Recovery of mode shapes from Continuous Scanning Laser Doppler Vibration Data: a Mode Matching Frequency Domain Approach

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ABSTRACT
The paper illustrates a method for processing, in a blind way, data obtained by Continuous Scanning Laser Doppler Vibrometry (CSLDV). CSLDV makes it possible to measure the structure vibration joining together the spatial and time information. The vibration datum obtained from the laser, which continuously scans (over time and space) the structure under test, is in fact modulated by the Operational Deflection Shape (ODS) excited during the experiment. The idea that we propose in this paper is based on the fact that, if the mode shapes of the structure under test are known a priori, e.g. from a numerical model or from an analytical formulation, it is possible to settle a procedure that searches for similarities between those known mode shapes (the candidate mode shapes) and ODSs that actually modulate the signal. This procedure can be considered a pattern matching technique that makes it possible to identify the resonance frequency related to each ODS and the mode shapes that better match with ODSs excited. A detailed description of the algorithm is given in this paper.

Keywords: Laser Doppler Vibrometry, Continuous Scanning Laser Doppler Vibrometry, Pattern Matching, Vibration Testing, Mode Matching

1. Introduction

The Continuous Scanning Laser Doppler Vibrometry (CSLDV) method was introduced by Ewins et al. [1] as an alternative to conventional Scanning Laser Doppler Vibrometry. This characteristic rises from the possibility to recover Operational Deflection Shapes (ODSs) from a unique time history acquired by the Laser Doppler Vibrometer (LDV) while the laser beam scans, in a continuous way, all over the vibrating surface. With respect to Discrete Scanning Laser Doppler Vibrometry, CSLDV presents the following advantages:

- extremely high spatial resolution,
- compact data structure (e.g. a single time history contains both time and spatial information),
- limited duration of the experiment(e.g. acquisition time depends only on the required frequency resolution).

When a vibration measurement is performed by CSLDV, the time history appears as an amplitude modulated signal whose modulation is due by the Operational Deflection Shapes excited during the experiment.

At the beginning the CSLDV technique was applied only in single frequency excitation conditions, e.g. step sine testing. Such testing methodology, however, had the drawback of being extremely time-demanding. During the years several works have demonstrated its functionality even in case of broadband excitation, e.g. impact testing [2], broadband [3] and operational [4] excitation. The CSLDV technique presented by Ewins et al. grounds on the hypothesis that an ODS can be modelled by a polynomial. The polynomial's coeffcients are directly related to the sidebands that characterise the vibration spectrum of the CSLDV signal. The sidebands around the resonance frequency of a specific ODS are spaced by the laser beam scanning frequency. The number of sidebands are directly related to the ODS complexity. This characteristic spectrum will be referred hereafter as the sideband spectrum related to the ODS. The recognition of resonance frequencies and the recovery of ODSs in the processing of CSLDV data are performed, conventionally, starting from the visual inspection of the CSLDV output spectrum. This process has obviously to be performed by an expert experimentalist who is used to treat CSLDV data.

This paper presents a new philosophy for processing CSDLV data. The proposed approach reverses the point of view of the experimentalist who analyses CSLDV data. The standard approach proposed by Ewins et al. starts from CSLDV spectrum and identifies the dynamic characteristics of the structure under test (the ODSs) from the information extracted from the sideband patterns located within the spectrum. The technique proposed in this paper, on the contrary, reverses this approach. In fact, if a specific ODS produces a unique sideband pattern, then the identification of that pattern within the CSLDV spectrum proves that the same ODS was excited during the test. This assumption constitutes the basis of this new approach. It is therefore possible to create a set of sideband patterns starting from a set of ODSs (e.g. obtained theoretically, numerically, etc.) and look for those patterns within the CSDLV spectrum. The patterns that are recovered within the spectrum correspond to the ODSs that were effectively excited during the test. Basically the procedures can be depicted as a pattern matching technique that makes it possible the blind recognition of the resonance frequencies and the related ODSs of the structure under tests. A similar approach, analysing CSLDV time domain data, was presented by Castellini et al. [5].

2. Mode matching procedure in frequency domain

The aim of the method proposed in this paper is to exploit the knowledge of mode shapes or ODSs, which can be known a priori from other approaches like numerical analysis, analytical models, previous experimental testing, etc., to create a set of sideband patterns to be looked for within the CLSDV spectrum. The patterns that best match with those effectively present in the CSLDV spectrum enhance the actual ODSs that best resemble the ideal mode shapes/ODSs that constituted the input of the procedure. In practice, this technique both emulates and reverse what an expert experimentalist does, with the added value of making the approach automatic and blind. As the experimentalist looks for a sideband pattern within the CSLDV spectrum and recovers the ODS from it, the algorithm assign a sideband pattern (the kernel) to each mode shape/ODS constituting the initial set of data and perform a pattern matching procedure within the CSLDV spectrum. In such a way the pattern that best matches within the CSLDV spectrum indicates that an ODS that is similar to the corresponding mode shape/ODS that produced that kernel was excited during the test. At this point a clarification is needed. The CSLDV technique, applied in the conventional way, i.e. with single or multi-sine excitation, allows to extract only ODSs from the acquired signal. The mode matching procedure makes it possible to identify the candidate shapes that better match with the ODSs that effectively modulate the CSLDV signal. The procedure, therefore, does not extract neither ODSs nor mode shapes, but indicates which ODSs, among those excited, best resemble those constituting the set of candidate shapes. In this sense it is not inappropriate to state that the candidate shapes can be both mode shapes (e.g. obtained analytically or numerically) and ODSs (e.g. obtained experimentally). The shapes used from now on in this paper are mode shapes calculated from analytical models, and therefore the authors will refer to them only as mode shapes. Since the procedure looks for specific sideband patterns within the CSLDV spectrum, its output consists in:

- the natural frequencies corresponding to the central frequencies of the sideband pattern better matching with the candidate ones,
- a set of mode shapes that best matches with the ODSs that effectively modulate the measured signal.

As already said this technique reverses, in a certain way, the point of view of the traditional approach: it starts form the mode shapes, creates the sideband pattern and identifies those modes shapes, among the set of candidates, that best match with the actual ODSs that were excited during the test. The number of unknowns with respect to the traditional approach is therefore reduced, as reported in 1, where n indicates the mode shapes and m the degrees of freedom of the structure. A detailed description of the mode matching procedure is reported hereafter, divided in its four main steps.

Step1: CSLDV data collection

The full field vibration of a structure excited in a wide frequency range is measured by CSLDV. An 1D example will be treated in this paper but it can be easily generalised to 2D. The laser beam is made to scan sinusoidally along the whole length of the 1D structure, e.g. a beam, with a scanning frequency much lower than that of the first expected mode. The CSLDV output time history is amplitude modulated by the excited ODSs.

Step 2: Selection of a set of candidate mode shapes

The mode shapes are selected a priori starting from the knowledge of the structure geometry and constraints. Those mode shapes can be calculated analytically or via numerical models (e.g. FE models). If we consider a clamped-free beam, the mode shapes can be defined analytically using the formulation proposed in ([6]):

$$X_i = \left[\cosh(\lambda_i \frac{x}{L}) - \cos(\lambda_i \frac{x}{L})\right] - \sigma_i \left[\sinh(\lambda_i \frac{x}{L}) - \sin(\lambda_i \frac{x}{L})\right] \quad (1)$$

where

x is the coordinate position along the beam length

L, λ_i and σ_i are the non-dimensional frequency amplitude parameters.

The constraint of a real structure, however, can be partially known or even unknown. For this reason it is plausible to insert, in the set of candidate mode shapes, modes that are obtained with different types of constraints (e.g. free-free, clamped-free, etc.).

Step 3: Sideband spectrum kernel synthesis

The basic assumption of the procedure is that each candidate mode shape, obtained from Equation 1, can be fitted by a polynomial of degree p and coefficients V_R:

$$\mathrm{M}(x) = \sum_{n=0}^{P} V_R x^n \quad (2)$$

This polynomial, *M(x)*, would therefore modulate the amplitude of the time history obtained from a CSLDV measurement when the laser beam scans along the vibrating structure. The real and imaginary coefficients (A_R and A_I) of the sidebands that characterise the spectrum of the thus amplitude-modulated vibration signal, are related to the polynomial coefficients (V_R) by means of the Chebyshev matrix ($[T]^{-1}$), [7], in accordance to:

$$\{A_R\} = [T]^{-1}\{V_R\}, \{A_I\} = [T]^{-1}\{V_I\} \quad (3)$$

The pattern of these sidebands (*kernel*) identifies a unique mode shape, and therefore it can be exploited as a template for a pattern matching procedure (the procedure is described in the following) that aims at identifying the presence of that mode in the spectrum of the CSLDV vibration signal. Each kernel is reconstructed from the sideband amplitude coefficients, obtained from A_R and A_I, according to:

$$A = \sqrt{\left(A_R^2 + A_I^2\right)} \quad (4)$$

It is important, at this point, to enhance the fact that, even though the candidates mode shape can be amplitude normalised when they are obtained, for instance, from analytical equations as 1, the corresponding sideband spectrum is not. This happens because the energy of the mode shape is spread over several sidebands, see Figure 1. It is thus clear that, increasing the complexity of the shape, which means considering mode shapes of higher spatial order, the number of sidebands increases and the amplitude of the kernel decreases accordingly.

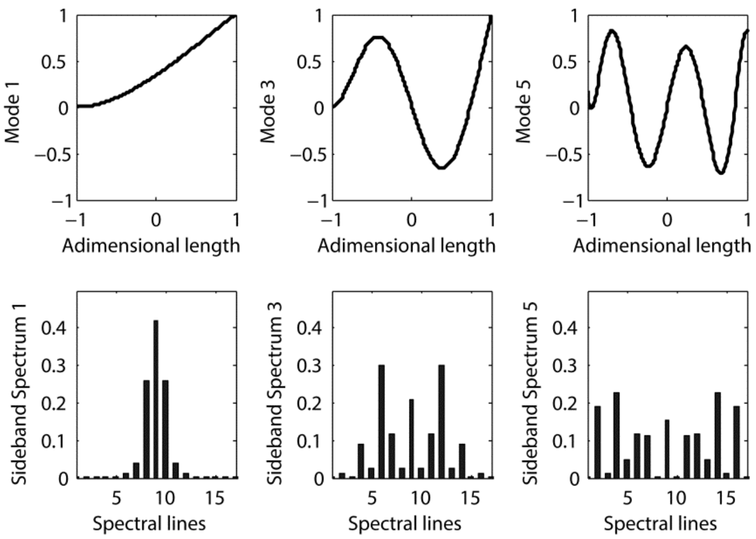

Fig. 1 Mode shapes and related sidebands spectra

Step 4: Sideband spectra matching

The kernels that identify the complete set of candidate mode shapes are then compared with the CSLDV experimental amplitude spectrum in order to find those ODSs that are effectively recognisable from the measured signal. The identification step follows a pattern matching procedure based on the minimisation of the Euclidean distance di between the template (each kernel) and the signal (the measured CSLDV amplitude spectrum). The method is based on the sliding window approach, that is a typical brute force method in Time-series Subsequence Matching ([8], [9]).

$$d_i = \left\| \frac{K}{\max(K)} - \frac{W_i}{\max(W_i)} \right\| \tag{5}$$

Where

$W_i = S_{(i\,:\,\Omega Laser\,:\,i+N)}$	is the sliding window of the signal S
$\|.\|$	represents the L^2 Euclidean distance
N	is the number of spectral lines of the kernel K
M	is the number of spectral lines of the signal spectrum S
i = 1: M	represents the position (frequency) of the sliding window
$\max(K)$	is the maximum value in the kernel K
$\max(W_i)$	is the maximum of amplitude spectrum S within the ith window of length N
Ωs	is the scan frequency adopted in the CSLDV measurement
d_f	is the frequency resolution adopted in the CSLDV measurement

The portion of the amplitude spectrum of the CSLDV measured signal (the sliding window W_i) is extracted from the whole spectrum and compared with the template (*kernel*). The distance metric (Euclidean) is stored and the sliding window is shifted ahead by one spectral line. Only the amplitude values at the discrete spectral lines that are multiples of Ω_S are considered within the sliding window when calculating the distance to the kernel. Indeed, these are the only significant lines that contain the information related to the ODSs. Moreover, this approach both improves the computational effort and makes the procedure less sensitive to noise. Once the all amplitude spectrum is scanned, the sliding window showing the minimum distance for a certain kernel is identified and the central frequency of that window extracted accordingly.

The procedure is repeated for the whole set of candidate mode shapes, until the contribution of each mode shape is determined in terms of ODS and central frequency. The mode shapes that are not present with their equivalent ODSs within the CSLDV amplitude spectrum show high values of Euclidean Distance and central frequencies that are well far from the spectrum significant range.

3. Algorithm testing on simplified data

The performances of the proposed approach were evaluated on a simple analytical model in order to deal with a well-controlled test-case. The algorithm was applied to two synthesised signals, Signal 1 and Signal 2 of Figure2, representing the sideband spectra obtained from two different mode shapes. Only one mode shape was assumed as the candidate mode shape. The kernel representing this mode shape is reported in the top plot of Figure 2.

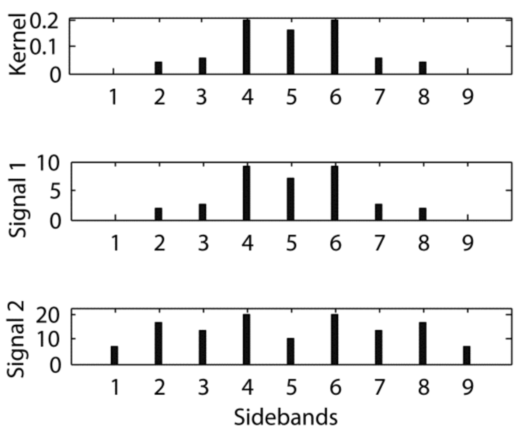

Fig. 2 Kernel and virtual signals synthesised sidebands spectra

That kernel represents the template for the pattern matching procedure when analysing the two synthesised signals. The similarity between the kernel and Signal 1 is clear and therefore it would be expected the algorithm to converge to a distance close to zero when the kernel is compared to Signal 1. The same would not happen for Signal 2.

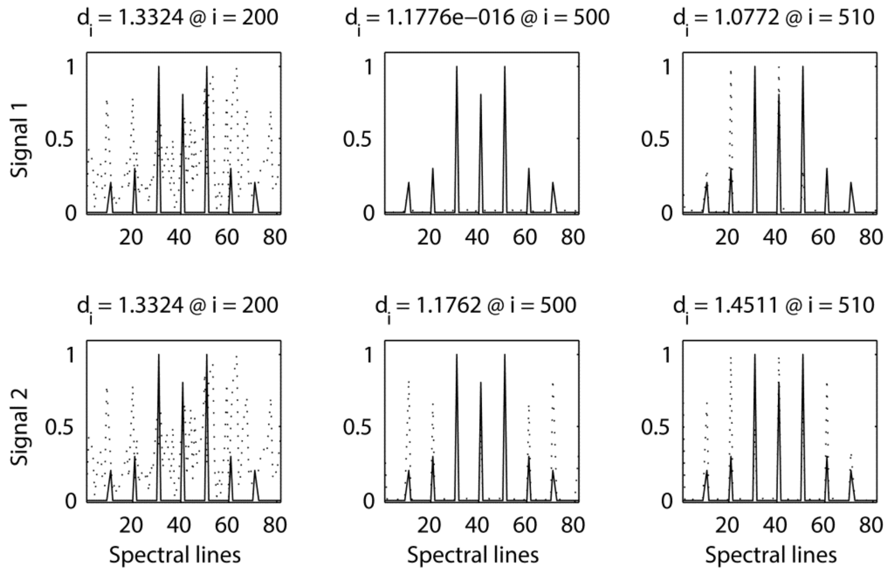

Fig. 3 Superimposition of the kernel (solid line) with the sliding signals (dashed line) at different spectral line position and their distance.

Figure 3 shows the process of sliding windowing of both Signal 1 (dashed line on top plot) and Signal 2 (dashed line on bottom plot) superimposed to the kernel (solid line). When the analysis window does not contain the sideband spectrum, for instance at the 200th spectral line, the signal is completely dissimilar to the kernel and the distance di is far from zero, see plots in the first column of Figure 3. When the analysis window contains the characteristic sideband spectrum, for instance at the 500th spectral line, the comparison between the kernel and the portion of the signal within the analysis window is even more significant. With reference to the 500th spectral line, it can be seen that the kernel almost coincides with Signal 1 (distance 0.2) but not coincides with Signal 2 (distance 1.3). Since the relative distance between the kernel and Signal 1 assumes the minimum value the procedure thus recognise the candidate mode shape as present in Signal 1. Let's assume the analysis window is centred on a spectral line that does not coincide with the central frequency of the sideband pattern, but coincides with a sideband of the spectrum. This situation happens at the 510th spectral line. In this case, the distance is still low but higher than the value assumed when the analysis window is centred at the central frequency of the sideband spectrum.

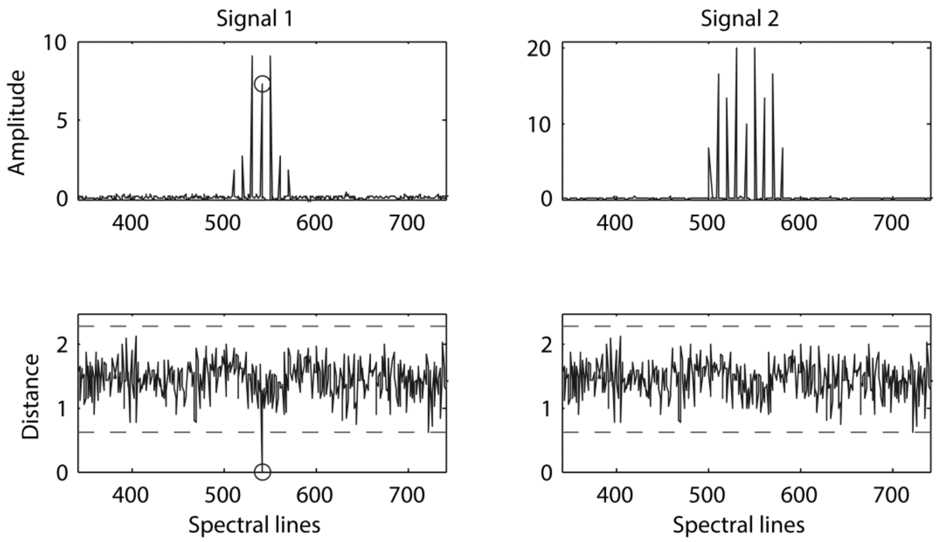

Fig. 4 Euclidean distance trend for Signal 1 (left) and Signal 2 (right).

Figure 4 illustrates the trend of the L^2 distance with the spectral line sliding for both Signal 1 (left) and Signal 2 (right). The bottom left plot clearly shows a minimum of the Euclidean distance exactly at the spectral line coinciding with the central frequency of Signal 1, the 500^{th} spectral line evidenced with a circle in Figure. The bottom plots of Figure 4 report, plotted in dashed lines, the standard deviation bands (coverage factor of 3) referred to the distribution of the Euclidean distance. It is clear that, when the kernel shape is matched at the 500^{th} spectral line, the metric is well below the threshold represented by the standard deviation bands.

4. Conclusion

This paper presents an alternative method for the processing of experimental data measured by CSLDV aiming at overcoming the main drwaback of such testing procedure. The standard approach indeed grounds on the experience and the knowledge of the experimentalist, who, starting from the visual inspection of the CSLDV spectrum, becomes able to extract the ODSs of the structure. The technique proposed is a sort of ”blind” data processing and does not requires anymore that data be handled by a CSLDV expert experimentalist. The main assumption is based on the awareness that a mode shape produces a sideband pattern that is unique in nature. Therefore, it is possible to create a set of candidate sideband patterns, directly related to a set of candidate mode shapes/ODSs (this depending on the way these mode shapes/ODSs are obtained, if analytically, numerically or from previous experiments) and look for those patterns within the CSDLV spectrum exploiting a pattern matching approach. Those patterns that are found within the spectrum represent the mode shapes/ODSs the best match with the ODSs that were effectively excited during the vibration test.
The method has been applied on synthesised data and its effectiveness tested in recognising ”true” and ”fake” candidate mode shapes which are present or not in the synthesised signal.

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