasible set is defined as

$$K \equiv \{(x, \lambda, \pi, \rho) \in R_+^{H(n_P+n_G+n_k+n_l)}, \text{ and } \underline{\pi}^h_{(i,k)} \leq \pi^h_{(i,k)} \leq \bar{\pi}^h_{(i,k)}, \forall h, i, k,$$
$$\text{and } 0 \leq \rho^h_{(j,l)} \leq \bar{\rho}^h_{(j,l)}, \forall h, j, l\}.$$

Definition 5.1: Multiproduct, Multicountry Spatial Price Equilibrium Conditions Under Trade Measures

A multiproduct, multicountry product flow, quota Lagrange multiplier, supply price, and demand price pattern $(x^*, \lambda^*, \pi^*, \rho^*) \in K$ *is an essential product spatial price* *equilibrium under trade measures of tariffs, quotas, supply price floors and ceilings, and demand price ceilings, if the following conditions hold: For all essential products* h *and for all groups* G^h_g *for all* g, h, *and for all pairs of supply and demand markets in the countries:* $(i,j),(k,l) \in G^h_g$, *and all paths* $p \in P^{(i,k)}_{(j,l)}$, *for all* i,j,k,l*:*

$$\pi^{h*}_{(i,k)} + C^h_p(x^*) + \tau^h_{ij} + \lambda^*_{G^h_g} \begin{cases} = \rho^{h*}_{(j,l)}, & \text{if} \quad x^{h*}_p > 0, \\ \geq \rho^{h*}_{(j,l)}, & \text{if} \quad x^{h*}_p = 0, \end{cases} \tag{5.4}$$

$$\lambda^*_{G^h_g} \begin{cases} \geq 0, & \text{if} \quad \sum_{p \in P_{G^h_g}} x^{h*}_p = \bar{Q}_{G^h_g}, \\ = 0, & \text{if} \quad \sum_{p \in P_{G^h_g}} x^{h*}_p < \bar{Q}_{G^h_g}; \end{cases} \tag{5.5}$$

for all products h *and all supply markets* k *in the countries* i*:*

$$s^h_{(i,k)}(\pi^*) \begin{cases} \leq \sum_{p \in P^{(i,k)}} x^{h*}_p, & \text{if} \quad \pi^{h*}_{(i,k)} = \bar{\pi}^h_{(i,k)}, \\ = \sum_{p \in P^{(i,k)}} x^{h*}_p, & \text{if} \quad \underline{\pi}^h_{(i,k)} < \pi^{h*}_{(i,k)} < \bar{\pi}^h_{(i,k)}, \\ \geq \sum_{p \in P^{(i,k)}} x^{h*}_p, & \text{if} \quad \pi^{h*}_{(i,k)} = \underline{\pi}^h_{(i,k)}, \end{cases} \tag{5.6}$$

plus, for all products h *and all demand markets* l *in the countries* j*:*

$$d^h_{(j,l)}(\rho^*) \begin{cases} \geq \sum_{p \in P_{(j,l)}} x^{h*}_p, & \text{if} \quad \rho^{h*}_{(j,l)} = \bar{\rho}^h_{(j,l)}, \\ = \sum_{p \in P_{(j,l)}} x^{h*}_p, & \text{if} \quad 0 < \rho^{h*}_{(j,l)} < \bar{\rho}^h_{(j,l)}, \\ \leq \sum_{p \in P_{(j,l)}} x^{h*}_p, & \text{if} \quad \rho^{h*}_{(j,l)} = 0. \end{cases} \tag{5.7}$$

For paths not belonging to any group associated with a quota, we have that (5.4) holds with the quota Lagrange multiplier removed and (5.5) also excised.

In the case of the complete prohibition of exports of an essential product h from a country i, as in the case of, for example, certain medical products such as PPEs, ventilators, etc., one could excise the associated paths from the country to other countries for such products. Alternatively, one could define the appropriate group and set the quota for the group equal to 0. On the other hand, and, interestingly, there have also been import bans instituted by several countries due to COVID-19 to reduce the spread of the disease. For example, in that case, if a country j institutes an import ban on a product in its country then all paths from other countries for that product to demand markets in country j would not be used; in effect, they would be eliminated from the network topology for that product. One could also set the quota equal to 0 for the defined group.

According to equilibrium conditions (5.4), there will be a positive flow of an essential product on a path, in equilibrium, if the supply price at the supply market in a country at which the path originates at plus the unit path cost of transporting the essential product on the path plus the tariff levied plus the quota Lagrange multiplier associated with the group on which the quota is imposed (and the path connects a pair of origin and destination nodes in it) is equal to the price the consumers are willing to pay at the demand market in the destination country. If, on the other hand, the above supply price plus the unit path transportation cost plus tariff plus quota Lagrange multiplier exceeds the price the consumers are willing to pay, then the product flow on that path will be equal to zero. Conditions (5.5), in turn, reveal that, in equilibrium, if the quota is reached by the path flows associated with the group and product that the quota is imposed on, then the equilibrium Lagrange multiplier for that group and product is positive; otherwise, the Lagrange multiplier is equal to zero.

Equilibrium conditions (5.6) state that if the equilibrium supply price for an essential product at a supply market in a country is greater than the imposed supply price lower bound, and less than the imposed upper bound, then the supply of the product at the supply market in the country is equal to the product flows out. If the supply price is equal to the imposed supply market lower bound in the country, then there could be excess supply. On the other hand, if the supply price for a product is equal to the imposed supply price upper bound, then the supply at equilibrium could be lower than the shipments out. Equilibrium conditions (5.7) state that, if the demand price at a demand market in a country for an essential product is equal to the imposed demand market price upper bound, then the demand can exceed the product inflows to that demand market in that country of the medial product. If the equilibrium demand market price is greater than zero and less than the imposed demand price upper bound for the product at the demand market in the country, then the demand for the product at the demand market in the country is equal to the product inflows of that essential product. Finally, if the equilibrium demand price is equal to zero for the product at a demand market in a country, then the demand can be lower.

The above spatial price equilibrium conditions provide a unified framework for a multiplicity of trade measures for essential product supplies in the COVID-19 pandemic. They are extensions of classical spatial price equilibrium conditions due to Samuelson (1952) and Takayama and Judge (1971); see Nagurney (1999, 2006) for many references to spatial price equilibrium models and the more recent work of Nagurney, Besik, and Dong (2019). For example, the above model includes multiple products (of great importance in the case of medical supplies), and concomitant handling of multiproduct tariffs and group quotas, along with the inclusion of supply price floors and ceilings plus demand price ceilings, all on a general network. Furthermore, it is identified how to handle complete prohibition of essential product exports or even imports, including those of food products.

The above spatial price equilibrium framework enables decision-makers and policy-makers to evaluate the impacts of different trade measures on product prices and product flows. The impacts of the tightening of trade restrictions can be ascertained, once the model is solved, as well as the impacts of trade liberalizations, such as the reduction in tariffs and the lifting of import or export bans.

Now the variational inequality formulation of the above equilibrium conditions is provided in a Theorem.

Theorem 5.1: Variational Inequality Formulation of the Multiproduct, Multicountry Essential Product Spatial Equilibrium Conditions Under Trade Measures

A multiproduct, multicountry product flow, quota Lagrange multiplier, supply price, and demand price pattern $(x^*, \lambda^*, \pi^*, \rho^*) \in K$ is an essential product spatial price equilibrium under trade measures of tariffs, quotas, supply price floors and ceilings, and demand price ceilings, according to Definition 5.1, if and only if it satisfies the variational inequality problem:

$$\sum_{h=1}^{H}\sum_{g=1}^{n_{G^h}}\sum_{(i,j)\in G^h_g}\sum_{k\in O_i}\sum_{l\in D_j}\sum_{p\in P^{(i,k)}_{(j,l)}}\left[\pi^{h*}_{(i,k)}+C^h_p(x^*)+\tau^h_{ij}+\lambda^*_{G^h_g}-\rho^{h*}_{(j,l)}\right]\times\left[x^h_p-x^{h*}_p\right]$$

$$+\sum_{h=1}^{H}\sum_{g=1}^{n_{G^h}}\sum_{(i,j)\notin G^h_g}\sum_{k\in O_i}\sum_{l\in D_j}\sum_{p\in P^{(i,k)}_{(j,l)}}\left[\pi^{h*}_{(i,k)}+C^h_p(x^*)+\tau^h_{ij}-\rho^{h*}_{(j,l)}\right]\times\left[x^h_p-x^{h*}_p\right]$$

$$+\sum_{h=1}^{H}\sum_{g=1}^{n_{G^h}}\left[\bar{Q}_{G^h_g}-\sum_{p\in P_{G^h_g}}x^{h*}_p\right]\times\left[\lambda_{G^h_g}-\lambda^*_{G^h_g}\right]$$

$$+\sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{k=1}^{k_i}\left[s^h_{(i,k)}(\pi^*)-\sum_{p\in P^{(i,k)}}x^{h*}_p\right]\times\left[\pi^h_{(i,k)}-\pi^{h*}_{(i,k)}\right]$$

$$+\sum_{h=1}^{H}\sum_{j=1}^{n}\sum_{l=1}^{l_j}\left[\sum_{p\in P_{(j,l)}} x_p^{h*} - d_{(j,l)}^h(\rho^*)\right] \times \left[\rho_{(j,l)}^h - \rho_{(j,l)}^{h*}\right] \geq 0, \quad \forall (x,\lambda,\pi,\rho) \in K. \quad (5.8)$$

Proof: First necessity is established, that is, it should be proved that if the vector $(x^*, \lambda^*, \pi^*, \rho^*) \in K$ satisfies the spatial price equilibrium conditions according to Definition 5.1, then it also satisfies variational inequality (5.8). Note that, for a fixed path $p \in P_{(j,l)}^{(i,k)}$ with $(i,j), (k,l) \in G_g^h$, essential product $h \in H$, and group G_g^h, (5.4) implies that:

$$\left[\pi_{(i,k)}^{h*} + C_p^h(x^*) + \tau_{ij}^h + \lambda_{G_g^h}^* - \rho_{(j,l)}^{h*}\right] \times \left[x_p - x_p^*\right] \geq 0, \quad \forall x_p \geq 0. \tag{5.9}$$

Indeed, according to (5.4), if $x_p^* > 0$, then $\left[\pi_{(i,k)}^{h*} + C_p^h(x^*) + \tau_{ij}^h + \lambda_{G_g^h}^* - \rho_{(j,l)}^{h*}\right] = 0$, so (5.9) holds. Also, if $x_p^* = 0$, then $\left[\pi_{(i,k)}^{h*} + C_p^h(x^*) + \tau_{ij}^h + \lambda_{G_g^h}^* - \rho_{(j,l)}^{h*}\right] \geq 0$, and, since, due to the nonnegativity assumption on the path flows, $\left[x_p - x_p^*\right] \geq 0$, the product of these two terms is also nonnegative and (5.9) holds. Inequality (5.9) is independent of path p, essential product h, and group G_g^h; therefore, summation over all essential products, groups, and paths, yields:

$$\sum_{h=1}^{H}\sum_{g=1}^{n_{G^h}}\sum_{(i,j)\in G_g^h}\sum_{k\in O_i}\sum_{l\in D_j}\sum_{p\in P_{(j,l)}^{(i,k)}} \left[\pi_{(i,k)}^{h*} + C_p^h(x^*) + \tau_{ij}^h + \lambda_{G_g^h}^* - \rho_{(j,l)}^{h*}\right] \times \left[x_p - x_p^*\right] \geq 0$$

$$\forall x \in K. \tag{5.10}$$

Using similar arguments, it can be concluded that

$$\sum_{h=1}^{H}\sum_{g=1}^{n_{G^h}}\sum_{(i,j)\notin G_g^h}\sum_{k\in O_i}\sum_{l\in D_j}\sum_{p\in P_{(j,l)}^{(i,k)}} \left[\pi_{(i,k)}^{h*} + C_p^h(x^*) + \tau_{ij}^h - \rho_{(j,l)}^{h*}\right] \times \left[x_p - x_p^*\right] \geq 0, \quad \forall x \in K. \tag{5.11}$$

Now, (5.5) implies that, for fixed g and h, if $\lambda^*_{G^h_g}$ satisfies (5.5), then:

$$\left[\bar{Q}_{G^h_g} - \sum_{p \in P_{G^h_g}} x^{h*}_p\right] \times \left[\lambda_{G^h_g} - \lambda^*_{G^h_g}\right] \geq 0, \quad \forall \lambda_{G^h_g} \geq 0. \tag{5.12}$$

Summation of (5.12) over all h and g yields:

$$\sum_{h=1}^{H} \sum_{g=1}^{n_{G^h}} \left[\bar{Q}_{G^h_g} - \sum_{p \in P_{G^h_g}} x^{h*}_p\right] \times \left[\lambda_{G^h_g} - \lambda^*_{G^h_g}\right] \geq 0, \quad \forall \lambda \in K. \tag{5.13}$$

Also, (5.6) implies that, if $\pi^{h*}_{(i,k)}$ satisfies (5.6), then:

$$\left[s^h_{(i,k)}(\pi^*) - \sum_{p \in P^{(i,k)}} x^{h*}_p\right] \times \left[\pi^h_{(i,k)} - \pi^{h*}_{(i,k)}\right] \geq 0, \quad \forall \underline{\pi}^h_{(i,k)} \leq \pi^h_{(i,k)} \leq \bar{\pi}^h_{(i,k)}. \tag{5.14}$$

But inequality (5.14) holds for any h, i, and k, so summation over all h, i, and k yields:

$$\sum_{h=1}^{H} \sum_{i=1}^{n} \sum_{k=1}^{k_i} \left[s^h_{(i,k)}(\pi^*) - \sum_{p \in P^{(i,k)}} x^{h*}_p\right] \times \left[\pi^h_{(i,k)} - \pi^{h*}_{(i,k)}\right] \geq 0, \quad \forall \pi \in K. \tag{5.15}$$

Analogously, (5.7) implies that if $\rho^{h*}_{(j,l)}$ satisfies (5.7), then:

$$\left[\sum_{p \in P_{(j,l)}} x^{h*}_p - d^h_{(j,l)}(\rho^*)\right] \times \left[\rho^h_{(j,l)} - \rho^{h*}_{(j,l)}\right] \geq 0, \quad 0 \leq \rho^h_{(j,l)} \leq \bar{\rho}^h_{(j,l)}. \tag{5.16}$$

Since inequality (5.16) holds for any h, j, and l, it can be concluded that summation over all h, j, and l yields:

$$\sum_{h=1}^{H} \sum_{j=1}^{n} \sum_{l=1}^{l_j} \left[\sum_{p \in P_{(j,l)}} x^{h*}_p - d^h_{(j,l)}(\rho^*)\right] \times \left[\rho^h_{(j,l)} - \rho^{h*}_{(j,l)}\right] \geq 0, \quad \forall \rho \in K. \tag{5.17}$$

Now, summing up (5.10), (5.11), (5.13), (5.15), and (5.17) gives us variational inequality (5.8) and necessity has been established.

Now sufficiency should be established, that is, if $(x^*, \lambda^*, \pi^*, \rho^*) \in K$ satisfies variational inequality (5.8) then it also satisfies the spatial price equilibrium conditions (5.4) - (5.7).

Let $\pi^h_{(i,k)} = \pi^{h*}_{(i,k)}, \forall(h,i,k)$, $\rho^h_{(j,l)} = \rho^{h*}_{(j,l)}, \forall(h,j,l)$, $\lambda_{G^h_g} = \lambda^*_{G^h_g}, \forall(h,g)$, and $x_p = x^*_p, \forall p \neq q$. Substitution of these values into (5.8), yields:

$$\left[\pi^{h*}_{(i,k)} + C^h_q(x^*) + \tau^h_{ij} + \lambda^*_{G^h_g} - \rho^{h*}_{(j,l)}\right] \times \left[x_q - x^*_q\right] \geq 0, \quad \forall x_q \geq 0. \tag{5.18}$$

Inequality (5.18) implies that, if $x^*_q = 0$, then $\left[\pi^{h*}_{(i,k)} + C^h_q(x^*) + \tau^h_{ij} + \lambda^*_{G^h_g} - \rho^{h*}_{(j,l)}\right] \geq 0$, and if $x^*_q > 0$, then $\left[\pi^{h*}_{(i,k)} + C^h_q(x^*) + \tau^h_{ij} + \lambda^*_{G^h_g} - \rho^{h*}_{(j,l)}\right] = 0$. Since this condition holds for any q in a group, we conclude that (5.18) implies that equilibrium conditions (5.4) must hold for any path in a group.

Similarly, let $\pi^h_{(i,k)} = \pi^{h*}_{(i,k)}, \forall(h,i,k)$, $\rho^h_{(j,l)} = \rho^{h*}_{(j,l)}, \forall(h,j,l)$, $x_p = x^*_p, \forall p \neq q$, and $(i,j),(k,l) \notin \cup_g G^h_g$. Substituting these values into (5.8), we obtain:

$$\left[\pi^{h*}_{(i,k)} + C^h_q(x^*) + \tau^h_{ij} - \rho^{h*}_{(j,l)}\right] \times \left[x_q - x^*_q\right] \geq 0, \quad \forall x_q \geq 0. \tag{5.19}$$

And (5.19) implies that equilibrium conditions (5.4) must hold for any path q not in a group.

By setting now $\pi^h_{(i,k)} = \pi^{h*}_{(i,k)}, \forall(h,i,k)$, $\rho^h_{(j,l)} = \rho^{h*}_{(j,l)}, \forall(h,j,l)$, $x_p = x^*_p, \forall p$, and $\lambda_{G^h_g} = \lambda^*_{G^h_g}, \forall(h,g) \neq (s,t)$ in variational inequality (8), we get:

$$\left[\bar{Q}_{G^t_s} - \sum_{p \in P_{G^t_s}} x^{t*}_p\right] \times \left[\lambda_{G^t_s} - \lambda^*_{G^t_s}\right] \geq 0, \quad \forall \lambda_{G^t_s} \geq 0. \tag{5.20}$$

(5.20), in turn, implies that equilibrium conditions (5.5) must hold for any s and t.

Analogously, now let $\rho^h_{(j,l)} = \rho^{h*}_{(j,l)}, \forall (h,j,l)$, $x_p = x^*_p, \forall p$, $\lambda_{G^h_g} = \lambda^*_{G^h_g}, \forall (h,g)$, and $\pi^h_{(i,k)} = \pi^{h*}_{(i,k)}, \forall (h,i,k) \neq (r,u,w)$. Substitution into (5.8) yields:

$$\left[s^w_{ru}(\pi^*) - \sum_{p \in P^{(r,u)}} x^{w*}_p \right] \times [\pi^w_{ru} - \pi^{w*}_{ru}] \geq 0, \quad \forall \underline{\pi}^w_{ru} \leq \pi^w_{ru} \leq \bar{\pi}^w_{ru}. \tag{5.21}$$

(5.21) implies that equilibrium conditions (5.6) must hold for any r, u, and w.

Finally, let's set $x_p = x^*_p, \forall p$, $\lambda_{G^h_g} = \lambda^*_{G^h_g}, \forall (h,g)$, $\pi^h_{(i,k)} = \pi^{h*}_{(i,k)}, \forall (h,i,k)$, and $\rho^h_{(j,l)} = \rho^{h*}_{(j,l)}, \forall (h,j,l) \neq (e,o,v)$. Substitution of these values into variational inequality (5.8) yields:

$$\left[\sum_{p \in P_{(e,o)}} x^{v*}_p - d^v_{eo}(\rho^*) \right] \times [\rho^v_{eo} - \rho^{v*}_{eo}] \geq 0, \quad 0 \leq \rho^v_{eo} \leq \bar{\rho}^v_{eo}. \tag{5.22}$$

And (5.22) implies that equilibrium conditions (5.7) must hold for any e, o, and v. □

Now, using arguments as in Chapter 2, I put variational inequality (5.8) into the standard form (2.15). $\mathcal{N} = H(n_P + n_G + n_k + n_l)$ for this model. Let $X \equiv (x, \lambda, \pi, \rho)$ and $\mathcal{K} \equiv K$. Also, I define $F(X) \equiv (F^1(X), F^2(X), F^3(X), F^4(X), F^5(X))$, where the components of $F^1(X)$ correspond to the n_P elements with a typical p element as preceding the first multiplication sign in (5.8); the components of $F^2(X)$ correspond to the n_P elements with a typical such element as immediately preceding the second multiplication sign; the components of $F^3(X)$ correspond to the n_G elements with a typical gh element as preceding the third multiplication sign, and so on.

5.1.1 Illustrative Examples

In order to fix ideas, now several illustrative examples are provided, the solutions to which can be determined analytically.

Baseline Illustrative Example

First a single country with a single supply market and a single demand market is considered, with the network topology depicted in Figure 5.2.

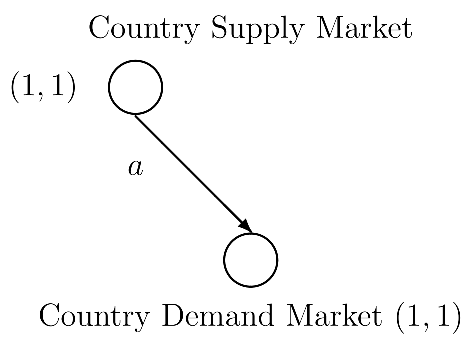

Figure 5.2. Network Topology for the Baseline Illustrative Example

For simplicity, there is a single path consisting of a single link joining supply market node $(1, 1)$ with demand market node $(1, 1)$. There is a single essential product and, hence, for simplicity sake, the superscript $h = 1$ is suppressed in the functional notation. The supply function is given by:

$$s_{(1,1)}(\pi) = 5\pi_{(1,1)} + 5,$$

and the demand function is given by

$$d_{(1,1)}(\rho) = -\rho_{(1,1)} + 22.$$

The path joining the supply market node with the demand market node is denoted by p_1 and it consists of link a, with a unit cost of $c_a = f_a + 1$ and, therefore, the path cost $C_{p_1} = x_{p_1} + 1$.

Making use of the spatial price equilibrium conditions in Definition 5.1, and noting that this example has no imposed trade measures, it is clear that the spatial price equilibrium product path flow and supply price and demand price pattern is:

$$x^*_{p_1} = 10, \quad \pi^*_{(1,1)} = 1, \quad \rho^*_{(1,1)} = 12,$$

since $C_{p_1}(x^*) = 11$ and, hence, we have that: $\pi^*_{(1,1)} + C_{p_1}(x^*) = \rho^*_{(1,1)}$. Furthermore, $s_{(1,1)}(\pi^*) = x^*_{p_1} = d_{(1,1)}(\rho^*)$.

Baseline Illustrative Example with Supply Price Floor Added

Now a supply price floor of $\underline{\pi}_{(1,1)} = 2$ is imposed in the above example. A government may do that to support, for example, farmers in terms of the minimum price for their product. Again, referring to the spatial price equilibrium conditions in Definition 5.1, it is straightforward to determine the spatial equilibrium pattern. Specifically, it yields:

$$x^*_{p_1} = 9.5, \quad \pi^*_{(1,1)} = 2, \quad \rho^*_{(1,1)} = 12.5.$$

Furthermore, $C_{p_1}(x^*) = 10.5$, and, therefore, $\pi^*_{(1,1)} + C_{p_1}(x^*) = \rho^*_{(1,1)} = 12.5$. Also, $s_{(1,1)}(\pi^*) = 15 > x^*_{p_1} = 9.5$ and $d_{(1,1)}(\rho^*) = 9.5 = x^*_{p_1}$.

Baseline Illustrative Example with Supply Price Floor Plus Demand Price Ceiling Added

Now to the previous example, with the supply price floor, a demand price ceiling of $\bar{\rho}_{(1,1)} = 10$ is imposed. The new spatial price product flow, supply price and demand price equilibrium pattern is:

$$x^*_{p_1} = 7, \quad \pi^*_{(1,1)} = 2, \quad \rho^*_{(1,1)} = 10,$$

and also: $C_{p_1}(x^*) = 8$, so that $\pi^*_{(1,1)} + C_{p_1}(x^*) = \rho^*_{(1,1)}$; $s_1(\pi^*) = 15 > x^*_{p_1} = 7$ and $d_{(1,1)}(\rho^*) = 12 > x^*_{p_1} = 7$.

Baseline Illustrative Example with Supply Price Floor Plus Demand Price Ceiling Plus New Supply Market in Another Country

Now the impact of the addition of a new supply market in another country is considered. The network topology is as in Figure 5.3. The data are as in the immediately preceding example but with the following additions. The supply price function at the supply market in the second country is: $s_{(2,1)}(\pi) = \pi_{(1,2)} + 1$. The cost on link b and, hence, on path $p_2 = (b)$ is $c_b(f) = f_b + 1$ and $C_{p_2}(x) = x_{p_2} + 1$, respectively. Making use of the equilibrium conditions in Definition 5.1 yields the following equilibrium pattern:

$$x^*_{p_1} = 7, \quad x^*_{p_2} = 5, \quad \pi^*_{(1,1)} = 2, \quad \pi^*_{(2,1)} = 4, \quad \rho^*_{(1,1)} = 10.$$

Also: $s_{(1,1)}(\pi^*) = 15 > x^*_{p_1} = 7$; $s_{(2,1)}(\pi^*) = 5 = x^*_{p_2}$ and $d_{(1,1)}(\rho^*) = 12 = x^*_{p_1} + x^*_{p_2}$. Moreover: $\pi^*_{(1,1)} + C_{p_1}(x^*) = \rho^*_{(1,1)}$ and $\pi^*_{(2,1)} + C_{p_2}(x^*) = \rho^*_{(1,1)}$.

Baseline Illustrative Example with Supply Price Floor Plus Demand Price Ceiling Plus Supply Market in Another Country Plus Added Tariff

The final illustrative example has the same data as the immediately preceding example but now the impact of an addition of a tariff $\tau_{21} = 2$ is considered. The new equilibrium pattern is:

$$x^*_{p_1} = 7, \quad x^*_{p_2} = 4, \quad \pi^*_{(1,1)} = 2, \quad \pi^*_{(2,1)} = 3, \quad \rho^*_{(1,1)} = 10.$$

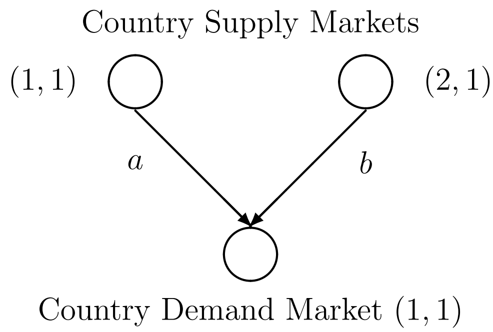

Figure 5.3. Network Topology for the Illustrative Example with Two Supply Markets, Each in a Different Country

In addition: $s_{(1,1)}(\pi^*) = 15 > x^*_{p_1} = 7$; $s_{(2,1)}(\pi^*) = 5 = x^*_{p_2}$ and $d_{(1,1)}(\rho^*) = 12 > x^*_{p_1} + x^*_{p_2} = 11$. Moreover: $\pi^*_{(1,1)} + C_{p_1}(x^*) = \rho^*_{(1,1)}$ and $\pi^*_{(2,1)} + C_{p_2}(x^*) + \tau_{12} = \rho^*_{(1,1)}$. In this example, as in the preceding ones, the spatial price equilibrium conditions according to Definition 5.1 hold precisely.

5.2. The Algorithm and Qualitative Properties

The algorithm that is utilized to solve the variational inequality (5.8), governing the multiproduct, multicountry spatial price equilibrium model with trade measures, is the modified projection method. I now provide the explicit formulae for the variables induced by the modified projection method for the model at a given iteration for Step 1. Similar explicit formulae can be determined accordingly for Step 2 (cf. (3.25) and (3.26) in Chapter 3).

Explicit Formulae for Step 1 for the Essential Product Flows on Paths with Quotas

For all h, g, for all $(i, j) \in G^h_g$, and for all (k, l), for each path $p \in P^{(i,k)}_{(j,l)}$, compute:

$$\hat{x}_p^{\tau} = \max\{0, x_p^{\tau-1} - \beta(\pi_{(i,k)}^{h(\tau-1)} + C_p^h(x^{\tau-1}) + \tau_{ij}^h + \lambda_{G_g^h}^{\tau-1} - \rho_{(j,l)}^{h(\tau-1)})\}; \tag{5.23}$$

Explicit Formulae for Step 1 for the Essential Product Flows on Paths without Quotas

For all h, g, for all $(i,j) \notin \cup_g G_g^h$, and for all (k,l), for each path $p \in P_{(j,l)}^{(i,k)}$, compute:

$$\hat{x}_p^{h\tau} = \max\{0, x_p^{h\tau-1} - \beta(\pi_{(i,k)}^{h(\tau-1)} + C_p^h(x^{\tau-1}) + \tau_{ij}^h - \rho_{(j,l)}^{h(\tau-1)})\}; \tag{5.24}$$

Explicit Formulae for Step 1 for the Quota Lagrange Multipliers

For all h and for all g, for each group G_g^h, compute:

$$\hat{\lambda}_{G_g^h}^{\tau} = \max\{0, \lambda_{G_g^h}^{\tau-1} - \beta(\bar{Q}_{G_g^h} - \sum_{p \in P_{G_g^h}} x_p^{h(\tau-1)})\}; \tag{5.25}$$

Explicit Formulae for Step 1 for the Supply Prices

For all h, for all i, and for all $\forall k$, for each h, i, k, compute:

$$\hat{\pi}_{(i,k)}^{h\tau} = \max\{\underline{\pi}_{(i,k)}^h, \min\{\pi_{(i,k)}^{h(\tau-1)} - \beta(s_{(i,k)}^h(\pi^{\tau-1}) - \sum_{p \in P_{G_g^h}} x_p^{h(\tau-1)}), \bar{\pi}_{(i,k)}^h\}\}; \tag{5.26}$$

Explicit Formulae for Step 1 for the Demand Prices

For all h, for all j, and for all l, for each h, j, l, compute:

$$\hat{\rho}_{(j,l)}^{h\tau} = \max\{0, \min\{\rho_{(j,l)}^{h(\tau-1)} - \beta(\sum_{p \in P_{G_g^h}} x_p^{h(\tau-1)} - d_{(j,l)}^h(\rho^{\tau-1})), \bar{\rho}_{(j,l)}^h\}\}. \tag{5.27}$$

5.2.1 Qualitative Properties

In this subsection, I discuss qualitative properties of the function $F(X)$ for this model required for convergence of the modified projection method. Also an existence result is provided.

In the following proposition it is shown that if the product supply functions, minus the product demand functions, and the product link unit transportation cost functions are monotone in their respective vectors of variables, then the function $F(X)$ is monotone in (x, λ, π, ρ). In the following proof an alternative variational inequality to the one (5.8), which is in link flows, is utilized. Let $K^{11} \equiv \{\pi | \pi \in R_+^{H(n_k)}; \underline{\pi}_{(i,k)}^h \leq \pi_{(i,k)}^h \leq \bar{\pi}_{(i,k)}^h, \forall h, i, k\}$; $K^{12} \equiv \{\rho | \rho \in R_+^{H(n_l)}; 0 \leq \rho_{(j,l)}^h \leq \bar{\rho}_{(j,l)}^h, \forall h, j, l\}$; $K^{13} \equiv \{(x, f) | (x, f), \{(5.2) \text{ holds}\}$. Hence, let's define the feasible set $K^1 \equiv \{(x, f, \lambda, \pi, \rho) | x \in R_+^{H(n_P)}, \lambda \in R_+^{H(n_G)}, \pi \in R_+^{H(n_k)}, \rho \in R_+^{H(n_l)}; \underline{\pi}_{(i,k)}^h \leq \pi_{(i,k)}^h \leq \bar{\pi}_{(i,k)}^h, \forall h, i, k; 0 \leq \rho_{(j,l)}^h \leq \bar{\rho}_{(j,l)}^h, \forall h, j, l, \text{ and } (5.2) \text{ holds}\} \equiv K^{11} \times K^{12} \times K^{13}$.

Proposition 5.1: Monotonicity of $F(X)$

Assume that the vector of product supply functions $s(\pi)$, minus the vector of product demand functions $d(\rho)$, and the vector of link product unit transportation cost functions $c(f)$ are all monotone as follows:

$$\langle s(\pi^1) - s(\pi^2), \pi^1 - \pi^2 \rangle \geq 0, \quad \forall \pi^1, \pi^2 \in K^{11}, \tag{5.28a}$$

$$-\langle d(\rho^1) - d(\rho^2), \rho^1 - \rho^2 \rangle \geq 0, \quad \forall \rho^1, \rho^2 \in K^{12}, \tag{5.28b}$$

$$\langle c(f^1) - c(f^2), f^1 - f^2 \rangle \geq 0, \quad \forall f^1, f^2 \in K^{13}. \tag{5.28c}$$

Then, the function that enters variational inequality (5.8), as in standard form $F(X)$, is monotone, with respect to the product flow vector x, the quota Lagrange multiplier vector λ, the vector of product supply prices π, and the vector of minus product demand prices ρ, that is, X.

Proof: Using (5.2) and (5.3), one can have the following:

$$\sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k\in O_i}\sum_{l\in D_j}\sum_{p\in P_{(j,l)}^{(i,k)}} C_p^h(x^*)\times\left[x_p^h-x_p^{h*}\right]=\sum_{h=1}^{H}\sum_{p\in P} C_p^h(x^*)\times\left[x_p^h-x_p^{h*}\right]$$

$$=\sum_{h=1}^{H}\sum_{p\in P}\left[\sum_{a\in L} c_a^h(f^*)\delta_{ap}\right]\times\left[x_p^h-x_p^{h*}\right]=\sum_{h=1}^{H}\sum_{a\in L} c_a^h(f^*)\times\left[\sum_{p\in P}\delta_{ap}x_p^h-\sum_{p\in P}\delta_{ap}x_p^{h*}\right]$$

$$=\sum_{h=1}^{H}\sum_{a\in L} c_a^h(f^*)\times\left[f_a^h-f_a^{h*}\right]. \tag{5.29}$$

Now, using (5.29), variational inequality (5.8) can be rewritten as follows:

$$\sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k\in O_i}\sum_{l\in D_j}\sum_{p\in P_{(j,l)}^{(i,k)}}\left[\pi_{(i,k)}^{h*}+\tau_{ij}^h-\rho_{(j,l)}^{h*}\right]\times\left[x_p^h-x_p^{h*}\right]$$

$$+\sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k\in O_i}\sum_{l\in D_j}\sum_{p\in P_{(j,l)}^{(i,k)}}\left[C_p^h(x^*)\right]\times\left[x_p^h-x_p^{h*}\right]$$

$$+\sum_{h=1}^{H}\sum_{g=1}^{n_{G^h}}\sum_{(i,j)\in G_g^h}\sum_{k\in O_i}\sum_{l\in D_j}\sum_{p\in P_{(j,l)}^{(i,k)}}\left[\lambda_{G_g^h}^*\right]\times\left[x_p^h-x_p^{h*}\right]$$

$$+\sum_{h=1}^{H}\sum_{g=1}^{n_{G^h}}\left[\bar{Q}_{G_g^h}-\sum_{p\in P_{G_g^h}} x_p^{h*}\right]\times\left[\lambda_{G_g^h}-\lambda_{G_g^h}^*\right]$$

$$+\sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{k=1}^{k_i}\left[s_{(i,k)}^h(\pi^*)-\sum_{p\in P^{(i,k)}} x_p^{h*}\right]\times\left[\pi_{(i,k)}^h-\pi_{(i,k)}^{h*}\right]$$

$$+\sum_{h=1}^{H}\sum_{j=1}^{n}\sum_{l=1}^{l_j}\left[\sum_{p\in P_{(j,l)}} x_p^{h*}-d_{(j,l)}^h(\rho^*)\right]\times\left[\rho_{(j,l)}^h-\rho_{(j,l)}^{h*}\right]$$

$$=\sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{k=1}^{k_i}\sum_{p\in P_{(j,l)}^{(i,k)}}\left[\pi_{(i,k)}^{h*}\right]\times\left[x_p^h-x_p^{h*}\right]+\sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{l_j}\sum_{p\in P_{(j,l)}^{(i,k)}}\left[\tau_{ij}^h-\rho_{(j,l)}^{h*}\right]\times\left[x_p^h-x_p^{h*}\right]$$

$$+\sum_{h=1}^{H}\sum_{a\in L} c_a^h(f^*)\times\left[f_a^h-f_a^{h*}\right]+\sum_{h=1}^{H}\sum_{g=1}^{n_{G^h}}\sum_{G_g^h}\sum_{p\in P_{G_g^h}}\left[\lambda_{G_g^h}^*\right]\times\left[x_p^h-x_p^{h*}\right]$$

$$+\sum_{h=1}^{H}\sum_{g=1}^{n_{G^h}}\left[\bar{Q}_{G_g^h}-\sum_{p\in P_{G_g^h}}x_p^{h*}\right]\times\left[\lambda_{G_g^h}-\lambda^*_{G_g^h}\right]+\sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{k=1}^{k_i}\left[s^h_{(i,k)}(\pi^*)\right]\times\left[\pi^h_{(i,k)}-\pi^{h*}_{(i,k)}\right]$$

$$+\sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{k=1}^{k_i}\left[-\sum_{p\in P^{(i,k)}}x_p^{h*}\right]\times\left[\pi^h_{(i,k)}-\pi^{h*}_{(i,k)}\right]+\sum_{h=1}^{H}\sum_{j=1}^{n}\sum_{l=1}^{l_j}\sum_{p\in P_{(j,l)}}x_p^{h*}\times\left[\rho^h_{(j,l)}-\rho^{h*}_{(j,l)}\right]$$

$$+\sum_{h=1}^{H}\sum_{j=1}^{n}\sum_{l=1}^{l_j}\left[-d^h_{(j,l)}(\rho^*)\right]\times\left[\rho^h_{(j,l)}-\rho^{h*}_{(j,l)}\right]\geq 0,\quad\forall(x,f,\pi,\lambda,\rho)\in K^1. \tag{5.30}$$

One can now establish that $F(X)$ is monotone. For any $X^1=(x^1,\lambda^1,\pi^1,\rho^1)\in K, X^2=(x^2,\lambda^2,\pi^2,\rho^2)\in K$,

$$\langle F(X^1)-F(X^2),X^1-X^2\rangle=\langle F(x^1,\lambda^1,\pi^1,\rho^1)-F(x^2,\lambda^2,\pi^2,\rho^2),\begin{bmatrix}x^1-x^2\\ \lambda^1-\lambda^2\\ \pi^1-\pi^2\\ \rho^1-\rho^2\end{bmatrix}\rangle$$

$$=\sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{k=1}^{k_i}\left[\pi^{h1}_{(i,k)}-\pi^{h2}_{(i,k)}\right]\times\left[\sum_{p\in P^{(i,k)}_{(j,l)}}x_p^{h1}-\sum_{p\in P^{(i,k)}_{(j,l)}}x_p^{h2}\right]$$

$$+\sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{l_j}\left[-\rho^{h1}_{(j,l)}+\rho^{h2}_{(j,l)}\right]\times\left[\sum_{p\in P^{(i,k)}_{(j,l)}}x_p^{h1}-\sum_{p\in P^{(i,k)}_{(j,l)}}x_p^{h2}\right]$$

$$+\sum_{h=1}^{H}\sum_{a\in L}\left[c^h_a(f^1)-c^h_a(f^2)\right]\times\left[f_a^{h1}-f_a^{h2}\right]$$

$$+\sum_{h=1}^{H}\sum_{g=1}^{n_{G^h}}\left[\lambda^1_{G_g^h}-\lambda^2_{G_g^h}\right]\times\left[\sum_{p\in P_{G_g^h}}x_p^{h1}-\sum_{p\in P_{G_g^h}}x_p^{h2}\right]$$

$$+\sum_{h=1}^{H}\sum_{g=1}^{n_{G^h}}\left[-\sum_{p\in P_{G_g^h}}x_p^{h1}+\sum_{p\in P_{G_g^h}}x_p^{h2}\right]\times\left[\lambda^1_{G_g^h}-\lambda^2_{G_g^h}\right]$$

$$+\sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{k=1}^{k_i}\left[s^h_{(i,k)}(\pi^1)-s^h_{(i,k)}(\pi^2)\right]\times\left[\pi^{h1}_{(i,k)}-\pi^{h2}_{(i,k)}\right]$$

$$
+\sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{k=1}^{k_i}\left[-\sum_{p\in P^{(i,k)}} x_p^{h1} + \sum_{p\in P^{(i,k)}} x_p^{h2}\right] \times \left[\pi_{(i,k)}^{h1} - \pi_{(i,k)}^{h2}\right]
$$

$$
+\sum_{h=1}^{H}\sum_{j=1}^{n}\sum_{l=1}^{l_j}\left[\sum_{p\in P_{(j,l)}} x_p^{h1} - \sum_{p\in P_{(j,l)}} x_p^{h2}\right] \times \left[\rho_{(j,l)}^{h1} - \rho_{(j,l)}^{h2}\right]
$$

$$
+\sum_{h=1}^{H}\sum_{j=1}^{n}\sum_{l=1}^{l_j}\left[-d_{(j,l)}^{h}(\rho^1) + d_{(j,l)}^{h}(\rho^2)\right] \times \left[\rho_{(j,l)}^{h1} - \rho_{(j,l)}^{h2}\right]
$$

$$
= \sum_{h=1}^{H}\sum_{a\in L}\left[c_a^h(f^1) - c_a^h(f^2)\right] \times \left[f_a^{h1} - f_a^{h2}\right] + \sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{k=1}^{k_i}\left[s_{(i,k)}^{h}(\pi^1) - s_{(i,k)}^{h}(\pi^2)\right]
$$

$$
\times \left[\pi_{(i,k)}^{h1} - \pi_{(i,k)}^{h2}\right] - \sum_{h=1}^{H}\sum_{j=1}^{n}\sum_{l=1}^{l_j}\left[d_{(j,l)}^{h}(\rho^1) - d_{(j,l)}^{h}(\rho^2)\right] \times \left[\rho_{(j,l)}^{h1} - \rho_{(j,l)}^{h2}\right]. \tag{5.31}
$$

With the assumptions $(5.28a, b, c)$ on the product supply functions, minus the product demand functions, and the link product unit transportation cost functions, it can be concluded that expression (5.31) is greater than or equal to zero. Therefore, $F(X)$ is monotone.

Remark 5.1

$F(X)$ is Lipschitz continuous for this model provided that the link product unit transportation cost functions, the product supply functions, and the product demand functions have bounded first order derivatives (see also Dong, Zhang, and Nagurney (2004) and Li and Nagurney (2015)).

Only monotonicity and Lipschitz continuity of $F(X)$ are required for the convergence of the modified projection method, provided a solution exists.

Remark 5.2

Observe that equilibrium conditions (5.4) and (5.5) may be re-expressed without the use of the Lagrange multiplier vector λ^*, leading to a variational inequality in path flows, supply prices, and demand prices and over a feasible set that includes the quota constraints. Since the link unit cost functions and the supply and the demand functions are all assumed to be continuous, and since the supply prices and the demand prices are bounded and the product flows are as well under the assumption that there are quotas imposed on all product flows (and these may actually be very large), existence of a solution is guaranteed under the classical theory of variational inequalities (see Kinderlehrer and Stampacchia (1980)).

5.3. Numerical Examples

In this section, algorithmically computed solutions to spatial price numerical examples are presented in order to show the types of insights that can be obtained, when trade measures are imposed. Furthermore, the examples yield not only qualitative information, but also quantitative results. The spatial price network topology is as in Figure 5.4. Specifically, a network with two countries, with a single supply market in each country and with two demand markets in each country is considered. The product being produced, shipped, and demanded is that of N95 masks. The prices, the price floors, and the price ceilings are all in a common currency. The modified projection method was implemented in FORTRAN and a Linux system at the University of Massachusetts Amherst used for the implementation of the algorithm and the solution of these numerical examples. The algorithm was initialized as follows. A demand of 100 was divided equally among the paths to each demand market and the initial demand market prices were set at 1 and the initial supply market prices were set at 0 (except for Examples 5.4 and 5.5 where the supply price at the supply market of Country 1 is initialized to the supply price floor of 20). The algorithm was deemed to have converged if the absolute value of the successive variable iterates differed by no more than 10^{-4}.

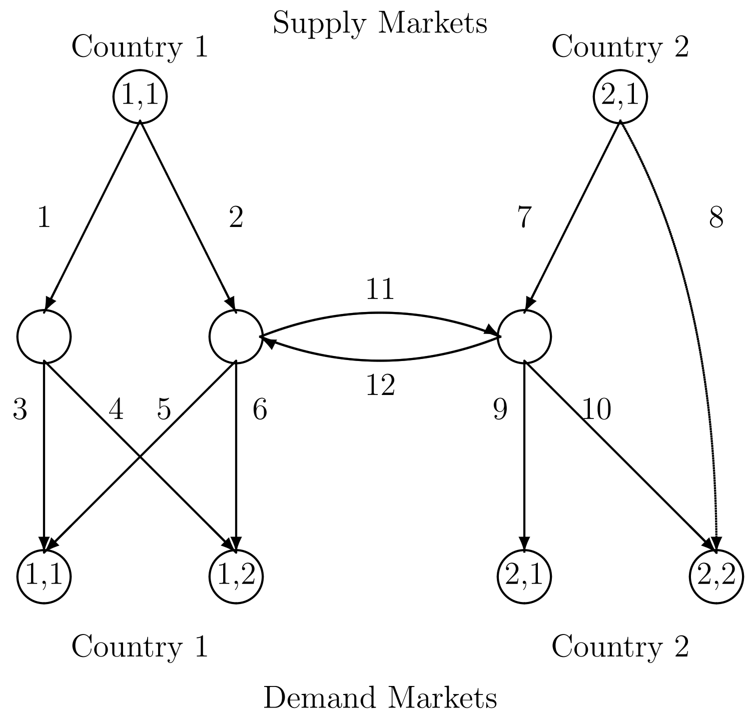

Figure 5.4. Network for the Spatial Price Equilibrium Examples Solved Algorithmically

These numerical examples are stylized but, nevertheless, are grounded in realistic data. Country 1, for example, is inspired by China and Country 2 by the United States. The unit of flow is a kilogram of N95 masks with a kilogram corresponding to 100-150 masks.

The cost of air, sea, and land transportation depends on various factors, but according to Freightos (2020) and WTO (2020b), air cargo shipping cost ranges from \$2 to \$4 per kilogram. In this network, I assume links 1, 2, 7, 8, 11, and 12 are air freight links. Links 11 and 12 are international long distance air shipping links between the two countries, so they have the highest unit shipping costs. Other links also have different rates depending on the distance between the origin and the destination.

The computed equilibrium path flows for Examples 5.1 through 5.5 are given in Table 5.2 and the computed equilibrium link flows in Table 5.3. The computed equilibrium prices, and the supplies and demands at the markets are reported according to their functional formulae in the numerical examples, as well as the associated flows out of the supply markets and associated flows into the demand markets. Please refer to equilibrium conditions (5.5), (5.6), and (5.7).

Example 5.1 - No Trade Measures

In order to be able to ascertain the impact of a specific trade measure, or combination thereof, first a numerical example in a pure form, that is, one in which no trade measures are imposed is considered.

The data for this example are as follows. The paths are:

$$p_1 = (1,3), \quad p_2 = (2,5), \quad p_3 = (7,12,5), \quad p_4 = (1,4), \quad p_5 = (2,6), \quad p_6 = (7,12,6),$$
$$p_7 = (2,11,9), \quad p_8 = (7,9), \quad p_9 = (2,11,10), \quad p_{10} = (7,10), \quad p_{11} = (8).$$

The supply functions at the supply markets are:

$$s^1_{(1,1)}(\pi) = 25\pi^1_{(1,1)} + 3000, \quad s^1_{(2,1)}(\pi) = 22\pi^1_{(2,1)} + 1000.$$

The demand functions at the demand markets are:

$$d^1_{(1,1)}(\rho) = -0.5\rho^1_{(1,1)} + 1800, \quad d^1_{(1,2)}(\rho) = -0.5\rho^1_{(1,2)} + 1500,$$
$$d^1_{(2,1)}(\rho) = -1.5\rho^1_{(2,1)} + 1000, \quad d^1_{(2,2)}(\rho) = -1.0\rho^1_{(2,2)} + 2500.$$

The link unit transportation cost functions are:

$$c_1^1(f) = .01f_1^1 + 3, \quad c_2^1(f) = .02f_2^1 + 2, \quad c_3^1(f) = .03f_3^1 + 2,$$

$$c_4^1(f) = .06f_4^1 + 1, \quad c_5^1(f) = .5f_5^1 + 1,$$

$$c_6^1(f) = .02f_6^1 + 2, \quad c_7^1(f) = .4f_7^1 + 4, \quad c_8^1(f) = .1f_8^1 + 3, \quad c_9^1(f) = .5f_9^1 + 1,$$

$$c_{10}^1(f) = .05f_{10}^1 + 1,$$

$$c_{11}^1(f) = .2f_{11}^1 + .1f_{12}^1 + 4, \quad c_{12}^1(f) = .3f_{12}^1 + .20f_{11}^1 + 5.$$

The computed equilibrium path flows and equilibrium link flows are reported in Tables 5.2 and 5.3, respectively. Now the computed equilibrium prices are reported. Observe that Country 2 exports no N95 masks to Country 1, since the equilibrium link flow on link 12 is equal to 0.00.

The computed equilibrium supply market prices are:

$$\pi_{(1,1)}^{1*} = 32.55, \quad \pi_{(2,1)}^{1*} = 48.57,$$

and the computed equilibrium demand market prices are:

$$\rho_{(1,1)}^{1*} = 109.16, \quad \rho_{(1,2)}^{1*} = 88.50, \quad \rho_{(2,1)}^{1*} = 397.14, \quad \rho_{(2,2)}^{1*} = 223.08.$$

The spatial price equilibrium conditions (5.5), (5.6), and (5.7) hold with excellent accuracy. Observe that the supply market price in Country 1 is lower than that at the supply market in Country 2; the same for the demand market prices in Country 1 as opposed to those in Country 2. Recall that the prices, which are in a common currency, are for a kilogram of N95 masks, which corresponds to 100-150 masks. Hence, the above prices are quite reasonable. Also, it is worth noting that all the supply market prices and all the demand markets prices are positive and none are at a value of 0. In fact, the flow of N95 masks out of Supply Market 1 is equal to 3813.82, and the flow out of supply market in Country 2 is equal to 2068.57; furthermore, the flows of N95 masks into the demand markets are, respectively: 1745.42, 1455.75, 404.30, and 2276.92. These values are precisely equal to their respective values according to the supply function and demand function formulae above (note equations (5.6) and (5.7) of the equilibrium conditions).

Example 5.2 - Data as in Example 5.1 but with a Tariff Imposed by Country 2 on Imports of N95 Masks from Country 1

Now the impact of the imposition of a unit tariff by Country 2 on the imports of N95 masks from Country 1 is evaluated. This, of course, allows to also compare the removal of such a tariff, which would correspond to Example 5.1. The data are as in Example 5.1 but with the following unit tariff: $\tau^1_{12} = 2$. Please refer to the computed equilibrium path flows and the equilibrium link flows, respectively, in Tables 5.2 and 5.3.

The computed equilibrium supply market prices are now:

$$\pi^{1*}_{(1,1)} = 32.35, \quad \pi^{1*}_{(2,1)} = 48.76,$$

and the equilibrium demand market prices are:

$$\rho^{1*}_{(1,1)} = 108.95, \quad \rho^{1*}_{(1,2)} = 88.24, \quad \rho^{1*}_{(2,1)} = 397.52, \quad \rho^{1*}_{(2,2)} = 223.57.$$

Clearly, under the imposed tariff, consumers at the two demand markets in Country 2 experience a higher price for the N95 masks than they did in Example 5.1, whereas consumers at demand markets in Country 1 experience reduced prices, as compared to the values in Example 5.1.

The supply of N95 masks at the supply market in Country 1 according to the functional formula is: 3808.80 and the supply at the supply market in Country 2 is: 2072.76. The demands at the demand markets for N95 masks are now, according to their functional formulae, respectively: 1745.24, 1455.89, 403.73, and 2276.43. Both the supply flows and the demand flows correspond precisely to the values of the respective function evaluated at the equilibrium prices (see equilibrium conditions (5.6) and (5.7)).

Sensitivity analysis is conducted to determine at what value of the tariff there would be zero flows of the N95 masks from Country 1 to Country 2. It is observed that when the tariff τ^1_{12} was 234 or higher the flow on link 11 was zero (and if it were 233 or lower there would be a positive volume of flow on link 11). Furthermore, under a unit tariff of 234, the computed equilibrium supply market prices were

$$\pi^{1*}_{(1,1)} = 9.15, \quad \pi^{1*}_{(2,1)} = 70.79,$$

and the computed equilibrium demand market prices were:

$$\rho^{1*}_{(1,1)} = 84.56, \quad \rho^{1*}_{(1,2)} = 57.89, \quad \rho^{1*}_{(2,1)} = 441.50, \quad \rho^{1*}_{(2,2)} = 280.34.$$

One can see that consumers in Country 2 suffer, in that, the higher the tariff that is levied by their country's government, the higher the demand market prices for the N95 masks.

The supply at Supply Market 1 in Country 1, as represented by the flow, at the unit tariff of 234, is equal to 3808.80 and that at the supply market in Country 2 equal to 2072.76; the demands at the demand markets, as represented by the flow, are, respectively: 1745.52, 1455.89, 403.73, and 2276.43. Again, as expected, since the equilibrium prices are not equal to 0, which would correspond, to price floors at that value, the above flow values coincide precisely with the corresponding functions evaluated at the computed equilibrium prices.

Example 5.3 - Data as in Example 5.2 but with Demand Price Bounds on Demand Markets in Country 2

In Example 5.3, the data remain as in Example 5.2, except that now demand price ceilings on the N95 masks are imposed at the demand markets in Country 2. The government in Country 2 is concerned about the prices of the N95 at its demand markets and sets the following demand market price ceilings:

$$\bar{\rho}^1_{(2,1)} = 200.00, \quad \bar{\rho}^1_{(2,2)} = 100.00.$$

The computed equilibrium flow patterns for Example 5.3 are reported in Tables 5.2 and 5.3. In addition, for completeness, and for comparison with the preceding example, now the computed equilibrium supply and demand market prices are reported:

$$\pi^{1*}_{(1,1)} = 16.61, \quad \pi^{1*}_{(2,1)} = 5.37,$$

and the computed equilibrium demand market prices:

$$\rho^{1*}_{(1,1)} = 92.40, \quad \rho^{1*}_{(1,2)} = 67.65, \quad \rho^{1*}_{(2,1)} = 200.00, \quad \rho^{1*}_{(2,2)} = 100.00.$$

The supply market price decreases at the supply market in Country 1, as compared to the supply market price in Example 5.2, and also decreases at the supply market in Country 2. The demand market prices decrease at the demand markets in Country 1, and also at the demand markets in Country 2, where they attain values at the imposed price ceilings.

The supply at Supply Market 1 in Country 1, as represented by the flow, is now equal to 3415.23, and that at the supply market in Country 2 is equal to 1118.04. The demands at the demand markets are, respectively: 1753.80, 1466.18, 217.90, and 1095.37. The flow values for the supply markets coincide with the respective functional values evaluated at the computed equilibrium prices. The flow values into the demand markets in Country 1 coincide with the corresponding demand functions evaluated at the computed equilibrium prices. However, since the equilibrium demand prices at both demand markets in Country 2 are at the imposed price ceiling (see also equilibrium condition (5.7)), the demand functions evaluated at the computed equilibrium prices, which are for the two demand markets in Country 2 equal to 700.00 and 2400, respectively, exceed the flow into the respective demand market.

As compared to Example 5.2, the flows of the N95 masks to demand markets in Country 2 decrease precipitously, and are at about 50%, which is detrimental to the health of those who require them as well as to the containment of contagion. Clearly, price ceilings at demand markets may reduce the purchase price at the demand market, but at the expense of volume of essential product. Interestingly, consumers at demand markets in Country 1 gain in terms of an increased volume of flow into their demand markets of the N95 masks, and at lower prices, as compared to the demand market prices in Example 5.2.

Example 5.4 - Data as in Example 5.3 but with a Supply Price Floor at the Supply Market of Country 2

Country 1 is concerned that the supply price of the N95 masks at its supply market is low, and, in order to protect producers, it has instituted a price floor of: $\underline{\pi}^1_{(1,1)} = 20$. The rest of the data for Example 5.4 is as in Example 5.3.

The computed equilibrium path flows and equilibrium link flows are reported in Tables 5.2 and 5.3, respectively.

The computed equilibrium supply market prices are now:

$$\pi^{1*}_{(1,1)} = 20.00, \quad \pi^{1*}_{(2,1)} = 5.40,$$

and the equilibrium demand market prices are:

$$\rho^{1*}_{(1,1)} = 95.69, \quad \rho^{1*}_{(1,2)} = 70.82, \quad \rho^{1*}_{(2,1)} = 200.00, \quad \rho^{1*}_{(2,2)} = 100.00.$$

Also, the supply at Supply Market 1 in Country 1, according to the supply function formula, is now equal to 3500.00, whereas the flow out is equal to 3398.90. The supply at the supply market in Country 2, according to the supply function formula, is equal to 1118.90, and this is exactly the value of the sum of flows out of this supply market. The demands at the demand markets according to the demand functions are, respectively: 1752.16, 1464.60, 700.00, and 2400.00, with the first two corresponding, respectively to the flow in to each demand market. The flow at Country 2's demand markets, in turn, are, respectively: 216.83 and 1084.22.

One can see that, at the supply market in Country 1, the supply price is at the supply price floor of 20 and that the supply market price at the supply market in Country 2 has risen, albeit slightly. The prices that the consumers pay at the two demand markets in Country 1 have also risen (as compared to their values in Example 5.3). Hence, although producers in Country 1 enjoy a higher price, consumers in Country 1 pay more for the N95 masks. As for the demand markets in Country 2, the demand market prices remain at the imposed price ceilings (as was the case in Example 5.3). Interestingly, although the supply price floor is enacted in Country 1 at its supply market, consumers at both demand markets in Country 2 experience a lower volume of N95 masks at their demand markets.

Example 5.5 - Data as in Example 5.4 but with an Export Quota of 100 Imposed by Country 1 on Country 2

In Example 5.5, the government of Country 1 is getting concerned about rising cases of COVID-19 and institutes an export quota for the N95 masks of 100. Please refer to Tables 5.2 and 5.3 for the computed equilibrium path flows and equilibrium link flows in Tables 5.2 and 5.3, respectively.

First, observe that, in Example 5.5, the quota of 100 is met, since the sum of the path flows on paths p_7 and p_9 is equal to 100; equivalently, one can see that, given the network topology for this set of problems, the equilibrium link flow on link 11 is 100.00. The associated computed Lagrange multiplier is $\lambda^{1*} = 21.11$, when the group notation is suppressed.

The computed equilibrium supply market prices are now:

$$\pi^{1*}_{(1,1)} = 20.00, \quad \pi^{1*}_{(2,1)} = 5.65,$$

and the equilibrium demand market prices are:

$$\rho^{1*}_{(1,1)} = 95.47, \quad \rho^{1*}_{(1,2)} = 69.80, \quad \rho^{1*}_{(2,1)} = 200.00, \quad \rho^{1*}_{(2,2)} = 100.00.$$

Also, the flow of N95 masks out of Supply Market 1 in Country 1, according to the supply function, is now equal to 3500.00, whereas the flow out is equal to 3317.37.

The supply at the supply market in Country 2, according to the supply function formula, is equal to 1124.27, and this is exactly the value of the sum of flows out of this supply market.

The demands at the demand markets according to the demand functions are, respectively: 1752.27, 1465.20, 700.00, and 2400.00, and correspond, respectively, for the first two demand markets to the flow to each demand market. In the case of the demand markets in Country 2, the flow into its demand markets is: 210.07 and 1014.20, respectively. The supply flows of the N95 masks in both countries now decrease, which is detrimental to the health of those who require them as well as to the containment of contagion. The demand markets in Country 2 receive fewer of the N95 masks than in Example 5.4. The prices of N95 masks in the demand markets in Country 2 are bounded by the price ceilings but the price in the supply market of Country 2 has increased.

Path	**Equilibrium Path Flows**				
	Ex. 5.1	Ex. 5.2	Ex. 5.3	Ex. 5.4	Ex. 5.5
p_1	1664.01	1663.97	1661.00	1659.09	1657.04
p_2	81.41	81.55	92.80	93.06	95.23
p_3	0.00	0.00	0.00	0.00	0.00
p_4	504.46	503.58	434.68	431.94	417.77
p_5	951.29	952.30	1031.50	1032.65	1047.33
p_6	0.00	0.00	0.00	0.00	0.00
p_7	266.93	256.04	130.97	125.58	91.30
p_8	137.36	138.69	86.93	91.24	118.77
p_9	345.71	342.35	64.27	56.56	8.70
p_{10}	216.17	216.04	114.77	111.71	92.02
p_{11}	1715.03	1718.04	916.33	915.94	913.48

Table 5.2. Equilibrium Path Flows for Examples 5.1-5.5

Link	Equilibrium Link Flows				
	Ex. 5.1	Ex. 5.2	Ex. 5.3	Ex. 5.4	Ex. 5.5
1	2168.47	2167.56	2095.68	2091.03	2074.81
2	1645.34	1641.24	1319.55	1307.86	1242.56
3	1664.01	1663.97	1661.00	1659.09	1657.04
4	504.46	503.58	434.68	431.94	417.77
5	81.41	81.55	92.80	93.06	95.23
6	951.29	952.30	1031.50	1032.65	1047.33
7	353.54	354.72	201.71	202.95	210.79
8	1715.03	1718.04	916.33	915.94	913.48
9	404.30	403.73	217.90	216.83	210.07
10	561.88	558.39	179.04	168.28	100.72
11	612.64	607.39	195.24	182.15	100.00
12	0.00	0.00	0.00	0.00	0.00

Table 5.3. Equilibrium Link Flows for Examples 5.1-5.5

These numerical examples illustrate the impacts of trade measures not only on the country imposing the trade measure(s) but also on other countries. Interestingly, it may so happen that a trade measure imposed by a country, hoping to help its citizens, may actually adversely affect its consumers, but may positively affect those in another country.

5.4. Summary and Conclusions

The COVID-19 pandemic has disrupted lives and economies around the globe and has resulted in both a health and an economic disaster. Many governments of different countries, seeking to protect their citizens, along with essential workers, has instituted a variety of trade measures on essential products in the pandemic. Examples of such trade measures include: tariffs, quotas, as well as price supports in the form of price floors and ceilings.

This chapter introduces a multiproduct, multicountry spatial price equilibrium model, which has both product flows, as well as supply market prices and demand market prices as variables. It also handles multiple paths, consisting of multiple numbers of links between supply markets and demand markets. The governing equilibrium conditions, in the presence of the trade measures, are stated and a variational inequality formulation is derived, which is then used to obtain qualitative results as well as solutions to illustrative examples and numerical examples, with the latter solved via an implemented algorithm.

The modeling and algorithmic framework enables policy makers and decision-makers to quantify the impacts of different trade measures, both individually or jointly, and to ascertain who may benefit and who may lose. For example, the computed solutions to numerical examples reveal that unexpected results may occur. Hence, a country may think that it is benefiting its own consumers, but actually helps those in another country and not its own. In constructing a general, computable spatial price equilibrium model in both quantity and price variables and with a spectrum of trade instruments, this chapter has enriched the portfolio of spatial price equilibrium models which incorporate relevant policies in the form of trade measures, and which are being actively imposed now by governments around the world in the pandemic.

CHAPTER 6

CONCLUSIONS AND FUTURE RESEARCH

6.1. Conclusions

As a result of the COVID-19 pandemic, millions have died and the lives of billions have been disrupted with the global economy severely impacted. Medical supplies are among the many products crucial in fighting this global health disaster. However, various essential commodities including raw materials and medical items have been in short supply during the pandemic. As a result of the global demand for various medical supplies, including PPEs, fierce competition for such items developed, and the supply chain disruptions in the pandemic have only intensified the competition. My goal in this dissertation was to investigate the competition for medical and other essential items in supply chain economic networks during the pandemic. In this regard, in different chapters, I looked into various aspects of this competition, the behavior of decision-makers, and the effects of numerous factors on the outcome of pandemic disaster management efforts.

In Chapter 2, I developed an integrated financial and logistical game theory network model for disaster management that has several significant features. The framework adds to the literature on game theory and disaster relief as well as to the literature on variational inequalities with nonlinear constraints. Based on the methodology used in this chapter, competition among different decision-makers for a range of limited resources in the COVID-19 pandemic was examined. In Chapter 3, in order to shed light on the competition for medical supplies during the pandemic, I developed a Generalized Nash Equilibrium model. A supply chain network consisting of multiple supply points and multiple demand points with stochastic demand was established in this chapter. A variational inequality with variables being the product flows and Lagrange multipliers associated with the medical commodity supply capacities was formulated and the model studied both qualitatively and quantitatively using illustrative as well as more complex numerical examples.

In Chapter 4, I constructed a stochastic Generalized Nash Equilibrium model to address, for the first time, both the stages of preparedness and response in the resource competition among countries in the pandemic. The model includes the specific features of the pandemic, such as the uncertainty of the scenarios, limitations associated with product supply capacities, and varying parameters in different stages of pandemic disaster management. Using a two-stage stochastic model to look at specific features of the COVID-19 pandemic, this chapter provides new applications of game theory to disaster management.

In Chapter 5, a multiproduct, multicountry spatial price equilibrium model with multiple supply markets and multiple demand markets in each country is introduced. The model incorporates a variety of trade measures that can be imposed by different governments, such as tariffs, quotas, and price floors and ceilings, to assist decision-makers in assessing the effects of such policy instruments over the course of a pandemic. In addition to adding to the literature on spatial price equilibrium models, this chapter has particular relevance in the pandemic since governments have been implementing multiple trade measures.

Each chapter contains relevant examples inspired by the challenges that organizations and governments have been facing in the context of supply chain economic networks for disaster management during the COVID-19 pandemic. The solution of a plethora of numerical examples demonstrates the applications of and the insights that can be obtained from the proposed models and methods.

6.2. Future Research

In this section, I highlight some possible directions for future research.

6.2.1 Short-term Supply Chain Challenges

With the approval of effective vaccines and their widespread distribution and administration, hopes for a significant decline in the number of COVID-19 cases and deaths have been heightened. However, this is not the end of the pandemic story. In addition to the need for booster doses (LaFraniere (2022)), there are still concerns about the emergence of new variants and the need for new vaccines (Fay Cortez (2022)), which could cause additional competition for vaccines. Furthermore, about 40% of the world population has yet to be fully vaccinated (Holder (2022)).

In addition, the secondary crises that were predicted to plague the world as a consequence of this pandemic continue unabated. Global supply chains continue to face severe disruptions, with shortages of raw materials and even workers (Nagurney (2022)). Economic difficulties, due to high inflation have also become a serious secondary crisis. According to a recent study, rising prices of everyday items due to inflation, supply chain issues, and global uncertainty are the top three sources of stress in the United States (American Psychological Association (2022)). While consumer demand is high for certain products, the bottlenecks at ports, the shutdowns of economic sectors, and shortages in the labor force have all contributed to a significant supply deficit, and that well explains the sharp rise in inflation (Patton (2021)).

The above-noted challenges are being faced by governments, businesses, and many organizations. I intend to study the behavior of decision-makers in the face of such secondary crises resulting from the pandemic. For example, how to plan for the production and distribution of vaccines and treatments in future pandemics is a matter of complexity. On one hand, many believe that there may no longer be a need to invest heavily in this sector; on the other hand, we know that if governments neglect such investments, we will be ill-prepared for other variants and pandemics. Another challenge that needs to be addressed is global supply chain disruptions. I plan on studying how different governments, with different characteristics and tools at their disposal, can achieve their goals and lose less in terms of competition in severely compromised global supply chains.

6.2.2 Long-term Supply Chain Challenges

There are also significant long-term challenges in supply chain economic networks for governments and organizations to address after this historic pandemic. A survey shows that it is expected that most companies will pursue some form of regionalization over the next three years (Alicke, Barriball, and Trautwein (2021)). The long-term preparedness planning for similar possible future events is now more important than ever. Crises such as this pandemic, climate change, and even large-scale wars threaten global supply chains. Companies, organizations, and governments must face the challenge of increasing the resilience of their supply chain networks without sacrificing their competitiveness (Shih (2020)).

The long-term resiliency of supply chains has many dimensions and must be addressed from different angles. Of special relevance now is the modeling and analysis of strategic commodity supply chain networks. President Biden, in the Executive Order on America's supply chains, states that "the United States needs resilient, diverse, and secure supply chains to ensure our economic prosperity and national security" (The White House (2021)). The manufacturing capacity and the availability of critical goods and services are threatened by pandemics, climate-related events, geopolitical and economic competition, and other challenges (The White House (2021)).

Sustainability is another important issue that governments and businesses are faced with which has become one of the most significant goals of the world community in recent years. Global supply chains are at the core of this challenge (Howells (2021)). More than 80% of greenhouse gas emissions generated by a typical consumer company are coming from its supply chain (Bove and Swartz (2016)). The number one challenge to improving sustainability performance, according to UN Global Compact participants, is supply chain practices (UN Global Compact (2022)).

Supply chain sustainability was in the spotlight prior to the COVID-19 pandemic. Economic difficulties, the unstable geopolitical situation around the world, decreases in supply and increases in the price of fossil fuels, and competition for limited resources, are among issues that challenge the achievement of the sustainable supply chains goal. Utilizing technologies and innovations in transportation systems, transparency in information sharing, and collaboration with other stakeholders are among the steps that decision-makers must take to improve sustainability in supply chains. However, as mentioned earlier, another significant goal of organizations and governments is how to prepare for major global crises and to compete for limited resources. Pursuing these objectives simultaneously in highly competitive supply chain economic networks has certain complexities that I intend to study in the future.

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