A New Mathematical Model for Optimizing the Performance of Parallel and Discrete Event Simulation Systems

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Abstract

Null message algorithm is an important conservative time management protocol in parallel discrete event simulation systems for providing synchronization between the distributed computers with the capability of both avoiding and resolving the deadlock. However, the excessive generation of null messages prevents the widespread use of this algorithm. The excessive generation of null messages results due to an improper use of some of the critical parameters such as frequency of transmission and Lookahead values. However, if we could minimize the generation of null messages, most of the parallel discrete event simulation systems would be likely to take advantage of this algorithm in order to gain increased system throughput and minimum transmission delays. In this paper, a new mathematical model for optimizing the performance of parallel and distributed simulation systems is proposed. The proposed mathematical model utilizes various optimization techniques such as variance of null message elimination to improve the performance of parallel and distributed simulation systems. For the sake of simulation results, we consider both uniform and non-uniform distribution of Lookahead values across multiple output lines of an LP. Our experimental verifications demonstrate that an optimal NMA offers better scalability in parallel discrete event simulation systems if it is used with the proper selection of critical parameters.

1. INTRODUCTION

While there is a considerable literature exploring how poor selection of critical parameters might results poor performance of PDES systems [8, 12], surprisingly little work has examined how critical parameters impact on the performance of PDES systems. These research works indicate the strong relationship among many critical parameters such as Lookahead and frequency of transmission that one may use to quantify the impact of these parameters on the PDES performance. None of these research works, however, evaluate the determinants of the critical parameters to the performance of PDES systems. This paper presents a mathematical model to optimize the performance of PDES systems by minimizing the null message transmission across each LP using various optimization techniques.

In parallel discrete event simulation (PDES) systems, the distributed discrete events need to be tightly synchronized with each other in order to work simultaneously on different parts of a common task. However, if these discrete events are not properly synchronized, the performance of a PDES environment may degrade significantly [2]. Time management algorithms (TMA) are, therefore, required to ensure that the execution of a PDES is properly synchronized. In general, synchronization protocols can be categorized into two different families: conservative and optimistic. In optimistic algorithm, both deadlock detection and recovery occur at run time. However, if it is used in a wide range parallel network, each logical process (LP) may experience longer transmission delays at run time [13]. On the other hand, conservative protocols fundamentally maintain causality in event execution by strictly disallowing the processing of events out of timestamp order. In order to avoid and resolve the deadlock situations, each LP needs to exchange time stamp information with the other neighboring LPs [1, 3]. Examples of conservative mechanisms include Chandy, Misra and Byrant's NMP [6], and Peacock, Manning, and Wong [11] avoided deadlock through null messages.

Conservative TMA can be further classified as synchronous and asynchronous protocols [1]. Synchronous algorithm uses global synchronization mechanism to determine the minimum time stamp of each incoming event for an LP. On the other hand, NMA is an example of an asynchronous conservative algorithm that does not require global synchronizations. The primary problem associated with null messages is that if their timestamps are chosen inappropriately, the simulation becomes choked with null messages and performance suffers. Some intelligent approaches to null message generation include generation on demand [8], and generation after a time-out [5]. Some earlier research on discrete event simulation has focused on variants of null message protocol (NMP, with the objective of reducing the high null message overhead. For instance, Bain and Scott [4] attempt to simplify the communication topology to resolve the problem of transmitting redundant null messages due to low Lookahead cycles. Other recent developments [10] have focused on incorporating knowledge about the LP into the synchronization algorithms. Cota and Sargent [7] focused on the skew in simulation time between different LPs by exploiting knowledge about the LPs and the topology of the interconnections.

Although, much research has been done to evaluate the performance of conservative NMA for inefficiencies and transmission overhead [3, 8, 12], none of them suggest any potential optimization for the NMA. Reference [12] proposed a new approach that shows relationships between many parameters to quantify the performance of PDES system running under NMA. It has been shown that the selection of values for several critical parameters such as the values for Lookahead, null message ratio (NMR), and frequency of transmission plays an important role in the generation of null messages [12]. If these values are not properly chosen by a simulation designer, the result will be an excessive number of null messages across each LP. This situation gets more severe when the NMA needs to run to perform a detailed logistics simulation in a distributed environment to simulate a huge amount of data [9]. This paper presents a mathematical model that implements many optimization techniques to optimize the performance of NMA by minimizing the exchange of null messages across the LPs. A significant improvement is measured in the performance of PDES system in terms of reduced execution speed and transmission delays.

The rest of the paper is organized as follows. Section 2 presents the proposed mathematical model. Section 3 provides the implementation of various optimization techniques on the NMA for improving the performance of PDES systems. The numerical and simulation results are presented in Section 4. Finally, we conclude in Section 5.

2. PERFORMANCE OPTIMIZATION THROUGH PROPOSED MATHEMATICAL MODEL

Some of the important model variables, along with their definition, are listed in Table I. For the sake of mathematical model, we assume that the value of Lookahead may change during the execution of a Lookahead period. However, a sudden increase or decrease in the values of Lookahead during the simulation can not be accepted. In addition, we assume that each LP is initialized with a constant event arrival. However, as the simulation progresses, we use both uniform and non-uniform distribution of Lookahead values across multiple output lines of each LP. For the frequency of message transmission, we assume that all messages are equally distributed among the LPs. Unless otherwise stated, we use the term all messages to refer to both null and event messages. Finally, we assume that a fixed size message is transmitted between LPs.

TABLE I

System Parameter Definition

Parameter	Definition
P	Computation required for processing an event per second
ρ	Event arrival rate (events per second)
MRT	Minimum receiving time
MST	Minimum sending time
L	Lookahead
STA	Simulation time advancement
F_T	Frequency of transmission
T_{Null}	Timestamp of a null message
T_S	Current simulation of a LP
T_{Total}	Total simulation time in seconds

Our proposed mathematical model is based on the internal architecture of an LP as shown in Fig. 1 and Fig. 2. The advancement in simulation time can be defined as a ratio of performance to the event arrival rate. The number of event messages processed per second per LP is represented by ***P***, where as the occurrence of the number of events per simulation second is refereed as an *even arrival rate* and it is represented by ***ρ***. This leads us to the following mathematical expression of the relative speed for advancement:

$$(P)\left(\frac{T_S}{E_{Msg}}\right) \tag{1}$$

Taking this into account, we can give the following hypothesis for approximating the number of null messages transmitted per LP: "*If we assume that we have an average value of Lookahead (**L**) which associates with one of the output lines of an LP, then **P** can be approximated as*":

$$P \cong E_{Msg}\left(\frac{1}{L}\right) \tag{2}$$

Combining (1) and (2) yields the estimated number of null messages transmitted per LP that has only one output line as shown in (3).

$$Null_{(LP)} \triangleq E_{Msg}\left(\frac{1}{L}\right)\left(\frac{T_S}{E_{Msg}}\right) \triangleq T_S\left(\frac{1}{L}\right) \tag{3}$$

Furthermore, if we assume that we have ***O*** number of output lines attached with each LP with the uniform distribution of Lookahead value on each output line, then (3) can be further generalized for ***O*** number of output lines per LP as follows:

$$Null_{(LP)} \triangleq E_{Msg}\left(\frac{O}{L}\right)\left(\frac{T_S}{E_{Msg}}\right) \triangleq T_S\left(\frac{O}{L}\right) \tag{4}$$

where ***O*** represents the total output lines per LP.

It should be noted that (4) represents total number of null messages transmitted per LP via ***O*** number of output lines to the neighboring LPs when the distribution of ***L*** is assumed to be uniform per output lines. If we assume that we have ***m*** number of total LPs present in a system where each LP has ***O*** number of output lines, then this allows us to extend (4) and generalize it for ***m*** number of LPs present in distributed simulation as shown in (5). It can be evident that (5) gives total

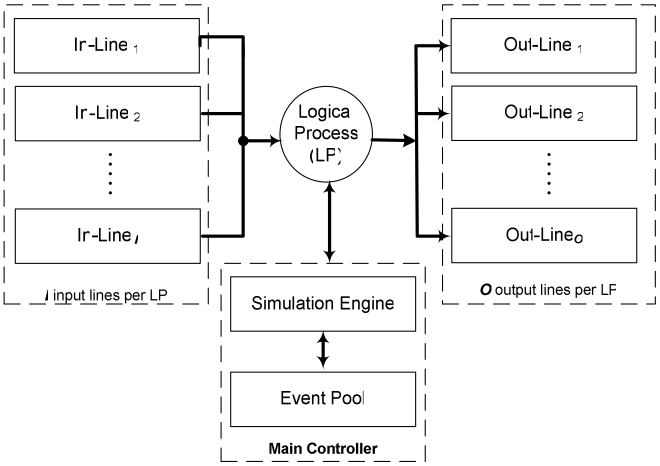

Fig.1. Internal architecture of an LP

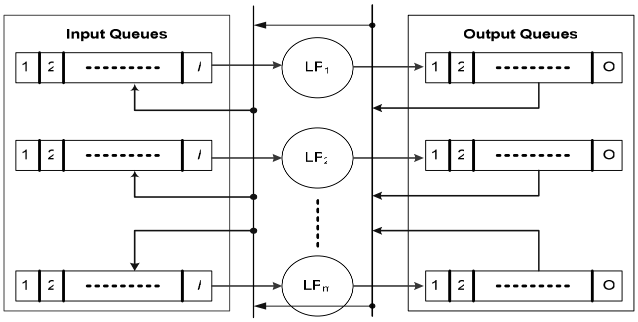

Fig.2. *m* number of logical processes with *I* number of input queues and *O* number of output queues per LP.

number of null messages exchange among all LPs present in the system.

$$Null_{(m-LP)} \triangleq E_{Msg}\left(\frac{O}{L}\right)m\left(\frac{T_S}{E_{Msg}}\right) \triangleq T_S\left(\frac{O}{L}\right)m \tag{5}$$

where the term O/L in (5) shows a uniform distribution of Lookahead value for ***O*** number of output lines per LP and the term m represents total number of LPs in the system.

The assumption of uniform distribution of Lookahead among ***O*** output lines of an LP simplifies the procedure for computing the number of null messages transmitted per LP to other neighboring LPs. However, the values for Lookahead may change during the execution of a Lookahead period that makes the uniform distribution assumption of Lookahead a little unrealistic. Based on this argument, we can rewrite (4) as:

$$Null_{(LP)} \triangleq \sum_{i=1}^{O}\left(\frac{E_{Msg}}{L_i}\right)\left(\frac{T_S}{E_{Msg}}\right) \triangleq \sum_{i=1}^{O} T_S\left(\frac{1}{L_i}\right) \tag{6}$$

It should be noted that (6) represents the total number of null messages transmitted per LP to other neighboring LPs via ***O*** number of output lines where each line can have a different Lookahead value.

If we assume that the model is partitioned into ***m*** number of total LPs where each LP can have at most ***O*** number of output lines, then this allows us to extend (6) and generalize it for ***m*** number of LPs. This generalization can be expressed in (7).

$$Null_{(m-LP)} \triangleq \sum_{h=1}^{m}\sum_{i=1}^{O}\left(\frac{E_{Msg}}{L_{hi}}\right)\left(\frac{T_S}{E_{Msg}}\right) \triangleq \sum_{h=1}^{m}\sum_{i=1}^{O} T_S\left(\frac{1}{L_{hi}}\right) \tag{7}$$

It can be evident that (7) gives total number of null messages exchange among all LPs present in the system.

3. PROPOSED OPTIMIZATION TECHNIQUES FOR NMA

In this section, we first derive a closed form mathematical expression for both frequency of transmission and the variance of null message elimination that can be further used to determine the reduction in the null message traffic in the presence of deadlock. The derived closed form expression uses the simple concept of frequency of transmission described in [12] to minimize the exchange of null messages across the LPs. In addition, we implement the optimization technique via variance of null message elimination.

3.1. Optimization Via Frequency of Transmission

Instead of sending null messages after processing each event on each output line of an LP, it should be transmitted with respect to a certain frequency of transmission. This frequency of transmission (F_T) is a fixed amount of time and it should be measured in simulation second per second. In other words, the Lookahead value which is associated with one or more output lines can be approximated as the frequency of transmission per output line of an LP. The above argument yields the following approximation for F_T in term of the Lookahead value.

$$\frac{E_{Msg}L}{F_T} \cong 2E_{Msg}\left(\frac{1}{L}\right) \Rightarrow L \cong \sqrt{2F_T} \tag{8}$$

Substituting the value of (8) into (3), we get,

$$Null_{(LP)} \triangleq E_{Msg}\left(\frac{1}{\sqrt{2F_T}}\right)\left(\frac{T_S}{E_{Msg}}\right) \triangleq T_S\left(\frac{1}{\sqrt{2F_T}}\right) \tag{9}$$

Equation (9) can be generalized for ***O*** *number of output lines* per LP when the numbers of null messages are generated with a certain frequency of transmission. In other words, the expected rate (i.e., F_T) at which null messages may generate per output line per LP can be roughly estimated as a percentage of the Lookahead values. This expected rate per output line per LP results (10) as follows:

$$Null_{(LP)} \triangleq E_{Msg}\left(\frac{O}{\sqrt{2F_T}}\right)\left(\frac{T_S}{E_{Msg}}\right) \triangleq T_S\left(\frac{O}{\sqrt{2F_T}}\right) \text{ where } \sqrt{F_T} \xrightarrow{\%} L \tag{10}$$

Equation (10) gives an estimated number of null messages transmitted by single LP that has ***O*** number of output lines where each line carry an equal percentage of the Lookahead value in terms of a fixed frequency of transmission per output line. In addition, if we assume that the system consists of ***m*** number of total LPs where each LP has fixed number of output lines, then (10) can be further extended for ***m*** number of LPs. This generalization results (11) as follows:

$$Null_{(m-LP)} \triangleq \left(E_{Msg} \times m\right)\left(O/\sqrt{2F_T}\right)\left(\frac{T_S}{E_{Msg}}\right) \triangleq T_S \times m\left(O/\sqrt{2F_T}\right) \qquad (11)$$
$$where\ \sqrt{F_T} \xrightarrow{\%} L$$

where the denominator of (11) (i.e., $O/\sqrt{F_T}$) represents a uniform rate of null message transmission per output line.

Based on (6), we can conclude that a non uniformity in null message algorithm results non linear generation of null messages. In other words, the approximation of null messages can be well optimized when a non uniform transmission rate is considered. Based on this argument, a mathematical expression can be derived for ***O*** number of output lines where each line may carry a different frequency of transmission.

$$Null_{(LP)} \triangleq \sum_{i=1}^{O}\left(\frac{E_{Msg}}{\sqrt{2F_{Ti}}}\right)\left(\frac{T_S}{E_{Msg}}\right) \triangleq \sum_{i=1}^{O} T_S\left(\frac{1}{\sqrt{2F_{Ti}}}\right) \qquad (12)$$
$$where\ \sqrt{F_{Ti}} \xrightarrow{\%} L_i$$

Furthermore, (12) can be further extended and generalized for ***m*** number of LPs where each LP can have at most ***O*** number of output lines. This generalization can be expressed in (13).

$$Null_{(m-LP)} \triangleq \sum_{k=1}^{m}\sum_{i=1}^{O}\left(E_{Msg}\Big/\sqrt{2F_{T(ki)}}\right)\left(T_S/E_{Msg}\right) \triangleq \sum_{k=1}^{m}\sum_{i=1}^{O} T_S\left(\frac{1}{\sqrt{2F_{T(ki)}}}\right) \qquad (13)$$
$$where\ \sqrt{F_{T(ki)}} \xrightarrow{\%} L_{(ki)}$$

3.2. Optimization Via Variance for Null Message Elimination in NMA

Also, in this scenario, it is essential to cancel out the unnecessary generation of null messages. To consider and analyze the effect of null message elimination on the performance of PDES systems, we introduce variance as a variable quantity. Variance represents the probability of cancellation of unnecessary null messages. The value of variance may exist between 0 and 1 (i.e., it can not be one, since 1 represents that all generated null messages cancelled with the maximum probability). It should also be subtracted from 1, so that we can show that increase in variance causes a decrease in the over all null messages where as a decrease in variance causes an increase in null messages. If we consider variance as 0, then it should give us the same results that we could achieve with out using variance. In order to reflect the variance of null message cancellation, we can rewrite (11) for ***m*** number of LPs with the uniform distribution of null message transmission per output line as follows:

$$Null_{(m-LP)} \triangleq \left(E_{Msg} \times m\right)\left(O/\sqrt{2F_T}\right)\left(\frac{T_S}{E_{Msg}}\right)(1-\sigma)$$
$$\triangleq T_S \times m\left(\frac{O}{\sqrt{2F_T}}\right)(1-\sigma)\ where\ \sqrt{F_T} \xrightarrow{\%} L\ and\ 0 \le \sigma < 1 \qquad (14)$$

where σ represents probability of null message cancellation.

The same concept of null message cancellation can be implemented with a simulation model where the Lookahead values are non-uniformly distributed among ***O*** number of output lines. This leads us to the following modification in (14):

$$Null_{(m-LP)} \triangleq \sum_{k=1}^{m}\sum_{i=1}^{O}\frac{E_{Msg}}{\sqrt{2F_{T(ki)}}}\left(\frac{T_S}{E_{Msg}}\right)(1-\sigma) \triangleq \sum_{k=1}^{m}\sum_{i=1}^{O} T_S\left(\frac{1}{\sqrt{2F_{T(ki)}}}\right)(1-\sigma) \qquad (15)$$
$$where\ \sqrt{F_{T(ki)}} \xrightarrow{\%} L_{(ki)}\ and\ 0 \le \sigma < 1$$

4. PERFORMANCE ANALYSIS OF THE PROPOSED MATHEMATICAL MODEL

For the sake of performance analysis, we simulate 6 different cases. The system is modeled in C++

4.1. CASE-I: Multiple Output Lines per LP

Using (4) [Null $_{(LP)}$ = Ts (O/L)], Fig.3 shows the null message transmission with the following simulation parameters: simulation time = 500 sec, *L* is uniformly distributed per output line (***O***). The number of output line may vary from 0 to 8 for all cases as show in Fig.3. Both numerical and simulation results present a comparison of null message transmission per LP versus multiple output lines.

4.2. CASE-II: Multiple LPs with Multiple Output Lines per LP

In CASE-II, we assume that we have multiple LPs with *O* output lines (fixed per LP). Let the output lines per LP is 4 with the simulation Time (*Ts*) of 500 sec. Using (5) $Null_{(m-LP)} \triangleq E_{Msg}\left(\frac{O}{L}\right)m\left(\frac{T_S}{E_{Msg}}\right) \triangleq T_S\left(\frac{O}{L}\right)m$, Fig.4 shows the null

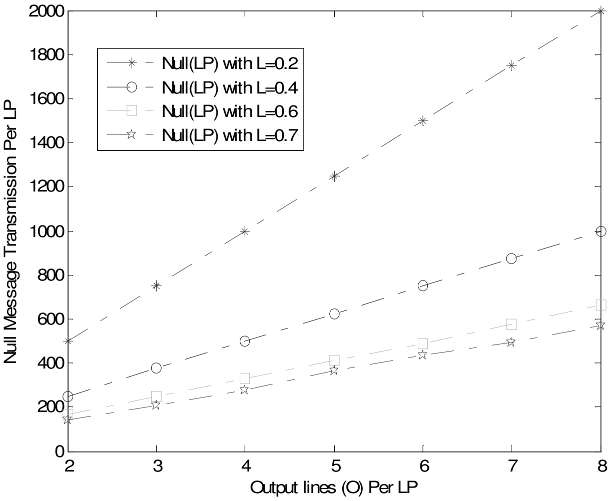

Fig3. Multiple output lines per LP versus null message transmission per LP

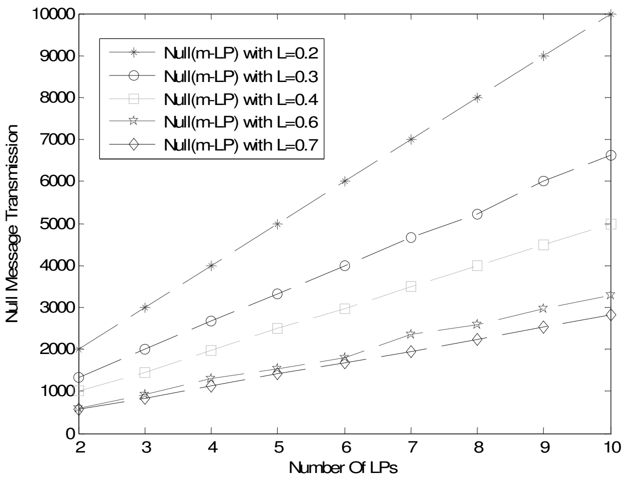

Fig4. Multiple LPs with Fixed Output Lines per LP versus null message transmission

message transmission with the following simulation parameters: simulation time = 500 sec, L is uniformly distributed per output lines (***O***), the output lines are assumed to be fixed for each LP (***O* = 4**). The numbers of LPs are varied from 1 to 10 as show in Fig.4.

4.3. CASE III: Multiple Output Lines per LP with Non-Uniform Distribution of Lookahead

For this simulation, we assume that we have single LP that has O number of output lines where each output line of an LP can have different value of Lookahead (L). Using (6), Fig.5 shows the null message transmission with the following simulation parameters: simulation time = 500 sec, L is non-uniformly distributed per output lines (***O***). The numbers of output lines are varied from 1 to 10 as show in Fig.5. Also, it should be noted that the value of Lookahead is chosen randomly within the range of 0 to 1 and assigned to each output line at run time.

4.4. CASE-IV: Multiple LPs with Multiple Fixed Output Lines where each Output line can have Different Lookahead Value

For this simulation, we assume that we have multiple LPs that can have fixed number of output lines where each line of an LP can have different value of Lookahead (L). Using (7), Fig.6 shows the null message transmission with the following simulation parameters: simulation time = 500 sec, L is non-uniformly distributed per output lines (***O***). The numbers of LPs are varied from 1 to 20 as show in Fig.6. Also, it should be noted that the value of ***m*** and ***O*** are both varying quantity for this particular scenario.

5. CONCLUSION

We have proposed a mathematical model to predict the optimum values of critical parameters that have great impact on the performance of NMA. The proposed mathematical model provides a quick and practical way for simulation designers to predict whether a simulation model has potential to perform well under NMA in a given simulation environment by giving the approximate optimal values of the critical parameters. We have experimentally verified that if critical parameters, specifically the Lookahead value, are chosen intelligently, we can limit the transmission of null messages among the LPs.

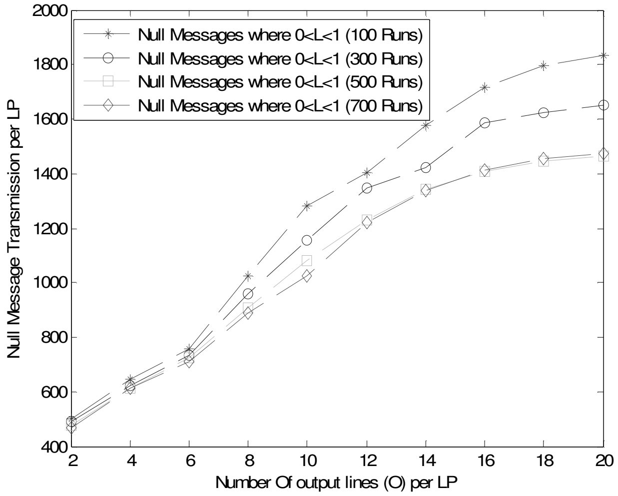

Fig5. Multiple Output Lines per LP with Non-Uniform Distribution of Lookahead versus null message transmission

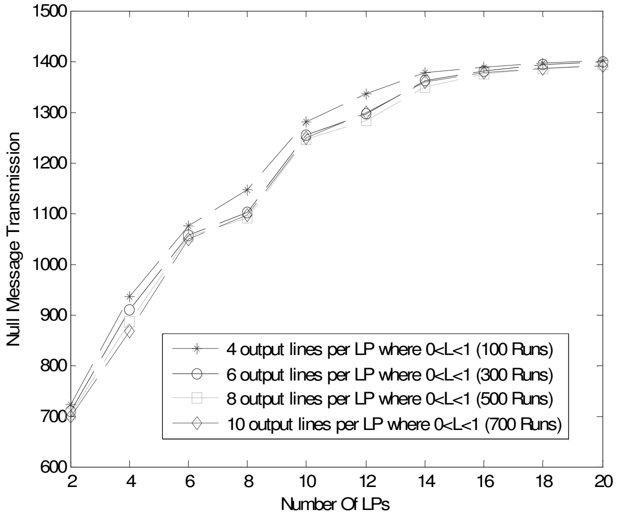

Fig.6. Multiple LPs and Multiple Fixed Output Lines with non-uniform distribution of Lookahead value versus null message transmission

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