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A PC-Based Simulator/Controller/Monitor Software for a Manipulators and Electromechanical Systems

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Abstract

General form application is a very important issue in industrial design. Prototyping a design helps in determining system parameters, ranges and in structuring better systems. Robotics is one of the industrial design fields in which prototyping is crucial for improved functionality. Developing an environment that enables optimal and flexible design using reconfigurable links, joints, actuators and sensors is essential for using robots in the education and industrial fields [4] [6]. We propose a PC-Based software package to control, monitor and simulate a generic 6-DOF (six degrees of freedom) robot including a spherical wrist. This package may be used as a black box for the design implementations or as white (detailed) box for learning about the basics of robotics and simulation technology.

1 Introduction

To design a complete and efficient robotics system there is a need for performing a sequence of cascaded tasks. The design task starts by determining the application of the robot, the performance requirements, and then determining the robot configuration and parameters suitable for that application. The physical design starts by gathering the parts and assembling the robot. Developing the required software (controller, simulator and monitor) elements is the next task. The next stage includes manipulator testing which determines the performance of the robot and the efficiency of the design.

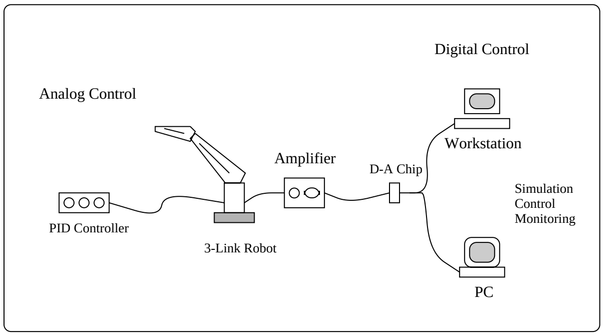

Figure 1: Typical robot control setup

Our aim is to build a complete PC-Based software package for control, monitoring and simulation of a 6-DOF manipulator, including a spherical wrist. The design will be independent of any existing specific robot parameters . The package will be an integration of several packages.

Figure 1 shows how such a PC-based robot could be controlled, possibly using different schemes.

The idea for this work came from a project we have done in a robotics class at University of Bridgeport. The project was to design a full integrated package to control, monitor, and simulate an SIR-1 robot. The SIR-1 robot is a 6 - DOF robot with a gripper. While working on the project, we continuously looked for the existence of similar prototyping packages on the market. We did a wide range search and exhaustive market survey for what was available. We searched a variety of papers, books,book chapters and web sites. We have also talked to a number of companies that manufacture manipulators. We found out that a reasonable progress has been done in the field, however, most of the prototyping is done for special or specific manipulators, with mainly

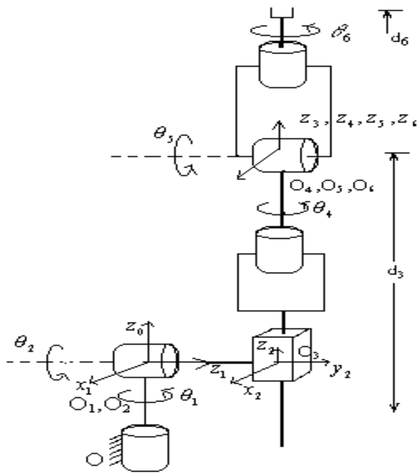

Figure 2: A physical six-link robot manipulator

numerical solutions. Unfortunately, PC-based controller/monitor/simulator packages for generic manipulators are not something common and sufficiently debated. Please visit the following URL's for more information:

www.bridgeport.edu/sobhdir/introb/node36.html

www.bridgeport.edu/sobhdir/introb/rep.html

www.bridgeport.edu/sobhdir/proj/wachter/

www.bridgeport.edu/sobhdir/proj/proto/paper.html

2 Background

The final design of the software package will be a collection of smaller packages. Each of these packages will be independent of any specific set of robot parameters. This can be done by making all calculations symbolically. Needless to say that will make the mathematics more difficult. By using mathematical application packages available nowadays such as Maple, Mathematica, Matlab and others, the job will be easier but not trivial. The next few sections give a theoretical background.

2.1 Forward kinematics

The standard Denavit-Hartenberg approach is being taken, figure 2 showing a physical six-link robot manipulator.

The *D-H* parameters for our prototype robot are shown in *Table 1.* The parameters for the last 3 links are constants with the exception of θ's, the joint variables and d_6 the offset parameter, which represents the offset distance between O_3 and the center of the wrist O.

The corresponding transformation matrix is

$$A_0^6 = A_1 A_2 A_3 A_4 A_5 A_6 \tag{1}$$

where

$$A_i = Rot_{z,\theta_i}, Trans_{z,d_i}, Trans_{x,a_i}, Rot_{x,\alpha_i} \tag{2}$$

$$A_i = \begin{bmatrix} c(\theta_i) & -s(\theta_i)c(\alpha_i) & s(\theta_i)s(\alpha_i) & a_i c(\theta_i) \\ s(\theta_i) & c(\theta_i)c(\alpha_i) & -c(\theta_i)s(\alpha_i) & a_i s(\theta_i) \\ 0 & s(\alpha_i) & c(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

2.2 Inverse Kinematics

Inverse kinematics solves for the joint angles given the desired position and orientation in Cartesian space. This is a more difficult problem than forward kinematics. The complexity of inverse kinematics can be described as follows, Given a $4x4$ homogeneous transformation which gives the required position and orientation

$$H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix} \tag{4}$$

where d and R are the given position and orientation of the tool frame relative to the origin. The homogeneous transformation matrix results in 12 nonlinear equations in 16-unknown variables $(a_1, a_2, a_3, \alpha_1, \alpha_2, \alpha_3, \theta_1 ... \theta_6, d_1, d_2, d_3, d_6)$.

$$T_{ij}(q_1, \ldots, q_6) = H_{ij} \tag{5}$$

Table 1: Symbolic DH parameters for the robot.

Link	a_i	α_i	d_i	θ_i
1	a_1	α_1	d_1	θ_1
2	a_2	α_2	d_2	θ_2
3	a_3	α_3	d_3	θ_3
4	0	-90	0	θ_4
5	0	$+90$	0	θ_5
6	0	0	d_6	θ_6

where $i = 1, 2, 3$, $j = 1, 2, 3, 4$.
For example, to find the corresponding joint variables $(\theta_1, \theta_2, d_3, \theta_4, \theta_5 \theta_6)$ for $RRP : RRR$ manipulator shown in *Figure 2* where

$$A_0^6 = \begin{bmatrix} e_{11} & e_{12} & e_{13} & d_x \\ e_{21} & e_{22} & e_{23} & d_y \\ e_{31} & e_{32} & e_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

we must solve 12 simultaneous set of nonlinear equations. See appendix 1. The first glance at a simple homogeneous transformation matrix eliminates the possibility of finding the solution by solving those 12 simultaneous set of nonlinear trigonometric equations. These equations are much too difficult to solve directly in closed form and therefore we need to develop efficient techniques that solves for the particular kinematics structure of the manipulator. To solve the inverse kinematics problem, closed form solution of the equations or a numerical solution could be used [8]. Closed form solution is preferable because in many applications where the manipulator supports or is to be supported by a sensory system, the results need to be supplied rapidly. Since inverse kinematics can result in a range of solutions rather than a unique one, finding a closed form will make it easy to implement the fastest possible sensory tracking algorithm.

The aim of this work is to try to find a closed solution for a prototype robot which is a general 3 - DOF robot having an arbitrary kinematic configuration connected to a spherical wrist. This closed form solution could be attained by different approaches. On possible approach is to decouple the inverse kinematics problem into two simpler problems, known respectively, as inverse position kinematics, and inverse orientation kinematics [1]. To put it in another way, for a six-DOF manipulator with a spherical wrist, the inverse kinematics problem may be separated into two simpler problems, by first finding the position of the intersection of the wrist axes, the center, and then finding the orientation of the wrist. Lets suppose that there are exactly six degrees of freedom and the last three joints axes intersect at a point O. We express the rotational and positional equations as

$$R_0^6(q_1, \dots, q_6) = R_{3\times3} \quad (7)$$

$$d_0^6(q_1, \dots, q_6) = d \quad (8)$$

where d and R are the given position and orientation of the tool frame relative to the origin. The assumption of a spherical wrist means that the axes $z_4, z_5 and z_6$ intersects at O and hence the origins O_4 and O_5 assigned by the D-H convention will always be at the wrist center O. The important point of this assumption for inverse kinematics is that the motion of the final three links about these axes will not change the position of O. The position of the wrist center is thus a function only of the first three joint variables. Since the origin of the tool frame O_6 is simply a translation by a distance d_6 along the z_5 axes from O_v, the vector O_{6v} relative to the frame $O_0X_0Y_0Z_0$ is

$$O_{6v} - O_v = -d_6 R_v k \quad (9)$$

where the $_v$ symbol denotes the vectorial notation; note that R is multiplied by k because it is a translation along z axes.
Suppose P_c denotes the vector from the origin of the base frame to the wrist center. Thus, in order to have the end-effector of the robot at the point d with the orientation of the wrist center O located at the point

$$P_c = d - d_6 R k \quad (10)$$

the orientation of the frame $O_0X_0Y_0Z_0$ with respect to the base be given by R. If the components of the end-effector position d are denoted d_x, d_y, d_z and the components of the wrist center P_c are denoted P_x, P_y, P_z then in this case the equation results in the relationship

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} d_x - d_6 r_{13} \\ d_y - d_6 r_{23} \\ d_z - d_6 r_{33} \end{bmatrix} \quad (11)$$

Using equation 10 we may find the values of the first three joint variable. Thus for this class of manipulators, the determination of the inverse kinematics can be summarized in 3 steps:

Step 1: Find q_1, q_2, q_3 such that the wrist center P_c is located at $P_c = d - d_6 k$

Step 2: Using the joint variables determined in Step 1, evaluate R_0^3 .

Step 3: Find a set of *Euler angles* corresponding to the rotation matrix $R_3^6 = (R_3^6)^{-1} R$

2.3 Velocity and Inverse Velocity Kinematics

In order to move the manipulator at constant velocity, or at any prescribed velocity, we must know the relationship between the velocity of the tool and the joint velocities. To calculate the velocity, the following Jacobian matrix should be constructed:

$$J = J_1 J_2 J_3 J_4 J_5 J_6 \tag{12}$$

where

$$J_i = \begin{bmatrix} z_{i-1} \times (O_n - O_{i-1}) \\ z_{i-1} \end{bmatrix} \tag{13}$$

if i is a revolute and

$$J_i = \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix} \tag{14}$$

if i is a prismatic,
where Z_i is the first three elements in 3^{rd} column of A_0^i and O_i is the first three elements in the 4^{th} column of A_0^i. Then forward velocity will be

$$\dot{X} = J(q)\dot{q} \tag{15}$$

The inverse velocity problem becomes one of solving the system of linear equations. The Inverse Velocity Kinematics will then be

$$\dot{q} = J^{-1}(q)\dot{X} \tag{16}$$

where the singularities will be discussed in the following section.

2.4 Acceleration and Inverse Acceleration Kinematics

Differentiating 15 yields the acceleration equation

$$\ddot{X} = J(q)\ddot{q} + \dot{J(q)}\dot{q} \tag{17}$$

By solving 17 for inverse acceleration, we find

$$\ddot{q} = J(q)^{-1}\ddot{X} - J(q)^{-1}\dot{J(q)}\dot{q} \tag{18}$$

2.5 Singularities

Singularities represents configurations from which certain directions of motion may be unattainable. It is possible to decouple the determination of a singular configurations for those manipulators with spherical wrist, into two simpler problems [9]. The first is to determine the arm singularities, that is , singularities resulting from motion of the arm, which consists of the first three or more links, while the second is to determine the wrist singularities resulting from motion of the spherical wrist. Suppose that n=*6*, that is, the manipulator consists of a 3-DOF arm with a 3 - DOF spherical wrist. In this case the Jacobian matrix is a 6x6 matrix and a configuration is singular if and only if

$$\det J(q) = 0 \tag{19}$$

if we now partition the Jacobian matrix into 3×3 blocks as

$$J = \begin{bmatrix} J_p & J_0 \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \tag{20}$$

then, since the final three joints are always revolute

$$J_0 = \begin{bmatrix} z_3 \times (O_6 - O_3) & z_4 \times (O_6 - O_4) & z_5 \times (O_6 - O_5) \\ z_3 & z_4 & z_5 \end{bmatrix} \tag{21}$$

Since the wrist axes intersect at a common point O, if we choose the coordinate frames so that $O_3 = O_4 = O_5 = O_6 = O$, then J_0 becomes

$$J_0 = \begin{bmatrix} 0 & 0 & 0 \\ z_3 & z_4 & z_5 \end{bmatrix} \tag{22}$$

and the $i-th$ column J_i of J_p is

$$J_i = \begin{bmatrix} z_{i-1} \times (O - O_{i-1}) \\ z_{i-1} \end{bmatrix} \tag{23}$$

if joint i is revolute and

$$J_i = \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix} \tag{24}$$

if joint i is prismatic. In this case the Jacobian matrix has the block triangular form

$$J = \begin{bmatrix} J_{11} & 0 \\ J_{21} & J_{22} \end{bmatrix} \tag{25}$$

with determinant

$$\det J = \det J_{11} \det J_{22} \tag{26}$$

where J_{11} and J_{22} are each $3x3$ matrices. J_{11} has $i-th$ column $z_{i-1} \times (O - O_{i-1})$ if joint i is revolute, and z_{i-1} if joint i is prismatic, while

$$J_{22} = \begin{bmatrix} z_3 & z_4 & z_5 \end{bmatrix} \tag{27}$$

2.6 Dynamics

Manipulator dynamics is concerned with the equation of motion, the way in which the manipulator moves in response to the torque applied by the actuators or external forces [7]. There are two problems related to manipulator dynamics that are important to solve:

- inverse dynamics in which the manipulator's equations of motion are solved for given motion to determine the generalized forces required for each joint (control stage) and
- direct dynamics in which the equations of motion are integrated to determine the generalized coordinate response to applied generalized forces (simulation stage).

The equation of motion for an n-axes manipulator are given by

$$Q = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + F(\dot{q}) + G(q) \qquad (28)$$

Where

q is the vector of generalized joint coordinates describing the pose of the manipulator

$\dot{q}$ is the vector of joint velocities

$\ddot{q}$ is the vector of joint accelerations

M is the symmetric joint-space inertia matrix, or manipulator inertia tensor

C describes *Coriolis* and centripetal effects

F describes viscous and Coulomb friction and is not generally considered part of rigid- body dynamics

G is the gravity loading

Q is the vector of generalized forces associated with generalized coordinates q

The equation may be derived via a number of techniques, including *Lagrangian* method [**?**]. Due to the enormous computational cost of this approach it is always difficult to compute manipulator torque for real-time control. To achieve real-time performance many approaches were suggested, including table lookup and approximation [3]. The most common approximation is to ignore

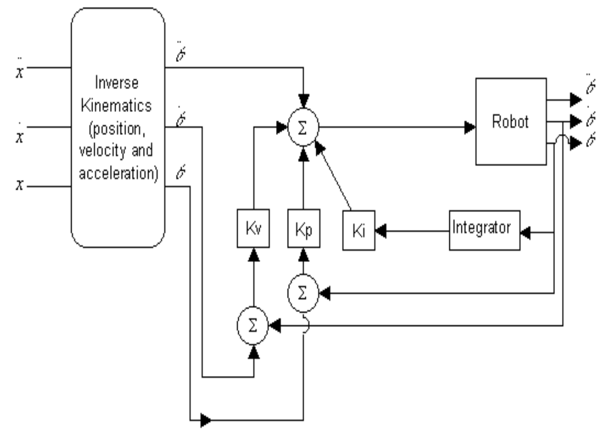

Figure 3: PID control loop

the velocity-dependent term C, since accurate positioning and high speed motion are exclusive in typical robot application. Practically, a *PID* controller might be a good option to achieve a real-time performance,

$$Q = \ddot{\theta}_d + k_v\dot{E} + k_pE + K_i\int Edt \qquad (29)$$

where k_v, k_p and k_i are the derivative, proportional and integral parameters respectively. See Figure 3 for a schematic of a PID control loop. The advantage of using a PID controller are the following:

- Simple to implement
- Suitable for a real-time control
- The behavior of the system can be controlled by changing the feedback gains

For a concrete decision on the controlling technique, often more details are needed, such as mechanics or motors, case in which the controller will be adjusted accordingly.

2.7 Simulation

To simulate the motion of a manipulator, we may use the simulation module by manipulating 28

$$\ddot{\theta} = M^{-1}(q)\left[Q - C(q, \dot{q})\dot{q} - F(\dot{q}) - G(q)\right] \qquad (30)$$

This represents the direct or integral or forward dynamic formulation giving joint motion in terms

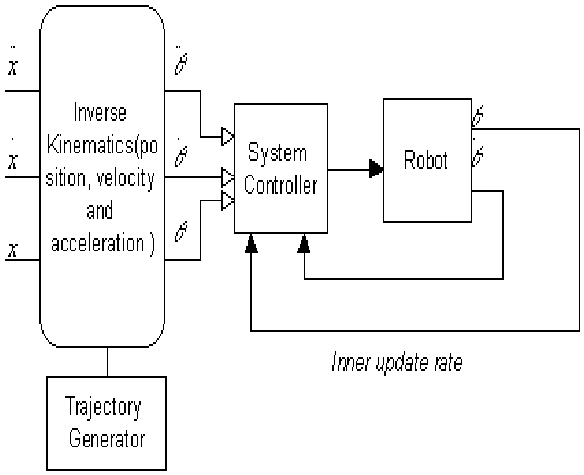

Figure 4: Simulation Loop

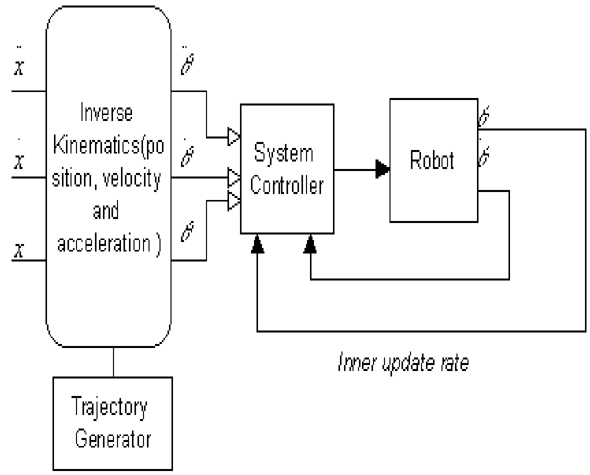

Figure 5: Trajectory Genrator integrated in the control loop

of input torque. $M(q)$ is the symmetric joint-space inertia matrix and for a 6-DOF manipulator M is an 6x6 symmetric matrix. $C(q, \dot{q})$ is the manipulator Coriolis/centripetal torque and for 6-DOF manipulator C will be a 6×1 matrix. $F(\dot{q})$ is the joint friction torque, where

$$F_i(t) = \begin{cases} B_i + \tau_i^-, \dot{\theta} < 0 \\ B_i + \tau_i^+, \dot{\theta} > 0 \end{cases} \tag{31}$$

Figure 4 shows the simulation loop.

2.8 Trajectory Generator

Trajectory generation describes the position velocity and acceleration of each link. This includes how the front user interfaces to describe the desired behavior of the manipulator. This could be a very complicated problem depending on the desired accuracy of the system. In some applications we might need to specify only the goal position, whereas in some application, we might need to specify the velocity with which the end effector should move. Since trajectory generation occurs at run time on a digital computer, the trajectory points are calculated at a certain rate, called the *path update rate*. One disadvantage of using a PID controller is a high update rate is required to achieve reasonable accuracy.

Our package role here is to calculate trajectory points which generate a smooth motion for the manipulator. The smoothness of motion is a very important issue due to physical considerations such as the required torque that causes this motion, the friction at the joints, and the frequency of update required to minimize the sampling error - cubic polynomial trajectories were used for the interpolation.

Figure 5 shows how trajectory generator can integrated in the control loop. It also shows two update rates, one is the inner update rate which update the system control with the actual joint position and velocity. Other updates the system control with the required joint values. The sampling of the two update rates can be different.

3 Package Outline

The package will handle a 6 - DOF robot which includes a spherical wrist. Thus, the first three joints could be revolute or prismatic which yields 8 different configurations of manipulators (i.e., XXX:RRR (X-denotes any type of link, R denotes a revolute link)). The user may use this package as a black box for design applications and as a white box for education and training purposes.

1	RRR:RRR
2	PRR:RRR
3	RPR:RRR
4	PPR:RRR
5	RPP:RRR
6	PRP:RRR
7	RPP:RRR
8	PPP:RRR

The first input is the manipulator configuration, see table 2. The package will ask for the DH parameters one after another. From the manipulator configuration, the package recognizes what the variables of the manipulator.

If the package is used as a white box, the package shows a menu with a list of tasks that the user may select.

[1]	Forward Kinematics
[2]	Inverse Kinematics
[3]	Velocity Kinematics
[4]	Inverse Velocity Kinematics
[5]	Acceleration Kinematics
[6]	Inverse Acceleration Kinematics
[7]	Dynamics
[8]	Simulation
[8]	Simulation
[10]	Trajectory Generation

Tasks are listed in table 3. The user then types the number of the task that it should be done. The package will ask for the other inputs required to do the calculations for the assigned job.

The black box includes
1. Full control loop implementation(PID & Dynamics based)
2. Full simulation loop
3. GUI with error analysis

The inputs and outputs can be summarized in the following sections.

3.1 Forward Kinematics

I/P: q_i where q_i will be θ_i if the joint is revolute, otherwise it will be d_i
O/P : $x = T_{14}$
$y = T_{24}$
$z = T_{34}$
$w_1 = \begin{bmatrix} T_{11} & T_{21} & T_{31} \end{bmatrix}$
$w_2 = \begin{bmatrix} T_{12} & T_{22} & T_{32} \end{bmatrix}$
$w_3 = \begin{bmatrix} T_{13} & T_{23} & T_{33} \end{bmatrix}$

where A is the transformation matrix.

3.2 Inverse Kinematics

I/P : x, y, z, w_1, w_2, w_3
O/P : q_i where $q_i = \theta_i$ if the i^{th} joint is revolute, and $q_i = d_i$ if it is prismatic.

3.3 Velocity Kinematics

I/P : $\theta_1, \theta_2, \ldots, \theta_6$ and $\dot{\theta}_1, \dot{\theta}_2, \ldots, \dot{\theta}_6$
O/P : $v_x, v_y, v_z, w_x, w_y, w_z$
where $\dot{\theta}_i$ is the i^{th} joint velocity, v is the linear tool velocity vector and w is an angular tool velocity vector.

3.4 Inverse Velocity Kinematics

I/P : $\theta_1, \theta_2, \ldots, \theta_6$ and $v_x, v_y, v_z, w_x, w_y, w_z$
O/P : $\dot{\theta}_1, \dot{\theta}_2, \ldots, \dot{\theta}_6$

3.5 Acceleration Kinematics

I/P : $\theta_1, \theta_2, \ldots, \theta_6$, $\dot{\theta}_1, \dot{\theta}_2, \ldots, \dot{\theta}_6$ ..and.. $\ddot{\theta}_1, \ddot{\theta}_2, \ldots \ddot{\theta}_6$

O/P :$a_x, a_y, a_z, \dot{w}_x, \dot{w}_y, \dot{w}_z$
where $\ddot{\theta}_i$ is the i^{th} joint acceleration, α is the linear tool acceleration and $\dot{w}$ is the angular tool acceleration.

3.6 Inverse Acceleration Kinematics

I/P : $\theta_1, \theta_2, \ldots, \theta_6$, $\dot{\theta}_1, \dot{\theta}_2, \ldots, \dot{\theta}_6$ and $a_x, a_y, a_z, \dot{w_x}, \dot{w_y}, \dot{w_z}$

O/P : $\ddot{\theta}_1, \ddot{\theta}_2, \ldots, \ddot{\theta}_6$

3.7 PID Controller and Trajectory Generation

I/P : $x, \dot{x}, \ddot{x}, k_p, k_v, k_i$
O/P : $\theta_i, \dot{\theta}_i, \ddot{\theta}_i$
Trajectory generator updates the controller with a new position the manipulator targets.

3.8 Simulation

I/P : $x, \dot{x}, \ddot{x}, M^{-1}(q), C(q, \dot{q}), F(\dot{q}), G(q)$
O/P :$\ddot{\theta}_i$

Dynamics parameters will be:
mass - mass of the link
rx - link COG with respect to the link coordinate frame
ry
rz
Ixx - elements of link inertia tensor about the link COG
Iyy

Izz
Ixy
Iyz
Ixz
Jm - armature inertia
G - reduction gear ratio/joint speed/link speed
B - viscous friction, motor referred
Tc+ - coulomb friction (positive rotation) motor referred
Tc- - coulomb friction (negative rotation) motor referred

4 Project Ideas and Progress

One target of the package is to find closed form solutions such that direct substitution are made when parameter are entered. That requires to determine what parameters should be variables and what should be constants [2]. Variables could be robot parameter configuration variables or state variables. The former are variables that define the structure of the manipulator, so they are constants for the same robot i.e. a's and α's . The latter describe the state of the robot (Joint Variable). Thus θ_i may be a state variable if i-th joint is revolute otherwise, it is a configuration variable. Same thing for d_i where it will be a state variable if the joint is prismatic. When the program is run, it will ask for the configuration of the robot (one of those listed in table 2). Then the program will decide what the robot configuration variables are and ask the user to enter them one after another. According to the task the program is asked to run, it will ask for the state variables. For example if the program is asked to calculate the Inverse Kinematics, the program will ask for the target Cartesian position and orientation to get the values of q's as an output. When the front user asks to do a task, the program calls the task handler. The task handler is a huge set of equations that are invoked when the front user enters the required input, and displays the results rapidly.

Figure 6 shows the task flow chart.
To find out the final and modified shape of equations for each task, a lot of math work has to be done. The next few sections give a few examples of how we managed to do the math chores.

Closed Form Solutions for Jacobian and Inverse Jacobian:

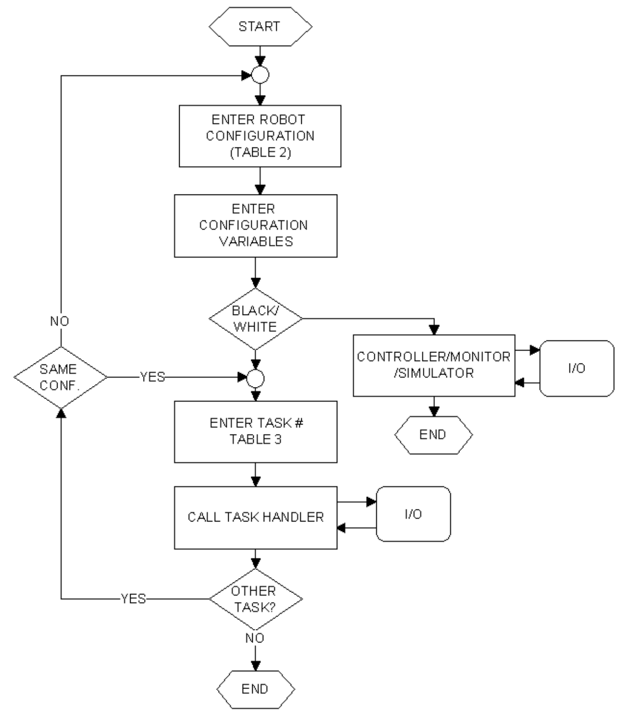

Figure 6: Task flow chart

Jacobian and Inverse Jacobian are fundamental objects to calculate the velocity and inverse velocity as shown in 15 and 16. We have used Maple as a math engine to find the Jacobian. It starts by defining the variables (a_i, α_i, θ_i and d_i). After that the program starts to calculate A_i's and then find the final transformation matrix
$A_0^6 = A_1 A_2 A_3 A_4 A_5 A_6$
To calculate the Jacobian, z_i's and O_i's should be involved. Maple computes them and stores them as variables. By now all the parameters required to evaluate the Jacobian are ready, we just let Maple find it out. The solved Jacobian contains a very large number of sines and cosines (inappropriate for using it directly as code and compile it) and this is passed through simplifications, Maple can handle the task. After that, Maple calculates the inverse Jacobian. To pass the result of the inverse Jacobian, it should be converted to an understandable and simplified C code. This job has to be done for each one of the eight robot configurations listed in table 1 . Appendix 2 shows how to use Maple to calculate the Jacobian matrix for RRP:RRR

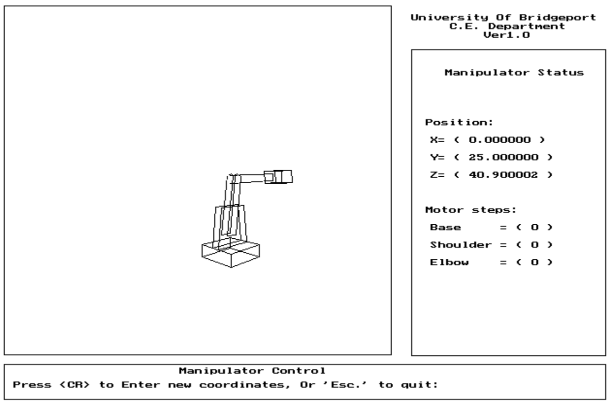

Figure 7: Monitoring Menu for SIR-1 Robot

Closed Form Solutions for $\frac{d}{dt}J(q)$*:*

The derivative of Jacobian is required to calculate the acceleration as indicated in 16. We have to derive Jacobeans for t as an exterior derivative. While the version of Maple we used does not do this job and Mathematica does, the output of Maple gets converted to be a suitable input for Mathematica. Then Mathematica differentiates the Jacobian and gets the output as collection of sines and cosines. Although both Mathematica and Maple can do the further simplification, we preferred Maple's output and so a conversion from Mathematica back to Maple is required.. Finally, a simplified C-code for the derivative of the Jacobian is found. This procedure is used for each configuration of the eight robot configurations. The derivative will be one part required to evaluate acceleration and inverse acceleration kinematics as shown in 17 and 18. Appendix 3 shows the derivative for RRP:RRR manipulator.

The two examples mentioned above show how it is tough in general to find closed form solutions, but by playing around with different tools, it may be reality. We keep doing this until we find final closed form solutions for the 6 - DOF robot including a spherical wrist. After we do all control and simulation parts, this package is supported with a 3 - D & 2 - D graphical monitoring system.

Figure 7 shows the monitoring system for SIR-1 robot as an example. The monitoring will be supported with a controller and a simulator.

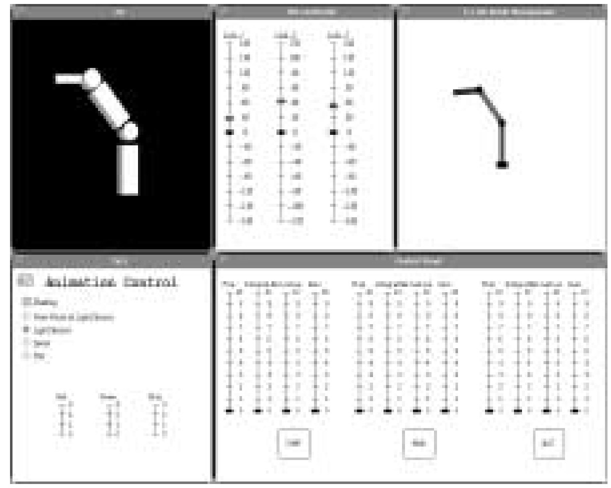

Figure 8: The interface window for the PID controller simulator

Figure 8 shows the interface window for the PID controller simulator.

5 Conclusion

A PC-Based software package for control, monitoring and simulation of a 6-DOF including a spherical wrist manipulator has been built. The design of the software package relies on symbolic calculations, The design is independent of specific robot parameters, has a generic character and can be used on various design implementations or for learning about the basics of robotics and simulation technology.

6 Appendix

Transformation Matrix for RRP : RRR with(linalg):
Warning, new definition for norm Warning, new definition for trace a1:=a1;

$$a1 := a1$$

a2:=a2;

$$a2 := a2$$

a3:=a3;

$$a3 := a3$$

a4:=0;

$$a4 := 0$$

a5:=0;

$$a5 := 0$$

a6:=0;

$$a6 := 0$$

d1:=d1;

$$d1 := d1$$

d2:=d2;

$$d2 := d2$$

d3:=d3;

$$d3 := d3$$

d4:=0;

$$d4 := 0$$

d5:=0;

$$d5 := 0$$

d6:=d6;

$$d6 := d6$$

alpha1:=alpha1;

$$\alpha 1 := \alpha 1$$

alpha2:=alpha2;

$$\alpha 2 := \alpha 2$$

alpha3:=alpha3;

$$\alpha 3 := \alpha 3$$

alpha4:=-Pi/2;

$$\alpha 4 := -\frac{1}{2}\pi$$

alpha5:=Pi/2;

$$\alpha 5 := \frac{1}{2}\pi$$

alpha6:=0;

$$\alpha 6 := 0$$

theta1:=theta1;

$$\theta 1 := \theta 1$$

theta2:=theta2;

$$\theta 2 := \theta 2$$

theta3:=theta3;

$$\theta 3 := \theta 3$$

theta4:=theta4;

$$\theta 4 := \theta 4$$

theta5:=theta5;

$$\theta 5 := \theta 5$$

theta6:=theta6;

$$\theta 6 := \theta 6$$

$A1 := matrix([[cos(theta1), -sin(theta1)*cos(alpha1), sin(theta1)* sin(alpha1), a1*cos(theta1)], [sin(theta1), cos(theta1)*cos(alpha1), -cos(theta1)*sin(alpha1), a1*sin(theta1)], [0, sin(alpha1), cos(alpha1), d1], [0, 0, 0, 1]]);$

$$A1 := \begin{bmatrix} \cos(\theta 1) & -\sin(\theta 1)\cos(\alpha 1) & \sin(\theta 1)\sin(\alpha 1) & \mathit{a1}\cos(\theta 1) \\ \sin(\theta 1) & \cos(\theta 1)\cos(\alpha 1) & -\cos(\theta 1)\sin(\alpha 1) & \mathit{a1}\sin(\theta 1) \\ 0 & \sin(\alpha 1) & \cos(\alpha 1) & \mathit{d1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A2 := matrix([[cos(theta2), -sin(theta2) * cos(alpha2), sin(theta2) * sin(alpha2), a2 * cos(theta2)], [sin(theta2), cos(theta2) * cos(alpha2), -cos(theta2) * sin(alpha2), a2 * sin(theta2)], [0, sin(alpha2), cos(alpha2), d2], [0, 0, 0, 1]]);$

$$A2 := \begin{bmatrix} \cos(\theta2) & -\sin(\theta2)\cos(\alpha2) & \sin(\theta2)\sin(\alpha2) & a2\cos(\theta2) \\ \sin(\theta2) & \cos(\theta2)\cos(\alpha2) & -\cos(\theta2)\sin(\alpha2) & a2\sin(\theta2) \\ 0 & \sin(\alpha2) & \cos(\alpha2) & d2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A3 := matrix([[cos(theta3), -sin(theta3) * cos(alpha3), sin(theta3) * sin(alpha3), a3 * cos(theta3)], [sin(theta3), cos(theta3) * cos(alpha3), -cos(theta3) * sin(alpha3), a3 * sin(theta3)], [0, sin(alpha3), cos(alpha3), d3], [0, 0, 0, 1]]);$

$$A3 := \begin{bmatrix} \cos(\theta3) & -\sin(\theta3)\cos(\alpha3) & \sin(\theta3)\sin(\alpha3) & a3\cos(\theta3) \\ \sin(\theta3) & \cos(\theta3)\cos(\alpha3) & -\cos(\theta3)\sin(\alpha3) & a3\sin(\theta3) \\ 0 & \sin(\alpha3) & \cos(\alpha3) & d3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A4 := matrix([[cos(theta4), -sin(theta4) * cos(alpha4), sin(theta4) * sin(alpha4), a4 * cos(theta4)], [sin(theta4), cos(theta4) * cos(alpha4), -cos(theta4) * sin(alpha4), a4 * sin(theta4)], [0, sin(alpha4), cos(alpha4), d4], [0, 0, 0, 1]]);$

$$A4 := \begin{bmatrix} \cos(\theta4) & 0 & -\sin(\theta4) & 0 \\ \sin(\theta4) & 0 & \cos(\theta4) & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A5 := matrix([[cos(theta5), -sin(theta5) * cos(alpha5), sin(theta5) * sin(alpha5), a5 * cos(theta5)], [sin(theta5), cos(theta5) * cos(alpha5), -cos(theta5) * sin(alpha5), a5 * sin(theta5)], [0, sin(alpha5), cos(alpha5), d5], [0, 0, 0, 1]]);$

$$A5 := \begin{bmatrix} \cos(\theta5) & 0 & \sin(\theta5) & 0 \\ \sin(\theta5) & 0 & -\cos(\theta5) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A6 := matrix([[cos(theta6), -sin(theta6) * cos(alpha6), sin(theta6) * sin(alpha6), a6 * cos(theta6)], [sin(theta6), cos(theta6) * cos(alpha6), -cos(theta6) * sin(alpha6), a6 * sin(theta6)], [0, sin(alpha6), cos(alpha6), d6], [0, 0, 0, 1]]);$

$$A6 := \begin{bmatrix} \cos(\theta6) & -\sin(\theta6) & 0 & 0 \\ \sin(\theta6) & \cos(\theta6) & 0 & 0 \\ 0 & 0 & 1 & d6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$AA := evalm(A1\& * A2\& * A3\& * A4\& * A5\& * A6) :$

The next 12 nonlinear equations should be solved to find out the Inverse Kinematics of the Manipulator;

e11:=simplify(AA[1,1]);

$$e11 := (((\%3\cos(\theta3) + \%2\sin(\theta3))\cos(\theta4) + (-\%3\sin(\theta3)\cos(\alpha3) + \%2\cos(\theta3)\cos(\alpha3) + \%1\sin(\alpha3))\sin(\theta4))\cos(\theta5)$$

$$+\,(-\%3\sin(\theta3)\sin(\alpha3)+\%2\cos(\theta3)\sin(\alpha3)-\%1\cos(\alpha3))\sin(\theta5))\cos(\theta6)+($$
$$-(\%3\cos(\theta3)+\%2\sin(\theta3))\sin(\theta4)$$
$$+\,(-\%3\sin(\theta3)\cos(\alpha3)+\%2\cos(\theta3)\cos(\alpha3)+\%1\sin(\alpha3))\cos(\theta4))\sin(\theta6)$$
$$\%1 :=$$
$$\cos(\theta1)\sin(\theta2)\sin(\alpha2)+\sin(\theta1)\cos(\alpha1)\cos(\theta2)\sin(\alpha2)+\sin(\theta1)\sin(\alpha1)\cos(\alpha2)$$
$$\%2 :=$$
$$-\cos(\theta1)\sin(\theta2)\cos(\alpha2)-\sin(\theta1)\cos(\alpha1)\cos(\theta2)\cos(\alpha2)+\sin(\theta1)\sin(\alpha1)\sin(\alpha2)$$
$$\%3 := \cos(\theta1)\cos(\theta2)-\sin(\theta1)\cos(\alpha1)\sin(\theta2)$$

e21:=simplify(AA[2,1]);

$$\begin{aligned}
e21 := {} & \cos(\theta6)\cos(\theta5)\cos(\theta4)\sin(\theta3)\cos(\theta1)\cos(\alpha1)\cos(\theta2)\cos(\alpha2)\\
& -\cos(\theta6)\cos(\theta5)\cos(\theta4)\sin(\theta3)\cos(\theta1)\sin(\alpha1)\sin(\alpha2)\\
& -\cos(\theta6)\cos(\theta5)\sin(\theta4)\sin(\theta3)\cos(\alpha3)\sin(\theta1)\cos(\theta2)\\
& -\cos(\theta6)\cos(\theta5)\sin(\theta4)\sin(\theta3)\cos(\alpha3)\cos(\theta1)\cos(\alpha1)\sin(\theta2)\\
& -\cos(\theta6)\cos(\theta5)\sin(\theta4)\cos(\theta3)\cos(\alpha3)\sin(\theta1)\sin(\theta2)\cos(\alpha2)\\
& +\cos(\theta6)\cos(\theta5)\sin(\theta4)\cos(\theta3)\cos(\alpha3)\cos(\theta1)\cos(\alpha1)\cos(\theta2)\cos(\alpha2)\\
& -\cos(\theta6)\cos(\theta5)\sin(\theta4)\cos(\theta3)\cos(\alpha3)\cos(\theta1)\sin(\alpha1)\sin(\alpha2)\\
& +\cos(\theta6)\cos(\theta5)\sin(\theta4)\sin(\alpha3)\sin(\theta1)\sin(\theta2)\sin(\alpha2)\\
& -\cos(\theta6)\cos(\theta5)\sin(\theta4)\sin(\alpha3)\cos(\theta1)\cos(\alpha1)\cos(\theta2)\sin(\alpha2)\\
& -\cos(\theta6)\cos(\theta5)\sin(\theta4)\sin(\alpha3)\cos(\theta1)\sin(\alpha1)\cos(\alpha2)\\
& -\cos(\theta6)\sin(\theta5)\sin(\theta3)\sin(\alpha3)\sin(\theta1)\cos(\theta2)\\
& -\cos(\theta6)\sin(\theta5)\sin(\theta3)\sin(\alpha3)\cos(\theta1)\cos(\alpha1)\sin(\theta2)\\
& -\cos(\theta6)\sin(\theta5)\cos(\theta3)\sin(\alpha3)\sin(\theta1)\sin(\theta2)\cos(\alpha2)\\
& +\cos(\theta6)\sin(\theta5)\cos(\theta3)\sin(\alpha3)\cos(\theta1)\cos(\alpha1)\cos(\theta2)\cos(\alpha2)\\
& -\cos(\theta6)\sin(\theta5)\cos(\alpha3)\sin(\theta1)\sin(\theta2)\sin(\alpha2)\\
& +\sin(\theta6)\sin(\theta4)\sin(\theta3)\sin(\theta1)\sin(\theta2)\cos(\alpha2)\\
& -\sin(\theta6)\sin(\theta4)\sin(\theta3)\cos(\theta1)\cos(\alpha1)\cos(\theta2)\cos(\alpha2)\\
& +\sin(\theta6)\sin(\theta4)\sin(\theta3)\cos(\theta1)\sin(\alpha1)\sin(\alpha2)\\
& -\sin(\theta6)\cos(\theta4)\sin(\theta3)\cos(\alpha3)\sin(\theta1)\cos(\theta2)\\
& -\sin(\theta6)\cos(\theta4)\sin(\theta3)\cos(\alpha3)\cos(\theta1)\cos(\alpha1)\sin(\theta2)\\
& -\sin(\theta6)\cos(\theta4)\cos(\theta3)\cos(\alpha3)\sin(\theta1)\sin(\theta2)\cos(\alpha2)\\
& +\sin(\theta6)\cos(\theta4)\cos(\theta3)\cos(\alpha3)\cos(\theta1)\cos(\alpha1)\cos(\theta2)\cos(\alpha2)\\
& -\sin(\theta6)\cos(\theta4)\cos(\theta3)\cos(\alpha3)\cos(\theta1)\sin(\alpha1)\sin(\alpha2)\\
& +\sin(\theta6)\cos(\theta4)\sin(\alpha3)\sin(\theta1)\sin(\theta2)\sin(\alpha2)\\
& -\sin(\theta6)\cos(\theta4)\sin(\alpha3)\cos(\theta1)\cos(\alpha1)\cos(\theta2)\sin(\alpha2)\\
& -\sin(\theta6)\cos(\theta4)\sin(\alpha3)\cos(\theta1)\sin(\alpha1)\cos(\alpha2)\\
& -\sin(\theta6)\sin(\theta4)\cos(\theta3)\sin(\theta1)\cos(\theta2)\\
& -\sin(\theta6)\sin(\theta4)\cos(\theta3)\cos(\theta1)\cos(\alpha1)\sin(\theta2)\\
& +\cos(\theta6)\sin(\theta5)\cos(\alpha3)\cos(\theta1)\cos(\alpha1)\cos(\theta2)\sin(\alpha2)\\
& +\cos(\theta6)\sin(\theta5)\cos(\alpha3)\cos(\theta1)\sin(\alpha1)\cos(\alpha2)
\end{aligned}$$

$$+\cos(\theta 6)\cos(\theta 5)\cos(\theta 4)\cos(\theta 3)\cos(\theta 1)\cos(\alpha 1)\sin(\theta 2)$$
$$+\cos(\theta 6)\cos(\theta 5)\cos(\theta 4)\cos(\theta 3)\sin(\theta 1)\cos(\theta 2)$$
$$-\cos(\theta 6)\cos(\theta 5)\cos(\theta 4)\sin(\theta 3)\sin(\theta 1)\sin(\theta 2)\cos(\alpha 2)$$
$$-\cos(\theta 6)\sin(\theta 5)\cos(\theta 3)\sin(\alpha 3)\cos(\theta 1)\sin(\alpha 1)\sin(\alpha 2)$$

e31:=simplify(AA[3,1]);

$$\begin{aligned}
e31 := {} & \cos(\theta 6)\cos(\theta 5)\cos(\theta 4)\sin(\alpha 1)\sin(\theta 2)\cos(\theta 3) \\
& + \cos(\theta 6)\cos(\theta 5)\cos(\theta 4)\sin(\theta 3)\sin(\alpha 1)\cos(\theta 2)\cos(\alpha 2) \\
& + \cos(\theta 6)\cos(\theta 5)\cos(\theta 4)\sin(\theta 3)\cos(\alpha 1)\sin(\alpha 2) \\
& - \cos(\theta 6)\cos(\theta 5)\sin(\theta 4)\sin(\alpha 1)\sin(\theta 2)\sin(\theta 3)\cos(\alpha 3) \\
& + \cos(\theta 6)\cos(\theta 5)\sin(\theta 4)\cos(\theta 3)\cos(\alpha 3)\sin(\alpha 1)\cos(\theta 2)\cos(\alpha 2) \\
& + \cos(\theta 6)\cos(\theta 5)\sin(\theta 4)\cos(\theta 3)\cos(\alpha 3)\cos(\alpha 1)\sin(\alpha 2) \\
& - \cos(\theta 6)\cos(\theta 5)\sin(\theta 4)\sin(\alpha 3)\sin(\alpha 1)\cos(\theta 2)\sin(\alpha 2) \\
& + \cos(\theta 6)\cos(\theta 5)\sin(\theta 4)\sin(\alpha 3)\cos(\alpha 1)\cos(\alpha 2) \\
& - \cos(\theta 6)\sin(\theta 5)\sin(\alpha 1)\sin(\theta 2)\sin(\theta 3)\sin(\alpha 3) \\
& + \cos(\theta 6)\sin(\theta 5)\cos(\theta 3)\sin(\alpha 3)\sin(\alpha 1)\cos(\theta 2)\cos(\alpha 2) \\
& + \cos(\theta 6)\sin(\theta 5)\cos(\theta 3)\sin(\alpha 3)\cos(\alpha 1)\sin(\alpha 2) \\
& + \cos(\theta 6)\sin(\theta 5)\cos(\alpha 3)\sin(\alpha 1)\cos(\theta 2)\sin(\alpha 2) \\
& - \cos(\theta 6)\sin(\theta 5)\cos(\alpha 3)\cos(\alpha 1)\cos(\alpha 2) - \sin(\theta 6)\sin(\theta 4)\sin(\alpha 1)\sin(\theta 2)\cos(\theta 3) \\
& - \sin(\theta 6)\sin(\theta 4)\sin(\theta 3)\sin(\alpha 1)\cos(\theta 2)\cos(\alpha 2) \\
& - \sin(\theta 6)\sin(\theta 4)\sin(\theta 3)\cos(\alpha 1)\sin(\alpha 2) \\
& - \sin(\theta 6)\cos(\theta 4)\sin(\alpha 1)\sin(\theta 2)\sin(\theta 3)\cos(\alpha 3) \\
& + \sin(\theta 6)\cos(\theta 4)\cos(\theta 3)\cos(\alpha 3)\sin(\alpha 1)\cos(\theta 2)\cos(\alpha 2) \\
& + \sin(\theta 6)\cos(\theta 4)\cos(\theta 3)\cos(\alpha 3)\cos(\alpha 1)\sin(\alpha 2) \\
& - \sin(\theta 6)\cos(\theta 4)\sin(\alpha 3)\sin(\alpha 1)\cos(\theta 2)\sin(\alpha 2) \\
& + \sin(\theta 6)\cos(\theta 4)\sin(\alpha 3)\cos(\alpha 1)\cos(\alpha 2)
\end{aligned}$$

e12:=simplify(AA[1,2]);

$$\begin{aligned}
e12 := {} & \sin(\theta 6)\cos(\theta 5)\cos(\theta 4)\sin(\theta 3)\sin(\theta 1)\cos(\alpha 1)\cos(\theta 2)\cos(\alpha 2) \\
& - \sin(\theta 6)\cos(\theta 5)\cos(\theta 4)\sin(\theta 3)\sin(\theta 1)\sin(\alpha 1)\sin(\alpha 2) \\
& + \sin(\theta 6)\cos(\theta 5)\sin(\theta 4)\sin(\theta 3)\cos(\alpha 3)\cos(\theta 1)\cos(\theta 2) \\
& - \sin(\theta 6)\cos(\theta 5)\sin(\theta 4)\sin(\theta 3)\cos(\alpha 3)\sin(\theta 1)\cos(\alpha 1)\sin(\theta 2) \\
& + \sin(\theta 6)\cos(\theta 5)\sin(\theta 4)\cos(\theta 3)\cos(\alpha 3)\cos(\theta 1)\sin(\theta 2)\cos(\alpha 2) \\
& + \sin(\theta 6)\cos(\theta 5)\sin(\theta 4)\cos(\theta 3)\cos(\alpha 3)\sin(\theta 1)\cos(\alpha 1)\cos(\theta 2)\cos(\alpha 2) \\
& - \sin(\theta 6)\cos(\theta 5)\sin(\theta 4)\cos(\theta 3)\cos(\alpha 3)\sin(\theta 1)\sin(\alpha 1)\sin(\alpha 2) \\
& - \sin(\theta 6)\cos(\theta 5)\sin(\theta 4)\sin(\alpha 3)\cos(\theta 1)\sin(\theta 2)\sin(\alpha 2) \\
& + \sin(\theta 6)\sin(\theta 5)\cos(\alpha 3)\sin(\theta 1)\sin(\alpha 1)\cos(\alpha 2) \\
& + \cos(\theta 6)\cos(\theta 4)\cos(\theta 3)\cos(\alpha 3)\sin(\theta 1)\sin(\alpha 1)\sin(\alpha 2) \\
& + \cos(\theta 6)\cos(\theta 4)\sin(\theta 3)\cos(\alpha 3)\sin(\theta 1)\cos(\alpha 1)\sin(\theta 2) \\
& - \cos(\theta 6)\cos(\theta 4)\cos(\theta 3)\cos(\alpha 3)\cos(\theta 1)\sin(\theta 2)\cos(\alpha 2)
\end{aligned}$$

$$
\begin{aligned}
&- \cos(\theta 6) \cos(\theta 4) \cos(\theta 3) \cos(\alpha 3) \sin(\theta 1) \cos(\alpha 1) \cos(\theta 2) \cos(\alpha 2) \\
&- \sin(\theta 6) \cos(\theta 5) \sin(\theta 4) \sin(\alpha 3) \sin(\theta 1) \cos(\alpha 1) \cos(\theta 2) \sin(\alpha 2) \\
&- \sin(\theta 6) \cos(\theta 5) \sin(\theta 4) \sin(\alpha 3) \sin(\theta 1) \sin(\alpha 1) \cos(\alpha 2) \\
&+ \sin(\theta 6) \sin(\theta 5) \sin(\theta 3) \sin(\alpha 3) \cos(\theta 1) \cos(\theta 2) \\
&- \sin(\theta 6) \sin(\theta 5) \sin(\theta 3) \sin(\alpha 3) \sin(\theta 1) \cos(\alpha 1) \sin(\theta 2) \\
&+ \sin(\theta 6) \sin(\theta 5) \cos(\theta 3) \sin(\alpha 3) \cos(\theta 1) \sin(\theta 2) \cos(\alpha 2) \\
&+ \sin(\theta 6) \sin(\theta 5) \cos(\theta 3) \sin(\alpha 3) \sin(\theta 1) \cos(\alpha 1) \cos(\theta 2) \cos(\alpha 2) \\
&- \sin(\theta 6) \sin(\theta 5) \cos(\theta 3) \sin(\alpha 3) \sin(\theta 1) \sin(\alpha 1) \sin(\alpha 2) \\
&+ \sin(\theta 6) \sin(\theta 5) \cos(\alpha 3) \cos(\theta 1) \sin(\theta 2) \sin(\alpha 2) \\
&+ \sin(\theta 6) \sin(\theta 5) \cos(\alpha 3) \sin(\theta 1) \cos(\alpha 1) \cos(\theta 2) \sin(\alpha 2) \\
&- \sin(\theta 6) \cos(\theta 5) \cos(\theta 4) \cos(\theta 3) \cos(\theta 1) \cos(\theta 2) \\
&+ \sin(\theta 6) \cos(\theta 5) \cos(\theta 4) \cos(\theta 3) \sin(\theta 1) \cos(\alpha 1) \sin(\theta 2) \\
&+ \cos(\theta 6) \cos(\theta 4) \sin(\alpha 3) \cos(\theta 1) \sin(\theta 2) \sin(\alpha 2) \\
&+ \cos(\theta 6) \cos(\theta 4) \sin(\alpha 3) \sin(\theta 1) \cos(\alpha 1) \cos(\theta 2) \sin(\alpha 2) \\
&+ \cos(\theta 6) \cos(\theta 4) \sin(\alpha 3) \sin(\theta 1) \sin(\alpha 1) \cos(\alpha 2) \\
&- \cos(\theta 6) \sin(\theta 4) \cos(\theta 3) \cos(\theta 1) \cos(\theta 2) \\
&+ \cos(\theta 6) \sin(\theta 4) \cos(\theta 3) \sin(\theta 1) \cos(\alpha 1) \sin(\theta 2) \\
&+ \cos(\theta 6) \sin(\theta 4) \sin(\theta 3) \cos(\theta 1) \sin(\theta 2) \cos(\alpha 2) \\
&+ \cos(\theta 6) \sin(\theta 4) \sin(\theta 3) \sin(\theta 1) \cos(\alpha 1) \cos(\theta 2) \cos(\alpha 2) \\
&- \cos(\theta 6) \sin(\theta 4) \sin(\theta 3) \sin(\theta 1) \sin(\alpha 1) \sin(\alpha 2) \\
&- \cos(\theta 6) \cos(\theta 4) \sin(\theta 3) \cos(\alpha 3) \cos(\theta 1) \cos(\theta 2) \\
&+ \sin(\theta 6) \cos(\theta 5) \cos(\theta 4) \sin(\theta 3) \cos(\theta 1) \sin(\theta 2) \cos(\alpha 2)
\end{aligned}
$$

e22:=simplify(AA[2,2]);

$$
\begin{aligned}
\mathit{e22} := &-\sin(\theta 6) \cos(\theta 5) \cos(\theta 4) \cos(\theta 3) \sin(\theta 1) \cos(\theta 2) \\
&- \sin(\theta 6) \cos(\theta 5) \cos(\theta 4) \cos(\theta 3) \cos(\theta 1) \cos(\alpha 1) \sin(\theta 2) \\
&+ \sin(\theta 6) \cos(\theta 5) \cos(\theta 4) \sin(\theta 3) \sin(\theta 1) \sin(\theta 2) \cos(\alpha 2) \\
&- \sin(\theta 6) \cos(\theta 5) \cos(\theta 4) \sin(\theta 3) \cos(\theta 1) \cos(\alpha 1) \cos(\theta 2) \cos(\alpha 2) \\
&+ \sin(\theta 6) \cos(\theta 5) \cos(\theta 4) \sin(\theta 3) \cos(\theta 1) \sin(\alpha 1) \sin(\alpha 2) \\
&+ \sin(\theta 6) \cos(\theta 5) \sin(\theta 4) \sin(\theta 3) \cos(\alpha 3) \sin(\theta 1) \cos(\theta 2) \\
&+ \sin(\theta 6) \cos(\theta 5) \sin(\theta 4) \sin(\theta 3) \cos(\alpha 3) \cos(\theta 1) \cos(\alpha 1) \sin(\theta 2) \\
&+ \sin(\theta 6) \cos(\theta 5) \sin(\theta 4) \cos(\theta 3) \cos(\alpha 3) \sin(\theta 1) \sin(\theta 2) \cos(\alpha 2) \\
&- \sin(\theta 6) \cos(\theta 5) \sin(\theta 4) \cos(\theta 3) \cos(\alpha 3) \cos(\theta 1) \cos(\alpha 1) \cos(\theta 2) \cos(\alpha 2) \\
&+ \sin(\theta 6) \cos(\theta 5) \sin(\theta 4) \cos(\theta 3) \cos(\alpha 3) \cos(\theta 1) \sin(\alpha 1) \sin(\alpha 2) \\
&- \sin(\theta 6) \cos(\theta 5) \sin(\theta 4) \sin(\alpha 3) \sin(\theta 1) \sin(\theta 2) \sin(\alpha 2) \\
&+ \sin(\theta 6) \cos(\theta 5) \sin(\theta 4) \sin(\alpha 3) \cos(\theta 1) \cos(\alpha 1) \cos(\theta 2) \sin(\alpha 2) \\
&+ \sin(\theta 6) \cos(\theta 5) \sin(\theta 4) \sin(\alpha 3) \cos(\theta 1) \sin(\alpha 1) \cos(\alpha 2) \\
&+ \sin(\theta 6) \sin(\theta 5) \sin(\theta 3) \sin(\alpha 3) \sin(\theta 1) \cos(\theta 2) \\
&+ \sin(\theta 6) \sin(\theta 5) \sin(\theta 3) \sin(\alpha 3) \cos(\theta 1) \cos(\alpha 1) \sin(\theta 2) \\
&+ \sin(\theta 6) \sin(\theta 5) \cos(\theta 3) \sin(\alpha 3) \sin(\theta 1) \sin(\theta 2) \cos(\alpha 2)
\end{aligned}
$$

$$- \sin(\theta6) \sin(\theta5) \cos(\theta3) \sin(\alpha3) \cos(\theta1) \cos(\alpha1) \cos(\theta2) \cos(\alpha2)$$
$$+ \sin(\theta6) \sin(\theta5) \cos(\theta3) \sin(\alpha3) \cos(\theta1) \sin(\alpha1) \sin(\alpha2)$$
$$+ \sin(\theta6) \sin(\theta5) \cos(\alpha3) \sin(\theta1) \sin(\theta2) \sin(\alpha2)$$
$$- \sin(\theta6) \sin(\theta5) \cos(\alpha3) \cos(\theta1) \cos(\alpha1) \cos(\theta2) \sin(\alpha2)$$
$$- \sin(\theta6) \sin(\theta5) \cos(\alpha3) \cos(\theta1) \sin(\alpha1) \cos(\alpha2)$$
$$- \cos(\theta6) \sin(\theta4) \cos(\theta3) \sin(\theta1) \cos(\theta2)$$
$$- \cos(\theta6) \sin(\theta4) \cos(\theta3) \cos(\theta1) \cos(\alpha1) \sin(\theta2)$$
$$+ \cos(\theta6) \sin(\theta4) \sin(\theta3) \sin(\theta1) \sin(\theta2) \cos(\alpha2)$$
$$- \cos(\theta6) \sin(\theta4) \sin(\theta3) \cos(\theta1) \cos(\alpha1) \cos(\theta2) \cos(\alpha2)$$
$$+ \cos(\theta6) \sin(\theta4) \sin(\theta3) \cos(\theta1) \sin(\alpha1) \sin(\alpha2)$$
$$- \cos(\theta6) \cos(\theta4) \sin(\theta3) \cos(\alpha3) \sin(\theta1) \cos(\theta2)$$
$$- \cos(\theta6) \cos(\theta4) \sin(\theta3) \cos(\alpha3) \cos(\theta1) \cos(\alpha1) \sin(\theta2)$$
$$- \cos(\theta6) \cos(\theta4) \cos(\theta3) \cos(\alpha3) \sin(\theta1) \sin(\theta2) \cos(\alpha2)$$
$$+ \cos(\theta6) \cos(\theta4) \cos(\theta3) \cos(\alpha3) \cos(\theta1) \cos(\alpha1) \cos(\theta2) \cos(\alpha2)$$
$$- \cos(\theta6) \cos(\theta4) \cos(\theta3) \cos(\alpha3) \cos(\theta1) \sin(\alpha1) \sin(\alpha2)$$
$$+ \cos(\theta6) \cos(\theta4) \sin(\alpha3) \sin(\theta1) \sin(\theta2) \sin(\alpha2)$$
$$- \cos(\theta6) \cos(\theta4) \sin(\alpha3) \cos(\theta1) \cos(\alpha1) \cos(\theta2) \sin(\alpha2)$$
$$- \cos(\theta6) \cos(\theta4) \sin(\alpha3) \cos(\theta1) \sin(\alpha1) \cos(\alpha2)$$

e:32=simplify(AA[3,2]);

$$32 = -\sin(\theta6) \cos(\theta5) \cos(\theta4) \sin(\alpha1) \sin(\theta2) \cos(\theta3)$$
$$- \sin(\theta6) \cos(\theta5) \cos(\theta4) \sin(\theta3) \sin(\alpha1) \cos(\theta2) \cos(\alpha2)$$
$$- \sin(\theta6) \cos(\theta5) \cos(\theta4) \sin(\theta3) \cos(\alpha1) \sin(\alpha2)$$
$$+ \sin(\theta6) \cos(\theta5) \sin(\theta4) \sin(\alpha1) \sin(\theta2) \sin(\theta3) \cos(\alpha3)$$
$$- \sin(\theta6) \cos(\theta5) \sin(\theta4) \cos(\theta3) \cos(\alpha3) \sin(\alpha1) \cos(\theta2) \cos(\alpha2)$$
$$- \sin(\theta6) \cos(\theta5) \sin(\theta4) \cos(\theta3) \cos(\alpha3) \cos(\alpha1) \sin(\alpha2)$$
$$+ \sin(\theta6) \cos(\theta5) \sin(\theta4) \sin(\alpha3) \sin(\alpha1) \cos(\theta2) \sin(\alpha2)$$
$$- \sin(\theta6) \cos(\theta5) \sin(\theta4) \sin(\alpha3) \cos(\alpha1) \cos(\alpha2)$$
$$+ \sin(\theta6) \sin(\theta5) \sin(\alpha1) \sin(\theta2) \sin(\theta3) \sin(\alpha3)$$
$$- \sin(\theta6) \sin(\theta5) \cos(\theta3) \sin(\alpha3) \sin(\alpha1) \cos(\theta2) \cos(\alpha2)$$
$$- \sin(\theta6) \sin(\theta5) \cos(\theta3) \sin(\alpha3) \cos(\alpha1) \sin(\alpha2)$$
$$- \sin(\theta6) \sin(\theta5) \cos(\alpha3) \sin(\alpha1) \cos(\theta2) \sin(\alpha2)$$
$$+ \sin(\theta6) \sin(\theta5) \cos(\alpha3) \cos(\alpha1) \cos(\alpha2) - \cos(\theta6) \sin(\theta4) \sin(\alpha1) \sin(\theta2) \cos(\theta3)$$
$$- \cos(\theta6) \sin(\theta4) \sin(\theta3) \sin(\alpha1) \cos(\theta2) \cos(\alpha2)$$
$$- \cos(\theta6) \sin(\theta4) \sin(\theta3) \cos(\alpha1) \sin(\alpha2)$$
$$- \cos(\theta6) \cos(\theta4) \sin(\alpha1) \sin(\theta2) \sin(\theta3) \cos(\alpha3)$$
$$+ \cos(\theta6) \cos(\theta4) \cos(\theta3) \cos(\alpha3) \sin(\alpha1) \cos(\theta2) \cos(\alpha2)$$
$$+ \cos(\theta6) \cos(\theta4) \cos(\theta3) \cos(\alpha3) \cos(\alpha1) \sin(\alpha2)$$
$$- \cos(\theta6) \cos(\theta4) \sin(\alpha3) \sin(\alpha1) \cos(\theta2) \sin(\alpha2)$$
$$+ \cos(\theta6) \cos(\theta4) \sin(\alpha3) \cos(\alpha1) \cos(\alpha2)$$

e13:=simplify(AA[1,3]);

$$
\begin{aligned}
\mathit{e13} := {} & \sin(\theta 5)\cos(\theta 4)\cos(\theta 3)\cos(\theta 1)\cos(\theta 2) \\
& - \sin(\theta 5)\cos(\theta 4)\cos(\theta 3)\sin(\theta 1)\cos(\alpha 1)\sin(\theta 2) \\
& - \sin(\theta 5)\cos(\theta 4)\sin(\theta 3)\cos(\theta 1)\sin(\theta 2)\cos(\alpha 2) \\
& - \sin(\theta 5)\cos(\theta 4)\sin(\theta 3)\sin(\theta 1)\cos(\alpha 1)\cos(\theta 2)\cos(\alpha 2) \\
& + \sin(\theta 5)\cos(\theta 4)\sin(\theta 3)\sin(\theta 1)\sin(\alpha 1)\sin(\alpha 2) \\
& - \sin(\theta 5)\sin(\theta 4)\sin(\theta 3)\cos(\alpha 3)\cos(\theta 1)\cos(\theta 2) \\
& + \sin(\theta 5)\sin(\theta 4)\sin(\theta 3)\cos(\alpha 3)\sin(\theta 1)\cos(\alpha 1)\sin(\theta 2) \\
& - \sin(\theta 5)\sin(\theta 4)\cos(\theta 3)\cos(\alpha 3)\cos(\theta 1)\sin(\theta 2)\cos(\alpha 2) \\
& - \sin(\theta 5)\sin(\theta 4)\cos(\theta 3)\cos(\alpha 3)\sin(\theta 1)\cos(\alpha 1)\cos(\theta 2)\cos(\alpha 2) \\
& + \sin(\theta 5)\sin(\theta 4)\cos(\theta 3)\cos(\alpha 3)\sin(\theta 1)\sin(\alpha 1)\sin(\alpha 2) \\
& + \sin(\theta 5)\sin(\theta 4)\sin(\alpha 3)\cos(\theta 1)\sin(\theta 2)\sin(\alpha 2) \\
& + \sin(\theta 5)\sin(\theta 4)\sin(\alpha 3)\sin(\theta 1)\cos(\alpha 1)\cos(\theta 2)\sin(\alpha 2) \\
& + \sin(\theta 5)\sin(\theta 4)\sin(\alpha 3)\sin(\theta 1)\sin(\alpha 1)\cos(\alpha 2) \\
& + \cos(\theta 5)\sin(\theta 3)\sin(\alpha 3)\cos(\theta 1)\cos(\theta 2) \\
& - \cos(\theta 5)\sin(\theta 3)\sin(\alpha 3)\sin(\theta 1)\cos(\alpha 1)\sin(\theta 2) \\
& + \cos(\theta 5)\cos(\theta 3)\sin(\alpha 3)\cos(\theta 1)\sin(\theta 2)\cos(\alpha 2) \\
& + \cos(\theta 5)\cos(\theta 3)\sin(\alpha 3)\sin(\theta 1)\cos(\alpha 1)\cos(\theta 2)\cos(\alpha 2) \\
& - \cos(\theta 5)\cos(\theta 3)\sin(\alpha 3)\sin(\theta 1)\sin(\alpha 1)\sin(\alpha 2) \\
& + \cos(\theta 5)\cos(\alpha 3)\cos(\theta 1)\sin(\theta 2)\sin(\alpha 2) \\
& + \cos(\theta 5)\cos(\alpha 3)\sin(\theta 1)\cos(\alpha 1)\cos(\theta 2)\sin(\alpha 2) \\
& + \cos(\theta 5)\cos(\alpha 3)\sin(\theta 1)\sin(\alpha 1)\cos(\alpha 2)
\end{aligned}
$$

e23:=simplify(AA[2,3]);

$$
\begin{aligned}
\mathit{e23} := {} & \sin(\theta 5)\cos(\theta 4)\cos(\theta 3)\sin(\theta 1)\cos(\theta 2) \\
& + \sin(\theta 5)\cos(\theta 4)\cos(\theta 3)\cos(\theta 1)\cos(\alpha 1)\sin(\theta 2) \\
& - \sin(\theta 5)\cos(\theta 4)\sin(\theta 3)\sin(\theta 1)\sin(\theta 2)\cos(\alpha 2) \\
& + \sin(\theta 5)\cos(\theta 4)\sin(\theta 3)\cos(\theta 1)\cos(\alpha 1)\cos(\theta 2)\cos(\alpha 2) \\
& - \sin(\theta 5)\cos(\theta 4)\sin(\theta 3)\cos(\theta 1)\sin(\alpha 1)\sin(\alpha 2) \\
& - \sin(\theta 5)\sin(\theta 4)\sin(\theta 3)\cos(\alpha 3)\sin(\theta 1)\cos(\theta 2) \\
& - \sin(\theta 5)\sin(\theta 4)\sin(\theta 3)\cos(\alpha 3)\cos(\theta 1)\cos(\alpha 1)\sin(\theta 2) \\
& - \sin(\theta 5)\sin(\theta 4)\cos(\theta 3)\cos(\alpha 3)\sin(\theta 1)\sin(\theta 2)\cos(\alpha 2) \\
& + \sin(\theta 5)\sin(\theta 4)\cos(\theta 3)\cos(\alpha 3)\cos(\theta 1)\cos(\alpha 1)\cos(\theta 2)\cos(\alpha 2) \\
& - \sin(\theta 5)\sin(\theta 4)\cos(\theta 3)\cos(\alpha 3)\cos(\theta 1)\sin(\alpha 1)\sin(\alpha 2) \\
& + \sin(\theta 5)\sin(\theta 4)\sin(\alpha 3)\sin(\theta 1)\sin(\theta 2)\sin(\alpha 2) \\
& - \sin(\theta 5)\sin(\theta 4)\sin(\alpha 3)\cos(\theta 1)\cos(\alpha 1)\cos(\theta 2)\sin(\alpha 2) \\
& - \sin(\theta 5)\sin(\theta 4)\sin(\alpha 3)\cos(\theta 1)\sin(\alpha 1)\cos(\alpha 2) \\
& + \cos(\theta 5)\sin(\theta 3)\sin(\alpha 3)\sin(\theta 1)\cos(\theta 2) \\
& + \cos(\theta 5)\sin(\theta 3)\sin(\alpha 3)\cos(\theta 1)\cos(\alpha 1)\sin(\theta 2)
\end{aligned}
$$

$$+\cos(\theta 5)\cos(\theta 3)\sin(\alpha 3)\sin(\theta 1)\sin(\theta 2)\cos(\alpha 2)$$
$$-\cos(\theta 5)\cos(\theta 3)\sin(\alpha 3)\cos(\theta 1)\cos(\alpha 1)\cos(\theta 2)\cos(\alpha 2)$$
$$+\cos(\theta 5)\cos(\theta 3)\sin(\alpha 3)\cos(\theta 1)\sin(\alpha 1)\sin(\alpha 2)$$
$$+\cos(\theta 5)\cos(\alpha 3)\sin(\theta 1)\sin(\theta 2)\sin(\alpha 2)$$
$$-\cos(\theta 5)\cos(\alpha 3)\cos(\theta 1)\cos(\alpha 1)\cos(\theta 2)\sin(\alpha 2)$$
$$-\cos(\theta 5)\cos(\alpha 3)\cos(\theta 1)\sin(\alpha 1)\cos(\alpha 2)$$

e33:=simplify(AA[3,3]);

$$\begin{aligned}
e33 :=\ &\sin(\theta 5)\cos(\theta 4)\sin(\alpha 1)\sin(\theta 2)\cos(\theta 3)\\
&+\sin(\theta 5)\cos(\theta 4)\sin(\theta 3)\sin(\alpha 1)\cos(\theta 2)\cos(\alpha 2)\\
&+\sin(\theta 5)\cos(\theta 4)\sin(\theta 3)\cos(\alpha 1)\sin(\alpha 2)\\
&-\sin(\theta 5)\sin(\theta 4)\sin(\alpha 1)\sin(\theta 2)\sin(\theta 3)\cos(\alpha 3)\\
&+\sin(\theta 5)\sin(\theta 4)\cos(\theta 3)\cos(\alpha 3)\sin(\alpha 1)\cos(\theta 2)\cos(\alpha 2)\\
&+\sin(\theta 5)\sin(\theta 4)\cos(\theta 3)\cos(\alpha 3)\cos(\alpha 1)\sin(\alpha 2)\\
&-\sin(\theta 5)\sin(\theta 4)\sin(\alpha 3)\sin(\alpha 1)\cos(\theta 2)\sin(\alpha 2)\\
&+\sin(\theta 5)\sin(\theta 4)\sin(\alpha 3)\cos(\alpha 1)\cos(\alpha 2)+\cos(\theta 5)\sin(\alpha 1)\sin(\theta 2)\sin(\theta 3)\sin(\alpha 3)\\
&-\cos(\theta 5)\cos(\theta 3)\sin(\alpha 3)\sin(\alpha 1)\cos(\theta 2)\cos(\alpha 2)\\
&-\cos(\theta 5)\cos(\theta 3)\sin(\alpha 3)\cos(\alpha 1)\sin(\alpha 2)-\cos(\theta 5)\cos(\alpha 3)\sin(\alpha 1)\cos(\theta 2)\sin(\alpha 2)\\
&+\cos(\theta 5)\cos(\alpha 3)\cos(\alpha 1)\cos(\alpha 2)
\end{aligned}$$

dx:=simplify(AA[1,4]);

$$\begin{aligned}
dx :=\ &a1\cos(\theta 1)-\sin(\theta 1)\cos(\alpha 1)\,a2\sin(\theta 2)+\cos(\theta 1)\,a2\cos(\theta 2)+\sin(\theta 1)\sin(\alpha 1)\,d2\\
&+d6\sin(\theta 5)\cos(\theta 4)\cos(\theta 3)\cos(\theta 1)\cos(\theta 2)\\
&-d6\sin(\theta 5)\cos(\theta 4)\cos(\theta 3)\sin(\theta 1)\cos(\alpha 1)\sin(\theta 2)\\
&-d6\sin(\theta 5)\cos(\theta 4)\sin(\theta 3)\cos(\theta 1)\sin(\theta 2)\cos(\alpha 2)\\
&-d6\sin(\theta 5)\cos(\theta 4)\sin(\theta 3)\sin(\theta 1)\cos(\alpha 1)\cos(\theta 2)\cos(\alpha 2)\\
&+d6\sin(\theta 5)\cos(\theta 4)\sin(\theta 3)\sin(\theta 1)\sin(\alpha 1)\sin(\alpha 2)\\
&-d6\sin(\theta 5)\sin(\theta 4)\sin(\theta 3)\cos(\alpha 3)\cos(\theta 1)\cos(\theta 2)\\
&+d6\sin(\theta 5)\sin(\theta 4)\sin(\theta 3)\cos(\alpha 3)\sin(\theta 1)\cos(\alpha 1)\sin(\theta 2)\\
&-d6\sin(\theta 5)\sin(\theta 4)\cos(\theta 3)\cos(\alpha 3)\cos(\theta 1)\sin(\theta 2)\cos(\alpha 2)\\
&-d6\sin(\theta 5)\sin(\theta 4)\cos(\theta 3)\cos(\alpha 3)\sin(\theta 1)\cos(\alpha 1)\cos(\theta 2)\cos(\alpha 2)\\
&+d6\sin(\theta 5)\sin(\theta 4)\cos(\theta 3)\cos(\alpha 3)\sin(\theta 1)\sin(\alpha 1)\sin(\alpha 2)\\
&+d6\sin(\theta 5)\sin(\theta 4)\sin(\alpha 3)\cos(\theta 1)\sin(\theta 2)\sin(\alpha 2)\\
&+d6\sin(\theta 5)\sin(\theta 4)\sin(\alpha 3)\sin(\theta 1)\cos(\alpha 1)\cos(\theta 2)\sin(\alpha 2)\\
&+d6\sin(\theta 5)\sin(\theta 4)\sin(\alpha 3)\sin(\theta 1)\sin(\alpha 1)\cos(\alpha 2)\\
&+d6\cos(\theta 5)\sin(\theta 3)\sin(\alpha 3)\cos(\theta 1)\cos(\theta 2)\\
&-d6\cos(\theta 5)\sin(\theta 3)\sin(\alpha 3)\sin(\theta 1)\cos(\alpha 1)\sin(\theta 2)\\
&+d6\cos(\theta 5)\cos(\theta 3)\sin(\alpha 3)\cos(\theta 1)\sin(\theta 2)\cos(\alpha 2)\\
&+d6\cos(\theta 5)\cos(\theta 3)\sin(\alpha 3)\sin(\theta 1)\cos(\alpha 1)\cos(\theta 2)\cos(\alpha 2)\\
&-d6\cos(\theta 5)\cos(\theta 3)\sin(\alpha 3)\sin(\theta 1)\sin(\alpha 1)\sin(\alpha 2)
\end{aligned}$$

$$+ d6 \cos(\theta 5) \cos(\alpha 3) \cos(\theta 1) \sin(\theta 2) \sin(\alpha 2)$$
$$+ d6 \cos(\theta 5) \cos(\alpha 3) \sin(\theta 1) \cos(\alpha 1) \cos(\theta 2) \sin(\alpha 2)$$
$$+ d6 \cos(\theta 5) \cos(\alpha 3) \sin(\theta 1) \sin(\alpha 1) \cos(\alpha 2) + a3 \cos(\theta 3) \cos(\theta 1) \cos(\theta 2)$$
$$- a3 \cos(\theta 3) \sin(\theta 1) \cos(\alpha 1) \sin(\theta 2) - a3 \sin(\theta 3) \cos(\theta 1) \sin(\theta 2) \cos(\alpha 2)$$
$$- a3 \sin(\theta 3) \sin(\theta 1) \cos(\alpha 1) \cos(\theta 2) \cos(\alpha 2) + a3 \sin(\theta 3) \sin(\theta 1) \sin(\alpha 1) \sin(\alpha 2)$$
$$+ d3 \cos(\theta 1) \sin(\theta 2) \sin(\alpha 2) + d3 \sin(\theta 1) \cos(\alpha 1) \cos(\theta 2) \sin(\alpha 2)$$
$$+ d3 \sin(\theta 1) \sin(\alpha 1) \cos(\alpha 2)$$

dy:=simplify(AA[2,4]);

$$dy := \sin(\theta 1)\, a2 \cos(\theta 2) + \cos(\theta 1) \cos(\alpha 1)\, a2 \sin(\theta 2) - \cos(\theta 1) \sin(\alpha 1)\, d2 + a1 \sin(\theta 1)$$
$$+ d6 \sin(\theta 5) \sin(\theta 4) \sin(\alpha 3) \sin(\theta 1) \sin(\theta 2) \sin(\alpha 2)$$
$$- d6 \sin(\theta 5) \sin(\theta 4) \sin(\alpha 3) \cos(\theta 1) \cos(\alpha 1) \cos(\theta 2) \sin(\alpha 2)$$
$$- d6 \sin(\theta 5) \sin(\theta 4) \sin(\alpha 3) \cos(\theta 1) \sin(\alpha 1) \cos(\alpha 2)$$
$$+ d6 \cos(\theta 5) \sin(\theta 3) \sin(\alpha 3) \cos(\theta 1) \cos(\alpha 1) \sin(\theta 2)$$
$$+ d6 \cos(\theta 5) \sin(\theta 3) \sin(\alpha 3) \sin(\theta 1) \cos(\theta 2)$$
$$+ d6 \cos(\theta 5) \cos(\theta 3) \sin(\alpha 3) \cos(\theta 1) \sin(\alpha 1) \sin(\alpha 2)$$
$$+ d6 \cos(\theta 5) \cos(\alpha 3) \sin(\theta 1) \sin(\theta 2) \sin(\alpha 2)$$
$$- d6 \cos(\theta 5) \cos(\alpha 3) \cos(\theta 1) \cos(\alpha 1) \cos(\theta 2) \sin(\alpha 2)$$
$$- d6 \cos(\theta 5) \cos(\alpha 3) \cos(\theta 1) \sin(\alpha 1) \cos(\alpha 2) + a3 \cos(\theta 3) \sin(\theta 1) \cos(\theta 2)$$
$$+ a3 \cos(\theta 3) \cos(\theta 1) \cos(\alpha 1) \sin(\theta 2) - a3 \sin(\theta 3) \sin(\theta 1) \sin(\theta 2) \cos(\alpha 2)$$
$$+ a3 \sin(\theta 3) \cos(\theta 1) \cos(\alpha 1) \cos(\theta 2) \cos(\alpha 2) - a3 \sin(\theta 3) \cos(\theta 1) \sin(\alpha 1) \sin(\alpha 2)$$
$$+ d3 \sin(\theta 1) \sin(\theta 2) \sin(\alpha 2) - d3 \cos(\theta 1) \cos(\alpha 1) \cos(\theta 2) \sin(\alpha 2)$$
$$- d3 \cos(\theta 1) \sin(\alpha 1) \cos(\alpha 2) + d6 \sin(\theta 5) \cos(\theta 4) \cos(\theta 3) \sin(\theta 1) \cos(\theta 2)$$
$$+ d6 \sin(\theta 5) \cos(\theta 4) \cos(\theta 3) \cos(\theta 1) \cos(\alpha 1) \sin(\theta 2)$$
$$- d6 \sin(\theta 5) \cos(\theta 4) \sin(\theta 3) \sin(\theta 1) \sin(\theta 2) \cos(\alpha 2)$$
$$+ d6 \sin(\theta 5) \cos(\theta 4) \sin(\theta 3) \cos(\theta 1) \cos(\alpha 1) \cos(\theta 2) \cos(\alpha 2)$$
$$- d6 \sin(\theta 5) \cos(\theta 4) \sin(\theta 3) \cos(\theta 1) \sin(\alpha 1) \sin(\alpha 2)$$
$$- d6 \sin(\theta 5) \sin(\theta 4) \sin(\theta 3) \cos(\alpha 3) \sin(\theta 1) \cos(\theta 2)$$
$$- d6 \sin(\theta 5) \sin(\theta 4) \sin(\theta 3) \cos(\alpha 3) \cos(\theta 1) \cos(\alpha 1) \sin(\theta 2)$$
$$- d6 \sin(\theta 5) \sin(\theta 4) \cos(\theta 3) \cos(\alpha 3) \sin(\theta 1) \sin(\theta 2) \cos(\alpha 2)$$
$$+ d6 \sin(\theta 5) \sin(\theta 4) \cos(\theta 3) \cos(\alpha 3) \cos(\theta 1) \cos(\alpha 1) \cos(\theta 2) \cos(\alpha 2)$$
$$+ d6 \cos(\theta 5) \cos(\theta 3) \sin(\alpha 3) \sin(\theta 1) \sin(\theta 2) \cos(\alpha 2)$$
$$- d6 \cos(\theta 5) \cos(\theta 3) \sin(\alpha 3) \cos(\theta 1) \cos(\alpha 1) \cos(\theta 2) \cos(\alpha 2)$$
$$- d6 \sin(\theta 5) \sin(\theta 4) \cos(\theta 3) \cos(\alpha 3) \cos(\theta 1) \sin(\alpha 1) \sin(\alpha 2)$$

dz:=simplify(AA[3,4]);

$$dz := d6 \sin(\theta 5) \cos(\theta 4) \sin(\alpha 1) \sin(\theta 2) \cos(\theta 3)$$
$$+ d6 \sin(\theta 5) \cos(\theta 4) \sin(\theta 3) \sin(\alpha 1) \cos(\theta 2) \cos(\alpha 2)$$
$$+ d6 \sin(\theta 5) \cos(\theta 4) \sin(\theta 3) \cos(\alpha 1) \sin(\alpha 2)$$
$$- d6 \sin(\theta 5) \sin(\theta 4) \sin(\alpha 1) \sin(\theta 2) \sin(\theta 3) \cos(\alpha 3)$$

$+ d6 \sin(\theta 5) \sin(\theta 4) \cos(\theta 3) \cos(\alpha 3) \sin(\alpha 1) \cos(\theta 2) \cos(\alpha 2)$
$+ d6 \sin(\theta 5) \sin(\theta 4) \cos(\theta 3) \cos(\alpha 3) \cos(\alpha 1) \sin(\alpha 2)$
$- d6 \sin(\theta 5) \sin(\theta 4) \sin(\alpha 3) \sin(\alpha 1) \cos(\theta 2) \sin(\alpha 2)$
$+ d6 \sin(\theta 5) \sin(\theta 4) \sin(\alpha 3) \cos(\alpha 1) \cos(\alpha 2)$
$+ d6 \cos(\theta 5) \sin(\alpha 1) \sin(\theta 2) \sin(\theta 3) \sin(\alpha 3)$
$- d6 \cos(\theta 5) \cos(\theta 3) \sin(\alpha 3) \sin(\alpha 1) \cos(\theta 2) \cos(\alpha 2)$
$- d6 \cos(\theta 5) \cos(\theta 3) \sin(\alpha 3) \cos(\alpha 1) \sin(\alpha 2)$
$- d6 \cos(\theta 5) \cos(\alpha 3) \sin(\alpha 1) \cos(\theta 2) \sin(\alpha 2) + d6 \cos(\theta 5) \cos(\alpha 3) \cos(\alpha 1) \cos(\alpha 2)$
$+ \sin(\alpha 1) \sin(\theta 2)\, a3 \cos(\theta 3) + a3 \sin(\theta 3) \sin(\alpha 1) \cos(\theta 2) \cos(\alpha 2)$
$+ a3 \sin(\theta 3) \cos(\alpha 1) \sin(\alpha 2) - d3 \sin(\alpha 1) \cos(\theta 2) \sin(\alpha 2) + d3 \cos(\alpha 1) \cos(\alpha 2)$
$+ \sin(\alpha 1)\, a2 \sin(\theta 2) + \cos(\alpha 1)\, d2 + d1$

This is Jacobian for RRP : RRR

file name : JRRP.mws

a1:=a1;

$$a1 := a1$$

a2:=a2;

$$a2 := a2$$

a3:=a3;

$$a3 := a3$$

a4:=0;

$$a4 := 0$$

a5:=0;

$$a5 := 0$$

a6:=0;

$$a6 := 0$$

d1:=d1;

$$d1 := d1$$

d2:=d2;

$$d2 := d2$$

d3:=d3;

$$d3 := d3$$

d4:=0;

$$d4 := 0$$

d5:=0;

$$d5 := 0$$

d6:=d6;

$$d6 := d6$$

alpha1:=alpha1;

$$\alpha 1 := \alpha 1$$

alpha2:=alpha2;

$$\alpha 2 := \alpha 2$$

alpha3:=alpha3;

$$\alpha 3 := \alpha 3$$

alpha4:=-Pi/2;

$$\alpha 4 := -\frac{1}{2}\pi$$

alpha5:=Pi/2;

$$\alpha 5 := \frac{1}{2}\pi$$

alpha6:=0;

$$\alpha 6 := 0$$

theta1:=theta1;

$$\theta 1 := \theta 1$$

theta2:=theta2;

$$\theta 2 := \theta 2$$

theta3:=theta3;

$$\theta 3 := \theta 3$$

theta4:=theta4;

$$\theta 4 := \theta 4$$

theta5:=theta5;

$$\theta 5 := \theta 5$$

theta6:=theta6;

$$\theta 6 := \theta 6$$

$A1 := matrix([[cos(theta1), -sin(theta1) * cos(alpha1), sin(theta1) * sin(alpha1), a1 * cos(theta1)], [sin(theta1), cos(theta1) * cos(alpha1), -cos(theta1) * sin(alpha1), a1 * sin(theta1)], [0, sin(alpha1), cos(alpha1), d1], [0, 0, 0, 1]]);$

$$A1 := \begin{bmatrix} \cos(\theta 1) & -\sin(\theta 1)\cos(\alpha 1) & \sin(\theta 1)\sin(\alpha 1) & a1\cos(\theta 1) \\ \sin(\theta 1) & \cos(\theta 1)\cos(\alpha 1) & -\cos(\theta 1)\sin(\alpha 1) & a1\sin(\theta 1) \\ 0 & \sin(\alpha 1) & \cos(\alpha 1) & d1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A2 := matrix([[cos(theta2), -sin(theta2) * cos(alpha2), sin(theta2) * sin(alpha2), a2 * cos(theta2)], [sin(theta2), cos(theta2) * cos(alpha2), -cos(theta2) * sin(alpha2), a2 * sin(theta2)], [0, sin(alpha2), cos(alpha2), d2], [0, 0, 0, 1]]);$

$$A2 := \begin{bmatrix} \cos(\theta 2) & -\sin(\theta 2)\cos(\alpha 2) & \sin(\theta 2)\sin(\alpha 2) & a2\cos(\theta 2) \\ \sin(\theta 2) & \cos(\theta 2)\cos(\alpha 2) & -\cos(\theta 2)\sin(\alpha 2) & a2\sin(\theta 2) \\ 0 & \sin(\alpha 2) & \cos(\alpha 2) & d2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A3 := matrix([[cos(theta3), -sin(theta3) * cos(alpha3), sin(theta3) * sin(alpha3), a3 * cos(theta3)], [sin(theta3), cos(theta3) * cos(alpha3), -cos(theta3) * sin(alpha3), a3 * sin(theta3)], [0, sin(alpha3), cos(alpha3), d3], [0, 0, 0, 1]]);$

$$A3 := \begin{bmatrix} \cos(\theta 3) & -\sin(\theta 3)\cos(\alpha 3) & \sin(\theta 3)\sin(\alpha 3) & a3\cos(\theta 3) \\ \sin(\theta 3) & \cos(\theta 3)\cos(\alpha 3) & -\cos(\theta 3)\sin(\alpha 3) & a3\sin(\theta 3) \\ 0 & \sin(\alpha 3) & \cos(\alpha 3) & d3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A4 := matrix([[cos(theta4), -sin(theta4) * cos(alpha4), sin(theta4) * sin(alpha4), a4 * cos(theta4)], [sin(theta4), cos(theta4) * cos(alpha4), -cos(theta4) * sin(alpha4), a4 * sin(theta4)], [0, sin(alpha4), cos(alpha4), d4], [0, 0, 0, 1]]);$

$$A4 := \begin{bmatrix} \cos(\theta 4) & 0 & -\sin(\theta 4) & 0 \\ \sin(\theta 4) & 0 & \cos(\theta 4) & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A5 := matrix([[cos(theta5), -sin(theta5) * cos(alpha5), sin(theta5) * sin(alpha5), a5 * cos(theta5)], [sin(theta5), cos(theta5) * cos(alpha5), -cos(theta5) * sin(alpha5), a5 * sin(theta5)], [0, sin(alpha5), cos(alpha5), d5], [0, 0, 0, 1]]);$

$$A5 := \begin{bmatrix} \cos(\theta 5) & 0 & \sin(\theta 5) & 0 \\ \sin(\theta 5) & 0 & -\cos(\theta 5) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
A6 := matrix([[cos(theta6), -sin(theta6) * cos(alpha6), sin(theta6) * sin(alpha6), a6 * cos(theta6)], [sin(theta6), cos(theta6) * cos(alpha6), -cos(theta6) * sin(alpha6), a6 * sin(theta6)], [0, sin(alpha6), cos(alpha6), d6], [0, 0, 0, 1]]);
```

$$A6 := \begin{bmatrix} \cos(\theta 6) & -\sin(\theta 6) & 0 & 0 \\ \sin(\theta 6) & \cos(\theta 6) & 0 & 0 \\ 0 & 0 & 1 & d6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
AA:=evalm(A1&*A2&*A3&*A4&*A5&*A6):
AA1:=A1:
AA2:=evalm(A1&*A2):
AA3:=evalm(A1&*A2&*A3):
AA4:=evalm(A1&*A2&*A3&*A4):
AA5:=evalm(A1&*A2&*A3&*A4&*A5):
AA6:=evalm(A1&*A2&*A3&*A4&*A5&*A6):
Z0:=vector([0,0,1]):
Z1:=vector([AA1[1,3],AA1[2,3],AA1[3,3]]):
Z2:=vector([AA2[1,3],AA2[2,3],AA2[3,3]]):
Z3:=vector([AA3[1,3],AA3[2,3],AA3[3,3]]):
Z4:=vector([AA4[1,3],AA4[2,3],AA4[3,3]]):
Z5:=vector([AA5[1,3],AA5[2,3],AA5[3,3]]):
O0:=vector([0,0,0]):
O1:=vector([AA1[1,4],AA1[2,4],AA1[3,4]]):
O2:=vector([AA2[1,4],AA2[2,4],AA2[3,4]]):
O3:=vector([AA3[1,4],AA3[2,4],AA3[3,4]]):
O4:=vector([AA4[1,4],AA4[2,4],AA4[3,4]]):
O5:=vector([AA5[1,4],AA5[2,4],AA5[3,4]]):
O6:=vector([AA6[1,4],AA6[2,4],AA6[3,4]]):
O60:=evalm(O6-O0):
O61:=evalm(O6-O1):
O62:=evalm(O6-O2):
O63:=evalm(O6-O3):
O64:=evalm(O6-O4):
O65:=evalm(O6-O5):
CP0:=crossprod(Z0,O60):
CP1:=crossprod(Z1,O61):
CP2:=crossprod(Z2,O62):
CP3:=crossprod(Z3,O63):
CP4:=crossprod(Z4,O64):
CP5:=crossprod(Z5,O65):
J:=matrix([[CP0[1], CP1[1], Z2[1], CP3[1], CP4[1], CP5[1]], [CP0[2], CP1[2], Z2[2], CP3[2], CP4[2], CP5[
J11:=J[1,1];
```

$$\begin{aligned} J11 := & -(((\%3\cos(\theta 3) + \%2\sin(\theta 3))\cos(\theta 4) \\ & + (-\%3\sin(\theta 3)\cos(\alpha 3) + \%2\cos(\theta 3)\cos(\alpha 3) + \%1\sin(\alpha 3))\sin(\theta 4))\sin(\theta 5) \\ & - (-\%3\sin(\theta 3)\sin(\alpha 3) + \%2\cos(\theta 3)\sin(\alpha 3) - \%1\cos(\alpha 3))\cos(\theta 5))\, d6 \\ & - \%3\, a3\cos(\theta 3) - \%2\, a3\sin(\theta 3) - \%1\, d3 - \sin(\theta 1)\, a2\cos(\theta 2) \\ & - \cos(\theta 1)\cos(\alpha 1)\, a2\sin(\theta 2) + \cos(\theta 1)\sin(\alpha 1)\, d2 - a1\sin(\theta 1) \end{aligned}$$

$\%1 :=$
$\sin(\theta 1)\sin(\theta 2)\sin(\alpha 2) - \cos(\theta 1)\cos(\alpha 1)\cos(\theta 2)\sin(\alpha 2) - \cos(\theta 1)\sin(\alpha 1)\cos(\alpha 2)$
$\%2 :=$
$-\sin(\theta 1)\sin(\theta 2)\cos(\alpha 2) + \cos(\theta 1)\cos(\alpha 1)\cos(\theta 2)\cos(\alpha 2) - \cos(\theta 1)\sin(\alpha 1)\sin(\alpha 2)$
$\%3 := \sin(\theta 1)\cos(\theta 2) + \cos(\theta 1)\cos(\alpha 1)\sin(\theta 2)$

J12:=J[1,2];

$$
\begin{aligned}
J12 := & -\cos(\theta 1)\sin(\alpha 1)((((\sin(\alpha 1)\sin(\theta 2)\cos(\theta 3) + \%5\sin(\theta 3))\cos(\theta 4) \\
& + (-\sin(\alpha 1)\sin(\theta 2)\sin(\theta 3)\cos(\alpha 3) + \%5\cos(\theta 3)\cos(\alpha 3) + \%4\sin(\alpha 3))\sin(\theta 4))\sin(\theta 5) \\
& - (-\sin(\alpha 1)\sin(\theta 2)\sin(\theta 3)\sin(\alpha 3) + \%5\cos(\theta 3)\sin(\alpha 3) - \%4\cos(\alpha 3))\cos(\theta 5))\,d6 \\
& + \sin(\alpha 1)\sin(\theta 2)\,a3\cos(\theta 3) + \%5\,a3\sin(\theta 3) + \%4\,d3 + \sin(\alpha 1)\,a2\sin(\theta 2) + \cos(\alpha 1)\,d2 \\
&) - \cos(\alpha 1)((((\%3\cos(\theta 3) + \%2\sin(\theta 3))\cos(\theta 4) \\
& + (-\%3\sin(\theta 3)\cos(\alpha 3) + \%2\cos(\theta 3)\cos(\alpha 3) + \%1\sin(\alpha 3))\sin(\theta 4))\sin(\theta 5) \\
& - (-\%3\sin(\theta 3)\sin(\alpha 3) + \%2\cos(\theta 3)\sin(\alpha 3) - \%1\cos(\alpha 3))\cos(\theta 5))\,d6 \\
& + \%3\,a3\cos(\theta 3) + \%2\,a3\sin(\theta 3) + \%1\,d3 + \sin(\theta 1)\,a2\cos(\theta 2) \\
& + \cos(\theta 1)\cos(\alpha 1)\,a2\sin(\theta 2) - \cos(\theta 1)\sin(\alpha 1)\,d2)
\end{aligned}
$$

$\%1 :=$
$\sin(\theta 1)\sin(\theta 2)\sin(\alpha 2) - \cos(\theta 1)\cos(\alpha 1)\cos(\theta 2)\sin(\alpha 2) - \cos(\theta 1)\sin(\alpha 1)\cos(\alpha 2)$
$\%2 :=$
$-\sin(\theta 1)\sin(\theta 2)\cos(\alpha 2) + \cos(\theta 1)\cos(\alpha 1)\cos(\theta 2)\cos(\alpha 2) - \cos(\theta 1)\sin(\alpha 1)\sin(\alpha 2)$
$\%3 := \sin(\theta 1)\cos(\theta 2) + \cos(\theta 1)\cos(\alpha 1)\sin(\theta 2)$
$\%4 := -\sin(\alpha 1)\cos(\theta 2)\sin(\alpha 2) + \cos(\alpha 1)\cos(\alpha 2)$
$\%5 := \sin(\alpha 1)\cos(\theta 2)\cos(\alpha 2) + \cos(\alpha 1)\sin(\alpha 2)$

J13:=J[1,3];

$$
\begin{aligned}
J13 := & \\
& \cos(\theta 1)\sin(\theta 2)\sin(\alpha 2) + \sin(\theta 1)\cos(\alpha 1)\cos(\theta 2)\sin(\alpha 2) + \sin(\theta 1)\sin(\alpha 1)\cos(\alpha 2)
\end{aligned}
$$

J14:=J[1,4];

$$
\begin{aligned}
J14 := & (\%3\sin(\theta 3)\sin(\alpha 3) - \%2\cos(\theta 3)\sin(\alpha 3) + \%1\cos(\alpha 3))((\\
& (\sin(\alpha 1)\sin(\theta 2)\cos(\theta 3) + \%5\sin(\theta 3))\cos(\theta 4) \\
& + (-\sin(\alpha 1)\sin(\theta 2)\sin(\theta 3)\cos(\alpha 3) + \%5\cos(\theta 3)\cos(\alpha 3) + \%4\sin(\alpha 3))\sin(\theta 4))\sin(\theta 5) \\
& - (-\sin(\alpha 1)\sin(\theta 2)\sin(\theta 3)\sin(\alpha 3) + \%5\cos(\theta 3)\sin(\alpha 3) - \%4\cos(\alpha 3))\cos(\theta 5))\,d6\, - \\
& (\sin(\alpha 1)\sin(\theta 2)\sin(\theta 3)\sin(\alpha 3) - \%5\cos(\theta 3)\sin(\alpha 3) + \%4\cos(\alpha 3))((\\
& (\%3\cos(\theta 3) + \%2\sin(\theta 3))\cos(\theta 4) \\
& + (-\%3\sin(\theta 3)\cos(\alpha 3) + \%2\cos(\theta 3)\cos(\alpha 3) + \%1\sin(\alpha 3))\sin(\theta 4))\sin(\theta 5) \\
& - (-\%3\sin(\theta 3)\sin(\alpha 3) + \%2\cos(\theta 3)\sin(\alpha 3) - \%1\cos(\alpha 3))\cos(\theta 5))\,d6
\end{aligned}
$$

$\%1 :=$
$\sin(\theta 1)\sin(\theta 2)\sin(\alpha 2) - \cos(\theta 1)\cos(\alpha 1)\cos(\theta 2)\sin(\alpha 2) - \cos(\theta 1)\sin(\alpha 1)\cos(\alpha 2)$

$$\%2 :=$$
$$-\sin(\theta 1)\sin(\theta 2)\cos(\alpha 2)+\cos(\theta 1)\cos(\alpha 1)\cos(\theta 2)\cos(\alpha 2)-\cos(\theta 1)\sin(\alpha 1)\sin(\alpha 2)$$
$$\%3 := \sin(\theta 1)\cos(\theta 2)+\cos(\theta 1)\cos(\alpha 1)\sin(\theta 2)$$
$$\%4 := -\sin(\alpha 1)\cos(\theta 2)\sin(\alpha 2)+\cos(\alpha 1)\cos(\alpha 2)$$
$$\%5 := \sin(\alpha 1)\cos(\theta 2)\cos(\alpha 2)+\cos(\alpha 1)\sin(\alpha 2)$$

J15:=J[1,5];

$$\begin{aligned}
J15 := {} & (-(\%3\cos(\theta 3)+\%2\sin(\theta 3))\sin(\theta 4) \\
& +(-\%3\sin(\theta 3)\cos(\alpha 3)+\%2\cos(\theta 3)\cos(\alpha 3)+\%1\sin(\alpha 3))\cos(\theta 4))((\\
& (\sin(\alpha 1)\sin(\theta 2)\cos(\theta 3)+\%5\sin(\theta 3))\cos(\theta 4) \\
& +(-\sin(\alpha 1)\sin(\theta 2)\sin(\theta 3)\cos(\alpha 3)+\%5\cos(\theta 3)\cos(\alpha 3)+\%4\sin(\alpha 3))\sin(\theta 4))\sin(\theta 5) \\
& -(-\sin(\alpha 1)\sin(\theta 2)\sin(\theta 3)\sin(\alpha 3)+\%5\cos(\theta 3)\sin(\alpha 3)-\%4\cos(\alpha 3))\cos(\theta 5))\,d6-(\\
& -(\sin(\alpha 1)\sin(\theta 2)\cos(\theta 3)+\%5\sin(\theta 3))\sin(\theta 4) \\
& +(-\sin(\alpha 1)\sin(\theta 2)\sin(\theta 3)\cos(\alpha 3)+\%5\cos(\theta 3)\cos(\alpha 3)+\%4\sin(\alpha 3))\cos(\theta 4))((\\
& (\%3\cos(\theta 3)+\%2\sin(\theta 3))\cos(\theta 4) \\
& +(-\%3\sin(\theta 3)\cos(\alpha 3)+\%2\cos(\theta 3)\cos(\alpha 3)+\%1\sin(\alpha 3))\sin(\theta 4))\sin(\theta 5) \\
& -(-\%3\sin(\theta 3)\sin(\alpha 3)+\%2\cos(\theta 3)\sin(\alpha 3)-\%1\cos(\alpha 3))\cos(\theta 5))\,d6
\end{aligned}$$
$$\%1 :=$$
$$\sin(\theta 1)\sin(\theta 2)\sin(\alpha 2)-\cos(\theta 1)\cos(\alpha 1)\cos(\theta 2)\sin(\alpha 2)-\cos(\theta 1)\sin(\alpha 1)\cos(\alpha 2)$$
$$\%2 :=$$
$$-\sin(\theta 1)\sin(\theta 2)\cos(\alpha 2)+\cos(\theta 1)\cos(\alpha 1)\cos(\theta 2)\cos(\alpha 2)-\cos(\theta 1)\sin(\alpha 1)\sin(\alpha 2)$$
$$\%3 := \sin(\theta 1)\cos(\theta 2)+\cos(\theta 1)\cos(\alpha 1)\sin(\theta 2)$$
$$\%4 := -\sin(\alpha 1)\cos(\theta 2)\sin(\alpha 2)+\cos(\alpha 1)\cos(\alpha 2)$$
$$\%5 := \sin(\alpha 1)\cos(\theta 2)\cos(\alpha 2)+\cos(\alpha 1)\sin(\alpha 2)$$

J16:=J[1,6];

$$J16 := 0$$

J21:=J[2,1];

$$\begin{aligned}
J21 := {} & (((\%3\cos(\theta 3)+\%2\sin(\theta 3))\cos(\theta 4) \\
& +(-\%3\sin(\theta 3)\cos(\alpha 3)+\%2\cos(\theta 3)\cos(\alpha 3)+\%1\sin(\alpha 3))\sin(\theta 4))\sin(\theta 5) \\
& -(-\%3\sin(\theta 3)\sin(\alpha 3)+\%2\cos(\theta 3)\sin(\alpha 3)-\%1\cos(\alpha 3))\cos(\theta 5))\,d6 \\
& +\%3\,a3\cos(\theta 3)+\%2\,a3\sin(\theta 3)+\%1\,d3+\cos(\theta 1)\,a2\cos(\theta 2) \\
& -\sin(\theta 1)\cos(\alpha 1)\,a2\sin(\theta 2)+\sin(\theta 1)\sin(\alpha 1)\,d2+a1\cos(\theta 1)
\end{aligned}$$
$$\%1 :=$$
$$\cos(\theta 1)\sin(\theta 2)\sin(\alpha 2)+\sin(\theta 1)\cos(\alpha 1)\cos(\theta 2)\sin(\alpha 2)+\sin(\theta 1)\sin(\alpha 1)\cos(\alpha 2)$$
$$\%2 :=$$
$$-\cos(\theta 1)\sin(\theta 2)\cos(\alpha 2)-\sin(\theta 1)\cos(\alpha 1)\cos(\theta 2)\cos(\alpha 2)+\sin(\theta 1)\sin(\alpha 1)\sin(\alpha 2)$$
$$\%3 := \cos(\theta 1)\cos(\theta 2)-\sin(\theta 1)\cos(\alpha 1)\sin(\theta 2)$$

J22:=J[2,2];

$$J22 := \cos(\alpha1)((((\%5\cos(\theta3) + \%4\sin(\theta3))\cos(\theta4)$$
$$+ (-\%5\sin(\theta3)\cos(\alpha3) + \%4\cos(\theta3)\cos(\alpha3) + \%3\sin(\alpha3))\sin(\theta4))\sin(\theta5)$$
$$- (-\%5\sin(\theta3)\sin(\alpha3) + \%4\cos(\theta3)\sin(\alpha3) - \%3\cos(\alpha3))\cos(\theta5))\,d6$$
$$+ \%5\,a3\cos(\theta3) + \%4\,a3\sin(\theta3) + \%3\,d3 + \cos(\theta1)\,a2\cos(\theta2)$$
$$- \sin(\theta1)\cos(\alpha1)\,a2\sin(\theta2) + \sin(\theta1)\sin(\alpha1)\,d2) - \sin(\theta1)\sin(\alpha1)((($$
$$(\sin(\alpha1)\sin(\theta2)\cos(\theta3) + \%2\sin(\theta3))\cos(\theta4)$$
$$+ (-\sin(\alpha1)\sin(\theta2)\sin(\theta3)\cos(\alpha3) + \%2\cos(\theta3)\cos(\alpha3) + \%1\sin(\alpha3))\sin(\theta4))\sin(\theta5)$$
$$- (-\sin(\alpha1)\sin(\theta2)\sin(\theta3)\sin(\alpha3) + \%2\cos(\theta3)\sin(\alpha3) - \%1\cos(\alpha3))\cos(\theta5))\,d6$$
$$+ \sin(\alpha1)\sin(\theta2)\,a3\cos(\theta3) + \%2\,a3\sin(\theta3) + \%1\,d3 + \sin(\alpha1)\,a2\sin(\theta2) + \cos(\alpha1)\,d2$$
$$)$$
$$\%1 := -\sin(\alpha1)\cos(\theta2)\sin(\alpha2) + \cos(\alpha1)\cos(\alpha2)$$
$$\%2 := \sin(\alpha1)\cos(\theta2)\cos(\alpha2) + \cos(\alpha1)\sin(\alpha2)$$
$$\%3 :=$$
$$\cos(\theta1)\sin(\theta2)\sin(\alpha2) + \sin(\theta1)\cos(\alpha1)\cos(\theta2)\sin(\alpha2) + \sin(\theta1)\sin(\alpha1)\cos(\alpha2)$$
$$\%4 :=$$
$$-\cos(\theta1)\sin(\theta2)\cos(\alpha2) - \sin(\theta1)\cos(\alpha1)\cos(\theta2)\cos(\alpha2) + \sin(\theta1)\sin(\alpha1)\sin(\alpha2)$$
$$\%5 := \cos(\theta1)\cos(\theta2) - \sin(\theta1)\cos(\alpha1)\sin(\theta2)$$

J23:=J[2,3];

$$J23 :=$$
$$\sin(\theta1)\sin(\theta2)\sin(\alpha2) - \cos(\theta1)\cos(\alpha1)\cos(\theta2)\sin(\alpha2) - \cos(\theta1)\sin(\alpha1)\cos(\alpha2)$$

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J65:=J[6,5];

$$J65 :=$$
$$-(\sin(\alpha1)\sin(\theta2)\cos(\theta3) + (\sin(\alpha1)\cos(\theta2)\cos(\alpha2) + \cos(\alpha1)\sin(\alpha2))\sin(\theta3))\sin(\theta4)$$
$$+ (-\sin(\alpha1)\sin(\theta2)\sin(\theta3)\cos(\alpha3)$$
$$+ (\sin(\alpha1)\cos(\theta2)\cos(\alpha2) + \cos(\alpha1)\sin(\alpha2))\cos(\theta3)\cos(\alpha3)$$
$$+ (-\sin(\alpha1)\cos(\theta2)\sin(\alpha2) + \cos(\alpha1)\cos(\alpha2))\sin(\alpha3))\cos(\theta4)$$

J66:=J[6,6];

$$
\begin{aligned}
J66 := & ((\sin(\alpha 1)\sin(\theta 2)\cos(\theta 3) + \%1\sin(\theta 3))\cos(\theta 4) + (\\
& -\sin(\alpha 1)\sin(\theta 2)\sin(\theta 3)\cos(\alpha 3) + \%1\cos(\theta 3)\cos(\alpha 3) \\
& + (-\sin(\alpha 1)\cos(\theta 2)\sin(\alpha 2) + \cos(\alpha 1)\cos(\alpha 2))\sin(\alpha 3))\sin(\theta 4))\sin(\theta 5) - (\\
& -\sin(\alpha 1)\sin(\theta 2)\sin(\theta 3)\sin(\alpha 3) + \%1\cos(\theta 3)\sin(\alpha 3) \\
& - (-\sin(\alpha 1)\cos(\theta 2)\sin(\alpha 2) + \cos(\alpha 1)\cos(\alpha 2))\cos(\alpha 3))\cos(\theta 5) \\
& \%1 := \sin(\alpha 1)\cos(\theta 2)\cos(\alpha 2) + \cos(\alpha 1)\sin(\alpha 2)
\end{aligned}
$$

Then Jacobian is converted to a C Code

```
C(J,optimized);
t1 = sin(theta1);
t2 = cos(theta2);
t4 = cos(theta1);
t5 = cos(alpha1);
t6 = t4*t5;
t7 = sin(theta2);
t9 = t1*t2+t6*t7;
t10 = cos(theta3);
t12 = t1*t7;
t13 = cos(alpha2);
t15 = t2*t13;
t17 = sin(alpha1);
t18 = t4*t17;
t19 = sin(alpha2);
t21 = -t12*t13+t6*t15-t18*t19;
t22 = sin(theta3);
t24 = t9*t10+t21*t22;
t25 = cos(theta4); t27 = t9*t22;
t28 = cos(alpha3);
t30 = t21*t10;
t33 = t2*t19;
t36 = t12*t19-t6*t33-t18*t13;
t37 = sin(alpha3);
t39 = -t27*t28+t30*t28+t36*t37;
t40 = sin(theta4);
t43 = sin(theta5);
t45 = t27*t37;
t46 = t30*t37;
t47 = t36*t28;
t49 = cos(theta5);
t51 = (t24*t25+t39*t40)*t43-(-t45+t46-t47)*t49;
t52 = t51*d6;
t54 = t9*a3*t10;
t56 = t21*a3*t22;
t57 = t36*d3;
t59 = t1*a2*t2;
t60 = a2*t7;
t61 = t6*t60;
```

```
t62 = t18*d2;
t65 = t17*t7;
t67 = t17*t2;
t70 = t67*t13+t5*t19;
t72 = t65*t10+t70*t22;
t76 = t70*t10;
t80 = -t67*t19+t5*t13;
t82 = -t65*t22*t28+t76*t28+t80*t37;
t87 = t65*t22*t37;
t88 = t76*t37;
t89 = t80*t28;
t92 = (t72*t25+t82*t40)*t43-(-t87+t88-t89)*t49;
t102 = t92*d6+t65*a3*t10+t70*a3*t22+t80*d3+t17*a2*t7+t5*d2;
t104 = t52+t54+t56+t57+t59+t61-t62;
t107 = t4*t7;
t109 = t1*t5;
t111 = t1*t17;
t113 = t107*t19+t109*t33+t111*t13;
t114 = t45-t46+t47;
t117 = t87-t88+t89;
t123 = -t24*t40+t39*t25;
t128 = -t72*t40+t82*t25;
t134 = t4*t2-t109*t7;
t139 = -t107*t13-t109*t15+t111*t19;
t141 = t134*t10+t139*t22;
t143 = t134*t22;
t145 = t139*t10;
t148 = -t143*t28+t145*t28+t113*t37;
t152 = t143*t37;
t153 = t145*t37;
t154 = t113*t28;
t157 = (t141*t25+t148*t40)*t43-(-t152+t153-t154)*t49;
t158 = t157*d6;
t160 = t134*a3*t10;
t162 = t139*a3*t22;
t163 = t113*d3;
t165 = t4*a2*t2;
t166 = t109*t60;
t167 = t111*d2;
t170 = t158+t160+t162+t163+t165-t166+t167;
t176 = t152-t153+t154;
t184 = -t141*t40+t148*t25;
J[0][0] = -t52-t54-t56-t57-t59-t61+t62-a1*t1;
J[0][1] = -t18*t102-t5*t104;
J[0][2] = t113;
J[0][3] = t114*t92*d6-t117*t51*d6;
J[0][4] = t123*t92*d6-t128*t51*d6;
J[0][5] = 0.0;
J[1][0] = t158+t160+t162+t163+t165-t166+t167+a1*t4;
J[1][1] = t5*t170-t111*t102;
```

J[1][2] = t36;
J[1][3] = t117*t157*d6-t176*t92*d6;
J[1][4] = t128*t157*d6-t184*t92*d6;
J[1][5] = 0.0;
J[2][0] = 0.0;
J[2][1] = t111*t104+t18*t170;
J[2][2] = t80;
J[2][3] = t176*t51*d6-t114*t157*d6;
J[2][4] = t184*t51*d6-t123*t157*d6;
J[2][5] = 0.0;
J[3][0] = 0.0;
J[3][1] = t111;
J[3][2] = 0.0;
J[3][3] = t176;
J[3][4] = t184;
J[3][5] = t157;
J[4][0] = 0.0;
J[4][1] = -t18;
J[4][2] = 0.0;
J[4][3] = t114;
J[4][4] = t123;
J[4][5] = t51;
J[5][0] = 1.0;
J[5][1] = t5;
J[5][2] = 0.0;
J[5][3] = t117;
J[5][4] = t128;
J[5][5] = t92;

$DJ[1,1] := -(a1 \ast cos(theta1) \ast dtheta1) - a2 \ast cos(theta1) \ast cos(theta2) \ast dtheta1 - a2 \ast cos(alpha1) \ast cos(theta1) \ast cos(theta2) \ast dtheta2 - d2 \ast dtheta1 \ast sin(alpha1) \ast sin(theta1) + a2 \ast cos(alpha1) \ast dtheta1 \ast sin(theta1) \ast sin(theta2) + a2 \ast dtheta2 \ast sin(theta1) \ast sin(theta2) - d3 \ast (cos(alpha2) \ast dtheta1 \ast sin(alpha1) \ast sin(theta1) + cos(alpha1) \ast cos(theta2) \ast dtheta1 \ast sin(alpha2) \ast sin(theta1) + cos(theta2) \ast dtheta2 \ast sin(alpha2) \ast sin(theta1) + cos(theta1) \ast dtheta1 \ast sin(alpha2) \ast sin(theta2) + cos(alpha1) \ast cos(theta1) \ast dtheta2 \ast sin(alpha2) \ast sin(theta2)) - a3 \ast cos(theta3) \ast (cos(theta1) \ast cos(theta2) \ast dtheta1 + cos(alpha1) \ast cos(theta1) \ast cos(theta2) \ast dtheta2 - cos(alpha1) \ast dtheta1 \ast sin(theta1) \ast sin(theta2) - dtheta2 \ast sin(theta1) \ast sin(theta2)) - dd3 \ast (-(cos(alpha2) \ast cos(theta1) \ast sin(alpha1)) - cos(alpha1) \ast cos(theta1) \ast cos(theta2) \ast sin(alpha2) + sin(alpha2) \ast sin(theta1) \ast sin(theta2)) - a3 \ast (-(cos(alpha1) \ast cos(alpha2) \ast cos(theta2) \ast dtheta1 \ast sin(theta1)) - cos(alpha2) \ast cos(theta2) \ast dtheta2 \ast sin(theta1) + dtheta1 \ast sin(alpha1) \ast sin(alpha2) \ast sin(theta1) - cos(alpha2) \ast cos(theta1) \ast dtheta1 \ast sin(theta2) - cos(alpha1) \ast cos(alpha2) \ast cos(theta1) \ast dtheta2 \ast sin(theta2)) \ast sin(theta3) - d6 \ast (-(cos(theta5) \ast (cos(theta3) \ast sin(alpha3) \ast (-(cos(alpha1) \ast cos(alpha2) \ast cos(theta2) \ast dtheta1 \ast sin(theta1)) - cos(alpha2) \ast cos(theta2) \ast dtheta2 \ast sin(theta1) + dtheta1 \ast sin(alpha1) \ast sin(alpha2) \ast sin(theta1) - cos(alpha2) \ast cos(theta1) \ast dtheta1 \ast sin(theta2) - cos(alpha1) \ast cos(alpha2) \ast cos(theta1) \ast dtheta2 \ast sin(theta2)) - cos(alpha3) \ast (cos(alpha2) \ast dtheta1 \ast sin(alpha1) \ast sin(theta1) + cos(alpha1) \ast cos(theta2) \ast dtheta1 \ast sin(alpha2) \ast sin(theta1) + cos(theta2) \ast dtheta2 \ast sin(alpha2) \ast sin(theta1) + cos(theta1) \ast dtheta1 \ast sin(alpha2) \ast sin(theta2) + cos(alpha1) \ast cos(theta1) \ast dtheta2 \ast sin(alpha2) \ast sin(theta2)) - sin(alpha3) \ast (cos(theta1) \ast cos(theta2) \ast dtheta1 + cos(alpha1) \ast cos(theta1) \ast cos(theta2) \ast dtheta2 - cos(alpha1) \ast dtheta1 \ast sin(theta1) \ast sin(theta2) - dtheta2 \ast sin(theta1) \ast sin(theta2)) \ast sin(theta3))) + cos(theta5) \ast dtheta5 \ast$

$(cos(theta4)*(cos(theta3)*(cos(theta2)*sin(theta1)+cos(alpha1)*cos(theta1)*sin(theta2))+$
$(cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2)-cos(theta1)*sin(alpha1)*sin(alpha2)-$
$cos(alpha2)*sin(theta1)*sin(theta2))*sin(theta3))+(cos(alpha3)*cos(theta3)*(cos(alpha1)*$
$cos(alpha2)*cos(theta1)*cos(theta2)-cos(theta1)*sin(alpha1)*sin(alpha2)-cos(alpha2)*$
$sin(theta1)*sin(theta2))+sin(alpha3)*(-(cos(alpha2)*cos(theta1)*sin(alpha1))-cos(alpha1)*$
$cos(theta1)*cos(theta2)*sin(alpha2)+sin(alpha2)*sin(theta1)*sin(theta2))-cos(alpha3)*$
$(cos(theta2)*sin(theta1)+cos(alpha1)*cos(theta1)*sin(theta2))*sin(theta3))*sin(theta4))+$
$dtheta5*(cos(theta3)*sin(alpha3)*(cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2)-$
$cos(theta1)*sin(alpha1)*sin(alpha2)-cos(alpha2)*sin(theta1)*sin(theta2))-cos(alpha3)*$
$(-(cos(alpha2)*cos(theta1)*sin(alpha1))-cos(alpha1)*cos(theta1)*cos(theta2)*sin(alpha2)+$
$sin(alpha2)*sin(theta1)*sin(theta2))-sin(alpha3)*(cos(theta2)*sin(theta1)+cos(alpha1)*$
$cos(theta1)*sin(theta2))*sin(theta3))*sin(theta5)+(cos(theta4)*dtheta4*(cos(alpha3)*$
$cos(theta3)*(cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2)-cos(theta1)*sin(alpha1)*$
$sin(alpha2)-cos(alpha2)*sin(theta1)*sin(theta2))+sin(alpha3)*(-(cos(alpha2)*cos(theta1)*$
$sin(alpha1))-cos(alpha1)*cos(theta1)*cos(theta2)*sin(alpha2)+sin(alpha2)*sin(theta1)*$
$sin(theta2))-cos(alpha3)*(cos(theta2)*sin(theta1)+cos(alpha1)*cos(theta1)*sin(theta2))*$
$sin(theta3))+cos(theta4)*(cos(theta3)*(cos(theta1)*cos(theta2)*dtheta1+cos(alpha1)*$
$cos(theta1)*cos(theta2)*dtheta2-cos(alpha1)*dtheta1*sin(theta1)*sin(theta2)-dtheta2*$
$sin(theta1)*sin(theta2))+(-(cos(alpha1)*cos(alpha2)*cos(theta2)*dtheta1*sin(theta1))-$
$cos(alpha2)*cos(theta2)*dtheta2*sin(theta1)+dtheta1*sin(alpha1)*sin(alpha2)*sin(theta1)-$
$cos(alpha2)*cos(theta1)*dtheta1*sin(theta2)-cos(alpha1)*cos(alpha2)*cos(theta1)*dtheta2*$
$sin(theta2))*sin(theta3))-dtheta4*(cos(theta3)*(cos(theta2)*sin(theta1)+cos(alpha1)*$
$cos(theta1)*sin(theta2))+(cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2)-cos(theta1)*$
$sin(alpha1)*sin(alpha2)-cos(alpha2)*sin(theta1)*sin(theta2))*sin(theta3))*sin(theta4)+$
$(cos(alpha3)*cos(theta3)*(-(cos(alpha1)*cos(alpha2)*cos(theta2)*dtheta1*sin(theta1))-$
$cos(alpha2)*cos(theta2)*dtheta2*sin(theta1)+dtheta1*sin(alpha1)*sin(alpha2)*sin(theta1)-$
$cos(alpha2)*cos(theta1)*dtheta1*sin(theta2)-cos(alpha1)*cos(alpha2)*cos(theta1)*dtheta2*$
$sin(theta2))+sin(alpha3)*(cos(alpha2)*dtheta1*sin(alpha1)*sin(theta1)+cos(alpha1)*$
$cos(theta2)*dtheta1*sin(alpha2)*sin(theta1)+cos(theta2)*dtheta2*sin(alpha2)*sin(theta1)+$
$cos(theta1)*dtheta1*sin(alpha2)*sin(theta2)+cos(alpha1)*cos(theta1)*dtheta2*sin(alpha2)*$
$sin(theta2))-cos(alpha3)*(cos(theta1)*cos(theta2)*dtheta1+cos(alpha1)*cos(theta1)*$
$cos(theta2)*dtheta2-cos(alpha1)*dtheta1*sin(theta1)*sin(theta2)-dtheta2*sin(theta1)*$
$sin(theta2))*sin(theta3))*sin(theta4))*sin(theta5));$

$DJ[1,2] := -(a1*cos(theta1)*dtheta1)-a2*cos(theta1)*cos(theta2)*dtheta1-a2*$
$cos(alpha1)*cos(theta1)*cos(theta2)*dtheta2-d2*dtheta1*sin(alpha1)*sin(theta1)+a2*$
$cos(alpha1)*dtheta1*sin(theta1)*sin(theta2)+a2*dtheta2*sin(theta1)*sin(theta2)-d3*$
$(cos(alpha2)*dtheta1*sin(alpha1)*sin(theta1)+cos(alpha1)*cos(theta2)*dtheta1*sin(alpha2)*$
$sin(theta1)+cos(theta2)*dtheta2*sin(alpha2)*sin(theta1)+cos(theta1)*dtheta1*sin(alpha2)*$
$sin(theta2)+cos(alpha1)*cos(theta1)*dtheta2*sin(alpha2)*sin(theta2))-a3*cos(theta3)*$
$(cos(theta1)*cos(theta2)*dtheta1+cos(alpha1)*cos(theta1)*cos(theta2)*dtheta2-cos(alpha1)*$
$dtheta1*sin(theta1)*sin(theta2)-dtheta2*sin(theta1)*sin(theta2))-dd3*(-(cos(alpha2)*$
$cos(theta1)*sin(alpha1))-cos(alpha1)*cos(theta1)*cos(theta2)*sin(alpha2)+sin(alpha2)*$
$sin(theta1)*sin(theta2))-a3*(-(cos(alpha1)*cos(alpha2)*cos(theta2)*dtheta1*sin(theta1))-$
$cos(alpha2)*cos(theta2)*dtheta2*sin(theta1)+dtheta1*sin(alpha1)*sin(alpha2)*sin(theta1)-$
$cos(alpha2)*cos(theta1)*dtheta1*sin(theta2)-cos(alpha1)*cos(alpha2)*cos(theta1)*dtheta2*$
$sin(theta2))*sin(theta3)-d6*(-(cos(theta5)*(cos(theta3)*sin(alpha3)*(-(cos(alpha1)*$
$cos(alpha2)*cos(theta2)*dtheta1*sin(theta1))-cos(alpha2)*cos(theta2)*dtheta2*sin(theta1)+$
$dtheta1*sin(alpha1)*sin(alpha2)*sin(theta1)-cos(alpha2)*cos(theta1)*dtheta1*sin(theta2)-$
$cos(alpha1)*cos(alpha2)*cos(theta1)*dtheta2*sin(theta2))-cos(alpha3)*(cos(alpha2)*$
$dtheta1*sin(alpha1)*sin(theta1)+cos(alpha1)*cos(theta2)*dtheta1*sin(alpha2)*sin(theta1)+$

cos(theta2)*dtheta2*sin(alpha2)*sin(theta1)+cos(theta1)*dtheta1*sin(alpha2)*sin(theta2)+cos(alpha1)*cos(theta1)*dtheta2*sin(alpha2)*sin(theta2))-sin(alpha3)*(cos(theta1)*cos(theta2)*dtheta1+cos(alpha1)*cos(theta1)*cos(theta2)*dtheta2-cos(alpha1)*dtheta1*sin(theta1)*sin(theta2)-dtheta2*sin(theta1)*sin(theta2))*sin(theta3)))+cos(theta5)*dtheta5*(cos(theta4)*(cos(theta3)*(cos(theta2)*sin(theta1)+cos(alpha1)*cos(theta1)*sin(theta2))+(cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2)-cos(theta1)*sin(alpha1)*sin(alpha2)-cos(alpha2)*sin(theta1)*sin(theta2))*sin(theta3))+(cos(alpha3)*cos(theta3)*(cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2)-cos(theta1)*sin(alpha1)*sin(alpha2)-cos(alpha2)*sin(theta1)*sin(theta2))+sin(alpha3)*(-(cos(alpha2)*cos(theta1)*sin(alpha1))-cos(alpha1)*cos(theta1)*cos(theta2)*sin(alpha2)+sin(alpha2)*sin(theta1)*sin(theta2))-cos(alpha3)*(cos(theta2)*sin(theta1)+cos(alpha1)*cos(theta1)*sin(theta2))*sin(theta3))*sin(theta4))+dtheta5*(cos(theta3)*sin(alpha3)*(cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2)-cos(theta1)*sin(alpha1)*sin(alpha2)-cos(alpha2)*sin(theta1)*sin(theta2))-cos(alpha3)*(-(cos(alpha2)*cos(theta1)*sin(alpha1))-cos(alpha1)*cos(theta1)*cos(theta2)*sin(alpha2)+sin(alpha2)*sin(theta1)*sin(theta2))-sin(alpha3)*(cos(theta2)*sin(theta1)+cos(alpha1)*cos(theta1)*sin(theta2))*sin(theta3))*sin(theta5)+(cos(theta4)*dtheta4*(cos(alpha3)*cos(theta3)*(cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2)-cos(theta1)*sin(alpha1)*sin(alpha2)-cos(alpha2)*sin(theta1)*sin(theta2))+sin(alpha3)*(-(cos(alpha2)*cos(theta1)*sin(alpha1))-cos(alpha1)*cos(theta1)*cos(theta2)*sin(alpha2)+sin(alpha2)*sin(theta1)*sin(theta2))-cos(alpha3)*(cos(theta2)*sin(theta1)+cos(alpha1)*cos(theta1)*sin(theta2))*sin(theta3))+cos(theta4)*(cos(theta3)*(cos(theta1)*cos(theta2)*dtheta1+cos(alpha1)*cos(theta1)*cos(theta2)*dtheta2-cos(alpha1)*dtheta1*sin(theta1)*sin(theta2)-dtheta2*sin(theta1)*sin(theta2))+(-(cos(alpha1)*cos(alpha2)*cos(theta2)*dtheta1*sin(theta1))-cos(alpha2)*cos(theta2)*dtheta2*sin(theta1)+dtheta1*sin(alpha1)*sin(alpha2)*sin(theta1)-cos(alpha2)*cos(theta1)*dtheta1*sin(theta2)-cos(alpha1)*cos(alpha2)*cos(theta1)*dtheta2*sin(theta2))*sin(theta3))-dtheta4*(cos(theta3)*(cos(theta2)*sin(theta1)+cos(alpha1)*cos(theta1)*sin(theta2))+(cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2)-cos(theta1)*sin(alpha1)*sin(alpha2)-cos(alpha2)*sin(theta1)*sin(theta2))*sin(theta3))*sin(theta4)+(cos(alpha3)*cos(theta3)*(-(cos(alpha1)*cos(alpha2)*cos(theta2)*dtheta1*sin(theta1))-cos(alpha2)*cos(theta2)*dtheta2*sin(theta1)+dtheta1*sin(alpha1)*sin(alpha2)*sin(theta1)-cos(alpha2)*cos(theta1)*dtheta1*sin(theta2)-cos(alpha1)*cos(alpha2)*cos(theta1)*dtheta2*sin(theta2))+sin(alpha3)*(cos(alpha2)*dtheta1*sin(alpha1)*sin(theta1)+cos(alpha1)*cos(theta2)*dtheta1*sin(alpha2)*sin(theta1)+cos(theta2)*dtheta2*sin(alpha2)*sin(theta1)+cos(theta1)*dtheta1*sin(alpha2)*sin(theta2)+cos(alpha1)*cos(theta1)*dtheta2*sin(alpha2)*sin(theta2))-cos(alpha3)*(cos(theta1)*cos(theta2)*dtheta1+cos(alpha1)*cos(theta1)*cos(theta2)*dtheta2-cos(alpha1)*dtheta1*sin(theta1)*sin(theta2)-dtheta2*sin(theta1)*sin(theta2))*sin(theta3))*sin(theta4))*sin(theta5));

DJ[1,3] := -(a1*cos(theta1)*dtheta1)-a2*cos(theta1)*cos(theta2)*dtheta1-a2*cos(alpha1)*cos(theta1)*cos(theta2)*dtheta2-d2*dtheta1*sin(alpha1)*sin(theta1)+a2*cos(alpha1)*dtheta1*sin(theta1)*sin(theta2)+a2*dtheta2*sin(theta1)*sin(theta2)-d3*(cos(alpha2)*dtheta1*sin(alpha1)*sin(theta1)+cos(alpha1)*cos(theta2)*dtheta1*sin(alpha2)*sin(theta1)+cos(theta2)*dtheta2*sin(alpha2)*sin(theta1)+cos(theta1)*dtheta1*sin(alpha2)*sin(theta2)+cos(alpha1)*cos(theta1)*dtheta2*sin(alpha2)*sin(theta2))-a3*cos(theta3)*(cos(theta1)*cos(theta2)*dtheta1+cos(alpha1)*cos(theta1)*cos(theta2)*dtheta2-cos(alpha1)*dtheta1*sin(theta1)*sin(theta2)-dtheta2*sin(theta1)*sin(theta2))-dd3*(-(cos(alpha2)*cos(theta1)*sin(alpha1))-cos(alpha1)*cos(theta1)*cos(theta2)*sin(alpha2)+sin(alpha2)*sin(theta1)*sin(theta2))-a3*(-(cos(alpha1)*cos(alpha2)*cos(theta2)*dtheta1*sin(theta1))-cos(alpha2)*cos(theta2)*dtheta2*sin(theta1)+dtheta1*sin(alpha1)*sin(alpha2)*sin(theta1)-cos(alpha2)*cos(theta1)*dtheta1*sin(theta2)-cos(alpha1)*cos(alpha2)*cos(theta1)*dtheta2*sin(theta2))*sin(theta3)-d6*(-(cos(theta5)*(cos(theta3)*sin(alpha3)*(-(cos(alpha1)*

$cos(alpha2)*cos(theta2)*dtheta1*sin(theta1)) - cos(alpha2)*cos(theta2)*dtheta2*sin(theta1) +$
$dtheta1*sin(alpha1)*sin(alpha2)*sin(theta1) - cos(alpha2)*cos(theta1)*dtheta1*sin(theta2) -$
$cos(alpha1)*cos(alpha2)*cos(theta1)*dtheta2*sin(theta2)) - cos(alpha3)*(cos(alpha2)*$
$dtheta1*sin(alpha1)*sin(theta1) + cos(alpha1)*cos(theta2)*dtheta1*sin(alpha2)*sin(theta1) +$
$cos(theta2)*dtheta2*sin(alpha2)*sin(theta1) + cos(theta1)*dtheta1*sin(alpha2)*sin(theta2) +$
$cos(alpha1)*cos(theta1)*dtheta2*sin(alpha2)*sin(theta2)) - sin(alpha3)*(cos(theta1)*$
$cos(theta2)*dtheta1 + cos(alpha1)*cos(theta1)*cos(theta2)*dtheta2 - cos(alpha1)*dtheta1*$
$sin(theta1)*sin(theta2) - dtheta2*sin(theta1)*sin(theta2))*sin(theta3))) + cos(theta5)*dtheta5*$
$(cos(theta4)*(cos(theta3)*(cos(theta2)*sin(theta1) + cos(alpha1)*cos(theta1)*sin(theta2)) +$
$(cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2) - cos(theta1)*sin(alpha1)*sin(alpha2) -$
$cos(alpha2)*sin(theta1)*sin(theta2))*sin(theta3)) + (cos(alpha3)*cos(theta3)*(cos(alpha1)*$
$cos(alpha2)*cos(theta1)*cos(theta2) - cos(theta1)*sin(alpha1)*sin(alpha2) - cos(alpha2)*$
$sin(theta1)*sin(theta2)) + sin(alpha3)*(-(cos(alpha2)*cos(theta1)*sin(alpha1)) - cos(alpha1)*$
$cos(theta1)*cos(theta2)*sin(alpha2) + sin(alpha2)*sin(theta1)*sin(theta2)) - cos(alpha3)*$
$(cos(theta2)*sin(theta1) + cos(alpha1)*cos(theta1)*sin(theta2))*sin(theta3))*sin(theta4)) +$
$dtheta5*(cos(theta3)*sin(alpha3)*(cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2) -$
$cos(theta1)*sin(alpha1)*sin(alpha2) - cos(alpha2)*sin(theta1)*sin(theta2)) - cos(alpha3)*$
$(-(cos(alpha2)*cos(theta1)*sin(alpha1)) - cos(alpha1)*cos(theta1)*cos(theta2)*sin(alpha2) +$
$sin(alpha2)*sin(theta1)*sin(theta2)) - sin(alpha3)*(cos(theta2)*sin(theta1) + cos(alpha1)*$
$cos(theta1)*sin(theta2))*sin(theta3))*sin(theta5) + (cos(theta4)*dtheta4*(cos(alpha3)*$
$cos(theta3)*(cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2) - cos(theta1)*sin(alpha1)*$
$sin(alpha2) - cos(alpha2)*sin(theta1)*sin(theta2)) + sin(alpha3)*(-(cos(alpha2)*cos(theta1)*$
$sin(alpha1)) - cos(alpha1)*cos(theta1)*cos(theta2)*sin(alpha2) + sin(alpha2)*sin(theta1)*$
$sin(theta2)) - cos(alpha3)*(cos(theta2)*sin(theta1) + cos(alpha1)*cos(theta1)*sin(theta2))*$
$sin(theta3)) + cos(theta4)*(cos(theta3)*(cos(theta1)*cos(theta2)*dtheta1 + cos(alpha1)*$
$cos(theta1)*cos(theta2)*dtheta2 - cos(alpha1)*dtheta1*sin(theta1)*sin(theta2) - dtheta2*$
$sin(theta1)*sin(theta2)) + (-(cos(alpha1)*cos(alpha2)*cos(theta2)*dtheta1*sin(theta1)) -$
$cos(alpha2)*cos(theta2)*dtheta2*sin(theta1) + dtheta1*sin(alpha1)*sin(alpha2)*sin(theta1) -$
$cos(alpha2)*cos(theta1)*dtheta1*sin(theta2) - cos(alpha1)*cos(alpha2)*cos(theta1)*dtheta2*$
$sin(theta2))*sin(theta3)) - dtheta4*(cos(theta3)*(cos(theta2)*sin(theta1) + cos(alpha1)*$
$cos(theta1)*sin(theta2)) + (cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2) - cos(theta1)*$
$sin(alpha1)*sin(alpha2) - cos(alpha2)*sin(theta1)*sin(theta2))*sin(theta3))*sin(theta4) +$
$(cos(alpha3)*cos(theta3)*(-(cos(alpha1)*cos(alpha2)*cos(theta2)*dtheta1*sin(theta1)) -$
$cos(alpha2)*cos(theta2)*dtheta2*sin(theta1) + dtheta1*sin(alpha1)*sin(alpha2)*sin(theta1) -$
$cos(alpha2)*cos(theta1)*dtheta1*sin(theta2) - cos(alpha1)*cos(alpha2)*cos(theta1)*dtheta2*$
$sin(theta2)) + sin(alpha3)*(cos(alpha2)*dtheta1*sin(alpha1)*sin(theta1) + cos(alpha1)*$
$cos(theta2)*dtheta1*sin(alpha2)*sin(theta1) + cos(theta2)*dtheta2*sin(alpha2)*sin(theta1) +$
$cos(theta1)*dtheta1*sin(alpha2)*sin(theta2) + cos(alpha1)*cos(theta1)*dtheta2*sin(alpha2)*$
$sin(theta2)) - cos(alpha3)*(cos(theta1)*cos(theta2)*dtheta1 + cos(alpha1)*cos(theta1)*$
$cos(theta2)*dtheta2 - cos(alpha1)*dtheta1*sin(theta1)*sin(theta2) - dtheta2*sin(theta1)*$
$sin(theta2))*sin(theta3))*sin(theta4))*sin(theta5));$

$DJ[1,4] := -(a1*cos(theta1)*dtheta1) - a2*cos(theta1)*cos(theta2)*dtheta1 - a2*$
$cos(alpha1)*cos(theta1)*cos(theta2)*dtheta2 - d2*dtheta1*sin(alpha1)*sin(theta1) + a2*$
$cos(alpha1)*dtheta1*sin(theta1)*sin(theta2) + a2*dtheta2*sin(theta1)*sin(theta2) - d3*$
$(cos(alpha2)*dtheta1*sin(alpha1)*sin(theta1) + cos(alpha1)*cos(theta2)*dtheta1*sin(alpha2)*$
$sin(theta1) + cos(theta2)*dtheta2*sin(alpha2)*sin(theta1) + cos(theta1)*dtheta1*sin(alpha2)*$
$sin(theta2) + cos(alpha1)*cos(theta1)*dtheta2*sin(alpha2)*sin(theta2)) - a3*cos(theta3)*$
$(cos(theta1)*cos(theta2)*dtheta1 + cos(alpha1)*cos(theta1)*cos(theta2)*dtheta2 - cos(alpha1)*$
$dtheta1*sin(theta1)*sin(theta2) - dtheta2*sin(theta1)*sin(theta2)) - dd3*(-(cos(alpha2)*$
$cos(theta1)*sin(alpha1)) - cos(alpha1)*cos(theta1)*cos(theta2)*sin(alpha2) + sin(alpha2)*$

$$
sin(theta1)*sin(theta2)) - a3*(-(cos(alpha1)*cos(alpha2)*cos(theta2)*dtheta1*sin(theta1)) -
cos(alpha2)*cos(theta2)*dtheta2*sin(theta1) + dtheta1*sin(alpha1)*sin(alpha2)*sin(theta1) -
cos(alpha2)*cos(theta1)*dtheta1*sin(theta2) - cos(alpha1)*cos(alpha2)*cos(theta1)*dtheta2*
sin(theta2))*sin(theta3) - d6*(-(cos(theta5)*(cos(theta3)*sin(alpha3)*(-(cos(alpha1)*
cos(alpha2)*cos(theta2)*dtheta1*sin(theta1)) - cos(alpha2)*cos(theta2)*dtheta2*sin(theta1) +
dtheta1*sin(alpha1)*sin(alpha2)*sin(theta1) - cos(alpha2)*cos(theta1)*dtheta1*sin(theta2) -
cos(alpha1)*cos(alpha2)*cos(theta1)*dtheta2*sin(theta2)) - cos(alpha3)*(cos(alpha2)*
dtheta1*sin(alpha1)*sin(theta1) + cos(alpha1)*cos(theta2)*dtheta1*sin(alpha2)*sin(theta1) +
cos(theta2)*dtheta2*sin(alpha2)*sin(theta1) + cos(theta1)*dtheta1*sin(alpha2)*sin(theta2) +
cos(alpha1)*cos(theta1)*dtheta2*sin(alpha2)*sin(theta2)) - (alpha3)*(cos(theta1)*cos(theta2)*
dtheta1 + cos(alpha1)*cos(theta1)*cos(theta2)*dtheta2 - cos(alpha1)*dtheta1*sin(theta1)*
sin(theta2) - dtheta2*sin(theta1)*sin(theta2))*sin(theta3))) + cos(theta5)*dtheta5*(cos(theta4)*
(cos(theta3)*(cos(theta2)*sin(theta1) + cos(alpha1)*cos(theta1)*sin(theta2)) + (cos(alpha1)*
cos(alpha2)*cos(theta1)*cos(theta2) - cos(theta1)*sin(alpha1)*sin(alpha2) - cos(alpha2)*
sin(theta1)*sin(theta2))*sin(theta3)) + (cos(alpha3)*cos(theta3)*(cos(alpha1)*cos(alpha2)*
cos(theta1)*cos(theta2) - cos(theta1)*sin(alpha1)*sin(alpha2) - cos(alpha2)*sin(theta1)*
sin(theta2)) + sin(alpha3)*(-(cos(alpha2)*cos(theta1)*sin(alpha1)) - cos(alpha1)*cos(theta1)*
cos(theta2)*sin(alpha2) + sin(alpha2)*sin(theta1)*sin(theta2)) - cos(alpha3)*(cos(theta2)*
sin(theta1) + cos(alpha1)*cos(theta1)*sin(theta2))*sin(theta3))*sin(theta4)) + dtheta5*
(cos(theta3)*sin(alpha3)*(cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2) - cos(theta1)*
sin(alpha1)*sin(alpha2) - cos(alpha2)*sin(theta1)*sin(theta2)) - cos(alpha3)*(-(cos(alpha2)*
cos(theta1)*sin(alpha1)) - cos(alpha1)*cos(theta1)*cos(theta2)*sin(alpha2) + sin(alpha2)*
sin(theta1)*sin(theta2)) - sin(alpha3)*(cos(theta2)*sin(theta1) + cos(alpha1)*cos(theta1)*
sin(theta2))*sin(theta3))*sin(theta5) + (cos(theta4)*dtheta4*(cos(alpha3)*cos(theta3)*
(cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2) - cos(theta1)*sin(alpha1)*sin(alpha2) -
cos(alpha2)*sin(theta1)*sin(theta2)) + sin(alpha3)*(-(cos(alpha2)*cos(theta1)*sin(alpha1)) -
cos(alpha1)*cos(theta1)*cos(theta2)*sin(alpha2) + sin(alpha2)*sin(theta1)*sin(theta2)) -
cos(alpha3)*(cos(theta2)*sin(theta1) + cos(alpha1)*cos(theta1)*sin(theta2))*sin(theta3)) +
cos(theta4)*(cos(theta3)*(cos(theta1)*cos(theta2)*dtheta1 + cos(alpha1)*cos(theta1)*
cos(theta2)*dtheta2 - cos(alpha1)*dtheta1*sin(theta1)*sin(theta2) - dtheta2*sin(theta1)*
sin(theta2)) + (-(cos(alpha1)*cos(alpha2)*cos(theta2)*dtheta1*sin(theta1)) - cos(alpha2)*
cos(theta2)*dtheta2*sin(theta1) + dtheta1*sin(alpha1)*sin(alpha2)*sin(theta1) - cos(alpha2)*
cos(theta1)*dtheta1*sin(theta2) - cos(alpha1)*cos(alpha2)*cos(theta1)*dtheta2*sin(theta2))*
sin(theta3)) - dtheta4*(cos(theta3)*(cos(theta2)*sin(theta1) + cos(alpha1)*cos(theta1)*
sin(theta2)) + (cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2) - cos(theta1)*sin(alpha1)*
sin(alpha2) - cos(alpha2)*sin(theta1)*sin(theta2))*sin(theta3))*sin(theta4) + (cos(alpha3)*
cos(theta3)*(-(cos(alpha1)*cos(alpha2)*cos(theta2)*dtheta1*sin(theta1)) - cos(alpha2)*
cos(theta2)*dtheta2*sin(theta1) + dtheta1*sin(alpha1)*sin(alpha2)*sin(theta1) - cos(alpha2)*
cos(theta1)*dtheta1*sin(theta2) - cos(alpha1)*cos(alpha2)*cos(theta1)*dtheta2*sin(theta2)) +
sin(alpha3)*(cos(alpha2)*dtheta1*sin(alpha1)*sin(theta1) + cos(alpha1)*cos(theta2)*dtheta1*
sin(alpha2)*sin(theta1) + cos(theta2)*dtheta2*sin(alpha2)*sin(theta1) + cos(theta1)*dtheta1*
sin(alpha2)*sin(theta2) + cos(alpha1)*cos(theta1)*dtheta2*sin(alpha2)*sin(theta2)) -
cos(alpha3)*(cos(theta1)*cos(theta2)*dtheta1 + cos(alpha1)*cos(theta1)*cos(theta2)*dtheta2 -
cos(alpha1)*dtheta1*sin(theta1)*sin(theta2) - dtheta2*sin(theta1)*sin(theta2))*sin(theta3))*
sin(theta4))*sin(theta5));
$$

$$
DJ[1,5] := -(a1*cos(theta1)*dtheta1) - a2*cos(theta1)*cos(theta2)*dtheta1 - a2*
cos(alpha1)*cos(theta1)*cos(theta2)*dtheta2 - d2*dtheta1*sin(alpha1)*sin(theta1) + a2*
cos(alpha1)*dtheta1*sin(theta1)*sin(theta2) + a2*dtheta2*sin(theta1)*sin(theta2) - d3*
(cos(alpha2)*dtheta1*sin(alpha1)*sin(theta1) + cos(alpha1)*cos(theta2)*dtheta1*sin(alpha2)*
sin(theta1) + cos(theta2)*dtheta2*sin(alpha2)*sin(theta1) + cos(theta1)*dtheta1*sin(alpha2)*
$$

$sin(theta2) + cos(alpha1) * cos(theta1) * dtheta2 * sin(alpha2) * sin(theta2)) - a3 * cos(theta3) * (cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta2 - cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) - dtheta2 * sin(theta1) * sin(theta2)) - dd3 * (-(cos(alpha2) * cos(theta1) * sin(alpha1)) - cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) - a3 * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 * sin(theta2)) * sin(theta3) - d6 * (-(cos(theta5) * (cos(theta3) * sin(alpha3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 * sin(theta2)) - cos(alpha3) * (cos(alpha2) * dtheta1 * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * dtheta1 * sin(alpha2) * sin(theta1) + cos(theta2) * dtheta2 * sin(alpha2) * sin(theta1) + cos(theta1) * dtheta1 * sin(alpha2) * sin(theta2) + cos(alpha1) * cos(theta1) * dtheta2 * sin(alpha2) * sin(theta2)) - sin(alpha3) * (cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta2 - cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) - dtheta2 * sin(theta1) * sin(theta2)) * sin(theta3))) + cos(theta5) * dtheta5 * (cos(theta4) * (cos(theta3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) + (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) * sin(theta3)) + (cos(alpha3) * cos(theta3) * (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) + sin(alpha3) * (-(cos(alpha2) * cos(theta1) * sin(alpha1)) - cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) - cos(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) + dtheta5 * (cos(theta3) * sin(alpha3) * (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) - cos(alpha3) * (-(cos(alpha2) * cos(theta1) * sin(alpha1)) - cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) - sin(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta5) + (cos(theta4) * dtheta4 * (cos(alpha3) * cos(theta3) * (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) + sin(alpha3) * (-(cos(alpha2) * cos(theta1) * sin(alpha1)) - cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) - cos(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) * sin(theta3)) + cos(theta4) * (cos(theta3) * (cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta2 - cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) - dtheta2 * sin(theta1) * sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 * sin(theta2)) * sin(theta3)) - dtheta4 * (cos(theta3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) + (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4) + (cos(alpha3) * cos(theta3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 * sin(theta2)) + sin(alpha3) * (cos(alpha2) * dtheta1 * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * dtheta1 * sin(alpha2) * sin(theta1) + cos(theta2) * dtheta2 * sin(alpha2) * sin(theta1) + cos(theta1) * dtheta1 * sin(alpha2) * sin(theta2) + cos(alpha1) * cos(theta1) * dtheta2 * sin(alpha2) * sin(theta2)) - cos(alpha3) * (cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta2 - cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) - dtheta2 * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) * sin(theta5));$

$DJ[1, 6] := -(a1 * cos(theta1) * dtheta1) - a2 * cos(theta1) * cos(theta2) * dtheta1 - a2 *$

$cos(alpha1) * cos(theta1) * cos(theta2) * dtheta2 - d2 * dtheta1 * sin(alpha1) * sin(theta1) + a2 *$
$cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) + a2 * dtheta2 * sin(theta1) * sin(theta2) - d3 *$
$(cos(alpha2) * dtheta1 * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * dtheta1 * sin(alpha2) *$
$sin(theta1) + cos(theta2) * dtheta2 * sin(alpha2) * sin(theta1) + cos(theta1) * dtheta1 * sin(alpha2) *$
$sin(theta2) + cos(alpha1) * cos(theta1) * dtheta2 * sin(alpha2) * sin(theta2)) - a3 * cos(theta3) *$
$(cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta2 - cos(alpha1) *$
$dtheta1 * sin(theta1) * sin(theta2) - dtheta2 * sin(theta1) * sin(theta2)) - dd3 * (-(cos(alpha2) *$
$cos(theta1) * sin(alpha1)) - cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) *$
$sin(theta1) * sin(theta2)) - a3 * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) -$
$cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) -$
$cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 *$
$sin(theta2)) * sin(theta3) - d6 * (-(cos(theta5) * (cos(theta3) * sin(alpha3) * (-(cos(alpha1) *$
$cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) +$
$dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) -$
$cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 * sin(theta2)) - cos(alpha3) * (cos(alpha2) *$
$dtheta1 * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * dtheta1 * sin(alpha2) * sin(theta1) +$
$cos(theta2) * dtheta2 * sin(alpha2) * sin(theta1) + cos(theta1) * dtheta1 * sin(alpha2) * sin(theta2) +$
$cos(alpha1) * cos(theta1) * dtheta2 * sin(alpha2) * sin(theta2)) - sin(alpha3) * (cos(theta1) *$
$cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta2 - cos(alpha1) * dtheta1 *$
$sin(theta1) * sin(theta2) - dtheta2 * sin(theta1) * sin(theta2)) * sin(theta3))) + cos(theta5) * dtheta5 *$
$(cos(theta4) * (cos(theta3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) +$
$(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) -$
$cos(alpha2) * sin(theta1) * sin(theta2)) * sin(theta3)) + (cos(alpha3) * cos(theta3) * (cos(alpha1) *$
$cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) *$
$sin(theta1) * sin(theta2)) + sin(alpha3) * (-(cos(alpha2) * cos(theta1) * sin(alpha1)) - cos(alpha1) *$
$cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) - cos(alpha3) *$
$(cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) +$
$dtheta5 * (cos(theta3) * sin(alpha3) * (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) -$
$cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) - cos(alpha3) *$
$(-(cos(alpha2) * cos(theta1) * sin(alpha1)) - cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) +$
$sin(alpha2) * sin(theta1) * sin(theta2)) - sin(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) *$
$cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta5) + (cos(theta4) * dtheta4 * (cos(alpha3) *$
$cos(theta3) * (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) *$
$sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) + sin(alpha3) * (-(cos(alpha2) * cos(theta1) *$
$sin(alpha1)) - cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) *$
$sin(theta2)) - cos(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) *$
$sin(theta3)) + cos(theta4) * (cos(theta3) * (cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) *$
$cos(theta1) * cos(theta2) * dtheta2 - cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) - dtheta2 *$
$sin(theta1) * sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) -$
$cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) -$
$cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 *$
$sin(theta2)) * sin(theta3)) - dtheta4 * (cos(theta3) * (cos(theta2) * sin(theta1) + cos(alpha1) *$
$cos(theta1) * sin(theta2)) + (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) *$
$sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4) +$
$(cos(alpha3) * cos(theta3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) -$
$cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) -$
$cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 *$
$sin(theta2)) + sin(alpha3) * (cos(alpha2) * dtheta1 * sin(alpha1) * sin(theta1) + cos(alpha1) *$
$cos(theta2) * dtheta1 * sin(alpha2) * sin(theta1) + cos(theta2) * dtheta2 * sin(alpha2) * sin(theta1) +$
$cos(theta1) * dtheta1 * sin(alpha2) * sin(theta2) + cos(alpha1) * cos(theta1) * dtheta2 * sin(alpha2) *$

$sin(theta2)) - cos(alpha3) * (cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) *$
$cos(theta2) * dtheta2 - cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) - dtheta2 * sin(theta1) *$
$sin(theta2)) * sin(theta3)) * sin(theta4)) * sin(theta5));$

$DJ[2,1] := d2 * cos(theta1) * dtheta1 * sin(alpha1) - a1 * dtheta1 * sin(theta1) - a2 * cos(theta2) *$
$dtheta1 * sin(theta1) - a2 * cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) - a2 * cos(alpha1) *$
$cos(theta1) * dtheta1 * sin(theta2) - a2 * cos(theta1) * dtheta2 * sin(theta2) + a3 * cos(theta3) *$
$(-(cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) -$
$cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) - cos(theta1) * dtheta2 * sin(theta2)) + dd3 *$
$(cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) +$
$cos(theta1) * sin(alpha2) * sin(theta2)) + d3 * (cos(alpha2) * cos(theta1) * dtheta1 * sin(alpha1) +$
$cos(alpha1) * cos(theta1) * cos(theta2) * dtheta1 * sin(alpha2) + cos(theta1) * cos(theta2) * dtheta2 *$
$sin(alpha2) - dtheta1 * sin(alpha2) * sin(theta1) * sin(theta2) - cos(alpha1) * dtheta2 * sin(alpha2) *$
$sin(theta1) * sin(theta2)) + a3 * (-(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) -$
$cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) +$
$cos(alpha2) * dtheta1 * sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) *$
$sin(theta2)) * sin(theta3) + d6 * (-(cos(theta5) * (cos(theta3) * sin(alpha3) * (-(cos(alpha1) *$
$cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) - cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 +$
$cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) + cos(alpha2) * dtheta1 * sin(theta1) * sin(theta2) +$
$cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) * sin(theta2)) - cos(alpha3) * (cos(alpha2) *$
$cos(theta1) * dtheta1 * sin(alpha1) + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta1 * sin(alpha2) +$
$cos(theta1) * cos(theta2) * dtheta2 * sin(alpha2) - dtheta1 * sin(alpha2) * sin(theta1) * sin(theta2) -$
$cos(alpha1) * dtheta2 * sin(alpha2) * sin(theta1) * sin(theta2)) - sin(alpha3) * (-(cos(theta2) *$
$dtheta1 * sin(theta1)) - cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) - cos(alpha1) * cos(theta1) *$
$dtheta1 * sin(theta2) - cos(theta1) * dtheta2 * sin(theta2)) * sin(theta3))) + cos(theta5) * dtheta5 *$
$(cos(theta4) * (cos(theta3) * (cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) +$
$(-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) -$
$cos(alpha2) * cos(theta1) * sin(theta2)) * sin(theta3)) + (cos(alpha3) * cos(theta3) * (-(cos(alpha1) *$
$cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) *$
$cos(theta1) * sin(theta2)) + sin(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) *$
$cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) - cos(alpha3) *$
$(cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) +$
$dtheta5 * (cos(theta3) * sin(alpha3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) +$
$sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) - cos(alpha3) *$
$(cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) +$
$cos(theta1) * sin(alpha2) * sin(theta2)) - sin(alpha3) * (cos(theta1) * cos(theta2) - cos(alpha1) *$
$sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta5) + (cos(theta4) * dtheta4 * (cos(alpha3) *$
$cos(theta3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) *$
$sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) + sin(alpha3) * (cos(alpha2) * sin(alpha1) *$
$sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) *$
$sin(theta2)) - cos(alpha3) * (cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) *$
$sin(theta3)) + cos(theta4) * (cos(theta3) * (-(cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha1) *$
$cos(theta2) * dtheta2 * sin(theta1) - cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) - cos(theta1) *$
$dtheta2 * sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) -$
$cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) +$
$cos(alpha2) * dtheta1 * sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) *$
$sin(theta2)) * sin(theta3)) - dtheta4 * (cos(theta3) * (cos(theta1) * cos(theta2) - cos(alpha1) *$
$sin(theta1) * sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) *$
$sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4) +$
$(cos(alpha3) * cos(theta3) * (-(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) -$
$cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) +$

$$cos(alpha2)*dtheta1*sin(theta1)*sin(theta2)+cos(alpha1)*cos(alpha2)*dtheta2*sin(theta1)*sin(theta2))+sin(alpha3)*(cos(alpha2)*cos(theta1)*dtheta1*sin(alpha1)+cos(alpha1)*cos(theta1)*cos(theta2)*dtheta1*sin(alpha2)+cos(theta1)*cos(theta2)*dtheta2*sin(alpha2)-dtheta1*sin(alpha2)*sin(theta1)*sin(theta2)-cos(alpha1)*dtheta2*sin(alpha2)*sin(theta1)*sin(theta2))-cos(alpha3)*(-(cos(theta2)*dtheta1*sin(theta1))-cos(alpha1)*cos(theta2)*dtheta2*sin(theta1)-cos(alpha1)*cos(theta1)*dtheta1*sin(theta2)-cos(theta1)*dtheta2*sin(theta2))*sin(theta3))*sin(theta4))*sin(theta5));$$

$$DJ[2,2] := -(sin(alpha1)*sin(theta1)*(a2*cos(theta2)*dtheta2*sin(alpha1)+a3*cos(theta2)*cos(theta3)*dtheta2*sin(alpha1)+dd3*(cos(alpha1)*cos(alpha2)-cos(theta2)*sin(alpha1)*sin(alpha2))+d3*dtheta2*sin(alpha1)*sin(alpha2)*sin(theta2)-a3*cos(alpha2)*dtheta2*sin(alpha1)*sin(theta2)*sin(theta3)+d6*(-(cos(theta5)*(-(cos(alpha3)*dtheta2*sin(alpha1)*sin(alpha2)*sin(theta2))-cos(alpha2)*cos(theta3)*dtheta2*sin(alpha1)*sin(alpha3)*sin(theta2)-cos(theta2)*dtheta2*sin(alpha1)*sin(alpha3)*sin(theta3)))+cos(theta5)*dtheta5*(cos(theta4)*(cos(theta3)*sin(alpha1)*sin(theta2)+(cos(alpha2)*cos(theta2)*sin(alpha1)+cos(alpha1)*sin(alpha2))*sin(theta3))+(cos(alpha3)*cos(theta3)*(cos(alpha2)*cos(theta2)*sin(alpha1)+cos(alpha1)*sin(alpha2))+(cos(alpha1)*cos(alpha2)-cos(theta2)*sin(alpha1)*sin(alpha2))*sin(alpha3)-cos(alpha3)*sin(alpha1)*sin(theta2)*sin(theta3))*sin(theta4))+dtheta5*(-(cos(alpha3)*(cos(alpha1)*cos(alpha2)-cos(theta2)*sin(alpha1)*sin(alpha2)))+cos(theta3)*(cos(alpha2)*cos(theta2)*sin(alpha1)+cos(alpha1)*sin(alpha2))*sin(alpha3)-sin(alpha1)*sin(alpha3)*sin(theta2)*sin(theta3))*sin(theta5)+(cos(theta4)*dtheta4*(cos(alpha3)*cos(theta3)*(cos(alpha2)*cos(theta2)*sin(alpha1)+cos(alpha1)*sin(alpha2))+(cos(alpha1)*cos(alpha2)-cos(theta2)*sin(alpha1)*sin(alpha2))*sin(alpha3)-cos(alpha3)*sin(alpha1)*sin(theta2)*sin(theta3))+cos(theta4)*(cos(theta2)*cos(theta3)*dtheta2*sin(alpha1)-cos(alpha2)*dtheta2*sin(alpha1)*sin(theta2)*sin(theta3))+(-(cos(alpha2)*cos(alpha3)*cos(theta3)*dtheta2*sin(alpha1)*sin(theta2))+dtheta2*sin(alpha1)*sin(alpha2)*sin(alpha3)*sin(theta2)-cos(alpha3)*cos(theta2)*dtheta2*sin(alpha1)*sin(theta3))*sin(theta4)-dtheta4*(cos(theta3)*sin(alpha1)*sin(theta2)+(cos(alpha2)*cos(theta2)*sin(alpha1)+cos(alpha1)*sin(alpha2))*sin(theta3))*sin(theta4))*sin(theta5))))-cos(theta1)*dtheta1*sin(alpha1)*(d2*cos(alpha1)+d3*(cos(alpha1)*cos(alpha2)-cos(theta2)*sin(alpha1)*sin(alpha2))+a2*sin(alpha1)*sin(theta2)+a3*cos(theta3)*sin(alpha1)*sin(theta2)+a3*(cos(alpha2)*cos(theta2)*sin(alpha1)+cos(alpha1)*sin(alpha2))*sin(theta3)+d6*(-(cos(theta5)*(-(cos(alpha3)*(cos(alpha1)*cos(alpha2)-cos(theta2)*sin(alpha1)*sin(alpha2)))+cos(theta3)*(cos(alpha2)*cos(theta2)*sin(alpha1)+cos(alpha1)*sin(alpha2))*sin(alpha3)-sin(alpha1)*sin(alpha3)*sin(theta2)*sin(theta3)))+(cos(theta4)*(cos(theta3)*sin(alpha1)*sin(theta2)+(cos(alpha2)*cos(theta2)*sin(alpha1)+cos(alpha1)*sin(alpha2))*sin(theta3))+(cos(alpha3)*cos(theta3)*(cos(alpha2)*cos(theta2)*sin(alpha1)+cos(alpha1)*sin(alpha2))+(cos(alpha1)*cos(alpha2)-cos(theta2)*sin(alpha1)*sin(alpha2))*sin(alpha3)-cos(alpha3)*sin(alpha1)*sin(theta2)*sin(theta3))*sin(theta4))*sin(theta5)))+cos(alpha1)*(d2*cos(theta1)*dtheta1*sin(alpha1)-a2*cos(theta2)*dtheta1*sin(theta1)-a2*cos(alpha1)*cos(theta2)*dtheta2*sin(theta1)-a2*cos(alpha1)*cos(theta1)*dtheta1*sin(theta2)-a2*cos(theta1)*dtheta2*sin(theta2)+a3*cos(theta3)*(-(cos(theta2)*dtheta1*sin(theta1))-cos(alpha1)*cos(theta2)*dtheta2*sin(theta1)-cos(alpha1)*cos(theta1)*dtheta1*sin(theta2)-cos(theta1)*dtheta2*sin(theta2))+dd3*(cos(alpha2)*sin(alpha1)*sin(theta1)+cos(alpha1)*cos(theta2)*sin(alpha2)*sin(theta1)+cos(theta1)*sin(alpha2)*sin(theta2))+d3*(cos(alpha2)*cos(theta1)*dtheta1*sin(alpha1)+cos(alpha1)*cos(theta1)*cos(theta2)*dtheta1*sin(alpha2)+cos(theta1)*cos(theta2)*dtheta2*sin(alpha2)-dtheta1*sin(alpha2)*sin(theta1)*sin(theta2)-cos(alpha1)*dtheta2*sin(alpha2)*sin(theta1)*sin(theta2))+a3*(-(cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2)*dtheta1)-cos(alpha2)*cos(theta1)*cos(theta2)*dtheta2+cos(theta1)*dtheta1*sin(alpha1)*sin(alpha2)+cos(alpha2)*dtheta1*sin(theta1)*sin(theta2)+cos(alpha1)*cos(alpha2)*dtheta2*sin(theta1)*sin(theta2))*$$

$$
sin(theta3) + d6 * (-(cos(theta5) * (cos(theta3) * sin(alpha3) * (-(cos(alpha1) * cos(alpha2) *
cos(theta1)*cos(theta2)*dtheta1) - cos(alpha2)*cos(theta1)*cos(theta2)*dtheta2+cos(theta1)*
dtheta1*sin(alpha1)*sin(alpha2)+cos(alpha2)*dtheta1*sin(theta1)*sin(theta2)+cos(alpha1)*
cos(alpha2) * dtheta2 * sin(theta1) * sin(theta2)) - cos(alpha3) * (cos(alpha2) * cos(theta1) *
dtheta1*sin(alpha1)+cos(alpha1)*cos(theta1)*cos(theta2)*dtheta1*sin(alpha2)+cos(theta1)*
cos(theta2)*dtheta2*sin(alpha2)-dtheta1*sin(alpha2)*sin(theta1)*sin(theta2)-cos(alpha1)*
dtheta2 * sin(alpha2) * sin(theta1) * sin(theta2)) - sin(alpha3) * (-(cos(theta2) * dtheta1 *
sin(theta1)) - cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) - cos(alpha1) * cos(theta1) *
dtheta1*sin(theta2) - cos(theta1)*dtheta2*sin(theta2))*sin(theta3))) + cos(theta5)*dtheta5*
(cos(theta4)*(cos(theta3)*(cos(theta1)*cos(theta2) - cos(alpha1)*sin(theta1)*sin(theta2)) +
(-(cos(alpha1)*cos(alpha2)*cos(theta2)*sin(theta1)) + sin(alpha1)*sin(alpha2)*sin(theta1) -
cos(alpha2)*cos(theta1)*sin(theta2))*sin(theta3)) + (cos(alpha3)*cos(theta3)*(-(cos(alpha1)*
cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) *
cos(theta1) * sin(theta2)) + sin(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) *
cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) - cos(alpha3) *
(cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) +
dtheta5 * (cos(theta3) * sin(alpha3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) +
sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) - cos(alpha3) *
(cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) +
cos(theta1) * sin(alpha2) * sin(theta2)) - sin(alpha3) * (cos(theta1) * cos(theta2) - cos(alpha1) *
sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta5) + (cos(theta4) * dtheta4 * (cos(alpha3) *
cos(theta3)*(-(cos(alpha1)*cos(alpha2)*cos(theta2)*sin(theta1)) + sin(alpha1)*sin(alpha2)*
sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) + sin(alpha3) * (cos(alpha2) * sin(alpha1) *
sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) *
sin(theta2)) - cos(alpha3) * (cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) *
sin(theta3)) + cos(theta4) * (cos(theta3) * (-(cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha1) *
cos(theta2)*dtheta2*sin(theta1) - cos(alpha1)*cos(theta1)*dtheta1*sin(theta2) - cos(theta1)*
dtheta2 * sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) -
cos(alpha2)*cos(theta1)*cos(theta2)*dtheta2+cos(theta1)*dtheta1*sin(alpha1)*sin(alpha2)+
cos(alpha2)*dtheta1*sin(theta1)*sin(theta2)+cos(alpha1)*cos(alpha2)*dtheta2*sin(theta1)*
sin(theta2)) * sin(theta3)) - dtheta4 * (cos(theta3) * (cos(theta1) * cos(theta2) - cos(alpha1) *
sin(theta1)*sin(theta2)) + (-(cos(alpha1)*cos(alpha2)*cos(theta2)*sin(theta1)) + sin(alpha1)*
sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4) +
(cos(alpha3) * cos(theta3) * (-(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) -
cos(alpha2)*cos(theta1)*cos(theta2)*dtheta2+cos(theta1)*dtheta1*sin(alpha1)*sin(alpha2)+
cos(alpha2)*dtheta1*sin(theta1)*sin(theta2)+cos(alpha1)*cos(alpha2)*dtheta2*sin(theta1)*
sin(theta2)) + sin(alpha3) * (cos(alpha2) * cos(theta1) * dtheta1 * sin(alpha1) + cos(alpha1) *
cos(theta1)*cos(theta2)*dtheta1*sin(alpha2)+cos(theta1)*cos(theta2)*dtheta2*sin(alpha2)-
dtheta1*sin(alpha2)*sin(theta1)*sin(theta2) - cos(alpha1)*dtheta2*sin(alpha2)*sin(theta1)*
sin(theta2)) - cos(alpha3) * (-(cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha1) * cos(theta2) *
dtheta2 * sin(theta1) - cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) - cos(theta1) * dtheta2 *
sin(theta2)) * sin(theta3)) * sin(theta4)) * sin(theta5)));
$$

$$
DJ[2,3] := cos(alpha2)*dtheta1*sin(alpha1)*sin(theta1)+cos(alpha1)*cos(theta2)*dtheta1*
sin(alpha2)*sin(theta1)+cos(theta2)*dtheta2*sin(alpha2)*sin(theta1)+cos(theta1)*dtheta1*
sin(alpha2) * sin(theta2) + cos(alpha1) * cos(theta1) * dtheta2 * sin(alpha2) * sin(theta2);
$$

$$
DJ[2,4] := -(d6 * (-(cos(theta3) * sin(alpha3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) *
sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2))) +
cos(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) *
sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) + sin(alpha3) * (cos(theta1) * cos(theta2) -
cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3)) * (-(cos(theta5) * (-(cos(alpha3) * dtheta2 *
$$

$sin(alpha1) * sin(alpha2) * sin(theta2)) - cos(alpha2) * cos(theta3) * dtheta2 * sin(alpha1) *$
$sin(alpha3) * sin(theta2) - cos(theta2) * dtheta2 * sin(alpha1) * sin(alpha3) * sin(theta3))) +$
$cos(theta5) * dtheta5 * (cos(theta4) * (cos(theta3) * sin(alpha1) * sin(theta2) + (cos(alpha2) *$
$cos(theta2) * sin(alpha1) + cos(alpha1) * sin(alpha2)) * sin(theta3)) + (cos(alpha3) * cos(theta3) *$
$(cos(alpha2) * cos(theta2) * sin(alpha1) + cos(alpha1) * sin(alpha2)) + (cos(alpha1) * cos(alpha2) -$
$cos(theta2) * sin(alpha1) * sin(alpha2)) * sin(alpha3) - cos(alpha3) * sin(alpha1) * sin(theta2) *$
$sin(theta3)) * sin(theta4)) + dtheta5 * (-(cos(alpha3) * (cos(alpha1) * cos(alpha2) - cos(theta2) *$
$sin(alpha1) * sin(alpha2))) + cos(theta3) * (cos(alpha2) * cos(theta2) * sin(alpha1) + cos(alpha1) *$
$sin(alpha2)) * sin(alpha3) - sin(alpha1) * sin(alpha3) * sin(theta2) * sin(theta3)) * sin(theta5) +$
$(cos(theta4) * dtheta4 * (cos(alpha3) * cos(theta3) * (cos(alpha2) * cos(theta2) * sin(alpha1) +$
$cos(alpha1) * sin(alpha2)) + (cos(alpha1) * cos(alpha2) - cos(theta2) * sin(alpha1) * sin(alpha2)) *$
$sin(alpha3) - cos(alpha3) * sin(alpha1) * sin(theta2) * sin(theta3)) + cos(theta4) * (cos(theta2) *$
$cos(theta3) * dtheta2 * sin(alpha1) - cos(alpha2) * dtheta2 * sin(alpha1) * sin(theta2) * sin(theta3)) +$
$(-(cos(alpha2) * cos(alpha3) * cos(theta3) * dtheta2 * sin(alpha1) * sin(theta2)) + dtheta2 *$
$sin(alpha1) * sin(alpha2) * sin(alpha3) * sin(theta2) - cos(alpha3) * cos(theta2) * dtheta2 *$
$sin(alpha1) * sin(theta3)) * sin(theta4) - dtheta4 * (cos(theta3) * sin(alpha1) * sin(theta2) +$
$(cos(alpha2) * cos(theta2) * sin(alpha1) + cos(alpha1) * sin(alpha2)) * sin(theta3)) * sin(theta4)) *$
$sin(theta5))) - d6 * (-(cos(theta3) * sin(alpha3) * (-(cos(alpha1) * cos(alpha2) * cos(theta1) *$
$cos(theta2) * dtheta1) - cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 *$
$sin(alpha1) * sin(alpha2) + cos(alpha2) * dtheta1 * sin(theta1) * sin(theta2) + cos(alpha1) *$
$cos(alpha2) * dtheta2 * sin(theta1) * sin(theta2))) + cos(alpha3) * (cos(alpha2) * cos(theta1) *$
$dtheta1 * sin(alpha1) + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta1 * sin(alpha2) + cos(theta1) *$
$cos(theta2) * dtheta2 * sin(alpha2) - dtheta1 * sin(alpha2) * sin(theta1) * sin(theta2) - cos(alpha1) *$
$dtheta2 * sin(alpha2) * sin(theta1) * sin(theta2)) + sin(alpha3) * (-(cos(theta2) * dtheta1 *$
$sin(theta1)) - cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) - cos(alpha1) * cos(theta1) *$
$dtheta1 * sin(theta2) - cos(theta1) * dtheta2 * sin(theta2)) * sin(theta3)) * (-(cos(theta5) *$
$(-(cos(alpha3) * (cos(alpha1) * cos(alpha2) - cos(theta2) * sin(alpha1) * sin(alpha2))) + cos(theta3) *$
$(cos(alpha2) * cos(theta2) * sin(alpha1) + cos(alpha1) * sin(alpha2)) * sin(alpha3) - sin(alpha1) *$
$sin(alpha3) * sin(theta2) * sin(theta3))) + (cos(theta4) * (cos(theta3) * sin(alpha1) * sin(theta2) +$
$(cos(alpha2) * cos(theta2) * sin(alpha1) + cos(alpha1) * sin(alpha2)) * sin(theta3)) + (cos(alpha3) *$
$cos(theta3) * (cos(alpha2) * cos(theta2) * sin(alpha1) + cos(alpha1) * sin(alpha2)) + (cos(alpha1) *$
$cos(alpha2) - cos(theta2) * sin(alpha1) * sin(alpha2)) * sin(alpha3) - cos(alpha3) * sin(alpha1) *$
$sin(theta2) * sin(theta3)) * sin(theta4)) * sin(theta5)) + d6 * (cos(alpha3) * (cos(alpha1) * cos(alpha2) -$
$cos(theta2) * sin(alpha1) * sin(alpha2)) - cos(theta3) * (cos(alpha2) * cos(theta2) * sin(alpha1) +$
$cos(alpha1) * sin(alpha2)) * sin(alpha3) + sin(alpha1) * sin(alpha3) * sin(theta2) * sin(theta3)) *$
$(-(cos(theta5) * (cos(theta3) * sin(alpha3) * (-(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) *$
$dtheta1) - cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) *$
$sin(alpha2) + cos(alpha2) * dtheta1 * sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 *$
$sin(theta1) * sin(theta2)) - cos(alpha3) * (cos(alpha2) * cos(theta1) * dtheta1 * sin(alpha1) +$
$cos(alpha1) * cos(theta1) * cos(theta2) * dtheta1 * sin(alpha2) + cos(theta1) * cos(theta2) *$
$dtheta2 * sin(alpha2) - dtheta1 * sin(alpha2) * sin(theta1) * sin(theta2) - cos(alpha1) * dtheta2 *$
$sin(alpha2) * sin(theta1) * sin(theta2)) - sin(alpha3) * (-(cos(theta2) * dtheta1 * sin(theta1)) -$
$cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) - cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) -$
$cos(theta1) * dtheta2 * sin(theta2)) * sin(theta3))) + cos(theta5) * dtheta5 * (cos(theta4) * (cos(theta3) *$
$(cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) + (-(cos(alpha1) * cos(alpha2) *$
$cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) *$
$sin(theta2)) * sin(theta3)) + (cos(alpha3) * cos(theta3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) *$
$sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) +$
$sin(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) *$
$sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) - cos(alpha3) * (cos(theta1) * cos(theta2) -$

$$
cos(alpha1) \ast sin(theta1) \ast sin(theta2)) \ast sin(theta3)) \ast sin(theta4)) + dtheta5 \ast (cos(theta3) \ast
sin(alpha3) \ast (-(cos(alpha1) \ast cos(alpha2) \ast cos(theta2) \ast sin(theta1)) + sin(alpha1) \ast sin(alpha2) \ast
sin(theta1) - cos(alpha2) \ast cos(theta1) \ast sin(theta2)) - cos(alpha3) \ast (cos(alpha2) \ast sin(alpha1) \ast
sin(theta1) + cos(alpha1) \ast cos(theta2) \ast sin(alpha2) \ast sin(theta1) + cos(theta1) \ast sin(alpha2) \ast
sin(theta2)) - sin(alpha3) \ast (cos(theta1) \ast cos(theta2) - cos(alpha1) \ast sin(theta1) \ast sin(theta2)) \ast
sin(theta3)) \ast sin(theta5) + (cos(theta4) \ast dtheta4 \ast (cos(alpha3) \ast cos(theta3) \ast (-(cos(alpha1) \ast
cos(alpha2) \ast cos(theta2) \ast sin(theta1)) + sin(alpha1) \ast sin(alpha2) \ast sin(theta1) - cos(alpha2) \ast
cos(theta1) \ast sin(theta2)) + sin(alpha3) \ast (cos(alpha2) \ast sin(alpha1) \ast sin(theta1) + cos(alpha1) \ast
cos(theta2) \ast sin(alpha2) \ast sin(theta1) + cos(theta1) \ast sin(alpha2) \ast sin(theta2)) - cos(alpha3) \ast
(cos(theta1) \ast cos(theta2) - cos(alpha1) \ast sin(theta1) \ast sin(theta2)) \ast sin(theta3)) + cos(theta4) \ast
(cos(theta3) \ast (-(cos(theta2) \ast dtheta1 \ast sin(theta1)) - cos(alpha1) \ast cos(theta2) \ast dtheta2 \ast
sin(theta1) - cos(alpha1) \ast cos(theta1) \ast dtheta1 \ast sin(theta2) - cos(theta1) \ast dtheta2 \ast sin(theta2)) +
(-(cos(alpha1) \ast cos(alpha2) \ast cos(theta1) \ast cos(theta2) \ast dtheta1) - cos(alpha2) \ast cos(theta1) \ast
cos(theta2) \ast dtheta2 + cos(theta1) \ast dtheta1 \ast sin(alpha1) \ast sin(alpha2) + cos(alpha2) \ast dtheta1 \ast
sin(theta1) \ast sin(theta2) + cos(alpha1) \ast cos(alpha2) \ast dtheta2 \ast sin(theta1) \ast sin(theta2)) \ast
sin(theta3)) - dtheta4 \ast (cos(theta3) \ast (cos(theta1) \ast cos(theta2) - cos(alpha1) \ast sin(theta1) \ast
sin(theta2)) + (-(cos(alpha1) \ast cos(alpha2) \ast cos(theta2) \ast sin(theta1)) + sin(alpha1) \ast sin(alpha2) \ast
sin(theta1) - cos(alpha2) \ast cos(theta1) \ast sin(theta2)) \ast sin(theta3)) \ast sin(theta4) + (cos(alpha3) \ast
cos(theta3) \ast (-(cos(alpha1) \ast cos(alpha2) \ast cos(theta1) \ast cos(theta2) \ast dtheta1) - cos(alpha2) \ast
cos(theta1) \ast cos(theta2) \ast dtheta2 + cos(theta1) \ast dtheta1 \ast sin(alpha1) \ast sin(alpha2) + cos(alpha2) \ast
dtheta1 \ast sin(theta1) \ast sin(theta2) + cos(alpha1) \ast cos(alpha2) \ast dtheta2 \ast sin(theta1) \ast sin(theta2)) +
sin(alpha3) \ast (cos(alpha2) \ast cos(theta1) \ast dtheta1 \ast sin(alpha1) + cos(alpha1) \ast cos(theta1) \ast
cos(theta2) \ast dtheta1 \ast sin(alpha2) + cos(theta1) \ast cos(theta2) \ast dtheta2 \ast sin(alpha2) - dtheta1 \ast
sin(alpha2) \ast sin(theta1) \ast sin(theta2) - cos(alpha1) \ast dtheta2 \ast sin(alpha2) \ast sin(theta1) \ast
sin(theta2)) - cos(alpha3) \ast (-(cos(theta2) \ast dtheta1 \ast sin(theta1)) - cos(alpha1) \ast cos(theta2) \ast
dtheta2 \ast sin(theta1) - cos(alpha1) \ast cos(theta1) \ast dtheta1 \ast sin(theta2) - cos(theta1) \ast dtheta2 \ast
sin(theta2)) \ast sin(theta3)) \ast sin(theta4)) \ast sin(theta5)) + d6 \ast (cos(alpha3) \ast dtheta2 \ast sin(alpha1) \ast
sin(alpha2) \ast sin(theta2) + cos(alpha2) \ast cos(theta3) \ast dtheta2 \ast sin(alpha1) \ast sin(alpha3) \ast
sin(theta2) + cos(theta2) \ast dtheta2 \ast sin(alpha1) \ast sin(alpha3) \ast sin(theta3)) \ast (-(cos(theta5) \ast
(cos(theta3) \ast sin(alpha3) \ast (-(cos(alpha1) \ast cos(alpha2) \ast cos(theta2) \ast sin(theta1)) + sin(alpha1) \ast
sin(alpha2) \ast sin(theta1) - cos(alpha2) \ast cos(theta1) \ast sin(theta2)) - cos(alpha3) \ast (cos(alpha2) \ast
sin(alpha1) \ast sin(theta1) + cos(alpha1) \ast cos(theta2) \ast sin(alpha2) \ast sin(theta1) + cos(theta1) \ast
sin(alpha2) \ast sin(theta2)) - sin(alpha3) \ast (cos(theta1) \ast cos(theta2) - cos(alpha1) \ast sin(theta1) \ast
sin(theta2)) \ast sin(theta3))) + (cos(theta4) \ast (cos(theta3) \ast (cos(theta1) \ast cos(theta2) - cos(alpha1) \ast
sin(theta1) \ast sin(theta2)) + (-(cos(alpha1) \ast cos(alpha2) \ast cos(theta2) \ast sin(theta1)) + sin(alpha1) \ast
sin(alpha2) \ast sin(theta1) - cos(alpha2) \ast cos(theta1) \ast sin(theta2)) \ast sin(theta3)) + (cos(alpha3) \ast
cos(theta3) \ast (-(cos(alpha1) \ast cos(alpha2) \ast cos(theta2) \ast sin(theta1)) + sin(alpha1) \ast sin(alpha2) \ast
sin(theta1) - cos(alpha2) \ast cos(theta1) \ast sin(theta2)) + sin(alpha3) \ast (cos(alpha2) \ast sin(alpha1) \ast
sin(theta1) + cos(alpha1) \ast cos(theta2) \ast sin(alpha2) \ast sin(theta1) + cos(theta1) \ast sin(alpha2) \ast
sin(theta2)) - cos(alpha3) \ast (cos(theta1) \ast cos(theta2) - cos(alpha1) \ast sin(theta1) \ast sin(theta2)) \ast
sin(theta3)) \ast sin(theta4)) \ast sin(theta5));
$$

$$
DJ[2,5] := -(d6 \ast (cos(theta4) \ast (cos(alpha3) \ast cos(theta3) \ast (-(cos(alpha1) \ast cos(alpha2) \ast
cos(theta2) \ast sin(theta1)) + sin(alpha1) \ast sin(alpha2) \ast sin(theta1) - cos(alpha2) \ast cos(theta1) \ast
sin(theta2)) + sin(alpha3) \ast (cos(alpha2) \ast sin(alpha1) \ast sin(theta1) + cos(alpha1) \ast cos(theta2) \ast
sin(alpha2) \ast sin(theta1) + cos(theta1) \ast sin(alpha2) \ast sin(theta2)) - cos(alpha3) \ast (cos(theta1) \ast
cos(theta2) - cos(alpha1) \ast sin(theta1) \ast sin(theta2)) \ast sin(theta3)) - (cos(theta3) \ast (cos(theta1) \ast
cos(theta2) - cos(alpha1) \ast sin(theta1) \ast sin(theta2)) + (-(cos(alpha1) \ast cos(alpha2) \ast cos(theta2) \ast
sin(theta1)) + sin(alpha1) \ast sin(alpha2) \ast sin(theta1) - cos(alpha2) \ast cos(theta1) \ast sin(theta2)) \ast
sin(theta3)) \ast sin(theta4)) \ast (-(cos(theta5) \ast (-(cos(alpha3) \ast dtheta2 \ast sin(alpha1) \ast sin(alpha2) \ast
sin(theta2)) - cos(alpha2) \ast cos(theta3) \ast dtheta2 \ast sin(alpha1) \ast sin(alpha3) \ast sin(theta2) -
$$

$$
cos(theta2)*dtheta2*sin(alpha1)*sin(alpha3)*sin(theta3)))+cos(theta5)*dtheta5*(cos(theta4)*
(cos(theta3)*sin(alpha1)*sin(theta2)+(cos(alpha2)*cos(theta2)*sin(alpha1)+cos(alpha1)*
sin(alpha2))*sin(theta3))+(cos(alpha3)*cos(theta3)*(cos(alpha2)*cos(theta2)*sin(alpha1)+
cos(alpha1)*sin(alpha2))+(cos(alpha1)*cos(alpha2)-cos(theta2)*sin(alpha1)*sin(alpha2))*
sin(alpha3)-cos(alpha3)*sin(alpha1)*sin(theta2)*sin(theta3))*sin(theta4))+dtheta5*
(-(cos(alpha3)*(cos(alpha1)*cos(alpha2)-cos(theta2)*sin(alpha1)*sin(alpha2)))+cos(theta3)*
(cos(alpha2)*cos(theta2)*sin(alpha1)+cos(alpha1)*sin(alpha2))*sin(alpha3)-sin(alpha1)*
sin(alpha3)*sin(theta2)*sin(theta3))*sin(theta5)+(cos(theta4)*dtheta4*(cos(alpha3)*
cos(theta3)*(cos(alpha2)*cos(theta2)*sin(alpha1)+cos(alpha1)*sin(alpha2))+(cos(alpha1)*
cos(alpha2)-cos(theta2)*sin(alpha1)*sin(alpha2))*sin(alpha3)-cos(alpha3)*sin(alpha1)*
sin(theta2)*sin(theta3))+cos(theta4)*(cos(theta2)*cos(theta3)*dtheta2*sin(alpha1)-
cos(alpha2)*dtheta2*sin(alpha1)*sin(theta2)*sin(theta3))+(-(cos(alpha2)*cos(alpha3)*
cos(theta3)*dtheta2*sin(alpha1)*sin(theta2))+dtheta2*sin(alpha1)*sin(alpha2)*sin(alpha3)*
sin(theta2)-cos(alpha3)*cos(theta2)*dtheta2*sin(alpha1)*sin(theta3))*sin(theta4)-dtheta4*
(cos(theta3)*sin(alpha1)*sin(theta2)+(cos(alpha2)*cos(theta2)*sin(alpha1)+cos(alpha1)*
sin(alpha2))*sin(theta3))*sin(theta4))*sin(theta5)))-d6*(-(cos(theta4)*dtheta4*(cos(theta3)*
(cos(theta1)*cos(theta2)-cos(alpha1)*sin(theta1)*sin(theta2))+(-(cos(alpha1)*cos(alpha2)*
cos(theta2)*sin(theta1))+sin(alpha1)*sin(alpha2)*sin(theta1)-cos(alpha2)*cos(theta1)*
sin(theta2))*sin(theta3)))+cos(theta4)*(cos(alpha3)*cos(theta3)*(-(cos(alpha1)*cos(alpha2)*
cos(theta1)*cos(theta2)*dtheta1)-cos(alpha2)*cos(theta1)*cos(theta2)*dtheta2+cos(theta1)*
dtheta1*sin(alpha1)*sin(alpha2)+cos(alpha2)*dtheta1*sin(theta1)*sin(theta2)+cos(alpha1)*
cos(alpha2)*dtheta2*sin(theta1)*sin(theta2))+sin(alpha3)*(cos(alpha2)*cos(theta1)*
dtheta1*sin(alpha1)+cos(alpha1)*cos(theta1)*cos(theta2)*dtheta1*sin(alpha2)+cos(theta1)*
cos(theta2)*dtheta2*sin(alpha2)-dtheta1*sin(alpha2)*sin(theta1)*sin(theta2)-cos(alpha1)*
dtheta2*sin(alpha2)*sin(theta1)*sin(theta2))-cos(alpha3)*(-(cos(theta2)*dtheta1*
sin(theta1))-cos(alpha1)*cos(theta2)*dtheta2*sin(theta1)-cos(alpha1)*cos(theta1)*dtheta1*
sin(theta2)-cos(theta1)*dtheta2*sin(theta2))*sin(theta3))-dtheta4*(cos(alpha3)*cos(theta3)*
(-(cos(alpha1)*cos(alpha2)*cos(theta2)*sin(theta1))+sin(alpha1)*sin(alpha2)*sin(theta1)-
cos(alpha2)*cos(theta1)*sin(theta2))+sin(alpha3)*(cos(alpha2)*sin(alpha1)*sin(theta1)+
cos(alpha1)*cos(theta2)*sin(alpha2)*sin(theta1)+cos(theta1)*sin(alpha2)*sin(theta2))-
cos(alpha3)*(cos(theta1)*cos(theta2)-cos(alpha1)*sin(theta1)*sin(theta2))*sin(theta3))*
sin(theta4)-(cos(theta3)*(-(cos(theta2)*dtheta1*sin(theta1))-cos(alpha1)*cos(theta2)*
dtheta2*sin(theta1)-cos(alpha1)*cos(theta1)*dtheta1*sin(theta2)-cos(theta1)*dtheta2*
sin(theta2))+(-(cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2)*dtheta1)-cos(alpha2)*
cos(theta1)*cos(theta2)*dtheta2+cos(theta1)*dtheta1*sin(alpha1)*sin(alpha2)+cos(alpha2)*
dtheta1*sin(theta1)*sin(theta2)+cos(alpha1)*cos(alpha2)*dtheta2*sin(theta1)*sin(theta2))*
sin(theta3))*sin(theta4))*(-(cos(theta5)*(-(cos(alpha3)*(cos(alpha1)*cos(alpha2)-
cos(theta2)*sin(alpha1)*sin(alpha2)))+cos(theta3)*(cos(alpha2)*cos(theta2)*sin(alpha1)+
cos(alpha1)*sin(alpha2))*sin(alpha3)-sin(alpha1)*sin(alpha3)*sin(theta2)*sin(theta3)))+
(cos(theta4)*(cos(theta3)*sin(alpha1)*sin(theta2)+(cos(alpha2)*cos(theta2)*sin(alpha1)+
cos(alpha1)*sin(alpha2))*sin(theta3))+(cos(alpha3)*cos(theta3)*(cos(alpha2)*cos(theta2)*
sin(alpha1)+cos(alpha1)*sin(alpha2))+(cos(alpha1)*cos(alpha2)-cos(theta2)*sin(alpha1)*
sin(alpha2))*sin(alpha3)-cos(alpha3)*sin(alpha1)*sin(theta2)*sin(theta3))*sin(theta4))*
sin(theta5))+d6*(cos(theta4)*(cos(alpha3)*cos(theta3)*(cos(alpha2)*cos(theta2)*sin(alpha1)+
cos(alpha1)*sin(alpha2))+(cos(alpha1)*cos(alpha2)-cos(theta2)*sin(alpha1)*sin(alpha2))*
sin(alpha3)-cos(alpha3)*sin(alpha1)*sin(theta2)*sin(theta3))-(cos(theta3)*sin(alpha1)*
sin(theta2)+(cos(alpha2)*cos(theta2)*sin(alpha1)+cos(alpha1)*sin(alpha2))*sin(theta3))*
sin(theta4))*(-(cos(theta5)*(cos(theta3)*sin(alpha3)*(-(cos(alpha1)*cos(alpha2)*cos(theta1)*
cos(theta2)*dtheta1)-cos(alpha2)*cos(theta1)*cos(theta2)*dtheta2+cos(theta1)*dtheta1*
sin(alpha1)*sin(alpha2)+cos(alpha2)*dtheta1*sin(theta1)*sin(theta2)+cos(alpha1)*
$$

$$
cos(alpha2) * dtheta2 * sin(theta1) * sin(theta2)) - cos(alpha3) * (cos(alpha2) * cos(theta1) *
dtheta1 * sin(alpha1) + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta1 * sin(alpha2) + cos(theta1) *
cos(theta2) * dtheta2 * sin(alpha2) - dtheta1 * sin(alpha2) * sin(theta1) * sin(theta2) - cos(alpha1) *
dtheta2 * sin(alpha2) * sin(theta1) * sin(theta2)) - sin(alpha3) * (-(cos(theta2) * dtheta1 *
sin(theta1)) - cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) - cos(alpha1) * cos(theta1) *
dtheta1 * sin(theta2) - cos(theta1) * dtheta2 * sin(theta2)) * sin(theta3))) + cos(theta5) * dtheta5 *
(cos(theta4) * (cos(theta3) * (cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) +
(-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) -
cos(alpha2) * cos(theta1) * sin(theta2)) * sin(theta3)) + (cos(alpha3) * cos(theta3) * (-(cos(alpha1) *
cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) *
cos(theta1) * sin(theta2)) + sin(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) *
cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) - cos(alpha3) *
(cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) +
dtheta5 * (cos(theta3) * sin(alpha3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) +
sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) - cos(alpha3) *
(cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) +
cos(theta1) * sin(alpha2) * sin(theta2)) - sin(alpha3) * (cos(theta1) * cos(theta2) - cos(alpha1) *
sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta5) + (cos(theta4) * dtheta4 * (cos(alpha3) *
cos(theta3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) *
sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) + sin(alpha3) * (cos(alpha2) * sin(alpha1) *
sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) *
sin(theta2)) - cos(alpha3) * (cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) *
sin(theta3)) + cos(theta4) * (cos(theta3) * (-(cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha1) *
cos(theta2) * dtheta2 * sin(theta1) - cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) - cos(theta1) *
dtheta2 * sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) -
cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) +
cos(alpha2) * dtheta1 * sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) *
sin(theta2)) * sin(theta3)) - dtheta4 * (cos(theta3) * (cos(theta1) * cos(theta2) - cos(alpha1) *
sin(theta1) * sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) *
sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4) +
(cos(alpha3) * cos(theta3) * (-(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) -
cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) +
cos(alpha2) * dtheta1 * sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) *
sin(theta2)) + sin(alpha3) * (cos(alpha2) * cos(theta1) * dtheta1 * sin(alpha1) + cos(alpha1) *
cos(theta1) * cos(theta2) * dtheta1 * sin(alpha2) + cos(theta1) * cos(theta2) * dtheta2 * sin(alpha2) -
dtheta1 * sin(alpha2) * sin(theta1) * sin(theta2) - cos(alpha1) * dtheta2 * sin(alpha2) * sin(theta1) *
sin(theta2)) - cos(alpha3) * (-(cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha1) * cos(theta2) *
dtheta2 * sin(theta1) - cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) - cos(theta1) * dtheta2 *
sin(theta2)) * sin(theta3)) * sin(theta4)) * sin(theta5)) + d6 * (cos(theta4) * (-(cos(alpha2) *
cos(alpha3) * cos(theta3) * dtheta2 * sin(alpha1) * sin(theta2)) + dtheta2 * sin(alpha1) * sin(alpha2) *
sin(alpha3) * sin(theta2) - cos(alpha3) * cos(theta2) * dtheta2 * sin(alpha1) * sin(theta3)) -
cos(theta4) * dtheta4 * (cos(theta3) * sin(alpha1) * sin(theta2) + (cos(alpha2) * cos(theta2) *
sin(alpha1) + cos(alpha1) * sin(alpha2)) * sin(theta3)) - dtheta4 * (cos(alpha3) * cos(theta3) *
(cos(alpha2) * cos(theta2) * sin(alpha1) + cos(alpha1) * sin(alpha2)) + (cos(alpha1) * cos(alpha2) -
cos(theta2) * sin(alpha1) * sin(alpha2)) * sin(alpha3) - cos(alpha3) * sin(alpha1) * sin(theta2) *
sin(theta3)) * sin(theta4) - (cos(theta2) * cos(theta3) * dtheta2 * sin(alpha1) - cos(alpha2) * dtheta2 *
sin(alpha1) * sin(theta2) * sin(theta3)) * sin(theta4)) * (-(cos(theta5) * (cos(theta3) * sin(alpha3) *
(-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) -
cos(alpha2) * cos(theta1) * sin(theta2)) - cos(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) +
cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) -
$$

$sin(alpha3) * (cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3))) +$
$(cos(theta4) * (cos(theta3) * (cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) +$
$(-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) -$
$cos(alpha2) * cos(theta1) * sin(theta2)) * sin(theta3)) + (cos(alpha3) * cos(theta3) * (-(cos(alpha1) *$
$cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) *$
$cos(theta1) * sin(theta2)) + sin(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) *$
$cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) - cos(alpha3) *$
$(cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) *$
$sin(theta5));$

$DJ[2, 6] := 0;$

$DJ[3, 1] := 0;$

$DJ[3, 2] := cos(theta1) * dtheta1 * sin(alpha1) * (-(d2 * cos(theta1) * sin(alpha1)) + a2 * cos(theta2) *$
$sin(theta1) + a2 * cos(alpha1) * cos(theta1) * sin(theta2) + a3 * cos(theta3) * (cos(theta2) * sin(theta1) +$
$cos(alpha1) * cos(theta1) * sin(theta2)) + d3 * (-(cos(alpha2) * cos(theta1) * sin(alpha1)) -$
$cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) + a3 *$
$(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) -$
$cos(alpha2) * sin(theta1) * sin(theta2)) * sin(theta3) + d6 * (-(cos(theta5) * (cos(theta3) * sin(alpha3) *$
$(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) -$
$cos(alpha2) * sin(theta1) * sin(theta2)) - cos(alpha3) * (-(cos(alpha2) * cos(theta1) * sin(alpha1)) -$
$cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) -$
$sin(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) * sin(theta3))) +$
$(cos(theta4) * (cos(theta3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) +$
$(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) -$
$cos(alpha2) * sin(theta1) * sin(theta2)) * sin(theta3)) + (cos(alpha3) * cos(theta3) * (cos(alpha1) *$
$cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) *$
$sin(theta1) * sin(theta2)) + sin(alpha3) * (-(cos(alpha2) * cos(theta1) * sin(alpha1)) - cos(alpha1) *$
$cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) - cos(alpha3) *$
$(cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) *$
$sin(theta5))) + cos(theta1) * sin(alpha1) * (d2 * cos(theta1) * dtheta1 * sin(alpha1) - a2 * cos(theta2) *$
$dtheta1 * sin(theta1) - a2 * cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) - a2 * cos(alpha1) *$
$cos(theta1) * dtheta1 * sin(theta2) - a2 * cos(theta1) * dtheta2 * sin(theta2) + a3 * cos(theta3) *$
$(-(cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) -$
$cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) - cos(theta1) * dtheta2 * sin(theta2)) + dd3 *$
$(cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) +$
$cos(theta1) * sin(alpha2) * sin(theta2)) + d3 * (cos(alpha2) * cos(theta1) * dtheta1 * sin(alpha1) +$
$cos(alpha1) * cos(theta1) * cos(theta2) * dtheta1 * sin(alpha2) + cos(theta1) * cos(theta2) * dtheta2 *$
$sin(alpha2) - dtheta1 * sin(alpha2) * sin(theta1) * sin(theta2) - cos(alpha1) * dtheta2 * sin(alpha2) *$
$sin(theta1) * sin(theta2)) + a3 * (-(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) -$
$cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) +$
$cos(alpha2) * dtheta1 * sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) *$
$sin(theta2)) * sin(theta3) + d6 * (-(cos(theta5) * (cos(theta3) * sin(alpha3) * (-(cos(alpha1) *$
$cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) - cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 +$
$cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) + cos(alpha2) * dtheta1 * sin(theta1) * sin(theta2) +$
$cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) * sin(theta2)) - cos(alpha3) * (cos(alpha2) *$
$cos(theta1) * dtheta1 * sin(alpha1) + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta1 * sin(alpha2) +$
$cos(theta1) * cos(theta2) * dtheta2 * sin(alpha2) - dtheta1 * sin(alpha2) * sin(theta1) * sin(theta2) -$
$cos(alpha1) * dtheta2 * sin(alpha2) * sin(theta1) * sin(theta2)) - sin(alpha3) * (-(cos(theta2) *$
$dtheta1 * sin(theta1)) - cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) - cos(alpha1) * cos(theta1) *$
$dtheta1 * sin(theta2) - cos(theta1) * dtheta2 * sin(theta2)) * sin(theta3))) + cos(theta5) * dtheta5 *$
$(cos(theta4) * (cos(theta3) * (cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) +$

$(-(cos(alpha1)*cos(alpha2)*cos(theta2)*sin(theta1))+sin(alpha1)*sin(alpha2)*sin(theta1)-$
$cos(alpha2)*cos(theta1)*sin(theta2))*sin(theta3))+(cos(alpha3)*cos(theta3)*(-(cos(alpha1)*$
$cos(alpha2)*cos(theta2)*sin(theta1))+sin(alpha1)*sin(alpha2)*sin(theta1)-cos(alpha2)*$
$cos(theta1)*sin(theta2))+sin(alpha3)*(cos(alpha2)*sin(alpha1)*sin(theta1)+cos(alpha1)*$
$cos(theta2)*sin(alpha2)*sin(theta1)+cos(theta1)*sin(alpha2)*sin(theta2))-cos(alpha3)*$
$(cos(theta1)*cos(theta2)-cos(alpha1)*sin(theta1)*sin(theta2))*sin(theta3))*sin(theta4))+$
$dtheta5*(cos(theta3)*sin(alpha3)*(-(cos(alpha1)*cos(alpha2)*cos(theta2)*sin(theta1))+$
$sin(alpha1)*sin(alpha2)*sin(theta1)-cos(alpha2)*cos(theta1)*sin(theta2))-cos(alpha3)*$
$(cos(alpha2)*sin(alpha1)*sin(theta1)+cos(alpha1)*cos(theta2)*sin(alpha2)*sin(theta1)+$
$cos(theta1)*sin(alpha2)*sin(theta2))-sin(alpha3)*(cos(theta1)*cos(theta2)-cos(alpha1)*$
$sin(theta1)*sin(theta2))*sin(theta3))*sin(theta5)+(cos(theta4)*dtheta4*(cos(alpha3)*$
$cos(theta3)*(-(cos(alpha1)*cos(alpha2)*cos(theta2)*sin(theta1))+sin(alpha1)*sin(alpha2)*$
$sin(theta1)-cos(alpha2)*cos(theta1)*sin(theta2))+sin(alpha3)*(cos(alpha2)*sin(alpha1)*$
$sin(theta1)+cos(alpha1)*cos(theta2)*sin(alpha2)*sin(theta1)+cos(theta1)*sin(alpha2)*$
$sin(theta2))-cos(alpha3)*(cos(theta1)*cos(theta2)-cos(alpha1)*sin(theta1)*sin(theta2))*$
$sin(theta3))+cos(theta4)*(cos(theta3)*(-(cos(theta2)*dtheta1*sin(theta1))-cos(alpha1)*$
$cos(theta2)*dtheta2*sin(theta1)-cos(alpha1)*cos(theta1)*dtheta1*sin(theta2)-cos(theta1)*$
$dtheta2*sin(theta2))+(-(cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2)*dtheta1)-$
$cos(alpha2)*cos(theta1)*cos(theta2)*dtheta2+cos(theta1)*dtheta1*sin(alpha1)*sin(alpha2)+$
$cos(alpha2)*dtheta1*sin(theta1)*sin(theta2)+cos(alpha1)*cos(alpha2)*dtheta2*sin(theta1)*$
$sin(theta2))*sin(theta3))-dtheta4*(cos(theta3)*(cos(theta1)*cos(theta2)-cos(alpha1)*$
$sin(theta1)*sin(theta2))+(-(cos(alpha1)*cos(alpha2)*cos(theta2)*sin(theta1))+sin(alpha1)*$
$sin(alpha2)*sin(theta1)-cos(alpha2)*cos(theta1)*sin(theta2))*sin(theta3))*sin(theta4)+$
$(cos(alpha3)*cos(theta3)*(-(cos(alpha1)*cos(alpha2)*cos(theta1)*cos(theta2)*dtheta1)-$
$cos(alpha2)*cos(theta1)*cos(theta2)*dtheta2+cos(theta1)*dtheta1*sin(alpha1)*sin(alpha2)+$
$cos(alpha2)*dtheta1*sin(theta1)*sin(theta2)+cos(alpha1)*cos(alpha2)*dtheta2*sin(theta1)*$
$sin(theta2))+sin(alpha3)*(cos(alpha2)*cos(theta1)*dtheta1*sin(alpha1)+cos(alpha1)*$
$cos(theta1)*cos(theta2)*dtheta1*sin(alpha2)+cos(theta1)*cos(theta2)*dtheta2*sin(alpha2)-$
$dtheta1*sin(alpha2)*sin(theta1)*sin(theta2)-cos(alpha1)*dtheta2*sin(alpha2)*sin(theta1)*$
$sin(theta2))-cos(alpha3)*(-(cos(theta2)*dtheta1*sin(theta1))-cos(alpha1)*cos(theta2)*$
$dtheta2*sin(theta1)-cos(alpha1)*cos(theta1)*dtheta1*sin(theta2)-cos(theta1)*dtheta2*$
$sin(theta2))*sin(theta3))*sin(theta4))*sin(theta5)))-dtheta1*sin(alpha1)*sin(theta1)*(a2*$
$cos(theta1)*cos(theta2)+d2*sin(alpha1)*sin(theta1)-a2*cos(alpha1)*sin(theta1)*sin(theta2)+$
$d3*(cos(alpha2)*sin(alpha1)*sin(theta1)+cos(alpha1)*cos(theta2)*sin(alpha2)*sin(theta1)+$
$cos(theta1)*sin(alpha2)*sin(theta2))+a3*cos(theta3)*(cos(theta1)*cos(theta2)-cos(alpha1)*$
$sin(theta1)*sin(theta2))+a3*(-(cos(alpha1)*cos(alpha2)*cos(theta2)*sin(theta1))+$
$sin(alpha1)*sin(alpha2)*sin(theta1)-cos(alpha2)*cos(theta1)*sin(theta2))*sin(theta3)+d6*$
$(-(cos(theta5)*(cos(theta3)*sin(alpha3)*(-(cos(alpha1)*cos(alpha2)*cos(theta2)*sin(theta1))+$
$sin(alpha1)*sin(alpha2)*sin(theta1)-cos(alpha2)*cos(theta1)*sin(theta2))-cos(alpha3)*$
$(cos(alpha2)*sin(alpha1)*sin(theta1)+cos(alpha1)*cos(theta2)*sin(alpha2)*sin(theta1)+$
$cos(theta1)*sin(alpha2)*sin(theta2))-sin(alpha3)*(cos(theta1)*cos(theta2)-cos(alpha1)*$
$sin(theta1)*sin(theta2))*sin(theta3)))+(cos(theta4)*(cos(theta3)*(cos(theta1)*cos(theta2)-$
$cos(alpha1)*sin(theta1)*sin(theta2))+(-(cos(alpha1)*cos(alpha2)*cos(theta2)*sin(theta1))+$
$sin(alpha1)*sin(alpha2)*sin(theta1)-cos(alpha2)*cos(theta1)*sin(theta2))*sin(theta3))+$
$(cos(alpha3)*cos(theta3)*(-(cos(alpha1)*cos(alpha2)*cos(theta2)*sin(theta1))+sin(alpha1)*$
$sin(alpha2)*sin(theta1)-cos(alpha2)*cos(theta1)*sin(theta2))+sin(alpha3)*(cos(alpha2)*$
$sin(alpha1)*sin(theta1)+cos(alpha1)*cos(theta2)*sin(alpha2)*sin(theta1)+cos(theta1)*$
$sin(alpha2)*sin(theta2))-cos(alpha3)*(cos(theta1)*cos(theta2)-cos(alpha1)*sin(theta1)*$
$sin(theta2))*sin(theta3))*sin(theta4))*sin(theta5)))+sin(alpha1)*sin(theta1)*(a2*cos(theta1)*$
$cos(theta2)*dtheta1+a2*cos(alpha1)*cos(theta1)*cos(theta2)*dtheta2+d2*dtheta1*sin(alpha1)*$

$$
sin(theta1) - a2 * cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) - a2 * dtheta2 * sin(theta1) *
sin(theta2) + d3 * (cos(alpha2) * dtheta1 * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) *
dtheta1 * sin(alpha2) * sin(theta1) + cos(theta2) * dtheta2 * sin(alpha2) * sin(theta1) + cos(theta1) *
dtheta1 * sin(alpha2) * sin(theta2) + cos(alpha1) * cos(theta1) * dtheta2 * sin(alpha2) * sin(theta2)) +
a3 * cos(theta3) * (cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) * cos(theta2) *
dtheta2 - cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) - dtheta2 * sin(theta1) * sin(theta2)) + dd3 *
(-(cos(alpha2) * cos(theta1) * sin(alpha1)) - cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) +
sin(alpha2) * sin(theta1) * sin(theta2)) + a3 * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 *
sin(theta1)) - cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) *
sin(theta1) - cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) *
cos(theta1) * dtheta2 * sin(theta2)) * sin(theta3) + d6 * (-(cos(theta5) * (cos(theta3) * sin(alpha3) *
(-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha2) * cos(theta2) *
dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) *
dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 * sin(theta2)) - cos(alpha3) *
(cos(alpha2) * dtheta1 * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * dtheta1 * sin(alpha2) *
sin(theta1) + cos(theta2) * dtheta2 * sin(alpha2) * sin(theta1) + cos(theta1) * dtheta1 * sin(alpha2) *
sin(theta2) + cos(alpha1) * cos(theta1) * dtheta2 * sin(alpha2) * sin(theta2)) - sin(alpha3) *
(cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta2 - cos(alpha1) *
dtheta1 * sin(theta1) * sin(theta2) - dtheta2 * sin(theta1) * sin(theta2)) * sin(theta3))) + cos(theta5) *
dtheta5 * (cos(theta4) * (cos(theta3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) *
sin(theta2)) + (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) *
sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) * sin(theta3)) + (cos(alpha3) * cos(theta3) *
(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) -
cos(alpha2) * sin(theta1) * sin(theta2)) + sin(alpha3) * (-(cos(alpha2) * cos(theta1) * sin(alpha1)) -
cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) -
cos(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) * sin(theta3)) *
sin(theta4)) + dtheta5 * (cos(theta3) * sin(alpha3) * (cos(alpha1) * cos(alpha2) * cos(theta1) *
cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) -
cos(alpha3) * (-(cos(alpha2) * cos(theta1) * sin(alpha1)) - cos(alpha1) * cos(theta1) * cos(theta2) *
sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) - sin(alpha3) * (cos(theta2) * sin(theta1) +
cos(alpha1) * cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta5) + (cos(theta4) * dtheta4 *
(cos(alpha3) * cos(theta3) * (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) *
sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) + sin(alpha3) * (-(cos(alpha2) *
cos(theta1) * sin(alpha1)) - cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) *
sin(theta1) * sin(theta2)) - cos(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) *
sin(theta2)) * sin(theta3)) + cos(theta4) * (cos(theta3) * (cos(theta1) * cos(theta2) * dtheta1 +
cos(alpha1) * cos(theta1) * cos(theta2) * dtheta2 - cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) -
dtheta2 * sin(theta1) * sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 *
sin(theta1)) - cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) *
sin(theta1) - cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) *
cos(theta1) * dtheta2 * sin(theta2)) * sin(theta3)) - dtheta4 * (cos(theta3) * (cos(theta2) * sin(theta1) +
cos(alpha1) * cos(theta1) * sin(theta2)) + (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) -
cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) * sin(theta3)) *
sin(theta4) + (cos(alpha3) * cos(theta3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 *
sin(theta1)) - cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) *
sin(theta1) - cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) *
cos(theta1) * dtheta2 * sin(theta2)) + sin(alpha3) * (cos(alpha2) * dtheta1 * sin(alpha1) * sin(theta1) +
cos(alpha1) * cos(theta2) * dtheta1 * sin(alpha2) * sin(theta1) + cos(theta2) * dtheta2 * sin(alpha2) *
sin(theta1) + cos(theta1) * dtheta1 * sin(alpha2) * sin(theta2) + cos(alpha1) * cos(theta1) * dtheta2 *
sin(alpha2) * sin(theta2)) - cos(alpha3) * (cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) *
$$

cos(theta1) * cos(theta2) * dtheta2 − cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) − dtheta2 * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) * sin(theta5)));

DJ[3, 3] := dtheta2 * sin(alpha1) * sin(alpha2) * sin(theta2);

DJ[3, 4] := d6 * (−(cos(theta3) * sin(alpha3) * (−(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) − cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) + cos(alpha2) * dtheta1 * sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) * sin(theta2))) + cos(alpha3) * (cos(alpha2) * cos(theta1) * dtheta1 * sin(alpha1) + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta1 * sin(alpha2) + cos(theta1) * cos(theta2) * dtheta2 * sin(alpha2) − dtheta1 * sin(alpha2) * sin(theta1) * sin(theta2) − cos(alpha1) * dtheta2 * sin(alpha2) * sin(theta1) * sin(theta2)) + sin(alpha3) * (−(cos(theta2) * dtheta1 * sin(theta1)) − cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) − cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) − cos(theta1) * dtheta2 * sin(theta2)) * sin(theta3)) * (−(cos(theta5) * (cos(theta3) * sin(alpha3) * (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) − cos(theta1) * sin(alpha1) * sin(alpha2) − cos(alpha2) * sin(theta1) * sin(theta2)) − cos(alpha3) * (−(cos(alpha2) * cos(theta1) * sin(alpha1)) − cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) − sin(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) * sin(theta3))) + (cos(theta4) * (cos(theta3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) + (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) − cos(theta1) * sin(alpha1) * sin(alpha2) − cos(alpha2) * sin(theta1) * sin(theta2)) * sin(theta3)) + (cos(alpha3) * cos(theta3) * (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) − cos(theta1) * sin(alpha1) * sin(alpha2) − cos(alpha2) * sin(theta1) * sin(theta2)) + sin(alpha3) * (−(cos(alpha2) * cos(theta1) * sin(alpha1)) − cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) − cos(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) * sin(theta5)) − d6 * (−(cos(theta3) * sin(alpha3) * (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) − cos(theta1) * sin(alpha1) * sin(alpha2) − cos(alpha2) * sin(theta1) * sin(theta2))) + cos(alpha3) * (−(cos(alpha2) * cos(theta1) * sin(alpha1)) − cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) + sin(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) * sin(theta3)) * (−(cos(theta5) * (cos(theta3) * sin(alpha3) * (−(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) − cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) + cos(alpha2) * dtheta1 * sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) * sin(theta2)) − cos(alpha3) * (cos(alpha2) * cos(theta1) * dtheta1 * sin(alpha1) + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta1 * sin(alpha2) + cos(theta1) * cos(theta2) * dtheta2 * sin(alpha2) − dtheta1 * sin(alpha2) * sin(theta1) * sin(theta2) − cos(alpha1) * dtheta2 * sin(alpha2) * sin(theta1) * sin(theta2)) − sin(alpha3) * (−(cos(theta2) * dtheta1 * sin(theta1)) − cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) − cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) − cos(theta1) * dtheta2 * sin(theta2)) * sin(theta3))) + cos(theta5) * dtheta5 * (cos(theta4) * (cos(theta3) * (cos(theta1) * cos(theta2) − cos(alpha1) * sin(theta1) * sin(theta2)) + (−(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) − cos(alpha2) * cos(theta1) * sin(theta2)) * sin(theta3)) + (cos(alpha3) * cos(theta3) * (−(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) − cos(alpha2) * cos(theta1) * sin(theta2)) + sin(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) − cos(alpha3) * (cos(theta1) * cos(theta2) − cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) + dtheta5 * (cos(theta3) * sin(alpha3) * (−(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) − cos(alpha2) * cos(theta1) * sin(theta2)) − cos(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) − sin(alpha3) * (cos(theta1) * cos(theta2) − cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta5) + (cos(theta4) * dtheta4 * (cos(alpha3) * cos(theta3) * (−(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) − cos(alpha2) * cos(theta1) *

$sin(theta2)) + sin(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) *$
$sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) - cos(alpha3) * (cos(theta1) *$
$cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3)) + cos(theta4) * (cos(theta3) *$
$(-(cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) -$
$cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) - cos(theta1) * dtheta2 * sin(theta2)) +$
$(-(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) - cos(alpha2) * cos(theta1) *$
$cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) + cos(alpha2) * dtheta1 *$
$sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) * sin(theta2)) *$
$sin(theta3)) - dtheta4 * (cos(theta3) * (cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) *$
$sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) *$
$sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4) + (cos(alpha3) *$
$cos(theta3) * (-(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) - cos(alpha2) *$
$cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) + cos(alpha2) *$
$dtheta1 * sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) * sin(theta2)) +$
$sin(alpha3) * (cos(alpha2) * cos(theta1) * dtheta1 * sin(alpha1) + cos(alpha1) * cos(theta1) *$
$cos(theta2) * dtheta1 * sin(alpha2) + cos(theta1) * cos(theta2) * dtheta2 * sin(alpha2) - dtheta1 *$
$sin(alpha2) * sin(theta1) * sin(theta2) - cos(alpha1) * dtheta2 * sin(alpha2) * sin(theta1) *$
$sin(theta2)) - cos(alpha3) * (-(cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha1) * cos(theta2) *$
$dtheta2 * sin(theta1) - cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) - cos(theta1) * dtheta2 *$
$sin(theta2)) * sin(theta3)) * sin(theta4)) * sin(theta5)) - d6 * (-(cos(theta3) * sin(alpha3) *$
$(-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha2) * cos(theta2) *$
$dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) *$
$dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 * sin(theta2))) +$
$cos(alpha3) * (cos(alpha2) * dtheta1 * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * dtheta1 *$
$sin(alpha2) * sin(theta1) + cos(theta2) * dtheta2 * sin(alpha2) * sin(theta1) + cos(theta1) * dtheta1 *$
$sin(alpha2) * sin(theta2) + cos(alpha1) * cos(theta1) * dtheta2 * sin(alpha2) * sin(theta2)) +$
$sin(alpha3) * (cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta2 -$
$cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) - dtheta2 * sin(theta1) * sin(theta2)) * sin(theta3)) *$
$(-(cos(theta5) * (cos(theta3) * sin(alpha3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) +$
$sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) - cos(alpha3) *$
$(cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) +$
$cos(theta1) * sin(alpha2) * sin(theta2)) - sin(alpha3) * (cos(theta1) * cos(theta2) - cos(alpha1) *$
$sin(theta1) * sin(theta2)) * sin(theta3))) + (cos(theta4) * (cos(theta3) * (cos(theta1) * cos(theta2) -$
$cos(alpha1) * sin(theta1) * sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) +$
$sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) * sin(theta3)) +$
$(cos(alpha3) * cos(theta3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) *$
$sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) + sin(alpha3) * (cos(alpha2) *$
$sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) *$
$sin(alpha2) * sin(theta2)) - cos(alpha3) * (cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) *$
$sin(theta2)) * sin(theta3)) * sin(theta4)) * sin(theta5)) + d6 * (-(cos(theta3) * sin(alpha3) *$
$(-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) -$
$cos(alpha2) * cos(theta1) * sin(theta2))) + cos(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) +$
$cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) +$
$sin(alpha3) * (cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3)) *$
$(-(cos(theta5) * (cos(theta3) * sin(alpha3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 *$
$sin(theta1)) - cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) *$
$sin(theta1) - cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) *$
$cos(theta1) * dtheta2 * sin(theta2)) - cos(alpha3) * (cos(alpha2) * dtheta1 * sin(alpha1) * sin(theta1) +$
$cos(alpha1) * cos(theta2) * dtheta1 * sin(alpha2) * sin(theta1) + cos(theta2) * dtheta2 * sin(alpha2) *$
$sin(theta1) + cos(theta1) * dtheta1 * sin(alpha2) * sin(theta2) + cos(alpha1) * cos(theta1) * dtheta2 *$

$sin(alpha2) \ast sin(theta2)) - sin(alpha3) \ast (cos(theta1) \ast cos(theta2) \ast dtheta1 + cos(alpha1) \ast$
$cos(theta1) \ast cos(theta2) \ast dtheta2 - cos(alpha1) \ast dtheta1 \ast sin(theta1) \ast sin(theta2) - dtheta2 \ast$
$sin(theta1) \ast sin(theta2)) \ast sin(theta3))) + cos(theta5) \ast dtheta5 \ast (cos(theta4) \ast (cos(theta3) \ast$
$(cos(theta2) \ast sin(theta1) + cos(alpha1) \ast cos(theta1) \ast sin(theta2)) + (cos(alpha1) \ast cos(alpha2) \ast$
$cos(theta1) \ast cos(theta2) - cos(theta1) \ast sin(alpha1) \ast sin(alpha2) - cos(alpha2) \ast sin(theta1) \ast$
$sin(theta2)) \ast sin(theta3)) + (cos(alpha3) \ast cos(theta3) \ast (cos(alpha1) \ast cos(alpha2) \ast cos(theta1) \ast$
$cos(theta2) - cos(theta1) \ast sin(alpha1) \ast sin(alpha2) - cos(alpha2) \ast sin(theta1) \ast sin(theta2)) +$
$sin(alpha3) \ast (-(cos(alpha2) \ast cos(theta1) \ast sin(alpha1)) - cos(alpha1) \ast cos(theta1) \ast cos(theta2) \ast$
$sin(alpha2) + sin(alpha2) \ast sin(theta1) \ast sin(theta2)) - cos(alpha3) \ast (cos(theta2) \ast sin(theta1) +$
$cos(alpha1) \ast cos(theta1) \ast sin(theta2)) \ast sin(theta3)) \ast sin(theta4)) + dtheta5 \ast (cos(theta3) \ast$
$sin(alpha3) \ast (cos(alpha1) \ast cos(alpha2) \ast cos(theta1) \ast cos(theta2) - cos(theta1) \ast sin(alpha1) \ast$
$sin(alpha2) - cos(alpha2) \ast sin(theta1) \ast sin(theta2)) - cos(alpha3) \ast (-(cos(alpha2) \ast cos(theta1) \ast$
$sin(alpha1)) - cos(alpha1) \ast cos(theta1) \ast cos(theta2) \ast sin(alpha2) + sin(alpha2) \ast sin(theta1) \ast$
$sin(theta2)) - sin(alpha3) \ast (cos(theta2) \ast sin(theta1) + cos(alpha1) \ast cos(theta1) \ast sin(theta2)) \ast$
$sin(theta3)) \ast sin(theta5) + (cos(theta4) \ast dtheta4 \ast (cos(alpha3) \ast cos(theta3) \ast (cos(alpha1) \ast$
$cos(alpha2) \ast cos(theta1) \ast cos(theta2) - cos(theta1) \ast sin(alpha1) \ast sin(alpha2) - cos(alpha2) \ast$
$sin(theta1) \ast sin(theta2)) + sin(alpha3) \ast (-(cos(alpha2) \ast cos(theta1) \ast sin(alpha1)) - cos(alpha1) \ast$
$cos(theta1) \ast cos(theta2) \ast sin(alpha2) + sin(alpha2) \ast sin(theta1) \ast sin(theta2)) - cos(alpha3) \ast$
$(cos(theta2) \ast sin(theta1) + cos(alpha1) \ast cos(theta1) \ast sin(theta2)) \ast sin(theta3)) + cos(theta4) \ast$
$(cos(theta3) \ast (cos(theta1) \ast cos(theta2) \ast dtheta1 + cos(alpha1) \ast cos(theta1) \ast cos(theta2) \ast$
$dtheta2 - cos(alpha1) \ast dtheta1 \ast sin(theta1) \ast sin(theta2) - dtheta2 \ast sin(theta1) \ast sin(theta2)) +$
$(-(cos(alpha1) \ast cos(alpha2) \ast cos(theta2) \ast dtheta1 \ast sin(theta1)) - cos(alpha2) \ast cos(theta2) \ast$
$dtheta2 \ast sin(theta1) + dtheta1 \ast sin(alpha1) \ast sin(alpha2) \ast sin(theta1) - cos(alpha2) \ast cos(theta1) \ast$
$dtheta1 \ast sin(theta2) - cos(alpha1) \ast cos(alpha2) \ast cos(theta1) \ast dtheta2 \ast sin(theta2)) \ast sin(theta3)) -$
$dtheta4 \ast (cos(theta3) \ast (cos(theta2) \ast sin(theta1) + cos(alpha1) \ast cos(theta1) \ast sin(theta2)) +$
$(cos(alpha1) \ast cos(alpha2) \ast cos(theta1) \ast cos(theta2) - cos(theta1) \ast sin(alpha1) \ast sin(alpha2) -$
$cos(alpha2) \ast sin(theta1) \ast sin(theta2)) \ast sin(theta3)) \ast sin(theta4) + (cos(alpha3) \ast cos(theta3) \ast$
$(-(cos(alpha1) \ast cos(alpha2) \ast cos(theta2) \ast dtheta1 \ast sin(theta1)) - cos(alpha2) \ast cos(theta2) \ast$
$dtheta2 \ast sin(theta1) + dtheta1 \ast sin(alpha1) \ast sin(alpha2) \ast sin(theta1) - cos(alpha2) \ast cos(theta1) \ast$
$dtheta1 \ast sin(theta2) - cos(alpha1) \ast cos(alpha2) \ast cos(theta1) \ast dtheta2 \ast sin(theta2)) + sin(alpha3) \ast$
$(cos(alpha2) \ast dtheta1 \ast sin(alpha1) \ast sin(theta1) + cos(alpha1) \ast cos(theta2) \ast dtheta1 \ast sin(alpha2) \ast$
$sin(theta1) + cos(theta2) \ast dtheta2 \ast sin(alpha2) \ast sin(theta1) + cos(theta1) \ast dtheta1 \ast sin(alpha2) \ast$
$sin(theta2) + cos(alpha1) \ast cos(theta1) \ast dtheta2 \ast sin(alpha2) \ast sin(theta2)) - cos(alpha3) \ast$
$(cos(theta1) \ast cos(theta2) \ast dtheta1 + cos(alpha1) \ast cos(theta1) \ast cos(theta2) \ast dtheta2 - cos(alpha1) \ast$
$dtheta1 \ast sin(theta1) \ast sin(theta2) - dtheta2 \ast sin(theta1) \ast sin(theta2)) \ast sin(theta3)) \ast sin(theta4)) \ast$
$sin(theta5));$

$DJ[3,5] := d6 \ast (-(cos(theta4) \ast dtheta4 \ast (cos(theta3) \ast (cos(theta1) \ast cos(theta2) -$
$cos(alpha1) \ast sin(theta1) \ast sin(theta2)) + (-(cos(alpha1) \ast cos(alpha2) \ast cos(theta2) \ast sin(theta1)) +$
$sin(alpha1) \ast sin(alpha2) \ast sin(theta1) - cos(alpha2) \ast cos(theta1) \ast sin(theta2)) \ast sin(theta3))) +$
$cos(theta4) \ast (cos(alpha3) \ast cos(theta3) \ast (-(cos(alpha1) \ast cos(alpha2) \ast cos(theta1) \ast cos(theta2) \ast$
$dtheta1) - cos(alpha2) \ast cos(theta1) \ast cos(theta2) \ast dtheta2 + cos(theta1) \ast dtheta1 \ast sin(alpha1) \ast$
$sin(alpha2) + cos(alpha2) \ast dtheta1 \ast sin(theta1) \ast sin(theta2) + cos(alpha1) \ast cos(alpha2) \ast dtheta2 \ast$
$sin(theta1) \ast sin(theta2)) + sin(alpha3) \ast (cos(alpha2) \ast cos(theta1) \ast dtheta1 \ast sin(alpha1) +$
$cos(alpha1) \ast cos(theta1) \ast cos(theta2) \ast dtheta1 \ast sin(alpha2) + cos(theta1) \ast cos(theta2) \ast dtheta2 \ast$
$sin(alpha2) - dtheta1 \ast sin(alpha2) \ast sin(theta1) \ast sin(theta2) - cos(alpha1) \ast dtheta2 \ast sin(alpha2) \ast$
$sin(theta1) \ast sin(theta2)) - cos(alpha3) \ast (-(cos(theta2) \ast dtheta1 \ast sin(theta1)) - cos(alpha1) \ast$
$cos(theta2) \ast dtheta2 \ast sin(theta1) - cos(alpha1) \ast cos(theta1) \ast dtheta1 \ast sin(theta2) - cos(theta1) \ast$
$dtheta2 \ast sin(theta2)) \ast sin(theta3)) - dtheta4 \ast (cos(alpha3) \ast cos(theta3) \ast (-(cos(alpha1) \ast$
$cos(alpha2) \ast cos(theta2) \ast sin(theta1)) + sin(alpha1) \ast sin(alpha2) \ast sin(theta1) - cos(alpha2) \ast$
$cos(theta1) \ast sin(theta2)) + sin(alpha3) \ast (cos(alpha2) \ast sin(alpha1) \ast sin(theta1) + cos(alpha1) \ast$

cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) − cos(alpha3) *
(cos(theta1) * cos(theta2) − cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4) −
(cos(theta3) * (−(cos(theta2) * dtheta1 * sin(theta1)) − cos(alpha1) * cos(theta2) * dtheta2 *
sin(theta1) − cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) − cos(theta1) * dtheta2 * sin(theta2)) +
(−(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) − cos(alpha2) * cos(theta1) *
cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) + cos(alpha2) * dtheta1 *
sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) * sin(theta2)) *
sin(theta3)) * sin(theta4)) * (−(cos(theta5) * (cos(theta3) * sin(alpha3) * (cos(alpha1) * cos(alpha2) *
cos(theta1) * cos(theta2) − cos(theta1) * sin(alpha1) * sin(alpha2) − cos(alpha2) * sin(theta1) *
sin(theta2)) − cos(alpha3) * (−(cos(alpha2) * cos(theta1) * sin(alpha1)) − cos(alpha1) * cos(theta1) *
cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) − sin(alpha3) * (cos(theta2) *
sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) * sin(theta3))) + (cos(theta4) * (cos(theta3) *
(cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) + (cos(alpha1) * cos(alpha2) *
cos(theta1) * cos(theta2) − cos(theta1) * sin(alpha1) * sin(alpha2) − cos(alpha2) * sin(theta1) *
sin(theta2)) * sin(theta3)) + (cos(alpha3) * cos(theta3) * (cos(alpha1) * cos(alpha2) * cos(theta1) *
cos(theta2) − cos(theta1) * sin(alpha1) * sin(alpha2) − cos(alpha2) * sin(theta1) * sin(theta2)) +
sin(alpha3) * (−(cos(alpha2) * cos(theta1) * sin(alpha1)) − cos(alpha1) * cos(theta1) * cos(theta2) *
sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) − cos(alpha3) * (cos(theta2) * sin(theta1) +
cos(alpha1) * cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) * sin(theta5)) − d6 *
(cos(theta4) * (cos(alpha3) * cos(theta3) * (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) −
cos(theta1) * sin(alpha1) * sin(alpha2) − cos(alpha2) * sin(theta1) * sin(theta2)) + sin(alpha3) *
(−(cos(alpha2) * cos(theta1) * sin(alpha1)) − cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) +
sin(alpha2) * sin(theta1) * sin(theta2)) − cos(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) *
cos(theta1) * sin(theta2)) * sin(theta3)) − (cos(theta3) * (cos(theta2) * sin(theta1) + cos(alpha1) *
cos(theta1) * sin(theta2)) + (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) − cos(theta1) *
sin(alpha1) * sin(alpha2) − cos(alpha2) * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) *
(−(cos(theta5) * (cos(theta3) * sin(alpha3) * (−(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) *
dtheta1) − cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) *
sin(alpha2) + cos(alpha2) * dtheta1 * sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 *
sin(theta1) * sin(theta2)) − cos(alpha3) * (cos(alpha2) * cos(theta1) * dtheta1 * sin(alpha1) +
cos(alpha1) * cos(theta1) * cos(theta2) * dtheta1 * sin(alpha2) + cos(theta1) * cos(theta2) *
dtheta2 * sin(alpha2) − dtheta1 * sin(alpha2) * sin(theta1) * sin(theta2) − cos(alpha1) * dtheta2 *
sin(alpha2) * sin(theta1) * sin(theta2)) − sin(alpha3) * (−(cos(theta2) * dtheta1 * sin(theta1)) −
cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) − cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) −
cos(theta1) * dtheta2 * sin(theta2)) * sin(theta3))) + cos(theta5) * dtheta5 * (cos(theta4) * (cos(theta3) *
(cos(theta1) * cos(theta2) − cos(alpha1) * sin(theta1) * sin(theta2)) + (−(cos(alpha1) * cos(alpha2) *
cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) − cos(alpha2) * cos(theta1) *
sin(theta2)) * sin(theta3)) + (cos(alpha3) * cos(theta3) * (−(cos(alpha1) * cos(alpha2) * cos(theta2) *
sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) − cos(alpha2) * cos(theta1) * sin(theta2)) +
sin(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) *
sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) − cos(alpha3) * (cos(theta1) * cos(theta2) −
cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) + dtheta5 * (cos(theta3) *
sin(alpha3) * (−(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) *
sin(theta1) − cos(alpha2) * cos(theta1) * sin(theta2)) − cos(alpha3) * (cos(alpha2) * sin(alpha1) *
sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) *
sin(theta2)) − sin(alpha3) * (cos(theta1) * cos(theta2) − cos(alpha1) * sin(theta1) * sin(theta2)) *
sin(theta3)) * sin(theta5) + (cos(theta4) * dtheta4 * (cos(alpha3) * cos(theta3) * (−(cos(alpha1) *
cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) − cos(alpha2) *
cos(theta1) * sin(theta2)) + sin(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) *
cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) − cos(alpha3) *

$(cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3)) + cos(theta4) *$
$(cos(theta3) * (-(cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha1) * cos(theta2) * dtheta2 *$
$sin(theta1) - cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) - cos(theta1) * dtheta2 * sin(theta2)) +$
$(-(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) - cos(alpha2) * cos(theta1) *$
$cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) + cos(alpha2) * dtheta1 *$
$sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) * sin(theta2)) *$
$sin(theta3)) - dtheta4 * (cos(theta3) * (cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) *$
$sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) *$
$sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4) + (cos(alpha3) *$
$cos(theta3) * (-(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) - cos(alpha2) *$
$cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) + cos(alpha2) *$
$dtheta1 * sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) * sin(theta2)) +$
$sin(alpha3) * (cos(alpha2) * cos(theta1) * dtheta1 * sin(alpha1) + cos(alpha1) * cos(theta1) *$
$cos(theta2) * dtheta1 * sin(alpha2) + cos(theta1) * cos(theta2) * dtheta2 * sin(alpha2) - dtheta1 *$
$sin(alpha2) * sin(theta1) * sin(theta2) - cos(alpha1) * dtheta2 * sin(alpha2) * sin(theta1) *$
$sin(theta2)) - cos(alpha3) * (-(cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha1) * cos(theta2) *$
$dtheta2 * sin(theta1) - cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) - cos(theta1) * dtheta2 *$
$sin(theta2)) * sin(theta3)) * sin(theta4)) * sin(theta5)) - d6 * (-(cos(theta4) * dtheta4 * (cos(theta3) *$
$(cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) + (cos(alpha1) * cos(alpha2) *$
$cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) *$
$sin(theta2)) * sin(theta3))) + cos(theta4) * (cos(alpha3) * cos(theta3) * (-(cos(alpha1) * cos(alpha2) *$
$cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 *$
$sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) -$
$cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 * sin(theta2)) + sin(alpha3) * (cos(alpha2) *$
$dtheta1 * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * dtheta1 * sin(alpha2) * sin(theta1) +$
$cos(theta2) * dtheta2 * sin(alpha2) * sin(theta1) + cos(theta1) * dtheta1 * sin(alpha2) * sin(theta2) +$
$cos(alpha1) * cos(theta1) * dtheta2 * sin(alpha2) * sin(theta2)) - cos(alpha3) * (cos(theta1) *$
$cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta2 - cos(alpha1) * dtheta1 *$
$sin(theta1) * sin(theta2) - dtheta2 * sin(theta1) * sin(theta2)) * sin(theta3)) - dtheta4 * (cos(alpha3) *$
$cos(theta3) * (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) *$
$sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) + sin(alpha3) * (-(cos(alpha2) * cos(theta1) *$
$sin(alpha1)) - cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) *$
$sin(theta2)) - cos(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) *$
$sin(theta3)) * sin(theta4) - (cos(theta3) * (cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) *$
$cos(theta1) * cos(theta2) * dtheta2 - cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) - dtheta2 *$
$sin(theta1) * sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) -$
$cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) -$
$cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) * cos(theta1) *$
$dtheta2 * sin(theta2)) * sin(theta3)) * sin(theta4)) * (-(cos(theta5) * (cos(theta3) * sin(alpha3) *$
$(-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) -$
$cos(alpha2) * cos(theta1) * sin(theta2)) - cos(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) +$
$cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) -$
$sin(alpha3) * (cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3))) +$
$(cos(theta4) * (cos(theta3) * (cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) +$
$(-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) -$
$cos(alpha2) * cos(theta1) * sin(theta2)) * sin(theta3)) + (cos(alpha3) * cos(theta3) * (-(cos(alpha1) *$
$cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) *$
$cos(theta1) * sin(theta2)) + sin(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) *$
$cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) - cos(alpha3) *$
$(cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) *$

$sin(theta5)) + d6 * (cos(theta4) * (cos(alpha3) * cos(theta3) * (-(cos(alpha1) * cos(alpha2) *$
$cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) *$
$sin(theta2)) + sin(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) *$
$sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) - cos(alpha3) * (cos(theta1) *$
$cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3)) - (cos(theta3) * (cos(theta1) *$
$cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta2) *$
$sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) *$
$sin(theta3)) * sin(theta4)) * (-(cos(theta5) * (cos(theta3) * sin(alpha3) * (-(cos(alpha1) *$
$cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) +$
$dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) -$
$cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 * sin(theta2)) - cos(alpha3) * (cos(alpha2) *$
$dtheta1 * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * dtheta1 * sin(alpha2) * sin(theta1) +$
$cos(theta2) * dtheta2 * sin(alpha2) * sin(theta1) + cos(theta1) * dtheta1 * sin(alpha2) * sin(theta2) +$
$cos(alpha1) * cos(theta1) * dtheta2 * sin(alpha2) * sin(theta2)) - sin(alpha3) * (cos(theta1) *$
$cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta2 - cos(alpha1) * dtheta1 *$
$sin(theta1) * sin(theta2) - dtheta2 * sin(theta1) * sin(theta2)) * sin(theta3))) + cos(theta5) * dtheta5 *$
$(cos(theta4) * (cos(theta3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) +$
$(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) -$
$cos(alpha2) * sin(theta1) * sin(theta2)) * sin(theta3)) + (cos(alpha3) * cos(theta3) * (cos(alpha1) *$
$cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) *$
$sin(theta1) * sin(theta2)) + sin(alpha3) * (-(cos(alpha2) * cos(theta1) * sin(alpha1)) - cos(alpha1) *$
$cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) - cos(alpha3) *$
$(cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) +$
$dtheta5 * (cos(theta3) * sin(alpha3) * (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) -$
$cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) - cos(alpha3) *$
$(-(cos(alpha2) * cos(theta1) * sin(alpha1)) - cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) +$
$sin(alpha2) * sin(theta1) * sin(theta2)) - sin(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) *$
$cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta5) + (cos(theta4) * dtheta4 * (cos(alpha3) *$
$cos(theta3) * (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) *$
$sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) + sin(alpha3) * (-(cos(alpha2) * cos(theta1) *$
$sin(alpha1)) - cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) *$
$sin(theta2)) - cos(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) *$
$sin(theta3)) + cos(theta4) * (cos(theta3) * (cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) *$
$cos(theta1) * cos(theta2) * dtheta2 - cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) - dtheta2 *$
$sin(theta1) * sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) -$
$cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) -$
$cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 *$
$sin(theta2)) * sin(theta3)) - dtheta4 * (cos(theta3) * (cos(theta2) * sin(theta1) + cos(alpha1) *$
$cos(theta1) * sin(theta2)) + (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) *$
$sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4) +$
$(cos(alpha3) * cos(theta3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) -$
$cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) -$
$cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 *$
$sin(theta2)) + sin(alpha3) * (cos(alpha2) * dtheta1 * sin(alpha1) * sin(theta1) + cos(alpha1) *$
$cos(theta2) * dtheta1 * sin(alpha2) * sin(theta1) + cos(theta2) * dtheta2 * sin(alpha2) * sin(theta1) +$
$cos(theta1) * dtheta1 * sin(alpha2) * sin(theta2) + cos(alpha1) * cos(theta1) * dtheta2 * sin(alpha2) *$
$sin(theta2)) - cos(alpha3) * (cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) *$
$cos(theta2) * dtheta2 - cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) - dtheta2 * sin(theta1) *$
$sin(theta2)) * sin(theta3)) * sin(theta4)) * sin(theta5));$

$DJ[3, 6] := 0;$

$DJ[4,1] := 0;$

$DJ[4,2] := cos(theta1) * dtheta1 * sin(alpha1);$

$DJ[4,3] := 0;$

$$DJ[4,4] := -(cos(theta3) * sin(alpha3) * (-(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) - cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) + cos(alpha2) * dtheta1 * sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) * sin(theta2))) + cos(alpha3) * (cos(alpha2) * cos(theta1) * dtheta1 * sin(alpha1) + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta1 * sin(alpha2) + cos(theta1) * cos(theta2) * dtheta2 * sin(alpha2) - dtheta1 * sin(alpha2) * sin(theta1) * sin(theta2) - cos(alpha1) * dtheta2 * sin(alpha2) * sin(theta1) * sin(theta2)) + sin(alpha3) * (-(cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) - cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) - cos(theta1) * dtheta2 * sin(theta2)) * sin(theta3);$$

$$DJ[4,5] := -(cos(theta4) * dtheta4 * (cos(theta3) * (cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) * sin(theta3))) + cos(theta4) * (cos(alpha3) * cos(theta3) * (-(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) - cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) + cos(alpha2) * dtheta1 * sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) * sin(theta2)) + sin(alpha3) * (cos(alpha2) * cos(theta1) * dtheta1 * sin(alpha1) + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta1 * sin(alpha2) + cos(theta1) * cos(theta2) * dtheta2 * sin(alpha2) - dtheta1 * sin(alpha2) * sin(theta1) * sin(theta2) - cos(alpha1) * dtheta2 * sin(alpha2) * sin(theta1) * sin(theta2)) - cos(alpha3) * (-(cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) - cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) - cos(theta1) * dtheta2 * sin(theta2)) * sin(theta3)) - dtheta4 * (cos(alpha3) * cos(theta3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) + sin(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) - cos(alpha3) * (cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4) - (cos(theta3) * (-(cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) - cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) - cos(theta1) * dtheta2 * sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) - cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) + cos(alpha2) * dtheta1 * sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4);$$

$$DJ[4,6] := -(cos(theta5) * (cos(theta3) * sin(alpha3) * (-(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) - cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) + cos(alpha2) * dtheta1 * sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) * sin(theta2)) - cos(alpha3) * (cos(alpha2) * cos(theta1) * dtheta1 * sin(alpha1) + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta1 * sin(alpha2) + cos(theta1) * cos(theta2) * dtheta2 * sin(alpha2) - dtheta1 * sin(alpha2) * sin(theta1) * sin(theta2) - cos(alpha1) * dtheta2 * sin(alpha2) * sin(theta1) * sin(theta2)) - sin(alpha3) * (-(cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) - cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) - cos(theta1) * dtheta2 * sin(theta2)) * sin(theta3))) + cos(theta5) * dtheta5 * (cos(theta4) * (cos(theta3) * (cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) * sin(theta3)) + (cos(alpha3) * cos(theta3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * sin(theta2)) + sin(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) - cos(alpha3) * (cos(theta1) * cos(theta2) - cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) +$$

dtheta5 * (cos(theta3) * sin(alpha3) * (−(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) − cos(alpha2) * cos(theta1) * sin(theta2)) − cos(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) − sin(alpha3) * (cos(theta1) * cos(theta2) − cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta5) + (cos(theta4) * dtheta4 * (cos(alpha3) * cos(theta3) * (−(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) − cos(alpha2) * cos(theta1) * sin(theta2)) + sin(alpha3) * (cos(alpha2) * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * sin(alpha2) * sin(theta1) + cos(theta1) * sin(alpha2) * sin(theta2)) − cos(alpha3) * (cos(theta1) * cos(theta2) − cos(alpha1) * sin(theta1) * sin(theta2)) * sin(theta3)) + cos(theta4) * (cos(theta3) * (−(cos(theta2) * dtheta1 * sin(theta1)) − cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) − cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) − cos(theta1) * dtheta2 * sin(theta2)) + (−(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) − cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) + cos(alpha2) * dtheta1 * sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) * sin(theta2)) * sin(theta3)) − dtheta4 * (cos(theta3) * (cos(theta1) * cos(theta2) − cos(alpha1) * sin(theta1) * sin(theta2)) + (−(cos(alpha1) * cos(alpha2) * cos(theta2) * sin(theta1)) + sin(alpha1) * sin(alpha2) * sin(theta1) − cos(alpha2) * cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4) + (cos(alpha3) * cos(theta3) * (−(cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) * dtheta1) − cos(alpha2) * cos(theta1) * cos(theta2) * dtheta2 + cos(theta1) * dtheta1 * sin(alpha1) * sin(alpha2) + cos(alpha2) * dtheta1 * sin(theta1) * sin(theta2) + cos(alpha1) * cos(alpha2) * dtheta2 * sin(theta1) * sin(theta2)) + sin(alpha3) * (cos(alpha2) * cos(theta1) * dtheta1 * sin(alpha1) + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta1 * sin(alpha2) + cos(theta1) * cos(theta2) * dtheta2 * sin(alpha2) − dtheta1 * sin(alpha2) * sin(theta1) * sin(theta2) − cos(alpha1) * dtheta2 * sin(alpha2) * sin(theta1) * sin(theta2)) − cos(alpha3) * (−(cos(theta2) * dtheta1 * sin(theta1)) − cos(alpha1) * cos(theta2) * dtheta2 * sin(theta1) − cos(alpha1) * cos(theta1) * dtheta1 * sin(theta2) − cos(theta1) * dtheta2 * sin(theta2)) * sin(theta3)) * sin(theta4)) * sin(theta5);

DJ[5, 1] := 0;

DJ[5, 2] := dtheta1 * sin(alpha1) * sin(theta1);

DJ[5, 3] := 0;

DJ[5, 4] := −(cos(theta3) * sin(alpha3) * (−(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) − cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) − cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) − cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 * sin(theta2))) + cos(alpha3) * (cos(alpha2) * dtheta1 * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * dtheta1 * sin(alpha2) * sin(theta1) + cos(theta2) * dtheta2 * sin(alpha2) * sin(theta1) + cos(theta1) * dtheta1 * sin(alpha2) * sin(theta2) + cos(alpha1) * cos(theta1) * dtheta2 * sin(alpha2) * sin(theta2)) + sin(alpha3) * (cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta2 − cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) − dtheta2 * sin(theta1) * sin(theta2)) * sin(theta3);

DJ[5, 5] := −(cos(theta4) * dtheta4 * (cos(theta3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) + (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) − cos(theta1) * sin(alpha1) * sin(alpha2) − cos(alpha2) * sin(theta1) * sin(theta2)) * sin(theta3))) + cos(theta4) * (cos(alpha3) * cos(theta3) * (−(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) − cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) − cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) − cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 * sin(theta2)) + sin(alpha3) * (cos(alpha2) * dtheta1 * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * dtheta1 * sin(alpha2) * sin(theta1) + cos(theta2) * dtheta2 * sin(alpha2) * sin(theta1) + cos(theta1) * dtheta1 * sin(alpha2) * sin(theta2) + cos(alpha1) * cos(theta1) * dtheta2 * sin(alpha2) * sin(theta2)) − cos(alpha3) * (cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta2 − cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) − dtheta2 * sin(theta1) * sin(theta2)) * sin(theta3)) − dtheta4 * (cos(alpha3) * cos(theta3) * (cos(alpha1) * cos(alpha2) *

$$cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) + sin(alpha3) * (-(cos(alpha2) * cos(theta1) * sin(alpha1)) - cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) - cos(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4) - (cos(theta3) * (cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta2 - cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) - dtheta2 * sin(theta1) * sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 * sin(theta2)) * sin(theta3)) * sin(theta4);$$

$$DJ[5,6] := -(cos(theta5) * (cos(theta3) * sin(alpha3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 * sin(theta2)) - cos(alpha3) * (cos(alpha2) * dtheta1 * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * dtheta1 * sin(alpha2) * sin(theta1) + cos(theta2) * dtheta2 * sin(alpha2) * sin(theta1) + cos(theta1) * dtheta1 * sin(alpha2) * sin(theta2) + cos(alpha1) * cos(theta1) * dtheta2 * sin(alpha2) * sin(theta2)) - sin(alpha3) * (cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta2 - cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) - dtheta2 * sin(theta1) * sin(theta2)) * sin(theta3))) + cos(theta5) * dtheta5 * (cos(theta4) * (cos(theta3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) + (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) * sin(theta3)) + (cos(alpha3) * cos(theta3) * (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) + sin(alpha3) * (-(cos(alpha2) * cos(theta1) * sin(alpha1)) - cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) - cos(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) + dtheta5 * (cos(theta3) * sin(alpha3) * (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) - cos(alpha3) * (-(cos(alpha2) * cos(theta1) * sin(alpha1)) - cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) - sin(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) * sin(theta3)) * sin(theta5) + (cos(theta4) * dtheta4 * (cos(alpha3) * cos(theta3) * (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) + sin(alpha3) * (-(cos(alpha2) * cos(theta1) * sin(alpha1)) - cos(alpha1) * cos(theta1) * cos(theta2) * sin(alpha2) + sin(alpha2) * sin(theta1) * sin(theta2)) - cos(alpha3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) * sin(theta3)) + cos(theta4) * (cos(theta3) * (cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta2 - cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) - dtheta2 * sin(theta1) * sin(theta2)) + (-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 * sin(theta2)) * sin(theta3)) - dtheta4 * (cos(theta3) * (cos(theta2) * sin(theta1) + cos(alpha1) * cos(theta1) * sin(theta2)) + (cos(alpha1) * cos(alpha2) * cos(theta1) * cos(theta2) - cos(theta1) * sin(alpha1) * sin(alpha2) - cos(alpha2) * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4) + (cos(alpha3) * cos(theta3) * (-(cos(alpha1) * cos(alpha2) * cos(theta2) * dtheta1 * sin(theta1)) - cos(alpha2) * cos(theta2) * dtheta2 * sin(theta1) + dtheta1 * sin(alpha1) * sin(alpha2) * sin(theta1) - cos(alpha2) * cos(theta1) * dtheta1 * sin(theta2) - cos(alpha1) * cos(alpha2) * cos(theta1) * dtheta2 * sin(theta2)) + sin(alpha3) * (cos(alpha2) * dtheta1 * sin(alpha1) * sin(theta1) + cos(alpha1) * cos(theta2) * dtheta1 * sin(alpha2) * sin(theta1) + cos(theta2) * dtheta2 * sin(alpha2) * sin(theta1) + cos(theta1) * dtheta1 * sin(alpha2) * sin(theta2) + cos(alpha1) * cos(theta1) * dtheta2 * sin(alpha2) * sin(theta2)) - cos(alpha3) * (cos(theta1) * cos(theta2) * dtheta1 + cos(alpha1) * cos(theta1) * cos(theta2) * dtheta2 - cos(alpha1) * dtheta1 * sin(theta1) * sin(theta2) - dtheta2 * sin(theta1) * sin(theta2)) * sin(theta3)) * sin(theta4)) *$$

$sin(theta5);$

$DJ[6,1] := 0;$

$DJ[6,2] := 0;$

$DJ[6,3] := 0;$

$DJ[6,4] := cos(alpha3) * dtheta2 * sin(alpha1) * sin(alpha2) * sin(theta2) + cos(alpha2) * cos(theta3) * dtheta2 * sin(alpha1) * sin(alpha3) * sin(theta2) + cos(theta2) * dtheta2 * sin(alpha1) * sin(alpha3) * sin(theta3);$

$DJ[6,5] := cos(theta4) * (-(cos(alpha2) * cos(alpha3) * cos(theta3) * dtheta2 * sin(alpha1) * sin(theta2)) + dtheta2 * sin(alpha1) * sin(alpha2) * sin(alpha3) * sin(theta2) - cos(alpha3) * cos(theta2) * dtheta2 * sin(alpha1) * sin(theta3)) - cos(theta4) * dtheta4 * (cos(theta3) * sin(alpha1) * sin(theta2) + (cos(alpha2) * cos(theta2) * sin(alpha1) + cos(alpha1) * sin(alpha2)) * sin(theta3)) - dtheta4 * (cos(alpha3) * cos(theta3) * (cos(alpha2) * cos(theta2) * sin(alpha1) + cos(alpha1) * sin(alpha2)) + (cos(alpha1) * cos(alpha2) - cos(theta2) * sin(alpha1) * sin(alpha2)) * sin(alpha3) - cos(alpha3) * sin(alpha1) * sin(theta2) * sin(theta3)) * sin(theta4) - (cos(theta2) * cos(theta3) * dtheta2 * sin(alpha1) - cos(alpha2) * dtheta2 * sin(alpha1) * sin(theta2) * sin(theta3)) * sin(theta4);$

$DJ[6,6] := -(cos(theta5) * (-(cos(alpha3) * dtheta2 * sin(alpha1) * sin(alpha2) * sin(theta2)) - cos(alpha2) * cos(theta3) * dtheta2 * sin(alpha1) * sin(alpha3) * sin(theta2) - cos(theta2) * dtheta2 * sin(alpha1) * sin(alpha3) * sin(theta3))) + cos(theta5) * dtheta5 * (cos(theta4) * (cos(theta3) * sin(alpha1) * sin(theta2) + (cos(alpha2) * cos(theta2) * sin(alpha1) + cos(alpha1) * sin(alpha2)) * sin(theta3)) + (cos(alpha3) * cos(theta3) * (cos(alpha2) * cos(theta2) * sin(alpha1) + cos(alpha1) * sin(alpha2)) + (cos(alpha1) * cos(alpha2) - cos(theta2) * sin(alpha1) * sin(alpha2)) * sin(alpha3) - cos(alpha3) * sin(alpha1) * sin(theta2) * sin(theta3)) * sin(theta4)) + dtheta5 * (-(cos(alpha3) * (cos(alpha1) * cos(alpha2) - cos(theta2) * sin(alpha1) * sin(alpha2))) + cos(theta3) * (cos(alpha2) * cos(theta2) * sin(alpha1) + cos(alpha1) * sin(alpha2)) * sin(alpha3) - sin(alpha1) * sin(alpha3) * sin(theta2) * sin(theta3)) * sin(theta5) + (cos(theta4) * dtheta4 * (cos(alpha3) * cos(theta3) * (cos(alpha2) * cos(theta2) * sin(alpha1) + cos(alpha1) * sin(alpha2)) + (cos(alpha1) * cos(alpha2) - cos(theta2) * sin(alpha1) * sin(alpha2)) * sin(alpha3) - cos(alpha3) * sin(alpha1) * sin(theta2) * sin(theta3)) + cos(theta4) * (cos(theta2) * cos(theta3) * dtheta2 * sin(alpha1) - cos(alpha2) * dtheta2 * sin(alpha1) * sin(theta2) * sin(theta3)) + (-(cos(alpha2) * cos(alpha3) * cos(theta3) * dtheta2 * sin(alpha1) * sin(theta2)) + dtheta2 * sin(alpha1) * sin(alpha2) * sin(alpha3) * sin(theta2) - cos(alpha3) * cos(theta2) * dtheta2 * sin(alpha1) * sin(theta3)) * sin(theta4) - dtheta4 * (cos(theta3) * sin(alpha1) * sin(theta2) + (cos(alpha2) * cos(theta2) * sin(alpha1) + cos(alpha1) * sin(alpha2)) * sin(theta3)) * sin(theta4)) * sin(theta5);$

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