Microlocal Analysis of Bistatic SAR Imaging

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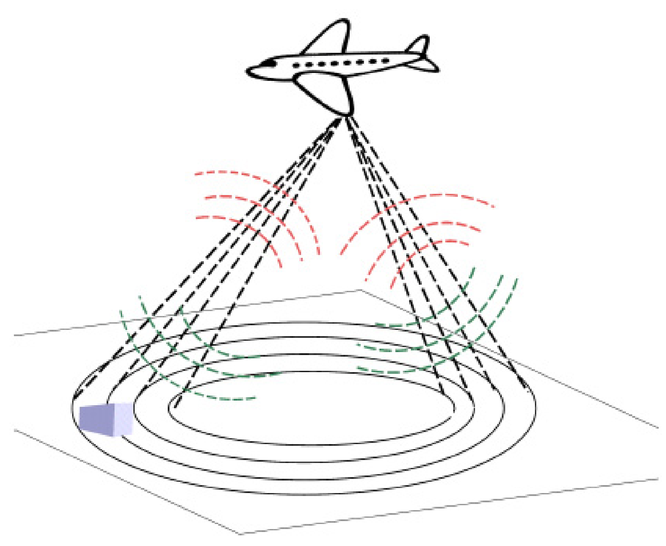
What is SAR imaging?

Synthetic Aperture Radar (SAR) imaging

- Region illuminated by electromagnetic (EM) waves from a moving airborne platform
- For each fixed antenna position, EM waves are sent for a time interval and the scattered waves measured
- Region imaged based on the measurement of scattered waves

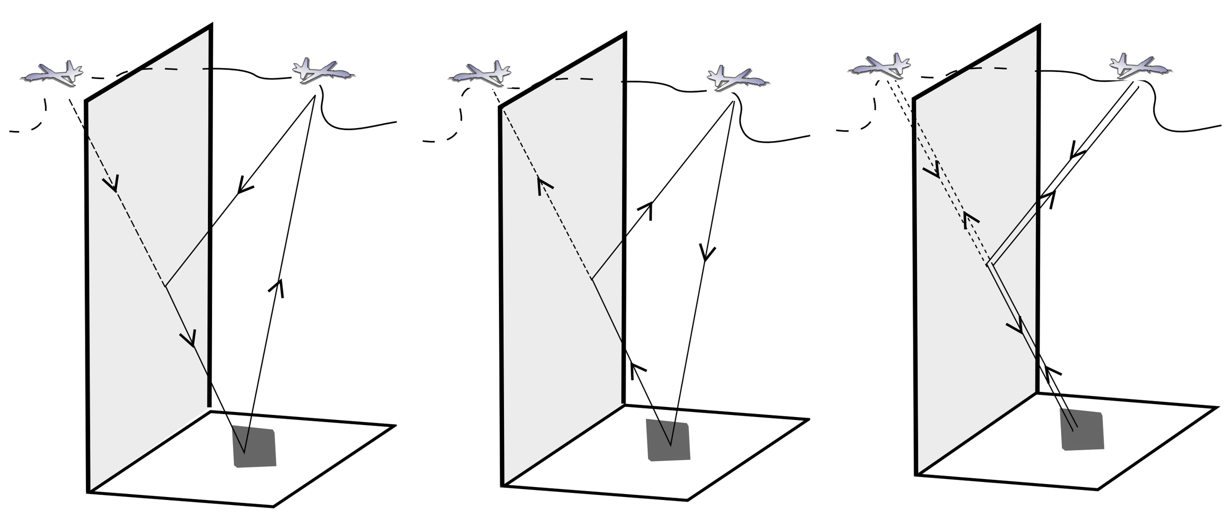


Monostatic and bistatic SAR

- Monostatic SAR – one moving platform has both transmitter and receiver
  The scattered data under simplifying assumptions modeled as integrals of a function over circles
- Bistatic SAR – transmitter and receiver follow independent trajectories
  The scattered data under simplifying assumptions modeled as integrals of a function over ellipses
  Ellipses – intersection with the ground of the ellipsoids of revolution with the transmitter and receiver locations as the foci

Advantages of bistatic SAR

- Receivers are passive. Can be flown in unsafe environment. Transmitters can be detected. Its movement is restricted to a safe environment.
- Some objects capable of beam steering, i.e., scatter signals in a direction different from the incoming one
- Bistatic data models arise in certain multipath data models



Main questions

- Reconstruct an image of the region with a bistatic radar imaging setup
- Reconstruct the singularities of the region
- The reconstruction operator introduces additional singularities
- Understand the strength of the added singularities in comparison to the true singularities

The mathematical model

$$\mathcal{F}V(s,t)=\int e^{-\mathrm{i}\omega(t-\frac{1}{c_0}R(x,s))}A(s,t,x,\omega)V(x)\mathrm{d}x\mathrm{d}\omega,$$

for $(s,t)\in\underbrace{(s_0,s_1)\times(t_0,t_1)}_{Y\text{ space}}$.

$\gamma_T(s)$ and $\gamma_R(s)$ – Trajectories of the transmitter and receiver, c_0 – Speed of light, s – slow time and t – fast time

$R(x,s)=|\gamma_T(s)-(x,0)|+|(x,0)-\gamma_R(s)|$ (bistatic distance)

$A(s,t,x,\omega)$ – Takes into account geometric spreading factors of the electromagnetic wave etc. Assume that A satisfies a decay estimate of order 2.

Analysis of the operator $\mathcal{F}$ for arbitrary transmitter and receiver trajectories is a hard problem

$\gamma_T(s)=(s+\alpha,0,h)$ and $\gamma_R(s)=(s-\alpha,0,h)$; α and h are positive constants

$\mathcal{F}$ is a Fourier integral operator of order $3/2$.

Analysis of singularities

Study the normal operator $\mathcal{F}^*\mathcal{F}$

This is not a ΨDO

We analyze the geometry of the canonical relation Λ of $\mathcal{F}$

Geometry of the canonical relation

To understand the microlocal mapping properties of $\mathcal{F}$ and $\mathcal{F}^*\mathcal{F}$, we consider the projections $\pi_L: T^*Y\times T^*X\to T^*Y$ and $\pi_R: T^*Y\times T^*X\to T^*X$ restricted to Λ. Here $X\subset\mathbb{R}^2$ is the object domain. We prove the following:

(a) The projection π_L restricted to Λ has a fold singularity on a submanifold of Σ.

(b) The projection π_R restricted to Λ has a blowdown singularity on the same submanifold as in (a).

The normal operator $\mathcal{F}^*\mathcal{F}$

- Let K be the kernel of $\mathcal{F}^*\mathcal{F}$. Then the wavefront set (WF) of K satisfies $\mathrm{WF}(K)'\subset\Delta\cup\widetilde{\Lambda}$. Here Δ contributes the true singularities of the object. $\widetilde{\Lambda}$ contributes the added singularities.
- We show that the normal operator $\mathcal{F}^*\mathcal{F}$ is not a ΨDO. Furthermore we characterize the strength of the added singularities and show that it is the same as that of the object singularities.
- Preprint available at `http://arxiv.org/abs/1008.0687`

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