Optimal Design of 3-Link Planar Manipulators using Grashof's Theorem

Sarosh Patel, Advisor: Dr. Tarek Sobh
Department of Computer Science & Engineering
University of Bridgeport, Bridgeport, CT

Abstract

The design of robotic manipulators is dictated by a set of pre-determined performance parameters and functionalities. These performance parameters are often defined in terms of the workspace dexterity, manipubility, and accuracy. These parameters can be used in the design process to optimize the manipulator configuration. In this work we present an algorithm for the optimal design of a three link planar manipulator using Grashof's Theorem.

Introduction

The problem of designing an optimal manipulator configuration is a very complex one, as the equations governing the motion of the end-effector in the workspace are non-linear and complex, often having no closed solutions. Prototyping methods such as kinematic synthesis and numerical optimization are very time consuming.

The inherent complexity of kinematic synthesis has helped in making a strong case in favor of rapid prototyping methods where manipulators are designed with very specific performance requirements or tasks point specifications. Rapid prototyping allows designers to spend more time in designing, simulating and evaluating the performance different manipulator configurations instead of solving mathematical models describing kinematics chains.

Dexterity Index

Dexterity index of a manipulator about a point in the workspace can be defined *as a measure of a manipulator to achieve varying orientations about that point*. In their work Kumar and Waldron introduced the parameter Dexterity index (D) as another measure for manipulator performance. They defined dexterous workspace *as a volume within which every point can be reached by the manipulator end-effector with any desired orientation.*

The orientation at any given point in the workspace can be represented in terms of the yaw (α), pitch (β) and roll (γ) angles as: $R_{xyz} = R_{x,\gamma} \, R_{y,\beta} \, R_{z,\alpha}$

All three of the angles are variable with the range $0 - 2\pi$ to provide all possible orientations. The dexterity index can be defined as the summation of the dexterity indices about each of the axis':

$$D = \frac{1}{3}(d_x + d_y + d_z) = \frac{1}{3}\left(\frac{\Delta\gamma}{2\pi} + \frac{\Delta\beta}{2\pi} + \frac{\Delta\alpha}{2\pi}\right)$$

where dx, dy, and dz are X, Y and Z dexterity indices
$\Delta\alpha, \Delta\beta$ and $\Delta\gamma$ are the range of possible yaw, pitch and roll angles about a point

Grashof's Theorem

Grashof proposed simple rule to judge the rotatability of links in four-link kinematic chains. Consider a four-link kinematic chain, comprising of four links a, b, c, and d as shown in the figure. Let a be the longest link and d be the short link in the chain, such that According to Grashof's criterion there exists at least one link that can fully revolve with respect to the other links if:

$$a + d \le b + c$$

i.e. the sum of the longest and the shortest link should be less than or equal to the sum of the other two links. And none of the links are fully revolute if:

$$a + d > b + c$$
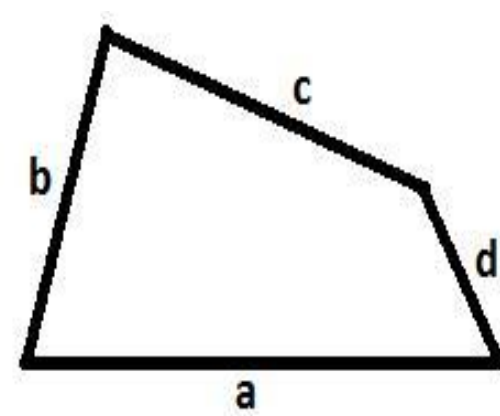
In this work Paul proved that this criterion was both necessary and sufficient condition for the existence of a fully rotatable link in the chain and such a mechanism is also known as a Grashof linkage. In a Grashof linkage the shortest link in the chain is the always fully revolvable with respect to the other links if the criterion is satisfied.

Design Optimization

In order to optimize the link lengths to achieve maximum dexterity in the area of interest or trajectory we propose the following algorithm.
1. Let dmax be the maximum distance from the base and dmin the minimum distance while following a trajectory such that
2. Let l3 be the shortest link in the manipulator. The length of is equal the minimum link length practically possible, this is determined by the other factors such as the size of the motors, loading on the manipulator.
3. Next we select l1 and l2 such that :

$$l_1 = \left(\frac{d_{max} + l_3}{2}\right) + 1; \quad l_2 = \left(\frac{d_{max} + l_3}{2}\right)$$

as dmax and l3 are finite and known and can easily be determined.

Design & Simulation

In order to design a 3-link manipulator having maximum dexterity in the region $10 < d < 20$, we follow the above listed steps. Accordingly, we have l1 =15; l2=14 and l3 =8.
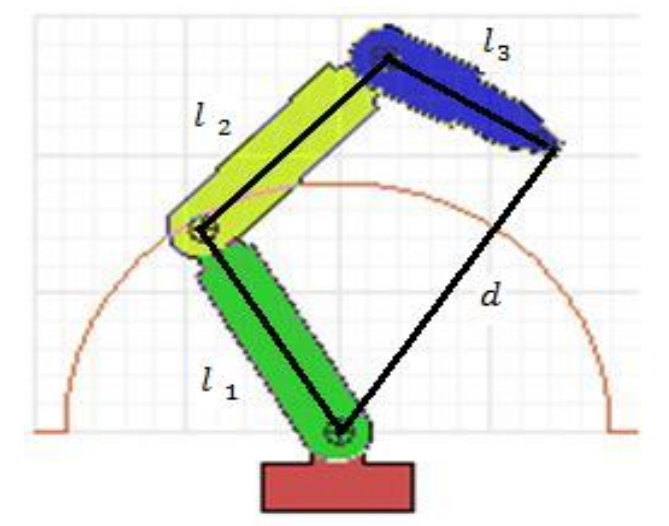
Case 1: Completely Revolute Joints

At points in the workspace where the Grashof's criterion is met the shortest link in the chain is completely revolute, therefore the end-effector can have infinite orientations about the point. As seen from the figures below the manipulator has maximum Dexterity Index (D) = 0.33 10 < d < 20, which was the design criteria.
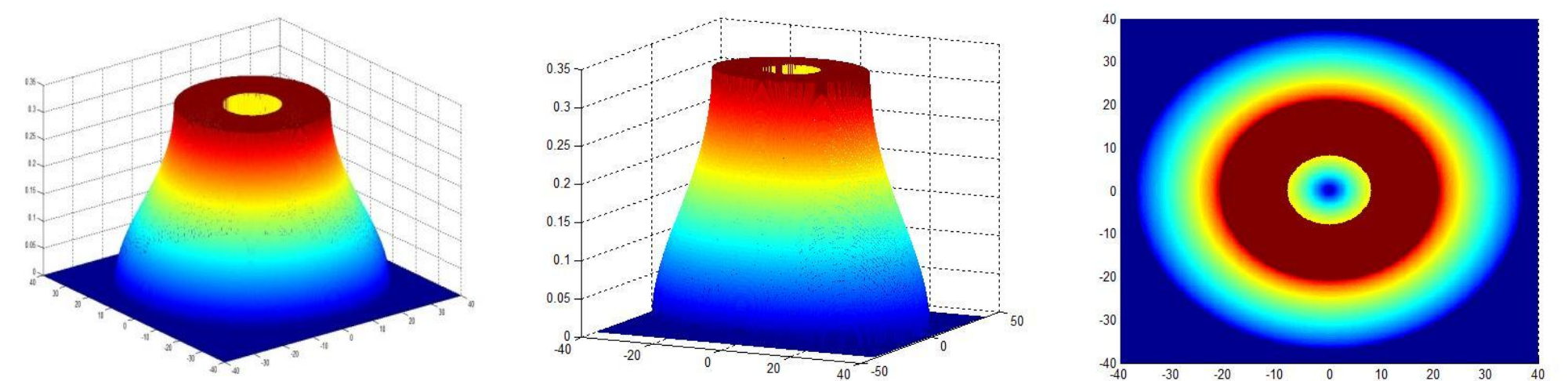
The dexterous workspace be divided into 4 as shown in the figure below:
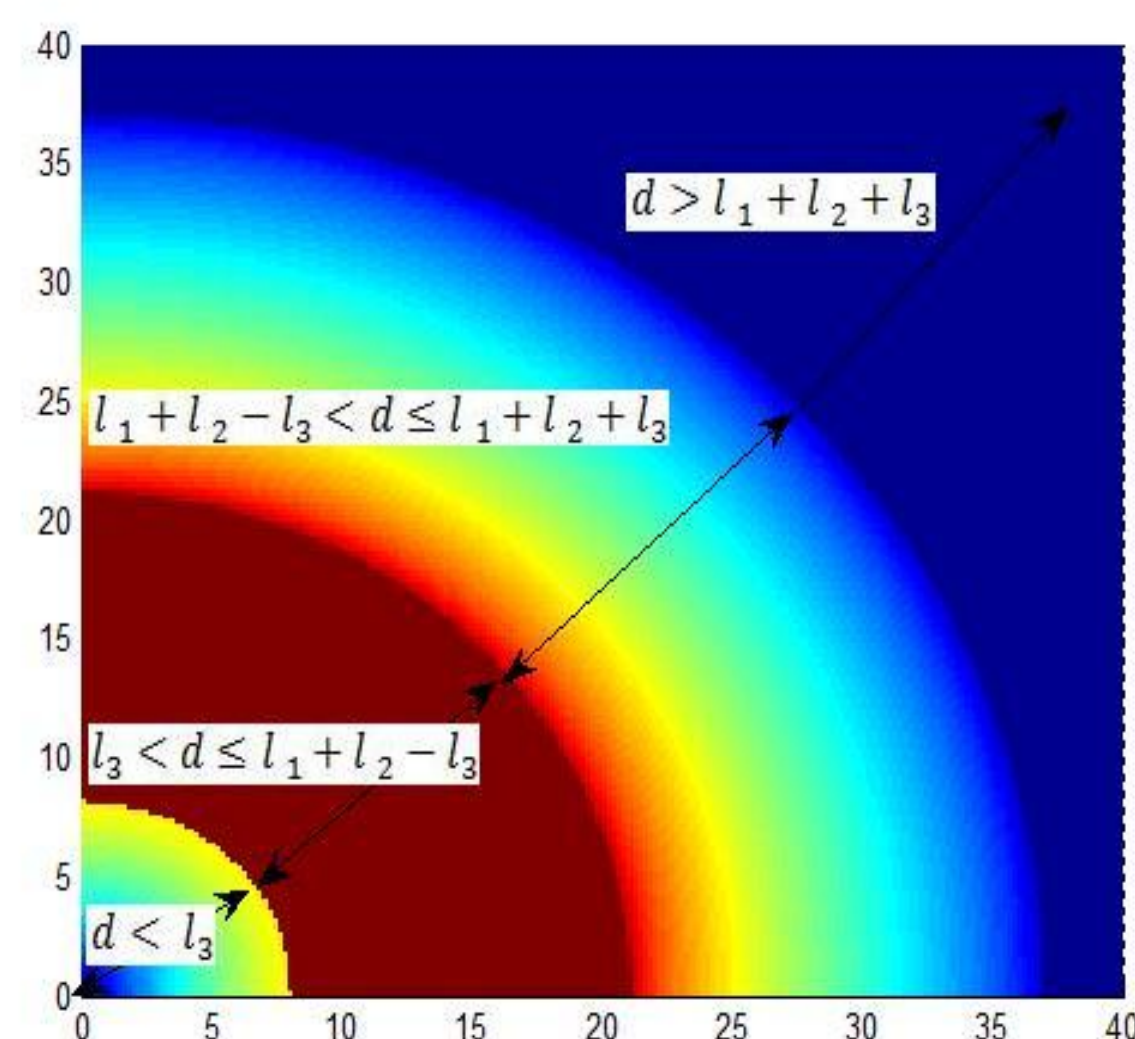
Case 2: Changed order of links

The dexterity of the manipulator is also a function of the relative positioning of the links. To prove in this, in this case we in this case we change the order of the links by swapping the lengths of links l2 and l3. As seen from the figure, with only the positioning of the links changed the size of the workspace remains the same but maximum dexterity region is diminished .
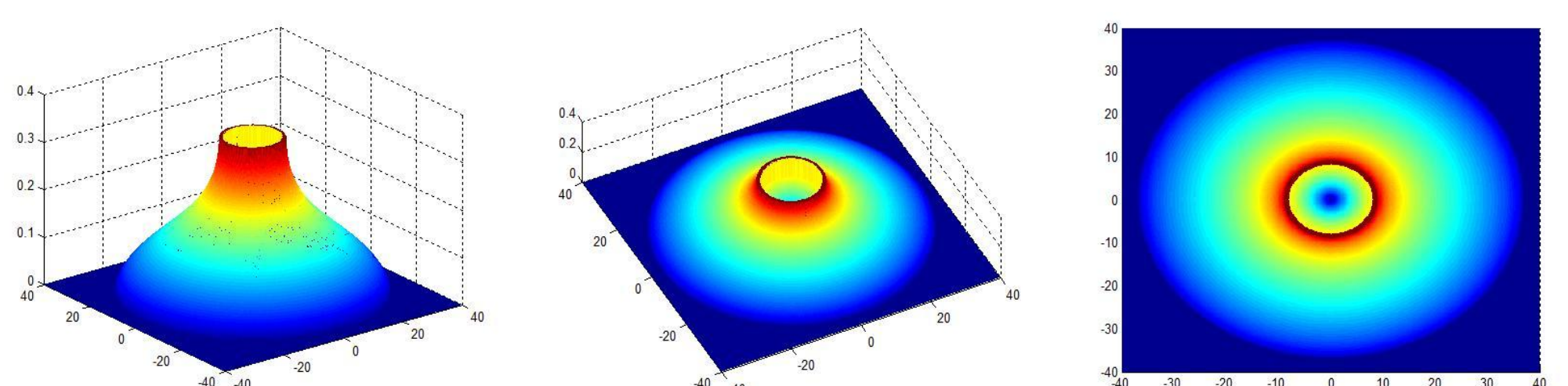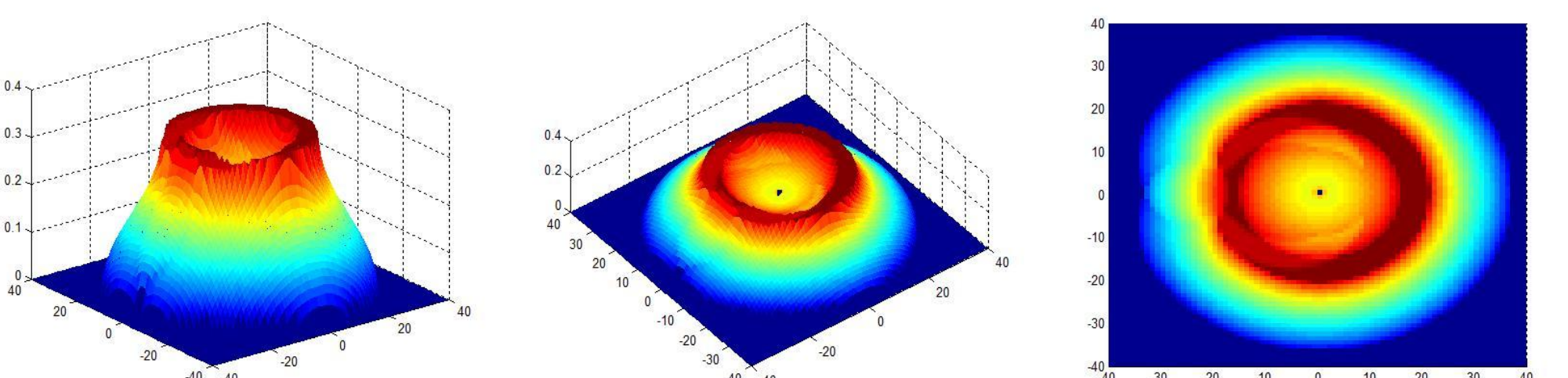
Case 3: Limited joint Angles

The rotation of joints is often limited due various mechanical and/or workspace constraints. In this case we consider a scenario where the range of the joint angles assumes practical values.

$$-150 \le \theta 1 \le 150; \; -150 \le \theta 2 \le 150 \text{ and } -175 \le \theta 3 \le 175$$

As seen from the figure the dexterity plot of the left half is distorted, this because of the joint limitations.

Conclusion

In this work we have proposed a simple algorithm to optimize the link lengths of a 3-Link planar manipulator. Using Grashoff's criterion, we optimize the link lengths such that the one of the links is has the freedom to be completely revolute. Dexterity plots generated by simulating the optimal manipulator configurations meet the design criteria. Our simulations also confirm that the manipulator has maximum dexterity area when the shortest link is the last link of the manipulator.