A Novel Implementation of the Perturbation Technique for Better Integration of NUTLs with Periodic Geometry

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***Abstract*—In this work, a novel implementation of the perturbative technique (PT) recently proposed in [1] for the solution of nonuniform transmission-lines (NUTLs) is presented. Unlike the original PT, the proposed method provides a $2n$-port S-parameter representation of the NUTL under analysis, which can be afterwards used in combination with different terminal conditions and/or cascaded with other $2n$-port networks. As an application example, an interdigital tabbed microstrip line terminated in SMA connectors and involving a bend discontinuity is solved by the proposed technique. The obtained predictions are validated versus those provided by a full-wave solver.**

***Index Terms*—tabbed lines, non-uniform transmission lines, scattering parameters, scattering transfer parameters, perturbative technique, bend discontinuities.**

I. Introduction

Simulation of high-speed interconnects on printed circuit boards (PCBs) often requires to model the line as a nonuniform transmission line (NUTL) due to either desired or undesired geometrical and/or material non-uniformity. Though full-wave simulation is a common practice to accurately evaluate the performance of such interconnects, resorting to multiconductor transmission-line (MTL) theory results to be computationally more effective as long as the assumption of transverse electromagnetic (TEM) propagation is satisfied (which is actually the case in a wide range of frequencies). The standard MTL-based approach to the solution of NUTLs foresees to subdivide the line into short line-sections with nearly-constant cross-sections. Every line section is then characterized at the output ports, and then cascaded with the other sections to evaluate voltages and currents at line terminals [2]. Accuracy and computational burden of the technique, known as Uniform Cascaded Section (UCS) method, strongly depend on the number of sub-sections.

To reduce the computational time, a perturbative technique (PT) was recently proposed in [1], [3], which treats line non-uniformity as a perturbation of a (average) uniform MTL. According to this approach, the original MTL equations with place-dependent per-unit-length (p.u.l.) matrices are suitably recast as the equations of a uniform line driven by equivalent distributed sources, accounting for line nonuniformity, which are iteratively updated to achieve accurate predictions at line terminals. Although this technique was proven to be computationally more efficient than the UCS method [1], [4]–[7], it offers less flexibility in terms of terminal conditions. As matter of fact, evaluation of voltages and currents at line terminals involves the specific set of loads at the terminals of the MTL under analysis. This introduces a twofold drawback. First, if terminal conditions change, the solution has to be repeated from scratches. Second, cascading the NUTL under analysis with other lumped/distributed networks is no longer possible.

In this paper, such a limitation is overcome by resorting to a $2n$-port characterization of the NUTL under analysis in terms of S-parameter matrices, whose entries are evaluated through the conventional PT by exploiting standard 50 Ω terminal conditions. The obtained $2n$-port S-parameter representation offers the advantage to be used in combination with different terminal conditions and/or cascaded with other lumped or distributed networks. Feasibility and prediction accuracy of the proposed technique is assessed versus full-wave simulation by an application example involving a tabbed differential microstrip line with tapered terminal sections and SMA connectors, and exhibiting a bend discontinuity.

Though not shown here due to limited space, this technique is also wide applicable to various emerging microstrip structures designed for far-end crosstalk (FEXT) elimination reduction, impedance control, and common-mode rejection (e.g., geometries in [8]–[10]), together with different types of connectors as far as the measurement or full-wave simulation result of S-parameters is available.

II. Limitations of Conventional PT

According to the original PT method, MTL voltages (and currents) are evaluated as the sum of different-order contributions, i.e,

$$\mathbf{V}(z) = \mathbf{V}_0(z) + \mathbf{V}_1(z) + \mathbf{V}_2(z) + \cdots \tag{1}$$

with iterative solution of the corresponding MTL equations as

$$\begin{aligned} &\frac{d}{dz}\mathbf{V}_0(z) + j\omega\bar{\mathbf{Z}}\mathbf{I}_0(z) = 0 \\ &\frac{d}{dz}\mathbf{V}_1(z) + \bar{\mathbf{Z}}\mathbf{I}_1(z) = -\mathbf{\Delta Z}(z)\mathbf{I}_0(z) \\ &\frac{d}{dz}\mathbf{V}_2(z) + \bar{\mathbf{Z}}\mathbf{I}_2(z) = -\mathbf{\Delta Z}(z)\mathbf{I}_1(z) \end{aligned} \tag{2}$$

Similar equations are cast for the currents:

$$\begin{aligned} &\mathbf{I}(z) = \mathbf{I}_0(z) + \mathbf{I}_1(z) + \mathbf{I}_2(z) + \cdots \\ &\frac{d}{dz}\mathbf{I}_0(z) + j\omega\bar{\mathbf{Y}}\mathbf{V}_0(z) = 0 \\ &\frac{d}{dz}\mathbf{I}_1(z) + \bar{\mathbf{Y}}\mathbf{V}_1(z) = -\mathbf{\Delta Y}(z)\mathbf{V}_0(z) \\ &\frac{d}{dz}\mathbf{I}_2(z) + \bar{\mathbf{Y}}\mathbf{V}_2(z) = -\mathbf{\Delta Y}(z)\mathbf{V}_1(z) \end{aligned} \tag{3}$$

In these equations, the right-hand side terms take the meaning of distributed sources and depend on the entries of the perturbation p.u.l. parameter matrices $\mathbf{\Delta Z}(z)$, $\mathbf{\Delta Y}(z)$ through voltages and currents evaluated at the previous step. Line solution is achieved iteratively by integrating the distributed sources along the line length.

Albeit the method is generally faster than the UCS, it is less versatile since the obtained solution is valid for a specific set of terminal constraints only. As a consequence: (a) the procedure must be repeated from scratches whenever the terminal loads change, and (b) it is not suitable to cascade different MTL sections. This latter issue becomes especially relevant if the MTL under analysis exhibits discontinuities (e.g., due to the presence of bends or connectors) or a periodic non-uniform geometry (as shown in this work).

III. Basic Principles of the Proposed Method

To overcome the limitations of the traditional PT, the whole interconnection is converted into the cascade connection of blocks, characterized in terms of chain or S-parameter matrices. In line with this idea, every n+1 NUTLs section is treated as an $2n$-port network, and fully characterized in terms of S-parameter matrices, whose entries are evaluated column-by-column by the PT.

The proposed solution procedure encompasses the following steps:

1) Starting from the left side, the entries of the i-th column ($i = 1, \ldots, n$) of the S-parameter matrix are evaluated by imposing $\hat{V}_{S,i} = 2\sqrt{Z_c}$ and $\hat{V}_{S,j} = 0\,(j \neq i)$ (Z_c being the reference impedance here set to 50 Ω), and by assuming the impedances of all ports equal to Z_c. The entries of the i-th column of the S-parameter matrix are evaluated starting from voltages and currents calculated at the $2n$ ports.
2) Repeat previous calculations for the other n-1 columns, by exchanging the position of the non-null voltage source.
3) Move to the right side, and repeat previous calculations for the other n columns of the matrix, by properly exchanging the position of the voltage source.

To interface the NUTL under analysis with additional lumped or distributed networks (preliminary characterized in terms of S-parameters through vector network analyzer measurement, full-wave simulation, or circuit representation), the obtained S-parameters are converted into T-parameter notation, according to the definition in Fig. 1(a), and

$$\begin{aligned} \mathbf{T} &= \begin{bmatrix} \mathbf{T}_{\text{I,I}} & \mathbf{T}_{\text{I,II}} \\ \mathbf{T}_{\text{II,I}} & \mathbf{T}_{\text{II,II}} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{S}_{\text{II,I}}^{-1} & -\mathbf{S}_{\text{II,I}}^{-1}\mathbf{S}_{\text{II,II}} \\ \mathbf{S}_{\text{I,I}}\mathbf{S}_{\text{II,I}}^{-1} & \mathbf{S}_{\text{I,II}} - \mathbf{S}_{\text{I,I}}\mathbf{S}_{\text{II,I}}^{-1}\mathbf{S}_{\text{II,II}} \end{bmatrix} \end{aligned} \tag{4}$$

The resulting T-parameter matrices are then cascaded in sequence (e.g., Fig. 1). Complex microstrip structures, involving repeated basic cells, are efficiently managed by taking advantage of their repetitive behavior. Other T-parameter definitions are also available in the literature (e.g., [11]), and the results can be deducted correspondingly. Once the T-parameter representation of the full system is available, terminal constraints (in the form of equivalent Thevenin or Norton representations) are incorporated to obtain voltages/currents at line terminals.

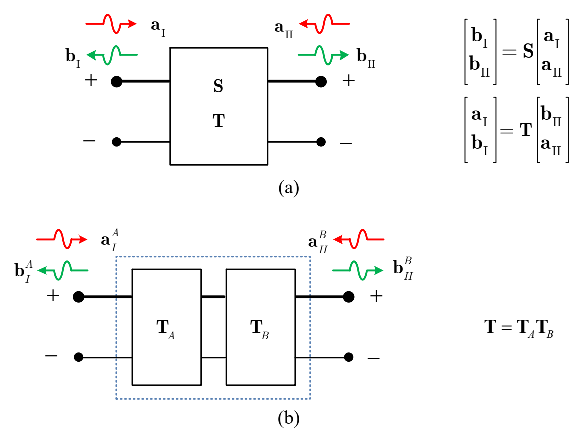

Fig. 1. (a) $2n$-port network representation in terms of S- and T-parameters; and (b) cascade connection of two T-parameter matrices. $\mathbf{a}$ and $\mathbf{b}$ are vectors of forward/reflected waves, respectively; subscripts I and II denote the left and right ends.

IV. Case Study

In this Section, the proposed approach is applied for the analysis of trapezoidal, tabbed differential microstrip lines [8], [12]–[14]. It is worth mentioning that the objective here is not to optimize the microstrip design, but just to provide an example of application of the proposed solution procedure.

As shown in Fig. 2, the microstrip under analysis is terminated into SMA connectors (in view of experimental characterization at the output ports) by means of tapered line-sections assuring smooth impedance transition. Furthermore, the two interdigital tabbed differential line sections are connected through a bend section, which causes undesired DM into CM conversion. In spite of the availability of circuit representation for the bend, e.g., [15], in this work such a

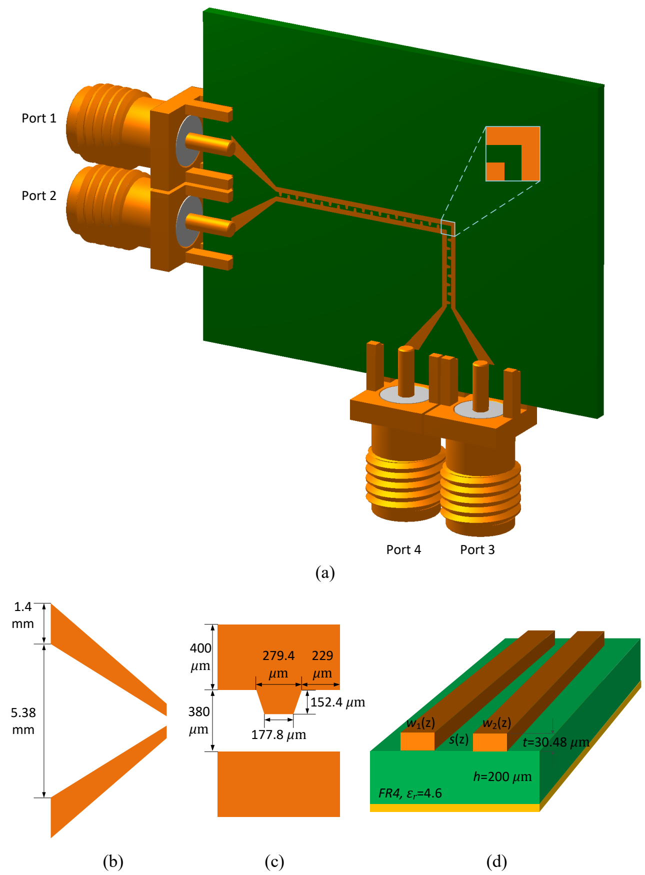

Fig. 2. (a) Sketch of the studied structure. (b)-(c) Geometric parameters for terminal tapered lines and a basic cell of interdigital lines. (d) Geometric parameters for PCB layers.

line discontinuity will be modelled by means of a "black-box" 4-port network whose S-parameters were extracted from full-wave simulation at the ports of the bend section.

The whole structure under analysis is modeled as the cascade connection of 4-by-4 T-parameter matrices, as shown in Fig. 3. To reduce the computational time, the interdigital tabbed traces incorporating interleaving trapezoidal tabs are further split into basic cells in Fig. 2(c), whose place-dependent p.u.l. parameters are evaluated at different cross-sections along the cell length. For the basic cell, pertinent S-parameters are firstly evaluated by solution of NUTL equations by the PT, in combination with standard 50 Ω terminal loads, and by converting the obtained S-parameter representation into T-parameter notation. Afterwards, the T-parameter matrix associated with the whole tabbed line is obtained by multiplication of the matrices pertinent to the basic cells. For instance, with reference to Fig. 3, the transmission matrix $\tilde{\mathbf{T}}_{\text{cell}}$ is preliminary obtained starting from $\mathbf{T}_{\text{cell}}$ by exploiting the transformation

$$\tilde{\mathbf{T}}_{\text{cell}} = \mathbf{T}_{\text{t}}\mathbf{T}_{\text{cell}}\mathbf{T}_{\text{t}}, \tag{5}$$

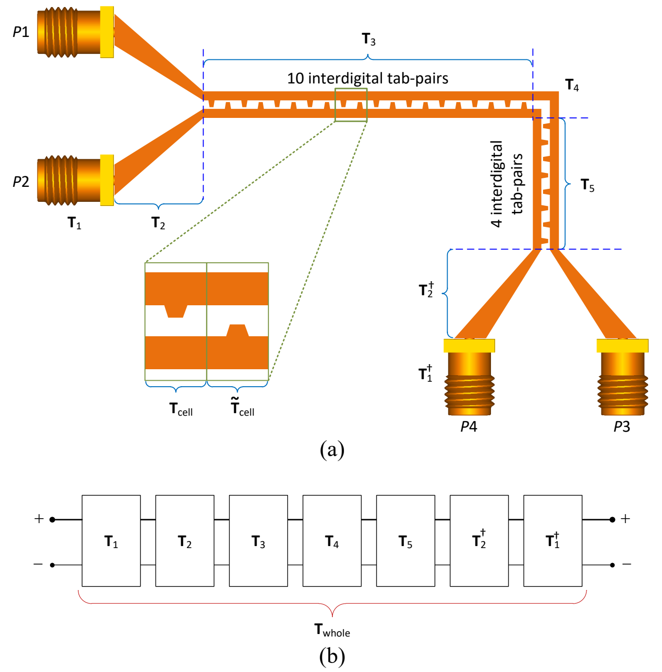

Fig. 3. (a) Subdivision of the microstrip line into subsections, and (b) line representation by cascaded T-parameter matrices.

where

$$\mathbf{T}_t = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Hence, matrix $\mathbf{T}_3$ is computed by simply cascading the obtained matrices as

$$\mathbf{T}_3 = \left(\mathbf{T}_{\text{cell}}\tilde{\mathbf{T}}_{\text{cell}}\right)^{10} = \left(\mathbf{T}_{\text{cell}}\mathbf{T}_{\text{t}}\right)^{20} \tag{6}$$

For the SMA connectors, the lumped circuit model shown in Fig. 4(a) of [16] is adopted, and its pertinent S/T parameters are extracted and exploited. Due to symmetry, the corresponding T-parameter matrices $\mathbf{T}_1^{\dagger}$, $\mathbf{T}_2^{\dagger}$ are obtained as

$$\mathbf{T}_i^{\dagger} = \mathbf{T}^{\dagger}\mathbf{T}_i^{-1}\mathbf{T}^{\dagger}, i = 1, 2 \tag{7}$$

with

$$\mathbf{T}^{\dagger} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

Eventually, the T-parameter representation of the entire structure (Fig. 3) is cast as

$$\mathbf{T}_{\text{whole}} = \mathbf{T}_1\mathbf{T}_2\mathbf{T}_3\mathbf{T}_4\mathbf{T}_5{\mathbf{T}_2}^{\dagger}{\mathbf{T}_1}^{\dagger} \tag{8}$$

With reference to the terminal conditions shown in Fig. 4, FE voltages predicted by the proposed approach are compared with those obtained by full-wave simulations with ADS [17] (reference solution) in Fig. 5. The observed good agreement in the whole frequency interval up to 10 GHz proves the accuracy of the proposed TL-based approached in spite of the strong non-uniformity affecting the structure under analysis. The frequency independent offset in the low-frequency range of the crosstalk voltages is due to the presence of the tabs. Here, such an effect was partially compensated in the MTL model by an extra capacitive term, as proposed in [13].

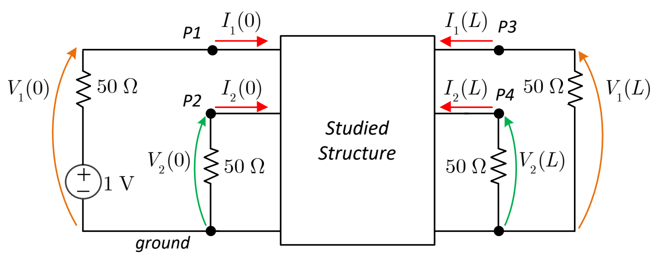

Fig. 4. Port constraints at the terminations of the microstrip under analysis.

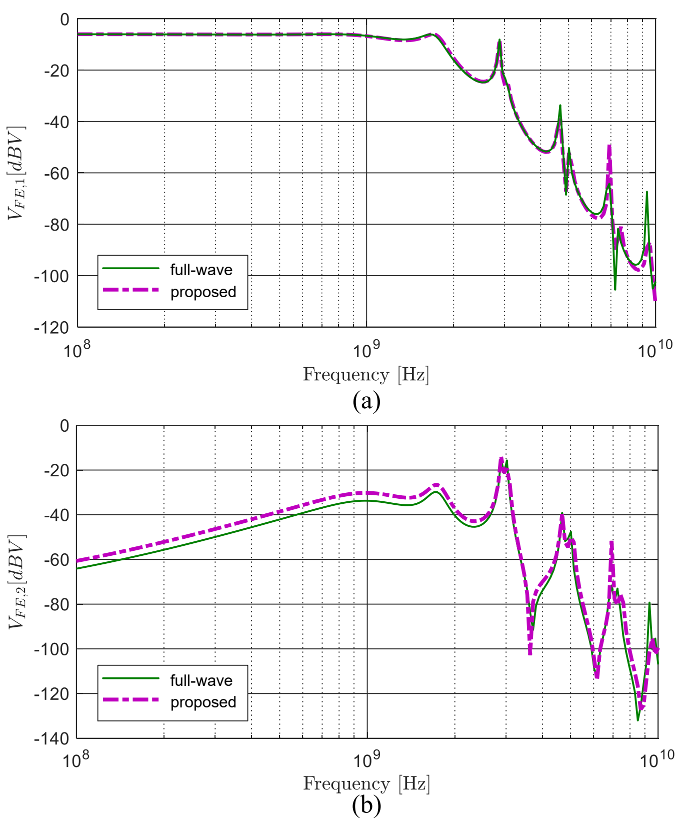

Fig. 5. FE voltages predicted by the full-wave solver and the proposed technique for (a) trace no. 1, and (b) trace no. 2.

The proposed technique allows investigating slightly modified versions of the original structure with limited additionally effort. Specifically, if the basic structure is kept as is but the number of tabs for each section is changed, (6) can be readily re-adapted to the new geometry without requiring additional computation. Moreover, if partial changes on the overall cascaded connection are performed (e.g., to account for additional discontinuities), the T-parameter representation in (8) can be easily modified accordingly. This modeling flexibility brings additional benefits when designing and assessing the performance of microstrip structures with periodic geometry.

V. Conclusion

In this paper, the perturbation technique originally proposed in [1] has been extended to provide a $2n$-port representation of a complex NUTL structure in terms of S- and T-parameter matrices. It has been shown that the limitations of the conventional PT can be overcome by using PT in combination of standard loads to iteratively solve the NUTL sub-problems required for the characterization of the NUTL under analysis in terms of S-parameters. The obtained representation can be easily cascaded with other lumped and/or distributed network to provide a $2n$-port representation of the whole structure. Feasibility and accuracy of the proposed TL-based technique was assessed versus full-wave simulation by solution of an interdigital tabbed microstrip line exhibiting a bend discontinuity, and equipped with SMA connectors. Future studies will be aimed at exploiting this solution method in different application scenarios (e.g., the chirped lines for common-mode rejection in [9], [10]), and in combination with statistical analyses aimed at investigating the sensitivity of transmission performance to slight variation of the specific geometry under test.

References

[1] P. Manfredi, D. De Zutter, and D. Vande Ginste, "Analysis of nonuniform transmission lines with an iterative and adaptive perturbation technique," *IEEE Trans. Electromagn. Compat.*, vol. 58, no. 3, pp. 859–867, 2016.

[2] C. R. Paul, *Analysis of multiconductor transmission lines*. John Wiley & Sons, 2008.

[3] M. Chernobryvko, D. Vande Ginste, and D. De Zutter, "A two-step perturbation technique for nonuniform single and differential lines," *IEEE Trans. Microw. Theory Techn.*, vol. 61, no. 5, pp. 1758–1767, 2013.

[4] F. Grassi, P. Manfredi, X. Liu, J. Sun, X. Wu, D. Vande Ginste, and S. A. Pignari, "Effects of undesired asymmetries and nonuniformities in differential lines," *IEEE Trans. Electromagn. Compat.*, vol. 59, no. 5, pp. 1613–1624, 2017.

[5] P. Manfredi, X. Wu, F. Grassi, D. Vande Ginste, and S. A. Pignari, "A perturbative approach to predict eye diagram degradation in differential interconnects subject to asymmetry and nonuniformity," in *Proc. IEEE 21st Workshop Signal Power Integrity*. IEEE, May 2017, pp. 1–4.

[6] X. Wu, F. Grassi, P. Manfredi, and D. Vande Ginste, "Perturbative analysis of differential-to-common mode conversion in asymmetric nonuniform interconnects," *IEEE Trans. Electromagn. Compat.*, vol. 60, no. 1, pp. 7–15, 2018.

[7] X. Wu, P. Manfredi, D. Vande Ginste, and F. Grassi, "A Hybrid Perturbative-Stochastic Galerkin Method for the Variability Analysis of Nonuniform Transmission Lines," *IEEE Trans. Electromagn. Compat.*, to be published, doi: 10.1109/TEMC.2019.2922407.

[8] M. Baldwin and V. Di Lello, "New techniques to address layout challenges of high-speed signal routing," Cadence, PCB West, Tech. Rep., 2016.

[9] P. Vélez, J. Bonache, and F. Martín, "Differential microstrip lines with common-mode suppression based on electromagnetic band-gaps (EBGs)," *IEEE Antennas Wireless Propag. Lett.*, vol. 14, pp. 40–43, 2015.

[10] P. Vélez, M. Valero, L. Su, J. Naqui, J. Mata-Contreras, J. Bonache, and F. Martín, “Enhancing common-mode suppression in microstrip differential lines by means of chirped and multi-tuned electromagnetic bandgaps,” *Microw. Opt. Technol. Lett.*, vol. 58, no. 2, pp. 328–332, 2016.

[11] P. A. Rizzi, *Microwave engineering: passive circuits*. Prentice Hall, 1988.

[12] R. K. Kunze, Y. Chu, Z. Yu, S. K. Chhay, M. Lai, and Y. Zhu, “Crosstalk mitigation and impedance management using tabbed lines,” *Intel white paper*, 2015.

[13] W. Jiang, X.-D. Cai, B. Sen, and G. Wang, “Equation-Based Solutions to Coupled, Asymmetrical, Lossy, and Nonuniform Microstrip Lines for Tab-Routing Applications,” *IEEE Trans. Electromagn. Compat.*, vol. 61, no. 2, pp. 548–557, 2018.

[14] X. Liu, X. Wu, F. Grassi, G. Spadacini, D. Vande Ginste, and S. A. Pignari, “Scattering parameters characterization of periodically nonuniform transmission lines with a perturbative technique,” *IEEE Trans. Electromagn. Compat.*, to be published, doi: 10.1109/TEMC.2020.2972844.

[15] X. Wu, F. Grassi, P. Manfredi, D. Vande Ginste, and S. A. Pignari, “Compensating Mode Conversion Due to Bend Discontinuities Through Intentional Trace Asymmetry,” *IEEE Trans. Electromagn. Compat.*, vol. 62, no. 2, pp. 617–621, 2020.

[16] F. Grassi and S. A. Pignari, “Characterization of the bulk current injection calibration-jig for probe-model extraction,” in *Proc. IEEE Int. Symp. Electromagn. Compat.* IEEE, 2010, pp. 344–347.

[17] Keysight Technologies, “ADS Product Documentation,” https://edadocs.software.keysight.com/display/support/ADS+Product+Documentation, (Accessed May 19, 2020).