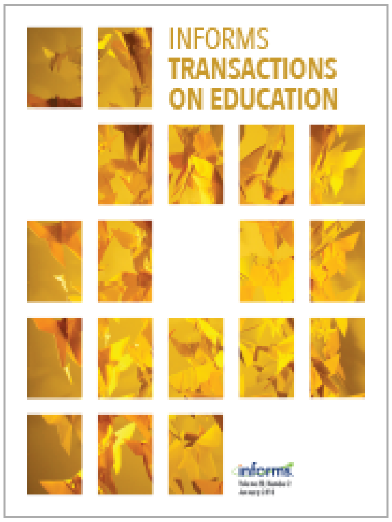

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Empowering Optimization Skills Through an Orienteering Competition

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Abstract. We report on a playful introduction of optimization in a high school. The final event of this experience has been an orienteering competition, in which the decision-making component plays a central role. This type of event has been very involving, and it boosted the interest of the students toward operations research. After summarizing the activities in class that preceded the competition, we describe in detail the organization aspects and the outcome. We also provide a simple AMPL code that compares the choices made by the participants with their optimal trajectory, that is, the one maximizing the score with the same traveled distance.

Supplemental Material: The online appendix is available at https://doi.org/10.1287/ited.2021.0243.

Keywords: active learning • high school • playful approach • orienteering competition

1. Introduction

Operations research and optimization are not widely known in Italy. They hardly appear in some of the Italian high school syllabi indications, and in the rare case in which they are present, they are seldom actually covered. However, the simple basic knowledge needed to understand many optimization problems gives the opportunity to arrive at an immediate application. Thus, the students can "touch" the solutions and propose creative approaches. This, overall, constitutes an extremely useful opportunity to make mathematics appealing to most of the students and to encourage their creativity and intuition. Nevertheless, very rarely, some "brave" teachers dare to cover optimization though it is widely recognized that it is an effective opportunity to let the students understand the power of mathematics in solving real problems (Ceselli and Righini 2018).

In the last years, we started an experiment in an Italian high school with the double intent of pointing out the creative component of mathematics and of introducing some optimization problems in a playful way (Fornasiero et al. 2017, Fornasiero and Malucelli 2020). An orienteering competition was the final event of this experience. The format of the competition was such that the decision-making component became fundamental. The same experiment has been transposed at the university level within an extracurricular course on soft skills at the Politecnico of Milan.[1]

The competition has been shown to be very appreciated and motivating among students, and we believe that it can be an important teaching tool for fostering the engagement and curiosity toward optimization and, more generally, toward applied mathematics.

In this paper, we briefly summarize our teaching experience, and we focus on the orienteering competition in order to give the basic elements to replicate the experiment.

2. Orienteering Competitions

Orienteering is a sport in which athletes from the starting line have to pass by a set of control points to be detected, with the help of a topographic map and a compass, and reach the finish. The topographic map is delivered at the moment of the start. There are two main types of orienteering competitions: *cross-country*, in which all control points must be reached in a specific order, and *score*, in which the visit sequence is free. In the cross-country competition, the winner is the athlete completing the sequence in the minimum time. In the second type of competition, the control points have a given score, and there is a time limit to complete the race. If the time limit is exceeded, penalties are introduced. The winner is the athlete maximizing the total score, that is, the sum of the control point scores minus the possible penalty. In this case, the athletes not only have to decide the optimal sequence, but in the presence of a very short time limit, they may also decide to select the control points to reach.

The combination of decision making under athletic stress is an important aspect in strategy sports, and it is thoroughly analyzed in Bennis and Pachur (2006). Orienteering is one of the sports in which the combination of athletic skills and clever decision making is essential. This aspect has been deeply studied in the specialized orienteering literature. For example, in Almeida (1997), Johansen (1997), Myrvold (1996), and Scarf (1998), a variety of decision-making aspects and evaluations are analyzed. However, usually, much of the attention is on cross-country competitions.

Orienteering competitions, besides being an Olympic sport, also have an important recreational value. They are often used with “team-building” purposes in companies, and they are used in schools to motivate and homogenize new classes.

Our purpose has been to motivate students with respect to applied mathematics and optimization in particular. We organized the competition in three different contexts. One has been in a high school class that could take advantage of an experimental approach to creativity and intuition over two years. In the second case, we condensed a similar experience during only one year. In the third case, we proposed the competition to a university recreational class about soft skills. The formula of the competition was similar in the three cases.

3. The Preliminary Activities

The activities were carried out in a second-year class of 24 students in an Italian high school in business and administration. Though the average level of mathematical skills was slightly above the average (both in comparison with the same classes of the same school and at the national level), the interest toward mathematics in general and the degree of engagement were poor. In the year preceding this experience, we carried out a brief course (10 sessions of one hour each) whose purpose was to motivate the students toward mathematics and to point out the creativity potential of the discipline. In this preliminary work, we have not introduced new concepts or methods. The main purpose has been to cancel the misbeliefs in mathematics maturated by the students in their career. Indeed, these misconceptions accumulated over the years often prevent students from getting involved and being engaged in applying mathematics and logic to the solution of new problems. In particular, most of the students were convinced that mathematics involves essentially repetitive and convoluted computations; it is far from reality; and for this reason, it is boring and useless. Thus, the preliminary activities were intended to convince the students that mathematics is creative as much as other disciplines and that, besides its rigor, it is based on intuition and creativity. Regaining the motivation in exploring mathematics from a different perspective allows students to have a new positive approach to learning new methods with renewed energies.

In the second year, we focused on optimization problems with the intent of making the students aware of optimization questions and letting them play with the problems rather than teaching some specific techniques.

We carried out the activities following an active learning and constructivist pedagogical model based on the *puzzle-based learning* method (Meyer et al. 2014). The puzzle-based learning method helps in refining reasoning skills, perseverance, and motivation in tackling problems. Thus, it consolidates the basic ingredients of problem solving.

The method considers puzzles that, without requiring any particular previous knowledge, are challenging and appealing for the students. In this context, the teacher plays the role of a guide in the community of learners (Brown and Campione 1994). The student remains the main actor in the learning process, becoming the constructor of the student’s own knowledge. Also, collaboration with classmates is a key element in the learning process (Vygotsky 1964, Vygotsky et al. 2013). Thus, the teacher, rather than showing how to solve problems in a predefined way, must keep up the attention and motivation, follow the students’ arguments, and accompany the students in constructing a solution. The teacher has to be open to accept solution methods that are different from what the teacher had in mind. In the activities, the main scope is not reaching a solution, but the attention is to the process that allows reaching it and the effort that must be put in. In other words, the role of the teacher can be compared with that of a sport coach, and the human and motivational skills are predominant with respect to the technical ones.

These activities, whose detailed description can be found in Fornasiero et al. (2017), have been carried out using drama, simple materials, and small rewards. Most of the time, they are very physical and require moving around and possibly getting out of the classroom. We focused on a variety of mathematical disciplines, such as logic, combinatorics, topology, probability, simulation, geometry, and graphs, even though they are not dealt with in a traditional academic way.

As for the optimization awareness construction, the activities involve several optimization problems arising in different fields, taking advantage of their ludic and challenging aspects. Also, in this case, we decided to motivate the students focusing on the application field in order to let the students accept the challenge and propose their solution approaches. The activity then involved a discussion phase in which, with the help of the teacher, some of the characteristics of the problems are analyzed. Then, the solution phase usually started, in which we tried to adopt the most intuitive and often "handicraft" approach to generate a good solution—sometimes the optimal one. A recap phase often followed in which the teacher discussed the solution found and tried to generalize the approach, highlighting the pros and cons.

These activities are described in detail in Fornasiero and Malucelli (2020). Over the course of 10 one-hour sessions, we addressed the following problems:

• The isoperimetric problem: We exploited the connection with history and recalled the foundation of Carthage in 814 BC. Given a fixed perimeter length, the problem consists of finding the shape that maximizes the area. We played with ropes and drew our conclusions.

• An introduction to linear programming: the cellphone assembly game: We introduced a simple production planning problem with two products and six resources. Given the "recipes" of two cellphone models, the problem was to decide how many cellphones to assemble for each model so as to maximize the overall economic return. The students received a kit containing a given number of cardboard rectangles of different colors and shapes, simulating the electronic components. We played with cardboard, and then, we discussed the optimality of the solution found with the help of a simple linear programming model and its geometric representation.

• An investment problem: continuous knapsack problem: We pretended to have a given budget to be invested in bonds to be selected from a given set. The duration of the investment was identical for each bond. At the end of the investing horizon, each bond, besides the invested amount, returned a coupon, whose amount was proportional to the investment. The available quantity of each bond was limited by a given amount. The problem consists in determining the quantities to be invested for each bond so as to maximize the total economic return. The problem was rather intuitive to the students, who immediately proposed the optimal greedy strategy.

• A fantasy sport game: zero–one knapsack problem: Taking advantage of the fact that some of the students in the class participate in a fantasy sport challenge, we introduced the zero–one knapsack problem. We considered a fixed amount of fantasy money to hire players. The data are completed by a projection of the performance p_i and a cost c_i for each player i in a given set. The problem consists in selecting a subset of players to buy, maximizing the overall projected performance and complying with the budget limit. After pointing out the similarities with the previous problem, we adapted the greedy strategy to the discrete case though we observed that it does not guarantee the optimality of the solution.

• The shortest path on a map: Given a map of the city, we considered the problem of finding the shortest walking distance path between an origin and a destination. This allowed us to introduce the graph notation, but then we decided to move to a very "physical" intuition of the problem. We reproduced a small subgraph derived from the map in a physical model made of strings (arcs) and clothespins (nodes). We then discussed how to take advantage of our physical model to find the solution. Someone observed that a way to find the shortest route is to rectify corners. This idea suggested stretching the physical graph between the clothespin of the origin and that of the destination, avoiding disconnecting strings from the clothespins. The rectilinear route corresponds to the shortest path from the origin to the destination. We observed that this operation was somehow counterintuitive because, to find the solution of a minimum problem, we actually perform a maximization by stretching the graph as much as possible. This vision gave us the opportunity to introduce the Dijkstra algorithm.

• The traveling salesman problem (TSP): We continued discussing shortest itinerary problems, adding the constraint of passing through each node of the graph exactly once and returning to the origin, thus introducing the TSP. To perceive the difficulty of the problem, we went to the computer laboratory and used a web application.[2] Even though it was designed to deal with the more general vehicle routing problem, it can be helpful to generate TSP instances. The application can visualize the solutions on a map, and it can be used to modify the solution in a very easy and interactive way. We generated an instance with 13 cities in Europe. Then, we opened a challenge among groups of two to three students for finding the shortest tour. This was done by starting from the same trivial solution and inviting the groups to improve it by making exchanges in the sequence. The length of the modified solutions have been evaluated with the help of the application.

4. A Mathematical Orienteering Competition

To conclude our trip in the optimization world, we organized our "mathematical orienteering" competition. The intent was to stimulate an optimization approach in defining the race strategy. For this reason, we adapted the standard orienteering competition format. First, we opted for a team competition with teams composed of two to three students. In our competition, we wanted to give more importance to the decision-making component with respect to the athletic one. Thus, we went for a score type of competition but not only. The scores were selected within a wide range, in comparison with the usual score competitions, and the time limit was fixed to a very short value with respect to the course extension such that even a well-trained runner would not have been able to complete the itinerary touching all control points without penalties. The short time limit was also intended so as not to penalize those teams including some non–well-trained runners. Moreover, instead of delivering the map at the moment of the start, we gave it five minutes before the start to each team to study their strategy. The gap between two successive starts was two minutes so that the decisions of a team could not influence those of the following one. Together with the map, we also gave a table with the scores of the control points and a rough estimate of the distance between any two control points.

In order to keep track of the actual control point selection and the sequence, we used free software.[3] This software allowed the teams to validate a control point by framing a QR code placed in the field with their smartphones.

At the end of the competition, we made two rankings. The first ranking considered the overall score of each team, that is, the total score of the collected control points minus the possible penalties. The second ranking took into account the actual sequence of control points and compared the actual score to the best possible choice that the team could make running the same distance.

In order to obtain such a comparison, we needed to solve an orienteering problem, this time in the mathematical optimization sense. The problem was first introduced by Golden et al. (1987). The problem is also mentioned in the literature as the selective traveling salesman problem (Laporte and Martello 1990) or as the single constraint maximum collection problem (Kataoka and Morito 1988). For a more detailed survey, we refer the reader to Vansteenwegen et al. (2011) and Vansteenwegen and Gunawan (2019).

4.1. Mathematical Model for the Optimal Control Point Selection and Sequencing

Among all possible formulations, we selected the simplest one. We selected that of Ascheuer et al. (2001) though it may be not the most efficient in terms of computation time. However, the small number of constraints and variables allows us to use any demo version of a commercial solver and provide the solution to the small instances that may arise in an orienteering competition.

Let indices $0, 1, 2, \dots, n$ represent the control points, in which zero is the start and n is the finish. Let S_i denote the score of control point i, d_{ij} the estimated distance between control points i and j $(i, j \in \{0, \dots, n\})$, and D the parameter indicating the limit in the total distance. With zero being the compulsory starting point, S_0 is set to zero. This parameter D varies depending on the actual sequence run by teams.

The model uses three sets of variables: zero–one variables x_{ij} $i, j \in \{0, \dots, n\})$ that are equal to one if control point i is immediately followed by control point j in the sequence; nonnegative variables u_i giving the position of control point i in the sequence if it is reached; and finally, we use a set of nonnegative variables y_{ij} $i, j \in \{0, \dots, n\})$ that equal u_i if the direct connection between control point i and control point j is used in the solution, thus, if $x_{ij} = 1$ and zero otherwise.

The model is

$$\max \sum_{i=1}^{n-1} \sum_{j=1}^{n} S_i x_{ij} \tag{1}$$

s.t.

$$\sum_{j=1}^{n} x_{0j} = 1, \tag{2}$$

$$\sum_{i=0}^{n-1} x_{in} = 1, \tag{3}$$

$$\sum_{i=0}^{n-1} x_{ik} = \sum_{j=1}^{n} x_{kj} \le 1 \qquad k = 1, \dots, n-1, \tag{4}$$

$$\sum_{i=0}^{n-1} \sum_{j=1}^{n} d_{ij} x_{ij} \le D, \tag{5}$$

$$\sum_{j=1}^{n} (y_{ji} + x_{ji}) = u_i = \sum_{j=1}^{n} y_{ij} \qquad i = 0, \dots, n, \tag{6}$$

$$y_{ij} \le n(x_{ij}) \qquad i, j = 0, \dots, n, \tag{7}$$

$$\sum_{i=0}^{n-1} x_{ni} = 0, \tag{8}$$

$$x_{ij} + x_{ji} \le 1 \qquad i, j = 0, \dots, n, \tag{9}$$

$$u_0 = 0, \tag{10}$$

$$x_{ij} \in \{0, 1\} \qquad i, j = 0, \dots, n. \tag{11}$$

The objective function maximizes the total score of the reached control points. Constraints (2) and (3) state that the sequence must start at zero and finish at n. Constraints (4) impose that, if a control point is reached, the sequence must continue and that the same control point can no longer appear in the sequence. Parametric Constraint (5) limits the total length of the sequence. Finally, Constraints (6) and (7) are the are flow-based subtour elimination inequalities adapted from the TSP formulation. We note that Constraints (6) imply that $x_{ii} =$

$0 \ \forall\, i \in 0, \ldots, n$. Constraint (8) imposes that the sequence must stop at n. Constraints (9) are meant to strengthen the model.

A simple implementation in AMPL of the model is presented in the online appendix, in which we also give the graphical output of the sequence to help the final analysis of the results.

5. One Example of a Mathematical Orienteering Competition

Here, we present the competition organized for a second-year class of the technical High School Bachelet of Ferrara. We took advantage of a public park close to the school that already hosted orienteering competitions. We set up a course with 15 control points and an additional control point 0, which served as the start and finish of the competition. The course is represented in Figure 1. The time limit has been fixed to 20 minutes and the penalty to one point for every 10 seconds exceeding the limit, rounded up.

The information about the scores and the distances between control points is summarized in Table 1. The distances were computed on the map, considering the presence of obstacles and impassable terrain. Thus, they are not the Euclidean distances between the points. The distance is expressed in centimeters on the map when it is printed on an A3 sheet.

Seven teams participated in the competition. The actual sequences and the scores of the groups are summarized in Table 2.

Table 1. Scores and Distances (in Centimeters on the Map)

Control point	Score	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	—	—	25	14	26	32	16	34	28	22	14	12	12	7	10	28	18
1	7		—	12	44	28	34	32	43	44	32	28	20	32	30	26	16
2	1			—	34	21	22	23	31	17	25	16	8	20	17	19	9
3	8				—	17	22	18	5	19	16	38	42	24	16	23	32
4	9					—	39	8	22	37	31	43	34	38	31	6	12
5	6						—	38	22	22	14	27	24	18	8	37	27
6	5							—	23	38	32	50	38	40	32	8	16
7	2								—	15	13	37	43	22	14	28	34
8	1									—	10	27	30	14	17	37	38
9	2										—	22	25	7	6	36	26
10	2											—	9	17	13	44	34
11	4												—	20	18	32	22
12	2													—	13	36	26
13	1														—	36	20
14	1															—	10
15	3																—

By analyzing the output of the competition, we observed that teams made different choices in both the target selection and the sequence to follow. In the final meeting in class, we ranked the teams according to the overall score as in a traditional orienteering competition. Besides that, we made also a second ranking that awarded the most optimized choice computed as the percentage difference between the actual score and the best score that the group could obtain running the same distance. This allowed us to rank the teams with respect to their optimization skills rather than with respect to their athletic ability, and we actually

Figure 1. Topographic Map of the Course

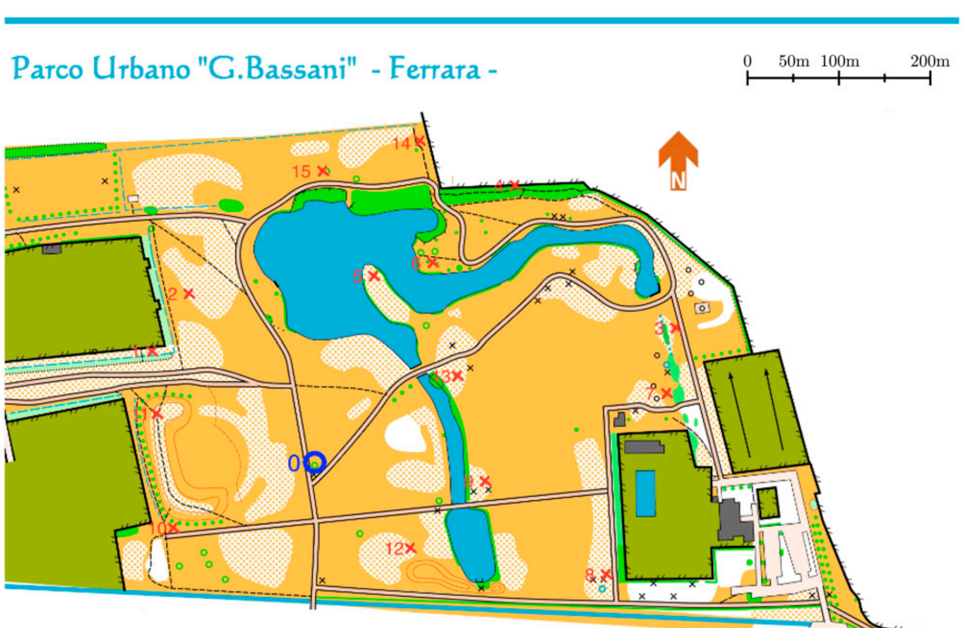


Table 2. Final Results

Team	Sequence	Time	Score − penalty	Optimal score	Gap, %	Distance
1	1, 15, 6, 4, 3	19′, 0″	32	38	15.8	108
2	11, 10, 12, 9, 13, 7	18′, 12″	13	32	59.8	93
3	11, 15, 6, 4, 5	20′, 42″	27-5	39	30.8	113
4	10, 11, 15, 4, 6, 3, 7, 13, 5, 9	18′, 12″	42	48	12.5	136
5	12, 9, 13, 7, 3, 6, 4, 15	19′, 5″	32	32	0.0	95
6	10,11, 15, 6, 13	20′, 0″	15	36	58.3	101
7	12, 9, 7, 3, 13, 10, 11, 1	22′, 26″	28–14	41	31.7	115

Note. The gap is computed as (optimal score − score)/(optimal score) and does not account for the penalty.

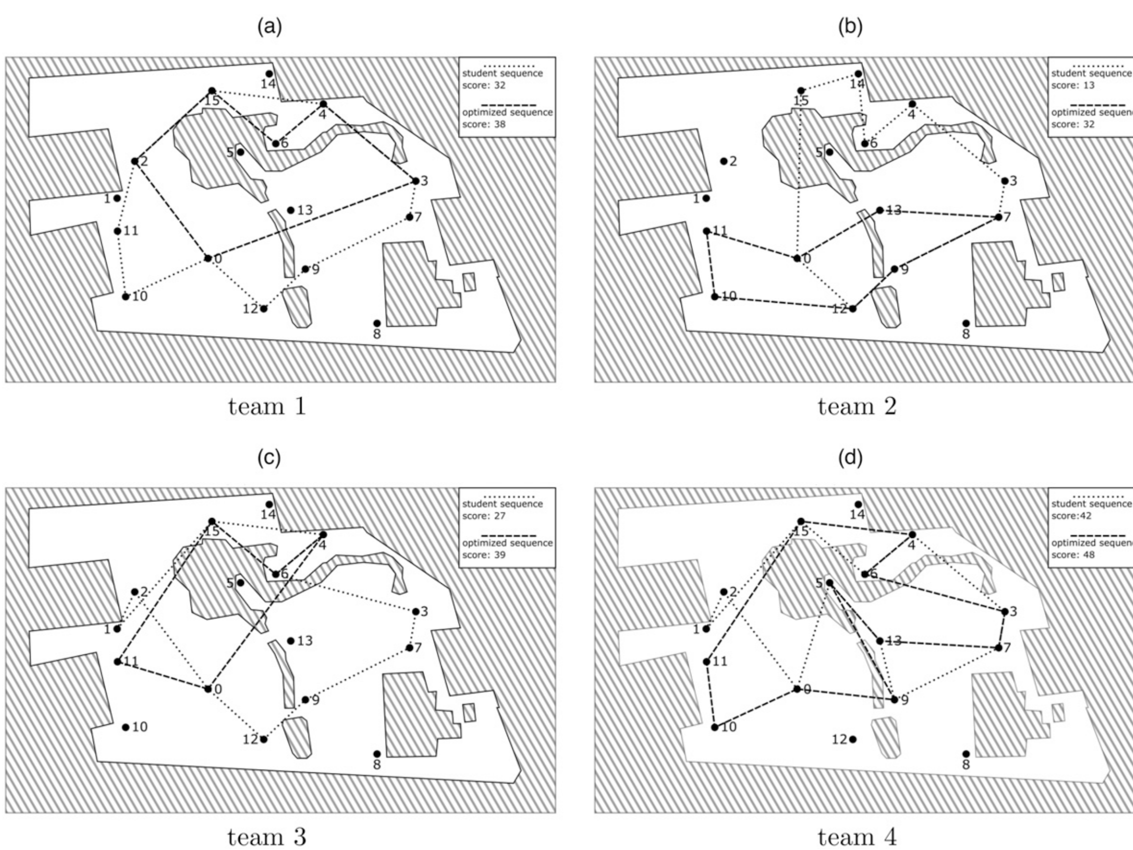
ended up with significantly different rankings. The mathematical model also allowed us to compare the actual target selection and itinerary with the optimal one.

Figure 2 shows the comparison between the itineraries of some teams and those obtained with the optimized model, indicating as maximum distance the actual distance run by the team. Note that the lines joining the points in the map are intended to represent the sequence of the control points and do not represent the actual path followed by the teams that had to circumvent the obstacles.

5.1. Instructions on How to Organize a Mathematical Orienteering Competition

The organization of the competition requires a not too large open space, possibly not too hilly and with some obstacles. The obstacles are needed to somehow hide the control points. Indeed, the choice of the control points could be affected by their visibility although the flatness of the course facilitates the distance computation between a pair of points and the approximate conversion into time. We do not recommend using indoor spaces because the management could be more complex. Our suggestion is to have not too may

Figure 2. Comparison Between the Student Path and the Optimized Path with the Same Distance



Notes. (a) Team 1. (b) Team 2. (c) Team 3. (d) Team 4.

control points (between 12 and 20) for two reasons. On the one hand, a high density of control points makes the choice of which to visit easier. On the other hand, too many control points make the solution of the mathematical model too time-consuming and not manageable during a class. In any case, we suggest testing the solution of the model before deploying the control points so as to be sure that the solution of the instance requires a reasonable time. Needless to say, another important detail is that the space should be traffic-free and possibly not too crowded to reduce the risk of accidents and interference. The control points should be signaled by traditional orienteering lanterns. However, some printed, colored A4 signs can be used as well if the school has no lanterns. Once the location of the control points has been decided, you need to compute all pairs' shortest paths. This requires a bit of work, especially if there are many obstacles. Indeed, in the presence of obstacles, it is not possible use the line-of-sight criterion and Euclidean distances, but some intermediate points must be added in the computation to take into account the circumvention of obstacles.

The running of the competition requires at least three supervising persons. We suggest having one person in charge of distributing the map and the distance table at the beginning, one to give the starting time, and one to keep track of the arrival times. As it emerged from the description of our experience, it is important to be able to reconstruct the exact sequence of control points for each team. There are two possibilities: using traditional punchers or a mobile app. With the traditional punching technique, each team receives a piece of cardboard with n numbered slots, and n is the number of control points. The team members have to use the puncher attached to the lantern, and punch their cardboard following the progression of the numbers. The punch of the first visited control point is done in slot number one and so on. In case punchers are not available, the control points can have a secret alphanumeric code that the team members have to copy in the slots of the cardboard. As an alternative, free orienteering applications are available as we indicated before. Instead of punching cardboard, the team members have to use their phones to frame a QR code printed on the sign of the control point. Should you decide on this technology, we recommend testing it before and letting the students practice in order to avoid loss of information or loss of time during the competition.

The final recommendation is to invite students not to rush to both avoid accidents and leave some blood in the brain to make clever decisions.

6. Conclusions

In this paper, we present the format of a sport competition that concludes a short course in optimization held in an Italian high school. The course had the main purpose of engaging the students in creative activities based on mathematics. In particular, we focused on optimization. Rather than introducing specific theoretical concepts or solution techniques, we wanted to play with some optimization problems, thus making the students aware of the optimization world. In the final event, we wanted to maintain the playful approach of the whole course. We organized a modified orienteering competition, and in the rankings, we took into account not only the athletic performance of the teams, but also their optimization skills in deciding the best strategy.

The perception of the students has been extremely positive. The vast majority of the students (95%) believed that the experience has been positive and worth repeating. Most of them (68%) said also that, after the course, their interest toward mathematics has much improved. Moreover, the few students who had an insufficient grade in math at the beginning of the year got a positive grade after the course.

We have been able to carry out a comparison with other classes of the same institute. All classes have about the same number of students (around 25). The number might slightly differ depending on the presence of students with special needs in the class. Indeed, a mathematics test is submitted to all classes at the beginning and end of the year. A direct comparison of the absolute values would not be very significant because, as we mentioned in the introduction, our class was among the best performing ones in the initial test. However, every class experiences an improvement of the score in the final test when compared with the initial one. We, thus, considered the improvement in the score of our class and compared it with that of the other classes. The class achieved an average grade improvement that is 10% greater than the improvement achieved by the other classes of the second year.

We think this experience can be replicated in other schools taking advantage of our experience. In the present paper, we give, to those who are willing to try, the main guidelines that we followed. In the online appendix, we give additional information about the optimization activities that preceded the orienteering competition. We are available to give more details to the interested teachers.

Acknowledgments

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Endnotes

[1] Please see http://www.peopleatdeib.polimi.it.

[2] Please see http://vrp.upf.edu.

[3] Please see http://iorienteering.com.

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