ESSAYS ON MACROECONOMIC DYNAMICS: LEARNING, UNCERTAINTY, AND HETEROGENEITY IN CREDIT CRISES

by

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Abstract

This thesis covers two research topics. Chapter 2 is an investigation into the properties of the equity risk premium and its relationships with uncertainty and macroeconomic fluctuations. A large literature suggests that the expected equity risk premium is countercyclical. Using a variety of different measures for this risk premium, we document that it also exhibits growth asymmetry, i.e. the risk premium rises sharply in recessions and declines much more gradually during the following recoveries. We show that a model with recursive preferences, in which agents cannot perfectly observe the state of current productivity, can generate the observed asymmetry in the risk premium. Key for this result are endogenous fluctuations in uncertainty which induce procyclical variations in agent's nowcast accuracy. In addition to matching moments of the risk premium, the model is also successful in generating the growth asymmetry in macroeconomic aggregates observed in the data, and in matching the cyclical relation between quantities and the risk premium.

Chapters 3 and 4 are an investigation into the distribution and dynamics of household debt. We present new empirical facts on the distributional dynamics of household debt around the Great Recession in the US using survey data from the Panel Study of Income Dynamics. We document that it is the 60% of households toward the middle of the income distribution that are responsible for the aggregate reduction of debt from the onset of the financial crisis in 2007 until 2015, not the 40% in the tails of the distribution. We extend the current class of heterogeneous-household models by explicitly tracking the distributions of gross debt and gross savings separately during a simulated credit crunch – instead of calculating exclusively the net financial positions of households as in the standard framework. The model successfully replicates the relative importance of the different income groups in the aggregate reduction of household debt. The results are driven by endogenous heterogeneity in the intertemporal utility cost of debt. In addition, the models provides new insights into the effectiveness of monetary policy when households are highly indebted. We show that collateralised debt is a stronger channel than liquid savings for the transmission of monetary policy.

PRIOR CIRCULATION

Chapter 2 of this thesis was first circulated in 2019 as a working paper (Görtz and Yeromonahos (2019)) with my main advisor Dr. Christoph Görtz under the name Asymmetries in Risk Premia, Macroeconomic Uncertainty and Business Cycles, as part of CESifo Working Paper Series (number 7959).

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Acronyms

BEA Bureau of Economic Analysis

CAPD Cyclically Adjusted Price-Dividend Ratio

CAPE Cyclically Adjusted Price-Earning Ratio

CES Constant Elasticity of Substitution

CPI Consumer Price Index

EIS Elasticity of Intertemporal Substutition

GDP Gross Domestic Product

HP filter Hodrick-Prescott filter

NIPA National Income and Product Accounts

OECD Organisation for Economic Co-operation and Development

PSID Panel Study of Income Dynamics

RBC Real Business Cycle

RRA Relative Risk Aversion

SPF Survey of Professional Forecasters

TFP Total Factor Productivity

UK United Kingdom

US United States

WWII Wold War II

Chapter 1

Introduction

The Great Recession has deeply affected our collective understanding of the macroeconomy. It has exposed the critical shortcomings of a representative-agent approach to many macroeconomic phenomenons, as well as the dangers of lacking accurate micro-foundations in macroeconomic theory. In addition, it has highlighted how the business and financial cycles are deeply intertwined. Policy makers around the world such as Yellen (2016) of the US Federal Reserve, Constâncio (2017) of the European Central Bank, and Kuroda (2017) of the Bank of Japan have insisted on the necessity to learn from this worldwide crisis and to update economic theory. As a response, macroeconomic research is undergoing a major evolution on three fronts. First, macroeconomic researchers are intensifying their efforts to understand the relationships between macroeconomic fluctuations and financial phenomenons. Second, it is increasingly common for macroeconomists to investigate micro-data, including survey data, to uncover important facts about the behaviour of firms and households and to construct theoretical models around these facts so as to better understand their implications in a macro context. Third, heterogeneous-agent models are no longer restricted to the study of cross-sectional phenomena as they used to be for several decades before the Great Recession, but they are now increasingly used to present novel insights into aggregate fluctuations as well. This thesis makes contributions to these three strands of the literature.

In chapter 2, we contribute to a growing literature jointly studying the behavior of the equity risk premium and macroeconomic dynamics. It is already well known that the equity risk premium is time-varying and countercyclical. At times of strong economic growth and financial optimism, agents require a small compensation for holding risky stocks instead of risk-free and short-term government bills. By opposition, during recessions and at times of high uncertainty, agents must be offered a stronger compensation if they are to be convinced to invest in the risky stock market. However, the literature has so far not addressed one important aspect of the equity risk premium: its asymmetric growth. Using US data post-WWII, we employ a variety of relevant empirical measures and we document that increases in the expected equity risk premium are sharp and short, while decreases are long and gradual. This asymmetry translates into a positively skewed distribution of the growth rate of the expected risk premium. We demonstrate the robustness of our findings by calculating the growth skewness across sub-samples, and also for different investment horizons.

We then build a model capable of replicating our empirical observations. The basis of the model is a standard real business cycle (RBC) model augmented with capital adjustment costs where the agents have recursive preferences of the Epstein and Zin (1989) type. This setup allows for the existence of sizable equity risk premium by disentangling the elasticity of intertemporal substitution (EIS) and the degree of relative risk aversion (RRA). In our empirical measures, expected risk premia incorporate information about future stock returns. This is the reason why they are time varying and may differ from realised ex-post risk premia. The mechanism in the model is similar. Households need to nowcast the state of Total Factor Productivity (TFP) because they only receive a noisy signal about current TFP. The Cobb-Douglas production function includes an additive noise term that the agents cannot observe separately from actual TFP. Productivity is itself represented by a two-state Markov process. The growth asymmetry of the risk premium in the model stems from procyclical variations in the quality of the signal agents receive about TFP, which in turn cause procyclical variations in agents' nowcast precision. At times of strong economic activity, when factors of production are used in large quantities, the signal on the state of productivity becomes stronger relative to the noise. Therefore, agents can learn the true state of productivity more easily when the business cycle is close to the peak. This implies that at the end of a phase of economic expansion, when productivity switches from its high state to its low state, agents are confident in their nowcast and the risk premium increases sharply as soon as the recession begins. By contrast, at the trough of the business cycle, when factors of productions are not used intensively, the signal is weak relative to the noise and agents cannot nowcast with high certainty the coming economic recovery. This implies that the risk premium falls slowly during the recovery.

The model successfully replicates the observed growth asymmetry of the risk premium as well as its relationship with the business cycle. Key to the success of the model are procyclical variations in agents' nowcasting precision, themselves resulting from endogenous procyclical variations of the signal-to-noise ratio. This mechanism finds strong support in the data. Using the Survey of Professional Forecasters (SPF), we document that the median absolute nowcast error for real GDP growth is countercyclical and higher when the economy contracts. We also finds the absolute median nowcast error is particularly large when the risk premium rises strongly. Defining uncertainty as the dispersion of GDP growth nowcast, we also find that uncertainty varies countercyclically and is particularly high when the economy contracts and when the risk premia are high. In addition, the model is successful in generating other widely documented business cycles stylised facts, such as the negative skewness of key macroeconomic aggregates. Recessions are sharp and short, but expansions are gradual and long. The model also successfully matches the countercyclical movements of risk premia.

This chapter makes a bridge between two stands of the literature. On the one hand, theoretical research in finance has mostly focused on endowment economies without studying the relationships between macroeconomic aggregate and risk premium. On the other hand, standard macroeconomic models do not give a significant role to the equity risk premium. In our model, we study the growth asymmetry of the risk premium and macroeconomic aggregates in a single setting. This is a contribution to a growing body of theoretical works that jointly study the business and financial cycles, such as Jermann (1998), Bekaert et al. (2009), Kaltenbrunner and Lochstoer (2010), Gourio (2013), Corradi et al. (2013) and Campbell et al. (2019).

This chapter is also related to the literature on the role played by beliefs about TFP for the dynamics of macroeconomic and financial aggregates, as in Beaudry and Portier (2006), Barsky and Sims (2011) and Cascaldi-Garcia and Vukotic (2019). Empirical studies have documented the close relationship between news about future TFP, stock prices and risk premia (Görtz et al. (2021)), and also the influence of expectations on output fluctuations (Milani (2011)). In addition, Enders et al. (2017) show that GDP nowcast errors in the SPF play a significant role and account for about 15% of GDP fluctuations. Therefore, the empirical evidence legitimise the key mechanism of our model where procyclical fluctuations in the precision of the signal lead to procyclical fluctuations in nowcast accuracy, and are ultimately responsible for the dynamics of macroeconomic and financial aggregates.

Our work is closely related to a literature that studies how variations in uncertainty lead to an asymmetric speed of learning, e.g. Veldkamp (2005), Boldrin and Levine (2001), Fajgelbaum et al. (2017). Finally, our mechanism for endogenous variations in the signal-to-noise ratio is closely related to Van Nieuwerburgh and Veldkamp (2006), Ordoñez (2013) and Saijo (2017). In these papers, endogenous variations in uncertainty imply state dependencies in the strength of agents' responses to shocks, leading to asymmetries in the growth of macroeconomic aggregates. We extend this framework to study asymmetries in the growth of risk premia as well.

Chapters 3 and 4 are part of the same project inspired by the deep evolution that theoretical macroeconomics models are currently undergoing. As observed by Kaplan and Violante (2018) in their review of the recent literature, a new framework for macroeconomic analysis is emerging. Representative-agent models have failed to provide convincing explanations for the Great Recession because they are not suited to study the distributional issues that played an essential role in the unfolding of the crisis. To address this gap in knowledge, heterogeneous-agents model are now being used in innovative ways. The most notable evolution is the emergence of a new branch of the literature that simultaneously studies distributional issues and aggregate fluctuations. These two topics used to be treated as separate research areas and were examined in separate frameworks for several decades before the Great Recession. Indeed, Krusell and Smith (1998) demonstrated that in standard heterogeneous-agent models, the dynamics of macro aggregates in response to a TFP shock were in fact generally equivalent to the predictions of representative-agent models. Since representative-agent models are easier to solve numerically, they used to be the instrument of choice for the study of macroeconomic fluctuations and stabilization policies. Complex heterogeneous-agent models were limited to areas of research where they were clearly inevitable, such as inequalities, economic mobility or other cross sectional topics. Modern heterogeneous-agent models are different from the older generation. They are designed specifically to explore the consequences of distributional issues on aggregate fluctuations. In particular, there is a growing consensus that heterogeneity in the composition of the portfolios of households is essential to understand the underlying sources of fluctuations in macroeconomic aggregates as well as the heterogeneity in the transmission of monetary policy. Chapters 3 and 4 make contributions to this field of the literature.

In Chapter 3, we present new empirical evidence on the distribution and dynamics of the portfolios of households. A growing body of empirical studies explore how the composition of the portfolio of households influences the response of the economy to aggregate economic shocks and to stabilization policies. Mian et al. (2013) find that more heavily levered household reduced their consumption more during the 2006-2009 housing collapse. Kaplan et al. (2014) reveal that households with less liquid savings adjust their consumption to a larger extent in response to transitory changes to their income. Cloyne and Surico (2017) document that households with a mortgage make large and significant adjustments to their consumption after a fiscal policy shock, while households without a mortgage do not adjust their consumption. Cloyne et al. (2020) show that the sign and the size of the response of households' consumption after a change in interest rates depend on households' net asset positions.

We contribute to this literature by analyzing survey data from the Panel Study of Income Dynamics (PSID) from a new angle. The innovation of our approach resides in the fact that we document the relative contributions of different income groups to the aggregate dynamics of household debt and savings separately, from 2001 to 2015. Our key new finding is that it is mainly the 60% of households around the middle of the income distribution that are responsible for the aggregate reduction of gross debt from the onset of the financial crisis in 2007 until 2015, not the 40% of households toward the tails. The literature has so far not studied the causes and implications of this heterogeneity in the deleveraging process. It is important to notice at this point that it is typical in the theoretical literature to focus only on net financial positions, defined as the difference between gross savings and gross debt. Using this measure, households toward the bottom of the income distribution appear to be solely responsible for the aggregate deleveraging. This is because poor households are the only ones to have more gross debt than gross savings (i.e. they are the only ones with negative net financial positions) at the start of the crisis. Therefore, our perception of who in the distribution of income is responsible for the dynamics of household debt crucially depends on whether debt is defined in net terms as is common in the existing literature, or in gross terms as we document for the first time in detail.

Consistently with Cloyne et al. (2020), we find that mortgage debt represents by far the largest share of total household debt and that close to half of the population has a mortgage. We also find that it is mortgages that drive the overall dynamics of total household debt for all income groups. Our key findings remain identical if we base our calculations on mortgage debt alone, or on the sum of mortgage debt plus other types of non-collateralised debt present in the survey, such as, e.g. credit card debt. As in Flodén and Lindé (2001), we find there is little income mobility in the PSID data, meaning that the migration of households across income groups does not drive our key results. Finally, we also find that bankruptcies are relatively rare and not concentrated in any particular income group. This is consistent with Adelino et al. (2018) and it implies that our key findings on the relative importance of different income groups are also not driven by bankruptcies.

The PSID is one of the very rare surveys that combines all the features necessary to conduct our analysis. It is one of the longest running and largest surveys in existence. It is widely used both in academic research and by policy makers, as reviewed by Moffitt and Zhang (2018) and Smeeding (2018). The survey is biennial and includes close to 67000 household-wave observations representative of the entire US population in the period from 2001 to 2015. Among other topics it includes detailed questions about households' income, savings and debt. Minimal cleaning is necessary because missing responses are rare. In our sample less than 3% of the relevant data are missing. The granularity of the data allow us to precisely construct the variables of interest according to standard definitions in the literature. In addition, given its complex design the PSID comes with a sampling error computation model necessary for the construction of confidence intervals robust to possible sampling errors around the estimated statistics. We use this feature to demonstrate the robustness of our results.

In Chapter 4, we build the first model capable of replicating the empirical observations we made on the relative contributions of different income groups to the aggregate dynamics of household gross debt during the Great Recession. We also discuss the implications for the transmission of monetary policy to households. We build on the framework of Guerrieri and Lorenzoni (2017) and add two features: an explicit distinction between gross debt and gross savings and a realistic structure of tax incentives for households with a mortgage. Households save for precautionary reasons because their future income is uncertain. They face unemployment risk as well as idiosyncratic productivity shocks affecting their labour income when they are employed. Households can also choose to accumulate collateralised debt (mortgages) to finance their investment in durable goods (houses). Households use a fraction of their stock of durable goods as collateral against their debt. The maximum loan-to-value ratio – i.e. maximum fraction of the stock of durable goods that can be used as collateral – is common to all households. Households also consume non-durable goods. They can deduct the interest paid on their debt from their taxable income – an arrangement known as the mortgage interest tax deduction in the US. We focus on collateralised debt in the model because we find in our empirical investigation that mortgages represent by far the largest share of total household debt, and because mortgage loans have already featured importantly in the literature, e.g. Iacoviello (2005), Guerrieri and Lorenzoni (2017), Garriga et al. (2017) and Kaplan et al. (2020)).

In steady state, the model replicates our empirical findings on the distribution of mortgage debt in 2007 at the beginning of the crisis. Poorer households tend to hold smaller stocks of durable goods, therefore their borrowing constraint is more often binding or close to be binding. In addition, debt represents a relatively heavier burden on their future budget and therefore on their intertemporal utility. For these reasons they hold on average smaller amounts of debt. Wealthier households with larger amounts of durable goods have slack borrowing constraints and can repay their debt at a lower cost in terms of intertemporal utility. They can afford to hold on average larger amounts of debt. The structure of taxes affects the aggregate quantity of debt. Without the mortgage interest tax deduction, we find that the aggregate quantity of debt is too low in comparison to the aggregate quantity of savings. In addition, the predicted net financial positions are also consistent with our empirical observations as well as with the predictions of existing models. Households toward the bottom of the income distribution are the only ones to hold on average higher amounts of gross debt than gross savings. Middle class households on the contrary have more gross savings than gross debt, while higher earners have even higher net savings.

The model is also successful in replicating our findings on the dynamics of the distribution of debt from 2007 to 2015. We progressively tighten the maximum loan-to-value ratio to simulate the fall in house prices. The model predicts a reduction of aggregate debt by 21%, exactly as in the data. The 60% of households toward the center of the income distribution explain 75% of this aggregate reduction of debt (against 72% in the data), while the 10% and 30% of households respectively at the upper and lower tails collectively explain only 25% of the aggregate dynamics (against 28% in the data).

The model also provides new insights into the transmission of monetary policy. We simulate an intervention of the Central Bank to cut interest rates on both debt and savings while the borrowing constraint remains untouched. We present two new findings. First, debt and savings represent two cumulative channels for the transmission of monetary policy. For a given amount of debt, households with larger savings increase their consumption proportionally more in response to a cut in interest rates. And it is also true that for a given amount of savings, households with a larger debt increase their consumption proportionally more in response to a cut in interest rates. Empirical studies such as Cloyne et al. (2020), Kim and Lim (2020), Vissing-Jørgensen (2002) and Kaplan et al. (2014) have documented each of these effects separately. Our theoretical model is the first where both can be observed simultaneously. Second, we observe that the debt channel is stronger than the savings channel. Households with larger quantities of debt relative to the total size of their portfolio – defined as the sum of savings plus debt – react more strongly to the monetary policy intervention.

This chapter contributes to the deep evolution macroeconomic theory has been undergoing since the aftermath of the Great Recession. Modern macro models such as Guerrieri and Lorenzoni (2017), Kaplan et al. (2018), Auclert (2019), Kaplan et al. (2020) or Benigno et al. (2020) include sophisticated micro-founded mechanisms necessary to replicate the heterogeneity of households' portfolios as documented in the empirical literature. The main innovation of these models is to show how balance-sheet-driven heterogeneity among households with incomplete markets plays a key role in the transmission of aggregate shocks and monetary policy. Households may have access to a single asset as in, e.g. Guerrieri and Lorenzoni (2017), or to two different assets as in, e.g. Kaplan et al. (2018) where households can chose to invest in a liquid or in an illiquid asset. However, all the assets available to households in these models can typically be held indifferently in positive or in negative quantities. This implies that only the net financial positions of households are observable. Gross debt and gross savings are not modeled individually. Therefore, existing models replicate successfully the distribution of households' net financial positions, but they remain silent about the distributions and the dynamics of gross debt and gross savings. This branch of the literature does not explore yet the motivations of households who so often decide to simultaneously hold debt and savings in their portfolios. The implications of this behaviour for the response of the economy to aggregate shocks and the transmission of stabilization policies have so far remained unexplored. Our model fills this gap in the literature.

Chapter 2

Asymmetries in Risk Premia, Macroeconomic Uncertainty and Business Cycles[1]

2.1 Introduction

The most recent US investment and housing booms that ended abruptly in 2001 and 2007 have been associated with highly optimistic beliefs about profitability. In the former case, beliefs about profitability were linked to information technology and in the latter to house price gains. Both booms were associated with times of low uncertainty and saw long hikes in stock markets. Adjustments of beliefs about profitability resulted in sharp recessions, heightened uncertainty, and strong corrections in stock markets, e.g. Beaudry and Portier (2006) and Shiller (2007). In this situation, investors were less willing to bear financial risk, and for a given level of stock market risk, they required a higher compensation to hold stocks instead of a risk free short-term asset. Indeed, this equity risk premium increased sharply at the brink of both recessions, very much in contrast to the slow and gradual decline that could be observed during the preceding booms. This growth asymmetry shows in positively skewed distributions of risk premium growth — skewness is 0.64 and 1.00, respectively over these two business cycles.[2]

[1] This chapter was first circulated as a working paper in Görtz and Yeromonahos (2019).

[2] These skewness statistics over the two business cycles are significant with p-values of 0.065 and 0.017, respectively. The positively skewed distribution implies that positive changes in risk premia are more extreme

There is a large body of work in the finance literature on risk premia, which for example provides substantial empirical evidence that the equity risk premium varies over time and is countercyclical.[3] A recent and growing literature jointly studies both the behavior of risk premia and macroeconomic dynamics. We contribute to this body of work which has not considered the important asymmetric feature of risk premia. We first document the degree of asymmetry in the data and then develop a structural model with endogenous countercyclical variations in uncertainty that is consistent with this feature in the data. To the best of our knowledge this is the first academic work that tackles this issue.

We start by computing statistics on growth asymmetry for a variety of expected equity risk premium measures that have been found relevant in the literature. We document for the post-WWII US economy that growth rates of all risk premium measures exhibit positive skewness. This growth asymmetry is not only a salient feature over the entire sample, but it is also present across subsamples — which we specify to include two consecutive business cycles as defined by the NBER's Business Cycle Dating Committee — and for different investment horizons of risk premia. In particular, we construct risk premium measures using models based on the historical mean of realised stock market returns in excess of Treasury bond yields, and models based on predictive time series regressions of equity returns on selected fundamentals. We further employ direct risk premium measures based on responses of Chief Financial Officers recorded in the Duke CFO Global Business Outlook Survey. The broad support for growth asymmetry is remarkable given the substantial diversity in assumptions underlying the various employed risk premium measures.

We then build a structural model that is consistent with the empirically observed growth asymmetry in the risk premium. The core of the model is a standard real business cycle model with capital adjustment costs and preferences of the Epstein and Zin (1989) type. This setup disentangles relative risk aversion and elasticity of intertemporal substitution and gives rise to a risk premium. Our empirical measures for expected risk premia incorporate information about future returns, so that their value can differ from realised ex-post risk premia. In the model, deviations from fundamentals are possible, because agents need to form nowcasts about the state of TFP. Nowcasting is required as the otherwise standard Cobb-Douglas type production function includes an additive noise term and neither TFP nor the noise term can be observed separately at the time decisions

than negative changes.

[3]See, e.g. Fama and French (1989), Ferson and Harvey (1991), and Bekaert and Harvey (1995).

about production inputs are made.[4]

The key mechanism to generate growth asymmetry in the risk premium are endogenous changes in the degree of uncertainty about the state of productivity, which in turn induce procyclical variations in agent's nowcast precision. TFP follows a two-state Markov process and agents employ a Bayesian learning technology to form a nowcast about the state of current productivity. Intensified use of production inputs amplifies the signal on the state of productivity relative to the noise and results in endogenous procyclical variations of the signal-to-noise ratio. When production inputs are high, agent's nowcasts are relatively accurate as uncertainty about the state of productivity is low. The end of a boom, implied by a change from the high to the low productivity state, can be nowcasted with high accuracy and the risk premium will increase sharply. The reduced use of production inputs during the following recession leads to a lower signal-to-noise ratio and a situation of heightened uncertainty about the state of productivity. For this reason, agent's nowcasting accuracy increases only slowly during the following recovery. The associated gradual decline in uncertainty comes along with a slow and gradual decline in the risk premium.

The model is calibrated to match nowcast precision in the SPF. Overall, it is successful in generating the observed growth asymmetry in the risk premium and in matching the risk premium's relation with macroeconomic activity in the data. The model captures the empirically observed positive skewness in the risk premium, while a framework without the endogenous variation in nowcast accuracy does not imply a skewness substantially different from zero. The model's ability to generate growth asymmetry rests on the procyclicality of nowcast precision and the associated variations in uncertainty. This mechanism finds strong support in the data. Firstly, the median absolute nowcast error for real GDP growth from the SPF varies countercyclically and is notably heightened when the economy contracts. We also find this absolute median nowcast error is particularly high at times when the risk premium rises strongly. Secondly, we employ the dispersion in nowcasts for GDP growth from the SPF as a proxy for uncertainty to provide further corroborative evidence for the model mechanism. We report uncertainty varies countercyclically — in line with evidence in the literature, see, e.g. Bloom (2014) — and is notably heightened at times when the economy contracts and when risk premia are high. Endogenous procyclical variations in agent's nowcasting precision are crucial also for the model's ability to generate the well known stylised business cycle

[4] This setup is consistent with the nowcasting process of statistical agencies documented in Faust et al. (2005). They describe the preliminary nowcast to be the sum of the final GDP announcement and an additive noise term.

fact of negatively skewed growth rates in macroeconomic aggregates — i.e. expansions in economic activity are long and gradual while recessions are sharp and short. In addition to the skewness statistics, it is notable that our model is also successful in matching the countercyclical movements of risk premia observed in the data.

Our paper is related to several strands of the literature. There is a large body of work in the empirical finance literature on risk premia. Yet, existing theoretical work in finance has mostly been confined to endowment economies that do not consider feedback between time-varying risk premia and macroeconomic aggregates.[5] On the other hand, most standard macroeconomic models do not include a meaningful role for the risk premium. Our work links to a growing literature that jointly studies the behavior of macroeconomic aggregates and risk premia in bond or equity markets, e.g. Jermann (1998) and Kaltenbrunner and Lochstoer (2010). Gilchrist and Zakrajšek (2012) empirically document a close link between increases in the excess bond premium and a deterioration of macroeconomic conditions. Gourio (2013) develops a macroeconomic framework driven by variations in disaster risk that reproduces key features of corporate bond risk premia — such as their countercyclicality — and studies their implications for business cycles. Campbell et al. (2019) show how macroeconomic dynamics drive risk premia in bond and equity markets and Corradi et al. (2013) find that the level and volatility of fluctuations in the stock market are largely explained by business cycle factors. Bekaert et al. (2009) highlight the role of uncertainty for the countercyclical volatility of asset returns. We contribute to this literature by explaining the growth asymmetry in risk premia and macroeconomic aggregates.

Our paper is also related to work that highlights the importance of beliefs about current and future TFP for fluctuations in macroeconomic and financial aggregates, e.g. Beaudry and Portier (2006), Barsky and Sims (2011), Cascaldi-Garcia and Vukotic (2019). Görtz et al. (2021) document a close link between changes in expectations about future TFP, stock prices and risk premia. Risk premia incorporate expectations about future stock market returns and as such, they can differ from ex-post realizations. In our model, this can be the case as agents need to form nowcasts to learn about the current state of productivity. Milani (2011) highlights the relevance of expectations and learning for output fluctuations. He relaxes the rational expectations assumption to allow for agent's learning in a New Keynesian framework and estimates the model using forecast data from

[5] Jermann (1998) and Lettau and Uhlig (2000) stress that many asset pricing models which are successful in endowment economies do not generalise well to production economies.

the SPF. Enders et al. (2017) compute GDP nowcast errors based on the SPF and show that these are sizable and play a non-negligible role, accounting for up to 15% of output fluctuations.

While the above literature typically does not consider asymmetries, in our framework agents have to solve a signal extraction problem with time varying parameters to explain growth asymmetries in the data. In this respect, our work links closely to a literature that considers an asymmetric speed of learning and time variation in uncertainty, e.g. Veldkamp (2005), Boldrin and Levine (2001), Fajgelbaum et al. (2017). Our mechanism for endogenous variations in the signal-to-noise ratio is closely related to Van Nieuwerburgh and Veldkamp (2006) and Ordoñez (2013) who employ it to explain steepness asymmetry in macroeconomic aggregates observed at business cycle frequencies. A similar mechanism is used in Saijo (2017). In this paper, agents learn about the efficiency of investments in an environment where uncertainty varies endogenously and has adverse effects on economic activity. Common across this literature is that endogenous variations in uncertainty imply state dependencies in the strength of agents' responses to shocks. While also our model relies on such a type of mechanism, the studies above use it to explain empirical facts related to macroeconomic aggregates. We add to this literature by studying asymmetries in risk premia.

The remainder of the paper is structured as follows. Section 2.2 provides an overview about the data. In Section 2.3 we provide details on the estimation of risk premium measures and document their growth asymmetry. Section 2.4 describes the model and Section 2.5 the calibration and computational details. Section 2.6 discusses the model mechanism that gives rise to asymmetries and results from simulations. Section 2.7 concludes.

2.2 Data

We construct measures for US risk premia over a horizon from 1957Q3 to 2019Q2. For comparability with the existing literature, we follow the common practice and use the S&P 500 as a measure for equity prices and treasury yields for the risk-free rate, e.g. Graham and Harvey (2007). Quarterly time series for the S&P 500 index are from Robert Shiller's website. Since it has become standard in the empirical literature to estimate risk premia with a horizon of one year and shorter, we consider investment horizons of one and four quarters.[6] Consistent with the respective horizon, we either use the 3-Month Treasury Bill rate (TB3MS) or the 1-Year Treasury Constant Maturity

[6]See for example Goyal and Welch (2008), Lettau and Ludvigson (2001a), Lettau and Ludvigson (2001b).

rate (DGS1) as measures for the risk-free rate which are obtained from the Board of Governors of the Federal Reserve System.

For the fundamentals in the regression based method to estimate risk premia, we use the cyclically adjusted price-earning ratio (CAPE) available from Robert Shiller's website. As an alternative fundamental, we compute the cyclically adjusted price-dividend ratio (CAPD) based on data from the same source. Consistent with Shiller's cyclical adjustment to the price earnings-ratio, we compute the CAPD as the current real price of equity divided by the average of dividends over the previous ten years. The real price of equity is defined as the S&P 500 index deflated with the consumer price index (CPI).

The US Bureau of Economic Analysis provides time series for real gross domestic product (GDPC1), real gross private domestic investment (GPDIC1), and real personal consumption expenditures (PCECC96). These series are quarterly, seasonally adjusted, and in billions of chained 2012 Dollars. Hours worked by all persons in the non-farm business sector (HOANBS) is available from the US Bureau of Labour Statistics. This source also provides a time series of civilian non-institutional population (CNP16OV) used to express the above macroeconomic aggregates in per-capita terms.

2.3 Empirical evidence on risk premia

In this section, we estimate risk premia using a variety of models that have been found relevant in the literature. We then document that all measures for risk premia exhibit growth asymmetry.

The equity risk premium is the compensation required to make agents indifferent at the margin between investing in a risky market portfolio and a risk-free bond. Formally, the equity risk premium at time t over investment horizon k, $ERP_{t,t+k}$, is defined as the difference between the expected return on equity, $R^e_{t,t+k}$, and the risk-free rate, $R^f_{t,t+k}$, over horizon k,

$$E_t[ERP_{t,t+k}] = E_t[R^e_{t,t+k}] - R^f_{t,t+k}. \tag{2.1}$$

The term $R^f_{t,t+k}$, as it is risk-free, is known at time t, while the future expected performance of the stock market is not. Investors can only observe with certainty the past returns of the stock market up to time t, and can use the information available to form expectations.

To compute the risk premium in equation (2.1), a variety of methods have been suggested in the literature. Duarte and Rosa (2015) provide an extensive overview about the most widely used models and classify these in five categories. We will estimate risk premia based on four models for investment horizon $k = 4$ and three models for investment horizon $k = 1$ which, according to Duarte and Rosa (2015)'s classification, are part of three of these categories. The first category comprises models based on the historical mean of realised equity premia, the second includes models that employ time series regressions and the third is based on survey data. Models in these three categories have the advantage that they rely on a minimum of assumptions, and importantly, allow us to compute long time series for risk premia. In addition, Goyal and Welch (2008) and Campbell and Thompson (2008) show that models based on the historical mean of realised equity premia and based on time series regressions are hard to improve upon in terms of out-of-sample predictability. The other two methods classified by Duarte and Rosa (2015) are undoubtedly very useful in other circumstances, but have substantial drawbacks for our purposes.[7] We now provide a brief overview over the models we employ to compute risk premia.

2.3.1 Historical mean of realised returns

This method is the most straightforward of all approaches to compute the future risk premium from time t to $t+k$. Following Goyal and Welch (2008), it is simply the historical mean of realised stock market returns in excess of the risk-free rate over H periods preceding time t. This can be formalised as

$$ERP_{t,t+k} = \frac{1}{H}\sum_{h=0}^{H}(R^e_{t-k-h,t-h} - R^f_{t-k-h,t-h}).$$

We specify $H = t-k$ as in Goyal and Welch (2008) who use systematically all the available historical data since the beginning of the sample.

The validity of this method relies on the assumption about consistent behavior between past and future. This means the mean of excess returns should either be constant or very slow moving

[7] Models based on cross-sectional regressions, e.g. Adrian et al. (2013), impose tight restrictions on the estimation of risk premia and results are heavily dependent on the portfolios, state variables and risk factors used (Harvey et al. (2016)). While models in our three considered categories use information in real time where investors don't have information sets that include future realizations, this method uses full-sample regression estimates which is particularly problematic in our context with a focus on asymmetries. Risk premium estimates based on dividend discount models, e.g. Damodaran (2019), require additional strong assumptions, for example on the computation of future expected dividends and a discount rate for these dividends.

to avoid a systematic bias in the estimates. We verify that there is no trend in realised excess stock market returns using the augmented Dickey-Fuller test (for details see Appendix A.1).

2.3.2 Time series regressions

This method is based on the idea to utilise the relationship between time series of economic variables and stock market returns to predict future equity returns from a linear regression. One can then subtract the contemporaneous risk-free rate to recover an estimate of the risk premium, as in Fama and French (1988), Fama and French (2002) and Campbell and Thompson (2008). We estimate the following predictive regression

$$R^e_{t,t+k} = \alpha + \beta \cdot fundamental_t + \varepsilon_t, \tag{2.2}$$

where $fundamental_t$ represents a variable that theory and practice have found likely to drive future excess stock returns. This method links as directly as possible to equation (2.1) by computing the equity risk premium

$$E_t[ERP_{t,t+k}] = \hat{\alpha} + \hat{\beta} \cdot fundamental_t - R^f_{t,t+k}, \tag{2.3}$$

based on the estimates $\hat{\alpha}$ and $\hat{\beta}$ for α and β. Generally the literature relies on a single fundamental in this regression, as using several variables at once has been found to reduce model's out-of-sample accuracy. The fundamental used is typically a valuation ratio such as the price-dividend ratio or the price-earning ratio. These valuation ratios are known to be negatively correlated with future stock returns since the works of Rozeff (1984), Campbell and Shiller (1988a), and Campbell and Shiller (1988b).

We compute risk premia from two different models based on the above time series regressions and follow the detailed methodology in Campbell and Thompson (2008). The models differ in the variable used as fundamental, where we either employ Shiller's cyclically adjusted price-earning ratio (CAPE) or the cyclically adjusted price-dividend ratio (CAPD). For each quarter t in our sample, we estimate parameters in equation (2.2) based on a sample up to time $t-1$. The risk premium is then constructed according to equation (2.3) using an out-of-sample forecast. To estimate α and β we use a sample that begins 20 years prior to 1957Q3.[8] We further implement the two

[8] Our results are robust also to using a sample beginning in 1881Q1, when both fundamentals are first available.

restrictions suggested by Campbell and Thompson (2008), i.e. $\hat{\beta}$ must have the sign predicted by theory, otherwise it is replaced by zero, and the predicted risk premium must be positive, otherwise the historical mean is used as a predictor instead. Out-of-sample forecasts are produced for each quarter t from 1957Q3 to 2019Q2.

2.3.3 Survey based risk premium measures

The third method we consider to derive a measure for the risk premium is based on survey data. The Duke CFO Global Business Outlook Survey is the longest ongoing survey about the expected equity return (conducted quarterly since 2000Q2) in the United States.[9] Graham and Harvey (2018) then recover the 10-year ahead expected risk premium by subtracting the known risk-free Treasury bond annual yield to the median forecast of future S&P 500 annual returns. Since 2004Q1, the survey also includes a question on the expected return of the S&P 500 over the next year. We use the responses to this question to compute, analogously to Graham and Harvey (2018), the expected risk premium for an investment horizon of one year. Responses and questions based upon which we could construct risk premia with an investment horizon of one quarter are not available in this survey.

2.3.4 Asymmetries in risk premia

In this section, we show skewness statistics for the growth rate of the risk premium measures described above. In particular, we report results based on a model that relies on the historical mean of realised returns, results based on two time series regression models (using either the cyclically adjusted price-dividend or the price-earning ratio as fundamental), and results based on survey evidence. Table 2.1 summarises results based on each of the four models for an investment horizon of one year ($k = 4$). All four risk premium measures exhibit growth asymmetry which manifests in positively skewed distributions. Over the entire sample (1957Q3 - 2019Q2) the growth in the risk premium based on the historical average method and the two time series regression models has a skewness of 2.55, 0.15 and 0.14, respectively. The positive skewness implies that the risk

[9] Every quarter, on average about 350 Chief Financial Officers from a sample of representative US firms respond to the following question: “The current annual yield on a 10-year Treasury bond is x%. Please complete the following: Over the next 10 years, I expect the average annual S&P 500 return will be: ...%”. Here x% is replaced by the the actual yield on a 10-year Treasury bond at the time of the survey. A corresponding question is asked for a one-year investment horizon.

premium exhibits growth asymmetry: it declines gradually and rises much more sharply. This result is robust also when considering parts of our sample. Table 2.1 shows skewness statistics for subsamples designed to cover two business cycles from peak to peak as defined by the NBER's Business Cycle Dating Committee. It is evident that the vast majority of risk premia also exhibit a positive skewness over these subsamples. The risk premium measure based on survey evidence covers a much shorter sample, starting in 2004Q1. Nonetheless, its use is appealing to confirm our results since this measure is based on a very different methodology. Over the available sample, the survey based measure exhibits a skewness of 0.80 and hence also provides evidence for steepness asymmetry in the risk premium.

Next, we discuss skewness statistics at an investment horizon of one quarter ($k = 1$) for the three risk premium measures based on the historical average and time series regressions.[10] These are summarised in Table 2.2, where we again provide statistics over the entire sample as well as subsamples. Also results at the one quarter investment horizon document positive skewness over the entire sample and the majority of subsamples. While qualitatively consistent, quantitatively the degree of skewness varies considerably across measures. The risk premium based on the historical average method implies a skewness of 0.12 while the measures based on time series exhibit skewness of 0.25 and 1.12. These quantitative differences are not surprising — and consistent with findings for first and second moments in the literature, e.g. Duarte and Rosa (2015) — in light of the substantial diversity in assumptions and the underlying methodologies to derive the risk premium measures. Given this, it is striking that all considered measures feature positive skewness. Overall, this section provides broad evidence that the risk premium — measured in a variety of ways and at different investment horizons — exhibits growth asymmetries: declines are long and gradual and rises are sharp and short. Appendix A.2 presents the values of the risk-premia over the time period, for both investment horizons, calculated in the different ways.

[10]The Duke CFO Global Business Outlook Survey does not include a question that corresponds to the one quarter investment horizon. Based on this survey, Graham and Harvey (2018) provide a risk premium measure for a 10 year investment horizon though. Skewness for growth in this measure is 0.151 over a 2000Q2-2019Q2 sample.

Table 2.1: Skewness statistics for growth in different measures of risk premia based on an investment horizon of one year

	Time series (fundamental = CAPE)	Time series (fundamental = CAPD)
1957Q3 - 2019Q2	0.15 (0.324)	0.14 (0.367)
1957Q3 - 1969Q4	1.64 (0.000)	1.99 (0.000)
1969Q4 - 1980Q1	0.08 (0.804)	0.65 (0.080)
1980Q1 - 1990Q3	0.39 (0.251)	0.13 (0.704)
1990Q3 - 2007Q4	-2.41 (0.000)	-1.34 (0.000)
2007Q4 - 2019Q2	1.85 (0.000)	2.51 (0.000)
	Historical average	
1957Q3 - 2019Q2	2.55 (0.000)	
1957Q3 - 1969Q4	-1.98 (0.000)	
1969Q4 - 1980Q1	1.89 (0.000)	
1980Q1 - 1990Q3	-0.01 (0.968)	
1990Q3 - 2007Q4	1.20 (0.000)	
2007Q4 - 2019Q2	3.12 (0.000)	
	Survey	
2004Q1 - 2019Q2	0.80 (0.012)	
2007Q4 - 2019Q2	0.71 (0.042)	

Notes. 1957Q3-2019Q2 is the full sample for the historical mean and time series methods. Smaller sub-samples are constructed such as to cover two peak-to-peak cycles each as defined by the NBER's Business Cycle Dating Committee, with the exception of the last sub-sample that covers the time from the most recent peak. Survey results are available only from 2004Q1 to 2019Q2. "Historical average", refers to the expectations obtained using the historical average method. "Time series (fundamental = CAPE)" and "Time series (fundamental = CAPD)" refer to the expectations obtained using the time series regression method, using the CAPE and CAPD ratios respectively as fundamentals. "Survey" refers to a risk premium measure based on the Duke CFO Global Business Outlook Survey. Skewness statistics are calculated from the first difference of the logarithm of the risk premium. P-values, in parenthesis, are based on D'Agostino et al. (1990) and Royston (1991) test statistics.

Table 2.2: Skewness statistics for growth in different measures of risk premia based on an investment horizon of one quarter

	Time series (fundamental = CAPE)	Time series (fundamental = CAPD)
1957Q3 - 2019Q2	0.25 (0.104)	1.12 (0.000)
1957Q3 - 1969Q4	0.07 (0.821)	0.05 (0.886)
1969Q4 - 1980Q1	0.23 (0.495)	1.62 (0.000)
1980Q1 - 1990Q3	0.68 (0.053)	0.27 (0.426)
1990Q3 - 2007Q4	2.16 (0.000)	1.63 (0.000)
2007Q4 - 2019Q2	2.88 (0.000)	3.10 (0.000)
	Historical average	
1957Q3 - 2019Q2	0.12 (0.042)	
1957Q3 - 1969Q4	-0.17 (0.596)	
1969Q4 - 1980Q1	0.72 (0.043)	
1980Q1 - 1990Q3	0.26 (0.430)	
1990Q3 - 2007Q4	0.32 (0.237)	
2007Q4 - 2019Q2	-0.14 (0.661)	

Notes. 1957Q3-2019Q2 is the full sample for the historical mean and time series methods. Smaller sub-samples are constructed such as to cover two peak-to-peak cycles each as defined by the NBER's Business Cycle Dating Committee, with the exception of the last sub-sample that covers the time from the most recent peak. "Historical average", refers to the expectations obtained using the historical average method. "Time series (fundamental = CAPE)" and "Time series (fundamental = CAPD)" refer to the expectations obtained using the time series regression method, using the CAPE and CAPD ratios respectively as fundamentals. Survey results are not available for this horizon. Skewness statistics are calculated from the first difference of the logarithm of the risk premium. P-values, in parenthesis, are based on D'Agostino et al. (1990) and Royston (1991) test statistics.

2.4 The model

The core of our model is a representative-agent real business cycle (RBC) model which is extended with two key mechanisms. Firstly, households have recursive preferences of the Epstein and Zin (1989) type. It is well known that standard RBC models with Arrow-Pratt preferences and a reasonable degree of RRA fail to account for the existence of risk premia. This is due to the fact that the intertemporal elasticity of substitution (EIS) and the RRA are reciprocal of each other. A small EIS of the magnitude necessary to justify meaningful risk premia necessarily leads to an excessively large RRA. Recursive preferences separate the RRA and the EIS. Secondly, agents cannot directly observe productivity. Instead, they receive a noisy signal about previous period's productivity and use a Bayesian learning technology to form nowcasts. Agents' varying speed of learning over the business cycle is the key to match empirically observed asymmetries in risk premia and macroeconomic variables.

2.4.1 Production and technology

The economy comprises of a continuum of perfectly competitive identical firms with unit mass. Firms use the following Cobb-Douglas production function to produce output, y_t,

$$y_t = A_t k_t^{\alpha} l_t^{1-\alpha} + \nu_t, \qquad 0 < \alpha < 1, \tag{2.4}$$

by employing capital, k_t, and labour, l_t. Output further depends on a productivity shock, A_t, and an additive noise shock, ν_t. This production function is based on Van Nieuwerburgh and Veldkamp (2006) and is consistent with the work of Faust et al. (2005) who characterise the preliminary GDP announcement of statistical agencies as the sum of a final GDP announcement and a noise term. The productivity shock takes the form of a Markov process with two states, high and low $A_t = \{A_t^H, A_t^L\}\ \forall\, t$, and a standard deviation σ_A. The Markov chain is ergodic and has a symmetric transition matrix, Π, to ensure any asymmetry in the resulting model dynamics is endogenous. The noise shock is independent and identically normal distributed with zero mean and standard deviation σ_ν.

The assumptions about agent's information set are such that — even though they know the underlying shock processes — they cannot separately observe the productivity and noise shock.

Further, agents make decisions about production inputs before they know the level of output since both shocks are realised only at the end of each period. To make an informed decision about production inputs, agents use a Bayesian learning technology to infer the level of current period's productivity based on their noisy observation of output in the previous period.[11]

Both, firms as well as households have the same belief about current productivity since all agents have the same information set and have access to the same Bayesian updating technology. In the following sections, we will discuss optimal decision making of firms and households, given their beliefs about productivity, and show how these agents employ the Bayesian learning technology to update their beliefs.

2.4.2 Firms

Firms enter the period with knowledge about their capital stock. They use Bayesian updating, to be described in detail below, to form a belief about productivity at the beginning of the period. Given this information, firms decide about labour demand and investment, where the latter determines next period's capital stock. Firms own the capital stock, rather than rent it from households, but issue shares and pay out dividends.

At the beginning of the period, after firms have formed a belief about productivity, they expect cash flow, $\tilde{f}_t$, to be

$$\tilde{f}_t = \tilde{A}_t k_t^{\alpha} {l_t^d}^{1-\alpha} + \tilde{\nu}_t - w_t l_t^d - i_t,$$

where $\tilde{A}_t$ denotes the beliefs about productivity and $\tilde{\nu}_t$ the belief about the noise. Following the discussion in the section above, agent's expectation about the noise, $\tilde{\nu}_t$, is zero. w_t denotes the real wage, l_t^d stands for labour demand, and i_t for investment. In general, notation $\tilde{x}_t$ indicates agent's belief about a particular variable. This belief is formed at the beginning of the current period, t, given the information set at the beginning of the current period, $\mathcal{I}_t$, such that $\tilde{x}_t = \mathbb{E}_t[x_t \mid \mathcal{I}_t]$. Then, x_t denotes the realization of this variable at the end of period t.

[11]These timing assumptions are consistent with nowcasting in public policy institutions. Bok et al. (2017) document that the New York Fed Staff Nowcast for GDP on the last quarter is only observable at about the beginning of the next quarter. They also describe that nowcasts and forecasts are based on surveys and limited number of reporting units, i.e. they filter.

Firms have to respect their investment financing constraint

$$i_t = \tilde{y}_t - w_t l_t^d - \tilde{d}_t s_t^s + p_t(s_{t+1}^s - s_t^s), \tag{2.5}$$

where the difference between s_{t+1}^s and s_t^s represents the supplied number of shares to be traded at price p_t between firms and households. Expected dividends, $\tilde{d}_t$, communicated to the households at the beginning of the period, are given by

$$\tilde{d}_t = \frac{\tilde{y}_t - w_t l_t^d - i_t + p_t(s_{t+1}^s - s_t^s)}{s_t^s}. \tag{2.6}$$

Actual dividends are paid out at the end of the period and will absorb the effects of incorrect beliefs and balance out the investment financing constraint. Note that realised cash flow,

$$f_t = A_t k_t^\alpha l_t^{d\,1-\alpha} + \nu_t - w_t l_t^d - i_t,$$

will differ from expected cash flow most of the time as they include realised productivity as well as the realization of the noise term.

The law of motion for capital is

$$k_{t+1} = \left[(1-\delta) + \Phi\left(\frac{i_t}{k_t}\right)\right] k_t, \tag{2.7}$$

where δ is the depreciation rate and the capital adjustment cost function $\Phi\left(\frac{i_t}{k_t}\right)$ is positive and concave. The concavity implies that large changes in the investment ratio are more expensive than gradual adjustments. As in Hayashi (1982) and Jermann (1998), the adjustment cost has the functional form

$$\Phi\left(\frac{i_t}{k_t}\right) = \frac{a_1}{1-\chi}\left(\frac{i_t}{k_t}\right)^{1-\chi} + a_2, \qquad \chi > 1,$$

where χ is the elasticity of the investment ratio with respect to Tobin's q and parameters a_1 and a_2 ensure costs are zero in the steady state. The use of these capital adjustment costs allows us to derive the expression for the return on equity as shown in Appendix A.6.

Firms maximise their value, which is equivalent to the sum of discounted expected cash flow

$$\max_{l_t^d, i_t, k_{t+1}} \mathbb{E}_t \left[\sum_{j=0}^{+\infty} m_{t,t+j} \left(A_{t+j} k_{t+j}^{\alpha} l^{d\,1-\alpha}_{t+j} - w_{t+j} l_{t+j}^d - i_{t+j} \right) \Bigg| \mathcal{I}_t \right], \tag{2.8}$$

where $m_{t,t+j}$ is the household's discount factor to be specified in the next section. We maximise equation (2.8) with respect to i_t, k_{t+1} and l_t^d subject to equation (2.7) and the constraints $i_t \geq 0$, $k_t \geq 0$ to obtain the first order conditions

$$q_t = \frac{1}{\Phi'(i_t/k_t)}, \tag{2.9}$$

$$q_t = \mathbb{E}_t \left\{ m_{t,t+1} \left[A_{t+1} l^{d\,1-\alpha}_{t+1} \alpha k_{t+1}^{\alpha-1} - \frac{i_{t+1}}{k_{t+1}} + q_{t+1} \left(1 - \delta + \Phi\left(\frac{i_{t+1}}{k_{t+1}} \right) \right) \right] \Bigg| \mathcal{I}_t \right\}, \tag{2.10}$$

$$w_t = (1-\alpha) \tilde{A}_t l_t^{d\,-\alpha} k_t^{\alpha}, \tag{2.11}$$

where q_t denotes the Lagrange multiplier and can be interpreted as Tobin's q. Equation (2.9) determines the real price of investment and equation (2.10) determines optimal investment. The labour supply function (2.11) states that the real wage is equal to the expected marginal productivity of labour, since the actual marginal productivity is unobservable.

2.4.3 Households

There is a continuum of identical households with unit mass. At the beginning of each period, households decide how much labour to supply and how many shares to buy. Based on their expected cash flow, firms also inform households on the amount of dividends, $\tilde{d}_t$, they expect to pay. When firms observe their realised cash flow at the end of the period, they pay dividends, d_t, which may differ from the expected dividends. At this point, households update their views about their income which they subsequently use for consumption, c_t. In other words, consumption expenditures absorb any unexpected realizations due to incorrect beliefs to satisfy the households' budget at the end of the period. At the beginning of the period, the households' expected budget constraint is

$$\tilde{c}_t + p_t(s_{t+1}^d - s_t^d) = w_t l_t^s + \tilde{d}_t s_t^d, \tag{2.12}$$

where $\tilde{c}_t$ is the expected consumption level, labour supply is l_t^s, and the difference between s_{t+1}^d and s_t^d represents the demand for the number of new shares.

Households have preferences as in Epstein and Zin (1989) so that recursive utility is a CES aggregate of their period utility function and a certainty equivalent for next period utility,

$$U_t = \left[(1-\beta) u_t(\tilde{c}_t, l_t^s)^{1-\frac{1}{\psi}} + \beta \left(\mathbb{E}_t \left[U_{t+1}^{1-\gamma} \,\middle|\, \mathcal{I}_t \right] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}}, \tag{2.13}$$

where $\psi > 1$ is the elasticity of intertemporal substitution, $\gamma \in [0, +\infty) \setminus \{1\}$ is the relative risk aversion, and $\beta \in (0,1)$ is the discount factor. Period utility takes the form

$$u_t(\tilde{c}_t, l_t^s) = \tilde{c}_t^\kappa (1 - l_t^s)^{1-\kappa}, \tag{2.14}$$

with $\kappa \in (0,1)$ which controls labour supply.

Households maximise equation (2.13) subject to (2.12) and to the interiority conditions $\tilde{c}_t \geq 0$, $c_t \geq 0$ and $0 \leq l_t^s \leq 1$. We obtain the following labour supply function from the household's maximization problem (details are provided in Appendix A.3)

$$\tilde{c}_t = \frac{\kappa}{1-\kappa} (1 - l_t^s) w_t, \tag{2.15}$$

which provides an intratemporal link between labour supply, the real wage and the beginning of period belief about consumption. Combining the first order condition with respect to s_{t+1}^d and the envelope Theorem for s_t^d (shown in Appendix A.3) we obtain the Lucas equation

$$1 = \mathbb{E}_t \left[m_{t,t+1} \frac{d_{t+1} + p_{t+1}}{p_t} \,\middle|\, \mathcal{I}_t \right], \tag{2.16}$$

where

$$m_{t,t+1} = \beta \left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t \left[U_{t+1}^{1-\gamma} \,\middle|\, \mathcal{I}_t \right]} \right)^{1-\frac{1}{\theta}} \left(\frac{c_{t+1}}{\tilde{c}_t} \right)^{\frac{\kappa(1-\gamma)}{\theta}-1} \left(\frac{1 - l_{t+1}^s}{1 - l_t^s} \right)^{\frac{(1-\gamma)(1-\kappa)}{\theta}}, \tag{2.17}$$

is the stochastic discount factor, using $\theta := (1-\gamma)/(1-\frac{1}{\psi})$. The risk-free rate between period t and $t+1$ is thus defined as

$$R_{t,t+1}^f = \frac{1}{\mathbb{E}_t[m_{t+1,t} \mid \mathcal{I}_t]}, \tag{2.18}$$

and the expected return on equity between period t and $t+1$ is

$$\mathbb{E}_t[R^e_{t,t+1} \mid \mathcal{I}_t] = \mathbb{E}_t\left[\frac{d_{t+1}+p_{t+1}}{p_t}\,\middle|\,\mathcal{I}_t\right]. \tag{2.19}$$

Then the expected risk premium is given by

$$\mathbb{E}_t[ERP_{t,t+1} \mid \mathcal{I}_t] = \mathbb{E}_t[R^e_{t,t+1} \mid \mathcal{I}_t] - R^f_{t,t+1}.$$

Note that, as for example in Heer and Maußner (2012), we do not explicitly take into account equation (2.5) in the maximization programme of the representative firm. This is because irrespective of the choice of labour and investment, it is always possible to find a combination of dividends and number of shares that satisfies equation (2.5). Since we also do not impose a specific dividend policy, we cannot directly compute the return on equity based on dividends and share prices. However, as shown in Appendix A.6, we can recover the expected return on equity to be

$$\mathbb{E}_t\left[\frac{d_{t+1}+p_{t+1}}{p_t}\,\middle|\,\mathcal{I}_t\right] = \mathbb{E}_t\left[\frac{q_{t+1}k_{t+2}+y_{t+1}-w_{t+1}l_{t+1}-i_{t+1}}{q_t k_{t+1}}\,\middle|\,\mathcal{I}_t\right]. \tag{2.20}$$

Variables on the right hand side of this equation can be recovered using household's and firm's programmes, given expectations about future productivity. We will discuss in the next section how these expectations about productivity can be formed. Using the right hand side of equation (2.20) to compute the expected return on equity has the advantage that it limits the state space of the dynamic programming problem and thereby keeps our computational problem tractable.

2.4.4 Bayesian learning

We now turn to a description of the Bayesian learning mechanism which agents use to form a belief about current technology, $\tilde{A}_t$. Information set $\mathcal{I}_t$ contains all information available to the agents at the beginning of period t

$$\mathcal{I}_t := \{y^{t-1}, c^{t-1}, d^{t-1}, p^t, w^t, l^{d^t}, l^{s^t}, i^t, k^t, s^{d^{t+1}}, s^{s^{t+1}}\},$$

where x^t denotes the history of variable x up to time t. The technology and the noise shocks are never individually observed, but agents have information about their underlying processes. This includes the transition matrix, Π, which consists of the probabilities of a state change as detailed in Section 2.4.1.

Agents use the following Bayesian filter to forecast A_t given information set $\mathcal{I}_t$

$$P(A_{t-1} = A^H \mid \mathcal{I}_t) = \frac{\phi(y_{t-1} \mid A^H, \mathcal{I}_{t-1}) P(A_{t-1} = A^H \mid \mathcal{I}_{t-1})}{\phi(y_{t-1} \mid A^H, \mathcal{I}_{t-1}) P(A_{t-1} = A^H \mid \mathcal{I}_{t-1}) + \phi(y_{t-1} \mid A^L, \mathcal{I}_{t-1}) P(A_{t-1} = A^L \mid \mathcal{I}_{t-1})}, \quad (2.21)$$

$$[P(A_t = A^H \mid \mathcal{I}_t), P(A_t = A^L \mid \mathcal{I}_t)] = [P(A_{t-1} = A^H \mid \mathcal{I}_t), P(A_{t-1} = A^L \mid \mathcal{I}_t)]\mathbf{\Pi}. \quad (2.22)$$

This filter comprises a Bayesian updating formula, equation (2.21), and an adjustment for the possibility of a state change, equation (2.22), where ϕ is the normal probability density function. In equation (2.21) Bayes' law gives the posterior probability at time t for productivity to be in a high state in the previous period. The reciprocal posterior probability for a low state, $P(A_{t-1} = A^L|\mathcal{I}_t)$, is obtained analogously. Then, agents adjust for the possibility of a state change from period $t-1$ to t using equation (2.22) by multiplying the vector of posterior probabilities with the transition matrix, to obtain a prior belief about the current state of productivity. Agents can subsequently form a belief about the productivity level in the current period by multiplying the vector of priors with the vector of productivity states

$$\tilde{A}_t = [P(A_t = A^H \mid \mathcal{I}_t), P(A_t = A^L \mid \mathcal{I}_t)][A^H, A^L]'. \quad (2.23)$$

Note that for agents to compute the risk-free rate and the return on equity they need to form expectations about several variables in period $t+1$. To do so, they need to estimate the probability that productivity will be in the high or low state in $t+1$, given their beliefs about the current state of productivity, $P(A_t = A^H \mid \mathcal{I}_t)$ and $P(A_t = A^L \mid \mathcal{I}_t)$. Then they multiply the vector of prior probabilities with the transition matrix,

$$[P(A_{t+1} = A^H \mid \mathcal{I}_t), P(A_{t+1} = A^L \mid \mathcal{I}_t)] = [P(A_t = A^H \mid \mathcal{I}_t), P(A_t = A^L \mid \mathcal{I}_t)]\mathbf{\Pi}, \quad (2.24)$$

so that agents employ part of the learning technology analogously to the case described above.

2.4.5 Equilibrium and social planner problem

Equilibrium. At the end of each period the equilibrium in the decentralised economy presented above is a sequence of quantities $\{c_t, l_t^s, l_t^d, i_t, d_t, k_t, s_t^d, s_t^d\}_{t=0}^{\infty}$ and prices $\{w_t, p_t\}_{t=0}^{\infty}$, given k_0, s_0, and A_0, such that the problem of firms is solved, the problem of households is solved, the markets for goods, labour and firm's shares clear

$$y_t = i_t + c_t, \qquad l_t^s = l_t^d = l_t, \qquad s_t^s = s_t^d = s_t.$$

Social planner problem. The decentralised economy has a social planner analogue which can be solved in a recursive fashion. At the beginning of each period the planner maximises the utility of the representative household, equation (2.13), subject to the capital accumulation constraint (2.7), the aggregate resource constraint $\tilde{y}_t = i_t + \tilde{c}_t$ (which is the combination of the households' budget constraint (2.12) and the firms' investment financing constraint (2.5)), and the interiority conditions $c_t \geq 0$, $\tilde{c}_t \geq 0$, $0 \leq l_t \leq 1$, $k_t \geq 0$ and $i_t \geq 0$.

The benevolent planner enters the period with knowledge about two state variables: the capital stock, k_t, and a belief about current period's productivity, $\tilde{A}_t$. The belief is established by using the Bayesian updating mechanism in equations (2.21)-(2.23). Given these state variables, the planner chooses hours worked, l_t, and investment, i_t, which then implies beliefs for the levels of output, $\tilde{y}_t$, and consumption, $\tilde{c}_t$. The planner uses this information together with the technology (2.24) to derive the risk-free rate, the expected return on equity and subsequently the risk premium. Then, the actual productivity shock A_t and the noise ν_t are realised, but not observed separately. The realization of these shocks implies that the planner can observe the actual level of output, y_t, which will typically differ from the belief about output, $\tilde{y}_t$. Subsequently, actual consumption, c_t is realised as a residual.

Formally, the planner solves the following Bellman equation, where V denotes the value function:

$$V(k_t, \tilde{A}_t) = \max_{l_t, i_t, k_{t+1}} \left[(1-\beta)(\tilde{c}_t^{\kappa}(1-l_t)^{1-\kappa})^{\frac{1-\gamma}{\theta}} + \beta(\mathbb{E}_t V^{1-\gamma}(k_{t+1}, \tilde{A}_{t+1} \,\Big|\, \mathcal{I}_t))^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}$$

$$\text{s.t. } k_{t+1} = \left(1 - \delta + \Phi\left(\frac{i_t}{k_t}\right)\right) k_t,$$

$$\tilde{c}_t = \tilde{y}_t - i_t,$$

$$i_t \geq 0, \quad k_t \geq 0, \quad \tilde{c}_t \geq 0, \quad c_t \geq 0, \quad 0 \leq l_t \leq 1, \quad \text{and } k_0, \ A_0 \text{ given},$$

where

$$\Phi\left(\frac{i_t}{k_t}\right) = \frac{b_1}{1-\kappa}\left(\frac{i_t}{k_t}\right)^{1-\kappa} + b_2 \qquad \text{and} \qquad \tilde{y}_t = \tilde{A}_t k_t^{\alpha} l_t^{1-\alpha} + \tilde{\nu},$$

and the updating rules (2.21)-(2.24) are taken as given.

The social planner equilibrium is achievable in the decentralised economy since the planner uses information that is available to all agents at no cost, the constraints and first order conditions of the planner are consistent with those of the agents, technology is convex and the preferences are insatiable.[12]

2.5 Calibration and computation

2.5.1 Calibration

Table 2.3 summarises the parameter values used to calibrate the model. Consistent with the empirical sections above the model is calibrated at quarterly frequency. Several values are standard in the literature. We calibrate the share of capital in production, $\alpha = 0.36$, the discount factor, $\beta = 0.98$, and the capital depreciation rate, $\delta = 0.025$, e.g. Kydland and Prescott (1982). We set the steady state labour supply to 1/3 which then implies $\kappa = 0.37$ for equation (2.15) to hold in steady state. The capital adjustment cost parameter is set to $\chi = 4$, consistent with the value in Jermann (1998). The two parameters related to the adjustment costs, a_1 and a_2, ensure zero capital adjustment costs in steady state and can be expressed as functions of other parameters (derivations of their functional forms are shown in Appendix A.7).

Given these parameters, we calibrate the elasticity of intertemporal substitution, ψ, to be 0.01, so that the mean risk premium in the model matches the equivalent moment in the data. This calibration is also consistent with the empirical estimates in Yogo (2004) and Gomes and Paz (2011) for the elasticity of intertemporal substitution. We use the risk premium based on the

[12] It is important to note that the formulation with a social planning economy rules out agent's active experimentation. In our setup there is no feedback between actions and beliefs and learning is passive. This is a common assumption in the literature, see, e.g. Van Nieuwerburgh and Veldkamp (2006), Ordoñez (2013) and Saijo (2017). Active learning would invalidate the Welfare Theorems in the social planning economy and hence there would be no decentralised counterpart to the planner's equilibrium. The passive learning is reflected in the planner's recursive problem above: the state variables, including beliefs about productivity, are determined before optimal production decisions are made, after which subsequently beliefs are updated again. This process can be repeated until beliefs about productivity coincide with its actual realization.

historical average measure as a benchmark to calibrate our model as it relies on a minimum of assumptions while at the same time we observe a long time series. We consider this measure at an investment horizon of one quarter, which is consistent with the setup of our model. While the model is calibrated to match the level of the mean risk premium in the data (0.063 vs. 0.069), it is reassuring that given the above parameters the model also delivers levels for the expected return on equity (0.103 vs. 0.088) and the risk-free rate (0.042 vs. 0.045) that are comparable to their data equivalents.[13] Based on Caldara et al. (2012), we calibrate the degree of relative risk aversion, γ, to be 5 as our benchmark, which is also in line with the value used in Gourio (2012). Empirical evidence on the degree of relative risk aversion is scarce. For robustness we verify $\gamma \in \{1, 10\}$ which does not significantly alter our results.[14]

The model's learning technology relies on three parameters that require calibrating. We set the states of the two-state Markov chain to be $A^H = 1 + 0.032$ and $A^L = 1 - 0.032$ so that the standard deviation of the technology process, σ_A, is consistent with the findings in Cooley and Prescott (1995) and Fernald (2019) based on estimates of Solow residuals. Note that the distance between the two states matters for the volatility of the process, but since we evaluate deviations from the steady state, the absolute level of the technology process is not important. Let p_{ij} denote the probability for a change from state $i = \{H, L\}$ to state $j = \{H, L\}$, then the ergodicity of the Markov chain implies $p_{ij} \in (0, 1)$ and $p_{iH} + p_{iL} = 1$. In combination with the symmetry assumption on the transition matrix this implies $p_{LH} = p_{HL}$ and $p_{HH} = p_{LL} = (1 - p_{LH})$. Hence, the autocorrelation for technology can be pinned down by the probability of a state change, p_{LH}, which we set to 0.05. This implies an auto-correlation for productivity of 0.95, which is consistent with the estimate in Cooley and Prescott (1995), and gives an autocorrelation for output in the model (0.93) that is in line with the corresponding statistic in the data (0.84). Finally, we calibrate the standard deviation of the noise shock to be $\sigma_\nu = 0.01$, so that our model matches the negative correlation between the median absolute nowcast error for GDP growth and real GDP growth in the data.[15] The variance of the noise shock affects the signal-to-noise ratio and thereby determines the speed of learning. If

[13] The difference between statistics for the expected return on equity and the risk-free rate do not exactly match the ones provided for the risk premium. The reason is that the risk premium is computed for every period before the average is taken across all simulations.

[14] These results are presented in Appendix A.8.

[15] The nowcast data is from the SPF. The SPF provides quarterly nowcasts over a horizon 1968Q4-2019Q2. The nowcasts are on GNP growth, up to 1991Q4, and GDP growth, from 1992Q1. Throughout the paper we compute nowcast errors using the corresponding series for GNP and GDP growth from the Bureau of Economic Analysis.

Table 2.3: Calibrated parameters

Description	Parameter	Value
Income share of capital	α	0.36
Discount factor	β	0.98
Depreciation rate of capital	δ	0.025
Probability of state change in transition matrix	p_{LH}	0.05
Standard deviation of productivity shock	σ_A	0.032
Standard deviation of noise shock	σ_ν	0.01
Relative risk aversion	γ	5
Elasticity of intertemporal substitution	ψ	0.01
Capital adjustment cost parameter	χ	4
Period utility parameter	κ	0.37

the volatility of the noise shock is too large, it becomes impossible to extract any information from the signal received. If the volatility of the noise shock is too small, it becomes straightforward to infer real productivity and learning is trivial. Our value for σ_ν is between these extreme cases so that learning is neither impossible nor trivial.

2.5.2 Computational details

We solve the model using Value Function Iteration. Epstein and Zin (1989) show that a version of the contraction mapping theorem still holds with recursive preferences. The algorithm requires the choice of two grids, for hours and for capital. We use 1000 grid points for capital and 500 grid points for hours. The upper and lower bounds of the grids are equal to 125% and 75% of the respective steady state values of the variables. These values ensure that the choices of the representative agent are not constrained by the boundaries, while maintaining a high grid density for precision of the solution. During simulations we do not visit the grid points at the boundaries of the state space. Consumption and the belief about consumption do not require a specific grid, as their values can be recovered using the grids for capital and labour. We use the policy functions to simulate 500 time series of 248 quarters after 50 periods are discarded. This is consistent with the length of the time horizon in the empirical sections above.

2.6 Results

2.6.1 Nowcasting, uncertainty and asymmetry

We have documented in Section 2.3.4 that there is substantial growth asymmetry across a variety of measures for the risk premium. In this section, we show that our model can explain this asymmetry due to endogenous variations in agent's nowcasting precision about productivity.[16] The key for this mechanism is the formulation for output (2.4), which consists of the product of TFP and the function of production inputs, as well as the additive noise term. Agents employ output realised at the end of the previous period in the Bayesian learning technology to infer the current state of productivity. When production inputs are low, agents learn slowly about productivity because the noise variance is relatively large in comparison to the variance of the signal. A recession is hence a time of high uncertainty and low nowcast accuracy. During a recovery, intensified use of production inputs amplifies changes in technology so that the variance of the signal increases. Given our assumption of a constant noise variance this implies a rising signal-to-noise ratio during a recovery. This decline in uncertainty raises nowcast precision so that agent's speed of learning increases with output and the risk premium declines gradually. At the peak, a situation of low uncertainty and high output, a decline in the state of productivity can be observed relatively precisely. The result is a strong negative adjustment in production inputs, an increase in uncertainty, and a sharp rise in the risk premium. Hence, procyclical fluctuations in the signal-to-noise ratio lead to endogenous variations in nowcasting accuracy which generate asymmetries in the risk premium and the other macroeconomic aggregates.

Important for the functionality of the learning mechanism is a procyclical signal-to-noise ratio resulting from endogenous variations in nowcast accuracy and the degree of uncertainty.[17] Empirical evidence supports this model mechanism. We employ the median absolute nowcast error for GDP growth from the Survey of Professional Forecasters as a measure for nowcast accuracy and hence the speed of learning. Table 2.4 shows results from a regression of this median absolute nowcast

[16] Enders et al. (2017) find that productivity shocks have a statistically and economically significant impact on nowcast errors and report evidence for Granger causality. They also investigate potential links between a variety of other non-technology shocks and nowcast errors, but cannot find significant effects of such shocks on nowcast accuracy.

[17] In Section 2.4.1 we assumed the variance of the noise to be constant. This assumption has been made for simplicity to keep the computational problem tractable. In principle, we can relax this assumption so that the noise variance can even rise when the use of production inputs increases. As long as it rises at a rate less than $k_t^{\alpha} l_t^{1-\alpha}$, this still guarantees a procyclical signal-to-noise ratio.

Table 2.4: Nowcast accuracy, economic contractions and surges in risk premia

	Negative GDP growth dummy	GDP growth rate	Positive risk premium growth dummy
$\hat{\alpha}$	1.436 (0.000)	2.160 (0.000)	0.753 (0.034)
$\hat{\beta}$	2.537 (0.000)	-0.177 (0.000)	1.638 (0.000)
Adjusted R^2	0.23	0.09	0.02

Results of the time series regression $y_t = \alpha + \beta x_t + \varepsilon$ where y_t is the median absolute nowcast error for real GDP growth from the SPF, and x_t is either the quarter-on-quarter growth rate of real GDP, or a dummy variable equal to one when the quarter-on-quarter growth of real GDP is negative, or a dummy variable equal to one when the quarter-on-quarter growth rate of risk premium exceeds 2%. The sample is limited to 1968Q4-2019Q2 by the availability of the SPF. P-values are reported in parenthesis.

error on either GDP growth or a dummy indicating a contraction of the economy. We find a positive relationship between GDP growth and nowcast accuracy and we document that nowcast errors are particularly large during contractions. Our results based on nowcasting accuracy are consistent with findings in the literature on procyclical forecast precision, see, e.g. evidence in Jaimovich and Rebelo (2009) based on the Livingston Survey. Table 2.4 also provides evidence on the relationship between nowcast accuracy and growth of expected risk premia. We regress the median absolute nowcast error for GDP growth on a dummy for large increases in risk premia — indicating a quarter-on-quarter growth rate of expected risk premia of at least to 2%.[18] The result of this regression indicates that surges in risk premia coincide with times of a slow speed of learning.

Next, we turn to regressions where we employ the dispersion of nowcasts for GDP growth from the SPF as dependent variable. Dispersion is defined as the difference between the 75th and the 25th percentile of the projections for quarter-on-quarter growth. Disagreement of private sector expectations, as reported in the SPF, are a widely used proxy for uncertainty.[19] Considering GDP growth as independent variable, Table 2.5 reveals a negative link between output growth and uncertainty. A regression with a dummy — indicating times when the economy contracts — as independent variable corroborates this finding, reporting a significant positive relationship between contractions and nowcast dispersion. Our results on the adverse link between uncertainty and

[18] This is a conservative classification for a period to exhibit a large increase in the risk premium. The dummy is one in 71 out of a total of 203 quarters. Results in Tables 2.4 and 2.5 are robust also when we apply a tighter threshold that includes surges in risk premia above about 4%. This implies the dummy is unity in 25 periods which is the same number of quarters covered by the dummy indicating a contraction in GDP.

[19] See, e.g. Bachmann et al. (2013).

Table 2.5: Uncertainty, economic contractions and surges in risk premia

	Negative GDP growth dummy	GDP growth rate	Positive risk premium growth dummy
$\hat{\alpha}$	1.267 (0.000)	1.570 (0.000)	1.352 (0.000)
$\hat{\beta}$	1.139 (0.000)	-0.069 (0.000)	0.369 (0.034)
Adjusted R^2	0.19	0.05	0.02

Results of the time series regression $y_t = \alpha + \beta x_t + \varepsilon$ where y_t is the dispersion of individual nowcasts for real GDP growth from the SPF, and x_t is either the quarter-on-quarter growth rate of real GDP, or a dummy variable equal to one when the quarter-on-quarter growth of real GDP is negative, or a dummy variable equal to one when the quarter-on-quarter growth rate of risk premium exceeds 2%. The dispersion of nowcasts is measured as the difference between the 75th percentile and the 25th percentile of the nowcasts for quarter-on-quarter GDP growth nowcasts, expressed in annualised percentage points. The sample is limited to 1968Q4-2019Q2 by the availability of the SPF. P-values are reported in parenthesis.

economic activity are consistent with findings in the literature (see, e.g. Bachmann et al. (2013), Bloom (2014)). Using the dummy for strong surges in risk premia as independent variable shows that risk premia are heightened at times of high uncertainty. Investors tend to be more uncertain about the current and future state of the economy during economic contractions which requires compensation through higher risk premia. Our results are consistent with evidence in Corradi et al. (2013) who report the volatility of risk premia to be strongly countercyclicyal and with Baker et al. (2012) who use firm level data to document that uncertainty raises stock price volatility.

The evidence from Tables 2.4 and 2.5 corroborates the model assumption of a procyclical signal-to-noise ratio. It further provides empirical support for the key elements to generate growth asymmetry in risk premia, as it implies a link between uncertainty, strong risk premium growth, the state of the business cycle and variations in the speed of learning.

2.6.2 Asymmetries in the model

We now evaluate the model's ability to explain the risk premium's growth asymmetry observed in the data. We report moments for the risk premium based on the historical average method at the one-quarter investment horizon as this measure has been employed to calibrate the model. Table 2.6 reports a selection of moments for the risk premium in the data (Panel A) and implied by the model (Panel B). Second moments are computed based on the cyclical components of HP(1600) filtered series. The appropriate transformation to detect growth asymmetry, as shown in Sichel (1993), is by computing the skewness from log first-differences. The model matches the countercyclicality of the risk premium and the autocorrelation observed in the data rather well. Also the risk premium's volatility relative to output volatility is reasonably close to the statistic reported in the data. Most notable however is that the model is able to generate positive skewness in the risk premium (0.156) that comes close to the one observed in the data (0.122). The positive skewness implies that increases in the risk premium are larger than decreases. Together with the observed negative correlation with output, this is consistent with the mechanism outlined in Section 2.6.1 above: the risk premium declines gradually during a recovery and increases sharply when a recession occurs.

It is interesting to contrast this result with statistics based on a model without the learning mechanism. The difference to the baseline model is that the state of productivity is revealed at the beginning of the period. The corresponding moments are shown in Table 2.6, Panel C. While the baseline model can generate the empirically observed positive skewness in the risk premium rather well, the model without the learning mechanism fails to generate this asymmetry. Skewness in this model is not substantially different from zero; in fact it is slightly negative (-0.054). Concerning the risk premium statistics, this is the main difference to the baseline model with learning. The model without learning nevertheless implies a countercyclical risk premium, correctly ranks the risk premium to be more volatile than output, and generates a positive autocorrelation, albeit the latter is somewhat weaker than in the data.

We also report in Table 2.6 moments for macroeconomic aggregates. Overall, our baseline model (as well as the model without learning) matches the corresponding volatilities and correlations in the data reasonably well. Consumption is less volatile and investment more volatile than output.[20] A well known issue of real business cycle models is the low volatility of hours worked which is larger than the volatility of output in the data.[21] Our baseline model is able to replicate the negative skewness of macroeconomic aggregates observed in the data. The growth asymmetry in macroeconomic aggregates implies that increases are long and gradual and declines are short and sharp. This is a well documented feature of business cycles. It is consistent with the evidence in Van Nieuwerburgh and Veldkamp (2006), Görtz and Tsoukalas (2013) and Ordoñez (2013) who employ signal

[20] To make learning non-trivial the variance of the noise shock needs to be large enough to disguise the true technology state. This however implies an unrealistically low autocorrelation and high volatility of output. We follow Van Nieuwerburgh and Veldkamp (2006) to resolve this conflict between learning and output volatility and report all moments for the model's output based on a filtered series given public information available at the end or the period, i.e. the persistent component of end-of-period output $\hat{y}_t = \mathbb{E}_t[A_t \mid \mathcal{I}_{t+1}]k_t^{\alpha}l_t^{1-\alpha}$. They show that $\hat{y}$ can be interpreted as revised data which is typically collected by data agencies who would like to report $y_t - \nu_t$ but don't observe the noise. For the national income accounts to balance also consumption must be filtered so that $\hat{y}_t = \hat{c}_t - i_t$.

[21] A remedy discussed in the literature can, e.g. be to use the indivisible labour approach of Hansen (1985).

extraction processes similar to ours to generate asymmetries in macroeconomic aggregates. Görtz and Tsoukalas (2013) report asymmetry in these variables is a salient feature of business cycles across G7 countries and Ordoñez (2013) show it is stronger even in countries with less developed financial sectors. Given the above discussion, it is not surprising that the model without learning fails also to generate the growth asymmetry in macroeconomic aggregates. Skewness of output and consumption is close to zero. Investment and hours are only slightly negatively skewed, of about the size of one standard error, and by far not as much as in the data. It is hence apparent that the learning mechanism is crucial to align the model outcomes with the empirically observed asymmetry in the risk premium and the macroeconomic aggregates.

2.7 Conclusion

The expected risk premium on equity is the expected excess return above the risk-free rate that investors require as compensation for the higher uncertainty associated with risky assets. We estimate a variety of measures for the expected risk premium on equity for the post-WWII US economy based on models that have been found relevant in the literature. We document these measures exhibit growth asymmetry in the sense that increases in the risk premium are sharp and short while declines are more gradual and long. We show this positive skewness is a salient feature of risk premium growth at different investment horizons and over different subsamples. A real business cycle model with Epstein-Zin preferences is consistent with this fact in the data. We demonstrate that the key mechanism to generate growth asymmetry in risk premia are procyclical variations in nowcast accuracy due to endogenous changes in the degree of uncertainty about productivity. This mechanism finds support in the data using measures for uncertainty and nowcast precision from the Survey of Professional Forecasters. In addition to matching the growth asymmetry in risk premia, the model is also successful in generating the empirically observed countercyclicality of risk premia and the negative skewness in macroeconomic aggregates.

Table 2.6: Key moments of the risk premium and macroeconomic aggregates

	Relative std deviation	Correlation with output	1st order auto-cor.	Skewness
Panel A: US data				
Risk premium	2.177	-0.486	0.772	0.122
Output	1.000	1.000	0.836	-0.523
Investment	4.373	0.898	0.817	-0.697
Hours	1.226	0.854	0.909	-0.965
Consumption	0.795	0.872	0.862	-0.672
Panel B: Baseline model (with learning)				
Risk premium	2.972 (0.028)	-0.511 (0.006)	0.640 (0.007)	0.156 (0.028)
Output	1.000 (0.000)	1.000 (0.000)	0.932 (0.003)	-0.591 (0.058)
Investment	2.038 (0.007)	0.735 (0.005)	0.890 (0.004)	-0.461 (0.063)
Hours	0.209 (0.001)	0.696 (0.006)	0.859 (0.004)	-0.453 (0.067)
Consumption	0.944 (0.004)	0.911 (0.001)	0.825 (0.007)	-0.137 (0.040)
Panel C: Model without learning				
Risk premium	3.676 (0.057)	-0.456 (0.006)	0.185 (0.007)	-0.054 (0.018)
Output	1.000 (0.000)	1.000 (0.000)	0.921 (0.004)	-0.029 (0.036)
Investment	1.992 (0.005)	0.983 (0.001)	0.908 (0.004)	-0.059 (0.049)
Hours	0.205 (0.001)	0.933 (0.003)	0.883 (0.000)	-0.071 (0.054)
Consumption	0.725 (0.002)	0.989 (0.000)	0.923 (0.003)	0.042 (0.046)

Values reported in parentheses are standard errors. The sample in panel A is 1957Q3 - 2019Q2. Statistics shown for the risk premium in Panel A are based on the historical average measure with one quarter investment horizon. The models in panels B and C are simulated 500 times over 298 periods after which the first 50 periods are discarded. Second moments are calculated based on percentage deviations from HP(1600) filter trend. Skewness is calculated from log first-differenced series.

Chapter 3

Distributional Dynamics of Mortgage Debt in the Great Recession: New Empirical Observations

3.1 Introduction

Households' balance sheets have been the focus of a number of recent and influential empirical studies in macroeconomics. These studies explore how the composition of the portfolio of households influences their responses to macroeconomic shocks and policy interventions, and the consequences for macroeconomic aggregates. The objective of many of these investigations is to improve our understanding of the underlying factors driving the aggregate dynamics of the economy. They are often motivated by observations made during the Great Recession. The emerging consensus is that balance-sheet-driven heterogeneity among households played a key role in the dynamics of the economy during this crisis, and is also essential to fully understand the channels through which the transmission of monetary policy to households operates. Given the nature of the questions they aim to answer, these studies cannot simply rely on aggregate data. Household-level panel data set are essential to observe the distribution and the evolution of balance sheets and consumption. Various forms of heterogeneity have been considered in these studies. Mian et al. (2013) find that more heavily levered households have a stronger marginal propensity to consume out of housing wealth. They base their observations on the analysis of micro-data from two private companies:

R.L. Polk (auto sales) and MasterCard Advisors, as well as on statistics from the Internal Revenue Service. Kaplan et al. (2014) reveal that households with less liquid portfolios have a larger marginal propensity to consume, which, in turn, implies a larger contraction of their consumption following the destruction of wealth caused by the collapse of house prices during the Great Recession. Their analysis relies on data from the Survey of Consumer Finances and the PSID. Cloyne and Surico (2017) document that the consumption of households with a mortgage is more sensitive to fiscal policy shocks than the consumption of households without a mortgage. They use data from the Living Costs and Food survey in the US and the British Household Panel Survey in the UK. Cloyne et al. (2020) explore the Consumer Survey Expenditure in the US and the Living Costs and Food Survey in the UK. and provide evidence that the consumption of households with negative net financial positions is more sensitive to changes in interest rates compared to households with positive net financial positions.

Noticeably, one aspect of the balance sheet of households remains unexplored. Although it is frequent for households to simultaneously hold variable quantities of debt and savings, the literature has so far not specifically investigated the consequences of the coexistence of these two types of assets. Instead, existing empirical studies focus either on one type of asset at a time, or on net positions. This structure of the empirical literature is reflected even in state-of-the-art theoretical models of household debt. Modern models such as Guerrieri and Lorenzoni (2017), Kaplan et al. (2018), Auclert (2019), Kaplan et al. (2020) or Benigno et al. (2020) include sophisticated mechanisms that provide convincing additional insights into the facts documented by these empirical studies. However, they also fail to differentiate between net and gross positions. The quantities of gross debt and gross savings in households' portfolios are not calculated. Only their net positions are known. Therefore, as in the empirical literature, these models do not discuss the consequences of the coexistence of debt and savings in households' portfolios.

In this chapter we contribute to the literature by exploring survey data from the PSID in an innovative way. We explicitly decompose the balance sheets of households in terms of gross debt and gross savings and we present new facts on their distributional dynamics. We use a methodology that enables us to sort households into four distinct income groups and to evaluate the relative importance of these groups in the aggregate dynamics of savings and debt around the Great Recession, from 2001 to 2015. Our key new finding is that it is mainly the 60% of households around the center of the income distribution that are responsible for the aggregate reduction of gross debt from the onset of the financial crisis in 2007 until 2015, not the 40% of households toward the tails. This result is not visible if we do not decompose portfolios in terms of gross debt and gross savings. Instead, in terms of net positions, only households toward the bottom the income distribution appear to be indebted, i.e. they are the only ones to have more gross debt than gross savings. We find that mortgage debt is by far the most important type of debt, accounting for 84% of total household gross debt at the peak in 2007. The results remain robust to the inclusion of other types of debt considered in the survey, such as, e.g. vehicle loans or credit card debt. The migration of households across groups over time does not affect the results, since we find there is in fact little income mobility over the period of interest. This is consistent with the findings of Flodén and Lindé (2001) who also use PSID data. Our observations on the relative importance of the four income groups are also not driven by bankruptcies, since they are relatively evenly distributed instead of being concentrated on a particular segment of the income distribution, as evidenced by Adelino et al. (2018).

Very specific data are necessary to conduct our analysis. Panel data are essential since we need to exploit the cross sectional dimension as well as the time dimension so as to quantify not only the heterogeneity of the gross financial positions of households, but also their dynamics. We also need the data set to simultaneously contain information on households' income, as well as on their assets and their liabilities, over an extended period of time. Finally, it must be possible to identify the same households across the successive waves of the survey. The PSID is one of very few major survey programmes satisfying all these criteria. It is biennial, so we can use eight waves of answers from 2001 to 2015, for a total of 66945 household-wave observations. The level of detail of the questions allow us to construct the variables of interest in a precise and consistent manner across the eight waves and to match widespread definitions of households' income, debt and savings adopted in the literature and by policy making institutions. The low rate of missing responses means the necessary cleaning of the data is minimal. In total, we are forced to drop only 2.92% of the total number of household-wave observations because of missing data. In addition, given its complex design the PSID comes with a sampling error computation model necessary for the construction of confidence intervals robust to possible sampling errors around the estimated statistics. We use this feature to demonstrate the robustness of our results. The PSID survey is frequently used in academic research as well as in policy making institutions, as documented in the reviews of Moffitt and Zhang (2018) and Smeeding (2018).

The chapter is constructed as follows. Section 3.2 presents the data, explains the construction of the variables and discusses income mobility and default. Section 3.3 presents the results and the robustness checks. Section 3.4 concludes.

3.2 Data

3.2.1 The Panel Study of Income Dynamics

The PSID is a survey conducted on a biennial basis by the University of Michigan and is the longest running household survey in the world according to its authors. This ongoing survey started in 1968 with a representative sample of US families. These families and their descendants are interviewed for each new round of the survey. In addition, families are regularly added to ensure the sample remains representative for the US population. Families are surveyed, among other things, about their income, savings and mortgage debt. We focus on the eight biennial survey waves between 2001 and 2015 that cover the expansion prior to the 2007 financial crisis and the recovery after this crisis. The size of the dataset increases from 7409 households in 2007 to 9051 households in 2015, and contains a total of 66945 household-wave observations. All variables used are expressed in 2009 US Dollars.

Similar to other major survey programmes, the PSID is set up as a multi-staged/complex design survey. The sample incorporates special design features such as stratification, clustering and differential selection probabilities. These features must be taken into account when estimating descriptive statistics and their sampling errors. Standard statistical analysis based on the assumption of simple random sampling and independence of observations would underestimate the variance of survey estimates of descriptive statistics and also lead to biased (too narrow) confidence intervals. For this reason, in line with the recommendation of the PSID, we follow Rust (1985) and Wolter (1985) and rely on Taylor Series Approximations to calculate the robust variances of the estimated statistics. This requires the use of a sampling error computation model to link the estimation programme with the complex structure of the sample by allocating respondents into their respective strata and clusters. The format of the sampling error computation model must be compatible with the estimation method (Taylor Series Approximation) and the estimation programme (Stata). This allows us to derive confidence intervals robust to sampling errors around estimated statistics. We follow the recommendation of the PSID and use their provided sampling error computation model.

3.2.2 Construction of the variables

We focus our analysis on four key household variables: income, gross mortgage debt, gross savings, and net savings – defined as gross savings minus gross debt. We define and construct these variables as follows.[1]

We use a measure of household income consistent with the definition of personal income from the Bureau of Economic Analysis (BEA). It is the sum of all types of income: labour and capital incomes from all the family unit members, plus transfer income, retirement and social security income, before taxes. It excludes capital gains from changes in stock prices. We use this variable to sort households in four distinct income groups. These groups include households up to the 30th percentile of the income distribution (low income group), households above the 30th and up to the 70th percentile of the income distribution (middle income group), households above the 70th and up to the 90th percentile (high income group), and households above the 90th percentile (top income group). The groups are redefined for every wave of the survey. Table 3.1 reports the income thresholds (at the 30th, 70th and 90th percentiles) as well as the average income of each group, for each wave of the survey.

Table 3.1: Income thresholds and averages within groups (in 2009 Dollars)

	Thresholds			Averages by income groups			
	P30	P70	P90	Low	Middle	High	Top
2001	32070	79989	144350	18279	53173	105984	268191
2003	30277	77906	139625	17067	51326	102850	238794
2005	29877	79150	144477	16777	51423	104726	256539
2007	30614	79807	150307	16770	52349	107422	264696
2009	29981	79948	143906	16941	51506	105538	260025
2011	28673	75551	137782	15515	49303	100827	238829
2013	28270	77272	139652	15253	49916	102680	249710
2015	27863	77918	143505	15162	49450	104202	243689

Data in 2009 US Dollars, rounded at the closest Dollar.

We define gross mortgage debt as the remaining principal on mortgage loans at the time of the survey. If a given household holds several mortgages simultaneously – be it on the same property

[1] Appendix B.1 contains the exact references to the 21 survey questions used to calculate households' income, the 16 questions relevant to calculate total and mortgage debt, and the four questions for savings.

or on different properties – we construct our measure of mortgage debt as the sum of the remaining principals on all of these loans. While detailed data on other debt categories are available for the last three waves only, e.g. credit card debt or student loans[2], questions on these had not been included in prior waves of the survey, and such other types account for a relatively small proportion of total debt. Figure 3.1 shows that mortgage debt accounts in the survey by far for the largest share of aggregate household debt — more than 80% on average. Figure 3.2 illustrates the composition of household debt for the four income groups separately. We can verify that mortgages represent the largest share of total household debt for all income groups as well. Furthermore, mortgage debt is the real driver of the dynamics of total household debt. In the full sample, the increase of the average amount of mortgage debt from 2001 to 2007 represents 98.3% of the increase of the average amount of total household debt. After 2007, the reduction of the average amount of mortgage debt until 2015 represents 109% of the reduction of the average amount of total debt. In other words, the deleveraging observed during the financial crisis is entirely driven by mortgages. Other types of non-collateralised debt actually increased slightly after 2007. Therefore, we decide to focus on mortgage debt. This type of debt is collateralizable and relies on specific mechanisms that we will study in depth. Collateralised debt has featured importantly in the recent literature, notably in Cloyne and Surico (2017), Garriga et al. (2017), Guerrieri and Lorenzoni (2017) and Cloyne et al. (2020). As a robustness check, we show our results nevertheless continue to hold also when pooling together all types of debt.

We define gross savings as the sum of all investments in all the assets considered in the survey. The exhaustive list is: savings accounts, money market funds, certificates of deposit, government bonds, treasury bills, cash value in life insurance policies, private annuities, Individual Retirement Accounts, shares of stock in publicly traded firms, stock mutual funds, investment trusts. Note that some assets are counted as a form of savings early in life, but become a source of income and are counted as such later on. For instance, retirement plans are counted as a form of savings as long as households are working and depositing money into these accounts, however, pensions received by retired household members are counted as income.

Finally, we define net savings as the simple difference between gross savings and gross debt.

[2]Since 2011 households are asked specifically about seven different types of debt: mortgages, vehicle debt, student debt, credit card debt, medical bills, legal bills and loans from relatives, plus a category for any other debt of unspecified nature. Before 2011, households are asked specifically about mortgage debt and vehicle loans only, plus a vast category for all other unspecified debt.

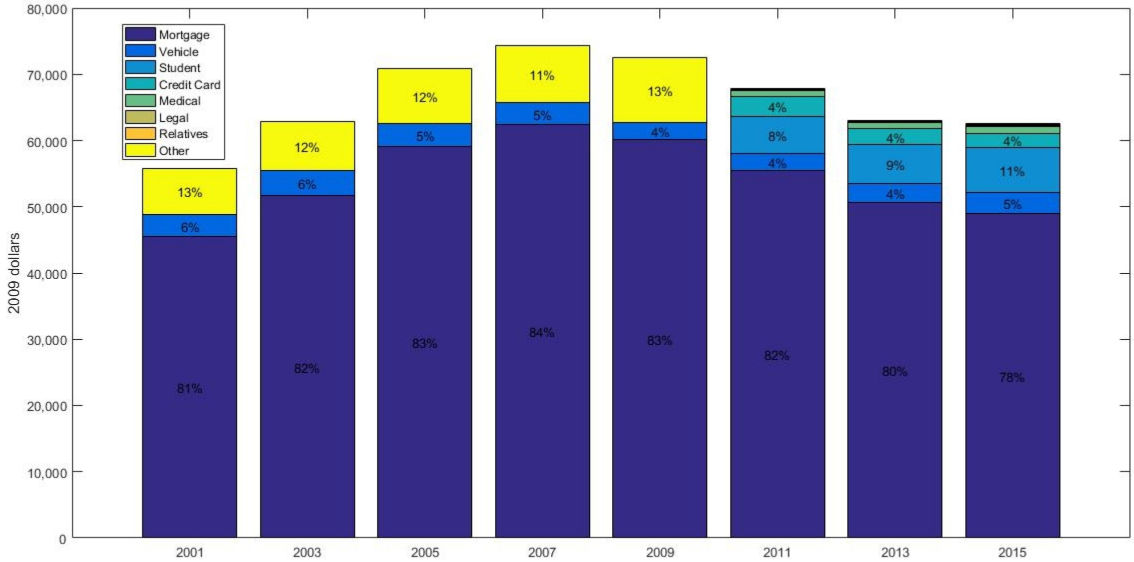


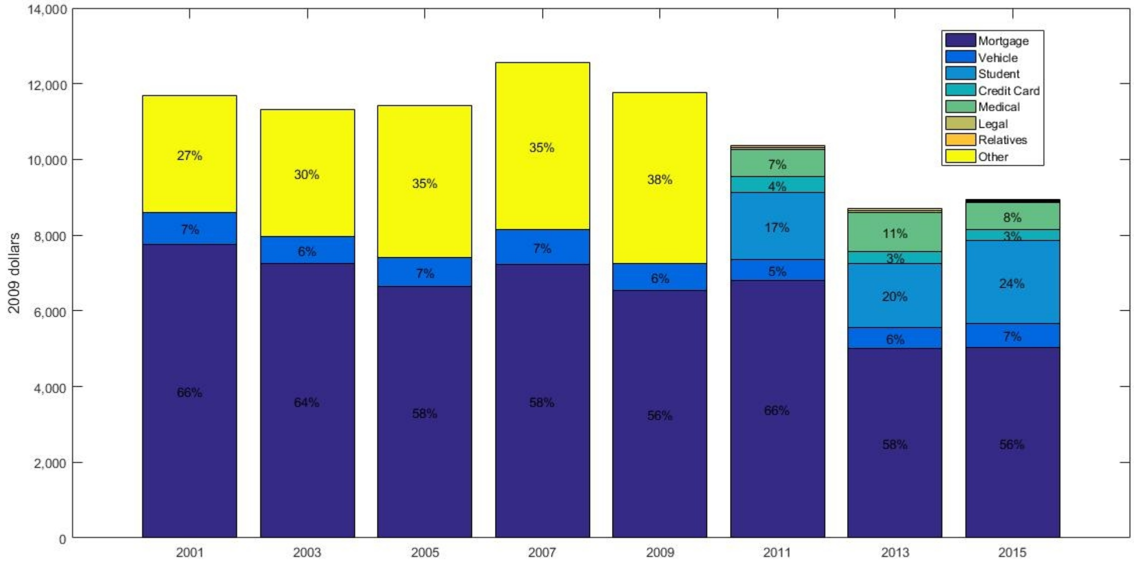
Figure 3.1: Average composition of household debt, whole sample, by year

Therefore, net savings are positive for a household holding larger amounts of gross savings than gross debt, and negative for a household holding more gross debt than gross savings. Naturally, it is possible for two households to hold similar amounts of net savings, but vastly different amounts of gross savings. This implies that there is a priori no systematic reason why the distributions and dynamics of the net and gross variables should be identical. In fact, as we document, the distinction between net and gross variables critically affects our understanding of the relative importance of the different income groups in the overall dynamics of savings and debt.

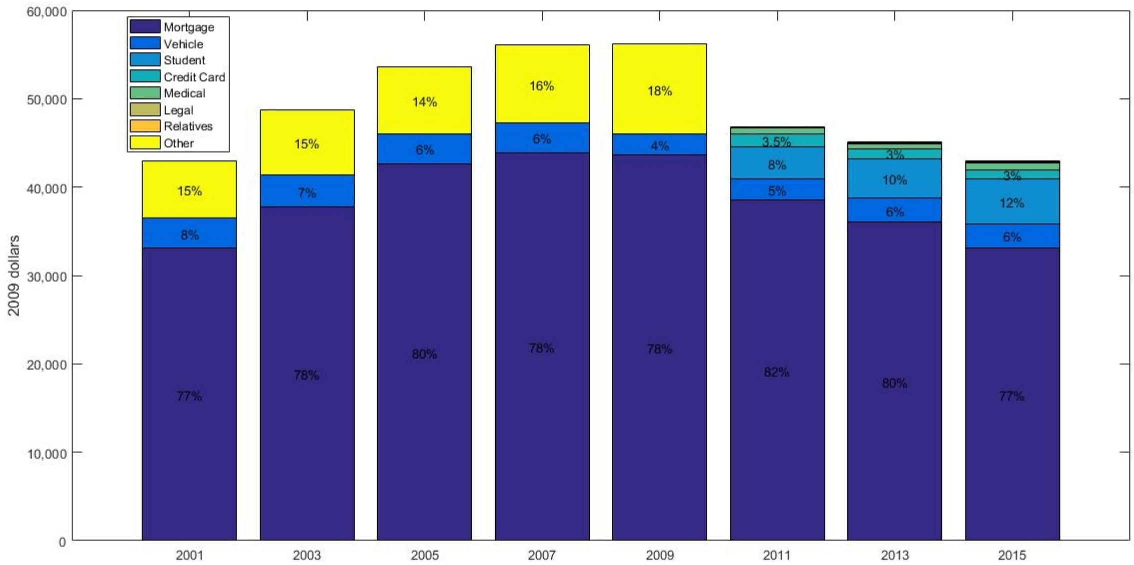
3.2.3 Missing responses

The authors of the PSID apply imputation techniques when households do not report their income – or some of the components of their income – so that there are no missing values for income in the publicly available data set.[3] However, no imputations are made when households do not answer questions relative to mortgage debt. Across the eight rounds of the survey between 2001 and 2015, 97.08% of household-wave observations do not contain missing information on mortgage debt. Missing values are distributed relatively evenly across survey waves and are not concentrated in a particular segment of the income distribution. Households that do not provide information on

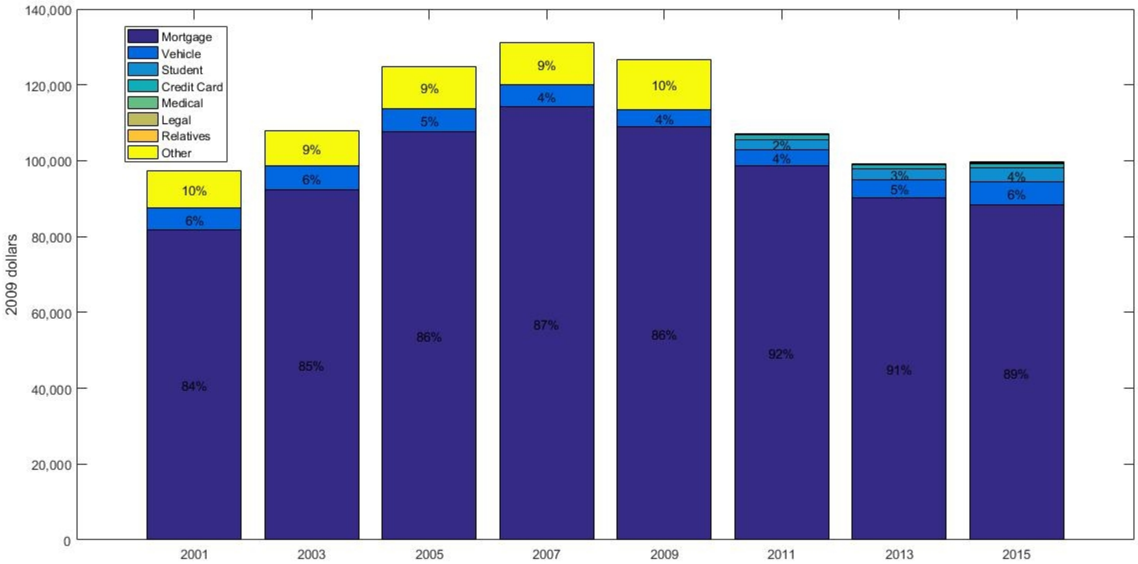
[3]There is no marker available that identifies the imputed values.



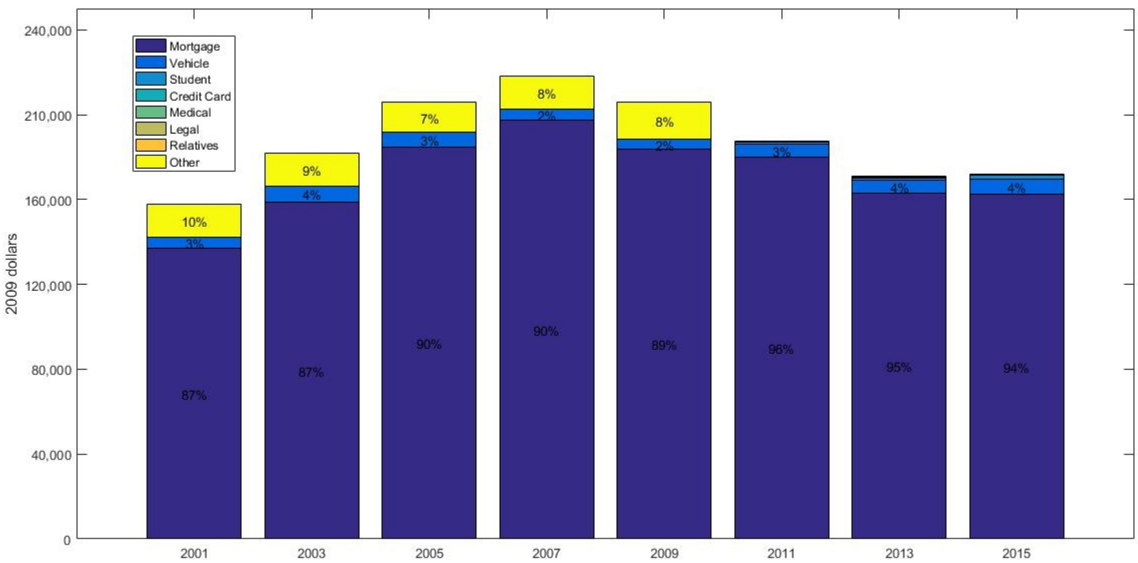
(a) Average composition of household debt, P0-P30



(b) Average composition of household debt, P30-P70



(c) Average composition of household debt, P70-P90



(d) Average composition of household debt, P90-P100

Figure 3.2: Composition of household debt, by income groups

mortgage debt in a particular survey wave are not considered in this particular wave. No imputations are made when households do not answer questions relative to their savings either. Across the eight waves, 91.05% of household-wave observations do not contain missing information on any of the assets households can invest in. Households are kept in the sample even if they fail to answer a question about one of the components of their savings, and missing values are set to zero.

The final dataset after cleaning missing mortgage responses contains eight biennial survey wave with 64990 household-wave responses.

3.2.4 Income mobility and defaults

We are about to present new empirical findings on the relative contributions of the four income groups to the dynamics of aggregate household gross debt, gross savings, and net savings. In order to offer an accurate interpretation of these facts, we need to ensure that we have a complete understanding of the factors that could weight on the distributional dynamics of these variables. The two relevant factors are the mobility of households across income groups and the frequency of defaults. We examine each of these factors in turn before presenting the results.

1. Income mobility. If households were very mobile in the distribution of income, observed differences in the dynamics of the four income groups could be the endogenous consequence of the migration of households between these groups over time. We provide evidence that there is, in fact, very little mobility between income groups over the period of interest. Table 3.2 reports the percentage of households in each wave that were in a different income group in the previous wave. Overall, we find that over 97% of households do not migrate between income groups from any given wave to the next. This tends to indicate that income mobility does not explain the relative importance of the different groups in the build-up and reduction of debt that we document below. This is consistent with findings in Flodén and Lindé (2001) who find incomes in the PSID data are highly persistent – they estimate an autocorrelation of 0.967. We retain these households in our baseline results as exclusion comes at the cost of reduced sample representativeness and the sampling error computation model suffers from a loss of precision if too many observations are dropped from the sample.

Table 3.2: Percentages of households who were in a different income group in the previous wave

		Decomposition by income group			
	Total sample	Low income	Middle income	High income	Top income
2003	2.83 %	0.25	0.39	0.87	1.32
2005	3.12 %	0.28	0.43	0.96	1.45
2007	2.89 %	0.26	0.40	0.89	1.34
2009	2.76 %	0.25	0.38	0.85	1.28
2011	2.74 %	0.25	0.38	0.84	1.27
2013	2.81 %	0.25	0.39	0.86	1.31
2015	2.85 %	0.26	0.39	0.88	1.32

Overall, for each wave of the survey, we find that about 3% of households or less belonged to a different income group the wave before. Decomposing by income groups, we observe that high and top earners play a marginally more important role in the overall income mobility in the sample in comparison to low and middle earners.

2. Defaults. If defaults were numerous and concentrated within a specific segment of the income distribution, this could affect the relative contributions of the four income groups to the dynamics of debt, in particular during the deleveraging phase after 2007. We find evidence that defaults on mortgage debt are rare in the survey and do not play a significant role in the results we are about to present. The last four waves of the survey include a question to identify households defaulting on their mortgages. Households are asked if their banks have started the process of foreclosing on their home because they have defaulted on their mortgage.[4] In 2009, 1.08% of households who had a mortgage declared their banks had begun the foreclosure process. This represented 0.47% of the whole 2009 sample. In 2011, bankrupt households represented 1.28% of indebted households and 0.49% of the total population, in 2013 they represented 0.98% of indebted households and 0.38% of the total population, and in 2015 they represented 0.56% of indebted households and 0.20% of the total population. These cases of foreclosure are distributed relatively evenly over the distribution of income. Thus, defaults are not treated as a relevant factor to explain the relative importance of different income groups in the dynamics of household debt.[5] This view is supported, for instance, by

[4]The exact reference of this question is reported in Appendix B.1.

[5]These percentages are lower than other estimates in the literature based on different data sources. For instance, Mayer et al. (2009) report that by the third quarter of 2008, the share of seriously delinquent mortgages had surged to 5.2%, using data from the the Mortgage Bankers Association. This difference suggests that survey respondents may be reluctant to admit when they are bankrupt. Note that since 2009, the survey also includes a question to identify households who do not describe themselves as bankrupt, but admit having some difficulties making their mortgage payments. In 2009, 6.17% of households declared having difficulties making their payments. This percentage was 6.42% in 2011, 5.40% in 2013 and 3.48% in

the findings of Adelino et al. (2018) who argue that low income households do not default more often than middle or higher income households. The addition or removal of these bankrupt households makes no discernible difference to the point estimates of the key statistics we present below. We decide again to retain these households in our baseline results so as to maintain a representative sample and preserve the precision of the sampling error computation model.

3.3 Empirical findings

We begin with the presentation of a new empirical observation on the relative importance of the different income groups in the reduction of gross household debt after 2007. We find that households toward the center of the income distribution contribute far more to the dynamics of aggregate gross debt than households in the upper and lower tails of the distribution. Although this observation is not accounted for in the existing literature, we argue this is a crucial feature of the medium-term dynamics of the US economy. In the next chapter, we build a model capable of reproducing this fact and we investigate its consequences for the transmission of monetary policy. We continue the presentation of our empirical findings with a discussion of the dynamics of gross and net savings. We show that the results we obtain with the PSID data are consistent with the findings of other authors based on different sources of data. This brings reassurance on the quality of our empirical results.

3.3.1 Main finding: mortgage debt dynamics

Figure 3.3 shows the point estimate of the average mortgage debt for each wave (the blue dots), as well as 95% confidence intervals around the mean (the interval bars) which account for possible sampling errors in the complex design survey. We first discuss the aggregate gross debt dynamics over our full sample in the first plot of Figure 3.3, before we dissect these by income groups in the other four plots. From 2001 to 2007, the average mortgage debt increases by about 27%. The confidence interval around the 2001 point estimate does not overlap with the confidence interval for 2007. This means we can affirm with high confidence that the increase of the average amount of mortgage debt observed in the survey is significant and is not simply the result of sampling errors. From the onset of the financial crisis in 2007 to 2015, households deleverage substantially so that the

2015.

mean mortgage debt declines by about 21%. This evolution is again significant, since the confidence intervals for 2007 and 2015 do not overlap either. This pattern in aggregate household debt has been well documented in the literature, e.g. Mian and Sufi (2014).

The other plots of Figure 3.3 shows the mortgage debt dynamics for four distinct groups of the income distribution. The debt dynamics are substantially different across these four income groups. Households in the middle and high income groups significantly increase their debt up to 2007 by about 33% and 40% respectively, and significantly reduce their mortgage debt by 25% and 23% respectively from the onset of the financial crisis to 2015. This pattern is consistent with the first plot of the figure for the aggregate data. In fact, the aggregate pattern is largely driven by the dynamics of medium and high income groups. The low income group does not show a significant deleveraging between 2007 and 2015; the confidence intervals of the means in 2007 and 2015 overlap for this income group. The top income group shows a deleveraging just at the limit of statistical significance; the lower bound of the confidence interval for 2007 coincides with the upper bound of the confidence interval for 2015. This is the new empirical fact we are documenting: the deleveraging after the financial crisis is driven by the 60% of households in the middle of the distribution rather than the 40% that are located in the tails of the income distribution.

Modern heterogeneous-agent models of households debt such as Guerrieri and Lorenzoni (2017), Garriga et al. (2017) Garriga, Kydland and Sustek (2017), Kaplan et al. (2018), Auclert (2019), Kaplan et al. (2020) or Benigno et al. (2020) do not address this fact. They do not explicitly differentiate between gross debt and gross savings. Other models of the class of Iacoviello (2005) such as Campbell and Hercowitz (2009) or Justiniano et al. (2019) do not address this fact either; they include two representative households, one patient (with a low discount rate) who is exclusively a saver, and one impatient (with a large discount rate) who is exclusively a debtor. All of these models typically predict that poorer households are the only ones to be indebted in net terms. This prediction is accurate, and we find corroborative evidence of this fact in the PSID data, as discussed below. However, these models are silent about the distributional dynamics of gross debt. In the next chapter, we develop a model with an explicit distinction between gross debt and gross savings and we discuss the key mechanisms that help replicate our empirical findings. We also discuss how the explicit distinction between gross debt and gross savings provides new insights into the transmission of monetary policy to households.

The first plot of Figure 3.4 presents the percentage of households with a mortgage from 2001

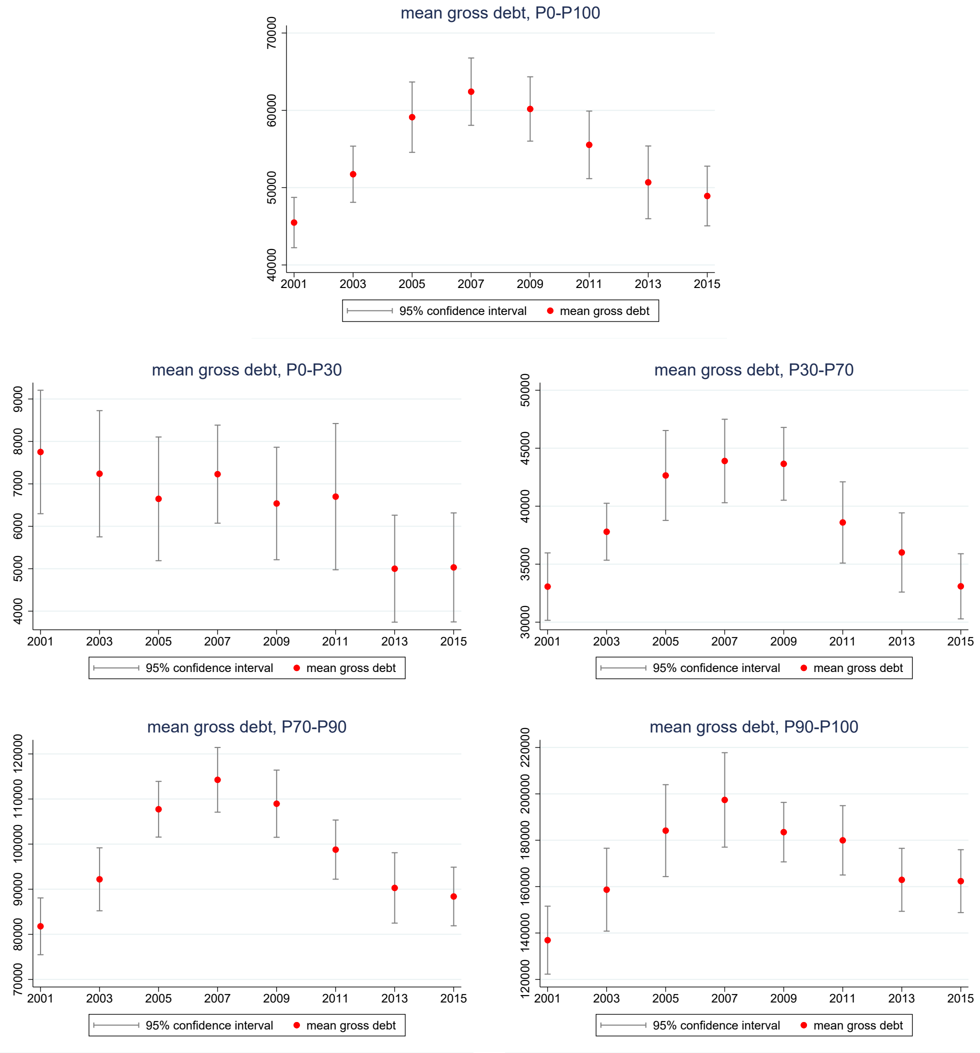


Figure 3.3: Averages of the individual amounts of mortgage debt, by income group, 2001-2015

to 2015 in the whole sample. From 2001 to 2007, the percentage of households with a mortgage remains stable. The increase in mortgage debt prior to the financial crisis is clearly explained by the intensive margin; it is the average size of the mortgage per indebted household that increases, not the share of households with a mortgage. From 2007 to 2015, there is a statistically significant but small reduction of the percentage of households with a mortgage. Overall, there is a reduction of just under six percentage points of the share of indebted households. Again, it is mostly the intensive margin that explains the reduction of aggregate mortgage debt after the financial crisis. The other plots of Figure 3.4 show the the evolution of the percentage of indebted households for the four distinct income groups. We can verify that it is mostly or exclusively the intensive margin that explains the dynamics of mortgage debt in all income groups separately. The evolution of the percentage of households with a mortgage is not significant for any group between 2001 and 2007, and there is only a modest decline between 2007 and 2015 for the middle and high income groups.

3.3.2 Robustness check

Figures 3.5 depicts the mean amount of total household debt, for the whole population and for each income group. All types of debt available in the survey are pooled together in this figure. Inevitably, the mean amount of total household debt is strictly larger than the mean amount of mortgage debt alone from figure 3.3. However, the dynamics are identical in both figures for all income groups. We observe a significant increase of the average amount of total gross debt by about 33% from 2001 to 2007, followed by a significant reduction by 18% from 2007 to 2015. This aggregate pattern is again consistent with the pattern of the middle income and high income groups who increase their mean total gross debt by 31% and 35% respectively before the crisis, and deleverage by 19% and 17% respectively after the crisis. The top income group continues to display a deleveraging after 2007 that is only at the border of statistical significance, while the mean amount of total gross debt of the low income group stagnates from 2001 to 2015. Therefore, our choice to focus on mortgage debt instead of total households debt does not affect our key finding that the deleveraging after the financial crisis is driven by the 60% of households in the middle of the distribution rather than the 40% that are located in the tails of the income distribution.

Figure 3.6 represents the percentage of households holding any form of gross debt. The variations of this percentage are minimal. Within all income groups, the percentage of indebted households

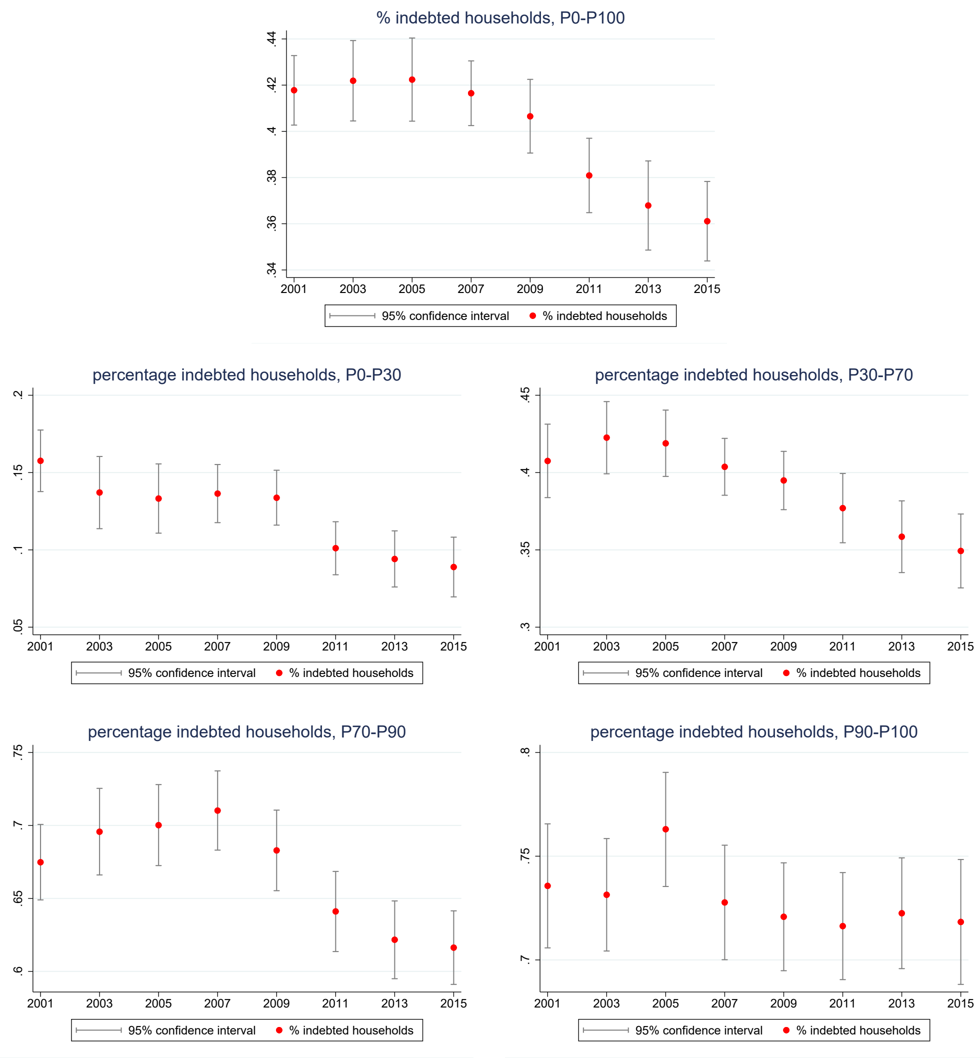


Figure 3.4: Percentages of households with a mortgage, by income group, 2001-2015

never varies by more than five percentage points over the whole period from 2001 to 2015. What is more, this variation of the percentage of indebted households could be the result of sampling errors since the confidence intervals almost always overlap. This confirms that the extensive margin plays either a limited role or no role at all in the dynamics of household debt within all income groups.

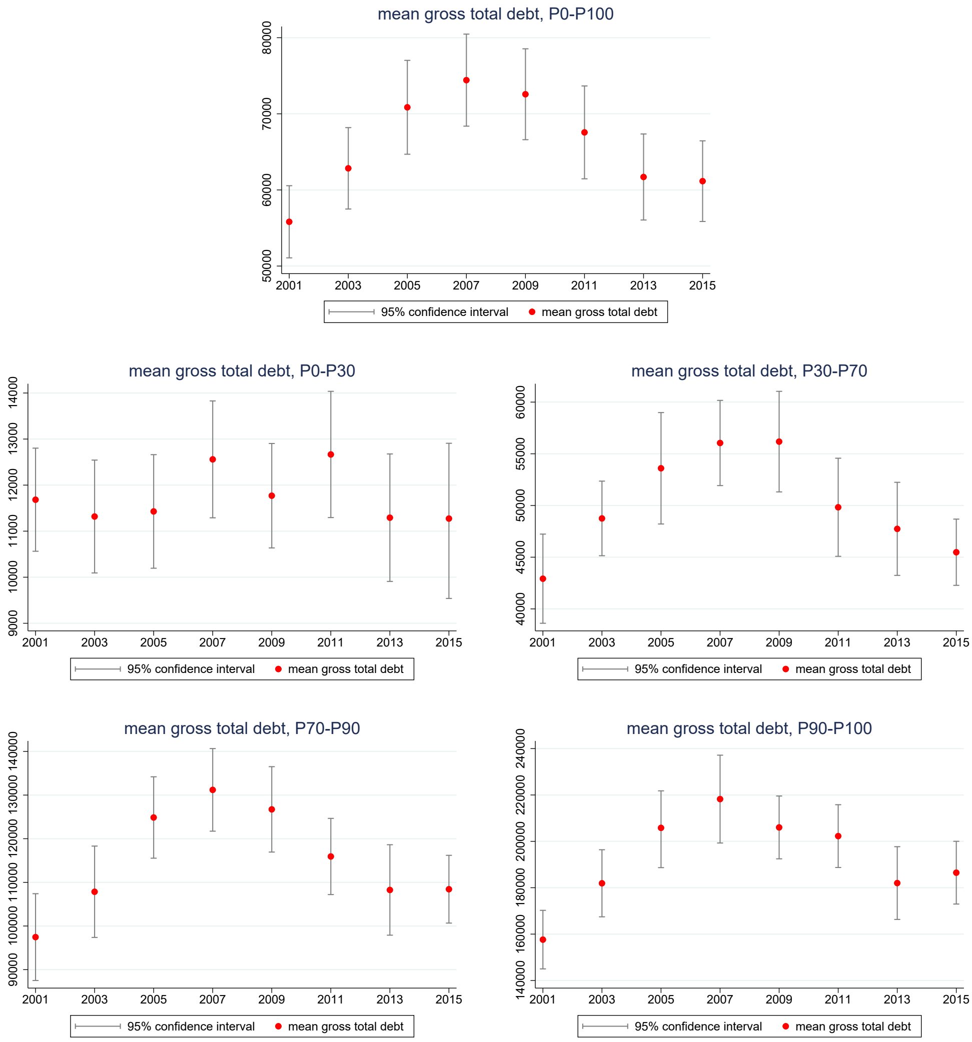


Figure 3.5: Averages of the individual amounts of gross debt, all types of debt pooled together, by income group, 2001-2015

3.3.3 Additional findings: gross savings dynamics

Figure 3.7 represents the mean amount of gross savings. At the aggregate level, we observe a relatively smooth increase of the mean amount of gross savings – except for a short depression around 2009 – of about 40% from 2001 to 2015. This aggregate dynamics is largely the result of the middle and high income groups who increase their mean holding of gross savings by about

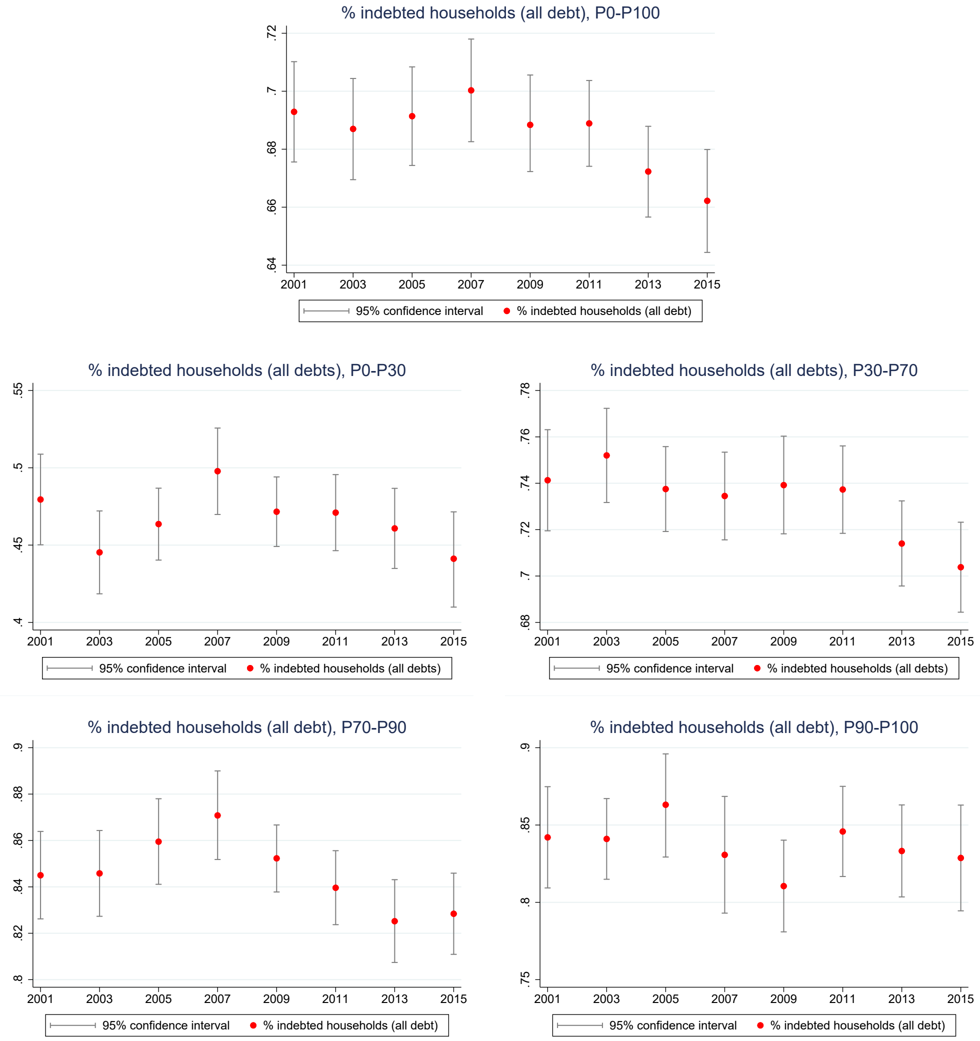


Figure 3.6: Percentages of households holding any form of gross debt, by income group, 2001-2015

70% and 96% respectively, while the top income groups increases its mean amount of savings by only 17%, and the low income group stagnates. Similarly to gross debt, Figure 3.8 confirms that any change in gross savings must be explained by the intensive margin since the percentage of households holding savings stagnates from 2001 to 2015. Within all income groups, the percentage of households holding gross savings never varies by more than six percentage points over the whole period from 2001 to 2015, which is not statistically significant.

These findings are consistent with the empirical work of Cooper (2013) who also finds that gross savings fell at the onset of the crisis in 2007, and started to increase again only after 2009. This evidence supports the suitability of our methodology for the analysis of the PSID data.

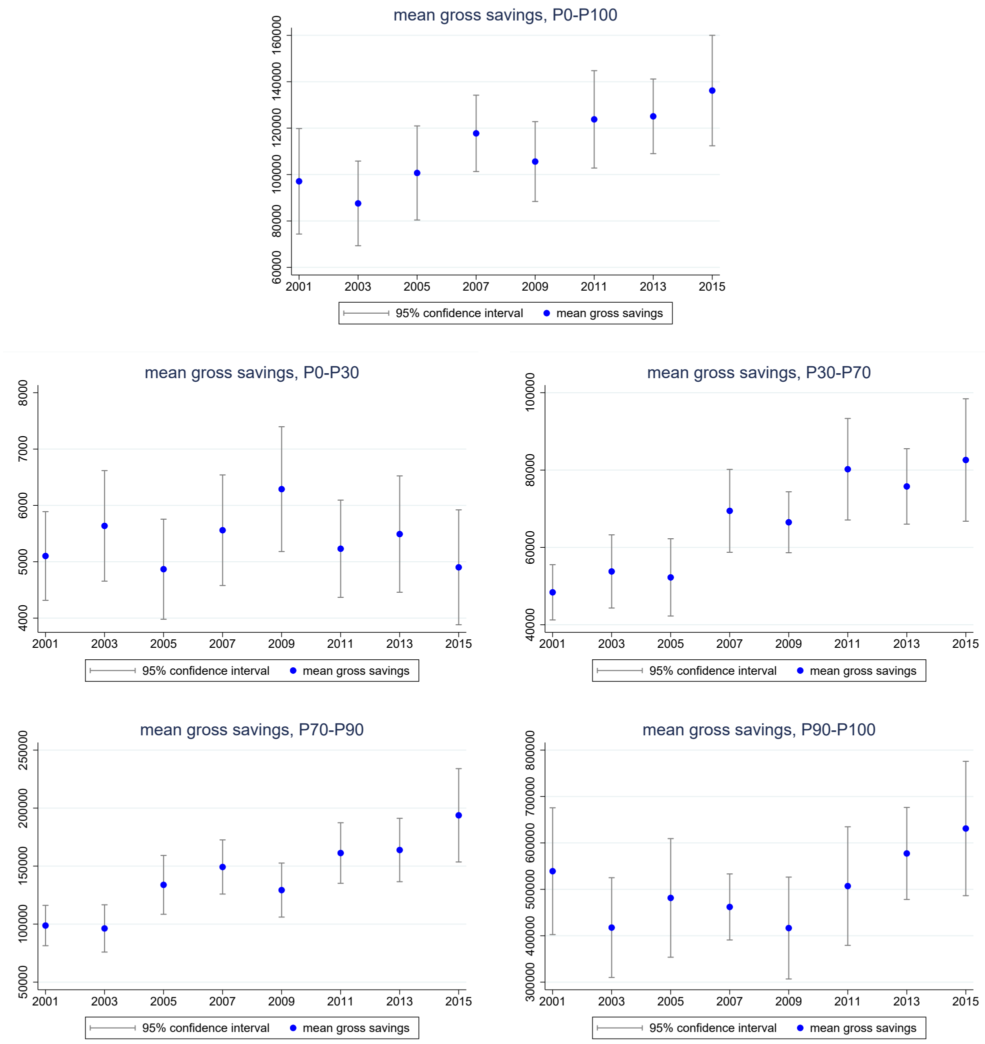


Figure 3.7: Averages of the individual amounts of gross savings, by income group, 2001-2015

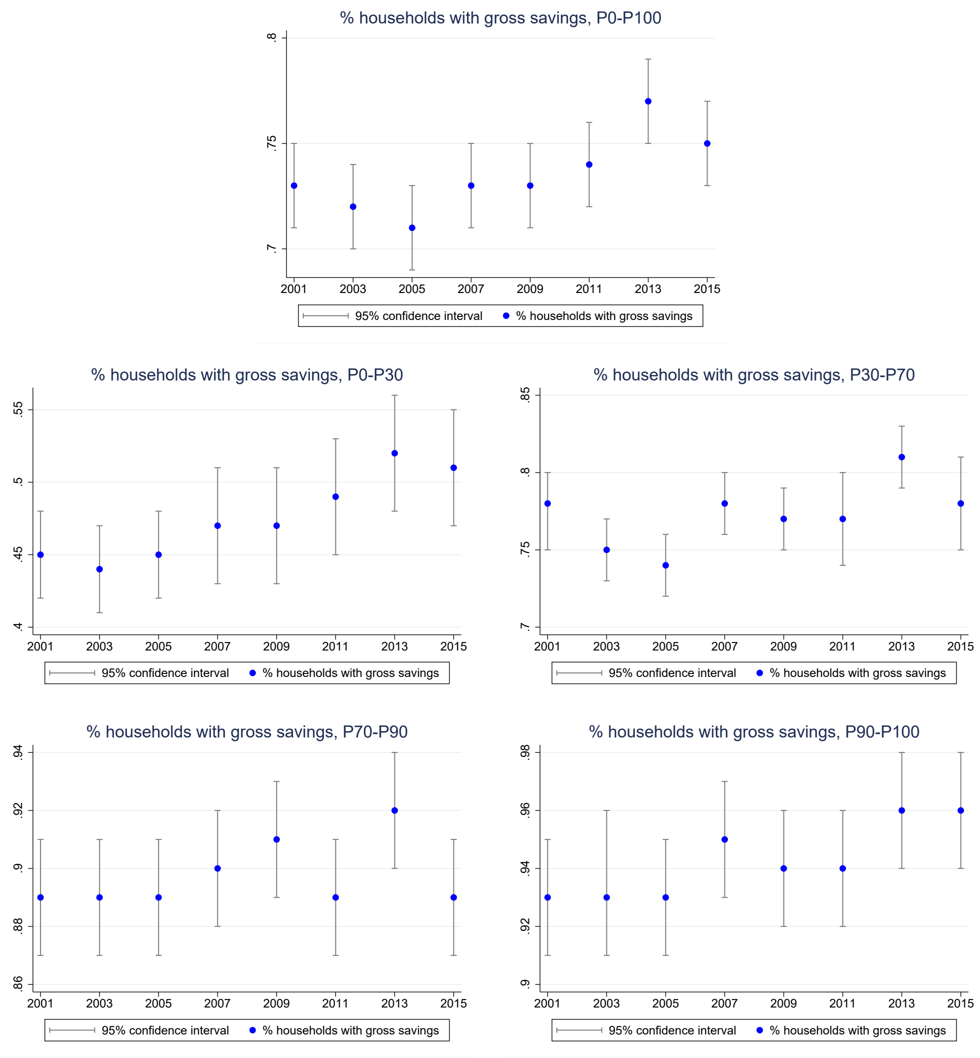


Figure 3.8: Percentages of households holding gross savings, by income group, 2001-2015

3.3.4 Additional findings: net savings dynamics

Figure 3.9 represents the mean amount of net savings, i.e. the average difference between gross savings and gross mortgage debt. Overall, there is a statistically significant and large increase of the mean amount of net savings after the financial crisis by about 110%. This pattern is consistent with the dynamics of the middle, high and top income groups. The reaction of households in the low income group is less significant, but they are also the only ones consistently holding small but negative net savings on average before the financial crisis, and move to zero net savings after the crisis. This observation also consistent with the findings of Cooper (2013). The implication of this fact is that in net terms, households toward the lower tail of the distribution are the only ones to be indebted, and therefore are entirely responsible for the reduction of aggregate net debt during the credit crunch. By contrast, in gross terms, it is the majority of households in the middle of the distribution that are responsible for the reduction of aggregate debt, not the households in the tails. We show in the next chapter that this explicit distinction between gross and net debt provides new insights into the medium-term dynamics of the economy and is important to evaluate the transmission of monetary policy to households.

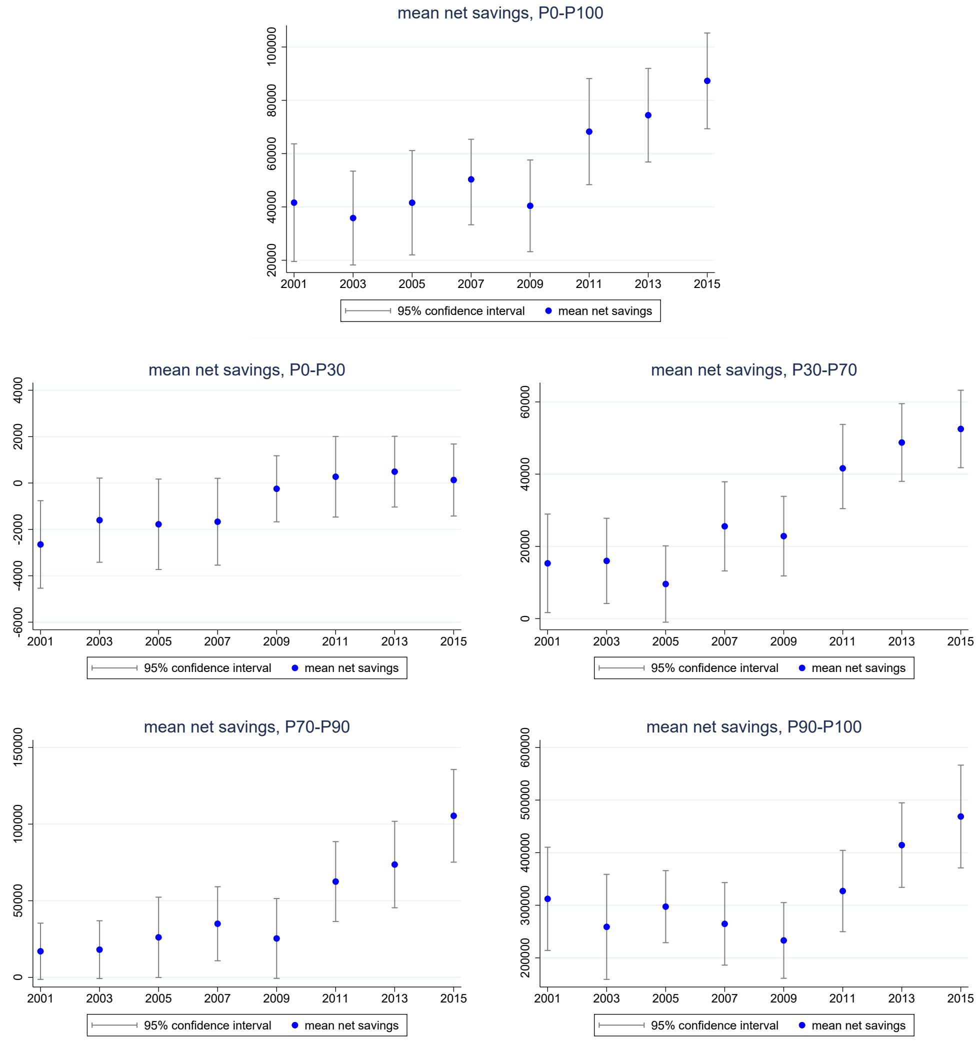


Figure 3.9: Averages of the individual amounts of net savings, by income group, 2001-2015

3.4 Conclusion

Using survey data from the PSID, we document new facts on the relative importance of different income groups in the dynamics of household gross debt around the 2007 financial crisis in the US. Our key finding is that the deleveraging after the financial crisis is driven by the 60% of households in the middle of the income distribution, rather than by the 40% that are located in the tails of the income distribution. We identify mortgages as the main type of debt driving this observation and show that it represents by far the largest share of total household debt, at the aggregate level as well as for all income groups separately. Nevertheless, our observation continues to hold if we pool other non-collateralizable types of debt together with mortgages. We show that our key result is not driven by defaults or by endogeneous mobility of households in the income distribution. We also find that it is the intensive rather than the extensive margin that explains the dynamics of both gross debt and gross savings.

Our key finding is not accounted for in the existing literature. Existing theoretical models of household debt focus strictly on net savings. They accurately predict that households toward the bottom of the income distribution are the only ones holding negative net savings, and we also find evidence supporting this prediction in the PSID data. However, existing models are silent about the distribution and dynamics of gross household debt. In the next chapter, we build an heterogeneous-household model with an explicit distinction between collateralised gross debt and gross savings. We present the mechanisms of the model that are necessary to replicate our key empirical finding and we discuss the insights this distinction provides, in particular concerning the transmission of monetary policy.

Chapter 4

Distributional Dynamics of Mortgage Debt in the Great Recession: Theoretical Modelling and Implications for Monetary Policy

4.1 Introduction

In standard macroeconomic theory, changes in interest rates affect the consumption of the representative household through intertemporal substitution. However, the Great Recession and the subsequent recovery have revealed important heterogeneity in the behavior of households located in different segments of the income distribution (see, e.g. the review of Kaplan and Violante (2018)). This observation has led many authors to revisit the standard theory and to investigate other mechanisms. In recent years, an increasing number of theoretical papers have been exploring how the transmission of monetary policy is affected by balance sheet differences across households, e.g. Serdar et al. (2017), Kaplan et al. (2018), Auclert (2019), Campbell and Hercowitz (2019), Bilbiie (2020) and Benigno et al. (2020). Inspired by a burgeoning empirical literature, e.g. Mian et al. (2013), Kaplan et al. (2014), Cloyne and Surico (2017), Cloyne et al. (2020), these theoretical papers include sophisticated mechanisms that help replicate the observed heterogeneity in households' portfolios. The types of portfolio heterogeneity considered in the literature are varied. They include distinctions between liquid and illiquid assets, nominal and real returns, fixed or variable interest rates. The emerging consensus is that these differences in portfolio compositions are responsible for differences in households' marginal propensities to consume out of transitory income or wealth shocks, and therefore explain why households react differently to the same aggregate shocks affecting their income and wealth either directly or though general equilibrium mechanisms.

Despite the rapid growth of the literature, one important aspect of households' balance sheets remains unexplored. Existing models do not explicitly allow households to simultaneously hold debt and savings. The various assets can be held indifferently in positive or in negative quantities, and only the net position is represented. This implies that existing models remain silent on the distribution and dynamics of gross debt and gross savings. We contribute to the literature by building the first model capable of accurately reproducing the distribution and dynamics of gross household debt during and after the 2007 financial crisis in the US, as documented in the previous chapter. The model is closely related to the framework of Guerrieri and Lorenzoni (2017), with a crucial addition: it allows for an explicit distinction between gross debt and gross savings. In the model, households have access to two distinct one-period assets; one for savings, the other representing collateralised debt. Households save for precautionary reasons because they face unemployment risk as well as idiosyncratic productivity shocks affecting their income when they are employed. Households can also choose to accumulate debt to finance their investment in durable goods. If they do so, a fraction of the durable goods they buy is used as collateral, as in a mortgage. Households also consume non-durable goods. Taxes paid by households are proportional to the sum of their labour income (if they are employed) or unemployment benefits (if they are unemployed) plus the interest earned on their savings, minus the interest paid on their debt. This arrangement of taxes reflects the rule of the Internal Revenue Service known as the mortgage interest tax deduction. We decide to focus on collateralised debt in the model because mortgage debt represents the vast majority of household debt in the data, and because mortgage loans have already featured importantly in the literature, e.g. Iacoviello (2005), Guerrieri and Lorenzoni (2017), Garriga et al. (2017) and Kaplan et al. (2020).

We first calibrate and solve the model so as to reproduce the state of the US economy in 2007. The steady state predictions of the model are consistent with our empirical observations from the PSID data. Poorer households hold smaller amounts of debt on average. They tend to hold smaller stocks of durable goods to use as collateral, therefore they face tighter borrowing constraints. In addition, debt has a higher cost in terms of intertemporal utility for poor households. Wealthier households by opposition can afford larger amount of debt because they also hold larger quantities of durable goods and because debt incurs a lower intertemporal utility cost for them. While the distribution of gross debt depends on the heterogeneity of the intertemporal utility cost of debt, the aggregate amount of debt in the economy is affected by the mortgage interest tax deduction.

We then conduct two exercises to explore the new insights offered by the explicit distinction between gross debt and gross savings. In the first exercise, we simulate the fall of house prices after 2007 in the US by tightening the maximum loan-to-value ratio – i.e. by lowering the maximum fraction of the stock of durable goods that can be used as collateral. We compute the transition of the economy during the credit crunch from 2007 to 2015. The model predicts a reduction of aggregate debt by 21%, exactly as in the data. Crucially, the 60% of households toward the center of the income distribution explain 75% of this aggregate reduction of debt (against 72% in the data), while the 10% and 30% of households respectively in the upper and lower tails collectively explain only 25% of the aggregate dynamics (against 28% in the data). In the second exercise, we simulate a monetary policy experiment. The central bank progressively cuts the interest rates while the maximum loan-to-value ratio remains constant. Unsurprisingly, the model predicts an increase of aggregate consumption because of the intertemporal substitution effect. However the response of households is heterogeneous and depends on the composition of their portfolios. First, gross debt and savings represent two cumulative channels for the transmission of monetary policy to households. For a given amount of debt, households with more savings react more strongly to a cut in interest rates. Similarly, for a given amount of savings, households with more debt also react more strongly. These two effects have been documented separately in the literature (see, e.g. Cloyne et al. (2020), Kim and Lim (2020), Vissing-Jørgensen (2002), Kaplan et al. (2014)), but our model is the first to illustrate both at once. Second, we find that debt is a stronger channel than savings. Households with larger mortgages as a percentage of the sum of their gross savings savings and gross debt are more sensitive to interest rate changes.

This chapter is related to a recent branch of the literature, as discussed in details in Kaplan and Violante (2018), that uses heterogeneous-agent models to combine the study of cross-sectional phenomena and aggregate dynamics in a single setting. For several decades before the Great Recession, it was widely accepted that in most cases the aggregate dynamics of heterogeneous and representative-agent models were practically identical, as formally demonstrated by Krusell and

Smith (1998). This paradigm changed in the aftermath of the Great Recession, when new models were designed specifically to explore the role that household heterogeneity played in the unfolding of the crisis and in the response of the economy to monetary and fiscal policies. Such papers include Oh and Reis (2012), Werning (2015), McKay and Reis (2016), Krueger et al. (2016), McKay et al. (2016), Den Haan et al. (2018) and Bayer et al. (2019). They all have in common to illustrate how aggregate dynamics are, in fact, critically affected by the shape of the distributions of income and wealth. The model presented in this chapter fits in this literature since we explore how the response of aggregate consumption to monetary policy are directly affected by the distribution of households' debt and savings, something a representative-agent model could not do.

The rest of the chapter is organised as follows. Section 4.2 presents the model, describes its equilibrium and its calibration. Section 4.3 presents the predictions of the model in steady state, during the simulated credit crunch and the the policy experiment, and discusses the model mechanisms. Section 4.4 concludes.

4.2 The model

4.2.1 Presentation of the model

The model comprises of a continuum of infinitely lived heterogeneous households of mass one. Households are subject to idiosyncratic productivity shocks and face unemployment risk. They receive income from either utilizing labour as a factor of production or, when unemployed, they receive tax financed benefit payments from the government. Households consume non-durable and durable goods. While non-durables consumption can be financed from income and savings, investments in durable goods can additionally be financed via collateralised debt. The model is closely related to the framework in Guerrieri and Lorenzoni (2017) and extends it by explicitly allowing for a distinct role of households' gross debt and gross savings.

Households' preferences are summarised in the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_{it}, h_{it}, k_{it}), \qquad 0 < \beta < 1,$$

where c_{it} stands for household i's consumption of non-durable goods in period t, k_{it} consumption of the services provided by durable goods, and h_{it} for hours worked. The discount factor is denoted by β and $\mathbb{E}$ is the expectations operator. As in Guerrieri and Lorenzoni (2017), household's period utility function is given by

$$U_{it}(c_{it}, h_{it}, k_{it}) = \frac{(c_{it}^{\theta} k_{it}^{1-\theta})^{1-\gamma}}{1-\gamma} + \psi \frac{(1-h_{it})^{1-\eta}}{1-\eta},$$

so that it is isoelastic in leisure and a consumption bundle. The latter is a Cobb-Douglas aggregate of non-durable and durable consumption goods. The coefficient on non-durables is $\theta \in (0, 1)$, γ is the coefficient on relative risk aversion and η determines the curvature on utility from leisure. The parameter ψ allows us to determine the shares of leisure and hours worked in the steady state.

Households are subject to employment risk. If employed, they use a linear technology to produce consumption goods in a perfectly competitive market

$$y_{it} = z_{it} h_{it}.$$

Durable and non-durable consumption goods are produced with the same technology, so the relative price of durables is unity. z_{it} is an idiosyncratic productivity shock that follows an ergodic Markov chain that can take values $\{z^0, z^1, ..., z^N\}$. If a household is unemployed, they are assumed to have productivity $z^0 = 0$, but receive unemployment benefit ν_t from the government. While households face uncertainty about their employment status in period t, the probabilities for becoming unemployed, $\pi_{e,u}$, and for moving out of unemployment, $\pi_{u,e}$, are known parameters and common across households. When households move out of unemployment, they draw a productivity level from the unconditional distribution of z.

In every period, households can decide about their hours of work, their consumption of durable and non-durable goods as well as about their level of savings and debt. However, their decisions are subject to a number of constraints. The household budget constraint is given by

$$\frac{s_{it+1}}{1+r_t^s} - \frac{d_{it+1}}{1+r_t^d} + f(k_{it+1}, k_{it}) + c_{it} + \tilde{\tau}_{it} \leq s_{it} - d_{it} + \mathcal{I}_{it}\nu_t + (1-\mathcal{I}_{it})y_{it},$$

where $s_{i,t} > 0$ denotes savings in bonds and $d_{i,t} > 0$ stands for collaterised debt. These financial vehicles are associated with gross interest rates r_t^s and r_t^d, respectively. We impose a non-negative interest rate wedge, ϖ, so that $r_t^d = r_t^s + \varpi$. It is charged by a not further specified financial intermediary as commission to facilitate household's borrowing. The indicator variable $\mathcal{I}_{it}$ is unity if the household is unemployed and is zero otherwise. Households pay proportional income taxes

$$\tilde{\tau}_{it} = \tau \left(\mathcal{I}_{it}\nu_t + (1 - \mathcal{I}_{it})y_{it} + r_{t-1}^s s_{it} - r_{t-1}^d d_{it} \right),$$

where τ is the income tax rate. Tax is paid on labour and savings income after deducting any interest payments for collateralised debt.[1]

The change in the stock of durable goods is captured by the function

$$f(k_{it+1}, k_{it}) = \begin{cases} k_{it+1} - k_{it} + \delta k_{it} & \text{if} \quad k_{it+1} \geq k_{it} \\ (1 - \zeta)(k_{it+1} - k_{it}) + \delta k_{it} & \text{if} \quad k_{it+1} < k_{it}. \end{cases}$$

The depreciation rate is denoted by δ. Households can invest in the stock of durable goods, but can also disinvest subject to a proportional cost ζ.[2]

When households incur debt they are subject to the borrowing constraint

$$d_{it+1} \leq \phi_{t+1} k_{it+1},$$

so that they can borrow only up to a fraction ϕ_{t+1} of the stock of collateral. Time variation in ϕ_{t+1} implies shifts in the tightness of the borrowing constraint. For the moment, we consider ϕ_{t+1} to be constant. In Section 4.3.2 below, we will define a law of motion and discuss the effects of a tightening in the borrowing constraint. Households are further constrained in that they can borrow

[1] As in Kaplan et al. (2018) the income tax is the only levy in the model. Its design is consistent with the rules applied by the US Internal Revenue Service which imply that labour income and any interest from taxable forms of savings are taxed after deducting interest payments on mortgages.

[2] We assume that $1 - \zeta > \delta$ so that the household can always liquidate part of the durable stock to cover for depreciation. Part of the literature takes into account that purchases of durable goods can be lumpy and associated with various fixed adjustment costs, see, e.g. Leahy and Zeira (2005). Abstracting from such fixed costs has the advantage that it keeps the household's problem concave which eases tractability.

collateralised debt only if they invest in durable goods at the same time.[3] Formally,

$$\begin{cases} d_{it+1} \geq 0 & \text{if} \quad k_{it+1} > k_{it}(1-\delta) \\ d_{it+1} = 0 & \text{if} \quad k_{it+1} \leq k_{it}(1-\delta). \end{cases}$$

The government chooses the unique income tax rate τ_t, pays unemployment benefits and issues bonds B_{t+1} to balance its budget. Government bonds are the only source of liquid assets in the economy. Then the government budget constraint must satisfy

$$(1 + r^s_{t-1})B_t + u\nu_t = B_{t+1} + \int \tilde{\tau}_{it} d\Psi_t(s_{it}, d_{it}, k_{it}, z_{it}),$$

where B_{t+1} is the aggregate supply of bonds, $\Psi_t(s_{it}, d_{it}, k_{it}, z_{it})$ is the joint distribution of all four state variables, and $\int \tilde{\tau}_{it} d\Psi_t(s_{it}, d_{it}, k_{it}, z_{it})$ represents the total income tax revenue. The share of unemployed households is denoted by u. We assume that unemployment benefits are constant at ν and the government's bond supply adjusts to ensure a balanced budget. So while households can, via a financial intermediary, directly borrow from and lend to each other, government's supply of bonds provides an additional vehicle for households to save. We assume the government guarantees for the financial intermediary so that there is no difference between savings vehicles and they are subject to the same interest rate.

4.2.2 Equilibrium

An equilibrium is a sequence of consumption policies $\{C_{it}(s_{it}, d_{it}, k_{it}, z_{it})\}$, working hours policies $\{H_{it}(s_{it}, d_{it}, k_{it}, z_{it})\}$, savings policies $\{S_{it+1}(s_{it}, d_{it}, k_{it}, z_{it})\}$, debt policies $\{D_{it+1}(s_{it}, d_{it}, k_{it}, z_{it})\}$, durable investment policies $\{K_{it+1}(s_{it}, d_{it}, k_{it}, z_{it})\}$, a sequence of joint distributions for savings, debt, durable goods and productivity levels $\{\Psi_t(s_{it}, d_{it}, k_{it}, z_{it})\}$, as well as sequences of interest rates $\{r^s_t\}$, $\{r^d_t\}$, income taxes $\{\tilde{\tau}_{it}\}$, and the aggregate quantity of liquid assets $\{B_t\}$, such that, given the initial distribution Ψ_0:

[3]This constraint applies as according to US tax law, households are allowed to deduct their mortgage interest from their taxable income only if their mortgage debt is used to invest in a house. If households use their existing house as collateral to obtain a loan but don't buy a new house, they are not allowed to use the mortgage interest tax deduction. By imposing this last constraint, we ensure that mortgage interest are always deductible.

1. $C_{it}(s_{it}, d_{it}, k_{it}, z_{it})$, $H_{it}(s_{it}, d_{it}, k_{it}, z_{it})$, $S_{it+1}(s_{it}, d_{it}, k_{it}, z_{it})$, $D_{it+1}(s_{it}, d_{it}, k_{it}, z_{it})$, $K_{it+1}(s_{it}, d_{it}, k_{it}, z_{it})$ are optimal given $\{r_t^s\}$, $\{r_t^d\}$, and $\{\tau_{it}\}$.

2. $\Psi_t(s_{it}, d_{it}, k_{it}, z_{it})$ is consistent with the policies for the choice variables.

3. Households' budget and borrowing constraints are satisfied.

4. The government's budget constraint is satisfied.

5. The market for liquid assets clears

$$\int s_{it+1} d\Psi_t(s_{it}, d_{it}, k_{it}, z_{it}) = B_{t+1} + \int d_{it+1} d\Psi_t(s_{it}, d_{it}, k_{it}, z_{it}).$$

4.2.3 Recursive programming problem

The model includes the three endogenous state variables, s_{it}, d_{it} and k_{it}, and the exogenous state variable z_{it}. Given these, households make choices on durable and non-durable consumption, hours worked, savings and collateralised borrowing. The Bellman operator for households takes the form

$$\begin{aligned} V(s_{it}, d_{it}, k_{it}, z_{it}) = & \max_{c_{it}, h_{it}, s_{it+1}, d_{it+1}, k_{it+1}} U(c_{it}, k_{it}, h_{it}) \\ & + \lambda_{it} \left(\mathcal{I}_{it}\nu + (1 - \mathcal{I}_{it}) z_{it} h_{it} - f(k_{it+1}, k_{it}) + s_{it} - d_{it} - \frac{s_{it+1}}{1 + r_t^s} + \frac{d_{it+1}}{1 + r_t^d} - c_{it} - \tilde{\tau}_{it} \right) \\ & + \mu_{it}(\phi_{t+1} k_{it+1} - d_{it+1}) + \beta \left\{ \mathbb{E}_t \left[V(s_{it+1}, d_{it+1}, k_{it+1}, z_{it+1}) | z_{it} \right] \right\}, \end{aligned}$$

where $V(\cdot)$ is the value function and λ_{it} and μ_{it} are two multipliers. The budget constraint is always binding since households are insatiable. By contrast, the borrowing constraint is only occasionally binding. By the complementary slackness condition:

$$\mu_{it} = 0 \text{ if } d_{it+1} < \phi_{t+1} k_{it+1} \quad \text{and} \quad \mu_{it} > 0 \text{ if } d_{it+1} = \phi_{t+1} k_{it+1}.$$

It further holds that

$$c_{it} > 0, \quad k_{it} > 0, \quad d_{it} \geq 0, \quad s_{it} \geq 0, \quad 0 < h_{it} < 1, \quad \text{and} \quad s_{i0}, d_{i0}, k_{i0}, z_{i0} \quad \text{are given.}$$

General equilibrium is achieved as follows. We first make a guess on the interest rate. Given this guess, we solve the problem of households and recover the policy functions for the five choices $(s_{it+1}, d_{it+1}, k_{it+1}, c_{it}, n_{it})$. Given these policies and the known transition probabilities for z, we can calculate the joint stationary distribution of the five state variables $\Psi_t(s_{it}, d_{it}, k_{it}, z_{it})$. We then recover households' aggregate demand for bonds, defined in condition 5 of the general equilibrium. We also derive the aggregate supply of bonds necessary to satisfy the budget constraint of the government, given aggregate tax income and the sum of benefits payed to all unemployed households. Finally, we verify if the aggregate demand and supply match. If they do not not match, we adjust the interest rate and repeat the procedure. The demand of bonds from households is far more sensitive to changes in the interest rate than the supply from the government.

This problem can only be solved numerically. In total, the model includes three endogeneous state variables (s_{it}, d_{it}, k_{it}) plus one exogeneous state variable (z_{it}), as well as two control variables (c_{it}, n_{it}). The presence of an occasionally binding borrowing constraint on gross debt implies that a global solution method is necessary to derive an accurate solution to the model. The solution algorithm we build is similar to Hintermaier and Koeniger (2010) and is partially based on the endogenous grid point method of Carroll (2006). These complexities push the model at the current computational frontier of the economic literature. We provide the further details on the employed procedure in appendix C.1.

4.2.4 Calibration

Table 1 reports our baseline calibration. The model is calibrated so that the steady state is consistent with the characteristics of the 2007 US economy, at the quarterly frequency. This will allow us to study the deleveraging after 2007 documented in Section 4.3.2.

The time discount factor β is chosen so that the safe yearly interest rate on savings is equal to 2.5%. There is no inflation in the model. This return is in line with the interest rate on the yearly Treasury inflation-indexed securities issued in 2007. The coefficient of relative risk aversion is $\gamma = 4$. This is a standard choice found in the relevant literature, for instance in Kaplan and Violante (2014), Guerrieri and Lorenzoni (2017) or Luetticke (2018). The curvature of the utility function from leisure η is chosen such that the average Frisch elasticity of hours of work to the wage is equal to one. The coefficient of leisure in utility ψ is chosen such that households work 40% of their time endowment, consistently with the evidence of Nekarda and Ramey (2013). The parameter θ represents the approximate ratio of non-durable consumption to total consumption.[4] We choose $\theta = 0.7$ to match the average ratio of non-durable to total consumption from the National Income and Product Accounts (NIPA) of the BEA from 2000 to 2010.

Productivity shocks – excluding the unemployment risk – follow the same Markov chain for all agents, with common auto-correlation ρ and common constant variance σ_ε. The Markov process is discretised in 12 points as in Rouwenhorst (1995). We set $\rho = 0.967$ and $\sigma_\varepsilon = 0.017$ so as to match the same moments of the persistent component of the stochastic wage process estimated by Flodén and Lindé (2001), who also use PSID data. Using the Current Population Survey, Shimer (2005) estimates the probability to transition from employment to unemployment is 0.057, and the probability to transition from unemployment to employment is 0.882, at the quarterly frequency. Shimer (2005) also finds unemployment benefits are equal on average to 40% of labour income. We use these values in our calibration.

The quarterly depreciation rate of durable goods is $\delta = 0.0129$ and is equal to the average depreciation rate from the NIPA Fixed Assets Tables from 2000 to 2008, as in Hall (2011). The degree of illiquidity of durable goods depends on ζ, which represents the transaction cost proportional to the quantity of durable goods being sold. We choose $\zeta = 0.15$ as in Guerrieri and Lorenzoni (2017).

We set the maximum loan-to-value ratio to $\phi = 0.9$. This corresponds to the upper bound of the range of mortgage loan-to-value ratios observed in the US in 2007 according to Demyanyk and Van Hemert (2011). The intermediation cost is $\varpi = 0.01$ annually, as in Guerrieri and Lorenzoni (2017). Finally, the income tax rate $\tau = 0.135$ is chosen to replicate the average income tax rate of US households according to the Organisation for Economic Co-operation and Development (OECD) in 2007.

Overall, our choices are based on the same targets and use the same sources as Guerrieri and Lorenzoni (2017) for the calibration of the extended version of their model.

[4] θ would be the exact ratio of non-durable consumption to total consumption if the collateral constraint either did not exist, or was always binding. The fact that this constraint is occasionally binding affects the shadow price of durable goods.

Table 4.1: Calibration of the parameters

Parameter	Explanation	Value	Target/Source
β	Discount factor	0.9711	Annual savings interest = 2.5% (yearly return on Treasury inflation-indexed securities issued in 2007)
γ	Coefficient of relative risk aversion	4	Kaplan and Violante (2014), Guerrieri and Lorenzoni (2017), Luetticke (2018)
η	Curvature of utility from leisure	1.5	Average Frisch elasticity = 1
ψ	Coefficient on leisure in utility	0.2	Average hours worked = 0.4 (Nekarda and Ramey (2013))
θ	Coefficient on non-durables	0.7	Ratio of non-durable to total consumption, NIPA tables (2000-2010 average, Hall (2011))
δ	Durables depreciation rate	0.0129	NIPA fixed assets tables, ratio of depreciation to net stock (2000-2008 average, Hall 2011)
ζ	Proportional loss on durable sales	0.15	Guerrieri and Lorenzoni (2017)
ρ	Persistence of productivity shock	0.967	Persistence of wage process in Flodén and Lindé (2001)
σ_ϵ	Variance of productivity shock	0.017	Variance of wage process in Flodén and Lindé (2001)
$\pi_{e,u}$	Transition to unemployment	0.057	Shimer (2005)
$\pi_{u,e}$	Transition to employment	0.882	Shimer (2005)
ν	Annual unemployment benefits	0.536	40% of average labour income Shimer (2005)
ϕ	Maximum loan-to-value ratio	0.90	Demyanyk and Van Hemert (2011)
τ	Income tax rate	0.135	Average US federal income tax rate (OECD.Stat table I.5)
ϖ	Intermediation cost	0.01	Guerrieri and Lorenzoni (2017)

4.3 Results

We first solve the steady state of the model and provide a description of households' behaviour in this initial equilibrium. In particular, we explain why households decide to hold different amounts of gross savings and gross debt simultaneously. Next, we tighten the borrowing constraint to simulate the credit crunch post-2007 and we describe the dynamics of the economy. We discuss the mechanisms in the model that help replicate our empirical findings. Finally, we make a monetary policy experiment to further illustrate the relevance of the explicit distinction between gross debt and gross savings in the model.

4.3.1 Initial steady state

Note than in an economy with heterogeneous households, the steady state is a state where the distribution of all the variables of the model – and therefore the aggregate quantities – remain constant. Accordingly, we remove the time subscripts for aggregate variables and for the interest rate in our discussion about the steady state. However, individual households are never static. They continue to receive idiosyncratic productivity shocks that make them move up or down the distribution of income, adjust their individual consumption, hours of work, investment, debt and savings. Therefore, we preserve time subscripts for all household-level variables.

Policies in steady state

In figure 4.1, we plot in turn each of the variables of the model averaged over the whole population of households, as a function of households' income in steady state. We begin with a discussion of the most notable results concerning the savings, debt and durable goods policies.

The first subplot of figure Figure 4.1 represents the choice of durable goods as a function of income. We observe a type of break in the durable goods policy. Holdings of durable goods are low among poorer households and become suddenly much higher among middle class and wealthier households. Poor households are trapped with low amounts of durable goods. They cannot afford the investment costs that durable goods incur, so they need a mortgage to invest. But since they hold low stocks of collateral, they cannot obtain a large mortgage. By contrast, middle class households can support the investment cost and accumulate durable goods, therefore they have enough collateral to obtain larger mortgages if necessary and continue to invest. The second subplot

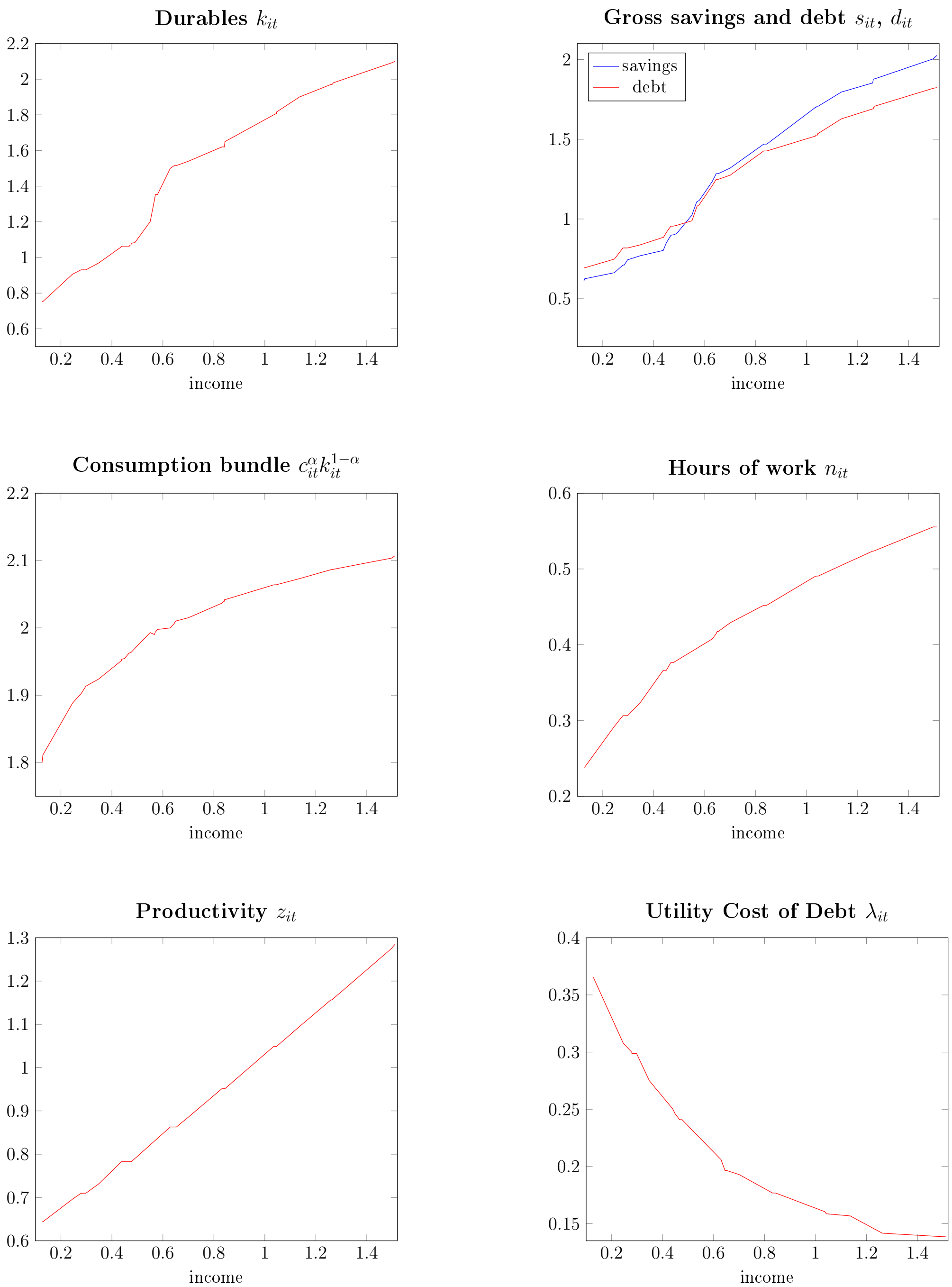


Figure 4.1: Description of the steady state

Plots of all the variables of the model as a function of households' income. The plots for hours of work and for productivity exclude unemployed households.

of Figure 4.1 represents the gross debt and savings policies. Considering that durable goods serve as collateral for mortgage debt, it is unsurprising that the debt policy resembles the durable goods policy. Debt holdings are smaller among poorer households, but suddenly higher among middle class and wealthier households. In the same plot, we can see that although poorer households hold low amounts of debt, they hold even lower amounts of savings. Poor households therefore hold negative net savings. This is consistent with empirical evidence reported earlier. Middle class households are close to zero net savings, while wealthier households have larger positive net savings. This result on the distribution of net savings is also consistent with Guerrieri and Lorenzoni (2017) who find that low productivity households tend to hold negative net savings while high productivity households hold positive net savings. We explain below why households decide to hold gross debt and gross savings simultaneously.

The policies for consumption and hours of work are without surprise for this type of model. In the third plot, we define consumption as in the utility function of households, i.e. as a Cobb-Douglas aggregate of durable and non-durable goods. We observe that the consumption policy is increasing concave in income, as is standard in models where households save part of their income for precautionary motive, see Carroll and Kimball (1996). By examining the fourth and fifth plots, we observe that wealthier households are those working longer hours with a higher productivity. This seems to indicate that the income effect dominates the substitution effect. A higher productivity – and therefore a higher wage – encourages households to work longer hours, as in Guerrieri and Lorenzoni (2017).

Heterogeneous portfolios

To understand why households might want to hold different amounts of gross debt and savings simultaneously, consider the trade-off they are facing. On the one hand, they want to save their income for precautionary reasons because they face the risk of unemployment. Their value function is increasing in s_{it}. On the other hand, they want to spend their income to invest in housing because this increases their utility. Their value function is increasing in k_{it} as well. However, their income is finite. Households must decide if they allocate their income more towards savings, or more towards housing investment. Mortgages are an option to mitigate this trade-off. Thanks to mortgages, households can immediately invest in housing without spending their savings. But debt is costly. It needs to be repayed with interest and it makes households' budget constraint tighter in the future.

Therefore, we can express the cost of debt in households' dynamic maximization programme in terms of lost intertemporal utility. Thereafter, we refer to this cost as the intertemporal utility cost of debt. The intertemporal utility cost of debt is time varying and idiosyncratic. Its magnitude plays a key role in the decision of households to finance their housing investment with a mortgage, or by spending their savings, or with a combination of both. We can illustrate the intertemporal utility cost of debt by calculating the envelope condition with respect to d_{it}. We obtain

$$\frac{\partial V(s_{it}, d_{it}, k_{it}, z_{it})}{\partial d_{it}} = -\lambda_{it} + \lambda_{it}\tau r^d. \tag{4.1}$$

If at time t the stock of debt of household i was increased marginally, the value function of household i would decrease by -λ_{it} + $\lambda\tau_t r^d_{t-1}$. The magnitude of this utility cost of debt is increasing in λ_{it}. We obtain an expression for λ_{it} by calculating the first order condition of households' maximization programme with respect to consumption:

$$\frac{\partial U(c_{it}, k_{it}, h_{it})}{\partial c_{it}} = \lambda_{it}. \tag{4.2}$$

We give a graphical representation of the utility cost of debt as a function of income in steady state in the last plot of figure 4.1. λ_{it} is quickly decreasing convex and becomes almost flat for higher earners. This implies that poor households face the highest utility cost of debt. The investment cost also makes durable goods too expensive without a mortgage. For these two reasons, poor households have simultaneously low amounts of debt and small stocks of durable goods. Concerning middle class households, their utility cost of debt is quickly declining and they can also more easily afford the investment cost. Therefore they invest much more in durable goods and hold much larger amounts of debt, while preserving positive net savings for precautionary reasons. Finally, wealthy households face a very lost utility cost of debt. Therefore, they are encouraged to rely more on mortgages to finance their durables investment while also maintaining large positive net savings.

Note that the structure of taxes has a critical influence on the total amount of gross debt in the economy. From (4.1) we can observe that the mortgage interest tax deduction mitigates the negative marginal impact of debt on the value function, for all households. This is an encouragement for all households to hold more debt. The structure of taxes helps to get the right overall amount of debt in the economy relative to the amount of savings, while the heterogeneity in the utility cost of debt through λ_{it} helps to get the right distribution of debt.

4.3.2 Credit crisis

In this section, we simulate a tightening of the borrowing constraint and replicate the credit crunch post 2007. We argue that the speed, magnitude and heterogeneity of the deleveraging process predicted by the model are consistent with the empirical evidence. We also discuss the key model mechanisms that explain these results.

Tightening of the borrowing constraint

To simulate the credit crunch after 2007, we progressively tighten the borrowing constraint by reducing ϕ_t following the law of motion

$$\phi_t = \phi - \Delta\phi \cdot t,$$

where ϕ is the maximum loan-to-value ratio in the initial steady state, ϕ_t is the maximum loan-to-value ratio in quarter t. Households perfectly anticipate this law of motion as in Guerrieri and Lorenzoni (2017). They know the future value of the borrowing constraint one quarter early.

House prices in the US started falling in 2007, and their trough was in 2012. In 2015, house prices had not yet fully recovered. As an illustration of the evolution of house prices, Figure 4.3 plots the S&P/Case-Shiller US National Home Price Index from 2001 to 2020. In line with this observation, since durable goods in our model are a proxy for housing, we decide the tightening of the borrowing constraint also lasts for 20 quarters in the model after the initial equilibrium. However, the impact of the credit crunch continues to be felt after 2012; we can see in PSID data that the reduction of mortgage debt continues until 2015. Therefore, in the model, we continue to simulate the dynamics of the economy for 12 additional quarters after we stop tightening the borrowing constraint. Overall, we simulate the economy for 32 quarters. The borrowing constraint is progressively tightened during the first 20 quarters and remains constant during the last 12 quarters, as illustrated by Figure 4.2. We chose $\Delta = 0.008$ – the depth of the credit crunch – such that the predicted aggregate mortgage debt falls by exactly 21% over these 32 quarters, as in the data.

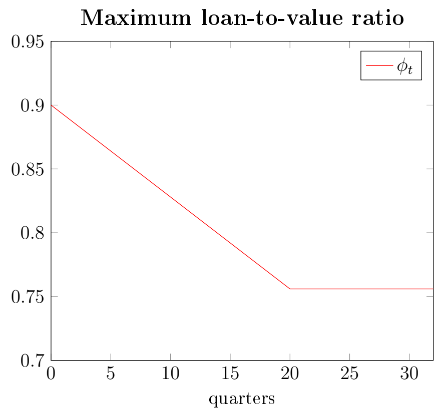


Figure 4.2: Dynamics of the maximum loan-to-value ratio

The maximum loan-to-value ratio is steadily tightened during the first 20 quarters of the transition, and remains constant for the remaining 12 quarters.

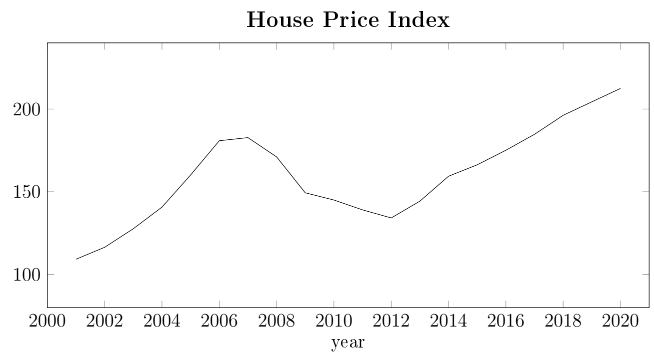


Figure 4.3: S&P/Case-Shiller US National Home Price Index

Index = 100 in 2000. House process were at the lowest in 2012 after the peak of 2007. In 2015, house prices had not yet fully recovered.

Computation

We solve the general equilibrium of the model for each of the 32 quarters. In order to remain in general equilibrium throughout, the interest rates on debt and savings are allowed to change, although the interest wedge remains constant. The aggregate quantity of net savings B_t is also allowed to change. At the beginning of each quarter during the transition, we know the current stock of liquid assets inherited from the previous quarter, the past interest rates on gross debt and savings, the past joint distribution of the state variables, and the current level of the maximum loan-to-value ratio. We make a guess on the interest rates and calculate the policy functions using the procedure described in appendix C.1. Given the new policy functions and the known past joint distribution of the state variables, we recover the new joint distribution of the state variables. We then recover aggregate quantities and verify if the market for liquid assets (gross savings and gross debt) is in equilibrium. If not, we adjust the interest rates and repeat until convergence to general equilibrium.

Predictions of the model

1. Debt dynamics. Figure 4.4 compares the predictions of the model regarding the dynamics of mortgage debt with the empirical observations from the previous chapter, during the deleveraging phase from 2007 to 2015. We first discuss the aggregate dynamics and then dissect these by income groups. The first plot of Figure 4.4 compares the empirical estimates of the average amounts of mortgage debt over the whole sample – with the corresponding 95% confidence intervals – to the predictions of the model. The prediction of the model in the initial steady state are scaled such that the average amount of debt exactly matches the empirical point estimate in 2007. All the following predictions of the model, for all dates and all income groups, are then scaled by the same factor, so that the results of the model are expressed in the same units as the empirical evidence. Since the tightening of the borrowing constraint is calibrated so as to replicate the magnitude of the aggregate deleveraging observed in the data, the prediction of the model at the aggregate level also matches the empirical point estimate for 2015 precisely. The model also successfully replicates the transition of the average amount of mortgage loans from 2007 to 2015, since the predictions of the model consistently fall within the empirical confidence intervals.

The other plots of Figure 4.4 illustrate the dynamics of mortgage debt for the other income groups. The model is again successful in replicating the dynamics of the middle and high income groups. The predictions fall almost always within the confidence intervals of the empirical point estimates. Although the model tends to overestimate the average amounts of debt among low income households and underestimate the amount of debt among top earners, the predicted dynamics are consistent with the data for these two groups as well. In particular, the predictions of the model are consistent with our key empirical finding that the aggregate reduction of debt is mostly due to the behaviour of the 60% of households in the middle and high income groups. Together, these households account for about 75% of the aggregate reduction of debt in the model, compared to about 72% in the survey. Therefore, the remaining 40% of households at the tails of the distribution explain only about one quarter of the aggregate dynamics.

We now discuss the mechanism in the model responsible for the observed heterogeneity of the deleveraging process. Let $D_{it+1}(s_{it}, d_{it}, k_{it}, z_{it}, \phi_{t+1}, \tau)$ denote the optimal debt policy of household i, i.e. the optimal choice of d_{it+1} as a function of the state variables in t, the anticipated maximum loan-to-value ratio in $t+1$, and the constant tax rate τ. Let I_{it} denote household's i taxable income at time t such that $I_{it} = \mathcal{I}_{it}\nu_t + (1-\mathcal{I}_{it})y_{it} + r^s_{t-1}s_{it} - r^d_{t-1}d_{it}$. D_{it+1} satisfies

$$\frac{\partial D_{it+1}}{\partial \phi_{t+1}} = k_{it+1}\left(1 - \frac{\partial D_{it+1}}{\partial \tau}\frac{1}{(1+r^d_t)I_{it}}\right). \tag{4.3}$$

Proof of this result is presented in appendix C.2. This expression illustrates how an anticipated tightening of the borrowing constraint affects the desired amount of mortgage debt, as a function of income. We can observe that if households invest little in durable goods, i.e. have a low k_{it+1}, their choice of debt will not be greatly influenced by the tightening of the borrowing constraint. We can verify that in the extreme case when households hold zero durable goods, their choice of mortgage debt is not affected at all by the credit crunch. This result is unsurprising considering that households are not allowed to be indebted if they do no hold any collateral. This explains why poorer households with low stocks of durable goods do not deleverage significantly during the credit crunch. Intuitively, poor households tend to hold small amounts of durable goods as well as small amounts of debt to begin with, so they have little room to deleverage. Assuming households have a larger stock of durables, the strength of their reaction to a tightening of their borrowing constraint depends on their taxable income I_{it}. If the taxable income is large, the right hand side of equation

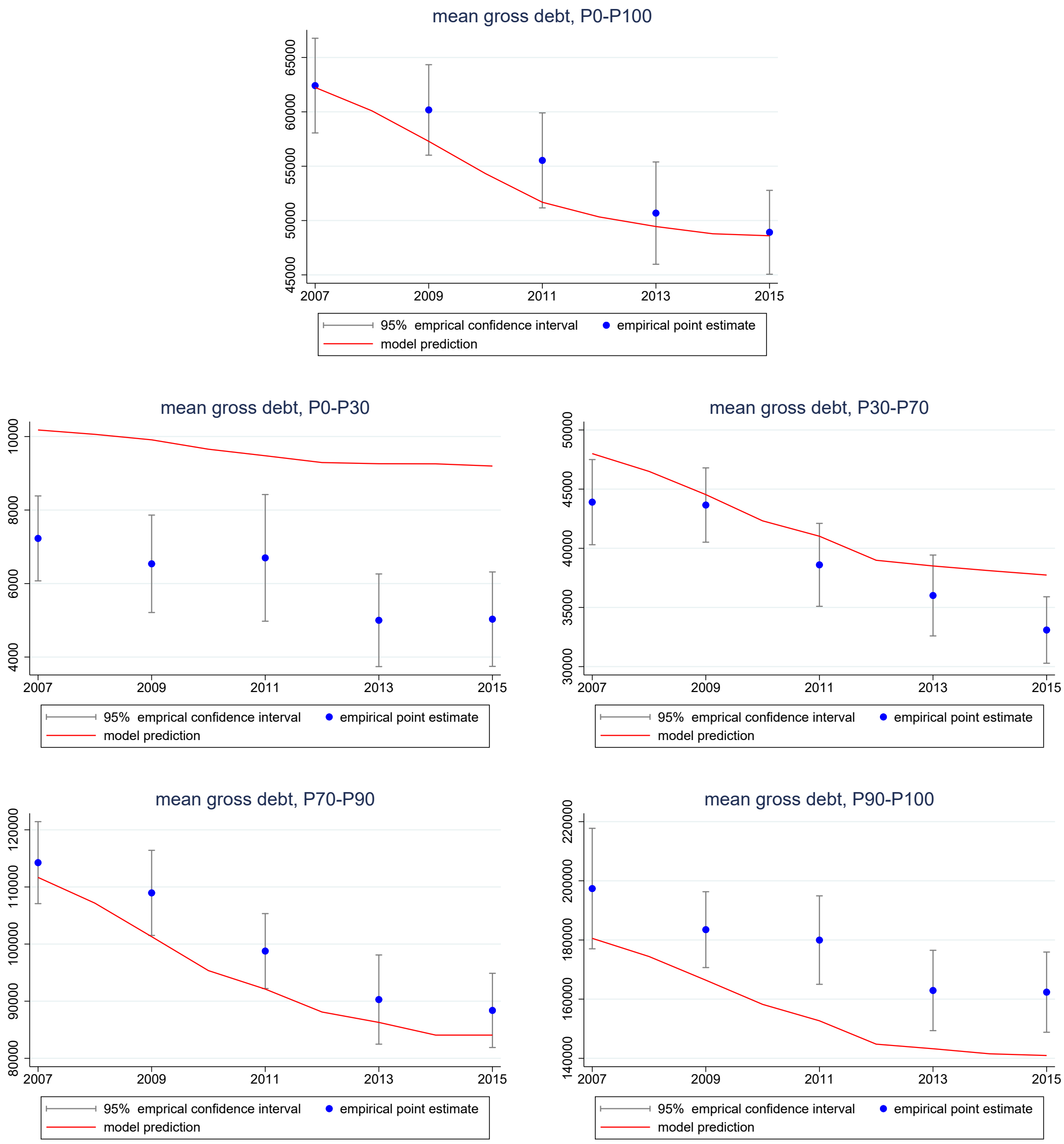


Figure 4.4: Averages of the individual amounts of mortgage debt, model vs. data, by income class, 2001-2015

4.3 is large as well, meaning the reaction to a change in the maximum loan-to-value ratio is large.

This results predicts that wealthier households should cut their debt by a larger absolute amount. This is verified both in the survey data and in the model. It does not predict, however, that wealthier households should deleverage by a larger percentage. In fact, top earners deleverage more than any other groups in absolute term, but as a percentage of their 2007 peak, they deleverage only by about half as much as middle and high earners.

2. Consumption dynamics. Figure 4.5 represents the general equilibrium response of consumption during the credit crunch in percentage deviation from the initial steady state. We first discuss the consumption dynamics at the aggregate level, before comparing the responses of the low and top income groups.

By the end of the 32 quarters, aggregate consumption declines by 3.75%. The dynamics we observe are the results of two opposing forces: the exogeneous reduction of the maximum loan-to-value ratio, and the endogeneous reduction of the interest rates. On the one hand, the contraction of the borrowing constraint pushes households to prioritise the repayment of their debt, and thus to cut their consumption. On the other hand, the endogeneous reduction of the interest rates leads to intertemporal substitution encouraging households to consume more in the present and less in the future. In the general equilibrium response, the former effect dominates the latter.[5] We find that the two critical parameters that determine which effect dominates are ψ, the coefficient on leisure in utility, and Δ which controls the depth of the contraction of the borrowing constraint. If we were to choose a value for ψ many orders of magnitude larger than in the baseline calibration and a Δ at least half lower, we would observe that the substitution effect dominates. However, this would also imply that households work far less than 40% of their total time endowment, as documented in the empirical study of Nekarda and Ramey (2013), and that households deleverage by only half the proportion documented in the PSID data. We conclude the baseline calibration is well suited

[5]Note that the optimality conditions of the model presented in Appendix C.1 impose a non-increasing relationship between hours of work and consumption. This implies that we cannot obtain co-movement of consumption and hours of work in this model, because of the shape of households' preferences. This is a well known weakness of this class of models, and the same problem can be observed for instance in Guerrieri and Lorenzoni (2017). A possibility to obtain a simultaneous decline of aggregate consumption and hours of work would be to augment the model with an independent production sector, as in Kaplan et al. (2018). The fall of demand for consumption goods during the credit crunch would be met by a reduction for labour demand, and thus a fall of working hours. However, this would make the model less tractable, without providing clear additional insights into the dynamics of household debt, which is our primary object of study.

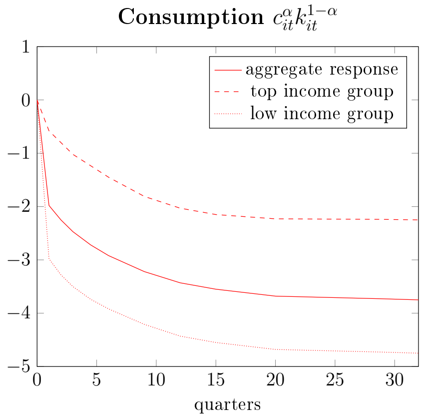


Figure 4.5: Heterogeneous response of consumption to the credit crunch

Percentage deviation from initial steady state, during the credit crunch. Solid line: aggregate response. Dashed line: top income group response. Dotted line: low income group response.

for the medium-term analysis of the economy with this class of model. Later, we implement a policy experiment where the borrowing constraint remains constant, but the Central Bank decides to progressively cut the interest rates. We will again discuss the dynamics of consumption in this exercise where only the substitution effect comes into play.

Decomposing the aggregate dynamics by income groups, we observe that the response is qualitatively the same for all households, although the magnitude differs. Low income households react more strongly than top income households, relative to their respective levels in the initial equilibrium. By the end of the credit crunch, top income households cut their consumption by only 2.23%, compared to 4.76% for low income households. The middle class and high earners are in between. Note that these results are consistent with the existing literature. In their related model of household debt, Guerrieri and Lorenzoni (2017) find that during a credit crunch, poorer households are predicted to cut their consumption proportionally more than wealthy households. This is because poorer households have a higher marginal propensity to consume, implied by the concavity of the consumption function. Therefore, they react more strongly when the reduction of the borrowing constraint tightens their budget.

3. Interest rates. The model predicts a fall of the interest rates. Since the net demand for liquid assets falls during the credit crunch, a reduction of the interest rates is necessary to maintain

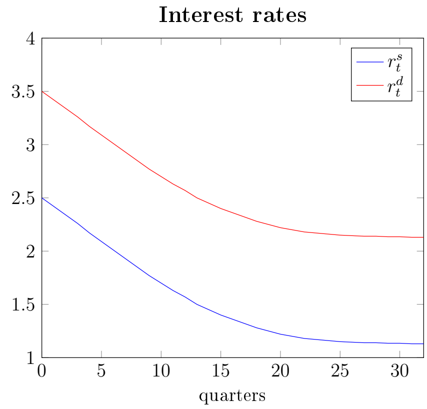


Figure 4.6: Dynamics of the interest rates

The wedge is kept constant, so both interest rates fall by 1.37 percentage points during the credit crunch. The interest rates become constant only after the last two quarters of the transition.

the market for liquid assets in equilibrium. Figure 4.6 illustrates the dynamics of the interest rates during the transition/credit crunch. We maintain a constant wedge, so the interest rates on gross savings and gross debt follow identical dynamics; they fall by 1.37 percentage points over the 32 quarters. Their fall is faster at the beginning of the transition when the borrowing is being tightened, and slower toward the end of the transition, in particular during the last 12 quarters when the maximum loan-to-value ratio remains constant. The effect of the tightening of the borrowing constraint is long lasting since the interest rates perfectly stabilise only during the last two quarters of the transition.

4.3.3 Monetary policy transmission with heterogeneous portfolios

To further demonstrate the importance of modeling separately gross debt and gross savings, we simulate a monetary policy experiment. We demonstrate that a fall of the nominal interest rates generates an increase of aggregate consumption, but the exact strength of individual consumption responses is heterogeneous and directly depends on the relative amounts of gross debt and gross savings – and not only net savings – that households hold.

Design of the experiment

In this exercise, we focus on the impact of changes in nominal interest rates – the most common instrument of monetary policy – on consumption – the largest component of GDP. The Central Bank centralises all borrowing and lending operations involving the government and households. The problem of households is otherwise unchanged and all parameters remain identical. We assume the central bank decides to implement the same path for the interest rates on debt and savings as the one described in Figure 4.6. However, in this exercise, we maintain the maximum loan-to-value ratio constant at the initial level. Note that for a given path of the maximum loan-to-value ratio and everything else being constant, there is a unique path for the interest rates that is compatible with general equilibrium as defined in section 4.2. Therefore, if the Central Bank decides to impose the same path for the interest rates as in Figure 4.6 but ϕ remains constant, general equilibrium as defined previously will be impossible. In order to maintain general equilibrium, we replace conditions 4 and 5 of the general equilibrium by the condition that the Central Bank commits to either buying or selling the necessary quantities of liquid assets so as to satisfy the budget constraint of the government and clear the market for liquid assets at the chosen interest rates.

Results of the experiment

1. Consumption dynamics. Figure 4.7 represents the responses of consumption during the policy experiment in percentage deviation from the initial steady state. At the aggregate level, we observe an increase of consumption by 6.92%. Households are encouraged by the cuts in interest rates to consume more in the present and to work and save less for future consumption. Qualitatively, this is the opposite of what we observed during the credit crunch. The magnitude of the responses is also stronger than during the credit crunch. This is because the intertemporal substitution is now the only effect at play, and it is now a direct effect. This contrasts with the credit crunch studied earlier where the fall of the interest rates was only the result of a general equilibrium reaction of the economy that partially compensated the direct effect of the borrowing constraint.

Decomposing the aggregate dynamics, we observe the response of households in the top income group is stronger than the response of households in the low income group. Top earners increase their consumption by 8.41%, against only 4.93% for low earners. The literature provides empirical evidence that the elasticity of intertemporal substitution of households – and therefore the mag-

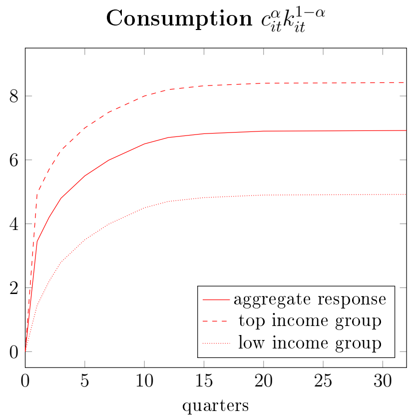


Figure 4.7: Heterogeneous response of consumption to the policy experiment

Percentage deviation from initial steady state, during the policy experiment. Solid line: aggregate response. Dashed line: top income group response. Dotted line: low income group response.

nitude of the response of households to cuts in the interest rates – directly depends on how much gross savings they hold, and on the type of savings. For instance, Vissing-Jørgensen (2002) finds that households with no gross savings do not react to cuts in interest rates, using data from the US Consumer Expenditure Survey. Similarly, Kaplan et al. (2014) find that one third of households in the US population are close to zero liquid gross savings and do not react to direct monetary policy interventions, using the PSID data. In our model, we can verify that households with the lowest liquid gross savings have the weakest reaction (toward the bottom of the income distribution), while households with the largest liquid gross savings have the strongest reaction (toward the top of the income distribution). Therefore, we can confirm that a large elasticity of intertemporal substitution coincides with large holdings of gross savings. However, it is too early to conclude that the transmission of monetary policy operates only or even mainly through gross savings in the model. Large holdings of gross savings also coincide with large holdings of gross debt and net savings. Further investigation is necessary to determine the relative importance of these different potential channels for the transmission of monetary policy.

2. Monetary policy transmission. We propose a strategy to identify the relevant channels for the transmission of monetary policy to households' consumption. We run the policy experiment and use the results to simulate a panel dataset of 32 quarters and 300 households representative of the whole population of the model. We then estimate a pooled OLS regression on the panel sample

$$\Delta ln(c_{it-h,t}) = \beta \; s_{it-h} + \gamma \; d_{it-h} + \varepsilon_{it}, \tag{4.4}$$

where $\Delta ln(c_{it-h,t})$ represents the log difference of the consumption level of household i between $t - h$ and t. Consumption continues to be defined as a Cobb-Douglas aggregate of durable and non-durable goods. As discussed previously, we know given the path of the interest rates that $\Delta ln(c_{it-h,t})$ is always positive irrespective of the choice of the time horizon h. To be consistent with our previous discussion of the empirical literature, we anticipate $\hat{\beta}$ to be positive and significant since we expect the substitution effect to be felt more strongly by households with larger existing stocks of gross savings. We do not include an intercept since the literature has already demonstrated that households with no assets are not interest-sensitive. We want to verify the sign and significance of $\hat{\gamma}$. This will tell us if mortgage debt also has a significant predictive power over the response of consumption. Table 4.2 presents the regression results for selected values of h. After controlling for gross savings, the response of consumption during the monetary policy intervention is consistently stronger at any horizon among households with larger mortgages. In fact, mortgages are a stronger predictor than savings. To complete these results, we create a variable $p_{it} = \frac{d_{it}}{s_{it}+d_{it}}$ reflecting the composition of the portfolio of household i in t. When gross debt represents a larger percentage of the sum of gross debt plus gross savings, p_{it} gets closer to one. We then estimate a pooled OLS regression on the panel sample

$$\Delta ln(c_{it-h,t}) = \kappa \; s_{it-h} + \lambda \; p_{it-h} + \varepsilon_{it}. \tag{4.5}$$

The results are presented in Table 4.3. We observe that the response of consumption is systematically stronger among households with a larger amount of debt relative to their absolute net savings.

Empirical papers such as Cloyne et al. (2020), Kim and Lim (2020) or theoretical models such as the model of Garriga et al. (2017) also demonstrate that mortgage debt can be a strong predictor the response of consumption to cuts in interest rates. However, these papers do not consider the joint effect of savings and mortgages. Our model contributes to the literature by illustrating for the first time how gross savings and gross mortgage debt can jointly affect the response of households,

Table 4.2: Determinants of consumption response during the policy experiment

coefficient	h=8	h=16	h = 24	h = 32
$\hat{\beta}$	.0200	.0332	.0480	.0362
	(.0000)	(.0000)	(.0000)	(.0000)
$\hat{\gamma}$	.0638	.0716	.0620	.0752
	(.0000)	(.0000)	(.0000)	(.0000)
R^2	.8874	.8533	.8453	0.8408

Results of the regression $\Delta ln(c_{it-h,t}) = \beta\ s_{it-h} + \gamma\ d_{it-h} + \varepsilon_{it}$.
A significant and positive $\hat{\gamma}$ means that the response of consumption at any horizon to the monetary policy experiment is stronger among households with large amounts of mortgage debt, ceteris paribus.
p-values in parentheses.

Table 4.3: Determinants of consumption response during the policy experiment, cont'd

coefficient	h=8	h=16	h = 24	h = 32
$\hat{\kappa}$	.0173	.0411	.0290	.0343
	(.0000)	(.0000)	(.0000)	(.0000)
$\hat{\lambda}$	.0187	.0129	.0178	.0158
	(.0000)	(.0000)	(.0000)	(.0000)
R^2	.8198	.7897	.7801	0.7645

Results of the regression $\Delta ln(c_{it-h,t}) = \kappa\ s_{it-h} + \lambda\ p_{it-h} + \varepsilon_{it}$.
A significant and positive $\hat{\lambda}$ means that the response of consumption at any horizon to the monetary policy experiment is stronger among households with larger amounts of gross debt relative to the sum of their gross debt plus gross savings, ceteris paribus.
p-values in parentheses.

and how the composition of households' portfolios – specifically the quantity of mortgage debt relative to the sum of gross debt plus gross savings – also affects the magnitude of the response of consumption.

4.4 Conclusion

We build a heterogeneous-agent model to study the dynamics of household debt during the 2007 financial crisis in the US, and during the subsequent economic recovery until 2015. The model is closely related to state-of-the-art models of household debt of the class of Guerrieri and Lorenzoni

(2017), with a crucial addition: it allows for an explicit distinction between gross debt and gross savings. Households can also deduct their mortgage interest payments from their taxable income.

We first solve the steady state of the model calibrated to reproduce the state of the US economy in 2007. The model predicts that all households hold significant amounts of gross debt and gross savings simultaneously. The amounts of both gross debt and gross savings held by households are increasing functions of income. Net savings (gross savings - gross debt) are negative only at the lower tail of the income distribution. All of these predictions are consistent with the evidence we gather from the PSID and with the literature. The aggregate amount of gross debt in the economy relative to gross savings is directly affected by the mortgage interest tax deduction, while the distribution of gross debt depends on heterogeneity in the intertemporal utility cost of debt.

We then conduct two exercises to explore the new insights offered by the explicit distinction between gross debt and gross savings. In the first exercise, we progressively reduce the fraction of the stock of durable goods that households are allowed to use as collateral. This tightening of the borrowing constraint is calibrated to reproduce the credit crunch in the US and the reduction of mortgage debt from 2007 to 2015. The predictions of the model regarding the dynamics of debt for the four income groups match our empirical observations from 2007 to 2015. There is a reduction of aggregate debt by 21%, exactly as in the data. The 60% of households toward the center of the income distribution explain 75% of this aggregate reduction of debt (against 72% in the data), while the 10% and 30% of households respectively at the upper and lower tails collectively explain only 25% of the aggregate dynamics (against 28% in the data). This is the first model of household debt capable of reproducing these empirical observations.

In the second exercise, we simulate a monetary policy experiment. The central bank progressively cuts interest rates, while the borrowing constraint remains constant. The transmission of monetary operates operates through two distinct channels; savings and debt. Our model is the first to illustrate both of these channels simultaneously. It reveals that the response of households' consumption to changes in the interest rates also depends on the relative quantities of savings and debt they hold. Households with larger mortgages as a percentage of the sum of their debt plus savings reacts more strongly. These findings contribute to a finer understanding of the factors conditioning the heterogeneous response of households to monetary policy interventions.

Chapter 5

Conclusion

This thesis contributes to three recent trends of the macroeconomic literature.

In Chapter 2, we contribute to the study of the relationships between the financial and the business cycles. Using a variety of empirical measures, we document the expected equity risk premium exhibits growth asymmetry for the post-WWII US economy; increases are sharp and short, while declines are long and gradual. This positive skewness is a robust feature. It is observable over different subsamples and at different investment horizons. We replicate this empirical observation in a real business cycle model augmented with Epstein-Zin preferences. Endogenous changes in the degree of uncertainty about productivity lead to procyclical variations in nowcast accuracy causing growth asymmetry in expected equity risk premia. Empirical evidence on uncertainty and nowcast precision from the Survey of Professional Forecasters support this mechanism. The model also successfully replicate the countercyclicality of the equity risk premium and the negative skewness in the growth of macroeconomic aggregates.

In Chapter 3, we exploit survey data to uncover new facts about the distributional dynamics of household debt. Using survey data from the PSID, we document new facts on the relative contribution of different income groups in the dynamics of household gross debt around the 2007 financial crisis in the US. Our key finding is that the deleveraging after the financial crisis is driven by the 60% of households in the middle of the income distribution, rather than by the 40% that are located in the tails of the distribution. Mortgage debt represents by far the majority of total household debt, and it is the main driver of the dynamics of debt in every segment of the income distribution. Our results on the relative importance of the different income groups are not explained by the mobility of households in the income distribution or by defaults. Existing theoretical models do not account for these facts because they focus on the net financial positions of household and do not track the distribution of gross debt.

In Chapter 4, we contribute to the emerging class of heterogeneous-agent models specifically designed to explore the role of distributional issues on aggregate dynamics. We build a heterogeneous-household model with an explicit distinction between savings and debt and with a realistic structure of tax incentive for households with a mortgage. The model successfully replicates for the first time the precise roles played distinct income groups during the credit crisis, with households in the middle of the distribution contributing proportionally far more to the aggregate reduction of mortgage debt than households in the tails of the distribution. The key mechanism for this result is endogenous heterogeneity in the intertemporal utility cost of debt. Finally, the model provides new insights into the transmission of monetary policy to household's consumption. We find that savings and mortgages are cumulative channels for the transmission of cuts in interest rates, and mortgages represent the stronger of the two channels.

Bibliography

Adelino, M., Schoar, A., and Severino, F. (2018). Dynamics of housing debt in the recent boom and Great Recession. *NBER macroeconomics annual*, 32(1):265–311.

Adrian, T., Crump, R. K., and Moench, E. (2013). Pricing the term structure with linear regressions. *Journal of Financial Economics*, 110(1):110–139.

Altug, S. and Labadie, P. (2008). *Asset pricing for dynamic economies.* Cambridge University Press, UK.

Auclert, A. (2019). Monetary policy and the redistribution channel. *American Economic Review*, 109(6):2333–2368.

Bachmann, R., Elstner, S., and Sims, E. R. (2013). Uncertainty and economic activity: evidence from business survey data. *American Economic Journal: Macroeconomics*, 5(2):217–249.

Baker, S. R., Bloom, N., and Davis, S. J. (2012). Has economic policy uncertainty hampered the recovery? *Chicago Booth Research Paper No. 12-06.*

Barsky, R. B. and Sims, E. R. (2011). News shocks and business cycles. *Journal of Monetary Economics*, 58(3):273–289.

Bayer, C., Luetticke, R., Pham-Dao, L., and Tjaden, V. (2019). Precautionary savings, illiquid assets, and the aggregate consequences of shocks to household income risk. *Econometrica*, 87(1):255–291.

Beaudry, P. and Portier, F. (2006). Stock prices, news, and economic fluctuations. *American Economic Review*, 96(4):1293–1308.

Bekaert, G., Engstrom, E., and Xing, Y. (2009). Risk, uncertainty, and asset prices. *Journal of Financial Economics*, 91(1):59–82.

Bekaert, G. and Harvey, C. R. (1995). Time-varying world market integration. *Journal of Finance*, 50(2):403–445.

Benigno, P., Eggertsson, G. B., and Romei, F. (2020). Dynamic debt deleveraging and optimal monetary policy. *American Economic Journal: Macroeconomics*, 12(2):310–350.

Bilbiie, F. O. (2020). The new Keynesian Cross. *Journal of Monetary Economics*, 114:90–109.

Bloom, N. (2014). Fluctuations in uncertainty. *Journal of Economic Perspectives*, 28(2):153–177.

Bok, B., Caratelli, D., Giannone, D., Sbordone, A., and Tambalotti, A. (2017). Macroeconomic nowcasting and forecasting with big data. *Federal Reserve Bank of New York Staff Report No. 830.*

Boldrin, M. and Levine, D. K. (2001). Growth cycles and market crashes. *Journal of Economic Theory*, 96(1-2):13–40.

Caldara, D., Fernández-Villaverde, J., Rubio-Ramírez, J. F., and Yao, W. (2012). Computing DSGE models with recursive preferences and stochastic volatility. *Review of Economic Dynamics*, 15(2):188–206.

Campbell, J. R. and Hercowitz, Z. (2009). Welfare implications of the transition to high household debt. *Journal of Monetary Economics*, 56(1):1–17.

Campbell, J. R. and Hercowitz, Z. (2019). Liquidity constraints of the middle class. *American Economic Journal: Economic Policy*, 11(3):130–155.

Campbell, J. Y., Pflueger, C., and Viceira, L. M. (2019). Macroeconomic drivers of bond and equity risks. *Harvard Business School Finance Working Paper No. 14-031.*

Campbell, J. Y. and Shiller, R. J. (1988a). The dividend-price ratio and expectations of future dividends and discount factors. *The Review of Financial Studies*, 1(3):195–228.

Campbell, J. Y. and Shiller, R. J. (1988b). Stock prices, earnings and expected dividends. *The Journal of Finance*, 43(3):661–676.

Campbell, J. Y. and Thompson, S. B. (2008). Predicting excess stock returns out of sample: Can anything beat the historical average? *The Review of Financial Studies*, 21(4):1509–1531.

Carroll, C. (2006). The method of endogenous grid points for solving dynamic stochastic optimization problems. *Economics Letters*, 91(3):312–320.

Carroll, C. D. and Kimball, M. S. (1996). On the concavity of the consumption function. *Econometrica*, 64(4):981–993.

Cascaldi-Garcia, D. and Vukotic, M. (2019). Patent-based news shocks. *The Warwick Economics Research Paper Series 1225.*

Cloyne, J., Ferreira, C., and Surico, P. (2020). Monetary policy when households have debt: new evidence on the transmission mechanism. *Review of Economic Studies*, 87(1):102–130.

Cloyne, J. S. and Surico, P. (2017). Household debt and the dynamic effects of income tax changes. *Review of Economic Studies*, 84(1):45–82.

Constâncio, V. (2017). Inequality and macroeconomic policies. *Speech at the Annual Congress of the European Economic Association, Lisbon, Portugal, August 22. https://www.ecb.europa.eu/press/key/date/2017/html/ecb.sp170822.en.html.*

Cooley, T. F. and Prescott, E. (1995). *Frontiers of business cycle research.* Princeton University Press, Princeton, NJ.

Cooper, D. H. (2013). Changes in U.S. household balance sheet behavior after the housing bust and Great Recession: evidence from panel data. *Boston FED, Research Department Public Policy Discussion Papers*, (No. 13-6).

Corradi, V., Distaso, W., and Mele, A. (2013). Macroeconomic determinants of stock volatility and volatility premiums. *Journal of Monetary Economics*, 60(2):203–220.

D'Agostino, R. B., Belanger, A., and D'Agostino, J. R. B. (1990). A suggestion for using powerful and informative tests of normality. *The American Statistician*, 44(4):316–321.

Damodaran, A. (2019). Equity risk premiums (erp): determinants, estimation and implications, the 2019 edition. *New York University - Stern School of Business working paper.*

Demyanyk, Y. and Van Hemert, O. (2011). Understanding the subprime mortgage crisis. *Review of Financial Studies*, 24(6):1848–1881.

Den Haan, W. J., Rendahl, P., and Riegler, M. (2018). Unemployment (fears) and deflationary spirals. *Journal of the European Economic Association*, 16(5):1281–1349.

Duarte, F. and Rosa, C. (2015). The equity risk premium: A review of models. *Economic Policy Review*, 21(2):39–58.

Enders, Z., Kleemann, M., and Müller, G. (2017). Growth expectations, undue optimism, and short-run fluctuations. *Bundesbank Discussion Paper No. 11/2017.*

Epstein, L. G. and Zin, S. E. (1989). Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica*, 57(4):937–969.

Fajgelbaum, P. D., Schaal, E., and Taschereau-Dumouchel, M. (2017). Uncertainty traps. *The Quarterly Journal of Economics*, 132(4):1641–1692.

Fama, E. F. and French, K. R. (1988). Dividend yields and expected stock returns. *Journal of Financial Economics*, 22(1):3–25.

Fama, E. F. and French, K. R. (1989). Business conditions and expected returns on stocks and bonds. *Journal of Financial Economics*, 25(1):23–49.

Fama, E. F. and French, K. R. (2002). The equity premium. *The Journal of Finance*, 57(2):637–659.

Faust, J., Rogers, J., and Wright, J. (2005). News and noise in g-7 gdp announcements. *Journal of Money, Credit and Banking*, 37(3):403–419.

Fernald, J. G. (2012 - 2019). A quarterly, utilization-adjusted series on total factor productivity. *FRBSF working paper series 2012-19.*

Ferson, W. E. and Harvey, C. R. (1991). The variation of economic risk premiums. *Journal of Political Economy*, 99(2):385–416.

Flodén, M. and Lindé, J. (2001). Idiosyncratic risk in the United States and Sweden: Is there a role for government insurance? *Review of economic dynamics*, 4(2):406–437.

Garriga, C., Kydland, F. E., and Šustek, R. (2017). Mortgages and monetary policy. *The Review of Financial Studies*, 30(10):3337–3375.

Gilchrist, S. and Zakrajšek, E. (2012). Credit spreads and business cycle fluctuations. *American Economic Review*, 109(4):1692–1720.

Gomes, F. A. R. and Paz, L. S. (2011). Narrow replication of yogo (2004) estimating the elasticity of intertemporal substitution when instruments are weak. *Journal of Applied Econometrics*, 26(7):1215–1216.

Görtz, C. and Tsoukalas, J. D. (2013). Learning, capital-embodied technology and aggregate fluctuation. *Review of Economic Dynamics*, 16(4):708–723.

Görtz, C., Tsoukalas, J. D., and Zanetti, F. (2021). News shocks under financial frictions. *American Economic Journal: Macroeconomics*, forthcoming.

Görtz, C. and Yeromonahos, M. (2019). Asymmetries in risk premia, macroeconomic uncertainty and business cycles. *CESifo Working Paper Series*, (7959).

Gourio, F. (2012). Disaster risk and business cycles. *The American Economic Review*, 102(6):2734–2766.

Gourio, F. (2013). Credit risk and disaster risk. *American Economic Journal: Macroeconomics*, 5(3):1–34.

Goyal, A. and Welch, I. (2008). A comprehensive look at the empirical performance of equity premium prediction. *The Review of Financial Studies*, 21(4):1455–1508.

Graham, J. R. and Harvey, C. (2007). The equity risk premium in january 2007: Evidence from the Global CFO Outlook Survey. *Financial Risk Management*, 4:46–61.

Graham, J. R. and Harvey, C. (2018). The equity risk premium in 2018. *Duke University - Fuqa School of Business working paper.*

Guerrieri, V. and Lorenzoni, G. (2017). Credit crises, precautionary savings, and the liquidity trap. *Quarterly Journal of Economics*, 132(3):1427–1467.

Hall, R. E. (2011). The high sensitivity of economic activity to financial frictions. *Economic Journal*, 121(552):351–379.

Hansen, G. D. (1985). Indivisible labour and the business cycle. *Journal of Monetary Economics*, 16(3):309–328.

Harvey, C. R., Liu, Y., and Zhu, H. (2016). ...and the cross-section of expected returns. *Review of Financial Studies*, 29(1):5–69.

Hayashi, F. (1982). Tobin's marginal and and average q: a neoclassical interpretation. *Econometrica*, 50(1):213–224.

Heer, B. and Maußner, A. (2012). Asset returns, the business cycle and the labor market. *German Economic Review*, 14(3):372–397.

Hintermaier, T. and Koeniger, W. (2010). The method of endogenous grid points with occasionally binding constraints among endogenous variables. *Journal of Economic Dynamics & Control*, 34(10):2074–2088.

Iacoviello, M. (2005). House prices, borrowing constraints, and monetary policy in the business cycle. *American Economic Review*, 95(3):739–765.

Jaimovich, N. and Rebelo, S. (2009). Can news about the future drive the business cycle? *American Economic Review*, 99(4):1097–1118.

Jermann, U. J. (1998). Asset pricing in production economies. *Journal of Monetary Economics*, 41(2):257–275.

Justiniano, A., Primiceri, G. E., and Tambalotti, A. (2019). Credit supply and the housing boom. *Journal of Political Economy*, 127(3):1317–1351.

Kaltenbrunner, G. and Lochstoer, L. A. (2010). Long-run risk through consumption smoothing. *Review of Financial Studies*, 23(8):3190–3225.

Kaplan, G., Mitman, K., and Violante, G. L. (2020). The housing boom and bust: model meets evidence. *The Journal of political economy*, 128(9):3285–3345.

Kaplan, G., Moll, B., and Violante, G. (2018). Monetary policy according to hank. *American Economic Review*, 108(3):697–743.

Kaplan, G. and Violante, G. L. (2014). A model of the consumption response to fiscal stimulus payments. *Econometrica*, 82(4):1199–1239.

Kaplan, G. and Violante, G. L. (2018). Microeconomic heterogeneity and macroeconomic shocks. *Journal of Economic Perspective*, 32(3):167–194.

Kaplan, G., Violante, G. L., and Weidner, J. (2014). The wealthy hand-to-mouth. *Brookings Papers on Economic Activity*, pages 77–154.

Kim, Y. and Lim, H. (2020). Transmission of monetary policy in times of high household debt. *Journal of Macroeconomics*, 63(C).

Krueger, D., Mitman, K., and Perri, F. (2016). *Handbook of Macroeconomics*. Elsevier.

Krusell, P. and Smith, A. A. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106(5):867–896.

Kuroda, H. (2017). Opening remarks. *2017 BOJ–IMES Conference Hosted by the Institute for Monetary and Economic Studies, Bank of Japan. https://www.boj.or.jp/en/announcements/press/koen_2017/ko170524a.htm/.*

Kydland, F. E. and Prescott, E. C. (1982). Time to build and aggregate fluctuations. *Econometrica*, 50(6):1345–1370.

Leahy, J. V. and Zeira, J. (2005). The timing of purchases and aggregate fluctuations. *Review of Economic Studies*, 72(4):1127–1152.

Lettau, M. and Ludvigson, S. (2001a). Consumption, aggregate wealth, and expected stock returns. *Journal of Finance*, 56(3):815–850.

Lettau, M. and Ludvigson, S. (2001b). Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying. *Journal of Political Economy*, 109(6):1238–1288.

Lettau, M. and Uhlig, H. (2000). Can habit formation be reconciled with business cycle facts? *Review of Economic Dynamics*, 3(1):79–99.

Luetticke, R. (2018). Transmission of monetary policy with heterogeneity in household portfolios. *Discussion Papers 1819, Centre for Macroeconomics (CFM).*

Mayer, C., Pence, K., and Sherlund, S. M. (2009). The rise in mortgage defaults. *Journal of Economic Perspective*, 23(1):27–50.

McKay, A., Nakamura, E., and Steinsson, J. (2016). The power of forward guidance revisited. *American Economic Review*, 106(10):3133–3158.

McKay, A. and Reis, R. (2016). The role of automatic stabilizers in the U.S. business cycle. *Econometrica*, 84(1):141–195.

Mian, A., Rao, K., and Sufi, A. (2013). Household balance sheets, consumption, and the economic slump. *Quarterly Journal of Economics*, 128(4):1687–1727.

Milani, F. (2011). Expectation shocks and learning as drivers of the business cycle. *Economic Journal*, 121(552):379–399.

Moffitt, R. and Zhang, S. (2018). The PSID and income volatility: its record of seminal research and some new findings. *The annals of the American Academy of Political and Social Science*, 680(1):48–81.

Nekarda, C. J. and Ramey, V. A. (2013). The cyclical behavior of the price-cost markup. *NBER Working Paper 19099*.

Oh, H. and Reis, R. (2012). Targeted transfers and the fiscal response to the Great Recession. *Journal of Monetary Economics*, 59(S):50–64.

Ordoñez, G. (2013). The asymmetric effects of financial frictions. *Journal of Political Economy*, 121(5):844–896.

Rouwenhorst, K. G. (1995). *Frontiers of business cycle research*. Princeton University Press.

Royston, P. (1991). Estimating departure from normality. *Statistics in Medecine*, 10(8):1283–1293.

Rozeff, M. S. (1984). Dividend yields are equity risk premiums. *Journal of Portfolio Management*, 11(1):68–75.

Saijo, H. (2017). The uncertainty multiplier and business cycles. *Journal of Economic Dynamics and Control*, 18:1–26.

Serdar, O., Kurt, M., Fatih, K., and Aaron, H. (2017). Monetary policy, heterogeneity, and the housing channel. *Society for Economic Dynamics, 2017 Meeting Papers 1610.*

Shiller, R. J. (2007). Understanding recent trends in house prices and home ownership. *NBER Working Paper No. 13553.*

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review*, 95(1):25–49.

Sichel, D. E. (1993). Business cycle asymmetry: A deeper look. *Economic Inquiry*, 31(2):224–237.

Smeeding, T. M. (2018). The PSID in research and policy. *The annals of the American Academy of Political and Social Science*, 680(1):29–47.

Van Nieuwerburgh, S. and Veldkamp, L. (2006). Learning asymetries in real business cycles. *Journal of Monetary Economics*, 53(4):753–772.

Veldkamp, L. L. (2005). Slow boom, sudden crash. *Journal of Economic Theory*, 124(2):230–258.

Vissing-Jørgensen, A. (2002). Limited asset market participation and the elasticity of intertemporal substitution. *Journal of Political Economy*, 110(4):825–854.

Werning, I. (2015). Incomplete markets and aggregate demand. *NBER Working Papers 21448.*

Yellen, J. (2016). Macroeconomic research after the crisis. *Speech at the Federal Reserve Bank of Boston, Boston, Massachusetts, October 14, https://www.federalreserve.gov/newsevents/speech/yellen20161014a.htm.*

Yogo, M. (2004). Estimating the elasticity of intertemporal substitution when instruments are weak. *The Review of Economics and Statistics*, 86(3):797–810.

Appendices

A Chapter 2 appendices

A.1 Additional evidence on the estimation of risk premia

Tables 1 and 2 show that ADF tests overwhelmingly reject the null hypothesis of a unit root in realised ex-post risk premia at the yearly or quarterly horizon. These are computed as the ex-post difference between the stock market return and the risk-free rate. This test statistic validates the use of the historical mean method to compute the risk premium.

Table 1: ADF test results, yearly risk premia

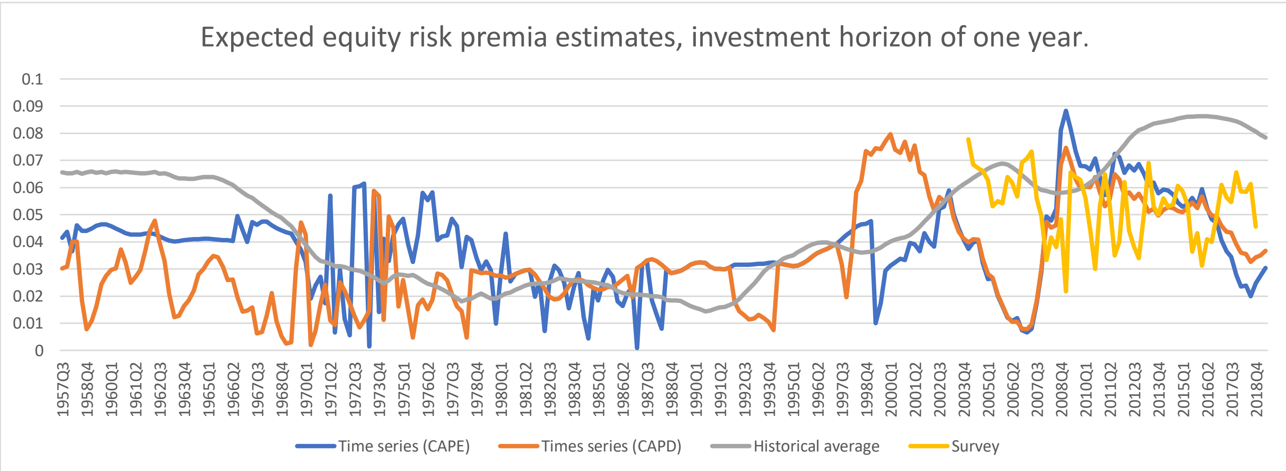
Test statistic	1% critical value	5% critical value	10% critical value
-5.247	-3.461	-2.880	-2.570
MacKinnon approximate p-value = 0.000			

Table 2: ADF test results, quarterly risk premia

Test statistic	1% critical value	5% critical value	10% critical value
-13.401	-3.461	-2.880	-2.570
MacKinnon approximate p-value = 0.000			

A.2 Time series plots

Figure 1 is a graphical representation of the values of all the risk premia estimates, at the yearly and quarterly investment horizon, calculated with the different methods.



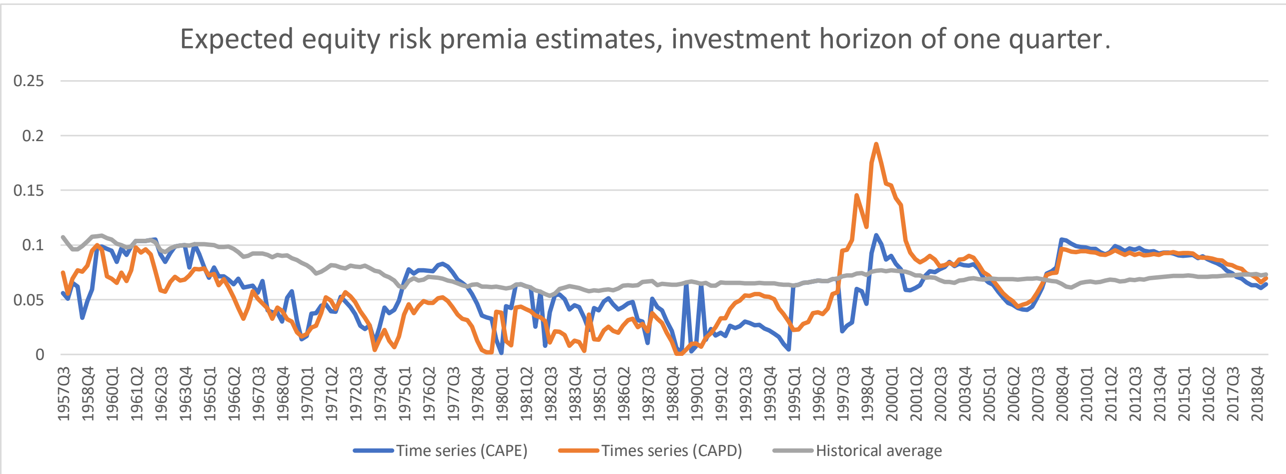


Figure 1: Equity risk premia estimates, yearly and quarterly investment horizons, all methods.

A.3 Household's optimality conditions

The recursive structure of the utility function immediately implies the Bellman equation

$$F(s_t^d, \tilde{A}_t) = \max_{s_{t+1}^d, l_t^s, \tilde{c}_t} W\left(u(\tilde{c}_t, l_t^s), \mu_t\right),$$

where

$$W\left(u(\tilde{c}_t, l_t^s), \mu_t\right) = \left[(1-\beta)u(\tilde{c}_t, l_t^s)^{\frac{1-\gamma}{\theta}} + \beta\mu_t^{\frac{1-\gamma}{\theta}}\right]^{\frac{\theta}{1-\gamma}},$$

with

$$\mu_t = \left(\mathbb{E}_t\left[F_{t+1}^{1-\gamma} \,\middle|\, \mathcal{I}_t\right]\right)^{\frac{1}{1-\gamma}}$$

and $\theta := \frac{1-\gamma}{1-\frac{1}{\psi}}$, subject to

$$\tilde{c}_t = w_t l_t^s + \tilde{d}_t s_t^d - p_t(s_{t+1}^d - s_t^d).$$

To ease notation in the following algebraic derivations, we use the simplified notations u_t to denote the period utility function $u(\tilde{c}_t, l_t)$, W_t to denote the CES aggregation $W\left(u(\tilde{c}_t, l_t^s), \mu_t\right)$, and F_t to denote the value function $F(s_t^d, \tilde{A}_t)$.

A.4 Derivation of the Lucas equation and the SDF

The first order condition with respect to s_{t+1} yields

$$\frac{\partial W_t}{\partial u_t}\frac{\partial u_t}{\partial \tilde{c}_t}\frac{\partial \tilde{c}_t}{\partial s_{t+1}^d} + \frac{\partial W_t}{\partial \mu_t}\frac{\partial \mu_t}{\partial \mathbb{E}_t[F_{t+1}^{1-\gamma} \mid \mathcal{I}_t]}\mathbb{E}_t\left[\frac{\partial F_{t+1}^{1-\gamma}}{\partial F_{t+1}}\frac{\partial F_{t+1}}{\partial s_{t+1}^d} \,\middle|\, \mathcal{I}_t\right] = 0.$$

The envelope theorem for s_t yields

$$\frac{\partial F_t}{\partial s_t^d} = \frac{\partial W_t}{\partial u_t}\frac{\partial u_t}{\partial \tilde{c}_t}\frac{\partial \tilde{c}_t}{\partial s_t^d}.$$

Combining both conditions, we obtain

$$\frac{\partial W_t}{\partial u_t}\frac{\partial u_t}{\partial \tilde{c}_t}\frac{\partial \tilde{c}_t}{\partial s_{t+1}^d} + \frac{\partial W_t}{\partial \mu_t}\frac{\partial \mu_t}{\partial \mathbb{E}_t[F_{t+1}^{1-\gamma} \mid \mathcal{I}_t]}\mathbb{E}_t\left[\frac{\partial F_{t+1}^{1-\gamma}}{\partial F_{t+1}}\frac{\partial W_{t+1}}{\partial u_{t+1}}\frac{\partial u_{t+1}}{\partial c_{t+1}}\frac{\partial c_{t+1}}{\partial s_{t+1}^d} \,\middle|\, \mathcal{I}_t\right] = 0. \quad \text{(A.1)}$$

We can then recover

$$\frac{\partial W_t}{\partial u_t} = F_t^{1-\frac{1-\gamma}{\theta}}(1-\beta)u_t^{\frac{1-\gamma}{\theta}-1},$$

$$\frac{\partial W_t}{\partial \mu_t} = \beta F_t^{1-\frac{1-\gamma}{\theta}} \left(\mathbb{E}_t[F_{t+1}^{1-\gamma} \,\middle|\, \mathcal{I}_t]\right)^{\frac{1}{\theta}-\frac{1}{1-\gamma}},$$

$$\frac{\partial \mu_t}{\partial \mathbb{E}_t[F_{t+1}^{1-\gamma} \mid \mathcal{I}_t]} = \frac{1}{1-\gamma}\mathbb{E}_t[F_{t+1}^{1-\gamma} \mid \mathcal{I}_t]^{\frac{1}{1-\gamma}-1},$$

$$\frac{\partial \tilde{c}_{t+1}}{\partial s_{t+1}^d} = \mathbb{E}_t[d_{t+1} + p_{t+1} \mid \mathcal{I}_t],$$

and

$$\frac{\partial \tilde{c}_t}{\partial s_{t+1}^d} = -p_t.$$

Plugging the last 5 equations into equation (A.1) yields after re-arranging

$$0 = \mathbb{E}_t \left[m_{t+1,t} \frac{d_{t+1} + p_{t+1}}{p_t} - 1 \,\middle|\, \mathcal{I}_t \right],$$

where

$$m_{t+1,t} = \beta \left(\frac{F_{t+1}^{1-\gamma}}{\mathbb{E}_t \left[F_{t+1}^{1-\gamma} \,\middle|\, \mathcal{I}_t \right]} \right)^{1-\frac{1}{\theta}} \left(\frac{u_{t+1}}{u_t} \right)^{\frac{1-\gamma}{\theta}-1} \frac{\frac{\partial u_{t+1}}{\partial c_{t+1}}}{\frac{\partial u_t}{\partial \tilde{c}_t}}.$$

These are the Lucas equation (2.16) and the stochastic discount factor (2.17).

A.5 Optimal labour supply

The first order condition with respect to l_t yields

$$\frac{\partial W_t}{\partial u_t} \left[\frac{\partial u_t}{\partial \tilde{c}_t} w_t + \frac{\partial u_t}{\partial l_t^s} \right] = 0. \tag{A.2}$$

We can then recover

$$\frac{\partial u_t}{\partial \tilde{c}_t} = \kappa \tilde{c}_t^{\kappa-1} (1 - l_t^s)^{1-\kappa}$$

and

$$\frac{\partial u_t}{\partial l_t^s} = -\tilde{c}_t^{\kappa} (1-\kappa)(1 - l_t^s)^{-\kappa}.$$

Plugging the last 2 equations into equation (A.2) yields after re-arranging

$$\tilde{c}_t = \frac{\kappa}{(1-\kappa)}(1 - l_t^s) w_t,$$

which is the labour supply function (2.15).

A.6 Derivation of the return on equity

Firm's expected cash flow at the beginning of the period is defined as

$$\tilde{f}_t = \tilde{d}_t s_t - p_t(s_{t+1} - s_t) = \tilde{y}_t - w_t l_t - i_t.$$

From equation (2.11), it holds that $w_t l_t = (1-\alpha)\tilde{y}_t$. Hence we can simplify the above equation for expected cash flow to become

$$\tilde{f}_t = \alpha\tilde{y}_t - i_t.$$

Using equations (2.10) and (2.7), and due to the specific capital adjustment costs we apply, we can write

$$\begin{aligned} q_t k_{t+1} =& \mathbb{E}_t \left\{ m_{t,t+1} \left[A_{t+1} l_{t+1}^{1-\alpha} \alpha k_{t+1}^{\alpha} - i_{t+1} + q_{t+1} \left(1 - \delta + \Phi\left(\frac{i_{t+1}}{k_{t+1}} \right) \right) k_{t+1} \right] \middle| \mathcal{I}_t \right\} \\ \Leftrightarrow q_t k_{t+1} =& \mathbb{E}_t \left[m_{t,t+1}(\alpha y_{t+1} - i_{t+1} + q_{t+1} k_{t+2}) \mid \mathcal{I}_t \right] \\ \Leftrightarrow q_t k_{t+1} =& \mathbb{E}_t \left[m_{t,t+1}(f_{t+1} + q_{t+1} k_{t+2}) \mid \mathcal{I}_t \right]. \end{aligned}$$

Iterating forward, we obtain

$$q_t k_{t+1} = \mathbb{E}_t \left[\sum_{i=1}^{+\infty} m_{t,t+i} f_{t+i} \middle| \mathcal{I}_t \right], \tag{A.3}$$

assuming that $\lim_{i\to+\infty} \mathbb{E}_t[m_{t,t+i} q_{t+i} k_{t+i+1} \mid \mathcal{I}_t] = 0$. Following Altug and Labadie (2008), the value of a firm on the stock market is equal its present value of future discounted cash flow. This allows us to rewrite equation (A.3) as

$$q_t k_{t+1} = p_t s_{t+1}.$$

Finally, using the above expressions, we can derive a formulation for the return on equity which depends on variables that have been pinned down uniquely in firm's and household's maximization problems

$$\begin{aligned} \mathbb{E}_t \left[\frac{d_{t+1} + p_{t+1}}{p_t} \middle| \mathcal{I}_t \right] =& \mathbb{E}_t \left[\frac{s_{t+1} d_{t+1} - p_{t+1}(s_{t+2} - s_{t+1}) + s_{t+2} p_{t+1}}{s_{t+1} p_t} \middle| \mathcal{I}_t \right] \\ =& \mathbb{E}_t \left[\frac{f_{t+1} + k_{t+2} q_{t+1}}{k_{t+1} q_t} \middle| \mathcal{I}_t \right] \\ =& \mathbb{E}_t \left[\frac{q_{t+1} k_{t+2} + y_{t+1} - w_{t+1} l_{t+1}^d - i_{t+1}}{q_t k_{t+1}} \middle| \mathcal{I}_t \right], \end{aligned}$$

which is equation (2.20) in the main body.

A.7 Derivation of functional forms for parameters a_1 and a_2

The parameters a_1 and a_2 are calibrated to ensure that adjustment costs are zero in steady state, so that steady state investment and Tobin's q are $i = \delta k$ and $q = 1$. From equations (2.7) and (2.9), we can see that the latter is satisfied if

$$\Phi(\delta) = \delta \text{ and } \Phi^{'}(\delta) = 1.$$

Given the functional form of Φ, this implies

$$\frac{a_1}{1-\chi}\delta^{1-\chi} + a_2 = \delta \text{ and } a_1\delta^{-\chi} = 1,$$

from where we deduce

$$a_1 = \delta^{\chi} \text{ and } a_2 = -\frac{\delta\chi}{1-\chi}.$$

A.8 Alternative risk aversion calibration

We replicate the main results summarised in Table 2.6 using two alternative calibrations for the coefficient of relative aversion. Instead of the baseline calibration $\gamma = 5$, we use $\gamma = 1$ in Table 3 and we use $\gamma = 10$ in Table 4. We can verify that the main quantitative predictions of the model are only marginally affected by these changes in calibration, while the qualitative interpretations remain identical.

Table 3: Key moments of the risk premium and macroeconomic aggregates, low relative risk aversion ($\gamma = 1$)

	Relative std deviation	Correlation with output	1st order auto-cor.	Skewness
Panel A: US data				
Risk premium	2.177	-0.486	0.772	0.122
Output	1.000	1.000	0.836	-0.523
Investment	4.373	0.898	0.817	-0.697
Hours	1.226	0.854	0.909	-0.965
Consumption	0.795	0.872	0.862	-0.672
Panel B: Baseline model (with learning)				
Risk premium	2.969 (0.026)	-0.509 (0.005)	0.645 (0.008)	0.155 (0.027)
Output	1.000 (0.000)	1.000 (0.000)	0.928 (0.003)	-0.602 (0.061)
Investment	2.033 (0.005)	0.737 (0.005)	0.892 (0.004)	-0.458 (0.062)
Hours	0.211 (0.001)	0.700 (0.007)	0.855 (0.004)	-0.453 (0.066)
Consumption	0.934 (0.004)	0.917 (0.001)	0.819 (0.005)	-0.138 (0.041)
Panel C: Model without learning				
Risk premium	3.678 (0.058)	-0.449 (0.004)	0.188 (0.008)	-0.053 (0.018)
Output	1.000 (0.000)	1.000 (0.000)	0.924 (0.004)	-0.028 (0.035)
Investment	1.995 (0.005)	0.983 (0.002)	0.911 (0.005)	-0.058 (0.049)
Hours	0.204 (0.001)	0.935 (0.003)	0.889 (0.001)	-0.069 (0.050)
Consumption	0.721 (0.002)	0.991 (0.001)	0.923 (0.003)	0.043 (0.045)

Values reported in parentheses are standard errors. The sample in panel A is 1957Q3 - 2019Q2. Statistics shown for the risk premium in Panel A are based on the historical average measure with one quarter investment horizon. The models in panels B and C are simulated 500 times over 298 periods after which the first 50 periods are discarded. Second moments are calculated based on percentage deviations from HP(1600) filter trend. Skewness is calculated from log first-differenced series.

Table 4: Key moments of the risk premium and macroeconomic aggregates, high relative risk aversion ($\gamma = 10$)

	Relative std deviation	Correlation with output	1st order auto-cor.	Skewness
Panel A: US data				
Risk premium	2.177	-0.486	0.772	0.122
Output	1.000	1.000	0.836	-0.523
Investment	4.373	0.898	0.817	-0.697
Hours	1.226	0.854	0.909	-0.965
Consumption	0.795	0.872	0.862	-0.672
Panel B: Baseline model (with learning)				
Risk premium	2.981 (0.030)	-0.510 (0.006)	0.639 (0.007)	0.159 (0.029)
Output	1.000 (0.000)	1.000 (0.000)	0.930 (0.003)	-0.594 (0.059)
Investment	2.042 (0.009)	0.733 (0.005)	0.877 (0.003)	-0.466 (0.068)
Hours	0.215 (0.003)	0.694 (0.006)	0.854 (0.003)	-0.459 (0.069)
Consumption	0.948 (0.005)	0.901 (0.001)	0.822 (0.006)	-0.141 (0.040)
Panel C: Model without learning				
Risk premium	3.679 (0.058)	-0.453 (0.005)	0.181 (0.007)	-0.052 (0.016)
Output	1.000 (0.000)	1.000 (0.000)	0.921 (0.004)	-0.031 (0.039)
Investment	1.992 (0.005)	0.983 (0.001)	0.908 (0.004)	-0.055 (0.047)
Hours	0.205 (0.001)	0.933 (0.003)	0.883 (0.000)	-0.070 (0.054)
Consumption	0.725 (0.002)	0.989 (0.000)	0.923 (0.003)	0.040 (0.044)

Values reported in parentheses are standard errors. The sample in panel A is 1957Q3 - 2019Q2. Statistics shown for the risk premium in Panel A are based on the historical average measure with one quarter investment horizon. The models in panels B and C are simulated 500 times over 298 periods after which the first 50 periods are discarded. Second moments are calculated based on percentage deviations from HP(1600) filter trend. Skewness is calculated from log first-differenced series.

B Chapter 3 appendices

B.1 Construction of the variables – survey questions

This appendix contains the list of the survey questions used to build the three variables of interest (income, savings and debt) and to identify the cases of foreclosure. Year 2009 is used as an example for clarity, but the same questions were asked during every wave of the survey.

Income The income variable in our empirical calculations is the sum of 21 items, when applicable. For instance, households without farm income would not be asked to report their net income from farming.

1. G1A Whether head or partner is farmer or rancher on current main job. G4. What was the net income from farming?
2. G5. Did you [or any one else in the family there] own a business at any time in 2009 or have a financial interest in any business enterprise? G11D/G11E. How much did you [or any one else in the family there] make at this business in 2009?
3. G12. Did you [or any one else in the family there] earn wages or salaries in 2009 from working on any jobs besides the unincorporated business we have just talked about]? G13/G52. How much did you [or any one else in the family there] earn altogether from wages or salaries in 2009, that is, before anything was deducted for taxes or other things?
4. G18A. I'm going to read you a list of other sources of income you [or any one else in the family there] might have had. Did you [or any one else in the family there] receive any other income in 2009 from professional practice or trade? G19A/G52Q. How much was it?
5. G25A. Did you [or any one else in the family there] receive any (other) income in 2009 from rent? G26A/G59A. How much was it?
6. G25B. Did you [or any one else in the family there] receive any other income in 2009 from dividends? G26B/G59B. How much was it?
7. G25C. Did you [or any one else in the family there] receive any other income in 2009 from interest? G26C/G59C. How much was it?
8. G25D. Did you [or any one else in the family there] receive any other income in 2009 from trust funds and royalties? G26D/G59D. How much was it?
9. G25E. Did you [or any one else in the family there] receive any income in 2009 from TANF (Tem-

porary Assistance for Needy Families) formerly called ADC or AFDC? G26E/G60B. How much was it?

10. G25F. Did you [or any one else in the family there] receive any income in 2009 from Supplemental Security Income? G26F/G60A4. How much was it?

11. G25G. Did you [or any one else in the family there] receive any income in 2009 from other welfare? G26D. How much was it?

12. G31. Did you [or any one else in the family there] or anyone else in the family there receive any income in 2009 from Social Security, such as disability, retirement or survivor's benefits? G34. How much was the total amount from Social Security?

13. G37A. Did you [or any one else in the family there] receive any income in 2009 from the Veteran's Administration, widow's or survivor's pension, service disability, or the GI bill? G38/G60G. How much was the total amount? Please include all amounts from all types of VA (Veteran's Administration) income you [or any one else in the family there] received in 2009.

14. G40. Did you [or any one else in the family there] receive any income in 2009 from other retirement pay, pensions, IRAs or annuities? G41A/G61A. How much was from retirement pay or pensions? G41B/G61C. How much was from annuities? G41C/G61G. How much was the other (retirement) income? G41D/G61E. How much was from IRAs?

15. G44A. Did you [or any one else in the family there] receive any income in 2009 from unemployment compensation? G45A/G54. How much was it?

16. G44A. Did you [or any one else in the family there] receive any income in 2009 from unemployment compensation? G45A/G57. How much was it?

17. G44C. Did you [or any one else in the family there] receive any income in 2009 from child support? G45C/G60C. How much was it?

18. G44D. Did you [or any one else in the family there] receive any income in 2009 from alimony or separate maintenance? G45D/G60E. How much was it?

19. G44E. Did you [or any one else in the family there] receive any help in 2009 from relatives? This must be from non-FU members. G45E/G62A. How much was it?

20. G44F. Did you [or any one else in the family there] receive any help in 2009 from non-relatives or friends? This must be from non-FU members. G45F/G62B. How much was it?

21. G44G. Did you [or any one else in the family there] receive any other income in 2009 from anything else? G45G1/G63A. What was that from? G45G/G63B. How much was the income from

that?

Savings The savings variable is the sum of the 4 items, when applicable.

1. W15. Do you [or any one else in the family there] have any shares of stock in publicly held corporations, stock mutual funds, or investment trusts–not including stocks in employer-based pensions or IRAs? W16. If you sold all that and paid off anything you owed on it, how much would you have?

2. W21. Do you [or any one else in the family there] have any money in private annuities or Individual Retirement Accounts (IRAs)? W22. How much would they be worth?

3. W27. Not including employer-based pensions or IRAs, do you [or any one else in the family there] have any money in any of the following: Checking or savings accounts, Money market funds, Certificates of deposit, Government bonds, or Treasury bills? W28. If you added up all such accounts for all of your family living there about how much would they amount to right now?

4. W33. Do you [or any one else in the family there] have any other savings or assets, such as cash value in a life insurance policy, a valuable collection for investment purposes, or rights in a trust or estate that you haven't already told us about? W34. If you sold that and paid off any debts on it, how much would you have?

Mortgages Our measure of mortgage debt corresponds to the sum of the remaining principals on all of the mortgages a household has, on the same property or on different properties.

1. A23. Do you [or any one else in the family there] have a mortgage or loan on this property?–first mortgage A24. About how much is the remaining principal on this loan?–first mortgage

2. A28. Do you [or any one else in the family there] also have a second mortgage?–second mortgage A24. About how much is the remaining principal on this loan?–second mortgage

Other debt Other debt are the sum of 7 items, when applicable, only available since 2011. Before 2011, these 8 different types of debt were pooled together in a single category.

1. F47. These next questions are about personal vehicles and transportation. Do you [or any one else in the family there] own or lease a car or other vehicle for personal use?–first vehicle F65. Did you borrow or finance part of the total price?–first vehicle F66. How much did you borrow, not including financing charges?–first vehicle

F65. Did you borrow or finance part of the total price?–second vehicle F66. How much did you borrow, not including financing charges?–second vehicle

2. W38A. Aside from the debts that we have already talked about, (like any mortgage on your main home (or/like) vehicle loans,) do you [or any one else in the family there] currently have any credit card or store card debt? Do not count new debt that will be paid off this month. W39A. If you added up all credit card and store card debts for all of your family living there, about how much would they amount to right now? Please do not count any new debt that will be paid off this month.

3. W38B. Do you [or any one else in the family there] currently have student loans? W39B1. If you added up all student loans for all of your family living there, about how much would they amount to right now?

4. W38B. Do you [or any one else in the family there] currently have medical bills? W39B2. If you added up all medical bills for all of your family living there, about how much would they amount to right now?

5. W38B. Do you [or any one else in the family there] currently have legal bills? W39B3. If you added up all legal bills for all of your family living there, about how much would they amount to right now?

6. W38B. Do you [or any one else in the family there] currently have loans from relatives? W39B4. If you added up all loans form relatives for all of your family living there, about how much would they amount to right now?

7. W38B. Do you [or any one else in the family there] currently have any other debts? W39B7. If you added up all other debts for all of your family living there, about how much would they amount to right now?

Foreclosure Two questions help identify bankrupt households.

1. A27c. Has your bank or lender started the process of foreclosing on your home?–first mortgage

2. A27c. Has your bank or lender started the process of foreclosing on your home?–second mortgage

C Chapter 4 appendices

C.1 Solution method

In this appendix, we present the procedure employed to solve the problem of households. The solution algorithm is similar to Hintermaier and Koeniger (2010). It is partially based on the endogenous grid point method of Carroll (2006), with additional adjustments to deal with an occasionally binding constraint on an endogenous state variable. In addition, we follow Guerrieri and Lorenzoni (2017) by computing the partial derivatives of the value function instead that of the policy functions as in Hintermaier and Koeniger (2010). We first derive all relevant optimality conditions and then present the algorithm.

Optimality conditions The Bellman operator for households takes the form

$$
\begin{aligned}
V(s_{it}, d_{it}, k_{it}, z_{it}) = & \max_{c_{it}, h_{it}, s_{it+1}, d_{it+1}, k_{it+1}} U(c_{it}, k_{it}, h_{it}) + \beta \left\{ \mathbb{E}_t \left[V(s_{it+1}, d_{it+1}, k_{it+1}, z_{it+1}) | z_{it} \right] \right\} \\
s.t. \quad & d_{it+1} \leq \phi_{t+1} k_{it+1} \\
& \frac{s_{it+1}}{1+r_t^s} - \frac{d_{it+1}}{1+r_t^d} + f(k_{it+1}, k_{it}) + c_{it} + \tilde{\tau}_{it} \leq s_{it} - d_{it} + \mathcal{I}_{it}\nu_t + (1-\mathcal{I}_{it}) h_{it} z_{it} \\
with \quad & \tilde{\tau}_{it} = \tau \left(\mathcal{I}_{it}\nu_t + (1-\mathcal{I}_{it}) h_{it} z_{it} + r_{t-1}^s s_{it} - r_{t-1}^d d_{it} \right)
\end{aligned}
$$

The first order conditions for c_{it} and h_{it} are respectively

$$U_c(c_{it}, k_{it}, h_{it}) = \lambda_{it}, \tag{C.1}$$

$$U_h(c_{it}, k_{it}, h_{it}) = -\lambda_{it} z_{it} + \lambda_{it} \tau z_{it} \quad if \quad \mathcal{I}_{it} = 0, \quad otherwise \;\; h_{it} = 0, \tag{C.2}$$

where U_c denote the partial derivative of the period utility function with respect to consumption and λ is the multiplier on the budget constraint. The first order conditions for s_{it+1}, d_{it+1} and k_{it+1} are respectively

$$-\frac{\lambda_{it}}{1+r_t^s} + \beta \mathbb{E}_t \left[V_s(s_{it+1}, d_{it+1}, k_{it+1}, z_{it+1}) | z_{it} \right] = 0, \tag{C.3}$$

$$\frac{\lambda_{it}}{1+r_t^d} - \mu_{it} + \beta \mathbb{E}_t \left[V_d(s_{it+1}, d_{it+1}, k_{it+1}, z_{it+1}) | z_{it} \right] = 0, \tag{C.4}$$

$$-\lambda_{it} f_{k_{t+1}}(k_{it+1}, k_{it}) + \mu_{it}\phi_{t+1} + \beta \mathbb{E}_t \left[V_k(s_{it+1}, d_{it+1}, k_{it+1}, z_{it+1}) | z_{it} \right] = 0, \tag{C.5}$$

where V_k denote the partial derivative of the value function with respect to durables, $f_{k_{t+1}}$ denotes the partial derivative of the durables investment cost function with respect to k_{it+1}, and μ is the multiplier on the borrowing constraint.

The envelope conditions for s_{it}, d_{it} and k_{it} are respectively

$$V_s(s_{it}, d_{it}, k_{it}, z_{it}) = \lambda_{it} - \lambda_{it}\tau_t r^s_{t-1}, \tag{C.6}$$

$$V_d(s_{it}, d_{it}, k_{it}, z_{it}) = -\lambda_{it} + \lambda_{it}\tau_t r^d_{t-1}, \tag{C.7}$$

$$V_k(s_{it}, d_{it}, k_{it}, z_{it}) = -\lambda_{it} f_{k_t}(k_{it+1}, k_{it}) + U_k(c_{it}, k_{it}, h_{it}). \tag{C.8}$$

The complementary slackness conditions for the borrowing constraint require

$$\mu_{it} = 0 \text{ if } d_{it+1} < \phi_{t+1}k_{it+1} \quad \text{and} \quad \mu_{it} > 0 \text{ if } d_{it+1} = \phi_{t+1}k_{it+1}. \tag{C.9}$$

Algorithm We start by making an initial guess for $V_s(s_{it}, d_{it}, k_{it}, z_{it})$, $V_d(s_{it}, d_{it}, k_{it}, z_{it})$ and for $V_k(s_{it}, d_{it}, k_{it}, z_{it})$. Our objective is to first find potentially optimal portfolios in the space $(s_{it+1}, d_{it+1}, k_{it+1})$ and the associated control variables (c_{it}, h_{it}) as a function of k_{it} and z_{it} only. Second, we make a backward step typical of the endogenous grid point method to find the corresponding state (s_{it}, d_{it}). Then we can update our guess for $V_s(s_{it}, d_{it}, k_{it}, z_{it})$, $V_d(s_{it}, d_{it}, k_{it}, z_{it})$ and $V_k(s_{it}, d_{it}, k_{it}, z_{it})$. We repeat the procedure until convergence of these three functions.

1. *Find a potential optimal portfolio* $(s_{it+1}, d_{it+1}, k_{it+1})$.

   Let k_{it} and k_{it+1} vary independently on the grid $\{k^0, ..., k^K\}$ and let d_{it+1} vary on the grid $\{d^0, ..., d^D\}$ subject to $d_{it+1} > 0$ iif $k_{it+1} > k_{it}(1-\delta)$ and $d_{it+1} \leq \phi_{t+1}k_{it+1}$ always.

   (a) If $d_{it+1} < \phi_{t+1}k_{it+1}$, $\mu_{it} = 0$.

   Find s_{it+1} that simultaneously satisfy C.3 and C.4 such that

$$- (1 + r^d_t)\mathbb{E}_t \left[V_d(s_{it+1}, d_{it+1}, k_{it+1}, z_{it+1}) | z_{it})\right] = \\ (1 + r^s_t)\mathbb{E}_t \left[V_s(s_{it+1}, d_{it+1}, k_{it+1}, z_{it+1}) | z_{it})\right].$$

Recover the value of the multiplier λ_{it} using C.3:

$$\lambda_{it} = \beta(1+r_t^s)\mathbb{E}_t\left[V_s(s_{it+1}, d_{it+1}, k_{it+1}, z_{it+1})|z_{it})\right].$$

Recover $\mathbb{E}_t\left[V_k(s_{it+1}, d_{it+1}, k_{it+1}, z_{it+1})|z_{it})\right]$ using C.5:

$$\mathbb{E}_t\left[V_k(s_{it+1}, d_{it+1}, k_{it+1}, z_{it+1})|z_{it})\right] = \lambda_{it} f_{k_{t+1}}(k_{it+1}, k_{it})\frac{1}{\beta}.$$

(b) If $d_{it+1} = \phi_{t+1}k_{it+1}$, $\mu_{it} > 0$.

Find s_{it+1} that simultaneously satisfies C.3 and C.4 such that

$$(1+r_t^d)\mathbb{E}_t\left[V_d(s_{it+1}, d_{it+1}, k_{it+1}, z_{it+1})|z_{it})\right] + \\ (1+r_t^s)\mathbb{E}_t\left[V_s(s_{it+1}, d_{it+1}, k_{it+1}, z_{it+1})|z_{it})\right] > 0.$$

Recover the value of the multiplier λ_{it} using C.3:

$$\lambda_{it} = \beta(1+r_t^s)\mathbb{E}_t\left[V_s(s_{it+1}, d_{it+1}, k_{it+1}, z_{it+1})|z_{it})\right].$$

Recover the value of the multiplier μ_{it} using C.4:

$$\mu_{it} = \frac{\lambda_{it}}{1+r_t^d} + \beta\mathbb{E}_t\left[V_d(s_{it+1}, d_{it+1}, k_{it+1}, z_{it+1})|z_{it})\right].$$

Recover $\mathbb{E}_t\left[V_k(s_{it+1}, d_{it+1}, k_{it+1}, z_{it+1})|z_{it})\right]$ using C.5:

$$\mathbb{E}_t\left[V_k(s_{it+1}, d_{it+1}, k_{it+1}, z_{it+1})|z_{it})\right] = \left(\lambda_{it+1} f_{k_{t+1}}(k_{it+1}, k_{it}) - \mu_{it}\phi_{t+1}\right)\frac{1}{\beta}.$$

2. *Find the corresponding control variables* (c_{it}, h_{it}).

Given the value of λ_{it} found in step 1, recover the choices c_{it} and h_{it} that solve C.1 and C.2 respectively

$$U_c(c_{it}, k_{it}, h_{it}) = \lambda_{it},$$

$$U_h(c_{it}, k_{it}, h_{it}) = -\lambda_{it}z_{it} + \lambda_{it}\tau z_{it} \quad if \quad \mathcal{I}_{it} = 0, \quad otherwise \quad h_{it} = 0.$$

3. *Find the corresponding state* (s_{it}, d_{it}).

   Find $s_{it}(1-\tau r^s_{t-1}) - d_{it}(1-\tau r^d_{t-1})$ – i.e. the level of net savings after taxes available at the beginning of period t – that solves the budget constraint given the state (k_{it}, z_{it}) and choices $(c_{it}, h_{it}, s_{it+1}, d_{it+1}, k_{it+1})$ identified in the previous steps:

$$s_{it}(1-\tau r^s_{t-1}) - d_{it}(1-\tau r^d_{t-1}) = \\ \frac{s_{it+1}}{1+r^s_t} - \frac{d_{it+1}}{1+r^d_t} + f(k_{it+1}, k_{it}) + c_{it} - (1-\tau)(1-\mathcal{I}_{it})h_{it}z_{it} - (1-\tau)\mathcal{I}_{it}\nu.$$

   Note that there is no constraint on how households can use their net savings available at the beginning of period t. Therefore, for a given k_{it} and z_{it}, any combination of s_{it} and d_{it} leading to the same amount of net savings after taxes such that $d_{it} \leq \phi_t k_{it}$ is compatible with the choices (c_{it}, h_{it}, s_{it+1}, d_{it+1}, k_{it+1}) recovered in steps 1 and 2.

   Note further that the quantity of net savings after taxes recovered in this step is unlikely to be exactly achievable using the points of the grids $\{s^0, ..., s^S\}$ and $\{d^0, ..., d^D\}$. Therefore, we rely on piecewise linear interpolation in the next step.

4. *Update* $V_s(s_{it}, d_{it}, k_{it}, z_{it}), V_d(s_{it}, d_{it}, k_{it}, z_{it}), V_k(s_{it}, d_{it}, k_{it}, z_{it})$.

   For each combination $(s_{it}, d_{it}, k_{it}, z_{it}, c_{it}, h_{it}, \lambda_{it}, s_{it+1}, d_{it+1}, k_{it+1})$ identified in the previous steps, update $V_s(s_{it}, d_{it}, k_{it}, z_{it})$, $V_d(s_{it}, d_{it}, k_{it}, z_{it})$ and $V_k(s_{it}, d_{it}, k_{it}, z_{it})$ using C.6, C.7 and C.8 respectively

$$V_s(s_{it}, d_{it}, k_{it}, z_{it}) = \lambda_{it} - \lambda_{it}\tau_t r^s_{t-1},$$

$$V_d(s_{it}, d_{it}, k_{it}, z_{it}) = -\lambda_{it} + \lambda_{it}\tau_t r^d_{t-1},$$

$$V_k(s_{it}, d_{it}, k_{it}, z_{it}) = -\lambda_{it} f_{k_t}(k_{it+1}, k_{it}) + U_k(c_{it}, k_{it}, h_{it}).$$

Steps 1 to 4 are repeated until convergence of the three functions $V_s(s_{it}, d_{it}, k_{it}, z_{it})$, $V_d(s_{it}, d_{it}, k_{it}, z_{it})$ and $V_k(s_{it}, d_{it}, k_{it}, z_{it})$.

C.2 Heterogeneous deleveraging

Let $D_{it+1}(s_{it}, d_{it}, k_{it}, z_{it}, \phi_{t+1}, \tau)$ denote the optimal debt policy in t, i.e. the optimal choice of d_{it+1} as a function of the state variables in t, the anticipated maximum loan-to-value ratio in $t+1$, and the constant tax rate τ. Denote ε any quantity larger than or equal to zero.

Proposition The debt policy satisfies:

$$\frac{\partial D_{it+1}}{\partial \phi_{t+1}} = k_{it+1}\left(1 - \frac{\partial D_{it+1}}{\partial \tau}\frac{1}{(1+r_t^d)I_{it}}\right). \tag{C.10}$$

where I_{it} is the taxable income:

$$I_{it} = \mathcal{I}_{it}\nu_t + (1-\mathcal{I}_{it})z_{it}h_{it} + r_{t-1}^s s_{it} - r_{t-1}^d d_{it}. \tag{C.11}$$

Proof To prove this proposition, we first need to show that

$$D_{it+1}(s_{it}, d_{it}, k_{it}, z_{it}, \phi_{t+1}, \tau) + \varepsilon k_{it+1} = D_{it+1}\left(s_{it}, d_{it}, k_{it}, z_{it}, \phi_{t+1}+\varepsilon, \tau + \frac{\varepsilon k_{it+1}}{(1+r_t^d)I_{it}}\right). \tag{C.12}$$

Equality C.12 holds because for given choices $(c_{it}, h_{it}, s_{it+1}, k_{it+1})$, the path of debt D_{it+1} augmented by the constant εk_{it+1} that is feasible with the states $(s_{it}, d_{it}, k_{it}, z_{it}, \phi_{t+1}, \tau)$ is also feasible with the states $\left(s_{it}, d_{it}, k_{it}, z_{it}, \phi_{t+1}+\varepsilon, \tau + \frac{\varepsilon k_{it+1}}{(1+r_t^d)I_{it}}\right)$. We can prove this by noting that the choices $(c_{it}, h_{it}, s_{it+1}, k_{it+1})$ satisfy the borrowing constraint

$$c_{it} = \mathcal{I}_{it}\nu + (1-\mathcal{I}_{it})z_{it}h_{it} - f(k_{it+1}, k_{it}) + s_{it} - d_{it} - \frac{s_{it+1}}{1+r_t^s} + \frac{d_{it+1}}{1+r_t^d} - \tau I_{it}$$

if and only if they satisfy the borrowing constraint

$$c_{it} = \mathcal{I}_{it}\nu + (1-\mathcal{I}_{it})z_{it}h_{it} - f(k_{it+1}, k_{it}) + s_{it} - d_{it} - \frac{s_{it+1}}{1+r_t^s} + \frac{d_{it+1}+\varepsilon k_{it+1}}{1+r_t^d} - \left(\tau + \frac{\varepsilon k_{it+1}}{(1+r_t^d)I_{it}}\right) I_{it}$$

and because $d_{it+1} \leq \phi_{t+1}k_{it+1}$ if and only if $d_{it+1} + \varepsilon k_{it+1} \leq (\phi_{t+1}+\varepsilon)k_{it+1}$.

We then obtain proposition C.11 by differentiating C.12 with respect to ε.