THE IMPACT OF SUPPLY CHAIN COORDINATION ON THE ENVIRONMENT

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By

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I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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Emission regulating mechanisms have been proposed by the policy makers to reduce the carbon emissions resulting from the industrial activities. We study the channel coordination problem of a two-level supply chain (i.e., a buyer and a vendor) under emission regulations. We first analyze a two-echelon chain that operates to meet the deterministic demand of a single product in the infinite horizon using a lot-for-lot policy under cap and trade, carbon tax and carbon cap policies. We analytically show and numerically illustrate that the average annual emissions of the system do not necessarily decrease when the buyer and the vendor make coordinated decisions. This implies coordination may not be good for the environment in terms of emissions related performance measures. We further extend our analysis under the emission regulating mechanisms mentioned above for a two-level supply chain in which the buyer operates to meet the stochastic demand of a single product. In both deterministic and stochastic demand settings, we propose coordination mechanisms including quantity discounts, fixed payments, carbon-credit sharing and carbon-credit price discounts that compensate the buyer's loss when the system's costs are minimized or profits are maximized.

Keywords: Environmental responsibility, environmental regulations, supply chain coordination.

ÖZET

TEDARİK ZİNCİRİ KOORDİNASYONUNUN ÇEVRE ÜZERİNE ETKİSİ

Bilgesu Çetinkaya
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Emisyon kontrol sistemleri endüstriyel faaliyetlerden kaynaklanan karbon salınımlarını azaltmak amacı ile tasarlanmıştır. Bu tezde bir satıcı ve bir perakendeciden oluşan iki basamaklı bir tedarik zincirindeki koordinasyon problemi emisyon düzenlemeleri altında çalışılmıştır. İlk olarak bir ürünün belirgin talebinin karşılanmaya çalışıldığı iki basamaklı bir tedarik zinciri, emisyon üst sınırı ve ticareti, karbon vergisi ve karbon üst sınırı politikaları altında analiz edilmiştir. Bu tedarik zincirinin sonsuz çevrende faaliyet gösterdiği ve satıcı ve perakendecinin bir siparişteki sipariş miktarlarının eşit olduğu varsayılmıştır. Sistemin yıllık ortalama emisyonlarının satıcı ve perakendecinin koordine karar verdiği her durumda azalmadığı analitik ve numerik olarak gösterilmiştir. Bu durum tedarik zinciri koordinasyonunun karbon emisyonları ile ilgili ölçütler altında iyi performans göstermeyebileceğine işaret etmektedir. İki basamaklı bir tedarik zincirinde yukarıda belirtilen emisyon kontrol sistemleri altında yapılan analiz perakendecinin rassal talep ile karşılaştığı durum için genişletilmiştir. Belirgin ve rassal talep durumlarının çalışıldığı modellerde satıcı-perakendeci sisteminin en iyi performansı istendiğinde, miktar indirimi, sabit ödenti, karbon kredisi paylaşımı ve karbon kredisi fiyat indiriminin de içinde bulunduğu koordinasyon mekanizmaları tasarlanmıştır.

Anahtar sözcükler: Çevresel sorumluluk, çevresel düzenlemeler, tedarik zinciri koordinasyonu.

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Chapter 1

Introduction

The levels of greenhouse gases in the atmosphere have increased due to human activities since the Industrial Revolution [1]. The World Meteorological Organization (WMO) [2] reported that the atmospheric concentrations of the greenhouse gases exhibited an upward and accelerating trend and reached a new record high in 2012. More specifically, the increase in the level of CO_2 was higher than its average increase over the past ten years [2]. The greenhouse gases slow or prevent the loss of heat to space, which makes the Earth warmer (i.e., the greenhouse effect) [3]. The greenhouse effect leads to an increase in the temperature of Earth's surface, which is known as the global warming [3]. It is reported that the measures of the climate warming effect increased by 32% between 1990-2012 [2]. Also, the global average temperature had risen by $0.6°C$ over the 20^{th} century due to increasing amount of greenhouse gases in the atmosphere [4].

According to European Environment Agency [5], the main sources of the greenhouse gases are fossil fuel burning (for electricity generation, transportation, industrial and household uses), agriculture, deforestation and land filling of waste in the member states of the European Union (EU). Also, the greenhouse gases are emitted mainly as a result of the activities of energy industries, transportation, residential and commercial uses, manufacturing, construction, industrial processes and agriculture in EU countries [5]. Carbon dioxide (CO_2) is the main greenhouse gas that is emitted as a result of the human activities [4].

Hence, it is responsible for 80% of the increase in the measures of the climate warming effect [2]. CO_2 is followed by methane (CH_4) and nitrous oxide (N_2O) [4].

In order to decrease the greenhouse gas (particularly CO_2) emissions, policy makers and international organizations have proposed agreements and regulations. In United States, guidelines provided by the Environmental Protection Agency (EPA) led to new regulations that put strict limits on the amount of carbon pollution generated by the power plants [6]. Also, additional regulations are proposed by EPA [7]. For instance, new regulations are proposed to reduce air pollution resulting from the activities of natural gas and oil industry [7]. Furthermore, transportation fuel must contain a minimum volume of renewable fuel due to the Renewable Fuel Standard (RFS) Program [7]. Similarly, in Europe, the European Commission proposed that at least 20% of the EU's budget for 2014-2020 should be spent on climate-relevant measures [8]. Moreover, the EU adopted new legislation in 2009, which sets compulsory emission reduction targets for new cars [9].

Apart from agreements and regulations, emission regulating mechanisms are proposed by policy makers. In this thesis, we consider three emission regulating mechanisms; emission trading system (i.e., cap-and-trade), carbon taxes and carbon caps. Under the cap-and-trade mechanism, the government sets a fixed quantity of emissions for each period (i.e., the cap) and firms are free to buy or sell allowances up to the level of the cap [10]. Currently, the emission trading systems (ETSs) are implemented in EU (EU ETS), Australia, New Zealand (NZ ETS), Northeastern United States, California (CA ETS), Québec and Tokyo (Tokyo ETS) [11]. ETSs are going to be implemented in Republic of Korea in 2015 and they are under development in countries including Brazil, China, India, Kazakhstan and Mexico [11]. The carbon tax mechanism puts a price on each tonne of the greenhouse gas (e.g., CO_2) emitted [12]. Finland, Netherlands, Norway, Sweden, Denmark, United Kingdom, Switzerland, Ireland, Australia, Costa Rica, Colorado, California, Québec and British Columbia are among the countries and states that have implemented a carbon tax [13]. Under the carbon cap mechanism, firms are allocated threshold values of carbon emissions that cannot be exceeded over a period [14].

In addition to the practices of the governments and international organizations, some industries and organizations take initiatives so as to reduce their greenhouse gas emissions voluntarily. In the United States, companies from private and public sectors partner with EPA to achieve emission reductions [15]. For example, Greenchill partnership, high-global-warming potential gases voluntary programs and methane reduction voluntary programs promote the reduction of greenhouse gas emissions [15]. Also, participants of Greenhouse Challenge in Australia reduced their emissions 14% below the business-as-usual levels [16]. In Japan, a voluntary emission trading scheme (Japan's Voluntary Emissions Trading Scheme, i.e., JVETS) was launched by the Ministry of Environment in 2005 [17]. The scheme supports voluntary CO_2 reduction activities by business operators to ensure their emission reduction targets with emissions trading [17].

While reducing their emissions and improving their environmental performances, the main objective of the firms is to reduce their costs and increase their profits. One way to achieve better economic performance is channel coordination among supply chain members. According to Simatupang et. al [18], firms in a supply chain collaborate to obtain mutual benefits due to increasing competition resulting from globalization, technological improvements and product diversity. The coordination mechanisms that are investigated most commonly include information flow among the supply chain members [19], logistics synchronization, incentive alignment, collective learning [18] and contracts that establish transfer payment schemes [20]. The accumulated research in this area suggests that coordination improves economic performance of the supply chain. To illustrate, Thomas et al. [21] argue that due to advances in information technology and logistics, firms can reduce operating costs by coordinating the planning of procurement, production and distribution. Similarly, Yu et al. [22] suggest that by coordinating different parties or forming partnerships between them, the supply chain members can benefit in terms of cost savings and inventory reductions.

Benjaafar et al. [23] suggest that emissions can be reduced by integrating carbon footprint considerations into decisions related to production, procurement and inventory management without significantly increasing cost. Combining this result with the notion of coordination provided in the previous paragraph, we examine the coordination in a two-level supply chain under an economic objective and carbon emissions considerations. We consider a system consisting of a buyer (retailer) and a vendor (manufacturer). In the first part of the thesis, we extend the EOQ model to account for this two-level supply chain (i.e., the buyer and the vendor) and the three emission regulating mechanisms described above in order to minimize the procurement, production and inventory holding costs. We examine the replenishment and inventory holding decisions of the buyer, and production and inventory holding decisions of the vendor. We propose two models (those are decentralized and centralized) for each emission regulating mechanism to find the order quantities that minimize the total cost of the buyer and the system.

In the second part of the thesis, we consider a two-level supply chain in which the buyer operates under the conditions of the classical newsvendor problem. We examine the replenishment decisions of the buyer and the system under carbon tax, cap-and-trade and carbon cap mechanisms. Similar to the first part, two models are proposed for each emission regulating mechanism to find the order quantities that maximize the expected profit of the buyer and the system. In both the first and the second parts of the thesis, we propose some coordination strategies including quantity discounts, carbon-credit sharing, carbon credit price discounts and fixed payments that compensate the buyer's loss resulting from the centralized optimal solution. Finally, we examine the impact of channel coordination on the optimal order quantities and on the cost (or expected profit) of the buyer, vendor, and the system under the EOQ model (or newsvendor problem) by numerical analyses.

Hence, this study contributes to the literature by investigating coordination issues in a two-level supply chain under emission regulating mechanisms (namely, cap-and-trade, carbon taxes and carbon cap mechanisms) under both deterministic and stochastic demand. Additionally, different from other studies, we propose coordination mechanisms that utilize carbon credit sharing and price discounts to compensate the buyer's loss while the best possible economic performance of the system is achieved.

Chapter 2

Literature Review

The literature related to this study are on carbon emissions management of a single firm, channel coordination in supply chains and channel coordination in supply chains with environmental efforts.

2.1 Studies on Carbon Emissions Management of a Single Firm

In the body of literature related to carbon emissions management of a single firm, some studies examine the decisions related to replenishment and inventory management. The papers that focus on this issue under the deterministic setting generally adapt the Economic Order Quantity (EOQ) framework.

In Hua et al. [24], the inventory management decisions of a firm under carbon emission trading mechanism (i.e., cap-and-trade system) and the impact of carbon cap and carbon price on replenishment decisions are investigated. The optimal order quantity of a single product that minimizes the total cost per unit time is found. It is assumed that the product demand is deterministic and the firm is allowed to change only the decisions related to replenishment. The EOQ model is updated under the cap-and-trade system by adding the emissions restriction as a constraint to the model. It is found that cap-and-trade system induces the firm to reduce its emissions and total cost simultaneously under some conditions related to carbon cap and carbon price.

Hua et al. [25] extend the analysis of Hua et al. [24] by analyzing the impact of carbon trade on the ordering and the pricing decisions of a firm under the same emissions structure. The objective is to maximize total profit per unit time where the replenishment quantity and the retail price are the decision variables. It is assumed that the demand decreases with increasing price and the marginal revenue is a strictly increasing function of price. Similar to [24], the EOQ model is updated under the cap-and-trade system where the emissions restriction is added as a constraint to the model. It is found that the optimal values of the order quantity, retail price, and the resulting amount of carbon emissions depend on the carbon price, but not on the carbon cap.

Different from Hua et al. [24] and Hua et al. [25], Chen et al. [26] study the inventory management decisions of a firm under carbon cap mechanism. The firm chooses the order quantity of a single product that minimizes the sum of fixed and variable ordering costs and inventory holding costs while ensuring that its emissions do not exceed the carbon cap. It is assumed that the product demand is known and the EOQ framework is adapted. Since emissions are also associated with procurement and inventory holding, the calculation of the amount of emissions follows the same structure as the calculation of average cost per unit time. It is proven that the cost function is flat while the emission function is steep around the cost-optimal solution. Hence, the benefit of emission reduction is greater than the relative increase in cost in this range. The study shows that it is possible to reduce carbon emissions by operational adjustments without significantly increasing cost in an inventory management system. The notion of emissions reduction without increasing costs considerably, is also extended to the facility location and newsvendor models.

Arslan and Türkay [27] extend the studies of Hua et al. [24] and Chen et al. [26] by incorporating social criteria into replenishment decisions of a single product under environmental criteria. The amount of greenhouse gas (GHG) emissions (i.e., the carbon footprint) of a firm is considered as the environmental criterion and amount of labor hours used by a firm is considered as the social criterion. In modeling environmental criterion, five approaches are formulated, which are direct accounting, carbon tax, direct cap, cap-and-trade and carbon offsets. Similar to Hua et al. [24] and Chen et al. [26], it is assumed that the demand of the product is deterministic and the EOQ framework is adapted. The calculation of the amount of emissions and labor hours follow the same structure as the calculation of average cost per unit time. The results of the paper show that cost-charging models do not give an initiative to reduce the amount of carbon emissions and labor hours. Thus, strict control of emissions and working hours is possible only when caps are exercised by regulatory agencies.

In Bouchery et al. [28], a multi-objective optimization model under economic and environmental objectives is formulated. The study extends the EOQ model to analyze the operational adjustment and the technology investment options under carbon cap and carbon tax mechanisms (i.e., the sustainable order quantity, SOQ, model). It is assumed that the technology investments reduce the emissions-related parameters. The calculation of the amount of emissions has the same structure as the calculation of average cost per unit time. It is proven that there exist threshold values for the cap and the unit emissions tax for the carbon cap and carbon tax mechanisms, respectively, that enable deciding between the operational adjustment and technology investment options.

Different from [24]-[28], Song and Leng [29] examine the production decision of a single product with stochastic demand under carbon emissions considerations. The optimal production quantity of a perishable product with stochastic demand is found where the objective is to maximize the total expected profits. The study extends the single-period (newsvendor) problem under carbon cap, carbon tax and cap-and-trade mechanisms. It is found that there are instances under cap-and-trade system in which both the firm's expected profits increase and its total emissions decrease. It is also shown that the carbon tax rate of a high-margin firm should be higher than the carbon tax rate of a low-margin firm for low-profit products so as to decrease emissions by a certain amount. However, the carbon tax rate of a low-margin firm should be higher than the carbon tax rate of a high-margin firm for high-profit products.

In literature related to carbon emissions management of a single firm, some studies examine the operational decisions of a firm including transport mode, route and product mix selection.

In Hoen et. al. [30], the transport mode among air, road, rail and water transportation which results in the least expected penalty, holding and transportation costs is selected to conduct all shipments of a single product with stochastic demand. The problem is formulated as an infinite horizon periodic review inventory model, where an order-up-to policy is used as the inventory policy. In order to reduce the carbon emissions resulting from transport, two different policies are considered. The first policy is to implement a constraint on the amount of carbon emissions and the second is to introduce an emission cost per ton of CO_2 emitted. Emissions for each transport mode is calculated using the Network for Transport and Environment (NTM) method. The results of the paper show that under the second policy, emissions cost is only a small part of the total cost under the current prices in the carbon market. Hence, road transport is selected most of the time and the second policy does not result in significant changes in transport mode selection. Implementing a constraint on emissions reduces the emissions by a larger fraction.

In Letmathe and Balakrishnan [14], the optimal product mix of a firm is found under emission regulating mechanisms using two different models. The first model assumes that each product has one operating procedure and it is formulated using linear programming. The second model assumes that each product has more than one operating procedure and it is formulated using mixed integer linear programming. The objective function of both models is to maximize profits. Also, in both models it is assumed that the demand of each product decreases with emissions. There can be multiple types of emissions. Emissions cap and emissions trading policies are used as the emission regulating mechanisms. In both of the policies, a penalty cost is paid for each unit of emission that does not exceed the cap, which is different from the other papers in emissions management literature.

In Kim et al. [31], a freight network is selected among truck-only and intermodal freight networks for each route connecting two cities. The intermodal freight networks are the combinations of different transport modes. The model is represented as a hub-and-spoke network. There are two types of nodes in the network, which are hub cities and flow cities. Also, there are two types of arcs, internal and external flows. The problem is formulated as an ideal multi-objective optimization problem in which minimization of freight costs and minimization of CO_2 emissions are the two objectives. There is a CO_2 emission quota for each route. The results of the study show that truck only and intermodal rail systems perform better in terms of freight costs. However, truck only system results in the highest CO_2 emissions. Rail-based intermodal and short-sea based intermodal systems give better results in terms of CO_2 emissions. Therefore, increasing intermodal systems' capacities would reduce emissions.

2.2 Studies on Channel Coordination in Supply Chains

In studies related to channel coordination in supply chains, most part of the research is built up on the single period (newsvendor) problem with two supply chain members.

In Cachon [20], a two-level supply chain (i.e., a supplier and a retailer) is studied where the retailer operates to meet the demand of a single product with stochastic demand. The newsvendor problem is extended so as to study the wholesale price contracts, buy back contracts, revenue sharing contracts, quantity flexibility contracts, sales rebate contracts and quantity discount contracts between the buyer and the vendor. It is found that the wholesale price contracts do not coordinate the channel while the others do.

Pasternack [32] studies the single period problem in a two-level supply chain (i.e., a manufacturer and a retailer) in which the retailer should meet the random demand of a perishable product. Possible pricing and return policies are examined so as to determine whether they provide a system optimal solution. It is proven that policies in which the manufacturer allows no returns or unlimited returns for full credit do not coordinate the channel. However, policies which allow unlimited returns for partial credit coordinate the channel for specific values of unit return credit and unit price paid by the retailer to the manufacturer.

Different from Cachon [20] and Pasternack [32], in Toptal and Çetinkaya [33], the single period coordination problem between a buyer and a vendor is extended to include the transportation costs, which include the fixed costs and stepwise freight costs. The buyer operates to meet the random demand of a single product with short life cycle and the vendor's production quantity is determined by the buyer's order quantity. Different from other studies, it is shown that the vendor's expected profit is not an increasing function of the buyer's order quantity since it also incurs the transportation costs. Also, the cases under which the vendor's profits increase/decrease with the buyer's order quantity are presented. Quantity discounts with economies and diseconomies of scale, fixed payments from the vendor to the buyer, vendor managed delivery arrangements and combinations of these are proposed as the coordination mechanisms. It is also analytically and numerically shown that such contracts can lead to win-win solutions and considerable monetary savings in terms of transportation costs and supply chain profits.

In some studies related to the coordination under uncertain demand, the coordination idea is extended to incorporate a second order from the retailer or a second production run by the manufacturer.

In Zhou and Li [34], similar to [20], [32] and [33], the newsvendor problem is extended to account for a two-level supply chain (i.e., a manufacturer and a retailer) which operates to meet the random demand of a single item. Different from these studies, if the demand is more than the order quantity, the retailer may choose to place a second order from the manufacturer to satisfy the demand depending on a breakeven quantity. Two models are proposed in which the order quantities that maximize the expected profit of the retailer and the supply chain are found, respectively. Full returns policy is proposed as a coordination strategy.

It is proven that the order quantity that maximizes the retailer's expected profit under the once ordering strategy is greater than or equal to the order quantity that maximizes the retailer's expected profit under the twice ordering strategy. It is also shown that the optimal expected profit of the retailer (system) under the twice ordering strategy is at least as good as the optimal expected profit of the retailer (system) under the once ordering strategy.

In Parlar and Weng [35], the coordination problem between a firm's manufacturing and a supply departments is studied. The manufacturing department operates to meet the random demand of a perishable product. Similar to Zhou and Li [34], if the demand exceeds the amount produced, manufacturer can initiate a second product run at a higher cost. Two models are proposed which are the models with and without coordination (i.e., with and without information exchange), where the objective is the maximization of the expected profit. The optimal production quantity and the amount of reserved material supplier keeps for the possible second run are determined. It is proven that the order quantity that maximizes the expected profit of the system does not depend on the amount of reserved material kept by the supplier for a possible second run. Additionally, the parameter values which lead to equal expected system profit under coordination and under independently made decisions are investigated.

Different from [20] and [32]-[35], Chen and Chen [36] examine the problem of coordination in a deterministic setting with multiple products. The retailer replenishes the stocks individually or jointly from the manufacturer on an EOQ basis. It is assumed that the production cycle of the manufacturer is an integer multiple of the replenishment cycle of the retailer and the procurement cycle of the manufacturer is an integer multiple of the production cycle. Four models are developed, which are individual item non-cooperative replenishment (policy I), joint item non-cooperative replenishment (policy II), individual item cooperative replenishment (policy III) and joint item cooperative replenishment policies (policy IV). It is shown that policy IV results in less channel cost than the others. Also, in some cases under policy III and policy IV, the retailer's cost increases when the channel cost decreases. In order to overcome this, quantity discount is given to the retailer. It is numerically shown that both the manufacturer and the retailer's costs decrease after the implementation of quantity discount mechanism.

2.3 Studies on Channel Coordination in Supply Chains with Environmental Efforts

Since environmental issues gained more importance over the last decade, studies related to channel coordination have been headed towards supply chains with environmental efforts. Among these studies, some of them incorporate carbon emissions management into decision making.

In Benjaafar et al. [23], a mixed linear integer programming model is developed that minimizes the replenishment, backorder and inventory holding costs of a firm over multiple periods under carbon cap, carbon tax, cap-and-trade and carbon offsets mechanisms. Also, multi-echelon extensions of the model are formulated, which are the models with and without collaboration. It is numerically observed that carbon constraints can increase the value of collaboration and the increase depends on the type of emission regulating mechanism. The collaboration is most effective under carbon cap mechanism. It is further observed that by introducing carbon caps along the supply chain, emissions can be decreased at lower costs. Also, it is numerically shown that if not all of the members of the chain collaborate, the costs and the emissions of the firms that do not participate in collaboration can increase.

Bouchery et al. [37] extend the EOQ model as an interactive multi-objective formulation under single and two-echelon settings. The model determines the optimal order size under economic, environmental (emissions) and social (injury rate) objectives by defining the Pareto optimal solutions. The results of the study indicate that operational adjustments effectively reduce emissions. It is further discussed that under emission regulating mechanisms that put a price on carbon emissions, the minimum amount of emissions cannot be reached. Therefore, imposing carbon caps is more effective in terms of reducing emissions.

In Jaber et al. [38], the manufacturer's production rate and number of shipments made by the manufacturer to the retailer in a production cycle are determined, where the objective is to minimize the sum of procurement, inventory holding and emission costs. The impacts of carbon taxes, cap-and-trade and emission penalties are examined. It is assumed that an emissions penalty is a fixed cost paid if the cap is exceeded; whereas, an emissions tax is paid per unit of emission. It is found that imposing only emission penalties is not effective in terms of reducing emissions and may lead to considerable amount of emissions. Also, it is shown that emission regulating mechanisms that integrate carbon taxes and emission penalties perform the best in terms of emissions reduction. It is further found that coordination decreases the supply chain costs; however, it does not decrease emission related costs.

Wahab et al. [39] extend the EOQ model to determine the optimal production-shipment policy for items with imperfect quality for a two-level closed loop supply chain. It is assumed that the percentage of items with imperfect quality is a random variable. The developed model studies the following three cases. In the first case and the second case, the buyer and the vendor are in the same and different countries, respectively. The third case incorporates fixed and variable carbon emission costs both for the buyer and the vendor. In the second case, the exchange rate between the countries is analyzed using a mean-reverting process. It is shown that including emission costs in the model decreases the optimal frequency of shipments. In the third case, it is further observed numerically that optimal shipment size can increase or decrease depending on the expected percentage of defective items.

In Chan et al. [40], the EOQ framework is used as a benchmark so as to study the coordination problem of a single vendor and multiple buyers under environmental issues. The model aims to maximize the utility resulting from cost, energy and raw materials waste for the vendor and air pollution (i.e., emissions) resulting from vendor-buyer transportation. The utility function is evaluated under independent optimization, synchronized cycles model and green optimization. Under independent optimization and synchronized cycles model, the cycle times that maximize the utility resulting from cost is found for each member of the chain and the whole chain, respectively. It is assumed that the cycle times of the buyers and the vendor are integer multiples of a basic cycle time. Under green optimization, the weighted utility resulting from cost and environmental performance measures are maximized. It is numerically illustrated that in comparison with independent optimization, cost utilities of the buyers and the vendor decrease and increase, respectively; whereas, utilities related to environmental performance measures increase under synchronized cycles model. Similarly, compared to synchronized cycles model, cost utilities of the buyers decrease; whereas, utilities related to environmental performance measures increase under green optimization. Cost utility of the vendor may increase or decrease depending on the weight assigned.

Finally, in the literature related to coordination in environmental supply chains, some studies also consider the pricing decisions where the consumers are willing to pay more to the environmental friendly products.

In Swami and Shah [41], the pricing decisions and the greening efforts which result in the maximum profit in a two-level supply chain are investigated. Centralized and decentralized models are developed, which maximize the profits of the whole chain and the retailer, respectively. It is assumed that demand linearly decreases with the retail price and linearly increases with the greening efforts of the manufacturer and the retailer. The channel coordination is achieved by a two-part tariff contract. Furthermore, it is analytically shown that total chain profit increases under the centralized model by more than 33% of the decentralized chain profit. It is numerically observed that the greening efforts are higher under the centralized model. Furthermore, the prices are lower (higher) under the centralized model for low (high) values of greening efforts.

In Zavanella et al. [42], a joint economic lot size model (JELS) that considers replenishment and inventory holding decisions of a single product under environmental considerations is developed. Similar to Swami and Shah [41], the demand rate is a decreasing linear function of the retail price and an increasing linear function of the product's environmental performance. A mathematical model that determines the vendor's production lot size, the number of shipments to the buyer, the selling price, and the amount invested by the vendor to improve environmental performance of the product is formulated. Under independent policy, the model is solved so that the buyer and the vendor maximize their profits separately; whereas, under integrated policy, the model is solved so that the total profit of the chain is maximized. It is numerically shown that under integrated policy, the total profit of the chain increases, the optimal retail price decreases and the environmental performance increases compared to independent policy.

In El Saadany et al. [43], a decision model is developed so as to examine the performance of a supply chain in terms of various quality characteristics. The retail price of the product, number of shipments made by the manufacturer to the retailer in a production cycle and the quality measure of the product are determined, where the objective is to maximize the supply chain profit. A quality function is used to optimize the quality measure of the product. It is assumed that quality measure is affected by product, process and environmental quality characteristics, each of which is assigned a weight in the quality function. It is further assumed that demand is a function of the quality and the price of the product. It is found that investments made to reduce environmental costs increases the total profit of the supply chain.

In Liu et al. [44], the competition between different manufacturers and between different retailers is analyzed. It is assumed the manufacturers produce partially substitutable products. A linear demand function is used, in which consumers are willing to pay higher prices for more environmental friendly products and the consumer environmental awareness level is a random variable. Also, a two-stage Stackelberg game is used to model the dynamics between the supply chain members. Three settings are considered, which are one manufacturer and one retailer, two manufacturers and one retailer, and two manufacturers and two retailers. It is found that as the environmental sensitivity of the consumers increase, the profits of the retailers and the manufacturer with more environmental friendly products increase. The profits of the manufacturer with less environmental friendly products increase if the manufacturing environment is not highly competitive.

Chapter 3

Supply Chain Coordination under Deterministic Demand and Cap-and-Trade Mechanism

3.1 Problem Definition under Deterministic Demand and Cap-and-Trade Mechanism

We consider a system which consists of a buyer (retailer) and a vendor (manufacturer). The buyer and the vendor operate to meet the deterministic demand of a single product in the infinite horizon using a lot-for-lot policy. That is, the quantity produced by the manufacturer at each setup is equal to the retailer's ordering lot size. Shortages are not allowed and the replenishment lead times are zero (or, equivalently, deterministic in this setting). The vendor incurs a setup cost of K_v monetary units at each production run, and the buyer incurs a fixed cost of K_b monetary units at each ordering. The vendor and the buyer are subject to cost rates h_v and h_b, respectively, for each unit held in the inventory for a unit time. It is important to note that the joint replenishment decisions in this setting have been previously studied by Banerjee and Burton [45] and Lu [46]. In this paper, we model the carbon emissions of the buyer and the vendor resulting from production and inventory related activities, and we study how the replenishment decisions can be coordinated under a cap-and-trade policy.

Under a cap-and-trade policy, both the buyer and the vendor have carbon caps (i.e., carbon emission quota per unit time). They emit carbon due to production/ordering setups, inventory holding and procurement. If the emissions per unit time of one the parties exceeds his/her cap, then he/she buys carbon credit at a rate of p_c^b monetary units for one unit carbon emission. If the emissions per unit time is below the cap, then excess amount of carbon credit is sold at a rate of p_c^s monetary units for unit carbon emission ($p_c^s \leq p_c^b$).

In order to arrive at a coordinated solution for the two-echelon system, we study two models; the decentralized model and the centralized model. In the decentralized model, buyer's independent replenishment decisions to minimize his/her cost per unit time determine the vendor's replenishment lot size. In the centralized model, buyer's and vendor's costs and constraints are simultaneously taken into account to find a quantity that minimizes the total system cost per unit time. Using the centralized solution as a benchmark, we develop mechanisms that utilize price discounts and carbon credit sharing to coordinate the system.

Before introducing the buyer's and the vendor's cost and emission functions, let us present the notation in Tables 3.1 and 3.2. Without any loss of generality, the time unit will be taken as a year in the rest of the thesis.

Under a cap-and-trade policy, buyer's average annual cost is given by

$$BC(Q, X_b) = \begin{cases} BC_1(Q, X_b) & \text{if} \quad X_b \leqslant 0 \\ \\ BC_2(Q, X_b) & \text{if} \quad X_b > 0, \end{cases} \tag{3.1}$$

where

$$BC_1(Q, X_b) = \frac{K_b D}{Q} + \frac{h_b Q}{2} + cD - p_c^b X_b, \tag{3.2}$$

and

$$BC_2(Q, X_b) = \frac{K_b D}{Q} + \frac{h_b Q}{2} + cD - p_c^s X_b. \tag{3.3}$$

Table 3.1: Problem Parameters and Decision Variables under Deterministic Demand and Cap-and-Trade Mechanism

Buyer's Parameters	
D	annual demand rate
K_b	fixed cost of ordering
h_b	cost of holding one unit inventory for a year
c	unit purchasing cost
f_b	fixed amount of carbon emission at each ordering
g_b	carbon emission amount due to holding one unit inventory for a year
e_b	carbon emission amount due to unit procurement

Vendor's Parameters	
P	annual production rate ($P > D$)
K_v	fixed cost per production run
h_v	cost of holding one unit inventory for a year
p_v	unit production cost
f_v	fixed amount of carbon emission at each production setup
g_v	carbon emission amount due to holding one unit inventory for a year
e_v	carbon emission amount due to producing one unit

Policy Parameters	
C_b	buyer's annual carbon emission cap
C_v	vendor's annual carbon emission cap
p_c^b	buying price of unit carbon emission
p_c^s	selling price of unit carbon emission

Decision Variables	
Q	buyer's order quantity (vendor's production lot size)
X_b	amount of carbon credit traded by the buyer
X_v	amount of carbon credit traded by the vendor
X_s	amount of carbon credit traded by the system in the centralized model with carbon credit sharing

Functions and Optimal Values of Decision Variables	
$BC(Q, X_b)$	buyer's average annual costs as a function of Q and X_b
$VC(Q, X_v)$	vendor's average annual costs as a function of Q and X_v
$TC(Q, X_b, X_v)$	total average annual costs as a function of Q, X_b and X_v ($TC(Q, X_b, X_v) = BC(Q, X_b) + VC(Q, X_v)$)
$SC(Q, X_s)$	total average annual costs of the buyer-vendor system in the centralized model with carbon credit sharing

Table 3.2: Problem Parameters and Decision Variables under Deterministic Demand and Cap-and-Trade Mechanism (Continued)

Functions and Optimal Values of Decision Variables (Continued)	
Q_d^*	optimal order quantity as a result of the decentralized model
Q_c^*	optimal order quantity as a result of the centralized model
Q_s^*	optimal order quantity as a result of the centralized model with carbon credit sharing

If the buyer buys carbon credit (i.e., X_b is negative), his/her annual cost function is given by Expression (3.2). If the buyer sells carbon credit (i.e., X_b is positive), his/her annual cost function is given by Expression (3.3). Note that, if the buyer neither sells nor buys carbon credit (i.e., $X_b = 0$), then $BC_1(Q, X_b) = BC_2(Q, X_b)$.

Buyer's average annual emission when Q units are ordered, amounts to

$$\frac{f_b D}{Q} + \frac{g_b Q}{2} + e_b D. \tag{3.4}$$

When no emission regulation policy is in place, $Q_d^0 = \sqrt{\frac{2K_b D}{h_b}}$ minimizes the buyer's annual costs and $\hat{Q}_d = \sqrt{\frac{2f_b D}{g_b}}$ minimizes his/her annual emissions.

Similar to Expression (3.1), vendor's annual cost is given by

$$VC\left(Q, X_v\right) = \begin{cases} VC_1\left(Q, X_v\right) & \text{if} \quad X_v \leqslant 0 \\ VC_2\left(Q, X_v\right) & \text{if} \quad X_v > 0, \end{cases} \tag{3.5}$$

where

$$VC_1\left(Q, X_v\right) = \frac{K_v D}{Q} + \frac{h_v D Q}{2P} + cD - p_c^b X_v, \tag{3.6}$$

and

$$VC_2\left(Q, X_v\right) = \frac{K_v D}{Q} + \frac{h_v D Q}{2P} + cD - p_c^s X_v. \tag{3.7}$$

If the vendor buys carbon credit (i.e., X_v is negative), his/her annual cost can be obtained by Expression (3.2), and if he/she sells carbon credit (i.e., X_v is positive), it can be obtained by Expression (3.3). If $X_v = 0$, then $VC_1(Q, X_v) =$

$VC_2(Q, X_v)$.

Vendor's average annual emission when he/she produces Q units at each setup, is

$$\frac{f_v D}{Q} + \frac{g_v DQ}{2P} + e_v D. \tag{3.8}$$

The decentralized model and the corresponding centralized model are then as follows:

Decentralized Model:

$$Min \quad BC(Q, X_b)$$

$$\text{s.t.} \quad \tfrac{f_b D}{Q} + \tfrac{g_b Q}{2} + e_b D + X_b = C_b,$$

$$Q \geq 0.$$

Centralized Model:

$$Min \quad TC(Q, X_b, X_v)$$

$$\text{s.t.} \quad \tfrac{f_b D}{Q} + \tfrac{g_b Q}{2} + e_b D + X_b = C_b,$$

$$\tfrac{f_v D}{Q} + \tfrac{g_v DQ}{2P} + e_v D + X_v = C_v,$$

$$Q \geq 0.$$

In the decentralized model presented above, buyer only considers his/her emission constraint to minimize $BC(Q, X_b)$. In the centralized model, the first and the second constraints belong to the buyer and the vendor, respectively. Since these constraints have to be satisfied at any feasible solution, with a slight change of notation, we will refer to the buyer's and the vendor's traded amounts of carbon credits for replenishing Q units by $X_b(Q)$ and $X_v(Q)$. Note that, $X_b(Q) = C_b - \frac{f_b D}{Q} - \frac{g_b Q}{2} - e_b D$ and $X_v(Q) = C_v - \frac{f_v D}{Q} - \frac{g_v DQ}{2P} - e_v D$. Buyer's optimal order quantity in the optimal solution of the decentralized model, Q_d^*, therefore, leads to $X_b(Q_d^*)$ and $X_v(Q_d^*)$ as the traded amounts of carbon credits by the buyer and the vendor. Similarly, in the optimal solution of the centralized model, the traded amounts of carbon credits by the buyer and the vendor are given by $X_b(Q_c^*)$ and $X_v(Q_c^*)$, respectively.

In order for this buyer-vendor system to achieve its maximum supply chain profitability, we will propose coordination mechanisms that entail carbon credit sharing. To this end, we introduce a third model that we refer to as the "centralized model with carbon credit sharing". In this model, it is assumed that if one party has excess carbon allowance, he/she can make it available to the other party who needs it. Therefore, the average annual costs of the buyer-vendor system under carbon credit sharing are given by

$$SC\left(Q, X_s\right) = \begin{cases} SC_1\left(Q, X_s\right) & \text{if} \quad X_s \leqslant 0 \\ \\ SC_2\left(Q, X_s\right) & \text{if} \quad X_s > 0, \end{cases} \tag{3.9}$$

where

$$SC_1(Q, X_s) = \frac{(K_b + K_v)D}{Q} + \frac{(h_b + \frac{h_v D}{P})Q}{2} + (c + p_v)D - p_c^b X_s, \tag{3.10}$$

and

$$SC_2(Q, X_s) = \frac{(K_b + K_v)D}{Q} + \frac{(h_b + \frac{h_v D}{P})Q}{2} + (c + p_v)D - p_c^s X_s. \tag{3.11}$$

Assuming carbon credit sharing is available, the centralized model is as follows:

Centralized Model with Carbon Credit Sharing:

$$\begin{aligned} &Min \quad SC(Q, X_s) \\ &\text{s.t.} \quad \tfrac{(f_b+f_v)D}{Q} + \tfrac{(g_b+\frac{g_v D}{P})Q}{2} + (e_b + e_v)D + X_s = C_b + C_v \\ &\qquad Q \geq 0. \end{aligned}$$

If the buyer-vendor system purchases carbon credit (i.e., X_s is negative), its annual cost function is presented in Expression (3.10). If the system sells carbon credit (i.e., X_s is positive), its annual cost function is presented in Expression (3.11). If the system neither purchases nor sells carbon credit (i.e., $X_s = 0$), then $SC_1(Q, X_s) = SC_2(Q, X_s)$.

Average annual emission of the system when the order size is Q units is given by

$$\frac{(f_b + f_v)D}{Q} + \frac{(g_b + \frac{g_v D}{P})Q}{2} + (e_b + e_v)D. \tag{3.12}$$

When no emission regulation policy is in place, $Q_c^0 = \sqrt{\frac{2(K_b+K_v)D}{h_b+\frac{h_v D}{P}}}$ minimizes the annual cost of the system and $\hat{Q}_c = \sqrt{\frac{2(f_b+f_v)D}{g_b+\frac{g_v D}{P}}}$ minimizes its annual emissions.

In addition, observe that, for any triplet $(Q, X_b(Q), X_v(Q))$, there exists a feasible point $(Q, X_s(Q))$ for the centralized model with carbon credit sharing, where $X_s(Q) = X_b(Q) + X_v(Q)$. Since $p_c^b \leq p_c^s$, $TC\,(Q, X_b(Q), X_v(Q))$ may not be equal to $SC\,(Q, X_s(Q))$. In fact, for any $Q \geq 0$ we have $SC\,(Q, X_s(Q)) \leq TC\,(Q, X_b(Q), X_v(Q))$. More specifically,

$$TC\,(Q, X_b(Q), X_v(Q)) - SC\,(Q, X_s(Q)) =$$

$$\begin{cases} (p_c^b - p_c^s) min\{-X_b(Q), X_v(Q)\} & \text{if} \quad X_b(Q) < 0 \text{ and } X_v(Q) > 0, \\ \\ (p_c^b - p_c^s) min\{X_b(Q), -X_v(Q)\} & \text{if} \quad X_b(Q) > 0 \text{ and } X_v(Q) < 0, \\ \\ 0 & \text{o.w.} \end{cases} \tag{3.13}$$

In the next section, we provide solution algorithms for the decentralized model and the centralized model with carbon credit sharing. We will use the solution of the latter as a benchmark to propose coordinated solutions based on discounting and carbon credit sharing mechanisms.

3.2 Analysis of the Decentralized Model and the Centralized Model with Carbon Credit Sharing under Deterministic Demand and Cap-and-Trade Mechanism

In this section, we provide an analysis of the decentralized model and the centralized model with carbon credit sharing to find Q_d^* and Q_s^*. Since the objective functions in the two models exhibit piecewise forms, we will propose algorithmic solutions based on some structural properties of the two problems.

3.2.1 Analysis of the Decentralized Model under Deterministic Demand and Cap-and-Trade Mechanism

As implied by Expression (3.1), $BC(Q, X_b)$ is given by either $BC_1(Q, X_b)$ or $BC_2(Q, X_b)$. In a feasible solution of the decentralized model, the buyer trades $X_b(Q)$ units of carbon credits. Therefore, for a feasible solution pair of Q and $X_b(Q)$, we have

$$BC_1\left(Q, X_b(Q)\right) = \frac{(K_b + p_c^b f_b)D}{Q} + \frac{(h_b + p_c^b g_b)Q}{2} + (c + p_c^b e_b)D - p_c^b C_b. \quad (3.14)$$

Note that, $BC_1\left(Q, X_b(Q)\right)$ is a strictly convex function of Q with a unique minimizer at

$$Q_{d1}^* = \sqrt{\frac{2(K_b + p_c^b f_b)D}{h_b + p_c^b g_b}}. \quad (3.15)$$

Likewise, for a feasible solution pair of Q and $X_b(Q)$, $BC_2\left(Q, X_b(Q)\right)$ can be rewritten as

$$BC_2\left(Q, X_b(Q)\right) = \frac{(K_b + p_c^s f_b)D}{Q} + \frac{(h_b + p_c^s g_b)Q}{2} + (c + p_c^s e_b)D - p_c^s C_b. \quad (3.16)$$

$BC_2\left(Q, X_b(Q)\right)$ is also a strictly convex function with a unique minimizer at

$$Q_{d2}^* = \sqrt{\frac{2(K_b + p_c^s f_b)D}{h_b + p_c^s g_b}}. \tag{3.17}$$

Lemma 1 *If $(C_b - e_b D) \leq \sqrt{2 g_b f_b D}$, then the buyer does not sell carbon credits at any order quantity, that is $X_b(Q) \leq 0$ for all Q, and $Q_d^* = Q_{d1}^*$.*

Proof: Using Expression (3.4), for any order quantity Q, the amount of traded carbon credits by the buyer is $X_b(Q) = C_b - \frac{f_b D}{Q} - \frac{g_b Q}{2} - e_b D$. Observe that $\hat{Q}_d$ minimizes $\frac{f_b D}{Q} + \frac{g_b Q}{2}$ with a minimum function value $\sqrt{2 f_b g_b D}$. That is,

$$\frac{f_b D}{Q} + \frac{g_b Q}{2} \geq \sqrt{2 f_b g_b D}$$

for all $Q \geq 0$. This implies

$$X_b(Q) \leq C_b - e_b D - \sqrt{2 f_b g_b D}.$$

Given that $(C_b - e_b D) \leq \sqrt{2 g_b f_b D}$, it turns out that $X_b(Q) \leq 0$ for all $Q \geq 0$. That is, the retailer does not sell carbon credits at any order quantity. In this case, Expression (3.1) implies that the retailer's inventory replenishment problem reduces to minimizing $BC_1(Q, X_b(Q))$ over $Q \geq 0$. As given by Expression (3.15), Q_{d1}^* is the optimal solution of this problem. ■

Lemma 1 and its proof imply that, if the annual cap is smaller than even the minimum annual emission possible by ordering decisions, then regardless of what quantity is ordered, the buyer has to purchase carbon credits. As discussed in Section 3.1, when $X_b(Q) = 0$, the buyer neither purchases nor sells carbon credits. If $(C_b - e_b D)^2 \geq 2 g_b f_b D$, there are two order quantities, which we refer to as Q_1 and Q_2, satisfying $X_b(Q) = 0$. In terms of the problem parameters, these quantities are given by

$$Q_1 = \frac{C_b - e_b D - \sqrt{(C_b - e_b D)^2 - 2 g_b f_b D}}{g_b} \tag{3.18}$$

and

$$Q_2 = \frac{C_b - e_b D + \sqrt{(C_b - e_b D)^2 - 2g_b f_b D}}{g_b}. \tag{3.19}$$

If $(C_b - e_b D)^2 > 2g_b f_b D$, we take Q_2 as the larger root, i.e., $Q_2 > Q_1$.

Lemma 2 *The buyer sells carbon credits (i.e., $X_b(Q) > 0$) only when $(C_b - e_b D) > \sqrt{2g_b f_b D}$ and $Q_1 < Q < Q_2$.*

Proof: From Lemma 1, we know that if $(C_b - e_b D) \leq \sqrt{2g_b f_b D}$, then the buyer does not sell carbon credits. Therefore, selling carbon credits is possible only when $(C_b - e_b D) > \sqrt{2g_b f_b D}$. Furthermore, under this condition, $X_b(Q) > 0$ should be satisfied. $X_b(Q) = C_b - \frac{f_b D}{Q} - \frac{g_b Q}{2} - e_b D > 0$ holds for order quantities Q such that $Q_1 < Q < Q_2$. Note that, as $(C_b - e_b D) > \sqrt{2g_b f_b D}$, both Q_1 and Q_2 are defined and $Q_1 < Q_2$. ■

Lemma 2 implies that in addition to the case of $(C_b - e_b D) \leq \sqrt{2g_b f_b D}$ suggested by Lemma 1, there are two cases that the retailer does not sell carbon credits; if $(C_b - e_b D) > \sqrt{2g_b f_b D}$ and $Q \leq Q_1$, and if $(C_b - e_b D) > \sqrt{2g_b f_b D}$ and $Q \geq Q_2$.

Lemma 3 *Depending on how $f_b h_b$ compares to $K_b g_b$, the following ordinal relations exist between Q^*_{d1} and Q^*_{d2}.*

- *If $f_b h_b > K_b g_b$, then $Q^*_{d1} > Q^*_{d2}$.*
- *If $f_b h_b = K_b g_b$, then $Q^*_{d1} = Q^*_{d2}$.*
- *If $f_b h_b < K_b g_b$, then $Q^*_{d1} < Q^*_{d2}$.*

Proof: We will prove the first part of the lemma. The proofs of the remaining two parts are similar.

Since $p^b_c \geq p^s_c$, $f_b h_b > K_b g_b$ implies that $(p^b_c - p^s_c) f_b h_b > (p^b_c - p^s_c) K_b g_b$. Adding $K_b h_b + p^b_c p^s_c f_b g_b$ to both sides of this inequality and after some rearrangement of terms, we have

$$(K_b + p_c^b f_b)(h_b + p_c^s g_b) > (K_b + p_c^s f_b)(h_b + p_c^b g_b).$$

The above expression can be rewritten as

$$\frac{(K_b + p_c^b f_b)}{(h_b + p_c^b g_b)} > \frac{(K_b + p_c^s f_b)}{(h_b + p_c^s g_b)},$$

which further implies

$$\sqrt{\frac{2(K_b + p_c^b f_b)D}{(h_b + p_c^b g_b)}} > \sqrt{\frac{2(K_b + p_c^s f_b)D}{(h_b + p_c^s g_b)}}.$$

Observe that the left hand side of the above inequality is Q_{d1}^* and the right hand side is Q_{d2}^*, and therefore, $Q_{d1}^* > Q_{d2}^*$. ■

In the next lemma, we present further properties of the retailer's problem in case of $(C_b - e_b D) > \sqrt{2 g_b f_b D}$.

Lemma 4 *When $(C_b - e_b D) > \sqrt{2 g_b f_b D}$, the following cases cannot be observed.*

- $Q_1 < Q_2 \leq Q_{d2}^* \leq Q_{d1}^*$
- $Q_{d1}^* \leq Q_{d2}^* \leq Q_1 < Q_2$.

Proof: Let us start with the first part of the lemma. Using Expression (3.17) and Expression (3.19), $Q_2 \leq Q_{d2}^*$ implies that

$$\frac{C_b - e_b D + \sqrt{(C_b - e_b D)^2 - 2 g_b f_b D}}{g_b} \leq \sqrt{\frac{2(K_b + p_c^s f_b)D}{h_b + p_c^s g_b}}.$$

Since $(C_b - e_b D) > \sqrt{2 g_b f_b D}$, the left hand side is positive. Therefore, taking the square of both sides leads to

$$\frac{(C_b - e_b D)^2 + (C_b - e_b D)\sqrt{(C_b - e_b D)^2 - 2 g_b f_b D} - g_b f_b D}{g_b} \leq \frac{(K_b g_b + p_c^s f_b g_b)D}{h_b + p_c^s g_b}$$

Due to Lemma 3, we know that having $Q^*_{d2} \leq Q^*_{d1}$ is possible only when $f_b h_b \geq K_b g_b$, which implies

$$\frac{(f_b h_b + p^s_c f_b g_b)D}{h_b + p^s_c g_b} \geq \frac{(K_b g_b + p^s_c f_b g_b)D}{h_b + p^s_c g_b}.$$

Combining the last two inequalities, we obtain

$$\frac{(C_b - e_b D)^2 + (C_b - e_b D)\sqrt{(C_b - e_b D)^2 - 2g_b f_b D} - g_b f_b D}{g_b} \leq \frac{(f_b h_b + p^s_c f_b g_b)D}{h_b + p^s_c g_b}.$$

Observe that, the right hand side of the above inequality reduces to $f_b D$. Multiplying both sides of the above expression by g_b and after some rearrangement of terms, it follows that

$$(C_b - e_b D)^2 - 2g_b f_b D \leq -(C_b - e_b D)\sqrt{(C_b - e_b D)^2 - 2g_b f_b D}.$$

Recall that, Q_1 and Q_2 were formed by considering the positive square root of the discriminant in $X_b(0)$, and Q_2 was defined as the larger root. Since $(C_b - e_b D) > \sqrt{2g_b f_b D}$, the above inequality cannot hold for the positive square root of $(C_b - e_b D)^2 - 2g_b f_b D$. Therefore, we cannot have $Q_1 < Q_2 \leq Q^*_{d2} \leq Q^*_{d1}$.

Now, let us continue with the second part of the lemma. Using Expression (3.17) and Expression (3.18), $Q^*_{d2} \leq Q_1$ implies that

$$\sqrt{\frac{2(K_b + p^s_c f_b)D}{h_b + p^s_c g_b}} \leq \frac{C_b - e_b D - \sqrt{(C_b - e_b D)^2 - 2g_b f_b D}}{g_b}.$$

Taking the square of both sides of this inequality leads to

$$\frac{(K_b + p^s_c f_b)D}{h_b + p^s_c g_b} \leq \frac{(C_b - e_b D)^2 - (C_b - e_b D)\sqrt{(C_b - e_b D)^2 - 2g_b f_b D} - g_b f_b D}{(g_b)^2},$$

which is equivalent to

$$\frac{(K_b g_b + p^s_c f_b g_b)D}{h_b + p^s_c g_b} \leq \frac{(C_b - e_b D)^2 - (C_b - e_b D)\sqrt{(C_b - e_b D)^2 - 2g_b f_b D} - g_b f_b D}{g_b}.$$

Based on Lemma 5, having $Q^*_{d2} \geq Q^*_{d1}$ suggests that $f_b h_b \leq K_b g_b$, which implies

$$\frac{(f_b h_b + p^s_c f_b g_b)D}{h_b + p^s_c g_b} \leq \frac{(C_b - e_b D)^2 - (C_b - e_b D)\sqrt{(C_b - e_b D)^2 - 2g_b f_b D} - g_b f_b D}{g_b}.$$

Observe that, the left hand side of the above inequality reduces to $f_b D$. Therefore, after some rearrangement of terms, it can be rewritten as

$$(C_b - e_b D)^2 - 2g_b f_b D \geq (C_b - e_b D)\sqrt{(C_b - e_b D)^2 - 2g_b f_b D}.$$

Again, the above inequality cannot hold for the positive square root of $(C_b - e_b D)^2 - 2g_b f_b D$. Therefore, we cannot have $Q^*_{d1} \leq Q^*_{d2} \leq Q_1 < Q_2$. ■

The first part of Lemma 4 implies that when $(C_b - e_b D) > \sqrt{2g_b f_b D}$, the case of $Q_1 < Q_2 \leq Q^*_{d2} = Q^*_{d1}$ cannot occur. Likewise, the second part implies that when $(C_b - e_b D) > \sqrt{2g_b f_b D}$, the case of $Q^*_{d1} = Q^*_{d2} \leq Q_1 < Q_2$ cannot take place. Combining this result with Lemma 3 further leads to the following implication: If $(C_b - e_b D) > \sqrt{2g_b f_b D}$ and $f_b h_b = K_b g_b$, the only possible ordering of Q_1, Q_2, Q^*_{d1} and Q^*_{d2} is $Q_1 < Q^*_{d1} = Q^*_{d2} < Q_2$. Because, having $(C_b - e_b D) > \sqrt{2g_b f_b D}$ implies $Q_2 > Q_1$, and it follows due to Lemma 3 that as $f_b h_b = K_b g_b$ we have $Q^*_{d1} = Q^*_{d2}$. Under these conditions, excluding the cases covered in Lemma 4 from further consideration, the only possible ordering that remains is $Q_1 < Q^*_{d1} = Q^*_{d2} < Q_2$.

Lemma 5 *If $(C_b - e_b D) > \sqrt{2g_b f_b D}$ and $f_b h_b = K_b g_b$, then $Q^*_d = Q^*_{d1} = Q^*_{d2}$.*

Proof: Under the conditions of the lemma, the only possible ordering of Q_1, Q_2, Q^*_{d1} and Q^*_{d2} is $Q_1 < Q^*_{d1} = Q^*_{d2} < Q_2$. In order to prove the lemma, we will consider three regions of Q separately; $Q \leq Q_1$, $Q_1 < Q < Q_2$, and $Q \geq Q_2$. Expression (3.1) and Lemma 2 together imply that if $(C_b - e_b D) > \sqrt{2g_b f_b D}$, for order quantities Q such that $Q_1 < Q < Q_2$, we have $BC\left(Q, X_b(Q)\right) = BC_2\left(Q, X_b(Q)\right)$; for order quantities Q such that $Q \leq Q_1$, we have $BC\left(Q, X_b(Q)\right) = BC_1\left(Q, X_b(Q)\right)$; for order quantities Q such that $Q \geq Q_2$, we have $BC\left(Q, X_b(Q)\right) = BC_1\left(Q, X_b(Q)\right)$.

Let us start with Q such that $Q_1 < Q < Q_2$ and $Q \neq Q^*_{d2}$. Since Q^*_{d2} is the unique minimizer of $BC_2(Q, X_b(Q))$ and $BC\left(Q, X_b(Q)\right) = BC_2\left(Q, X_b(Q)\right)$, it follows that

$$BC\left(Q, X_b(Q)\right) = BC_2\left(Q, X_b(Q)\right) > BC_2\left(Q^*_{d2}, X_b(Q^*_{d2})\right) = BC\left(Q^*_{d2}, X_b(Q^*_{d2})\right), \quad \forall Q \text{ s.t. } Q_1 < Q < Q_2 \text{ and } Q \neq Q^*_{d2}. \tag{3.20}$$

Now, let us continue with $Q \leq Q_1$. Recall that at Q_1, we have $BC_1\left(Q_1, X_b(Q_1)\right) = BC_2\left(Q_1, X_b(Q_1)\right)$. Since $BC_1\left(Q, X_b(Q)\right)$ is a strictly convex function with a unique minimizer Q^*_{d1}, and $Q \leq Q_1 < Q^*_{d1}$, it follows that

$$BC_1\left(Q, X_b(Q)\right) \geq BC_1\left(Q_1, X_b(Q_1)\right) = BC_2\left(Q_1, X_b(Q_1)\right).$$

Using the fact that $BC_2\left(Q, X_b(Q)\right)$ is a strictly convex function with a unique minimizer Q^*_{d2}, and $Q_1 \neq Q^*_{d2}$, we further have

$$BC_2\left(Q_1, X_b(Q_1)\right) > BC_2\left(Q^*_{d2}, X_b(Q^*_{d2})\right).$$

Combining the last two inequalities leads to

$$BC_1\left(Q, X_b(Q)\right) > BC_2\left(Q^*_{d2}, X_b(Q^*_{d2})\right),$$

which is equivalent to

$$BC\left(Q, X_b(Q)\right) > BC\left(Q^*_{d2}, X_b(Q^*_{d2})\right), \qquad \forall Q \text{ s.t. } Q \leq Q_1. \tag{3.21}$$

Finally, let us consider order quantities Q such that $Q \geq Q_2$. Recall that at Q_2, we have $BC_1\left(Q_2, X_b(Q_2)\right) = BC_2\left(Q_2, X_b(Q_2)\right)$. Since $BC_1\left(Q, X_b(Q)\right)$ is a strictly convex function with a unique minimizer Q^*_{d1}, and $Q^*_{d1} < Q_2 \leq Q$, it follows that

$$BC_1\left(Q, X_b(Q)\right) \geq BC_1\left(Q_2, X_b(Q_2)\right) = BC_2\left(Q_2, X_b(Q_2)\right).$$

Using the fact that $BC_2(Q, X_b(Q))$ is a strictly convex function with a unique minimizer Q^*_{d2}, and $Q_2 \neq Q^*_{d2}$, we further have

$$BC_2(Q_2, X_b(Q_2)) > BC_2(Q^*_{d2}, X_b(Q^*_{d2})).$$

Combining the last two inequalities leads to

$$BC_1(Q, X_b(Q)) > BC_2(Q^*_{d2}, X_b(Q^*_{d2})),$$

which, also, is equivalent to

$$BC(Q, X_b(Q)) > BC(Q^*_{d2}, X_b(Q^*_{d2})), \qquad \forall Q \text{ s.t. } Q \geq Q_2. \qquad (3.22)$$

Based on Expressions (3.20), (3.21) and (3.22), we conclude that $Q^* = Q^*_{d2}$. ■

Lemma 1 and Lemma 5 constitute parts of our solution algorithm for the retailer's decentralized replenishment problem. Lemma 1 suggests the solution in case of $(C_b - e_bD) \leq \sqrt{2g_bf_bD}$, and Lemma 5 provides the solution in case of $(C_b - e_bD) > \sqrt{2g_bf_bD}$ and $f_bh_b = K_bg_b$. At this point, there is one more case to be considered, that is, $(C_b - e_bD) > \sqrt{2g_bf_bD}$ and $f_bh_b \neq K_bg_b$. Before proceeding with a detailed analysis of this case, let us present a result which applies to the case of $(C_b - e_bD) > \sqrt{2g_bf_bD}$ in general.

Lemma 6 *When $(C_b - e_bD) > \sqrt{2g_bf_bD}$, we have $BC_1(Q, X_b(Q)) \leq BC_2(Q, X_b(Q))$ for all Q such that $Q_1 \leq Q \leq Q_2$, and $BC_1(Q, X_b(Q)) > BC_2(Q, X_b(Q))$ for all Q such that $Q < Q_1$ or $Q > Q_2$.*

Proof: Recall that $X_b(Q) = C_b - \frac{f_bD}{Q} - \frac{g_bQ}{2} - e_bD$, and $X_b(Q) = 0$ when $Q = Q_1$ and $Q = Q_2$. Furthermore, we have $X_b(Q) > 0$ for all Q s.t. $Q_1 < Q < Q_2$, and we have $X_b(Q) < 0$ for all Q s.t. $Q < Q_1$ and for all Q s.t. $Q > Q_2$. We will show that $BC_1(Q, X_b(Q)) \leq BC_2(Q, X_b(Q))$ if $Q \in [Q_1, Q_2]$. The proofs of the other parts of the lemma, which will be omitted, follow in a similar fashion.

Since $p_c^b \geq p_c^s$, it follows that

$$(p_c^b - p_c^s)\left(C_b - \frac{f_b D}{Q} - \frac{g_b Q}{2} - e_b D\right) \geq 0.$$

After adding $\frac{K_b D}{Q} + \frac{h_b Q}{2} + cD$ to both sides of the above inequality and rearranging the terms, we have

$$\begin{aligned} \tfrac{K_b D}{Q} + \tfrac{h_b Q}{2} + cD - p_c^b\left(C_b - \tfrac{f_b D}{Q} - \tfrac{g_b Q}{2} - e_b D\right) \;&\leq \tfrac{K_b D}{Q} + \tfrac{h_b Q}{2} + cD \\ &-p_c^s\left(C_b - \tfrac{f_b D}{Q} - \tfrac{g_b Q}{2} - e_b D\right), \end{aligned}$$

which implies $BC_1\left(Q, X_b(Q)\right) \leq BC_2\left(Q, X_b(Q)\right)$. ■

The above lemma will be used in the proofs of the next two results.

Lemma 7 *When* $(C_b - e_b D) > \sqrt{2 g_b f_b D}$ *and* $f_b h_b < K_b g_b$*, the following orderings among* Q_1*,* Q_2*,* Q_{d1}^**, and* Q_{d2}^* *cannot take place:*

- $Q_{d1}^* \leq Q_1 < Q_2 \leq Q_{d2}^*$,
- $Q_{d1}^* \leq Q_1 < Q_{d2}^* < Q_2$,
- $Q_{d1}^* < Q_{d2}^* \leq Q_1 < Q_2$.

Proof: We will prove the first two parts of the lemma. Note that the third part is a special case of $Q_{d1}^* \leq Q_{d2}^* \leq Q_1 < Q_2$ and is covered in Lemma 4.

Due to the strict convexity of $BC_2\left(Q, X_b(Q)\right)$ and the fact that Q_{d2}^* is its minimizer, having $Q_1 < Q_2 \leq Q_{d2}^*$ implies

$$BC_2\left(Q_1, X_b(Q_1)\right) > BC_2\left(Q_2, X_b(Q_2)\right).$$

At $Q = Q_1$ and $Q = Q_2$, we have $BC_1\left(Q, X_b(Q)\right) = BC_2\left(Q, X_b(Q)\right)$. Therefore, the above inequality is equivalent to the following:

$$BC_1\left(Q_1, X_b(Q_1)\right) > BC_1\left(Q_2, X_b(Q_2)\right). \tag{3.23}$$

However, due to the strict convexity of $BC_1(Q, X_b(Q))$ and Q^*_{d1} being its unique minimizer, having $Q^*_{d1} \leq Q_1 < Q_2$ would imply

$$BC_1(Q_1, X_b(Q_1)) < BC_1(Q_2, X_b(Q_2)). \tag{3.24}$$

Expression (3.23) and (3.24) contradict, therefore, it is not possible to have $Q^*_{d1} \leq Q_1 < Q_2 \leq Q^*_{d2}$.

Now, let us continue with the proof of the second part. Note that Lemma 6 and its proof imply

$$BC_1(Q^*_{d2}, X_b(Q^*_{d2})) < BC_2(Q^*_{d2}, X_b(Q^*_{d2}))$$

in case of $Q_1 < Q^*_{d2} < Q_2$. Furthermore, having $Q_1 < Q^*_{d2}$ leads to $BC_2(Q^*_{d2}, X_b(Q^*_{d2})) < BC_2(Q_1, X_b(Q_1))$ due to the strict convexity of $BC_2(Q, X_b(Q))$ and the fact that Q^*_{d2} is its minimizer. Combining this with the above inequality implies

$$BC_1(Q^*_{d2}, X_b(Q^*_{d2})) < BC_2(Q_1, X_b(Q_1)).$$

At $Q = Q_1$, we have $BC_2(Q_1, X_b(Q_1)) = BC_1(Q_1, X_b(Q_1))$. Therefore, the above expression is equivalent to

$$BC_1(Q^*_{d2}, X_b(Q^*_{d2})) < BC_1(Q_1, X_b(Q_1)). \tag{3.25}$$

However, due to the strict convexity of $BC_1(Q, X_b(Q))$ and Q^*_{d1} being its unique minimizer, having $Q^*_{d1} \leq Q_1 < Q^*_{d2}$ would imply

$$BC_1(Q^*_{d2}, X_b(Q^*_{d2})) > BC_1(Q_1, X_b(Q_1)). \tag{3.26}$$

As Expressions (3.25) and (3.26) contradict, it is not possible to have $Q^*_{d1} \leq Q_1 < Q^*_{d2} < Q_2$. ■

Notice that, since $(C_b - e_b D) > \sqrt{2g_b f_b D}$ and $f_b h_b < K_b g_b$ are the two conditions of Lemma 7, two common properties of the cases considered are $Q_1 < Q_2$ and $Q^*_{d1} < Q^*_{d2}$. Lemma 7 further leads to the result in the next corollary.

Corollary 1 *When $(C_b - e_b D) > \sqrt{2g_b f_b D}$ and $f_b h_b < K_b g_b$, the following orderings are possible:*

- $Q_1 < Q_2 \leq Q^*_{d1} < Q^*_{d2}$,
- $Q_1 < Q^*_{d1} < Q^*_{d2} \leq Q_2$,
- $Q_1 < Q^*_{d1} < Q_2 < Q^*_{d2}$.

Numerical instances to illustrate the cases in Corollary 1 are presented in Table 3.3. The first three examples of Table 3.3 correspond to the different cases of the corollary in the order they are presented. In the next lemma, we provide a similar result to Lemma 7, now for the case of $(C_b - e_b D) > \sqrt{2g_b f_b D}$ and $f_b h_b > K_b g_b$.

Table 3.3: Numerical Illustrations of Corollary 1 and Corollary 2 Given $D = 50$, $c = 12$ and $g_b = 0.5$

Example Index	K_b	h_b	f_b	e_b	p^c_b	p^c_s	C_b	Q^*_{d1}	Q^*_{d2}	Q_1	Q_2
1	900	1	40	5	7.5	6	300	158.944	168.819	55.279	144.721
2	500	1	90	5	7.5	6	350	157.28	161.245	51.676	348.324
3	900	1	40	5	7.5	6	303	158.944	168.819	49.114	162.886
4	100	1.2	90	5	2.5	2	320	115.175	112.815	100	180
5	40	3.2	90	4.5	2.5	2	304	77.169	72.375	74.549	241.451
6	40	3.2	90	4.5	2.5	2	300	77.169	72.375	82.918	217.082

Lemma 8 *When $(C_b - e_b D) > \sqrt{2g_b f_b D}$ and $f_b h_b > K_b g_b$, the following orderings among Q_1, Q_2, Q^*_{d1}, and Q^*_{d2} cannot take place:*

- $Q^*_{d2} < Q_1 < Q_2 \leq Q^*_{d1}$,
- $Q_1 \leq Q^*_{d2} < Q_2 \leq Q^*_{d1}$,

- $Q_1 < Q_2 \leq Q^*_{d2} < Q^*_{d1}$.

Proof: Similar to the proof of Lemma 7, we will prove the first two parts of the lemma. The third part is a special case of $Q_1 < Q_2 \leq Q^*_{d2} \leq Q^*_{d1}$ and is covered in Lemma 4.

Let us assume that the ordering in the first part of the lemma takes place. Due to Lemma 6, having $Q_2 \leq Q^*_{d1}$ implies $BC_1\left(Q^*_{d1}, X_b(Q^*_{d1})\right) \geq BC_2\left(Q^*_{d1}, X_b(Q^*_{d1})\right)$. Furthermore, it follows from the strict convexity of $BC_2\left(Q, X_b(Q)\right)$ that having $Q^*_{d2} < Q_1 < Q_2 \leq Q^*_{d1}$ leads to

$$BC_2\left(Q^*_{d1}, X_b(Q^*_{d1})\right) \geq BC_2\left(Q_2, X_b(Q_2)\right) > BC_2\left(Q_1, X_b(Q_1)\right),$$

and hence, $BC_1\left(Q^*_{d1}, X_b(Q^*_{d1})\right) > BC_2\left(Q_1, X_b(Q_1)\right)$. At $Q = Q_1$, we have $BC_2\left(Q, X_b(Q)\right) = BC_1\left(Q, X_b(Q)\right)$. Therefore, if the ordering is true as it is assumed, it would follow that $BC_1\left(Q^*_{d1}, X_b(Q^*_{d1})\right) > BC_1\left(Q_1, X_b(Q_1)\right)$, which contradicts with Q^*_{d1}'s being the minimizer of $BC_1\left(Q, X_b(Q)\right)$. Therefore, it is not possible to have $Q^*_{d2} < Q_1 < Q_2 \leq Q^*_{d1}$.

Let us continue with the proof of the second part by assuming that there exists an instance with this ordering. Due to Lemma 6, having $Q_1 \leq Q^*_{d2} < Q_2$ implies $BC_2\left(Q^*_{d2}, X_b(Q^*_{d2})\right) \geq BC_1\left(Q^*_{d2}, X_b(Q^*_{d2})\right)$. Furthermore, it follows from the strict convexity of $BC_1\left(Q, X_b(Q)\right)$ that having $Q^*_{d2} < Q_2 \leq Q^*_{d1}$ leads to

$$BC_1\left(Q^*_{d2}, X_b(Q^*_{d2})\right) > BC_1\left(Q_2, X_b(Q_2)\right) \geq BC_1\left(Q^*_{d1}, X_b(Q^*_{d1})\right),$$

and hence, $BC_2\left(Q^*_{d2}, X_b(Q^*_{d2})\right) > BC_1\left(Q^*_{d1}, X_b(Q^*_{d1})\right)$. Using Lemma 6 once again and the fact that $Q^*_{d1} \geq Q_2$, we must have $BC_1\left(Q^*_{d1}, X_b(Q^*_{d1})\right) \geq BC_2\left(Q^*_{d1}, X_b(Q^*_{d1})\right)$, which would imply $BC_2\left(Q^*_{d2}, X_b(Q^*_{d2})\right) > BC_2\left(Q^*_{d1}, X_b(Q^*_{d1})\right)$. However, this contradicts with the fact that Q^*_{d2} is the minimizer of $BC_2\left(Q, X_b(Q)\right)$. Therefore, it is not possible to have $Q_1 \leq Q^*_{d2} < Q_2 \leq Q^*_{d1}$. ■

Note that under the two conditions of Lemma 8, two common properties of the cases considered are $Q_1 < Q_2$ and $Q^*_{d1} > Q^*_{d2}$. Lemma 8 further leads to the result in the next corollary.

Corollary 2 *When* $(C_b - e_bD) > \sqrt{2g_bf_bD}$ *and* $f_bh_b > K_bg_b$*, the following orderings are possible:*

- $Q_1 \leq Q_{d2}^* < Q_{d1}^* < Q_2$,
- $Q_{d2}^* < Q_1 < Q_{d1}^* < Q_2$,
- $Q_{d2}^* < Q_{d1}^* \leq Q_1 < Q_2$.

Numerical instances to illustrate the cases in Corollary 2 are also presented in Table 3.3. The last three examples of Table 3.3 correspond to the different cases of the corollary in the order they are presented.

Combining our results in Lemma 1, Lemma 5, Corollary 1 and Corollary 2, we propose the following algorithm to find the retailer's optimal solution to the decentralized model, i.e., Q_d^*.

Algorithm 1: Solution of the Decentralized Model

1. If $(C_b - e_bD) \leq \sqrt{2g_bf_bD}$, then set $Q_d^* = Q_{d1}^*$.
2. If $(C_b - e_bD) > \sqrt{2g_bf_bD}$, then do the following:
   (a) If $f_bh_b = K_bg_b$, set $Q_d^* = Q_{d2}^*$.
   (b) If $f_bh_b < K_bg_b$, and
      i. if $Q_2 \leq Q_{d1}^*$, set $Q_d^* = Q_{d1}^*$,
      ii. else,
         A. if $Q_2 \geq Q_{d2}^*$, set $Q_d^* = Q_{d2}^*$,
         B. if $Q_2 < Q_{d2}^*$, set $Q_d^* = Q_2$.
   (c) If $f_bh_b > K_bg_b$, and
      i. if $Q_{d1}^* \leq Q_1$, set $Q_d^* = Q_{d1}^*$,
      ii. else,

A. if $Q^*_{d2} \geq Q_1$, set $Q^*_d = Q^*_{d2}$,

B. if $Q^*_{d2} < Q_1$, set $Q^*_d = Q_1$.

Theorem 1 *Algorithm 1 gives the optimal solution to the retailer's replenishment problem formulated in the Decentralized Model.*

Proof: The proof will follow based on considering the cases presented in Lemma 1, Lemma 5, Corollary 1 and Corollary 2.

Case 1: $(C_b - e_b D) \leq \sqrt{2 g_b f_b D}$

It follows due to Lemma 1 that in this case $Q^*_d = Q^*_{d1}$.

Case 2: $(C_b - e_b D) > \sqrt{2 g_b f_b D}$

We have the following three subcases ($f_b h_b = K_b g_b$, $f_b h_b < K_b g_b$ and $f_b h_b > K_b g_b$):

Case 2.1: $(C_b - e_b D) > \sqrt{2 g_b f_b D}$ and $f_b h_b = K_b g_b$

It follows due to Lemma 5 that in this case $Q^*_d = Q^*_{d2}$.

Case 2.2: $(C_b - e_b D) > \sqrt{2 g_b f_b D}$ and $f_b h_b < K_b g_b$

Corollary 1 implies the following three subcases: $Q_1 < Q_2 \leq Q^*_{d1} < Q^*_{d2}$, $Q_1 < Q^*_{d1} < Q^*_{d2} \leq Q_2$, $Q_1 < Q^*_{d1} < Q_2 < Q^*_{d2}$. We present a detailed proof for the first subcase. Since the proofs of the other subcases are similar, we present sketches of proofs for the others.

- *Case 2.2.1:* $Q_1 < Q_2 \leq Q^*_{d1} < Q^*_{d2}$ Note that the subcase of $Q_1 < Q_2 \leq Q^*_{d1} < Q^*_{d2}$ is distinguished from the other two by the fact that $Q_2 \leq Q^*_{d1}$. The proof will follow by considering three different regions of Q (those are $Q > Q_2$, $Q_1 \leq Q \leq Q_2$, $Q < Q_1$), and in each case by showing that $BC\left(Q_{d1}^*, X_b(Q_{d1}^*)\right) \leq BC\left(Q, X_b(Q)\right)$. Let us start with Q values such that $Q > Q_2$. Expression (3.1) and Lemma 2 imply that $BC\left(Q, X_b(Q)\right) = BC_1\left(Q, X_b(Q)\right)$. By definition, Q_{d1}^* is the minimizer of $BC_1\left(Q, X_b(Q)\right)$, therefore, $BC_1\left(Q, X_b(Q)\right) \geq BC_1\left(Q_{d1}^*, X_b(Q_{d1}^*)\right)$. Since Q_{d1}^* is also in the region of Q values considered (i.e., $Q_{d1}^* \geq Q_2$), this, in turn, is equivalent to $BC\left(Q, X_b(Q)\right) \geq BC\left(Q_{d1}^*, X_b(Q_{d1}^*)\right)$. Now, let us consider Q values such that $Q_1 \leq Q \leq Q_2$. Expression (3.1) and Lemma 2 imply that $BC\left(Q, X_b(Q)\right) = BC_2\left(Q, X_b(Q)\right)$. Since $BC_2\left(Q, X_b(Q)\right)$ is a strictly convex function with a unique minimizer Q_{d2}^* and $Q < Q_{d2}^*$, $BC_2\left(Q, X_b(Q)\right)$, and hence $BC\left(Q, X_b(Q)\right)$, is decreasing in this region. Therefore, $BC\left(Q, X_b(Q)\right) \geq BC\left(Q_2, X_b(Q_2)\right)$ for all Q such that $Q_1 \leq Q \leq Q_2$. Furthermore, we have $BC\left(Q_2, X_b(Q_2)\right) = BC_2\left(Q_2, X_b(Q_2)\right) = BC_1\left(Q_2, X_b(Q_2)\right)$ and $BC_1\left(Q_2, X_b(Q_2)\right) \geq BC_1\left(Q_{d1}^*, X_b(Q_{d1}^*)\right) = BC\left(Q_{d1}^*, X_b(Q_{d1}^*)\right)$. Hence, $BC\left(Q, X_b(Q)\right) \geq BC\left(Q_{d1}^*, X_b(Q_{d1}^*)\right)$. Finally, let us consider Q values such that $Q < Q_1$. Again, due to Expression (3.1) and Lemma 2, we know that $BC\left(Q, X_b(Q)\right) = BC_1\left(Q, X_b(Q)\right)$. Since $BC_1\left(Q, X_b(Q)\right)$ is a strictly convex function with a unique minimizer Q_{d1}^* and $Q < Q_{d1}^*$, $BC_1\left(Q, X_b(Q)\right)$, and hence $BC\left(Q, X_b(Q)\right)$, is decreasing in this region. Therefore, $BC\left(Q, X_b(Q)\right) > BC\left(Q_1, X_b(Q_1)\right)$ for all Q such that $Q < Q_1$. We have discussed above that $BC\left(Q, X_b(Q)\right)$ is decreasing over $Q_1 \leq Q \leq Q_2$, hence $BC\left(Q_1, X_b(Q_1)\right) > BC\left(Q_2, X_b(Q_2)\right)$. Combining the last two results implies $BC\left(Q, X_b(Q)\right) > BC\left(Q_2, X_b(Q_2)\right)$. We have also argued above that $BC\left(Q_2, X_b(Q_2)\right) \geq BC\left(Q_{d1}^*, X_b(Q_{d1}^*)\right)$. Therefore, we conclude $BC\left(Q, X_b(Q)\right) > BC\left(Q_{d1}^*, X_b(Q_{d1}^*)\right)$

- *Case 2.2.2:* $Q_1 < Q_{d1}^* < Q_{d2}^* \leq Q_2$

  We have $BC\left(Q, X_b(Q)\right) \geq BC\left(Q_{d2}^*, X_b(Q_{d2}^*)\right)$ for all $Q \in [Q_1, Q_2]$, because, $Q_1 < Q_{d2}^* \leq Q_2$ and $BC\left(Q, X_b(Q)\right) = BC_2\left(Q, X_b(Q)\right)$ in this region of Q values. Next, we use the facts that $BC\left(Q, X_b(Q)\right) = BC_1\left(Q, X_b(Q)\right)$ for all $Q \in (Q_2, \infty)$, $BC_1\left(Q, X_b(Q)\right)$ is increasing in this region, and $BC_1\left(Q_2, X_b(Q_2)\right) = BC_2\left(Q_2, X_b(Q_2)\right)$ to conclude that $BC\left(Q, X_b(Q)\right) > BC\left(Q_2, X_b(Q_2)\right)$. This further implies $BC\left(Q, X_b(Q)\right) >$

$BC\left(Q^*_{d2}, X_b(Q^*_{d2})\right)$ for all $Q \in (Q_2, \infty)$. Finally, using the facts that $BC\left(Q, X_b(Q)\right) = BC_1\left(Q, X_b(Q)\right)$ for all Q such that $Q < Q_1$, $BC_1\left(Q, X_b(Q)\right)$ is decreasing in this region, and $BC_1\left(Q_1, X_b(Q_1)\right) = BC_2\left(Q_1, X_b(Q_1)\right)$ to conclude that $BC\left(Q, X_b(Q)\right) > BC\left(Q_1, X_b(Q_1)\right)$. This further implies $BC\left(Q, X_b(Q)\right) > BC\left(Q^*_{d2}, X_b(Q^*_{d2})\right)$ for all Q such that $Q < Q_1$.

- *Case 2.2.3:* $Q_1 < Q^*_{d1} < Q_2 < Q^*_{d2}$

  We have $BC\left(Q, X_b(Q)\right) \geq BC\left(Q_2, X_b(Q_2)\right)$ for all $Q \in [Q_1, Q_2]$, because, $BC\left(Q, X_b(Q)\right) = BC_2\left(Q, X_b(Q)\right)$ and $Q_1 < Q_2 < Q^*_{d2}$ (implying that $BC_2\left(Q, X_b(Q)\right)$ is decreasing in this region of Q values). Next, we use the facts that $BC\left(Q, X_b(Q)\right) = BC_1\left(Q, X_b(Q)\right)$ for all $Q \in (Q_2, \infty)$ and $Q^*_{d1} < Q_2$ (implying that $BC_1\left(Q, X_b(Q)\right)$ is increasing in this region) to conclude that $BC\left(Q, X_b(Q)\right) > BC\left(Q_2, X_b(Q_2)\right)$. Finally, using the facts that $BC\left(Q, X_b(Q)\right) = BC_1\left(Q, X_b(Q)\right)$ for all Q such that $Q < Q_1$, and $Q_1 < Q^*_{d1}$ (implying that $BC_1\left(Q, X_b(Q)\right)$ is decreasing in this region), to conclude that $BC\left(Q, X_b(Q)\right) > BC\left(Q_1, X_b(Q_1)\right)$. Combining this with the fact that $BC\left(Q_1, X_b(Q_1)\right) > BC\left(Q_2, X_b(Q_2)\right)$ further leads to $BC\left(Q, X_b(Q)\right) > BC\left(Q_2, X_b(Q_2)\right)$ for all Q such that $Q < Q_1$.

Case 2.3: $(C_b - e_b D) > \sqrt{2 g_b f_b D}$ and $f_b h_b > K_b g_b$

Corollary 2 implies the following three subcases: $Q_1 \leq Q^*_{d2} < Q^*_{d1} < Q_2$, $Q^*_{d2} < Q_1 < Q^*_{d1} < Q_2$, $Q^*_{d2} < Q_1 < Q^*_{d1} < Q_2$, $Q^*_{d2} < Q^*_{d1} \leq Q_1 < Q_2$. A detailed proof will be omitted for this case as it follows by analyzing the different subcases as in the proof of Case 2.2. ∎

Finally, we present a further property of $BC(Q, X_b(Q))$, which is used in the proofs in Section 3.3.

Proposition 1 *$BC(Q, X_b(Q))$ is a convex function of Q.*

Proof: Suppose $C_b - e_bD \leq \sqrt{2g_bf_bD}$. Using Expression (3.1) and Lemma 1, $BC(Q, X_b(Q)) = BC_1(Q, X_b(Q))$ for all Q. Since $BC_1(Q, X_b(Q))$ is a convex function of Q, $BC(Q, X_b(Q))$ is also a convex function of Q if $C_b - e_bD \leq \sqrt{2g_bf_bD}$.

Suppose now that $C_b - e_bD > \sqrt{2g_bf_bD}$. It suffices to show $BC(\alpha Q_a + (1-\alpha)Q_b, X_b(\alpha Q_a + (1-\alpha)Q_b)) \leqslant \alpha BC(Q_a, X_b(Q_a)) + (1-\alpha)BC(Q_b, X_b(Q_b))$ for all $Q_a \geqslant 0$, $Q_b \geqslant 0$ and $\alpha\epsilon[0,1]$. Expression (3.1) and Lemma 2 imply that $BC(Q, X_b(Q)) = BC_1(Q, X_b(Q))$ if $Q \leq Q_1$ or $Q \geq Q_2$ and $BC(Q, X_b(Q)) = BC_2(Q, X_b(Q))$ if $Q_1 < Q < Q_2$. Also, using Lemma 6, $BC_1(Q, X_b(Q)) \leq BC_2(Q, X_b(Q))$ if $Q_1 \leq Q \leq Q_2$ and $BC_2(Q, X_b(Q)) > BC_1(Q, X_b(Q))$ if $Q < Q_1$ or $Q > Q_2$. Combining the last two results, we have $BC(Q, X_b(Q)) = \max\{BC_1(Q, X_b(Q)), BC_2(Q, X_b(Q))\}$ when $C_b - e_bD > \sqrt{2g_bf_bD}$. Since $BC_1(Q, X_b(Q))$ and $BC_2(Q, X_b(Q))$ are convex functions of Q, $BC_1(\alpha Q_a + (1-\alpha)Q_b, X_b(\alpha Q_a + (1-\alpha)Q_b)) \leq \alpha BC_1(Q_a, X_b(Q_a)) + (1-\alpha)BC_1(Q_b, X_b(Q_b))$ and $BC_2(\alpha Q_a + (1-\alpha)Q_b, X_b(\alpha Q_a + (1-\alpha)Q_b)) \leq \alpha BC_2(Q_a, X_b(Q_a)) + (1-\alpha)BC_2(Q_b, X_b(Q_b))$ for all $Q_a \geqslant 0$, $Q_b \geqslant 0$ and $\alpha\epsilon[0,1]$, when $C_b - e_bD > \sqrt{2g_bf_bD}$. Combining this with $BC(Q, X_b(Q)) = \max\{BC_1(Q, X_b(Q)), BC_2(Q, X_b(Q))\}$ we found above, we have $BC_1(\alpha Q_a + (1-\alpha)Q_b, X_b(\alpha Q_a + (1-\alpha)Q_b)) \leqslant \alpha BC(Q_a, X_b(Q_a)) + (1-\alpha)BC(Q_b, X_b(Q_b))$ and $BC_2(\alpha Q_a + (1-\alpha)Q_b, X_b(\alpha Q_a + (1-\alpha)Q_b)) \leqslant \alpha BC(Q_a, X_b(Q_a)) + (1-\alpha)BC(Q_b, X_b(Q_b))$. Hence, $BC(\alpha Q_a + (1-\alpha)Q_b, X_b(\alpha Q_a + (1-\alpha)Q_b)) = \max\{BC_1(\alpha Q_a + (1-\alpha)Q_b, X_b(\alpha Q_a + (1-\alpha)Q_b)), BC_2(\alpha Q_a + (1-\alpha)Q_b, X_b(\alpha Q_a + (1-\alpha)Q_b))\} \leq \alpha BC(Q_a, X_b(Q_a)) + (1-\alpha)BC(Q_b, X_b(Q_b))$. Thus, $BC(Q, X_b(Q))$ is also a convex function of Q if $C_b - e_bD > \sqrt{2g_bf_bD}$. ■

Next, we proceed with a similar analysis for the centralized model with carbon sharing.

3.2.2 Analysis of the Centralized Model with Carbon Credit Sharing under Deterministic Demand and Cap-and-Trade Mechanism

Similar to the analysis of the decentralized model, using Expression (3.9), $SC(Q, X_s)$ is presented as either $SC_1(Q, X_s)$ or $SC_2(Q, X_s)$. In a feasible solution of the centralized model with carbon credit sharing, the system trades $X_s(Q)$ units of carbon credits. For any $(Q, X_s(Q))$ pair, it turns out that

$$SC_1\left(Q, X_s(Q)\right) = \frac{\left(K_b + K_v + p_c^b(f_b + f_v)\right) D}{Q} + \frac{\left(h_b + \frac{h_v D}{Q} + p_c^b\left(g_b + \frac{g_v D}{P}\right)\right) Q}{2} + \left(c + p_v + p_c^b(e_b + e_v)\right) D - p_c^b(C_b + C_v). \quad (3.27)$$

The above expression is strictly convex in Q with a unique minimizer at

$$Q_{c1}^* = \sqrt{\frac{2\left(K_b + K_v + p_c^b(f_b + f_v)\right) D}{h_b + \frac{h_v D}{P} + p_c^b\left(g_b + \frac{g_v D}{P}\right)}}. \quad (3.28)$$

A similar expression can be derived for $SC_2\left(Q, X_s(Q)\right)$ and is given by

$$SC_2\left(Q, X_s(Q)\right) = \frac{\left(K_b + K_v + p_c^s(f_b + f_v)\right) D}{Q} + \frac{\left(h_b + \frac{h_v D}{Q} + p_c^s\left(g_b + \frac{g_v D}{P}\right)\right) Q}{2} + \left(c + p_v + p_c^s(e_b + e_v)\right) D - p_c^s(C_b + C_v). \quad (3.29)$$

$SC_2\left(Q, X_s(Q)\right)$ is also a strictly convex function with a unique minimizer at

$$Q_{c2}^* = \sqrt{\frac{2\left(K_b + K_v + p_c^s(f_b + f_v)\right) D}{h_b + \frac{h_v D}{P} + p_c^s\left(g_b + \frac{g_v D}{P}\right)}}. \quad (3.30)$$

Lemma 9 *If* $[C_b + C_v - (e_b + e_v)D] \leq \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$, *then the buyer-vendor system does not sell carbon credits at any order quantity, that is* $X_s(Q) \leq 0$ *for all* Q, *and* $Q_s^* = Q_{c1}^*$.

Proof: From Expression (3.12), the amount of traded carbon credits by the system after carbon credit sharing amounts to $X_s(Q) = C_b + C_v - \frac{(f_b+f_v)D}{Q} - \frac{(g_b+\frac{g_vD}{P})Q}{2} - (e_b+e_v)D$ for any order quantity Q. Note that $\hat{Q}_c$ minimizes $\frac{(f_b+f_v)D}{Q} + \frac{(g_b+\frac{g_vD}{P})Q}{2}$ with a minimum emissions amount of $\sqrt{2(f_b+f_v)(g_b+\frac{g_vD}{P})D}$, which leads to

$$\frac{(f_b+f_v)D}{Q} + \frac{(g_b+\frac{g_vD}{P})Q}{2} \geq \sqrt{2(f_b+f_v)\left(g_b+\frac{g_vD}{P}\right)D}$$

for all $Q \geq 0$. This implies

$$X_s(Q) \leq C_b + C_v - (e_b+e_v)D - \sqrt{2\left(g_b+\frac{g_vD}{P}\right)(f_b+f_v)D}.$$

Since $[C_b + C_v - (e_b+e_v)D] \leq \sqrt{2\left(g_b+\frac{g_vD}{P}\right)(f_b+f_v)D}$, the above expression implies $X_b(Q) \leq 0$ for all $Q \geq 0$. That is, the buyer-vendor system does not sell carbon credits at any order quantity. In this case, Expression (3.9) implies that the inventory replenishment problem of the system after carbon credit sharing reduces to minimizing $SC_1(Q, X_s(Q))$ over $Q \geq 0$. As given by Expression (3.28), Q^*_{c1} is the optimal solution of this problem. ■

Similar to the reasoning in the decentralized model, Lemma 9 and its proof imply that, if the annual cap of the system is smaller than even the minimum annual emission possible by ordering decisions, then the buyer-vendor system has to purchase carbon credits independent of the order quantity. As discussed in Section 3.1, when $X_s(Q) = 0$, the system neither sells nor buys carbon credits. If $[C_b + C_v - (e_b+e_v)D]^2 \geq 2\left(g_b+\frac{g_vD}{P}\right)(f_b+f_v)D$, there are two order quantities, namely Q_3 and Q_4, that satisfy $X_s(Q) = 0$. These quantities are given by the following two expressions:

$$Q_3 = \frac{C_b + C_v - (e_b+e_v)D - \sqrt{[C_b + C_v - (e_b+e_v)D]^2 - 2(g_b+\frac{g_vD}{P})(f_b+f_v)D}}{g_b+\frac{g_vD}{P}} \tag{3.31}$$

and

$$Q_4 = \frac{C_b + C_v - (e_b + e_v)D + \sqrt{[C_b + C_v - (e_b + e_v)D]^2 - 2(g_b + \frac{g_v D}{P})(f_b + f_v)D}}{g_b + \frac{g_v D}{P}}. \tag{3.32}$$

If $[C_b + C_v - (e_b + e_v)D]^2 \geq 2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D$, Q_4 is the larger root, that is, $Q_4 > Q_3$.

Lemma 10 *The system sells carbon credits (i.e., $X_s(Q) > 0$) only when $[C_b + C_v - (e_b + e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$ and $Q_3 < Q < Q_4$.*

Proof: Using Lemma 9, we know that if $[C_b + C_v - (e_b + e_v)D] \leq \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$, the system does not sell carbon credits. Hence, system can sell carbon credits only when $[C_b + C_v - (e_b + e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$. In addition, under this condition, $X_s(Q) > 0$ should be satisfied. $X_s(Q) = C_b + C_v - \frac{(f_b+f_v)D}{Q} - \frac{\left(g_b+\frac{g_v D}{P}\right)Q}{2} - (e_b + e_v)D > 0$ holds for all Q such that $Q_3 < Q < Q_4$. Notice that, as $[C_b + C_v - (e_b + e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$, both Q_3 and Q_4 are defined and $Q_3 < Q_4$. ■

Lemma 10 implies that there are two more cases that the system does not sell carbon credits; namely if $[C_b + C_v - (e_b + e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$ and $Q \leq Q_3$, and if $[C_b + C_v - (e_b + e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$ and $Q \geq Q_4$, in addition to the case of $[C_b + C_v - (e_b + e_v)D] \leq \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$ suggested by Lemma 9.

Lemma 11 *Depending on how $(f_b+f_v)(h_b+\frac{h_v D}{P})$ compares to $(K_b+K_v)(g_b+\frac{g_v D}{P})$, the following ordinal relations exist between Q^*_{c1} and Q^*_{c2}.*

- *If $(f_b + f_v)(h_b + \frac{h_v D}{P}) > (K_b + K_v)(g_b + \frac{g_v D}{P})$, then $Q^*_{c1} > Q^*_{c2}$.*
- *If $(f_b + f_v)(h_b + \frac{h_v D}{P}) = (K_b + K_v)(g_b + \frac{g_v D}{P})$, then $Q^*_{c1} = Q^*_{c2}$.*
- *If $(f_b + f_v)(h_b + \frac{h_v D}{P}) < (K_b + K_v)(g_b + \frac{g_v D}{P})$, then $Q^*_{c1} < Q^*_{c2}$.*

Proof: We will prove the first part of the lemma. The proofs of the remaining two parts are similar.

Since $p_c^b \geq p_c^s$ and $P > 0$, multiplying both sides of the inequality $(f_b+f_v)(h_b+\frac{h_vD}{P}) > (K_b+K_v)(g_b+\frac{g_vD}{P})$ with $P(p_c^b - p_c^s)$, we obtain $(p_c^b - p_c^s)(f_b+f_v)(h_bP+h_vD) > (p_c^b-p_c^s)(K_b+K_v)(g_bP+g_vD)$. Adding $(K_b+K_v)(h_bP+h_vD)+p_c^bp_c^s(g_bP+g_vD)(f_b+f_v)$ to both sides of this inequality and after some rearrangement of terms, we have

$$\begin{aligned}&[K_b+K_v+p_c^b(f_b+f_v)][h_bP+h_vD+p_c^s(g_bP+g_vD)] > \\ &[K_b+K_v+p_c^s(f_b+f_v)][h_bP+h_vD+p_c^b(g_bP+g_vD)].\end{aligned}$$

The above expression can be rewritten as

$$\frac{K_b+K_v+p_c^b(f_b+f_v)}{h_bP+h_vD+p_c^b(g_bP+g_vD)} > \frac{K_b+K_v+p_c^s(f_b+f_v)}{h_bP+h_vD+p_c^s(g_bP+g_vD)},$$

which further implies

$$\sqrt{\frac{2[K_b+K_v+p_c^b(f_b+f_v)]D}{h_b+\frac{h_vD}{P}+p_c^b\left(g_b+\frac{g_vD}{P}\right)}} > \sqrt{\frac{2[K_b+K_v+p_c^s(f_b+f_v)]D}{h_b+\frac{h_vD}{P}+p_c^b\left(g_b+\frac{g_vD}{P}\right)}}.$$

Notice that the left hand side of the above inequality is Q_{c1}^* and the right hand side is Q_{c2}^*, and therefore, $Q_{c1}^* > Q_{c2}^*$. ■

In Lemma 12, further properties of the system's problem in case of $([C_b + C_v - (e_b+e_v)D] > \sqrt{2\left(g_b+\frac{g_vD}{P}\right)(f_b+f_v)D}$ are presented.

Lemma 12 *When $([C_b + C_v - (e_b+e_v)D] > \sqrt{2\left(g_b+\frac{g_vD}{P}\right)(f_b+f_v)D}$, the following cases cannot be observed.*

- $Q_3 < Q_4 \leq Q_{c2}^* \leq Q_{c1}^*$
- $Q_{c1}^* \leq Q_{c2}^* \leq Q_3 < Q_4$.

Proof: The proof follows similar steps to the proof of Lemma 4 and is omitted. ■

When $[C_b + C_v - (e_b + e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$, the first and second parts of Lemma 12 imply that the cases of $Q_3 < Q_4 \leq Q^*_{c2} = Q^*_{c1}$ and $Q^*_{c1} = Q^*_{c2} \leq Q_3 < Q_4$ cannot take place, respectively. Combining this result with Lemma 11 leads to the following implication: If $[C_b + C_v - (e_b + e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$ and $(f_b + f_v)(h_b + \frac{h_v D}{P}) = (K_b + K_v)(g_b + \frac{g_v D}{P})$, the only possible ordering of Q_3, Q_4, Q^*_{c1} and Q^*_{c2} is $Q_3 < Q^*_{c1} = Q^*_{c2} < Q_4$. This is because having $[C_b + C_v - (e_b + e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$ implies $Q_4 > Q_3$, and it follows due to Lemma 11 that as $(f_b + f_v)(h_b + \frac{h_v D}{P}) = (K_b + K_v)(g_b + \frac{g_v D}{P})$ we have $Q^*_{c1} = Q^*_{c2}$. Under these conditions, excluding the cases covered in Lemma 12 from further consideration, the only possible ordering that remains is $Q_3 < Q^*_{c1} = Q^*_{c2} < Q_4$.

Lemma 13 *If* $[C_b + C_v - (e_b + e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$ *and* $(f_b + f_v)(h_b + \frac{h_v D}{P}) = (K_b + K_v)(g_b + \frac{g_v D}{P})$*, then* $Q^*_s = Q^*_{c1} = Q^*_{c2}$.

Proof: Under the conditions of the lemma, the only possible ordering of Q_3, Q_4, Q^*_{c1} and Q^*_{c2} is $Q_3 < Q^*_{c1} = Q^*_{c2} < Q_4$. In order to prove the lemma, we will consider three regions of Q separately; $Q \leq Q_3$, $Q_3 < Q < Q_4$, and $Q \geq Q_4$. Expression (3.9) and Lemma 10 together imply that if $[C_b + C_v - (e_b + e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$, for order quantities Q such that $Q_3 < Q < Q_4$, we have $SC\left(Q, X_b(Q)\right) = SC_2\left(Q, X_b(Q)\right)$; for order quantities Q such that $Q \leq Q_3$, we have $SC\left(Q, X_b(Q)\right) = SC_1\left(Q, X_b(Q)\right)$; for order quantities Q such that $Q \geq Q_4$, we have $SC\left(Q, X_b(Q)\right) = sC_1\left(Q, X_b(Q)\right)$. Since $SC(Q, X_s(Q))$ has the similar structural properties as $BC(Q, X_b(Q))$, the proof follows along the same lines of Lemma 5's proof and is omitted. ■

Lemma 9 and Lemma 13 constitute parts of our solution algorithm for the centralized problem with carbon credit sharing. Lemma 9 suggests the solution in case of $[C_b+C_v-(e_b+e_v)D] \leq \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$, and Lemma 13 provides the solution in case of $[C_b + C_v - (e_b + e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$ and $(f_b + f_v)(h_b + \frac{h_v D}{P}) = (K_b + K_v)(g_b + \frac{g_v D}{P})$. Additionally, we need to consider the case of $[C_b+C_v-(e_b+e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$ and $(f_b+f_v)(h_b+\frac{h_v D}{P}) \neq (K_b + K_v)(g_b + \frac{g_v D}{P})$. Before proceeding with a detailed analysis of this case, let us present a result which applies to the case of $[C_b + C_v - (e_b + e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$ in general.

Lemma 14 *When* $[C_b + C_v - (e_b + e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$*, we have* $SC_1(Q, X_s(Q)) \leq SC_2(Q, X_s(Q))$ *for all* Q *such that* $Q_3 \leq Q \leq Q_4$*, and* $SC_1(Q, X_s(Q)) > SC_2(Q, X_s(Q))$ *for all* Q *such that* $Q < Q_3$ *or* $Q > Q_4$*.*

Proof: Recall that $X_s(Q) = C_b + C_v - \frac{(f_b+f_v)D}{Q} - \frac{\left(g_b+\frac{g_v D}{P}\right)Q}{2} - (e_b + e_v)D$, and $X_s(Q) = 0$ when $Q = Q_3$ and $Q = Q_4$. Moreover, we have $X_s(Q) > 0$ for all Q s.t. $Q_3 < Q < Q_4$, and we have $X_s(Q) < 0$ for all Q s.t. $Q < Q_3$ and for all Q s.t. $Q > Q_4$. We will show that $SC_1(Q, X_s(Q)) \leq SC_2(Q, X_s(Q))$ if $Q \in [Q_3, Q_4]$. The proofs of the other parts of the lemma, which will be omitted, follow in a similar fashion.

Since $p_c^b \geq p_c^s$, it follows that

$$(p_c^b - p_c^s)\left[C_b + C_v - \frac{(f_b + f_v)D}{Q} - \frac{\left(g_b + \frac{g_v D}{P}\right)Q}{2} - (e_b + e_v)D\right] \geq 0.$$

After adding $\frac{(K_b+K_v)D}{Q} + \frac{\left(h_b+\frac{h_v D}{P}\right)Q}{2} + (c+p_v)D$ to both sides of the above inequality and rearranging the terms, we get

$$\frac{(K_b+K_v)D}{Q} + \frac{\left(h_b+\frac{h_v D}{P}\right)Q}{2} + (c + p_v)D - p_c^b[C_b + C_v - \frac{(f_b+f_v)D}{Q} - \frac{\left(g_b+\frac{g_v D}{P}\right)Q}{2}$$
$$-(e_b + e_v)D] \leq \frac{(K_b+K_v)D}{Q} + \frac{\left(h_b+\frac{h_v D}{P}\right)Q}{2} + (c + p_v)D - p_c^s[C_b + C_v$$
$$-\frac{(f_b+f_v)D}{Q} - \frac{\left(g_b+\frac{g_v D}{P}\right)Q}{2} - (e_b + e_v)D].$$

This implies $SC_1(Q, X_s(Q)) \leq SC_2(Q, X_s(Q))$. ■

Lemma 15 *When* $[C_b + C_v - (e_b + e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$ *and* $(f_b + f_v)(h_b + \frac{h_v D}{P}) < (K_b + K_v)(g_b + \frac{g_v D}{P})$*, the following orderings among* Q_3*,* Q_4*,* Q_{c1}^**, and* Q_{c2}^* *cannot take place:*

- $Q_{c1}^* \leq Q_3 < Q_4 \leq Q_{c2}^*$,

- $Q^*_{c1} \leq Q_3 < Q^*_{c2} < Q_4$,
- $Q^*_{c1} < Q^*_{c2} \leq Q_3 < Q_4$.

Proof: Since $SC(Q, X_s(Q))$ has the similar structural properties as $BC(Q, X_b(Q))$, the proof follows along the same lines of Lemma 7's proof and is omitted. ■

Note that, two common properties of the cases considered are $Q_3 < Q_4$ and $Q^*_{c1} < Q^*_{c2}$ under the two conditions of Lemma 15. Lemma 15 further leads to the result in Corollary 3.

Corollary 3 *When* $[C_b + C_v - (e_b + e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$ *and* $(f_b + f_v)(h_b + \frac{h_v D}{P}) < (K_b + K_v)(g_b + \frac{g_v D}{P})$*, the following orderings are possible:*

- $Q_3 < Q_4 \leq Q^*_{c1} < Q^*_{c2}$,
- $Q_3 < Q^*_{c1} < Q^*_{c2} \leq Q_4$,
- $Q_3 < Q^*_{c1} < Q_4 < Q^*_{c2}$.

Numerical instances to illustrate the cases in Corollary 3 are presented in Tables 3.4 and 3.5. The first three examples of Tables 3.4 and 3.5 correspond to the different cases of the corollary in the order they are presented.In the next lemma, we provide a similar result to Lemma 15, now for the case of $[C_b+C_v-(e_b+e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$ and $(f_b+f_v)(h_b+\frac{h_v D}{P}) > (K_b+K_v)(g_b+\frac{g_v D}{P})$.

Lemma 16 *When* $[C_b + C_v - (e_b + e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$ *and* $(f_b + f_v)(h_b+\frac{h_v D}{P}) > (K_b+K_v)(g_b+\frac{g_v D}{P})$*, the following orderings among* Q_3*,* Q_4*,* Q^*_{c1}*, and* Q^*_{c2} *cannot take place:*

- $Q^*_{c2} < Q_3 < Q_4 \leq Q^*_{c1}$,
- $Q_3 \leq Q^*_{c2} < Q_4 \leq Q^*_{c1}$,

Table 3.4: Numerical Illustrations of Corollary 3 and Corollary 4 Given $D = 50$, $c = 12$, $p_v = 8$, $g_b = 0.5$ and $g_v = 0.25$

Example Index	P	p_c^b	p_c^s	K_b	K_v	h_b	h_v	f_b	f_v	e_b	e_v	C_b	C_v
7	60	7.5	6.5	2000	2500	1	0.8	150	100	4.5	8	320	440
8	150	7.5	6	900	1000	1	0.5	40	135	5	7	300	450
9	150	7.5	6	4000	6000	1	0.5	40	135	5	7	303	448.2
10	150	7.5	6	20	30	1	0.5	40	135	5	7	303	400
11	55	2.5	2	40	100	3.2	3	90	95	4.5	6	304	360
12	55	2.5	2	40	100	3.2	3	90	95	4.5	6	300	350

Table 3.5: Numerical Illustrations of Corollary 3 and Corollary 4 Given $D = 50$, $c = 12$, $p_v = 8$, $g_b = 0.5$ and $g_v = 0.25$ (Continued)

Example Index	Q^*_{c1}	Q^*_{c2}	Q_3	Q_4
7	302.231	312.529	158.498	222.678
8	240.769	251.425	67.084	447.202
9	451.813	486.606	66.367	452.033
10	156.801	153.53	142.257	210.886
11	88.197	83.12	85.81	296.44
12	88.197	83.12	107.816	235.934

- $Q_3 < Q_4 \leq Q^*_{c2} < Q^*_{c1}$.

Proof: Since $SC(Q, X_s(Q))$ has the similar structural properties as $BC(Q, X_b(Q))$, the proof follows along the same lines of Lemma 8's proof and is omitted. ■

Notice that under the two conditions of Lemma 16, two common properties of the cases considered are $Q_3 < Q_4$ and $Q^*_{c1} > Q^*_{c2}$. Lemma 16 further leads to the result in Corollary 4.

Corollary 4 *When* $[C_b + C_v - (e_b + e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$ *and* $(f_b + f_v)(h_b + \frac{h_v D}{P}) > (K_b + K_v)(g_b + \frac{g_v D}{P})$*, the following orderings are possible:*

- $Q_3 \leq Q^*_{c2} < Q^*_{c1} < Q_4$,
- $Q^*_{c2} < Q_3 < Q^*_{c1} < Q_4$,
- $Q^*_{c2} < Q^*_{c1} \leq Q_3 < Q_4$.

Numerical instances to illustrate the cases in Corollary 4 are also presented in Tables 3.4 and 3.5. The last three examples of Tables 3.4 and 3.5 correspond to the different cases of the corollary in the order they are presented.

Similar to Algorithm 1, we propose the following algorithm to find the optimal solution to the centralized model with carbon credit sharing (i.e., Q_s^*), using our results in Lemma 9, Lemma 13, Corollary 3 and Corollary 4.

Algorithm 2: Solution of the Centralized Model with Carbon Credit Sharing

1. If $[C_b + C_v - (e_b + e_v)D] \leq \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$, then set $Q_s^* = Q_{c1}^*$.
2. If $[C_b + C_v - (e_b + e_v)D] > \sqrt{2\left(g_b + \frac{g_v D}{P}\right)(f_b + f_v)D}$, then do the following:
    (a) If $(f_b + f_v)(h_b + \frac{h_v D}{P}) = (K_b + K_v)(g_b + \frac{g_v D}{P})$, set $Q_s^* = Q_{c2}^*$.
    (b) If $(f_b + f_v)(h_b + \frac{h_v D}{P}) < (K_b + K_v)(g_b + \frac{g_v D}{P})$, and
        i. if $Q_4 \leq Q_{c1}^*$, set $Q_s^* = Q_{c1}^*$,
        ii. else,
            A. if $Q_4 \geq Q_{c2}^*$, set $Q_s^* = Q_{c2}^*$,
            B. if $Q_4 < Q_{c2}^*$, set $Q_s^* = Q_4$.
    (c) If $(f_b + f_v)(h_b + \frac{h_v D}{P}) > (K_b + K_v)(g_b + \frac{g_v D}{P})$, and
        i. if $Q_{c1}^* \leq Q_3$, set $Q_s^* = Q_{c1}^*$,
        ii. else,
            A. if $Q_{c2}^* \geq Q_3$, set $Q_s^* = Q_{c2}^*$,
            B. if $Q_{c2}^* < Q_3$, set $Q_s^* = Q_3$.

Theorem 2 *Algorithm 2 gives the optimal solution to the buyer-vendor system's replenishment problem under carbon credit sharing formulated in the Centralized Model with Carbon Credit Sharing.*

Proof: Since $SC(Q, X_s(Q))$ has the similar structural properties as $BC(Q, X_b(Q))$, the proof follows along the same lines of Theorem 1's proof, and therefore, it is omitted. ■

3.3 Coordination Mechanisms for the Two-Echelon System under Cap-and-Trade Mechanism

In this section, we present coordination mechanisms that help the buyer-vendor system to arrive at the system optimal solution by making the most efficient use of carbon credits. These coordination mechanisms assume that vendor has full information about the ordering behavior of the buyer, and the buyer orders from the current vendor as long as his/her costs as a result of the coordinated solution are not worse than those under the decentralized solution. The novelty of the proposed coordination mechanisms is that they make use of carbon credit sharing. Recall that in this setting, the purchasing price of one unit carbon credit is greater than or equal to its selling price (i.e., $p_c^b \geq p_c^s$). In settings where $p_c^b > p_c^s$, and one party is selling carbon credits while the other party is purchasing them, the system is actually loosing some opportunity due to the monetary value that the purchasing party pays to intermediary agencies (i.e., $p_c^b - p_c^s$ per unit carbon credit purchased). Therefore, the proposed coordination mechanisms, as part of sharing the extra benefits of the centralized solutions, entail the party who has extra carbon credits to pass them to the other one who would otherwise purchase at a larger price in the market. This way, we minimize the system's need to purchase carbon credits, and hence, to pay to the intermediary agencies.

Let us define the following additional piece of notation:

Table 3.6: Additional Notation Used in Coordination Mechanisms for the Two-Echelon System under Cap-and-Trade Mechanism

$\overline{BC}(Q)$:	Cost of the buyer after coordination if order size is Q units
$\overline{VC}(Q)$:	Cost of the vendor after coordination if order size is Q units

Theorem 3 *Suppose one of the following conditions holds.*

- $X_b(Q_s^*) \leqslant 0$ *and* $X_v(Q_s^*) \leqslant 0$

- $X_b(Q_s^*) \geqslant 0$ *and* $X_v(Q_s^*) \geqslant 0$

Hence, if $Q_d^* < Q_s^*$*, then the following coordination mechanism minimizes the buyer's cost with a minimum function value of* $BC(Q_d^*)$ *realized at* Q_s^**.*

$$\overline{BC}(Q) = \begin{cases} BC(Q, X_b(Q)) & \text{if} \quad Q < Q_s^* \\ \\ BC(Q, X_b(Q)) - d \times D & \text{if} \quad Q \geqslant Q_s^* \end{cases}$$

where $d = \frac{BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))}{D}$ *is the unit discount given by the vendor to the buyer.*

That is, when $Q_d^* < Q_s^*$*, if the vendor gives a unit discount* d *for order sizes greater than or equal to* Q_s^* *to the buyer,* Q_s^* *coordinates the channel.*

Proof: See Appendix A.1.1 for the proof. ■

Theorem 4 *Suppose one of the following conditions holds.*

- $X_b(Q_s^*) \leqslant 0$ *and* $X_v(Q_s^*) \leqslant 0$
- $X_b(Q_s^*) \geqslant 0$ *and* $X_v(Q_s^*) \geqslant 0$

If $Q_d^* > Q_s^*$*, then the following coordination mechanism minimizes the buyer's cost with a minimum function value of* $BC(Q_d^*)$ *realized at* Q_s^**.*

$$\overline{BC}(Q) = \begin{cases} BC(Q, X_b(Q)) - d \times D & \text{if} \quad Q \leqslant Q_s^* \\ \\ BC(Q, X_b(Q)) & \text{if} \quad Q > Q_s^* \end{cases}$$

where $d = \frac{BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))}{D}$ *is the unit discount given by the vendor to the buyer.*

That is, when $Q_d^* > Q_s^*$*, if the vendor gives a unit discount* d *for order sizes less than or equal to* Q_s^* *to the buyer,* Q_s^* *coordinates the channel.*

Proof: Proof is similar to the proof of Theorem 3. $Q_d^* < Q_s^*$ is replaced with $Q_d^* > Q_s^*$, $Q < Q_s^*$ is replaced with $Q > Q_s^*$ and $Q \geqslant Q_s^*$ is replaced with $Q \leqslant Q_s^*$. ■

Theorem 3 and Theorem 4 have the following implication. If both the buyer and the vendor buy/sell carbon credits, there cannot be carbon trade between the two parties. Hence, the coordination is achieved by giving the buyer a quantity discount.

Theorem 5 *Suppose the following conditions hold.*

- $X_b(Q_s^*) \leqslant 0$ *and* $X_v(Q_s^*) \geqslant 0$
- $p_c^b \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\} \geqslant BC(Q_s^*) - BC(Q_d^*)$

If $Q_d^* < Q_s^*$*, then the following coordination mechanism minimizes the buyer's cost with a minimum function value of* $BC(Q_d^*)$ *realized at* Q_s^*.

$$\overline{BC}(Q) = \begin{cases} BC(Q, X_b(Q)) \qquad \text{if} \quad Q < Q_s^* \\ \\ BC(Q, X_b(Q)) - p_c^b \times Y \\ +[BC(Q_d^*, X_b(Q_d^*)) - BC(Q_s^*, X_b(Q_s^*)) + p_c^b \times Y] \qquad \text{if} \quad Q \geqslant Q_s^* \end{cases}$$

where $Y = \min\{-X_b(Q_s^*), X_v(Q_s^*)\}$ *is the amount of carbon credits given for free by the vendor to the buyer and* $BC(Q_d^*, X_b(Q_d^*)) + p_c^b \times Y - BC(Q_s^*, X_b(Q_s^*))$ *is the amount of the fixed payment made by the buyer to the vendor.*

The sum of the costs of the buyer and the vendor after coordination is equal to the total cost of the system resulting from the centralized model with carbon credit sharing (i.e., $\overline{BC}(Q_s^*) + \overline{VC}(Q_s^*) = SC(Q_s^*, X_s(Q_s^*))$*).*

That is, when $Q_d^* < Q_s^*$*, if the vendor gives* $\min\{-X_b(Q_s^*), X_v(Q_s^*)\}$ *carbon credits for free to the buyer and the buyer makes a fixed payment of* $BC(Q_d^*, X_b(Q_d^*)) - BC(Q_s^*, X_b(Q_s^*)) + p_c^b \times Y$ *to the vendor for order sizes greater than or equal to* Q_s^**,* Q_s^* *coordinates the channel.*

Proof: See Appendix A.1.2 for the proof. ■

Theorem 6 *Suppose the following conditions hold.*

- $X_b(Q_s^*) \leqslant 0$ *and* $X_v(Q_s^*) \geqslant 0$
- $p_c^b \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\} \geqslant BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))$

If $Q_d^* > Q_s^*$*, then the following coordination mechanism minimizes the buyer's cost with a minimum function value of* $BC(Q_d^*)$ *realized at* Q_s^**.*

$$\overline{BC}(Q) = \begin{cases} BC(Q, X_b(Q)) - p_c^b \times Y \\ +[BC(Q_d^*, X_b(Q_d^*)) - BC(Q_s^*, X_b(Q_s^*)) + p_c^b \times Y] & \text{if} \quad Q \leqslant Q_s^* \\ \\ BC(Q, X_b(Q)) \qquad \text{if} \quad Q > Q_s^* \end{cases}$$

where $Y = \min\{-X_b(Q_s^*), X_v(Q_s^*)\}$ *is the amount of carbon credits given for free by the vendor to the buyer and* $BC(Q_d^*, X_b(Q_d^*)) + p_c^b \times Y - BC(Q_s^*, X_b(Q_s^*))$ *is the amount of the fixed payment made by the buyer to the vendor.*

The sum of the costs of the buyer and the vendor after coordination is equal to the total cost of the system resulting from the centralized model with carbon credit sharing (i.e., $\overline{BC}(Q_s^*) + \overline{VC}(Q_s^*) = SC(Q_s^*, X_s(Q_s^*))$*).*

That is, when $Q_d^* < Q_s^*$*, if the vendor gives* $\min\{-X_b(Q_s^*), X_v(Q_s^*)\}$ *carbon credits for free to the buyer and the buyer makes a fixed payment of* $BC(Q_d^*) - BC(Q_s^*) + p_c^b \times Y$ *to the vendor for order sizes less than or equal to* Q_s^**,* Q_s^* *coordinates the channel.*

Proof: Proof is similar to the proof of Theorem 5. $Q_d^* < Q_s^*$ is replaced with $Q_d^* > Q_s^*$, $Q < Q_s^*$ is replaced with $Q > Q_c^*$ and $Q \geqslant Q_s^*$ is replaced with $Q \leqslant Q_s^*$. ■

Theorem 5 and Theorem 6 have the following implication. If the buyer buys and the vendor sells carbon credits, vendor gives carbon credits for free to the buyer. The amount of shared credits is equal to the minimum of vendor sells and buyer buys. If monetary amount of given credits in terms of buying price is greater than the buyer's loss from the decentralized solution, buyer pays the difference to the vendor. The same amount of credit is not bought by the buyer and not sold by the vendor. Since $p_c^b > p_c^s$, the total cost of the system decreases.

Theorem 7 *Suppose the following conditions hold.*

- $X_b(Q_s^*) \leqslant 0$ *and* $X_v(Q_s^*) \geqslant 0$
- $p_c^b \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\} < BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))$

If $Q_d^* < Q_s^*$, *then the following coordination mechanism minimizes the buyer's cost with a minimum function value of* $BC(Q_d^*)$ *realized at* Q_s^*.

$$\overline{BC}(Q) = \begin{cases} BC(Q, X_b(Q)) & \text{if} \quad Q < Q_s^* \\ \\ BC(Q, X_b(Q)) - d \times D - p_c^b \times Y & \text{if} \quad Q \geqslant Q_s^* \end{cases}$$

where $d = [BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*)) - p_c^b \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\}]/D$ *is the unit discount given and* $Y = \min\{-X_b(Q_s^*), X_v(Q_s^*)\}$ *is the amount of carbon credits given for free by the vendor to the buyer.*

The sum of the costs of the buyer and the vendor after coordination is equal to the total cost of the system resulting from the centralized model with carbon credit sharing (i.e., $\overline{BC}(Q_s^*) + \overline{VC}(Q_s^*) = SC(Q_s^*, X_s(Q_s^*))$*).*

That is, when $Q_d^* < Q_s^*$, *if the vendor gives* $\min\{-X_b(Q_s^*), X_v(Q_s^*)\}$ *carbon credits for free and a unit discount* d *for order sizes greater than or equal to* Q_s^* *to the buyer,* Q_s^* *coordinates the channel.*

Proof: See Appendix A.1.3 for the proof. ■

Theorem 8 *Suppose the following conditions hold.*

- $X_b(Q_s^*) \leqslant 0$ *and* $X_v(Q_s^*) \geqslant 0$
- $p_c^b \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\} < BC(Q_s^*) - BC(Q_d^*)$

If $Q_d^* > Q_s^*$*, then the following coordination mechanism minimizes the buyer's cost with a minimum function value of* $BC(Q_d^*)$ *realized at* Q_s^**.*

$$\overline{BC}(Q) = \begin{cases} BC(Q, X_b(Q)) - d \times D - p_c^b \times Y & \text{if} \quad Q \leqslant Q_s^* \\ \\ BC(Q, X_b(Q)) & \text{if} \quad Q > Q_s^* \end{cases}$$

where $d = [BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*)) - p_c^b \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\}]/D$ *is the unit discount given and* $Y = \min\{-X_b(Q_s^*), X_v(Q_s^*)\}$ *is the amount of carbon credits given for free by the vendor to the buyer.*

The sum of the costs of the buyer and the vendor after coordination is equal to the total cost of the system resulting from the centralized model with carbon credit sharing (i.e., $\overline{BC}(Q_s^*) + \overline{VC}(Q_s^*) = SC(Q_s^*, X_s(Q_s^*))$*).*

That is, when $Q_d^* > Q_s^*$*, if the vendor gives* $\min\{-X_b(Q_s^*), X_v(Q_s^*)\}$ *carbon credits for free and a unit discount d for order sizes less than or equal to* Q_s^* *to the buyer,* Q_s^* *coordinates the channel.*

Proof: Proof is similar to the proof of Theorem 7. $Q_d^* < Q_s^*$ is replaced with $Q_d^* > Q_s^*$, $Q < Q_s^*$ is replaced with $Q > Q_s^*$ and $Q \geqslant Q_s^*$ is replaced with $Q \leqslant Q_s^*$. ■

Theorem 7 and Theorem 8 have the following implication. If the buyer buys and the vendor sells carbon credits, vendor gives carbon credits for free to the buyer. The amount of shared credits is equal to the minimum of vendor sells and buyer buys. If monetary amount of given credits in terms of buying price is less than the buyer's loss from the decentralized solution, remaining loss of the buyer is compensated by giving him/her a quantity discount. The same amount of credit is not bought by the buyer and not sold by the vendor. Since $p_c^b > p_c^s$, the total cost of the system decreases.

Theorem 9 *Suppose $X_b(Q_s^*) \geqslant 0$ and $X_v(Q_s^*) \leqslant 0$ holds.*

Hence, if $Q_d^* < Q_s^*$, then the following coordination mechanism minimizes the buyer's cost with a minimum function value of $BC(Q_d^*)$ realized at Q_s^*.

$$\overline{BC}(Q) = \begin{cases} BC(Q, X_b(Q)) & \text{if} \quad Q < Q_s^* \\ \\ BC(Q, X_b(Q)) - \bar{d} \times D + p_c^s \times Y & \text{if} \quad Q \geqslant Q_s^* \end{cases}$$

where $\bar{d} = [BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*)) + p_c^s \times \min\{X_b(Q_s^*), -X_v(Q_s^*)\}]/D$ is the unit discount given by the vendor to the buyer and $Y = \min\{X_b(Q_s^*), -X_v(Q_s^*)\}$ is the amount of carbon credits given for free by the buyer to the vendor.

The sum of the costs of the buyer and the vendor after coordination is equal to the total cost of the system resulting from the centralized model with carbon credit sharing (i.e., $\overline{BC}(Q_s^*) + \overline{VC}(Q_s^*) = SC(Q_s^*, X_s(Q_s^*))$).

That is, when $Q_d^* < Q_s^*$, if the buyer gives $Y = \min\{X_b(Q_s^*), -X_v(Q_s^*)\}$ carbon credits for free to the vendor and the vendor gives a unit discount $\bar{d}$ to the buyer for order sizes greater than or equal to Q_s^*, Q_s^* coordinates the channel.

Proof: See Appendix A.1.4 for the proof. ■

Theorem 10 *Suppose $X_b(Q_s^*) \geqslant 0$ and $X_v(Q_s^*) \leqslant 0$ holds.*

Hence, if $Q_d^* > Q_s^*$, then the following coordination mechanism minimizes the buyer's cost with a minimum function value of $BC(Q_d^*)$ realized at Q_s^*.

$$\overline{BC}(Q) = \begin{cases} BC(Q, X_b(Q)) - \bar{d} \times D + p_c^s \times Y & \text{if} \quad Q \leqslant Q_s^* \\ \\ BC(Q, X_b(Q)) & \text{if} \quad Q > Q_s^* \end{cases}$$

where $\bar{d} = [BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*)) + p_c^s \times \min\{X_b(Q_s^*), -X_v(Q_s^*)\}]/D$ *is the unit discount given by the vendor to the buyer and* $Y = \min\{X_b(Q_s^*), -X_v(Q_s^*)\}$ *is the amount of carbon credits given for free by the buyer to the vendor.*

The sum of the costs of the buyer and the vendor after coordination is equal to the total cost of the system resulting from the centralized model with carbon credit sharing (i.e., $\overline{BC}(Q_s^*) + \overline{VC}(Q_s^*) = SC(Q_s^*, X_s(Q_s^*))$*).*

That is, when $Q_d^* > Q_s^*$*, if the buyer gives* $\min\{X_b(Q_s^*), -X_v(Q_s^*)\}$ *carbon credits for free to the vendor and the vendor gives a unit discount* $\bar{d}$ *to the buyer for order sizes less than or equal to* Q_s^**,* Q_s^* *coordinates the channel.*

Proof: Proof is similar to the proof of Theorem 9. $Q_d^* < Q_s^*$ is replaced with $Q_d^* > Q_s^*$, $Q < Q_s^*$ is replaced with $Q > Q_s^*$ and $Q \geqslant Q_s^*$ is replaced with $Q \leqslant Q_s^*$. ■

Theorem 9 and Theorem 10 have the following implication. If the buyer sells and the vendor buys carbon credits, buyer gives carbon credits for free to the vendor. The amount of shared credits is equal to the minimum of vendor buys and buyer sells. Hence, the buyer's loss from the decentralized solution increases. The vendor compensates the buyer's loss by giving him/her a quantity discount. The same amount of credit is not bought by the buyer and not sold by the vendor. Since $p_c^b > p_c^s$, the total cost of the system decreases.

3.4 Numerical Analysis under Deterministic Demand and Cap-and-Trade Mechanism

3.4.1 Numerical Analysis of Decentralized and Centralized Emissions under Deterministic Demand and Cap-and-Trade Mechanism

In this section, we analyze the impact of channel coordination on the system's carbon emissions. For this purpose, let us define the following additional notation.

Table 3.7: Additional Notation Used in Numerical Analysis of Decentralized and Centralized Emissions under Deterministic Demand and Cap-and-Trade Mechanism

$TE(Q)$:	Average annual emissions of the system if order size is Q units
R:	Ratio of average annual emissions of the system resulting from the two solutions

Using Equation (3.12), the average annual emissions of the system resulting from the optimal solutions of decentralized model and centralized model with carbon credit sharing are respectively given by

$$TE(Q_d^*) = \frac{(f_b + f_v)D}{Q_d^*} + \frac{(g_b + \frac{g_v D}{P})Q_d^*}{2} + (e_b + e_v)D \tag{3.33}$$

$$TE(Q_s^*) = \frac{(f_b + f_v)D}{Q_s^*} + \frac{(g_b + \frac{g_v D}{P})Q_s^*}{2} + (e_b + e_v)D \tag{3.34}$$

Using Equations (3.33) and (3.34), we define the mathematical expression for R as follows.

$$R = \frac{TE(Q_s^*)}{TE(Q_d^*)} = \frac{\frac{(f_b+f_v)D}{Q_s^*} + \frac{(g_b+\frac{g_v D}{P})Q_s^*}{2} + (e_b + e_v)D}{\frac{(f_b+f_v)D}{Q_d^*} + \frac{(g_b+\frac{g_v D}{P})Q_d^*}{2} + (e_b + e_v)D} \tag{3.35}$$

We use R as a performance measure on the system's environmental quality under the centralized model with carbon credit sharing compared to its environmental performance under the decentralized model so as to determine whether the emissions increase or decrease with coordination. A value of $R > 1$ would be due to $TE(Q_s^*) > TE(Q_d^*)$, implying that the coordinated solution is not good for the environment. Similarly, a value of $R < 1$ implies that coordination is better for the environment in contrast to the uncoordinated solution. In what follows, we study the effect of each parameter on R under different combinations of parameter settings. Here, since c and p_v do not affect the solutions under the optimal solutions of the decentralized model and centralized model with carbon credit sharing (i.e., Q_d^* and Q_s^*) and average annual emissions, we do not include them in our analysis.

The parameter settings are constructed considering the relationships between D vs. P, p_c^b vs. p_c^s, K_b vs. K_v, h_b vs. h_v, f_b vs. f_v, g_b vs. g_v, e_b vs. e_v, and C_b vs. C_v. Three scenarios are constructed for each pair. In the first scenario average values of both parameters are considered, in the second scenario an extremely large value for the first parameter is considered and in the third scenario an extremely large value for the second parameter is considered (See Table 3.8). Also, the setting that incorporates average values for both parameters (i.e., the base parameter setting) is the same for each pair in Table 3.8. The values of the parameters for each setting can be seen in Tables 3.9, 3.10 and 3.11. For each combination of parameter settings, one of the parameters is changed around its base value over a large enough interval to observe the behavior of R. Since $p_c^s \leq p_c^b$, $h_v < h_b$ and $D < P$, p_c^s, h_v and D cannot take values larger than p_c^b, h_b and P, respectively. In all the numerical analysis, MATLAB is used.

Our observations regarding how R changes with varying values of each parameter, are summarized below. In the figures where the pattern of R exhibit "jumps" (i.e., where the pattern does not go smoothly), Q_d^* (Q_s^*) switches from Q_{d1}^* or Q_{d2}^* (Q_{c1}^* or Q_{c2}^*) to Q_1 or Q_2 (Q_3 or Q_4) or vice versa.

Table 3.8: Construction of Parameter Settings

D vs. P	Average values for D and P	D is extremely close to P	P is extremely larger than D
p_c^b vs. p_c^s	Average values for p_c^b and p_c^s	p_c^b is extremely larger than p_c^s	p_c^s is equal to p_c^b
K_b vs. K_v	Average values for K_b and K_v	K_b is extremely larger than K_v	K_v is extremely larger than K_b
h_b vs. h_v	Average values for h_b and h_v	h_b is extremely larger than h_v	h_v is extremely close to h_b
f_b vs. f_v	Average values for f_b and f_v	f_b is extremely larger than f_v	f_v is extremely larger than f_b
g_b vs. g_v	Average values for g_b and g_v	g_b is extremely large than g_v	g_v is extremely large than g_b
e_b vs. e_v	Average values for e_b and e_v	e_b is extremely large than e_v	e_v is extremely large than e_b
C_b vs. C_v	Average values for C_b and C_v	C_b is extremely large than C_v	C_v is extremely large than C_b

The Impact of Increasing D on R

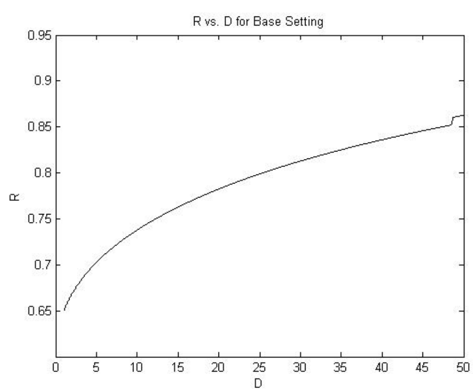

Figure 3.1: R vs. D for Base Setting

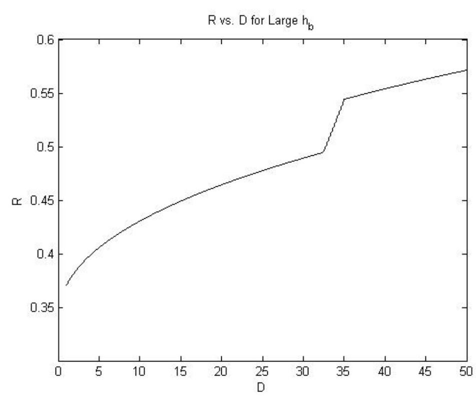

Figure 3.2: R vs. D for Large h_b

Since the optimal values of the unconstrained cost functions under both models (i.e., Q_{d1}^*, Q_{d2}^*, Q_{c1}^* and Q_{c2}^*), the order sizes at the boundary conditions (i.e., Q_1, Q_2, Q_3 and Q_4) and the order quantity that minimizes the average annual emissions of the system (i.e., the emission optimal solution, $\hat{Q}_c$) all depend on D, R does not exhibit a common behavior with increasing D. That is, the behavior of R depends on the specific parameter setting.

In the regions where $Q_d^* = Q_{d1}^*$ or $Q_d^* = Q_{d2}^*$ and $Q_s^* = Q_{c1}^*$ or $Q_s^* = Q_{c2}^*$, emissions under optimal solutions of both the decentralized model and centralized

Table 3.9: Parameter Values of the Settings in Table 3.8

Parameters	**Base Setting**	**Large D**	**Large P**	**Large p_c^b**	**Large p_c^s**
D	30	49.99	30	30	30
P	50	50	200	50	50
p_c^b	2.5	2.5	2.5	20	2.5
p_c^s	1.5	1.5	1.5	1.5	2.5
K_b	40	40	40	40	40
K_v	500	500	500	500	500
h_b	1.5	1.5	1.5	1.5	1.5
h_v	1.2	1.2	1.2	1.2	1.2
f_b	20	20	20	20	20
f_v	120	120	120	120	120
g_b	0.5	0.5	0.5	0.5	0.5
g_v	0.35	0.35	0.35	0.35	0.35
e_b	1	1	1	1	1
e_v	1.5	1.5	1.5	1.5	1.5
C_b	80	80	80	80	80
C_v	200	200	200	200	200

model with carbon credit sharing increase. This is because Q_d^*, Q_s^* and hence, the terms $(g_b + \frac{g_v D}{P})\frac{Q_d^*}{2}$ and $(g_b + \frac{g_v D}{P})\frac{Q_s^*}{2}$ increase with D. Using Equations (3.15), (3.17), (3.28) and (3.30), we can observe that the terms $\frac{(f_b+f_v)D}{Q_d^*}$ and $\frac{(f_b+f_v)D}{Q_s^*}$ also increase with D. Thus, the behavior of R depends on the amount of increase in $TE(Q_d^*)$ and $TE(Q_s^*)$.

In most of our parameter settings, we observed that R increases with D (Figures 3.1 and 3.2). However, at large values of g_v, R increases up to a point before

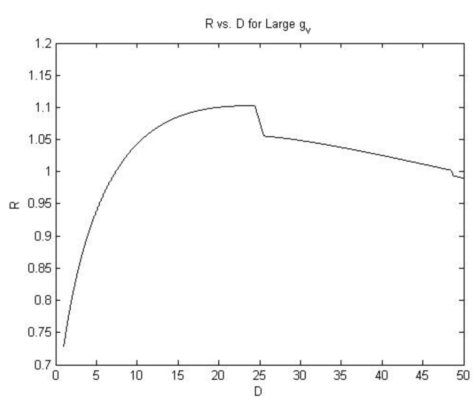

Figure 3.3: R vs. D for Large g_v

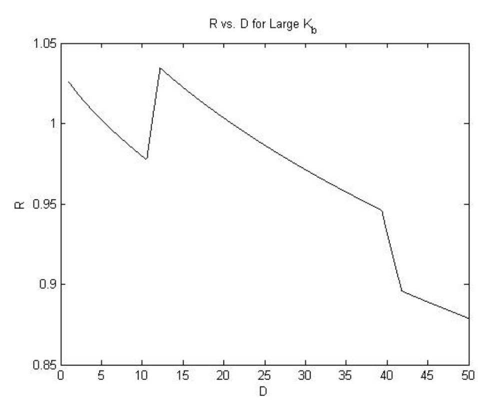

Figure 3.4: R vs. D for Large K_b

Table 3.10: Parameter Values of the Settings in Table 3.8 (Continued)

Parameters	Large K_b	Large K_v	Large h_b	Large h_v	Large f_b	Large f_v
D	30	30	30	30	30	30
P	50	50	50	50	50	50
p_c^b	2.5	2.5	2.5	2.5	2.5	2.5
p_c^s	1.5	1.5	1.5	1.5	1.5	1.5
K_b	8000	40	40	40	40	40
K_v	500	8000	500	500	500	500
h_b	1.5	1.5	20	1.5	1.5	1.5
h_v	1.2	1.2	1.2	1.49	1.2	1.2
f_b	20	20	20	20	2400	20
f_v	120	120	120	120	120	1800
g_b	0.5	0.5	0.5	0.5	0.5	0.5
g_v	0.35	0.35	0.35	0.35	0.35	0.35
e_b	1	1	1	1	1	1
e_v	1.5	1.5	1.5	1.5	1.5	1.5
C_b	80	80	80	80	80	80
C_v	200	200	200	200	200	200

it starts decreasing (Figure 3.3) as D increases. Also, at large values of K_b, R decreases with D (Figure 3.4). There can exist "jumps" depending on the existence of Q_1, Q_2, Q_3 and Q_4 (Figures 3.2, 3.3 and 3.4).

In Figures 3.1 and 3.2, we observed that coordinated solution remains to have better environmental quality (i.e., $R < 1$) at all values of D. On the other hand, Figures 3.3 and 3.4 involve cases where the environmental quality of the coordinated solution worsens briefly (i.e., $R < 1$).

The Impact of Increasing P on R

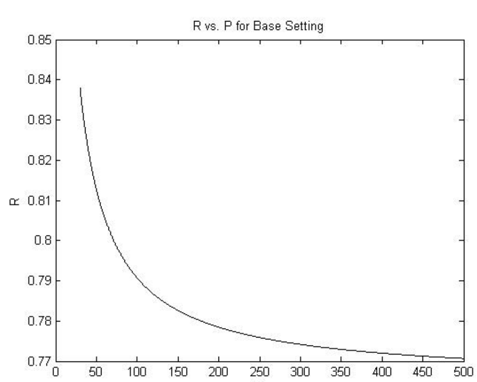

Figure 3.5: R vs. P for Base Setting

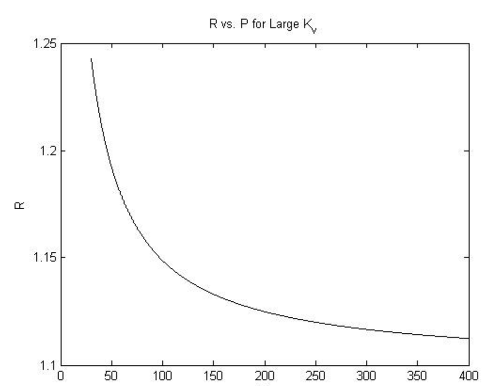

Figure 3.6: R vs. P for Large K_v

Table 3.11: Parameter Values of the Settings in Table 3.8 (Continued)

Parameters	**Large g_b**	**Large g_v**	**Large e_b**	**Large e_v**	**Large C_b**	**Large C_v**
D	30	30	30	30	30	30
P	50	50	50	50	50	50
p_c^b	2.5	2.5	2.5	2.5	2.5	2.5
p_c^s	1.5	1.5	1.5	1.5	1.5	1.5
K_b	400	40	40	40	40	40
K_v	500	500	500	500	500	500
h_b	1.5	1.5	1.5	1.5	1.5	1.5
h_v	1.2	1.2	1.2	1.2	1.2	1.2
f_b	20	20	20	20	20	20
f_v	120	120	120	120	120	120
g_b	12	0.5	0.5	0.5	0.5	0.5
g_v	0.35	10	0.35	0.35	0.35	0.35
e_b	1	1	30	1	1	1
e_v	1.5	1.5	1.5	30	1.5	1.5
C_b	80	80	80	80	4000	80
C_v	200	200	200	200	200	4000

Since Q_{d1}^*, Q_{d2}^*, Q_1 and Q_2 do not depend on P, Q_d^* does not change with increasing P. Using Equation (3.33), since the term $(g_b + \frac{g_v D}{P})\frac{Q_d^*}{2}$ decreases with P, $TE(Q_d^*)$ decreases with P. Also, Q_{c1}^*, Q_{c2}^* and $\hat{Q}_c$ increase with P. This implies the term $\frac{(f_b+f_v)D}{Q_s^*}$ decreases with P; however, the term $(g_b + \frac{g_v D}{P})\frac{Q_s^*}{2}$ may increase or decrease depending on the parameter setting. Hence, the behavior of R depends on the specific parameter setting.

In most of our parameter settings, we observed that R decreases with P (Figures 3.5 and 3.6). However, at large values of K_b, R increases with P (Figure 3.7).

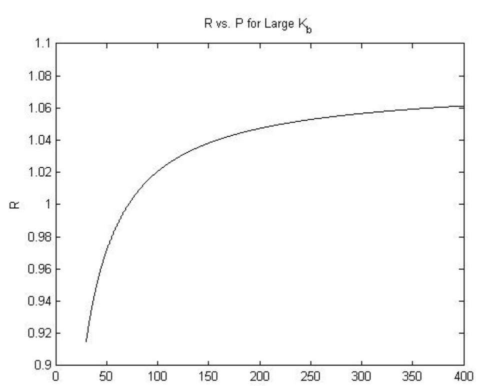

Figure 3.7: R vs. P for Large K_b

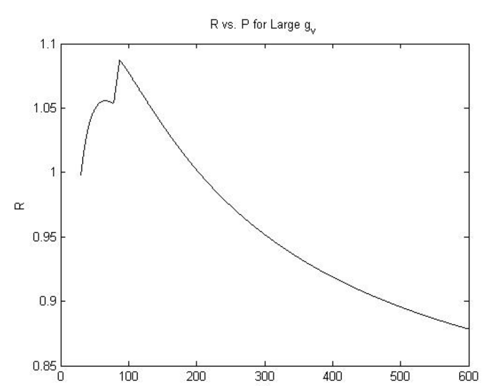

Figure 3.8: R vs. P for Large g_v

Also, for large values of g_v, R increases up to a point before it starts decreasing with P (Figure 3.8). We further observed that the change in R decreases with P and converges to zero (Figures 3.5, 3.6, 3.7 and 3.8).

The Impact of Increasing p_c^b on R

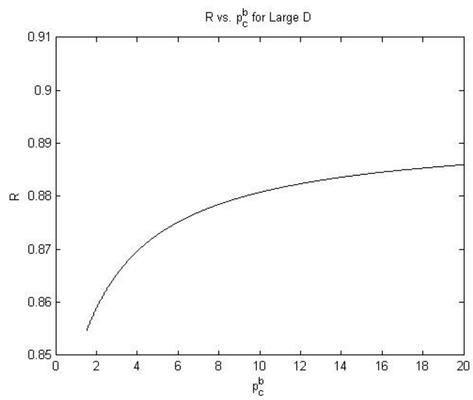

Figure 3.9: R vs. p_c^b for Large D

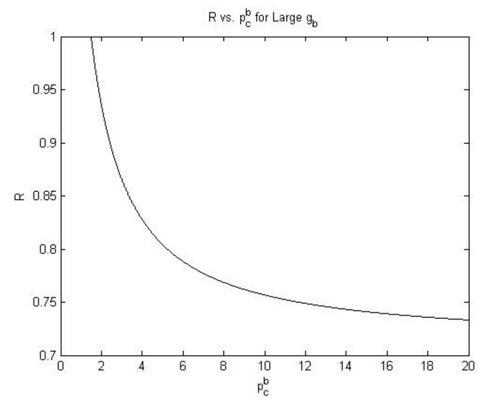

Figure 3.10: R vs. p_c^b for Large g_b

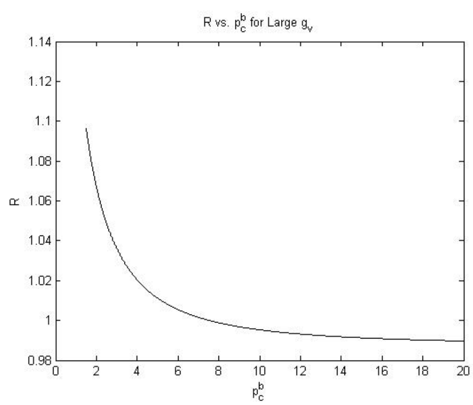

Figure 3.11: R vs. p_c^b for Large g_v

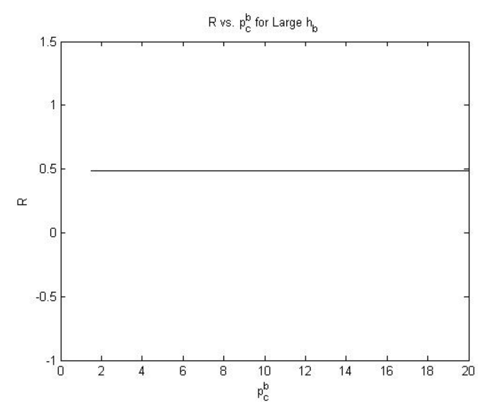

Figure 3.12: R vs. p_c^b for Large h_b

For our parameter settings, we observed that R can exhibit an increasing (Figure 3.9) or decreasing (Figures 3.10 and 3.11) pattern with increasing p_c^b. Also, as seen in Figure 3.12, there exist parameter settings under which R is constant.

In the figures where R is constant, we observed that $Q_d^* = Q_{d2}^*$ and $Q_s^* = Q_{c2}^*$ for all values of p_c^b under that parameter instance. This implies that the buyer (system) sells carbon credits under the solution of the decentralized model (centralized model with carbon credit sharing). Hence, R is not affected by p_c^b.

R can increase (decrease) under the following three cases. If $Q_d^* = Q_{d1}^*$ and $Q_s^* = Q_{c2}^*$ for all values of p_c^b under that parameter instance, since Q_{c2}^* is not affected by p_c^b, Q_{d1}^* should approach (diverge from) $\hat{Q}_c$ in order for $TE(Q_d^*)$ to decrease (increase). Similarly, if $Q_d^* = Q_{d2}^*$ and $Q_s^* = Q_{c1}^*$ for all values of p_c^b under that parameter instance, since Q_{d2}^* is not affected by p_c^b, Q_{c1}^* should diverge from (converge to) $\hat{Q}_c$ in order for $TE(Q_s^*)$ to increase (decrease). If $Q_d^* = Q_{d1}^*$ and $Q_s^* = Q_{c1}^*$ for all values of p_c^b under that parameter instance, Q_{d1}^* should converge to (diverge from) $\hat{Q}_c$ and Q_{c1}^* should diverge from (converge to) $\hat{Q}_c$. If both Q_{d1}^* and Q_{c1}^* converge to $\hat{Q}_c$, the decrease in $TE(Q_{d1}^*)$ should be by a greater amount. Similarly, if both Q_{d1}^* and Q_{c1}^* diverge from $\hat{Q}_c$, the increase in $TE(Q_{c1}^*)$ should be by a greater amount.

The Impact of Increasing p_c^s on R

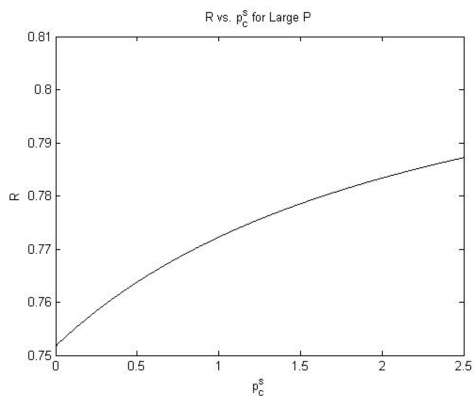

Figure 3.13: R vs. p_c^s for Large P

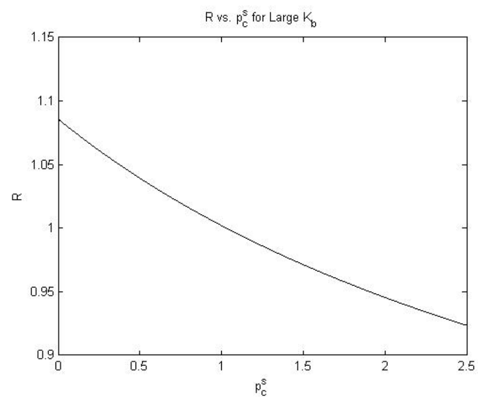

Figure 3.14: R vs. p_c^s for Large K_b

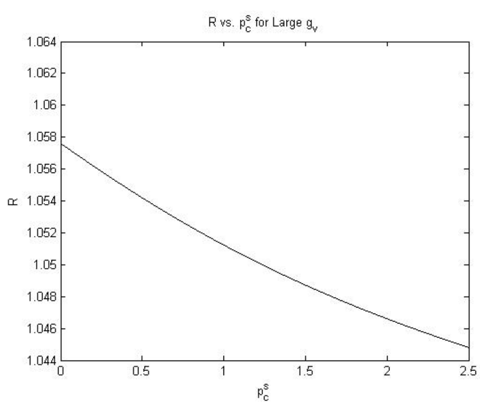

Figure 3.15: R vs. p_c^s for Large g_v

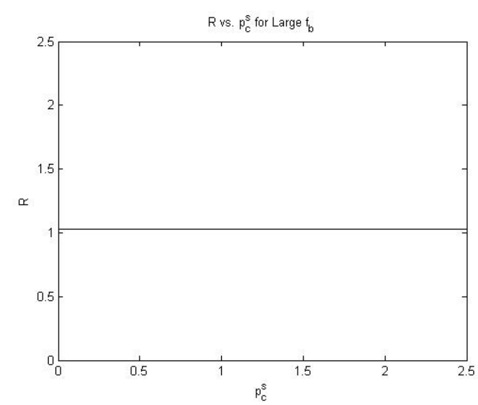

Figure 3.16: R vs. p_c^s for Large f_b

Similar to the analysis of increasing p_c^b, R can exhibit an increasing (Figure 3.13) or decreasing (Figures 3.14 and 3.15) pattern with increasing p_c^s. Also, as seen in Figure 3.16, there exist parameter settings under which R is constant.

The analysis under increasing p_c^s is similar to the analysis we presented under increasing p_c^b. If $Q_d^* = Q_{d1}^*$ and $Q_s^* = Q_{c1}^*$ for all values of p_c^b under that parameter instance (i.e., the buyer (system) buys carbon credits under the solution of the decentralized model (centralized model with carbon credit sharing)), R is not affected by p_c^s. R can decrease (increase) under the following three cases. If $Q_d^* = Q_{d1}^*$ and $Q_s^* = Q_{c2}^*$ for all values of p_c^s under that parameter instance, since Q_{d1}^* is not affected by p_c^s, Q_{c2}^* should approach (diverge from) $\hat{Q}_c$ in order for $TE(Q_s^*)$ to decrease (increase). Similarly, if $Q_d^* = Q_{d2}^*$ and $Q_s^* = Q_{c1}^*$ for all values of p_c^s under that parameter instance, since Q_{c1}^* is not affected by p_c^s, Q_{d2}^* should diverge from (converge to) $\hat{Q}_c$ in order for $TE(Q_d^*)$ to increase (decrease). If $Q_d^* = Q_{d2}^*$ and $Q_s^* = Q_{c2}^*$ for all values of p_c^s under that parameter instance, Q_{d2}^* should diverge from (converge to) $\hat{Q}_c$ and Q_{c2}^* should converge to (diverge from) $\hat{Q}_c$. If both Q_{d2}^* and Q_{c2}^* converge to $\hat{Q}_c$, the decrease in $TE(Q_{c2}^*)$ should be by a greater amount. Similarly, if both Q_{d2}^* and Q_{c2}^* diverge from $\hat{Q}_c$, the increase in $TE(Q_{d2}^*)$ should be by a greater amount.

The Impact of Increasing K_b on R

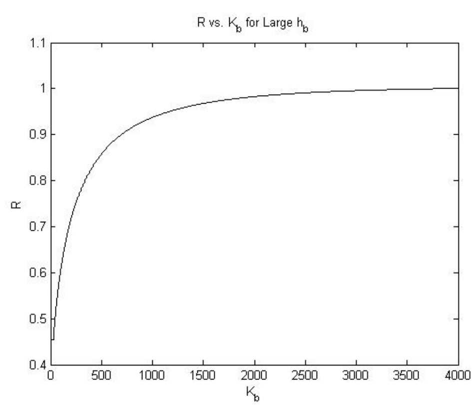

Figure 3.17: R vs. K_b for Large h_b

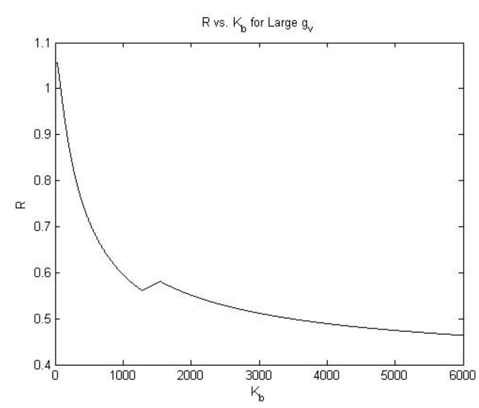

Figure 3.18: R vs. K_b for Large g_v

R exhibits increasing (Figure 3.17) or decreasing (Figure 3.18) patterns with increasing K_b. Also, for some parameter settings R increases up to a point before it starts decreasing (Figure 3.19). Here, the increasing or decreasing regions of R

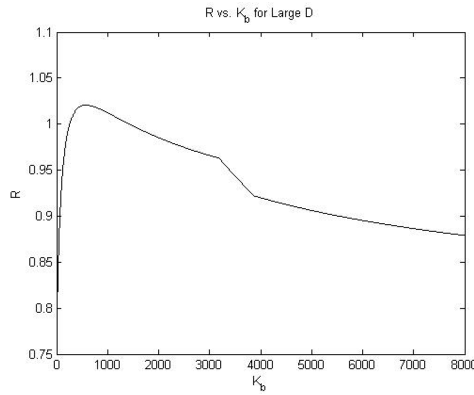

Figure 3.19: R vs. K_b for Large D

that appear to be approximately linear, one of $Q_d^* = Q_1$, $Q_d^* = Q_2$, $Q_s^* = Q_3$ and $Q_s^* = Q_4$ holds (Figures 3.18 and 3.19). Since Q_1, Q_2, Q_3 and Q_4 do not depend on K_b, either $TE(Q_d^*)$ or $TE(Q_c^*)$ is constant in these regions. Hence, R exhibits a sudden increasing or decreasing pattern.

Note that Q_{d1}^*, Q_{d2}^*, Q_{c1}^* and Q_{c2}^* increase with increasing K_b; whereas, $\hat{Q}_c$ is not affected by K_b. Under our parameter settings, we observed that R increases in the regions where $Q_d^* < \hat{Q}_c$ and $Q_s^* > \hat{Q}_c$ or $Q_d^* < \hat{Q}_c$ and $Q_s^* < \hat{Q}_c$. This is because for most parameter values, Q_d^* increases more rapidly than Q_s^*; hence, Q_d^* approaches more quickly to $\hat{Q}_c$ than Q_s^* does. We also observed that if $Q_d^* < \hat{Q}_c$ and $Q_s^* > \hat{Q}_c$ at the initial value of K_b, R does not start to decrease right after $Q_d^* = \hat{Q}_c$ due to the strict convexity of the emission function (see Equation (3.12)). That is, the change in total emissions is slower around $\hat{Q}_c$.

Similarly, we observed that R decreases in the regions where $Q_d^* > \hat{Q}_c$ and $Q_s^* > \hat{Q}_c$ as for most parameter values, Q_d^* increases more rapidly than Q_s^*. Hence, Q_d^* diverges from $\hat{Q}_c$ more rapidly than Q_s^* does.

Finally, R converges to a value as K_b increases (Figures 3.17, 3.18 and 3.19). This is because as K_b becomes larger, the amount of increase in Q_s^* converges to the amount of increase in Q_d^*.

The Impact of Increasing K_v on R

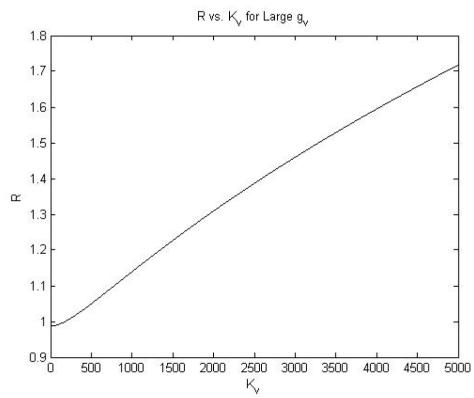

Figure 3.20: R vs. K_v for Large g_v

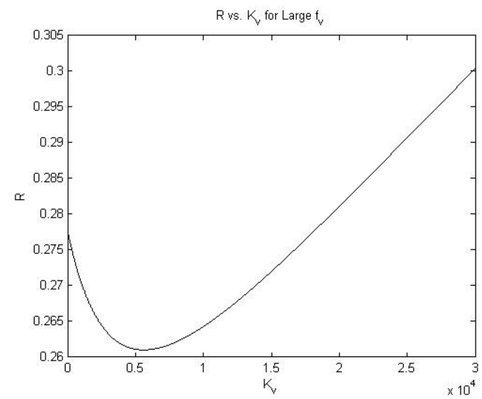

Figure 3.21: R vs. K_v for Large f_v

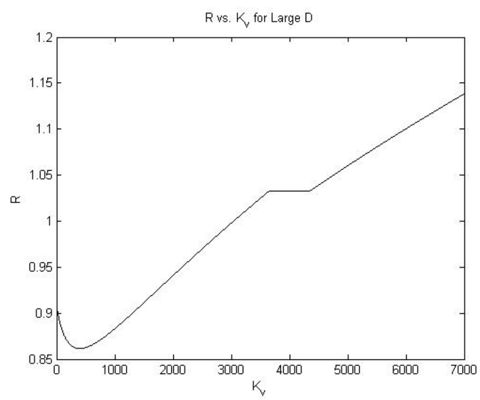

Figure 3.22: R vs. K_v for Large D

Note that $\hat{Q}_c$, Q^*_{d1}, Q^*_{d2}, Q_1 and Q_2 do not depend on K_v; hence, $TE(Q^*_d)$ is constant for all values of K_v. This implies that in the regions where $Q^*_s = Q_3$ or $Q^*_s = Q_4$, R is constant since the total emissions at Q_3 and Q_4 are equal to $C_b + C_v$. Also, if $Q^*_s > \hat{Q}_c$ at the initial value of K_v, R increases with K_v (Figure 3.20). This is because as Q^*_{c1} and Q^*_{c2} increase with K_v, Q^*_s diverges from $\hat{Q}_c$, implying $TE(Q^*_s)$ increases. Using the same reasoning, if $Q^*_s < \hat{Q}_c$ at the initial value of K_v, R decreases with K_v until $Q^*_s = \hat{Q}_c$ (Figures 3.21 and 3.22). After $Q^*_s = \hat{Q}_c$, R increases with K_v.

The Impact of Increasing h_b on R

At large values of h_b, R exhibits a decreasing pattern with further increasing h_b (Figures 3.23 and 3.24). Also, for some parameter settings R increases up to

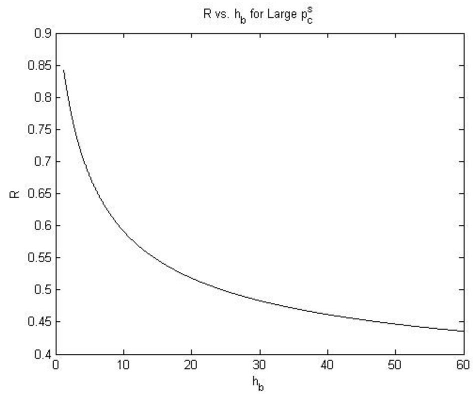

Figure 3.23: R vs. h_b for Large p_c^s

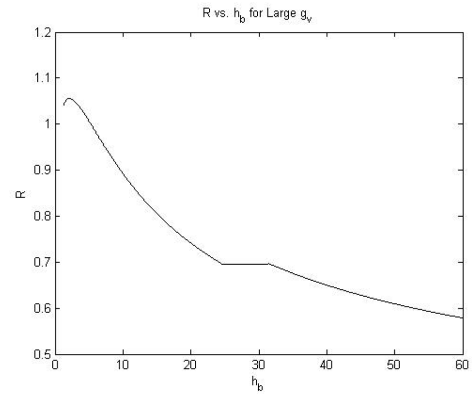

Figure 3.24: R vs. h_b for Large g_v

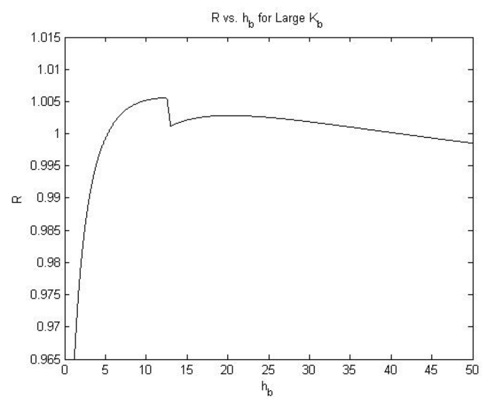

Figure 3.25: R vs. h_b for Large K_b

a point before it starts to decrease (Figure 3.25). As in the case of increasing K_b, the increasing or decreasing regions of R that appear to be approximately linear represent the regions where $Q_d^* = Q_1$, $Q_d^* = Q_2$, $Q_s^* = Q_3$ or $Q_s^* = Q_4$ (Figures 3.24 and 3.25).

Note that Q_{d1}^*, Q_{d2}^*, Q_{c1}^* and Q_{c2}^* decrease with increasing h_b; whereas, $\hat{Q}_c$ is not affected by h_b. Under our parameter settings, we observed that R decreases in the regions where $Q_d^* < \hat{Q}_c$ and $Q_s^* > \hat{Q}_c$ or $Q_d^* < \hat{Q}_c$ and $Q_c^* < \hat{Q}_c$. This is because for most parameter values, Q_d^* decreases more rapidly than Q_s^*; hence, Q_d^* diverges from $\hat{Q}_c$ more quickly than Q_s^* does. Similarly, R increases in the regions where $Q_d^* > \hat{Q}_c$ and $Q_s^* > \hat{Q}_c$ due to a similar reasoning.

We also observed that when $Q_d^* > \hat{Q}_c$ and $Q_s^* > \hat{Q}_c$, R can start decreasing before $Q_d^* = \hat{Q}_c$ due to the strict convexity of the emission function (see Equation

(3.12)). That is, the change in total emissions is slower around $\hat{Q}_c$.

Finally, R converges to a value with increasing h_b (Figures 3.23, 3.24 and 3.25). This is because as K_b becomes larger, the amount of increase in Q_s^* converges to the amount of increase in Q_d^*.

The Impact of Increasing h_v on R

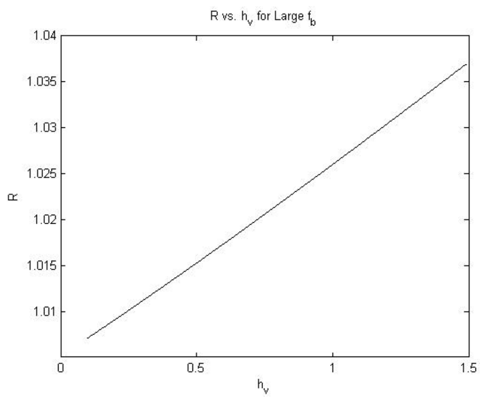

Figure 3.26: R vs. h_v for Large f_b

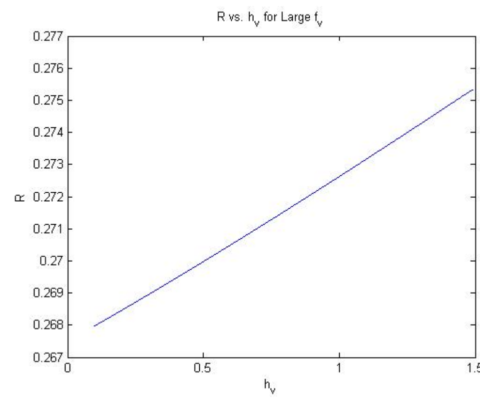

Figure 3.27: R vs. h_v for Large f_v

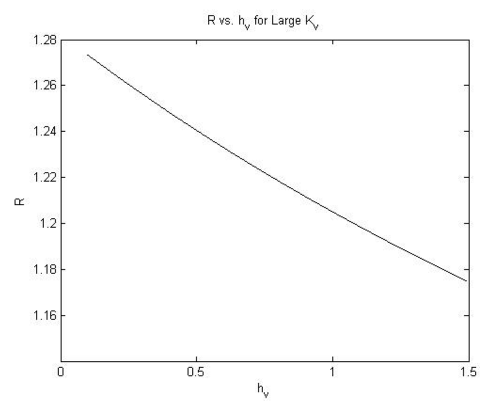

Figure 3.28: R vs. h_v for Large K_v

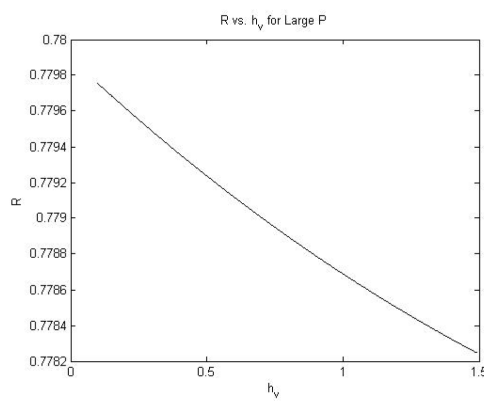

Figure 3.29: R vs. h_v for Large P

R exhibits an increasing (Figures 3.26 and 3.27) or decreasing (Figures 3.28 and 3.29) pattern with increasing h_v. Note here that $\hat{Q}_c$, Q_{d1}^*, Q_{d2}^*, Q_1 and Q_2 do not depend on h_v, implying $TE(Q_d^*)$ does not change with increasing h_v. Also, Q_{c1}^* and Q_{c2}^* decrease with h_v. Thus, if $Q_s^* < \hat{Q}_c$ ($Q_s^* > \hat{Q}_c$) at the initial value of h_v, Q_s^* will diverge from (converge to) $\hat{Q}_c$ as h_v increases, implying $TE(Q_c^*)$ will increase (decrease). Hence, R increases (decreases) with h_v if $Q_s^* < \hat{Q}_c$ ($Q_z^* > \hat{Q}_c$) at the beginning. Since we can increase h_v up to h_b, we did not observe a behavior under our parameter settings where R exhibits both increasing and decreasing patterns with respect to h_v over the same plot.

The Impact of Increasing f_b on R

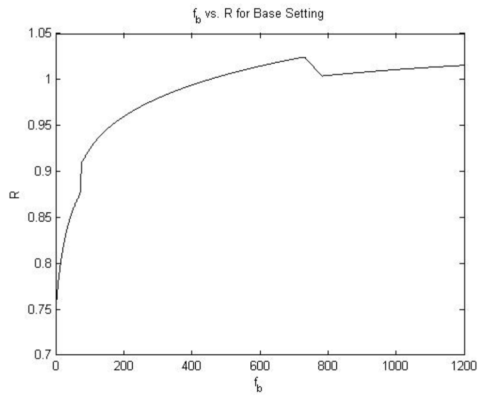

Figure 3.30: R vs. f_b for Base Setting

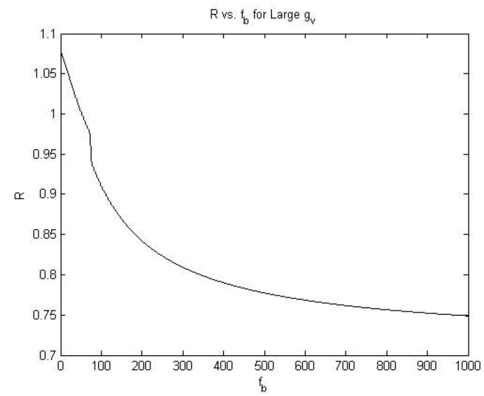

Figure 3.31: R vs. f_b for Large g_v

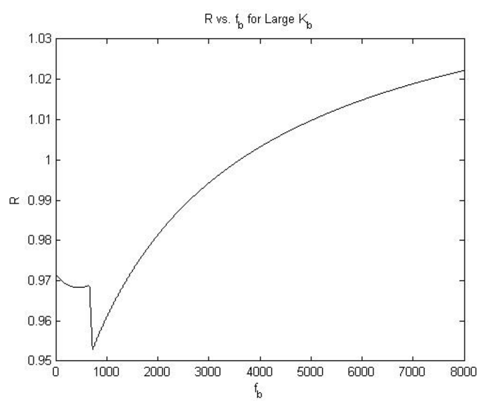

Figure 3.32: R vs. f_b for Large K_b

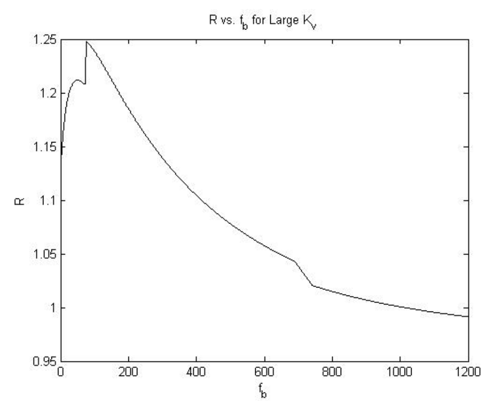

Figure 3.33: R vs. f_b for Large K_v

Since the optimal values of the unconstrained cost functions under both models (i.e., Q^*_{d1}, Q^*_{d2}, Q^*_{c1} and Q^*_{c2}), the order sizes at the boundary conditions (i.e., Q_1, Q_2, Q_3 and Q_4) and the emission optimal solution (i.e., $\hat{Q}_c$) all depend on f_b, R does not exhibit a common behavior with increasing f_b. That is, the behavior of R depends on the specific parameter setting. R can exhibit an increasing (Figure 3.30) or decreasing behavior (Figure 3.31) or both (Figures 3.32 and 3.33). There can exist "jumps" depending on the existence of Q_1, Q_2, Q_3 and Q_4 (See Figures 3.30, 3.31, 3.32 and 3.33).

In our parameter settings, we observed that for smaller values of g_v and K_v, R exhibits an increasing behavior for most values of f_b (Figures 3.30 and 3.32). Also, if f_b is increased to sufficiently large values, the changes in Q_d^*, Q_c^* and their total emission values $TE(Q_d^*)$ and $TE(Q_s^*)$ get smaller. Consequently, R exhibits a converging behavior (Figures 3.30, 3.31, 3.32 and 3.33).

The Impact of Increasing f_v on R

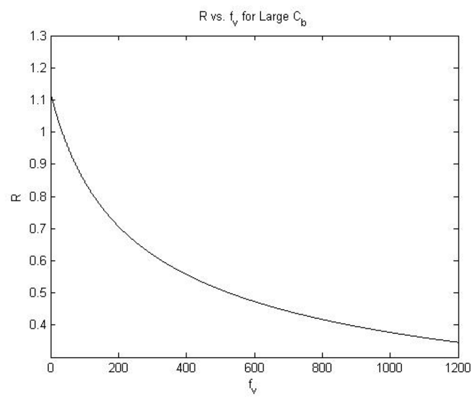

Figure 3.34: R vs. f_v for Large C_b

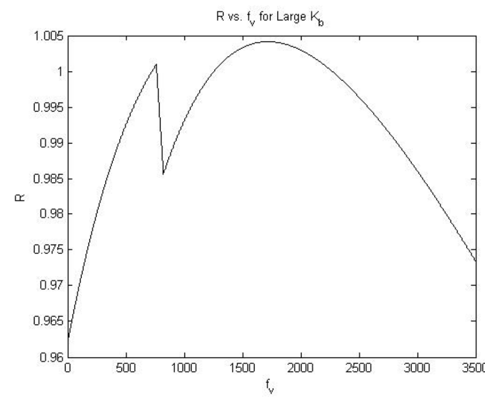

Figure 3.35: R vs. f_v for Large K_b

Since Q_{d1}^*, Q_{d2}^*, Q_1 and Q_2 do not depend on f_v, Q_d^* does not change with increasing f_v. Using Equation (3.33), since the term $\frac{(f_b+f_v)D}{Q_d^*}$ increases with f_v, $TE(Q_d^*)$ increases with f_v. Also, Q_{c1}^*, Q_{c2}^* and $\hat{Q}_c$ increase with f_v. This implies the term $(g_b + \frac{g_v D}{P})\frac{Q_s^*}{2}$ increases with f_v; however, the term $\frac{(f_b+f_v)D}{Q_s^*}$ may increase or decrease depending on the parameter setting. Hence, as in the case of varying f_b, the behavior of R depends on the specific parameter setting.

In most of our parameter settings, we observed that R exhibits a decreasing behavior with increasing f_v (Figure 3.34). However, for large values of K_b, R increases up to a value before it starts decreasing with f_v (Figure 3.35). The "jumps" in the graphs occur when $Q_s^* = Q_3$ or $Q_s^* = Q_4$, where $TE(Q_s^*) = C_b + C_v$ (Figure 3.35). Since $TE(Q_d^*)$ increases with f_v, R decreases.

The Impact of Increasing g_b on R

Similar to the case of increasing f_b, since Q_{d1}^*, Q_{d2}^*, Q_{c1}^*, Q_{c2}^*,Q_1, Q_2, Q_3 and Q_4 and $\hat{Q}_c$ all depend on g_b, the behavior of R depends on the specific parameter

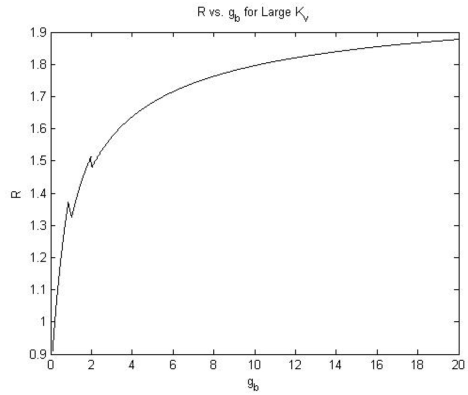

Figure 3.36: R vs. g_b for Large K_v

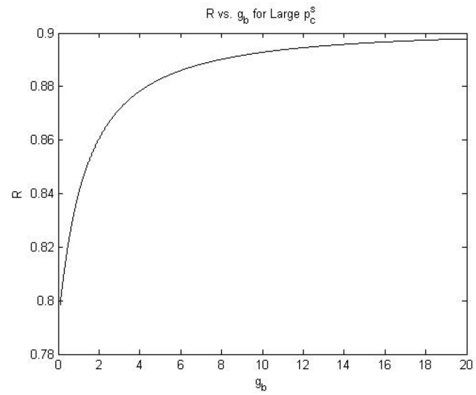

Figure 3.37: R vs. g_b for Large p_c^s

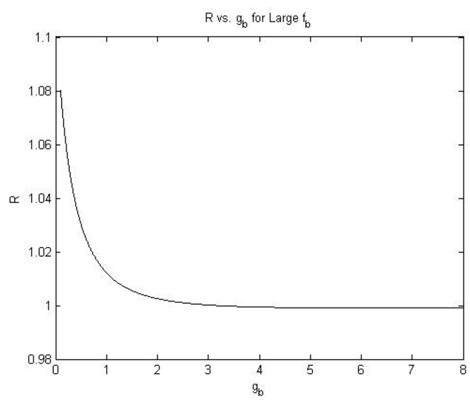

Figure 3.38: R vs. g_b for Large f_b

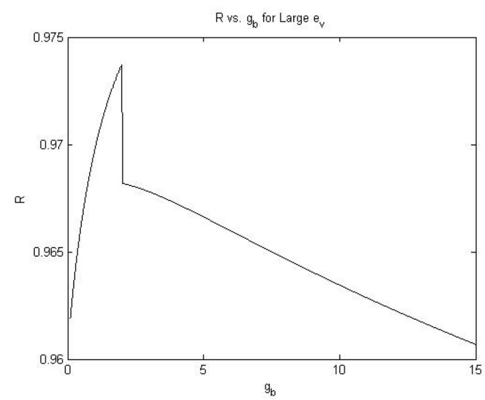

Figure 3.39: R vs. g_b for Large e_v

setting. R can exhibit an increasing (Figures 3.36 and 3.37) or decreasing behavior (Figure 3.38) or both (Figure 3.39). There can exist "jumps" depending on the existence of Q_1, Q_2, Q_3 and Q_4 (Figure 3.39).

In most of our parameter settings, we observed that R exhibits an increasing behavior with increasing g_b (Figures 3.36 and 3.37). However, for large values of e_b, e_v and f_b, R decreases with g_b (Figure 3.39). Also, if g_b is increased to sufficiently large values, the changes in Q_d^*, Q_s^* and their total emission values $TE(Q_d^*)$ and $TE(Q_s^*)$ get smaller. Consequently, R exhibits a converging behavior (Figures 3.36, 3.37, Figures 3.38 and 3.39).

The Impact of Increasing g_v on R

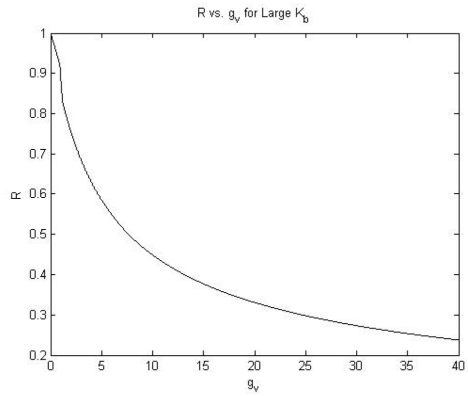

Figure 3.40: R vs. g_v for Large K_b

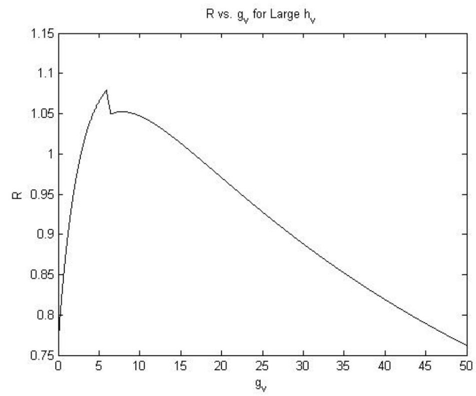

Figure 3.41: R vs. g_v for Large h_v

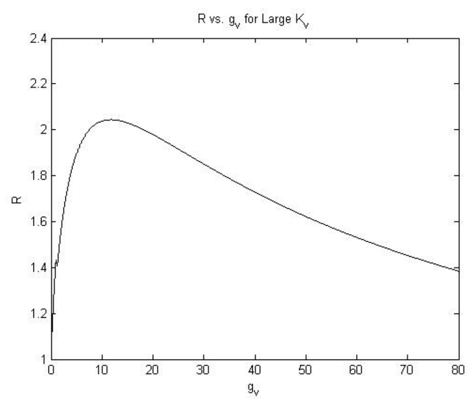

Figure 3.42: R vs. g_v for Large K_v

Since Q_{d1}^*, Q_{d2}^*, Q_1 and Q_2 do not depend on g_v, Q_d^* does not change with increasing g_v. Using Equation (3.33), since the term $(g_b + \frac{g_v D}{P})\frac{Q_d^*}{2}$ increases with g_v, $TE(Q_d^*)$ increases with g_v. Also, Q_{c1}^*, Q_{c2}^* and $\hat{Q}_c$ decrease with g_v. This implies the term $\frac{(f_b+f_v)D}{Q_s^*}$ increases with g_v; however, the term $(g_b + \frac{g_v D}{P})\frac{Q_s^*}{2}$ may increase or decrease depending on the parameter setting. Hence, as in the case of varying g_b, the behavior of R depends on the specific parameter setting.

For our parameter values, we observed the following behaviors of R. R decreases (Figure 3.40) or it increases up to a point before it starts to decrease (Figures 3.41 and 3.42). We observed that if Q_d^* is sufficiently larger than Q_d^*, the term $(g_b + \frac{g_v D}{P})\frac{Q_c^*}{2}$ increases to a greater extent with g_v. This results in a decreasing R. Since g_v is increased, Q_d^* is sufficiently larger than Q_s^* when K_b is large (Figure 3.40). The "jumps" in the graphs occur when $Q_s^* = Q_3$ or $Q_s^* = Q_4$, where $TE(Q_s^*) = C_b + C_v$ (Figures 3.40 and 3.41). Since $TE(Q_d^*)$ increases with g_v, R decreases.

The Impact of Increasing e_b on R

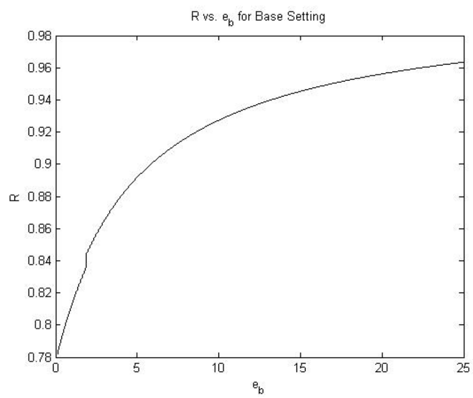

Figure 3.43: R vs. e_b for Base Setting

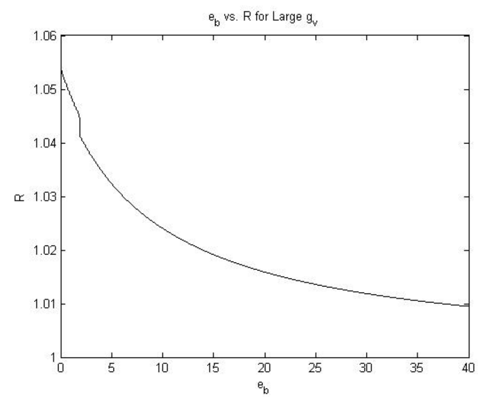

Figure 3.44: R vs. e_b for Large g_v

Notice that Q_{d1}^*, Q_{d2}^*, Q_{c1}^*, Q_{c2}^* and $\hat{Q}_c$ do not depend on e_b. In the regions where $Q_d^* = Q_{d1}^*$ or $Q_d^* = Q_{d2}^*$ and $Q_s^* = Q_{c1}^*$ or $Q_s^* = Q_{c2}^*$, the emissions under the solutions of decentralized model and the centralized model with carbon credit sharing increase by the same amount since the terms other than $(e_b + e_v)D$ of Equation (3.12) are constant. Hence, if $R < 1$ ($R > 1$) at the beginning R increases (decreases) with e_b. Also, notice that Q_1 and Q_3 increase; whereas, Q_2 and Q_4 decrease with e_b. Thus, in the regions where $Q_d^* = Q_1$, $Q_d^* = Q_2$, $Q_s^* = Q_3$ and $Q_s^* = Q_4$, R increases or decreases steeply. Suppose such steep increases and decreases are not observed. Then, if $R < 1$ at the initial value of e_b, $R < 1$ at every point of the plot (Figure 3.43). Similarly, if $R > 1$ at the initial value of e_b, $R < 1$ at every point of the plot (Figure 3.44).

The Impact of Increasing e_v on R

The analysis under increasing e_v is similar to the analysis we presented under increasing e_b and we observe the same results (Figures 3.45 and 3.46). Notice here that since Q_1 and Q_2 do not depend on e_v, we observe steep increases or decreases when $Q_s^* = Q_3$ or $Q_s^* = Q_4$.

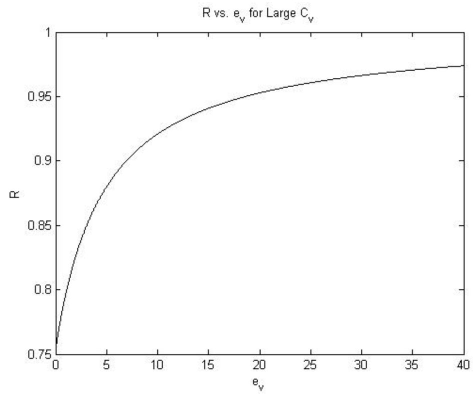

Figure 3.45: R vs. e_v for Large C_v

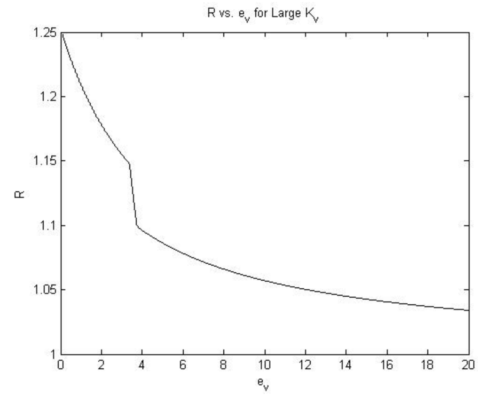

Figure 3.46: R vs. e_v for Large K_v

The Impact of Increasing C_b on R

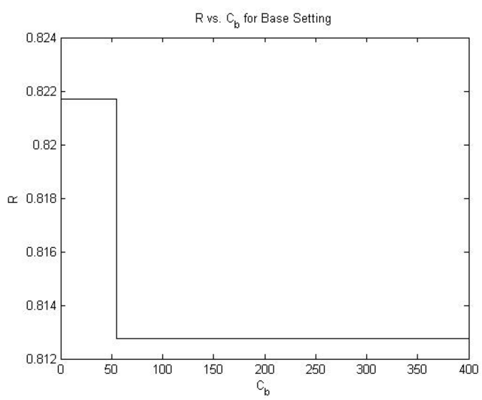

Figure 3.47: R vs. C_b for Base Setting

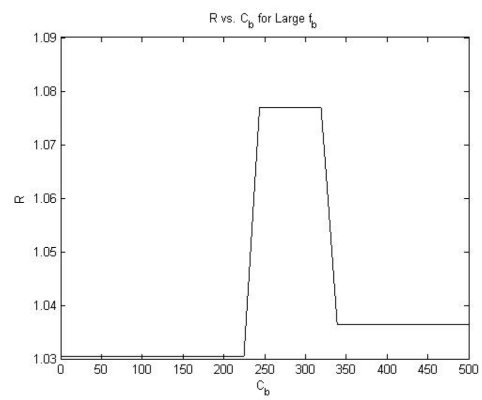

Figure 3.48: R vs. C_b for Large f_b

As seen in Figures 3.47, 3.48 and 3.49, the pattern of R with increasing C_b exhibits "jumps". Using Figure 3.47, we observed the following behavior of R. Suppose one of the following conditions is observed.

- Q_d^* changes from Q_{d1}^* to Q_{d2}^* or vice versa at some value of C_b.
- Q_c^* changes from Q_{c1}^* to Q_{c2}^* or vice versa at some value of C_b.

Then, if Q_d^* converges to (diverges from) $\hat{Q}_c$, R "jumps to" a higher (lower) value. Similarly, if Q_s^* converges to (diverges from) $\hat{Q}_c$, R "jumps to" a lower (higher) value.

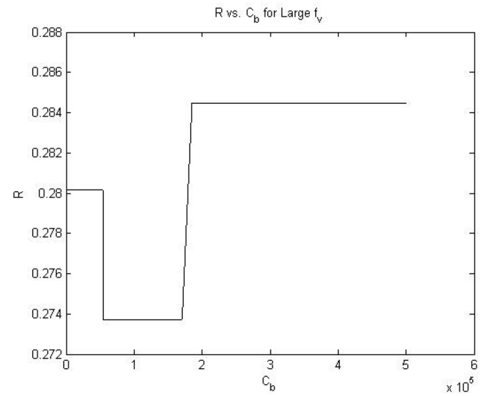

Figure 3.49: R vs. C_b for Large f_v

Using Figures 3.48 and 3.49, we observed the following behavior of R. Suppose one of the following conditions is observed.

- Q_d^* changes from Q_{d1}^* to Q_1 or Q_2 before switching to Q_{d2}^* or vice versa at some value of C_b.
- Q_c^* changes from Q_{c1}^* to Q_3 or Q_4 before switching to Q_{c2}^* or vice versa at some value of C_b.

Under the first condition, R may increase or decrease linearly. This is because when $Q_d^* = Q_1$ or $Q_d^* = Q_2$, the buyer's emissions increase as C_b increases under the decentralized optimal solution. However, the vendor's emission may increase or decrease. Under the second condition, R increases linearly. When $Q_s^* = Q_3$ or $Q_s^* = Q_4$, the total emissions of the system is equal to $C_b + C_v$, which increases with C_b. After Q_d^* (Q_s^*) switches from Q_1 or Q_2 (Q_3 or Q_4) to Q_{d1}^* or Q_{d2}^* (Q_{c1}^* or Q_{c2}^*), R is constant as Q_{d1}^*, Q_{d2}^*, Q_{c1}^* and Q_{c2}^* do not depend on C_b.

The Impact of Increasing C_v on R

The analysis under increasing C_v is similar to the analysis we presented under increasing C_b and we observe similar results (Figures 3.50 and 3.51). Note here that since Q_1, Q_2, Q_{d1}^* and Q_{d2}^* do not depend on C_v, Q_d^* does not change. Hence, the changes in R only result from the changes in Q_s^*.

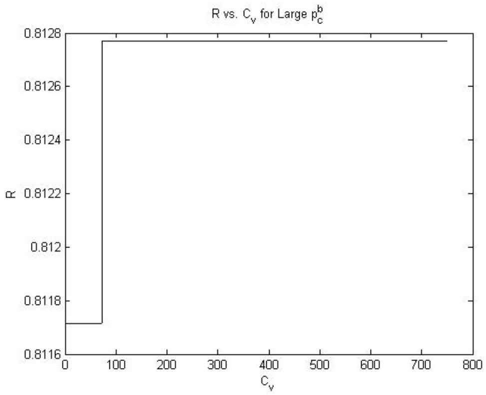

Figure 3.50: R vs. C_v for Large p_c^b

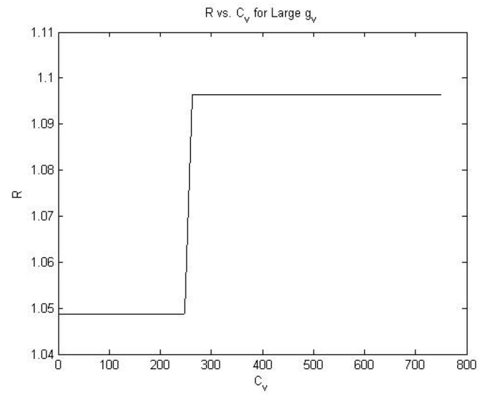

Figure 3.51: R vs. C_v for Large g_v

From Figures 3.1-3.51, we observe that R can exhibit an increasing or decreasing pattern. Additionally, R can increase above or decrease below 1 during the increase of each parameter. Therefore, we can conclude that channel coordination may not be good for the environment in terms of carbon emissions of the buyer-vendor system.

The administrative implications of our findings can be summarized as follows. In some parameter settings in which K_b or K_v is extremely large, R increases above 1. Similarly, while examining the effect of K_v on R, R increases above 1 for large values of K_v. In such cases, the policy maker (i.e., the government) can give some incentives to the supply chain members in order to decrease the cost of initiating a replenishment order (K_b) and the production setup cost (K_v) so that total emissions of the chain decrease with coordination. Also, while examining the effects of f_b, f_v, g_b and g_v, we observe that R increases above 1 in some parameter settings. The government can give incentives including environmental investments so that the emission parameters decrease; hence, the total emissions of the system decrease with coordination.

3.4.2 Numerical Illustration of Coordination Mechanisms Proposed under Deterministic Demand and Cap-and-Trade Mechanism

In this section eight numerical examples are presented to illustrate the coordination mechanisms proposed in Section 3.3. Each example corresponds to a specific case as seen in Table 3.12.

Table 3.12: Classification of the Numerical Illustrations of the Coordination Mechanisms in Section 3.3

Example Number	Q_d^* **vs** Q_s^*	$X_b(Q_d^*)$ **vs** $X_b(Q_s^*)$	$X_v(Q_d^*)$ **vs** $X_v(Q_s^*)$
13	$Q_d^* < Q_s^*$	$X_b(Q_d^*) < X_b(Q_s^*)$	$X_v(Q_d^*) < X_v(Q_s^*)$
14	$Q_d^* < Q_s^*$	$X_b(Q_d^*) < X_b(Q_s^*)$	$X_v(Q_d^*) > X_v(Q_s^*)$
8	$Q_d^* < Q_s^*$	$X_b(Q_d^*) > X_b(Q_s^*)$	$X_v(Q_d^*) < X_v(Q_s^*)$
15	$Q_d^* < Q_s^*$	$X_b(Q_d^*) > X_b(Q_s^*)$	$X_v(Q_d^*) > X_v(Q_s^*)$
16	$Q_d^* > Q_s^*$	$X_b(Q_d^*) < X_b(Q_s^*)$	$X_v(Q_d^*) < X_v(Q_s^*)$
17	$Q_d^* > Q_s^*$	$X_b(Q_d^*) < X_b(Q_s^*)$	$X_v(Q_d^*) > X_v(Q_s^*)$
18	$Q_d^* > Q_s^*$	$X_b(Q_d^*) > X_b(Q_s^*)$	$X_v(Q_d^*) < X_v(Q_s^*)$
19	$Q_d^* > Q_s^*$	$X_b(Q_d^*) > X_b(Q_s^*)$	$X_v(Q_d^*) > X_v(Q_s^*)$

During the remaining of our analysis in this section, the examples are arranged in order based on their index number. Recall that the parameter values of Example 8 is presented in Table 3.4 in Section 3.2.2. Also, the parameter values of Examples $13 - 19$ are presented in Table 3.13. The solutions of the decentralized model and centralized model with carbon credit sharing for each example are presented in Tables 3.14-3.17. The application of the coordination mechanisms proposed in Section 3.3 to each example is summarized in Table A.1 in Appendix A.2. Finally, the costs of the buyer and the vendor after coordination could be seen in Table A.2 in Appendix A.2.

Table 3.13: Parameter Values of Examples 13 − 19 Given $D = 50$, $c = 12$ and $p_v = 8$

Parameter	**Eg 13**	**Eg 14**	**Eg 15**	**Eg 16**	**Eg 17**	**Eg 18**	**Eg 19**
P	55	70	75	75	55	75	55
p_b^c	2.5	3	7.5	7.5	2.5	7.5	2.5
p_s^c	2	2.5	6	6	2	6	2
K_b	40	100	90	500	1000	185	330
K_v	100	300	1000	1000	100	300	100
h_b	3.2	2	2	1	3.2	1	3.2
h_v	3	0.5	0.8	0.8	3	0.8	3
f_b	90	55	90	90	90	90	90
f_v	95	60	60	60	95	60	95
g_b	0.5	0.5	1	0.5	0.5	0.5	0.5
g_v	0.25	1.75	1.75	1.75	0.25	1.75	0.25
e_b	4.5	5	5	5	4.5	5	4.5
e_v	6	6	6	6	6	6	6
C_b	304	350	345	350	300	350	300
C_v	350	400	400	400	350	400	350

Table 3.14: Optimal Order Quantities and Carbon Transfer Amounts of Examples 8 and 13 − 19 Resulting from the Decentralized Model

Example	Q_{d1}^*	Q_{d2}^*	Q_1	Q_2	$X_b(Q_d^*)$	$X_v(Q_d^*)$
8	158.944	168.819	55.279	144.721	−2.319	50.91
13	77.169	72.375	74.549	241.451	0	−22.188
14	87.014	85.485	29.706	370.294	46.459	11.478
15	89.736	88.741	90	100	−0.015	14.223
16	157.28	161.245	51.676	348.324	31.781	−12.665
17	165.916	167.616	82.918	217.082	6.249	2.614
18	134.556	134.629	51.676	348.324	32.918	−0.817
19	111.678	110.195	82.918	217.082	6.615	−5.628

Table 3.15: Decentralized Costs of Examples 8 and 13 − 19

Example	$BC(Q_d^*)$	$VC(Q_d^*)$	$TC(Q_d^*)$
8	979.983	422.365	1402.348
13	746.107	624.197	1370.304
14	627.826	562.04	1189.866
15	739.996	895.782	1635.778
16	644.981	848.072	1493.053
17	1153.989	653.169	1807.158
18	538.516	553.447	1091.963
19	912.817	609.709	1522.526

Table 3.16: Optimal Order Quantities and Carbon Transfer Amounts of Examples 8 and $13-19$ Resulting from the Centralized Model with Carbon Credit Sharing

Example	Q^*_{c1}	Q^*_{c2}	Q_3	Q_4	$X_b(Q^*_s)$	$X_v(Q^*_s)$
8	240.769	251.425	67.084	447.202	-20.811	62.677
13	88.197	83.12	99.758	254.992	5.929	-13.879
14	98.962	101.055	33.726	194.845	47.523	7.154
15	108.593	113.186	55.694	124.306	-1.351	7.47
16	136.768	144.254	46.515	193.485	32.742	-4.945
17	142.032	141.116	107.816	235.934	7.832	0.304
18	107.111	109.584	46.515	193.485	31.54	8.7
19	107.345	104.103	107.816	235.934	6.243	-6.448

Table 3.17: Costs of Examples 8 and $13-19$ Resulting from the Centralized Model with Carbon Credit Sharing

Example	$BC(Q^*_s)$	$VC(Q^*_s)$	$TC(Q^*_s)$	$SC(Q^*_s)$	$TC(Q^*_s)-SC(Q^*_s)$
8	1060.773	243.757	1304.531	1273.314	31.216
13	751.935	611.658	1363.592	1360.628	2.964
14	631.725	548.595	1180.32	1180.32	0
15	763.074	827.114	1590.188	1588.162	2.026
16	648.983	822.165	1471.148	1463.731	7.417
17	1164.439	627.255	1791.694	1791.694	0
18	549.964	513.905	1063.87	1063.87	0
19	912.976	609.079	1522.055	1518.934	3.121

Chapter 4

Supply Chain Coordination under Deterministic Demand and Carbon Tax or Carbon Cap Mechanisms

In this chapter, we revisit the two-echelon system introduced in Chapter 3, and we study the different decision making approaches under the existence of either the tax policy or the cap policy. We present our analysis and findings for the tax policy first. We continue with a similar discussion for the cap policy in this chapter.

4.1 Problem Definition and Analysis under Deterministic Demand and Carbon Tax Mechanism

4.1.1 Problem Definition under Deterministic Demand and Carbon Tax Mechanism

An external carbon tax is applied by the regulatory agencies. A linear tax schedule is adapted. That is, the buyer and the vendor pay a monetary amount for each unit of carbon emitted. We consider a general case in which the buyer's and the vendor's tax rates are different, allowing for settings where the parties operate in different geographical locations (e.g., different countries) and/or in different industries.

Two models are proposed, which are the decentralized and the centralized models. In the decentralized model, buyer decides the order quantity that minimizes his/her cost. In the centralized model, the order quantity that minimizes the total cost of the system (i.e., the total cost of the buyer and the vendor) is determined. The notation used in this section is summarized in Table 4.1.

In the decentralized model, buyer solves the following replenishment problem to decide the order quantity that minimizes his/her costs:

$$min \quad BC(Q) = \frac{(K_b + t_b f_b)D}{Q} + \frac{(h_b + t_b g_b)Q}{2} + (c + t_b e_b)D$$

$$Q \geq 0,$$

where $t_b f_b$ is the emission tax paid per replenishment, $t_b g_b$ is the emission tax paid per unit held in inventory per unit time and $t_b e_b$ is the emission tax paid per unit ordered by the buyer.

Table 4.1: Problem Parameters and Decision Variables under Deterministic Demand and Carbon Tax Mechanisms

Buyer's Parameters	
D	annual demand rate
K_b	fixed cost of ordering
h_b	cost of holding one unit inventory for a year
c	unit purchasing cost
f_b	fixed amount of carbon emission at each ordering
g_b	carbon emission amount due to holding one unit inventory for a year
e_b	carbon emission amount due to unit procurement

Vendor's Parameters	
P	annual production rate ($P > D$)
K_v	fixed cost per production run
h_v	cost of holding one unit inventory for a year
p_v	unit production cost
f_v	fixed amount of carbon emission at each production setup
g_v	carbon emission amount due to holding one unit inventory for a year
e_v	carbon emission amount due to producing one unit

Policy Parameters	
t_b	Cost of unit carbon emission to the buyer (i.e., carbon tax paid by the buyer for a unit emission)
t_v	Cost of unit carbon emission to the vendor (i.e., carbon tax paid by the vendor for a unit emission)

Decision Variables	
Q	buyer's order quantity (vendor's production lot size)

Functions and Optimal Values of Decision Variables	
$BC(Q)$	buyer's average annual costs as a function of Q
$VC(Q)$	vendor's average annual costs as a function of Q
$TC(Q)$	total average annual costs as a function of Q ($TC(Q) = BC(Q) + TC(Q)$)
Q_d^*	optimal order quantity as a result of the decentralized model
Q_c^*	optimal order quantity as a result of the centralized model

Vendor's average annual cost as a function of Q, is given by

$$VC(Q) = \frac{(K_v + t_v f_v)D}{Q} + \frac{(h_v + t_v g_v)QD}{2P} + (p_v + t_v e_v)D, \tag{4.1}$$

where $t_v f_v$ is the emission tax paid per production run, $t_v g_v$ is the emission tax paid per unit held in inventory per unit time and $t_v e_v$ is the emission tax paid per unit produced by the vendor.

In the centralized model, the order quantity that minimizes the total cost of the system (i.e, the total cost of the buyer and the vendor) is determined. In mathematical terms, the following problem is solved.

$$\begin{aligned} min \quad TC(Q) = & \frac{(K_b + K_v + t_b f_b + t_v f_v)D}{Q} + \frac{[h_b + t_b g_b + \frac{D}{P}(h_v + t_v g_v)]Q}{2} \\ & + (c + p_v + t_b e_b + t_v e_v)D \\ & Q \geq 0. \end{aligned}$$

4.1.2 Analysis of the Decentralized and the Centralized Models under Deterministic Demand and Carbon Tax Mechanism

In this section, we provide an analysis of the decentralized model and the centralized model under carbon tax mechanism to find the cost minimizing order quantities Q_d^* and Q_c^*, respectively. We also present some properties related to Q_d^*, Q_c^* and average annual tax amounts of the buyer and the vendor.

Let us define the average annual tax amounts of the buyer, vendor and the system, which are presented in Equations (4.2), (4.3) and (4.4), and defined in Table 4.2.

$$BT(Q) = \frac{t_b f_b D}{Q} + \frac{t_b g_b Q}{2} + t_b e_b D, \tag{4.2}$$

Table 4.2: Additional Notation for Carbon Tax Mechanism under Deterministic Demand

$BT(Q)$:	Average annual tax paid by the buyer as a function of order size Q
$VT(Q)$:	Average annual tax paid by the vendor as a function of order size Q
$TT(Q)$:	Average annual tax paid by the buyer-vendor system as a function of order size Q

$$VT(Q) = \frac{t_v f_v D}{Q} + \frac{t_v g_v Q D}{2P} + t_v e_v D, \tag{4.3}$$

$$TT(Q) = \frac{(t_b f_b + t_v f_v)D}{Q} + (t_b g_b + \frac{t_v g_v D}{P})\frac{Q}{2} + (t_b e_b + t_v e_v)D. \tag{4.4}$$

Notice here that Equation (4.4) is obtained by summing up Equations (4.2) and (4.3).

In Lemma 17 and Proposition 2, we present the order quantities that minimize the buyer's average annual taxes and costs, respectively.

Lemma 17 *The order quantity that minimizes the average annual taxes of the buyer (i.e., Q_d^t) is given by*

$$Q_d^t = \sqrt{\frac{2 f_b D}{g_b}}. \tag{4.5}$$

Proof: Since (4.2) is a strictly convex function in Q, Q_d^t is obtained from the first order condition. ■

As the average annual taxes are linearly proportional to average annual emissions, Q_d^t, as given by Expression (4.5), also minimizes the latter.

Proposition 2 *Buyer's optimal order quantity resulting from the decentralized model is given by*

$$Q_d^* = \sqrt{\frac{2(K_b + t_b f_b)D}{h_b + t_b g_b}}. \tag{4.6}$$

Proof: Since $BC(Q)$ is a strictly convex function in Q, Q_d^* is obtained from the first order condition. ■

Observe that, as t_b increases, the cost minimizing order quantity Q_d^* approaches to the emission optimal order quantity Q_d^t. We use Proposition 2 to find the minimum average annual cost of the buyer under the decentralized model and present it in the following corollary.

Corollary 5 *The average annual cost of the buyer under the optimal solution of the decentralized model is given by*

$$BC(Q_d^*) = \sqrt{2(K_b + t_b f_b)D(h_b + t_b g_b)} + (c + t_b e_b)D. \tag{4.7}$$

Proof: Obtained by plugging (4.6) in $BC(Q)$. ■

Similarly, the vendor's average annual cost under the decentralized model ($VC(Q_d^*)$) can be found by plugging Q_d^* in (4.1). Also, the total average annual cost of the system under the decentralized model is $TC(Q_d^*) = BC(Q_d^*) + VC(Q_d^*)$.

Similar to Lemma 17, we find the order quantity that minimizes the average annual taxes of the system and present it in the following lemma.

Lemma 18 *The order quantity that minimizes the average annual taxes of the system (i.e., Q_c^t) is given by*

$$Q_c^t = \sqrt{\frac{2(t_b f_b + t_v f_v)D}{t_b g_b + \frac{t_v g_v D}{P}}}. \tag{4.8}$$

Proof: Since (4.4) is a strictly convex function in Q, Q_c^t is obtained from the first order condition. ■

Since $BC(Q)$ and $TC(Q)$ follow a similar structure, the order quantity that minimizes the average annual cost of the system is found with a similar method used in Proposition 2. The centralized optimal solution is presented in the following theorem.

Theorem 11 *Buyer's optimal order quantity resulting from the centralized model is given by*

$$Q_c^* = \sqrt{\frac{2(K_b + K_v + t_b f_b + t_v f_v)D}{h_b + t_b g_b + (h_v + t_v g_v)\frac{D}{P}}}. \tag{4.9}$$

Proof: Since $TC(Q)$ is a strictly convex function in Q, Q_c^* is obtained from the first order condition. ■

Note that, as t_v gets larger, Q_c^* approaches to $\sqrt{\frac{2f_v D}{g_v}}$, which is the minimizer of Expression (4.3) (i.e., vendor's emission optimal order quantity). Similarly, as t_b gets larger, Q_c^* approaches to $\sqrt{\frac{2f_b D}{g_b}}$, which is the buyer's emission optimal order quantity. In the next corollary, we present the average annual cost of the system resulting from the optimal solution of the centralized model.

Corollary 6 *The total average annual cost of the system under the centralized model is given by*

$$\begin{aligned} TC(Q_c^*) = & \sqrt{2(K_b + K_v + t_b f_b + t_v f_v)D\left[h_b + t_b g_b + (h_v + t_v g_v)\frac{D}{P}\right]} \\ & + (c + p_v + t_b e_b + t_v e_v)D. \end{aligned} \tag{4.10}$$

Proof: Obtained by plugging (4.9) in $TC(Q)$. ■

Similarly, the buyer's average annual cost ($BC(Q_c^*)$) and the vendor's average annual cost ($VC(Q_c^*)$) under the centralized model can be found by plugging Q_c^* in $BC(Q)$ and (4.1), respectively.

In the next proposition, we present a further property of Q_d^* and Q_c^*.

Proposition 3 $Q_d^* \leqslant Q_c^*$ *if and only if* $\frac{K_b + t_b f_b}{h_b + t_b g_b} \leqslant \frac{K_v + t_v f_v}{h_v + t_v g_v}\frac{P}{D}$.

Proof: Using Equations (4.6) and (4.9), we have $Q_d^* \leqslant Q_c^*$ if and only if

$$\sqrt{\frac{2(K_b + t_b f_b)D}{h_b + t_b g_b}} \leqslant \sqrt{\frac{2(K_b + K_v + t_b f_b + t_v f_v)D}{h_b + t_b g_b + (h_v + t_v g_v)\frac{D}{P}}}.$$

Taking the square of both sides leads to

$$\frac{2(K_b + t_b f_b)D}{h_b + t_b g_b} \leqslant \frac{2(K_b + K_v + t_b f_b + t_v f_v)D}{h_b + t_b g_b + (h_v + t_v g_v)\frac{D}{P}}.$$

This, in turn, implies

$$K_b h_b + K_b t_b g_b + K_b h_v \tfrac{D}{P} + K_b t_v g_v \tfrac{D}{P} + t_b f_b h_b + t_b^2 f_b g_b + h_v t_b f_b \tfrac{D}{P} + t_b f_b t_v g_v \tfrac{D}{P}$$

$$\leqslant K_b h_b + K_v h_b + h_b t_b f_b + h_b t_v f_v + K_b t_b g_b + K_v t_b g_b + t_b^2 f_b g_b + t_b g_b t_v g_v.$$

After some cancelations and rearrangement of terms, we get

$$(K_b + t_b f_b)(h_v + t_v g_v)\frac{D}{P} \leqslant (K_v + t_v f_v)(h_b + t_b g_b).$$

This results in

$$\frac{K_b + t_b f_b}{h_b + t_b g_b} \leqslant \frac{K_v + t_v f_v}{h_v + t_v g_v}\frac{P}{D}.$$

■

Using Equations (4.2) and (4.3), we present the conditions under which the average annual taxes of the buyer and the vendor decrease under the centralized optimal solution in Propositions 4 and 5, respectively.

Proposition 4 *$BT(Q_c^*) \leqslant BT(Q_d^*)$ if and only if $(Q_d^* - Q_c^*)(2f_b D - g_b Q_c^* Q_d^*) \leqslant 0$.*

Proof: It follows from Equation (4.2), that $BT(Q_c^*) \leqslant BT(Q_d^*)$ if and only if

$$\frac{t_b f_b D}{Q_c^*} + \frac{t_b g_b Q_c^*}{2} + t_b e_b D \leqslant \frac{t_b f_b D}{Q_d^*} + \frac{t_b g_b Q_d^*}{2} + t_b e_b D.$$

After canceling some terms and taking others into common parentheses, we get

$$f_b D(\frac{1}{Q_c^*} - \frac{1}{Q_d^*}) + \frac{g_b}{2}(Q_c^* - Q_d^*) \leqslant 0.$$

This, in turn, implies

$$\frac{1}{2Q_c^* Q_d^*}(Q_d^* - Q_c^*)(2f_b D - g_b Q_c^* Q_d^*) \leqslant 0.$$

Since $\frac{1}{2Q_c^* Q_d^*} \geqslant 0$, the above expression reduces to

$$(Q_d^* - Q_c^*)(2f_b D - g_b Q_c^* Q_d^*) \leqslant 0.$$

■

Proposition 5 *$VT(Q_d^*) \geqslant VT(Q_c^*)$ if and only if $(Q_c^* - Q_d^*)(2f_v P - g_v Q_c^* Q_d^*) \geqslant 0$.*

Proof: Using Equation (4.3), we have $VT(Q_d^*) \geqslant VT(Q_c^*)$ if and only if

$$\frac{t_v f_v D}{Q_d^*} + \frac{t_v g_v Q_d^* D}{2P} + t_v e_v D \geqslant \frac{t_v f_v D}{Q_c^*} + \frac{t_v g_v Q_c^* D}{2P} + t_v e_v D.$$

The above inequality can further be reduced to

$$f_v D(\frac{1}{Q_d^*} - \frac{1}{Q_c^*}) + \frac{g_v D}{2P}(Q_d^* - Q_c^*) \geqslant 0.$$

This, in turn, implies

$$\frac{D}{2Q_c^* Q_d^*}(Q_c^* - Q_d^*)(2f_v P - g_v Q_c^* Q_d^*) \geqslant 0.$$

Since $\frac{D}{2Q_c^* Q_d^*} \geqslant 0$, the above expression reduces to

$$(Q_c^* - Q_d^*)(2f_v P - g_v Q_c^* Q_d^*) \geqslant 0.$$

■

Up until this point, we have taken the perspective of the buyer-vendor system in comparing the different solution approaches. We have obtained results on how the buyer's and the vendor's annual costs differ under the decentralized and centralized solutions. In the next two propositions, we take the perspective of the regulator or the government who collects taxes. We compare the average annual amount of taxes collected by the government under the decentralized and centralized solutions.

Proposition 6 *Suppose* $\frac{K_b+t_bf_b}{h_b+t_bg_b} \leqslant \frac{K_v+t_vf_v}{h_v+t_vg_v}\frac{P}{D}$.

i) If $\frac{t_bf_b+t_vf_v}{t_bg_b+\frac{t_vg_vD}{P}} \leqslant \frac{K_b+t_bf_b}{h_b+t_bg_b}$, *then the government collects no less taxes in the centralized solution than it does in the decentralized solution.*

ii) If $\frac{t_bf_b+t_vf_v}{t_bg_b+\frac{t_vg_vD}{P}} \geqslant \frac{K_b+K_v+t_bf_b+t_vf_v}{h_b+t_bg_b+(h_v+t_vg_v)\frac{D}{P}}$, *then the government collects no less taxes in the decentralized solution than it does in the centralized solution.*

Proof:

i) It follows from Proposition 3 that $\frac{K_b+t_bf_b}{h_b+t_bg_b} \leqslant \frac{K_v+t_vf_v}{h_v+t_vg_v}\frac{P}{D}$ is equivalent to $Q_d^* \leq Q_c^*$. Multiplying both sides of the inequality $\frac{t_bf_b+t_vf_v}{t_bg_b+\frac{t_vg_vD}{P}} \leqslant \frac{K_b+t_bf_b}{h_b+t_bg_b}$ with $2D$ and taking the square root of both sides, we obtain

$$\sqrt{\frac{2(t_bf_b+t_vf_v)D}{t_bg_b+\frac{t_vg_vD}{P}}} \leqslant \sqrt{\frac{2(K_b+t_bf_b)D}{h_b+t_bg_b}},$$

which implies $Q_c^t \leq Q_d^*$. Combining this result with the fact that $Q_d^* \leq Q_c^*$ implies $Q_c^t \leq Q_d^* \leq Q_c^*$. Since Expression (4.4) is a strictly convex function, this implies $TT(Q_c^*) \geq TT(Q_d^*)$. That is, in the centralized solution, the government collects at least as much taxes as it collects in the decentralized solution.

ii) Again, due to Proposition 3, we know that $\frac{K_b+t_bf_b}{h_b+t_bg_b} \leqslant \frac{K_v+t_vf_v}{h_v+t_vg_v}\frac{P}{D}$ implies $Q_d^* \leq Q_c^*$. Multiplying both sides of the inequality $\frac{t_bf_b+t_vf_v}{t_bg_b+\frac{t_vg_vD}{P}} \geqslant \frac{K_b+K_v+t_bf_b+t_vf_v}{h_b+t_bg_b+(h_v+t_vg_v)\frac{D}{P}}$

with $2D$ and taking the square root of both sides, we obtain

$$\sqrt{\frac{2(t_b f_b + t_v f_v)D}{t_b g_b + \frac{t_v g_v D}{P}}} \geqslant \sqrt{\frac{2(K_b + K_v + t_b f_b + t_v f_v)D}{h_b + t_b g_b + (h_v + t_v g_v)\frac{D}{P}}},$$

which is equivalent to $Q_c^t \geq Q_c^*$. Combining this result with the fact that $Q_d^* \leq Q_c^*$ implies $Q_c^t \geq Q_c^* \geq Q_d^*$. Since Expression (4.4) is a strictly convex function, this implies $TT(Q_d^*) \geq TT(Q_c^*)$. That is, in the decentralized solution, the government collects at least as much taxes as it collects in the centralized solution. ■

Proposition 7 *Suppose* $\frac{K_b+t_b f_b}{h_b+t_b g_b} > \frac{K_v+t_v f_v}{h_v+t_v g_v}\frac{P}{D}$.

i) If $\frac{t_b f_b+t_v f_v}{t_b g_b+\frac{t_v g_v D}{P}} \geqslant \frac{K_b+t_b f_b}{h_b+t_b g_b}$*, then the government collects more taxes in the centralized solution than it does in the decentralized solution.*

ii) If $\frac{t_b f_b+t_v f_v}{t_b g_b+\frac{t_v g_v D}{P}} \leqslant \frac{K_b+K_v+t_b f_b+t_v f_v}{h_b+t_b g_b+(h_v+t_v g_v)\frac{D}{P}}$*, then the government collects more taxes in the decentralized solution than it does in the centralized solution.*

Proof: The proof follows a similar structure to the proof of Proposition 6 and is omitted. ■

Proposition 6 and Proposition 7 imply that there are cases in which coordination of the buyer-vendor system may not be good from the perspective of a government or a regulator who wants to increase total annual average taxes collected. In Table 4.3, we present some numerical instances to illustrate our analytical results for the buyer-vendor coordination problem under the tax policy. In the last two columns of the table, we report the decentralized and the centralized optimum quantities. In Table 4.4, we present the buyer's, vendor's and the system's average annual taxes resulting from the decentralized and the centralized solutions to the examples in Table 4.3. Examples 20, 21 and 22 are to illustrate the first part of Proposition 6. As it can be observed from Table 4.4, in these examples, government collects more taxes in the centralized solution than it does in the decentralized solution (i.e., $TT(Q_c^*) > TT(Q_d^*)$). These examples differ in how the individual parties' average annual taxes change in the two solutions. For example, in Example 20, while $BT(Q_d^*) > BT(Q_c^*)$ and $VT(Q_d^*) < VT(Q_c^*)$, in Example 21, we have $BT(Q_d^*) < BT(Q_c^*)$ and $VT(Q_d^*) < VT(Q_c^*)$. The next three examples (those are Examples 23, 24, 25) illustrate the second part of Proposition 6. In these examples, government collects more taxes in the decentralized solution than it does in the centralized solution. Likewise, Examples 26, 27 and 28 illustrate the first part of Proposition 7. As it can be seen from Table 4.4, in these examples, government collects more taxes in the centralized solution. Finally, the second part of Proposition 7 is illustrated with Examples 29, 30 and 31, in which the average annual taxes collected by the government in the decentralized solution are more.

Since the average annual taxes collected by the government is linearly proportional to the total average annual emissions, the instances in which $TT(Q_c^*) > TT(Q_d^*)$ coincide with the cases where coordination is not good for the environment.

Table 4.3: Numerical Instances for Illustrating Analytical Results under the Tax Mechanism ($h_v = 1.5$, $c = 9$, $p_v = 6$, $e_b = 5$ and $e_v = 6$ in all instances)

Example Index	D	P	K_b	K_v	h_b	f_b	f_v	g_b	g_v	t_b	t_v	Q_d^*	Q_c^*
20	90	100	200	600	2	30	60	0.2	0.75	2	3	139.642	180.043
21	50	100	700	600	2	60	90	1	0.75	2	3	143.178	169.605
22	50	100	700	600	2	60	90	1	0.6	2	3	143.178	172.949
23	50	100	40	60	2	70	90	1	0.75	2	3	67.082	93.171
24	90	100	200	600	2	100	120	0.15	0.75	2	3	176.930	207.693
25	50	100	40	60	2	30	120	3	2	2	3	35.355	66.525
26	40	60	400	60	2	300	60	0.6	0.2	4	2	170.561	158.523
27	500	600	800	60	1.7	750	310	1	0.75	2	3	788.430	694.299
28	550	600	450	70	2	300	80	1.7	0.2	4	2	454.148	442.915
29	50	60	900	60	1.7	60	90	1	0.75	2	3	166.034	140.642
30	40	90	800	60	1.7	60	90	1	0.7	2	3	141.039	137.361
31	500	600	800	60	1.7	400	90	1	0.75	2	3	657.596	531.774

Table 4.4: Average Annual Taxes Resulting from the Decentralized and the Centralized Solutions of Instances in Table 4.3

Example Index	$BT\,(Q_d^*)$	$VT\,(Q_d^*)$	$TT\,(Q_d^*)$	$BT\,(Q_c^*)$	$VT\,(Q_c^*)$	$TT\,(Q_c^*)$
20	966.599	1877.399	2843.997	966.001	1892.272	2858.274
21	685.084	1074.826	1759.91	704.982	1075	1779.981
22	685.084	1058.718	1743.802	707.642	1055.885	1763.526
23	671.432	1138.98	1810.412	668.302	1097.303	1765.605
24	1028.275	1982.265	3010.539	1017.82	1986.289	3004.109
25	690.919	1462.15	2153.069	744.670	1270.363	2015.033
26	1286.098	530.884	1816.982	1293.023	531.416	1824.439
27	6739.688	10328.93	17068.62	6774.525	10320.65	17095.17
28	13997.37	6877.03	20874.4	13996.04	6879.885	20875.92
29	702.172	1136.966	1839.138	683.304	1127.84	1811.144
30	575.072	862.393	1437.465	572.305	862.727	1435.032
31	6265.872	9281.789	16087.66	6283.973	9752.405	16036.38

4.1.3 Coordination Mechanisms for the Two-Echelon System under Carbon Tax Mechanism

In this section, coordination mechanisms are proposed under carbon tax mechanism so that the buyer's loss from the centralized solution is compensated. Thus, the buyer is willing to order the optimal order quantity of the centralized model.

Let us define

$\overline{BC}(Q)$: Cost of the buyer after coordination as a function of order size Q

Theorem 12 *Suppose $Q_d^* < Q_c^*$. Also, suppose at least one of the following conditions holds.*

- $2f_bD - g_bQ_c^*Q_d^* \geqslant 0$
- $2f_vP - g_vQ_c^*Q_d^* \leqslant 0$

If $2f_bD - g_bQ_c^*Q_d^* \geqslant 0$ holds, $BT(Q_c^*) \leqslant BT(Q_d^*)$ from Proposition 4 as $Q_d^* < Q_c^*$. Thus, the buyer's average annual tax does not increase under centralized solution and he/she does not need to be compensated by paying his/her taxes.

Similarly, if $2f_vP - g_vQ_c^*Q_d^* \leqslant 0$ *holds,* $VT(Q_c^*) \geqslant VT(Q_d^*)$ *from from Proposition 5 as* $Q_d^* < Q_c^*$. *Thus, the vendor's average annual tax does not decrease under centralized solution and it is not plausible for the vendor to pay the buyer's taxes to compensate his/her loss.*

Then the following coordination mechanism minimizes the buyer's cost with a minimum function value of $BC(Q_d^*)$ *realized at* Q_c^*.

$$\overline{BC}(Q) = \begin{cases} BC(Q) & \text{if} \quad Q < Q_c^* \\ \\ BC(Q) - d \times D & \text{if} \quad Q \geqslant Q_c^* \end{cases}$$

where $d = \frac{BC(Q_c^*) - BC(Q_d^*)}{D}$ *is the unit discount given by the vendor to the buyer.*

That is, when $Q_d^* < Q_c^*$, *if the vendor gives a unit discount* d *for order sizes greater than or equal to* Q_c^* *to the buyer,* Q_c^* *coordinates the channel.*

Proof: See Appendix A.3.1 for the proof. ■

Next, we present a similar coordination mechanism, now for the case of $Q_d^* > Q_c^*$.

Theorem 13 *Suppose* $Q_d^* > Q_c^*$. *Also, suppose at least one of the following conditions holds.*

- $2f_bD - g_bQ_c^*Q_d^* \leqslant 0$
- $2f_vP - g_vQ_c^*Q_d^* \geqslant 0$

If $2f_bD - g_bQ_c^*Q_d^* \leqslant 0$ *holds,* $BT(Q_c^*) \leqslant BT(Q_d^*)$ *from Proposition 4 as* $Q_d^* > Q_c^*$. *Thus, the buyer's average annual tax does not increase under centralized solution and he/she does not need to be compensated by paying his/her taxes. Similarly, if* $2f_vP - g_vQ_c^*Q_d^* \geqslant 0$ *holds,* $VT(Q_c^*) \geqslant VT(Q_d^*)$ *from from Proposition 5 as* $Q_d^* > Q_c^*$. *Thus, the vendor's average annual tax does not decrease under* *centralized solution and it is not plausible for the vendor to pay the buyer's taxes to compensate his/her loss.*

Then the following coordination mechanism minimizes the buyer's cost with a minimum function value of $BC(Q_d^*)$ *realized at* Q_c^*.

$$\overline{BC}(Q) = \begin{cases} BC(Q) - d \times D & \text{if} \quad Q \leqslant Q_c^* \\ \\ BC(Q) & \text{if} \quad Q > Q_c^* \end{cases}$$

where $d = \frac{BC(Q_c^*) - BC(Q_d^*)}{D}$ *is the unit discount given by the vendor to the buyer.*

That is, when $Q_d^* > Q_c^*$, *if the vendor gives a unit discount d for order sizes less than or equal to* Q_c^* *to the buyer,* Q_c^* *coordinates the channel.*

Proof: Proof is similar to the proof of Theorem 12. $Q_d^* < Q_c^*$ is replaced with $Q_d^* > Q_c^*$, $Q < Q_c^*$ is replaced with $Q > Q_c^*$ and $Q \geqslant Q_c^*$ is replaced with $Q \leqslant Q_c^*$. ■

Theorem 12 and Theorem 13 have the following implication. When the buyer's average annual tax decreases (i.e., he/she does not need to be compensated by tax) and/or the vendor's average annual tax increases (i.e., it is not plausible for the vendor to pay buyer's tax to compensate his/her loss) under the centralized solution, channel coordination is achieved by giving quantity discount to the buyer.

Theorem 14 *Suppose* $\min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\} \geqslant BC(Q_c^*) - BC(Q_d^*)$. *That is, buyer's average annual tax increases and vendor's average annual tax decreases by an amount at least as large as the buyer's loss from the centralized solution.*

Hence, if $Q_d^* < Q_c^*$, then the following coordination mechanism minimizes the buyer's cost with a minimum function value of $BC(Q_d^*)$ realized at Q_c^*.

$$\overline{BC}(Q) = \begin{cases} BC(Q) & \text{if} \quad Q < Q_c^* \\ \\ BC(Q) + BC(Q_d^*) - BC(Q_c^*) & \text{if} \quad Q \geqslant Q_c^* \end{cases}$$

That is, when $Q_d^* < Q_c^*$, if the vendor pays $BC(Q_C^*) - BC(Q_d^*)$ amount of buyer's average annual tax for order sizes greater than or equal to Q_c^*, Q_c^* coordinates the channel.

Proof: See Appendix A.3.2 for the proof. ■

Next, we present a similar coordination mechanism, now for the case of $Q_d^* > Q_c^*$.

Theorem 15 *Suppose* $\min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\} \geqslant BC(Q_c^*) - BC(Q_d^*)$*. That is, buyer's average annual tax increases and vendor's average annual tax decreases by an amount at least as large as the buyer's loss from the centralized solution. Hence, if $Q_d^* > Q_c^*$, then the following coordination mechanism minimizes the buyer's cost with a minimum function value of $BC(Q_d^*)$ realized at Q_c^*.*

$$\overline{BC}(Q) = \begin{cases} BC(Q) + BC(Q_d^*) - BC(Q_c^*) & \text{if} \quad Q \leqslant Q_c^* \\ \\ BC(Q) & \text{if} \quad Q > Q_c^* \end{cases}$$

That is, when $Q_d^* > Q_c^*$, if the vendor pays $BC(Q_C^*) - BC(Q_d^*)$ amount of buyer's average annual tax for order sizes less than or equal to Q_c^*, Q_c^* coordinates the channel.

Proof: Proof is similar to the proof of Theorem 14. $Q_d^* < Q_c^*$ is replaced with $Q_d^* > Q_c^*$, $Q < Q_c^*$ is replaced with $Q > Q_c^*$ and $Q \geqslant Q_c^*$ is replaced with $Q \leqslant Q_c^*$. ■

Theorem 14 and Theorem 15 have the following implication. If the buyer's average annual tax increases and the vendor's average annual tax decreases and the decrease in the vendor's taxes is enough to compensate the buyer's loss from ordering the centralized optimal quantity, the vendor pays some amount of the buyer's taxes. The monetary value of the payment is equal to the difference between the buyer's costs in the centralized and the decentralized solutions.

Theorem 16 *Suppose $Q_d^* < Q_c^*$. Also, suppose the following conditions hold.*

- $2f_bD - g_bQ_c^*Q_d^* \leqslant 0$
- $2f_vP - g_vQ_c^*Q_d^* \geqslant 0$
- $\min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\} < BC(Q_c^*) - BC(Q_d^*)$

If $2f_bD - g_bQ_c^*Q_d^* \leqslant 0$ and $Q_d^* < Q_c^*$, $BT(Q_c^*) \geqslant BT(Q_d^*)$ from Proposition 4. Similarly, if $2f_vP - g_vQ_c^*Q_d^* \geqslant 0$ and $Q_d^* < Q_c^*$, $VT(Q_c^*) \leqslant VT(Q_d^*)$ from Proposition 5. Since $\min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\} \leqslant BC(Q_c^*) - BC(Q_d^*)$, either the vendor's gain in average annual taxes is not sufficient to compensate the buyer's loss in centralized solution or the buyer's loss is greater than his/her loss in average annual taxes in centralized solution.

Then the following coordination mechanism minimizes the buyer's cost with a minimum function value of $BC(Q_d^*)$ realized at Q_c^*.

$$\overline{BC}(Q) = \begin{cases} BC(Q) & \text{if} \quad Q < Q_c^* \\ BC(Q) - \bar{d} \times D - \min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\} & \text{if} \quad Q \geqslant Q_c^* \end{cases}$$

where $\bar{d} = [BC(Q_c^*) - BC(Q_d^*) - \min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\}]/D$ is the unit discount given and $\min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\}$ is the amount of buyer's average annual tax which is paid by the vendor.

That is, when $Q_d^* < Q_c^*$, if the vendor pays $\min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\}$ amount of vendor's annual tax and *gives a unit discount $\bar{d}$ for order sizes greater than or equal to Q_c^* to the buyer, Q_c^* coordinates the channel.*

Proof: See Appendix A.3.3 for the proof. ■

Next, we present a similar coordination mechanism, now for the case of $Q_d^* > Q_c^*$.

Theorem 17 *Suppose $Q_d^* > Q_c^*$. Also, suppose the following conditions hold.*

- $2f_bD - g_bQ_c^*Q_d^* \geqslant 0$
- $2f_vP - g_vQ_c^*Q_d^* \leqslant 0$
- $\min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\} < BC(Q_c^*) - BC(Q_d^*)$

If $2f_bD - g_bQ_c^*Q_d^* \geqslant 0$ and $Q_d^* > Q_c^*$, $BT(Q_c^*) \geqslant BT(Q_d^*)$ from Proposition 4. Similarly, if $2f_vP - g_vQ_c^*Q_d^* \leqslant 0$ and $Q_d^* > Q_c^*$, $VT(Q_c^*) \leqslant VT(Q_d^*)$ from Proposition 5. Since $\min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\} \leqslant BC(Q_c^*) - BC(Q_d^*)$, either the vendor's gain in average annual taxes is not sufficient to compensate the buyer's loss in centralized solution or the buyer's loss is greater than his/her loss in average annual taxes in centralized solution.

Then the following coordination mechanism minimizes the buyer's cost with a minimum function value of $BC(Q_d^*)$ realized at Q_c^*.

$$\overline{BC}(Q) = \begin{cases} BC(Q) - \bar{d} \times D - \min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if} \quad Q \leqslant Q_c^* \\ \\ BC(Q) \qquad \text{if} \quad Q > Q_c^* \end{cases}$$

where $\bar{d} = [BC(Q_c^*) - BC(Q_d^*) - \min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\}]/D$ is the unit discount given and $\min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\}$ is the amount of buyer's average annual tax which is paid by the vendor.

That is, when $Q_d^* < Q_c^*$, if the vendor pays $\min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\}$ *amount of vendor's annual tax and gives a unit discount $\bar{d}$ for order sizes less than or equal to Q_c^* to the buyer, Q_c^* coordinates the channel.*

Proof: Proof is similar to the proof of Theorem 16. $Q_d^* < Q_c^*$ is replaced with $Q_d^* > Q_c^*$, $Q < Q_c^*$ is replaced with $Q > Q_c^*$ and $Q \geqslant Q_c^*$ is replaced with $Q \leqslant Q_c^*$. ■

Theorem 16 and Theorem 17 have the following implication. If the buyer's average annual tax increases and the vendor's average annual tax decreases, the vendor pays some amount of the buyer's taxes. However, if the decrease in the vendor's taxes is not enough to compensate the buyer's loss from ordering the centralized optimal quantity or the increase in buyer's taxes is greater than his/her loss in average annual taxes, the remaining loss of the buyer is satisfied by quantity discount.

In Theorems 12 to 17, giving an amount more than the vendor's gain in taxes or the buyer's loss in taxes from the centralized solution to the buyer is not desired as it may not be practical in practice.

4.1.4 Numerical Analysis under Deterministic Demand and Carbon Tax Mechanism

In this section, coordination mechanisms proposed for carbon tax mechanism in Section 4.1.3 are illustrated with numerical examples. For this purpose, let us define the numerical instance in Table 4.5 in addition to the ones presented in Table 4.3. The decentralized and the centralized solutions of the instance and their resulting average annual taxes are presented in Table 4.6.

The classification of the examples illustrated in this section and their index numbers are presented in Table 4.7. Note here that the decentralized and the centralized solutions of the instances 20, 21, 23, 26, 29, 30 and 31 and their

Table 4.5: Additional Numerical Instance for Illustrating Coordination Mechanisms under the Tax Mechanism

Example Index	D	P	t_b	t_v	K_b	K_v	h_b	h_v	c	p_v	f_b	f_v	g_b	g_v	e_b	e_v
32	50	100	2	3	40	60	2	1.5	9	60	30	60	2	3	5	6

Table 4.6: The Decentralized and the Centralized Solutions and the Resulting Average Annual Taxes for the Instance in Table 4.5

Example Index	Q_d^*	$BT(Q_d^*)$	$VT(Q_d^*)$	$TT(Q_d^*)$
32	35.3553	690.9188	1207.5915	1898.5103
	Q_c^*	$BT(Q_c^*)$	$VT(Q_c^*)$	$TT(Q_c^*)$
	53.7924	717.1471	1207.5915	1865.1456

resulting average annual taxes are presented in Table 4.4 in Section 4.1.2. During the remaining of our analysis in this section, the examples are arranged in order based on their index number.

Table 4.7: Classification of Examples Illustrated for Coordination Mechanisms under Deterministic Demand and Carbon Tax Mechanism

Q_d^* **vs** Q_c^*	$BT(Q_d^*)$ **vs** $BT(Q_c^*)$	$VT(Q_d^*)$ **vs** $VT(Q_c^*)$	**Example Index**
$Q_d^* < Q_c^*$	$BT(Q_d^*) > BT(Q_c^*)$	$VT(Q_d^*) > VT(Q_c^*)$	23
$Q_d^* > Q_c^*$	$BT(Q_d^*) > BT(Q_c^*)$	$VT(Q_d^*) > VT(Q_c^*)$	29
$Q_d^* < Q_c^*$	$BT(Q_d^*) > BT(Q_c^*)$	$VT(Q_d^*) < VT(Q_c^*)$	20
$Q_d^* > Q_c^*$	$BT(Q_d^*) > BT(Q_c^*)$	$VT(Q_d^*) < VT(Q_c^*)$	30
$Q_d^* < Q_c^*$	$BT(Q_d^*) < BT(Q_c^*)$	$VT(Q_d^*) < VT(Q_c^*)$	21
$Q_d^* > Q_c^*$	$BT(Q_d^*) < BT(Q_c^*)$	$VT(Q_d^*) < VT(Q_c^*)$	26
$Q_d^* < Q_c^*$	$BT(Q_d^*) < BT(Q_c^*)$	$VT(Q_d^*) > VT(Q_c^*)$	32
$Q_d^* > Q_c^*$	$BT(Q_d^*) < BT(Q_c^*)$	$VT(Q_d^*) > VT(Q_c^*)$	31

The costs of the buyer, vendor and the system under centralized and the decentralized solutions for each example illustrated in this section are presented in Table 4.9. Also, the values of the expressions that indicate the increase or decrease in buyer's and vendor's taxes can be seen in Table 4.8. The application of coordination mechanisms proposed in Section 4.1.3 to each example is described in Appendix A.4 and summarized in Tables A.3 and A.4 in Appendix A.4.

Table 4.8: $2f_bD - g_bQ_c^*Q_d^*$ and $2f_vP - g_vQ_c^*Q_d^*$ Values of the Examples Illustrated for Coordinated Mechanisms under Deterministic Demand and Tax Mechanism

Example	$2f_bD - g_bQ_c^*Q_d^*$	$2f_vP - g_vQ_c^*Q_d^*$
20	371.6647045	−6856.257358
21	−18283.78322	−212.8374178
23	749.8936179	13312.42021
26	7777.377649	1792.459216
29	−17351.49985	−6713.624888
30	−14573.14417	2638.799083
31	50307.38886	−154269.4584
32	−2705.54042	8196.306386

Table 4.9: Costs of the Examples Illustrated for Coordinated Mechanisms under Deterministic Demand and Tax Mechanism Resulting from the Decentralized and Centralized Solutions

Example	$BC(Q_d^*)$	$VC(Q_d^*)$	$TC(Q_d^*)$	$BC(Q_c^*)$	$VC(Q_c^*)$	$TC(Q_c^*)$
20	2045.1418	2898.3592	4943.501	2056.0207	2853.7293	4909.75
21	1522.7128	1638.0468	3160.7596	1530.9483	1615.4828	3146.4311
23	1218.3282	1508.8569	2727.185	1232.9387	1464.4415	2697.3801
26	1910.4665	150.2354	2060.702	1912.4776	145.817	2058.2947
29	1564.3289	1558.8062	3123.135	1572.8104	1537.0724	3109.8828
30	1281.8429	1166.4223	2448.2652	1282.02501	1165.9859	2448.011
31	11933.105	13278.4076	25211.5126	11988.1801	13141.1793	25129.3595
32	1232.8427	1605.7025	2838.5452	1258.1194	1523.9407	2782.0601

4.2 Problem Definition and Analysis under Deterministic Demand and Carbon Cap Mechanism

4.2.1 Problem Definition under Deterministic Demand and Carbon Cap Mechanism

We consider the inventory problem introduced earlier, and now, we assume the existence of a cap mechanism. Under the carbon cap mechanism, the buyer and the vendor have carbon caps that are upper limits on the average annual carbon emissions. In the decentralized model, buyer decides the order quantity that minimizes his/her cost subject to the carbon emission constraints. In his/her independent decisions, if the buyer does not give any consideration to the vendor's emission constraint, then he/she may end up with an order quantity that is infeasible for the vendor to supply. Since this scenario is not desirable by the vendor, it is simply natural for the vendor to make his/her constraints be known by the buyer. Therefore, we assume that vendor passes the information regarding the feasible lot sizes he/she can provide to the buyer at each delivery before the buyer makes an ordering decision. In the centralized model, the order quantity that minimizes the total average annual cost of the system is minimized subject to both parties' constraints.

The notation used under carbon cap mechanism is similar to the notation used under carbon tax policy in Section 4.1.1, which could be seen in Table 4.1. The only difference is due to the policy parameters which are listed below.

Table 4.10: Additional Notation for Carbon Cap Mechanism under Deterministic Demand

Policy Parameters	
C_b	buyer's annual carbon emission cap
C_v	vendor's annual carbon emission cap

In the decentralized model, buyer implicitly solves the following problem:

$$min \quad BC(Q) = \frac{K_b D}{Q} + \frac{h_b Q}{2} + cD \tag{4.11}$$

$$s.t.$$

$$\frac{f_b D}{Q} + \frac{g_b Q}{2} + e_b D \leqslant C_b \tag{4.12}$$

$$\frac{f_v D}{Q} + \frac{g_v D Q}{2P} + e_v D \leqslant C_v \tag{4.13}$$

$$Q \geq 0.$$

Expression (4.13) is, in fact, the vendor's emission constraint. However, the buyer in this setting does not know vendor's specific parameters such as f_v, g_v, e_v, C_v. Instead, the vendor tells the buyer it is only possible for him/her to supply Q such that $Q_3 \leq Q \leq Q_4$, where Q_3 and Q_4 are the two quantities that satisfy $\frac{f_v D}{Q} + \frac{g_v DQ}{2P} + e_v D = C_v$. Mainly, all Q within the interval $[Q_3, Q_4]$ satisfy the vendor's emission constraint, and hence, feasible for the vendor to deliver. Similarly, Expression (4.12) implies that it is only feasible for the buyer to order Q, where $Q \in [Q_1, Q_2]$. Again, Q_1 and Q_2 are the two quantities that satisfy $\frac{f_b D}{Q} + \frac{g_b Q}{2} + e_b D = C_b$. Specific expressions for Q_1, Q_2, Q_3 and Q_4 are as given below:

$$Q_1 = \frac{C_b - e_b D - \sqrt{(e_b D - C_b)^2 - 2 g_b f_b D}}{g_b}, \tag{4.14}$$

$$Q_2 = \frac{C_b - e_b D + \sqrt{(e_b D - C_b)^2 - 2 g_b f_b D}}{g_b}, \tag{4.15}$$

$$Q_3 = \frac{P(C_v - e_v D) - \sqrt{[P(e_v D - C_v)]^2 - 2 g_v f_v P D^2}}{g_v D}, \tag{4.16}$$

$$Q_4 = \frac{P(C_v - e_v D) + \sqrt{[P(e_v D - C_v)]^2 - 2 g_v f_v P D^2}}{g_v D}. \tag{4.17}$$

The decentralized model then turns out to be:

$$min \quad BC(Q) = \frac{K_b D}{Q} + \frac{h_b Q}{2} + cD \tag{4.18}$$

$$s.t. \quad Q \geqslant \max\{Q_1, Q_3\} \tag{4.19}$$

$$Q \leqslant \min\{Q_2, Q_4\} \tag{4.20}$$

$$Q \geq 0.$$

Here, it is assumed that $\max\{Q_1, Q_3\} \leqslant \min\{Q_2, Q_4\}$ in order for the feasible region not to be empty.

Vendor's average annual cost as a function of Q is given by

$$VC(Q) = \frac{K_v D}{Q} + \frac{h_v DQ}{2P} + p_v D. \tag{4.21}$$

In the centralized model, the order quantity that minimizes the total cost of the system (i.e, the total cost of the buyer and the vendor) is determined. In mathematical terms, the following problem is solved.

$$min \quad TC(Q) = \frac{(K_b + K_v)D}{Q} + \left(h_b + \frac{h_v D}{P}\right)\frac{Q}{2} + (p_v + c)D \tag{4.22}$$

$$s.t. \quad Q \geqslant \max\{Q_1, Q_3\} \tag{4.23}$$

$$Q \leqslant \min\{Q_2, Q_4\} \tag{4.24}$$

$$Q \geq 0.$$

4.2.2 Analysis of the Decentralized Model and the Centralized Model under Deterministic Demand and Carbon Cap Mechanism

In this section, we provide an analysis of the decentralized model and the centralized model under carbon cap mechanism to find Q_d^* and Q_c^*. We further present some properties related to Q_d^* and Q_c^*.

We know from Expression (4.11) that the optimal order quantity under the decentralized model when the emission constraints are not considered (i.e., Q_d^0, namely, “the cost optimal order quantity” under the decentralized model) is

$$Q_d^0 = \sqrt{\frac{2K_b D}{h_b}}. \tag{4.25}$$

Since Expression (4.11) is a strictly convex function in Q, Q_d^0 is obtained from the first order condition.

Using the same reasoning, the emission optimal quantity is presented in the next lemma.

Lemma 19 *The order quantity that minimizes the total average annual emissions of the system (i.e., $\hat{Q}_d$, namely, "the emission optimal order quantity" under the decentralized model) is given by*

$$\hat{Q}_d = \sqrt{\frac{2(f_b + f_v)D}{g_b + \frac{g_v D}{P}}}. \tag{4.26}$$

Proof: Total average annual emissions of the system is obtained by summing the left hand sides of the Equations (4.12) and (4.13). Since the sum of two convex functions is also a convex function, $\hat{Q}_d$ is obtained from the first order condition. ■

Next, we provide the solution of the buyer's replenishment problem under carbon capacity constraints.

Theorem 18 *The optimal order quantity resulting from the decentralized model is given by*

$$Q_d^* = \begin{cases} \max\{Q_1, Q_3\} & \text{if} \quad Q_d^0 < \max\{Q_1, Q_3\} \\ Q_d^0 & \text{if} \quad \max\{Q_1, Q_3\} \leqslant Q_d^0 \leqslant \min\{Q_2, Q_4\} \\ \min\{Q_2, Q_4\} & \text{if} \quad Q_d^0 > \min\{Q_2, Q_4\} \end{cases} \tag{4.27}$$

Proof: From constraints (4.19) and (4.20), the optimal order quantity resulting from the decentralized model (i.e., Q_d^*) should satisfy $\max\{Q_1, Q_3\} \leqslant Q_d^* \leqslant \min\{Q_2, Q_4\}$. Hence, if $\max\{Q_1, Q_3\} \leqslant Q_d^0 \leqslant \min\{Q_2, Q_4\}$, $Q_d^* = Q_d^0$. If $Q_d^0 \leqslant \max\{Q_1, Q_3\}$, $Q_d^* = \max\{Q_1, Q_3\}$ since Expression (4.11) is a strictly convex function in Q and a lower value than $\max\{Q_1, Q_3\}$ will result in a higher cost. Similarly, if $Q_d^0 \geqslant \min\{Q_2, Q_4\}$, $Q_d^* = \min\{Q_2, Q_4\}$ since (4.11) is a strictly convex function in Q and a higher value than $\min\{Q_2, Q_4\}$ will result in a higher cost. ■

The average annual cost of the buyer ($BC(Q_d^*)$) and the vendor ($VC(Q_d^*)$) under the optimal solution of the decentralized model are obtained by plugging Expression (4.27) in Expressions (4.11) and (4.21), respectively. Also, the total average annual cost of the system under the optimal solution of the decentralized model is $TC(Q_d^*) = BC(Q_d^*) + VC(Q_d^*)$.

As $BC(Q)$ and $TC(Q)$ have identical structural properties and the constraints of the decentralized model and the centralized model are the same, a similar analysis can be done for the centralized model. As a result, the cost optimal order quantity, the emission optimal order quantity and the optimal solution under the centralized model are presented in Lemma 20, Lemma 21 and Theorem 19, respectively.

Lemma 20 *The optimal order quantity under the centralized model when emissions are not considered (i.e., Q_c^0, namely, "the cost optimal order quantity" under the centralized model) is given by*

$$Q_c^0 = \sqrt{\frac{2(K_b + K_v)D}{h_b + \frac{h_v D}{P}}}. \tag{4.28}$$

Proof: Since $TC(Q)$ is a strictly convex function in Q, Q_c^0 is obtained from the first order condition. ■

Lemma 21 *The order quantity that minimizes the total average annual emissions of the system (i.e., $\hat{Q}_c$, namely, "the emission optimal order quantity" under the centralized model) is given by*

$$\hat{Q}_c = \sqrt{\frac{2(f_b + f_v)D}{g_b + \frac{g_v D}{P}}}. \tag{4.29}$$

Proof: The proof is similar to the proof of Lemma 19 and is omitted. ■

Theorem 19 *The optimal order quantity resulting from the centralized model is given by*

$$Q_c^* = \begin{cases} \max\{Q_1, Q_3\} & \text{if} \quad Q_c^0 < \max\{Q_1, Q_3\} \\ Q_c^0 & \text{if} \quad \max\{Q_1, Q_3\} \leqslant Q_c^0 \leqslant \min\{Q_2, Q_4\} \\ \min\{Q_2, Q_4\} & \text{if} \quad Q_c^0 > \min\{Q_2, Q_4\} \end{cases} \tag{4.30}$$

Proof: Proof is similar to the proof of Theorem 18 and is omitted. ■

The average annual cost of the system under the optimal solution of the centralized model is obtained by plugging Expression (4.30) in the expression for $TC(Q)$. Similarly, the average annual cost of the buyer ($BC(Q_c^*)$) and the vendor ($VC(Q_c^*)$) under the optimal solution of the centralized model are obtained by plugging Expression (4.30) in Expressions (4.11) and (4.21), respectively.

We provide a further property of Q_d^* and Q_c^* in Proposition 8 presented below.

Proposition 8 $Q_d^* \leqslant Q_c^*$ *if one of the following conditions holds:*

i) $\frac{K_b}{K_v} \leq \frac{h_b P}{h_v D}$,

ii) $\frac{K_b}{K_v} > \frac{h_b P}{h_v D}$ *and* $Q_c^0 \geq \min\{Q_2, Q_4\}$,

iii) $\frac{K_b}{K_v} > \frac{h_b P}{h_v D}$ *and* $Q_d^0 \leq \max\{Q_1, Q_3\}$.

Proof:

i) Due to the fact that $\frac{K_b}{K_v} \leq \frac{h_b P}{h_v D}$, Expressions (4.25) and (4.28) jointly imply that $Q_d^0 \leq Q_c^0$. Now let us consider the three possible regions that Q_c^0 could fall into, which lead to different realizations of Q_c^*, as given by Theorem 19. Let us first assume that $Q_c^0 < \max\{Q_1, Q_3\}$. In this case, Theorem 19 implies $Q_c^* = \max\{Q_1, Q_3\}$. Since $Q_d^0 \leq Q_c^0$, it follows that $Q_d^0 < \max\{Q_1, Q_3\}$, which in turn leads us to conclude that $Q_d^* = \max\{Q_1, Q_3\}$ based on Theorem 18. Therefore, if $Q_c^0 < \max\{Q_1, Q_3\}$, we have $Q_c^* = Q_d^*$. Now, let us consider the second possible region that Q_c^0 could fall into; that is, let us assume $\max\{Q_1, Q_3\} \leq Q_c^0 \leq \min\{Q_2, Q_4\}$ so that $Q_c^* = Q_c^0$. Since $Q_d^0 \leq Q_c^0$, there are two possible realizations of Q_d^* depending on whether $\max\{Q_1, Q_3\} \leq Q_d^0 \leq Q_c^0$ or $Q_d^0 < \max\{Q_1, Q_3\}$. Theorem 18 implies that $Q_d^* = Q_d^0$ in the former case, and $Q_d^* = \max\{Q_1, Q_3\}$ in the latter. Recall that, if $\max\{Q_1, Q_3\} \leq Q_c^0 \leq \min\{Q_2, Q_4\}$, we have $Q_c^* = Q_c^0 \geq \max\{Q_1, Q_3\}$. Therefore, for both realizations of Q_d^*, it turns out that $Q_c^* \geq Q_d^*$. Lastly, let us consider the third possible region that Q_c^0 could fall into; that is, let us assume $Q_c^0 > \min\{Q_2, Q_4\}$. Theorem 19 implies that $Q_c^* = \min\{Q_2, Q_4\}$ in this case. Given $Q_d^0 \leq Q_c^0$, one the following can happen: $Q_c^0 \geq Q_d^0 > \min\{Q_2, Q_4\}$, $\min\{Q_2, Q_4\} \geq Q_d^0 \geq \max\{Q_1, Q_3\}$, $Q_d^0 < \max\{Q_1, Q_3\}$. Theorem 18 further implies that, $Q_d^* = \min\{Q_2, Q_4\}$ in the first case, $Q_d^* = Q_d^0$ in the second case, and $Q_d^* = \max\{Q_1, Q_3\}$ in the third case. Therefore, in all cases, it turns out that $Q_c^* \geq Q_d^*$.

ii) Due to the fact that $\frac{K_b}{K_v} > \frac{h_b P}{h_v D}$, Expressions (4.25) and (4.28) jointly imply that $Q_d^0 > Q_c^0$. Since $Q_c^0 \geq \min\{Q_2, Q_4\}$, it turns out that $Q_d^0 > \min\{Q_2, Q_4\}$. When Q_c^0 and Q_d^0 are greater than $\min\{Q_2, Q_4\}$, Theorem 18 and Theorem 19 help us to conclude that $Q_c^* = Q_d^* = \min\{Q_2, Q_4\}$.

iii) Since $\frac{K_b}{K_v} > \frac{h_b P}{h_v D}$, we again have $Q_d^0 > Q_c^0$. In addition, due to Theorem 18, having $Q_d^0 \leq \max\{Q_1, Q_3\}$ implies $Q_d^* = \max\{Q_1, Q_3\}$. Combining $Q_d^0 > Q_c^0$ with $Q_d^0 \leq \max\{Q_1, Q_3\}$ leads to $Q_c^0 < \max\{Q_1, Q_3\}$, which further implies $Q_c^* = \max\{Q_1, Q_3\}$ due to Theorem 19. Therefore, it turns out that $Q_c^* = Q_d^*$.

■

4.2.3 Coordination Mechanisms for the Two-Echelon System under Carbon Cap Mechanism

Similar to the coordination mechanisms proposed under carbon tax mechanism, coordination mechanisms are proposed under carbon cap mechanism so that the buyer's loss from the centralized solution is compensated. Since carbon trade is not allowed and a carbon tax is not paid to an external authority under carbon cap mechanism, the vendor cannot compensate the buyer's loss under the centralized solution by paying his/her expenses resulting from environmental regulations. Thus, the channel coordination is achieved by quantity discounts given by the vendor to the buyer under carbon cap mechanism. The notation used in this section is the same as the notation used in Section 4.1.3.

Theorem 20 *Suppose* $Q_d^* < Q_c^*$ *holds. Then the following coordination mechanism minimizes the buyer's cost with a minimum function value of* $BC(Q_d^*)$ *realized at* Q_c^*.

$$\overline{BC}(Q) = \begin{cases} BC(Q) & \text{if} \quad Q < Q_c^* \\ \\ BC(Q) - d \times D & \text{if} \quad Q \geqslant Q_c^* \end{cases}$$

where $d = \frac{BC(Q_c^*) - BC(Q_d^*)}{D}$ *is the unit discount given by the vendor to the buyer.*

That is, when $Q_d^* < Q_c^*$*, if the vendor gives a unit discount* d *for order sizes greater than or equal to* Q_c^* *to the buyer,* Q_c^* *coordinates the channel.*

Proof: See Appendix A.5.1 for the proof. ■

Next, we present a similar coordination mechanism, now for the case of $Q_d^* > Q_c^*$.

Theorem 21 *Suppose* $Q_d^* > Q_c^*$ *holds. Then the following coordination mechanism minimizes the buyer's cost with a minimum function value of* $BC(Q_d^*)$ *realized at* Q_c^*.

$$\overline{BC}(Q) = \begin{cases} BC(Q) - d \times D & \text{if} \quad Q \leqslant Q_c^* \\ \\ BC(Q) & \text{if} \quad Q > Q_c^* \end{cases}$$

where $d = \frac{BC(Q_c^*) - BC(Q_d^*)}{D}$ *is the unit discount given by the vendor to the buyer.*

That is, when $Q_d^* > Q_c^*$, if the vendor gives a unit discount d for order sizes less than or equal to Q_c^* to the buyer, Q_c^* coordinates the channel.

Proof: Proof is similar to the proof of Theorem 20. $Q_d^* < Q_c^*$ is replaced with $Q_d^* > Q_c^*$, $Q < Q_c^*$ is replaced with $Q > Q_c^*$ and $Q \geqslant Q_c^*$ is replaced with $Q \leqslant Q_c^*$. ■

4.2.4 Numerical Analysis under Deterministic Demand and Carbon Cap Mechanism

In this section sixteen numerical examples are illustrated for carbon cap policy. In each example the optimal order sizes of coordinated and uncoordinated models are calculated and the coordination mechanisms proposed in Section 4.2.3 are applied. Each example corresponds to a specific case as seen in Table 4.11. Let us define $Q_{cap1} = \max\{Q_1, Q_3\}$ and $Q_{cap2} = \min\{Q_2, Q_4\}$.

Here, in the examples where Q_d^* and Q_c^* are the same, one of $Q_d^* \leqslant Q_c^*$ and $Q_d^* \geqslant Q_c^*$ is satisfied only at equality. Also, the relationships between $BC(Q_d^*)$ and $BC(Q_c^*)$, $VC(Q_d^*)$ and $VC(Q_c^*)$, and $TC(Q_d^*)$ and $TC(Q_c^*)$ are not considered. Since $BC(Q)$ is a strictly convex function and the constraints of the decentralized and the centralized models are the same, we have $BC(Q_d^*) \leqslant BC(Q_c^*)$. Also, $VC(Q_c^*) \leqslant VC(Q_d^*)$ as the total cost decreases and cost of the buyer does not decrease under the centralized solution.

As stated in Section 4.2.3, since carbon trade is not allowed and a carbon tax is not paid to an external authority under carbon cap mechanism, the vendor cannot compensate the buyer's loss under the centralized solution by paying his/her expenses resulting from environmental regulations. Thus, the channel coordination is achieved by quantity discounts given by the vendor to the buyer.

The values of the parameters are presented in Tables 4.12 and 4.13. Also, the results of the centralized and the decentralized solutions for each example are presented Tables 4.14 and 4.15. The application of coordination mechanisms

Table 4.11: Classification of Examples for Carbon Cap Mechanism under Deterministic Demand

Example Index	Q_d^*	Q_c^*	Q_d^* **vs** Q_c^*
33	Q_{cap1}	Q_{cap1}	$Q_d^* = Q_c^*$
34	Q_{cap1}	Q_c^0	$Q_d^* < Q_c^*$
35	Q_{cap1}	Q_c^0	$Q_d^* = Q_c^*$
36	Q_{cap1}	Q_{cap2}	$Q_d^* < Q_c^*$
37	Q_{cap1}	Q_{cap2}	$Q_d^* = Q_c^*$
38	Q_d^0	Q_{cap1}	$Q_d^* = Q_c^*$
39	Q_d^0	Q_{cap1}	$Q_d^* > Q_c^*$
40	Q_d^0	Q_c^0	$Q_d^* < Q_c^*$
41	Q_d^0	Q_c^0	$Q_d^* > Q_c^*$
42	Q_d^0	Q_{cap2}	$Q_d^* < Q_c^*$
43	Q_d^0	Q_{cap2}	$Q_d^* = Q_c^*$
44	Q_{cap2}	Q_{cap1}	$Q_d^* = Q_c^*$
45	Q_{cap2}	Q_{cap1}	$Q_d^* > Q_c^*$
46	Q_{cap2}	Q_c^0	$Q_d^* = Q_c^*$
47	Q_{cap2}	Q_c^0	$Q_d^* > Q_c^*$
48	Q_{cap2}	Q_{cap2}	$Q_d^* = Q_c^*$

proposed in Section 4.2.3 to each example is described in Appendix A.6 and summarized in Table A.5 in Appendix A.6. In examples where some fields are marked with a minus (-) in Table A.5, the channel is already coordinated and $Q_d^* = Q_c^*$ as described in the previous paragraphs.

Table 4.12: Parameter Values of Examples 33-40

	Eg 33	**Eg 34**	**Eg 35**	**Eg 36**	**Eg 37**	**Eg 38**	**Eg 39**	**Eg 40**
D	50	50	50	50	50	50	50	50
P	150	150	150	150	150	55	55	150
K_b	40	40	21.25	45	45	231.05077	250	110
K_v	60	60	41.25	350	350	40	40	120
h_b	2	2	2	2	2	1.2	1.2	2
h_v	1.5	1.5	1.5	1.5	1.5	1	1	1.5
c	12	12	12	12	12	12	12	12
p_v	8	8	8	8	8	8	8	8
f_b	50	50	50	50	50.00017	90	90	50
f_v	60	60	60	70	70.00015	95	95	60
g_b	1	1	1	1	1.01667	0.5	0.5	1
g_v	0.75	0.75	0.75	0.75	0.75415	0.25	0.25	0.75
e_b	5.5	5.5	5.5	5.5	5.51667	5	5	5.5
e_v	7	6	6	7	7.01376	6	6	7
C_b	350	350	350	350	349.99967	320	320	350
C_v	400	400	400	400	399.99972	350	350	400

Table 4.13: Parameter Values of Examples 41-48

	Eg 41	**Eg 42**	**Eg 43**	**Eg 44**	**Eg 45**	**Eg 46**	**Eg 47**	**Eg 48**
D	50	50	50	50	50	50	50	50
P	55	150	150	96	100	55	55	150
K_b	400	200	250	150	150	492.64428	420	400
K_v	50	600	600	20	20	102.64429	30	300
h_b	1	2.5	2.5	2	2	1.2	1.2	1.5
h_v	0.8	2.3	2.3	1.8	1.8	1.1	1.1	1.2
c	12	12	12	12	12	12	12	12
p_v	8	8	8	8	8	8	8	8
f_b	40	50	50	56	56	95	95	50
f_v	135	60	60	55	55	100	100	60
g_b	2	1	0.5	1	1	0.5	0.5	1
g_v	2.5	0.75	0.75	0.75	0.75	0.25	0.25	0.75
e_b	5	5.5	5.5	5.5	5.5	5	5	5.5
e_v	7	7	7	7	7	6	6	7
C_b	550	350	350	350	350	320	320	360
C_v	600	400	400	400	400	400	400	400

Table 4.14: Solutions of the Decentralized Model for Examples 33-48

Example	Q_d^0	Q_{cap1}	Q_{cap2}	Q_d^*	$BC(Q_d^*)$	$VC(Q_d^*)$	$TC(Q_d^*)$
33	44.7214	73.5089	100	73.5089	700.7165	459.1886	1159.9051
34	44.7214	50	100	50	690	472.5	1162.5
35	32.596	50	100	50	671.25	453.75	1125
36	47.4342	90.4555	100	90.4555	715.3296	616.0792	1331.4088
37	47.4342	93.0437	93.0437	93.0437	717.2259	611.3446	1328.5705
38	138.7596	138.7596	180	138.7596	766.5115	477.486	1243.9975
39	144.3376	138.7596	180	144.3376	773.2051	479.4644	1252.6695
40	74.162	73.5089	100	74.162	748.324	499.4445	1247.7684
41	200	31.5143	188.4857	188.4857	800.3517	481.8039	1282.1555
42	89.4427	73.5089	100	89.4427	823.6068	769.6966	1593.3034
43	100	73.5089	100	100	850	738.3333	1588.3333
44	86.6025	80	80	80	773.75	450	1223.75
45	86.6025	77.556	80	80	773.75	448.5	1222.25
46	202.6171	115.5051	164.4949	164.4949	848.4415	513.4473	1361.8888
47	187.0829	115.5051	164.4949	164.4949	826.3605	491.3663	1317.7267
48	163.2993	73.5089	132.1699	132.1699	850.44781	539.9243	1390.3721

Table 4.15: Solutions of the Centralized Model for Examples 33-48

Example	Q_c^0	Q_{cap1}	Q_{cap2}	Q_c^*	$BC(Q_c^*)$	$VC(Q_c^*)$	$TC(Q_c^*)$
33	63.2456	73.5089	100	73.5089	700.7165	459.1886	1159.9051
34	63.2456	50	100	63.2456	694.8683	463.2456	1158.1139
35	50	50	100	50	671.25	453.75	1125
36	125.6981	90.4555	100	100	722.5	600	1322.5
37	125.6981	93.0437	93.0437	93.0437	717.2259	611.3446	1328.5704
38	113.3647	138.7596	180	138.7596	766.5115	477.486	1243.9975
39	117.2604	138.7596	180	138.7596	773.3396	477.486	1250.8256
40	95.9166	73.5089	100	95.9166	753.2581	486.5335	1239.7916
41	161.4083	31.5143	188.4851	161.4083	804.6135	474.1826	1278.7961
42	156.4922	73.5089	100	100	825	738.3333	1563.3333
43	161.3084	73.5089	100	100	850	738.3333	1588.3333
44	76.0739	80	80	80	773.75	450	1223.75
45	76.5641	77.556	80	77.556	774.2603	447.7941	1222.0544
46	164.4949	115.5051	164.4949	164.4949	848.4415	513.4473	1361.8888
47	143.0194	115.5051	164.4949	143.0194	832.6449	481.9978	1314.4265
48	191.9428	73.5089	132.1699	132.1699	850.4478	539.9243	1390.3721

Chapter 5

Problem Definition and Analysis under Stochastic Demand

5.1 Problem Definition under Stochastic Demand

In this section, we consider a system which consists of a buyer (retailer) and a vendor (manufacturer). The buyer operates to meet the random demand of a perishable product and the vendor simply makes the single order of the buyer available. The single period problem (i.e., the newsvendor problem) is used as a benchmark and this model is modified under carbon tax, cap-and-trade and carbon cap mechanisms. There is no order lead time under each policy. The buyer earns a unit revenue per product sold and incurs a cost per product purchased. He/she salvages each product that is not sold at a salvage value. The vendor earns a unit revenue and incurs a manufacturing cost per product purchased by the buyer. It is assumed that the shortage cost is incurred only by the buyer. For ease of modeling and analysis, the emissions-related parameters of the buyer and the vendor are measured on the product-unit basis. The notation used is summarized in Table 5.1.

Table 5.1: Problem Parameters and Decision Variables under Stochastic Demand

Parameters of the Buyer and the Vendor	
X:	Aggregate demand, which is a random variable with probability density function $f(x)$ and cumulative distribution function $F(x)$
p:	Selling price per unit at the buyer
c_v:	Manufacturing cost per item
c_b:	Price per unit paid by the buyer to the vendor
v:	Salvage value per unit
s:	Stockout cost incurred per unit by the buyer
e_b:	Variable emission amount released per unit ordered by the buyer
e_v:	Variable emission amount released per unit produced by the vendor

Policy Parameters	
t_b:	Emissions tax amount paid by the buyer per item ordered
t_v:	Emissions tax amount paid by the vendor per item produced
t_t:	Total emissions tax amount per item $(= t_b + t_v)$
p_c:	Carbon price per unit at the market
C_b:	Fixed carbon capacity of the buyer
C_v:	Fixed carbon capacity of the vendor
Q_b:	Carbon cap of the buyer
Q_v:	Carbon cap of the vendor

Decision Variables	
Q:	Order size
X_b:	Amount of carbon credits purchased/sold by the buyer
X_v:	Amount of carbon credits purchased/sold by the vendor

Functions and Optimal Values of Decision Variables	
$J_b(Q)$:	Expected profit of the buyer as a function of order size Q
$J_v(Q)$:	Expected profit of the vendor as a function of order size Q
$J_t(Q)$:	Total expected profit of the system as a function of order size Q
Q_d^*:	The optimal order quantity under the decentralized model
Q_c^*:	The optimal order quantity under the centralized model

Parameters Related to Coordination	
d:	Unit discount offered by the vendor to the buyer
d_c:	Unit carbon price discount offered by the vendor to the buyer

Since the emissions-related parameters of the buyer and the vendor are measured on the product-unit basis, $Q_b = C_b/e_b$ and $Q_v = C_v/e_v$ correspond to the carbon cap of the buyer and the vendor, respectively, on the product-unit basis.

In Sections 5.1.1, 5.1.2 and 5.1.3, we present the model formulations under carbon tax, cap-and-trade and carbon cap mechanisms, respectively.

5.1.1 Model Formulation under Stochastic Demand and Carbon Tax Mechanism

In this section, we use the newsvendor problem as a benchmark and update the model to account for carbon taxes as the emission regulating mechanism.

Under carbon tax mechanism, it is assumed that the shortage cost is incurred only by the buyer. The following properties are assumed to hold.

$$\begin{aligned} v < c_v + t_v < c_b \\ c_b + t_b < p \end{aligned} \tag{5.1}$$

In the decentralized model, the buyer decides the order quantity that maximizes his/her expected profit. Then the modified newsvendor model under the carbon tax mechanism is given by

$$\begin{aligned} max \quad J_b(Q) = -(c_b+t_b)Q + \int_0^Q [px+(Q-x)v]f(x)\,\mathrm{d}x + \int_Q^\infty [pQ-(x-Q)s]f(x)\,\mathrm{d}x \\ Q \geq 0. \end{aligned} \tag{5.2}$$

Similar to Expression (5.2), the decentralized expected profit of the vendor is given by

$$J_v(Q) = (c_b - c_v - t_v)Q. \tag{5.3}$$

In the centralized model, the order quantity that maximizes the total profit of the system (i.e, the total profit of the buyer and the vendor) is determined. Then, the modified newsvendor model under carbon tax mechanism is given by

$$max \quad J_t(Q) = -(c_v+t_t)Q + \int_0^Q [px+(Q-x)v]f(x)\,\mathrm{d}x + \int_Q^\infty [pQ-(x-Q)s]f(x)\,\mathrm{d}x \tag{5.4}$$

$$Q \geq 0.$$

5.1.2 Model Formulation under Stochastic Demand and Cap-and-Trade Mechanism

In this section, we provide the modified newsvendor problem that incorporates cap-and-trade system as the emission regulating mechanism. The additional notation used in this section is given by

Table 5.2: Additional Notation for Cap and Trade System under Stochastic Demand

$J_b(Q, X_b)$:	Expected profit of the buyer as a function of order size Q and his/her emission transfer quantity X_b
$J_t(Q, X_b, X_v)$:	Total expected profit of the system as a function of order size Q, buyer's emission transfer quantity X_b and vendor's emission transfer quantity X_v

As in carbon tax mechanism, it is assumed that the shortage cost is incurred only by the buyer. Also, similar to the assumptions (5.1) in the carbon tax mechanism, the following properties are assumed to hold.

$$\begin{aligned} v < c_v + p_c < c_b \\ c_b + p_c < p \end{aligned} \tag{5.5}$$

In the decentralized model, the buyer decides the order quantity that maximizes his/her expected profit. Then the modified newsvendor model under the cap-and-trade system is given by

$$\begin{aligned} max \quad J_b(Q, X_b) = -c_b Q + &\int_0^Q [px + (Q-x)v] f(x)\,\mathrm{d}x \\ + \int_Q^\infty [pQ - (x-Q)s] f(x)\,\mathrm{d}x &+ p_c X_b \end{aligned} \tag{5.6}$$

$$s.t. \quad Q + X_b = Q_b \tag{5.7}$$

$$Q \geq 0.$$

where Expression (5.7) is the emission balance constraint of the buyer. If X_b is positive (negative), the buyer sells (buys) carbon credit.

Similar to Expressions (5.6) and (5.7), the decentralized expected profit and the emission balance constraint of the vendor is given by

$$J_v(Q, X_v) = (c_b - c_v)Q + p_c X_v \tag{5.8}$$

$$s.t. \quad Q + X_v = Q_v \tag{5.9}$$

$$Q \geq 0.$$

If X_v is positive (negative), the vendor sells (buys) carbon credit.

From Expression (5.7),

$$X_b = Q_b - Q. \tag{5.10}$$

If we substitute Equation (5.10) into Expression (5.6), we get the unconstrained model given by

$$\begin{aligned} max \quad J_b(Q) = -(c_b + p_c)Q + p_c Q_b + &\int_0^Q [px + (Q-x)v] f(x)\,\mathrm{d}x \\ + \int_Q^\infty [pQ - (x-Q)s] f(x)\,\mathrm{d}x & \end{aligned} \tag{5.11}$$

$$Q \geq 0.$$

Similarly, the decentralized expected profit of the vendor in Equation (5.8) is updated as

$$J_v(Q) = (c_b - p_c - c_v)Q + p_c Q_v. \tag{5.12}$$

In the centralized model, the order quantity that maximizes the total profit of the system (i.e, the total profit of the buyer and the vendor) is determined. Then, the modified newsvendor model under the cap-and-trade system is given by

$$\begin{aligned} max \quad J_t(Q, X_b, X_v) = -c_v Q &+ \int_0^Q [px + (Q-x)v] f(x)\,\mathrm{d}x \\ &+ \int_Q^\infty [pQ - (x-Q)s] f(x)\,\mathrm{d}x + p_c(X_b + X_v) \end{aligned} \tag{5.13}$$

$$s.t. \quad Q + X_b = Q_b \tag{5.14}$$

$$Q + X_v = Q_v \tag{5.15}$$

$$Q \geq 0.$$

where Equations (5.14) and (5.15) are the emission balance constraints of the buyer and the vendor, respectively. If X_b is positive (negative), the buyer sells (buys) carbon credit. Similarly, if X_v is positive (negative), the vendor sells (buys) carbon credit.

The above model could be modified by summing Equations (5.14) and (5.15) up as follows.

$$\begin{aligned} max \quad J_t(Q, X_b, X_v) = -\, c_v Q &+ \int_0^Q [px + (Q-x)v] f(x)\,\mathrm{d}x \\ &+ \int_Q^\infty [pQ - (x-Q)s] f(x)\,\mathrm{d}x + p_c(X_b + X_v) \end{aligned} \tag{5.16}$$

$$s.t. \quad 2Q + X_b + X_v = Q_b + Q_v \tag{5.17}$$

$$Q \geq 0.$$

From Equation (5.17),

$$X_b + X_v = Q_b + Q_v - 2Q. \tag{5.18}$$

Substituting (5.18) into (5.16), we get the unconstrained model given by

$$\begin{aligned} max \quad J_t(Q) = &-(c_v + 2p_c)Q + p_c(Q_b + Q_v) + \int_0^Q [px + (Q-x)v] f(x)\,\mathrm{d}x \\ &+ \int_Q^\infty [pQ - (x-Q)s] f(x)\,\mathrm{d}x \end{aligned} \tag{5.19}$$

$$Q \geq 0.$$

5.1.3 Model Formulation under Stochastic Demand and Carbon Cap Mechanism

Similar to Section 5.1.1 and Section 5.1.2, in this section, we use the newsvendor problem as a benchmark and update the model to account for mandatory carbon capacities (i.e., carbon caps).

As in carbon tax and cap-and-trade mechanisms, it is assumed that the shortage cost is incurred only by the buyer. Also, the following property is assumed to hold.

$$v < c_v < c_b < p \tag{5.20}$$

In the decentralized model, the buyer decides the order quantity that maximizes his/her expected profit. Then the modified newsvendor model under the carbon cap mechanism is given by

$$max \quad J_b(Q) = -c_b Q + \int_0^Q [px + (Q-x)v] f(x)\,\mathrm{d}x + \int_Q^\infty [pQ - (x-Q)s] f(x)\,\mathrm{d}x \tag{5.21}$$

$$s.t. \quad Q \leqslant Q_b \tag{5.22}$$

$$Q \leqslant Q_v \tag{5.23}$$

$$Q \geq 0.$$

From Expressions (5.22) and (5.23), the emission levels of the buyer and the vendor cannot exceed the specified limits Q_b and Q_v, respectively.

In the centralized model, the order quantity that maximizes the total profit of the system (i.e, the total profit of the buyer and the vendor) is determined. Then, the modified newsvendor model under the carbon cap mechanism is given by

$$max \quad J_t(Q) = -c_v Q + \int_0^Q [px + (Q-x)v] f(x)\, \mathrm{d}x + \int_Q^\infty [pQ - (x-Q)s] f(x)\, \mathrm{d}x \tag{5.24}$$

$$s.t. \quad Q \leqslant Q_b \tag{5.25}$$

$$Q \leqslant Q_v \tag{5.26}$$

$$Q \geq 0.$$

From Expressions (5.25) and (5.26), the emission levels of the buyer and the vendor cannot the exceed specified limits Q_b and Q_v, respectively.

5.2 Analysis of the Decentralized and the Centralized Models under Stochastic Demand

5.2.1 Analysis of the Decentralized Model and the Centralized Model under Stochastic Demand and Carbon Tax Mechanism

In this section, we provide an analysis of the decentralized model and the centralized model under carbon tax mechanism to find the order quantities that maximize the expected profit of the buyer (i.e., Q_d^*) and the system (i.e., Q_c^*), respectively. We further provide some properties related to Q_d^* and Q_c^*.

Theorem 22 *The optimal order quantity for the decentralized model is given by*

$$Q_d^* = F^{-1}(\frac{p+s-c_b-t_b}{p+s-v}). \tag{5.27}$$

Proof: $J_b^{''}(Q) = (v-p-s)f(Q) \leqslant 0$ since $p > v$ by assumption and $f(Q) \geqslant 0$. Thus, $J_b(.)$ is a strictly concave function, implying the profit-maximizing value of Q can be found by checking the first order condition. Then

$$J_b^{'}(Q) = -(c_b + t_b) + pQf(Q) + \int_0^Q vf(x)\,\mathrm{d}x - pQf(Q) + \int_Q^\infty (p+s)f(x)\,\mathrm{d}x = 0.$$

This gives

$$F(Q_d^*) = \frac{p+s-c_b-t_b}{p+s-v}.$$

This, in turn, implies

$$Q_d^* = F^{-1}(\frac{p+s-c_b-t_b}{p+s-v}).$$

■

Since Equations (5.2) and (5.4) follow a similar structure, we can find the order quantity that maximizes the total expected profit of the system using the same reasoning of Theorem 22. That is,

Theorem 23 *The optimal order quantity for the centralized model is given by*

$$Q_c^* = F^{-1}(\frac{p+s-c_v-t_t}{p+s-v}). \tag{5.28}$$

Proof: $J_t^{''}(Q) = (v-p-s)f(Q) \leqslant 0$ since $p > v$ by assumption and $f(Q) \geqslant 0$. Thus, $J_t(.)$ is a strictly concave function, implying the profit-maximizing value of Q can be found by checking the first order condition. Then

$$J_t^{'}(Q) = -(c_v + t_t) + pQf(Q) + \int_0^Q vf(x)\,\mathrm{d}x - pQf(Q) + \int_Q^\infty (p+s)f(x)\,\mathrm{d}x = 0.$$

This gives

$$F(Q_c^*) = \frac{p + s - c_v - t_t}{p + s - v}.$$

This, in turn, implies

$$Q_c^* = F^{-1}(\frac{p + s - c_v - t_t}{p + s - v}).$$

■

We provide a further property related to Q_d^* and Q_c^* in the following proposition.

Proposition 9 *Under the assumptions (5.1) of the carbon tax mechanism, $Q_d^* < Q_c^*$ is always satisfied.*

Proof: It follows from Equations (5.27) and (5.28) that $Q_d^* < Q_c^*$ if and only if

$$F^{-1}(\frac{p + s - c_b - t_b}{p + s - v}) < F^{-1}(\frac{p + s - c_v - t_t}{p + s - v}).$$

Since $F(.)$ is a nondecreasing function, the above expression implies

$$\frac{p + s - c_b - t_b}{p + s - v} < \frac{p + s - c_v - t_t}{p + s - v}.$$

This gives

$$p^2 + ps - pv + ps + ss - sv - pc_b - c_b s + c_b v - pt_b - st_b + t_b v <$$
$$p^2 + ps - pc_v - pt_t + ps + ss - sc_v - st_t - pv - sv + c_v v + t_t v.$$

After canceling terms out and taking common parentheses, we get

$$c_b > c_v + t_v.$$

From assumptions (5.1), this always holds. Hence, $Q_d^* < Q_c^*$ is always satisfied.
■

5.2.2 Analysis of the Decentralized Model and the Centralized Model under Stochastic Demand and Cap-and-Trade Mechanism

Similar to Section 5.2.1, in this section, we present an analysis of the decentralized model and the centralized model to find Q_d^* and Q_c^* under the cap-and-trade system. We further provide some properties related to Q_d^* and Q_c^*.

Theorem 24 *The optimal order quantity for the decentralized model is given by*

$$Q_d^* = F^{-1}(\frac{p + s - c_b - p_c}{p + s - v}). \tag{5.29}$$

Proof: The proof is similar to the proof of Theorem 22. t_b is replaced with p_c. ■

As Equations (5.11) and (5.19) have identical structural properties, the order quantity that maximizes the total expected profit of the system can be found using the same reasoning of Theorem 24. That is,

Theorem 25 *The optimal order quantity for the centralized model is given by*

$$Q_c^* = F^{-1}(\frac{p + s - c_v - 2p_c}{p + s - v}). \tag{5.30}$$

Proof: The proof is similar to the proof of Theorem 23. t_b and t_v are replaced with p_c. ■

We provide a further property related to Q_d^* and Q_c^* in Proposition 10.

Proposition 10 *Under the assumptions (5.5) of the cap-and-trade mechanism, $Q_d^* < Q_c^*$ is always satisfied.*

Proof: It follows from Equations (5.29) and (5.30) that $Q_d^* < Q_c^*$ if and only if

$$F^{-1}(\frac{p+s-c_b-p_c}{p+s-v}) < F^{-1}(\frac{p+s-c_v-2p_c}{p+s-v}).$$

Since $F(.)$ is a nondecreasing function, the above expression implies

$$\frac{p+s-c_b-p_c}{p+s-v} < \frac{p+s-c_v-2p_c}{p+s-v}.$$

This gives

$$c_v + p_c < c_b.$$

From assumptions (5.5), this always holds. Hence, $Q_d^* < Q_c^*$ is always satisfied. ■

5.2.3 Analysis of the Decentralized Model and the Centralized Model under Stochastic Demand and Carbon Cap Mechanism

As in Sections 5.2.1 and 5.2.2, in this section, we present an analysis of the decentralized model and the centralized model to find Q_d^* and Q_c^* under the carbon cap mechanism. We further provide some properties related to Q_d^* and Q_d^*.

From Expression (5.21), the optimal order quantity under the decentralized model when the emissions are not considered, (i.e., Q_d^0, namely, the order quantity that maximizes the expected profits of the buyer) is

$$Q_d^0 = F^{-1}(\frac{p+s-c_b}{p+s-v}). \tag{5.31}$$

Since Expression (5.21) is a strictly concave function in Q, Q_d^0 is obtained from the first order condition.

Next, we provide the solution of the buyer's expected profit maximization problem under carbon capacity constraints.

Theorem 26 *The optimal order quantity for the decentralized model is given by*

$$Q_d^* = \min\{Q_b, Q_v, Q_d^0\}. \quad (5.32)$$

where Q_d^0 is the cost optimal order quantity of the decentralized model given in equation (5.31) and Q_b and Q_v are the carbon emission quotas of the buyer and the vendor, respectively.

Proof: Since Expression (5.21) is a strictly concave function in Q, temporarily ignoring the constraints (5.22) and (5.23), we get Q_d^0 as the optimal order quantity from the first order condition. Taking constraints (5.22) and (5.23) into consideration, we obtain $Q_d^* = \min\{Q_b, Q_v, Q_d^0\}$. ■

As Equations 5.21 and 5.24 have identical structural properties and the constraints of the decentralized model and the centralized model are the same, a similar analysis is repeated for the centralized model. The profit maximizing order quantity and the optimal solution under the centralized model are presented in Equation (5.33) and Theorem 27, respectively.

From Expression (5.24), the optimal order quantity under the centralized model when the emissions are not considered (i.e., Q_c^0, namely, the order quantity that maximizes the expected profits of the system) is

$$Q_c^0 = F^{-1}\left(\frac{p+s-c_v}{p+s-v}\right). \quad (5.33)$$

Since Expression (5.24) is a strictly concave function in Q, Q_c^0 is obtained from the first order condition.

Theorem 27 *The optimal order quantity for the centralized model is given by*

$$Q_c^* = \min\{Q_b, Q_v, Q_c^0\}. \quad (5.34)$$

where Q_c^0 is the cost optimal order quantity of the centralized model given in Equation (5.33) and Q_b and Q_v are the carbon emission quotas of the buyer and the vendor, respectively.

Proof: The proof is similar to the proof of Theorem 26 and is omitted. ■

In Proposition 11, we present a property related to the order quantities that maximize the expected profits of the buyer (i.e., Q_d^0) and the system (i.e., Q_c^0), respectively. Then we use Proposition 11 to characterize a result related to Q_d^* and Q_c^*.

Proposition 11 *Under the assumptions (5.20) of the carbon cap mechanism, $Q_d^0 < Q_c^0$ is always satisfied.*

Proof: It follows from Equations (5.31) and (5.33) that $Q_d^0 < Q_c^0$ if and only if

$$F^{-1}(\frac{p+s-c_b}{p+s-v}) < F^{-1}(\frac{p+s-c_v}{p+s-v}).$$

Since $F(.)$ is a nondecreasing function, the above expression implies

$$\frac{p+s-c_b}{p+s-v} < \frac{p+s-c_v}{p+s-v}.$$

This gives

$$c_v < c_b.$$

From assumptions (5.20), this always holds. Hence, $Q_d^0 < Q_c^0$ is always satisfied.
■

Proposition 11 will be used in the proof of the next proposition.

Proposition 12 *Under the assumptions (5.20) of the carbon cap mechanism, $Q_d^* \leqslant Q_c^*$ is always satisfied.*

Proof: Let us define $Q_{cap} = \min\{Q_b, Q_v\}$.

i) Suppose $Q_d^0 \leqslant Q_{cap}$ and $Q_c^0 \leqslant Q_{cap}$. Then from Theorem 26, $Q_d^* = Q_d^0$ and from Theorem 27, $Q_c^* = Q_c^0$. From Proposition 11, $Q_d^0 < Q_c^0$ is always satisfied. Hence, $Q_d^* = Q_d^0 < Q_c^* = Q_c^0 \leqslant Q_{cap}$.

ii) Suppose $Q_d^0 \leqslant Q_{cap}$ and $Q_c^0 > Q_{cap}$. Then from Theorem 26, $Q_d^* = Q_d^0$ and from Theorem 27, $Q_c^* = Q_{cap}$. Hence, $Q_d^* = Q_d^0 \leqslant Q_c^* = Q_{cap}$.

iii) Suppose $Q_d^0 > Q_{cap}$ and $Q_c^0 > Q_{cap}$. Then from Theorem 26, $Q_d^* = Q_{cap}$ and from Theorem 27, $Q_c^* = Q_{cap}$. Hence, $Q_d^* = Q_c^* = Q_{cap}$.

■

5.3 Coordination Mechanisms Proposed under Stochastic Demand

In this section, coordination mechanisms are proposed under carbon tax, cap-and-trade and carbon cap mechanisms so that the buyer's loss from the centralized solution is compensated. Thus, the buyer is willing to order the optimal order quantity of the centralized model.

The additional notation used in this section is presented in Table 5.3 as follows.

Table 5.3: Additional Notation Used in Coordination Mechanisms Proposed under Stochastic Demand

$\overline{J}_b(Q)$:	The expected profit of the buyer after the implementation of the coordination strategy when the order size is Q units
$J_b(Q, c_b)$:	The expected profit of the buyer when the order size is Q units and the price per unit paid by the buyer to the vendor is c_b
Q_{crd}:	The value of the order size that maximizes the expected profit of the buyer after coordination
$Q(c_b)$:	The order size as a function of the price per unit paid by the buyer to the vendor
$X_b(Q)$:	Amount of carbon credit bought (sold) by the buyer when the order size is Q units
$X_v(Q)$:	Amount of carbon credit bought (sold) by the vendor when the order size is Q units

5.3.1 Coordination Mechanisms Proposed under Stochastic Demand and Carbon Tax Mechanism

Under stochastic demand and carbon tax mechanism, channel coordination is achieved either by quantity discounts given by the vendor to the buyer and a fixed payment made by one party to the other (Theorem 28) or only quantity discounts given by the vendor to the buyer (Theorem 29).

Theorem 28 *Under a unit discount of $d = c_b - (c_v + t_v)$ offered by the vendor to the buyer and a fixed payment of $J_b(Q_c^*, c_b - d) - J_b(Q_d^*, c_b)$ made by the buyer to the vendor for the order sizes greater than or equal to Q_c^*, the buyer stays in a no worse situation by ordering Q_c^* units.*

Proof: See Appendix B.1.1 for the proof. ■

Theorem 29 *Under a unit discount of $d = \frac{J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b)}{Q_c^*}$ offered by the vendor to the buyer for the order sizes greater than or equal to Q_c^*, the buyer stays in a no worse situation by ordering Q_c^* units.*

Proof: See Appendix B.1.2 for the proof. ■

5.3.2 Coordination Mechanisms Proposed under Stochastic Demand and Cap-and-Trade Mechanism

Under stochastic demand and cap-and-trade system, in addition to quantity discounts, channel coordination can be achieved if the vendor gives carbon credits or carbon price discounts to the buyer under certain conditions.

Theorem 30 *Suppose one of the following conditions holds.*

- $X_b(Q_c^*) = Q_c^* - Q_b \geqslant 0$

- $X_v(Q_c^*) = Q_c^* - Q_v \leqslant 0$

Then under a unit discount of $d = c_b - (c_v + p_c)$ offered by the vendor to the buyer and a fixed payment of $J_b(Q_c^*, c_b - d) - J_b(Q_d^*, c_b)$ made by the buyer to the vendor for the order sizes greater than or equal to Q_c^*, the buyer stays in a no worse situation by ordering Q_c^* units.

Proof: Suppose $X_b(Q_c^*) \geqslant 0$. Thus, the buyer sells carbon credit under the centralized solution and he/she does not need to be compensated by giving him/her carbon credit. Suppose $X_v(Q_c^*) \leqslant 0$. Thus, the vendor buys carbon credit and he/she does not have any carbon credit to give to the buyer to compensate his/her loss. Hence, the channel coordination can be achieved by a quantity discount and/or a fixed payment. The remaining steps of the proof are similar to the proof of Theorem 28 and are omitted. ■

Theorem 31 *Suppose one of the following conditions holds.*

- $X_b(Q_c^*) = Q_c^* - Q_b \geqslant 0$
- $X_v(Q_c^*) = Q_c^* - Q_v \leqslant 0$

Then under a unit discount of $d = \frac{J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b)}{Q_c^*}$ offered by the vendor to the buyer for the order sizes greater than or equal to Q_c^*, the buyer stays in a no worse situation by ordering Q_c^* units.

Proof: Suppose $X_b(Q_c^*) \geqslant 0$. Thus, the buyer sells carbon credit under the centralized solution and he/she does not need to be compensated by giving him/her carbon credit. Suppose $X_v(Q_c^*) \leqslant 0$. Thus, the vendor buys carbon credit and he/she does not have any carbon credit to give to the buyer to compensate his/her loss. Hence, the channel coordination can be achieved by a quantity discount and/or a fixed payment. The remaining steps of the proof are similar to the proof of Theorem 29 and are omitted. ■

Theorem 30 and Theorem 31 have the following implication. When the buyer sells carbon credit (i.e., he does not need extra credit) or the vendor buys carbon credit (i.e., he does not have any credit to give to the buyer to compensate his loss), channel coordination is achieved either by quantity discounts given by the vendor to the buyer and a fixed payment made by one party to the other (Theorem 30) or only quantity discounts given by the vendor to the buyer (Theorem 31).

Theorem 32 *Suppose the following conditions hold.*

- $X_b(Q_c^*) = Q_c^* - Q_b \leqslant 0, \quad X_v(Q_c^*) = Q_c^* - Q_v \geqslant 0$
- $p_c \times \min\{-X_b(Q_c^*), X_v(Q_c^*)\} \geqslant J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b)$

Then if $[J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b)]/p_c$ *amount of carbon credit is given by the vendor to the buyer for the order sizes greater than or equal to* Q_c^**, the buyer stays in a no worse situation by ordering* Q_c^* *units.*

Proof: See Appendix B.2.1 for the proof. ■

Theorem 33 *Suppose the following conditions hold.*

- $X_b(Q_c^*) = Q_c^* - Q_b \leqslant 0, \quad X_v(Q_c^*) = Q_c^* - Q_v \geqslant 0$
- $p_c \times \min\{-X_b(Q_c^*), X_v(Q_c^*)\} \geqslant J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b)$
- $[J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b)]/\min\{-X_b(Q_c^*), X_v(Q_c^*)\} \leqslant p_c$

Then if a unit carbon price discount of $d_c = [J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b)]/$ $\min\{-X_b(Q_c^*), X_v(Q_c^*)\}$ *is given by the vendor to the buyer for the order sizes greater than or equal to* Q_c^**, the buyer stays in a no worse situation by ordering* Q_c^* *units.*

Proof: See Appendix B.2.2 for the proof. ■

Theorem 32 and Theorem 33 have the following implication. If the buyer buys and the vendor sells carbon credit and if the monetary amount of the credit sold by the vendor is enough to compensate the buyer's loss from ordering the centralized optimal quantity, vendor gives carbon credits for free or carbon price discounts to the buyer. The monetary value of the given credits or carbon price discounts is equal to the difference between the buyer's cost in the centralized and the decentralized solutions.

Theorem 34 *Suppose the following conditions hold.*

- $X_b(Q_c^*) = Q_c^* - Q_b \leqslant 0, \quad X_v(Q_c^*) = Q_c^* - Q_v \geqslant 0$
- $p_c \times \min\{-X_b(Q_c^*), X_v(Q_c^*)\} < J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b)$

Then if $\min\{-X_b(Q_c^*), X_v(Q_c^*)\}$ *amount of carbon credit and a unit discount* $d = c_b - (c_v + p_c)$ *are given by the vendor to the buyer and a fixed payment of* $J_b(Q_c^*, c_b - d) + p_c \times \min\{-X_b(Q_c^*), X_v(Q_c^*)\} - J_b(Q_d^*, c_b)$ *is made by the buyer to the vendor for the order sizes greater than or equal to* Q_c^**, the buyer stays in a no worse situation by ordering* Q_c^* *units.*

Proof: See Appendix B.2.3 for the proof. ■

Theorem 35 *Suppose the following conditions hold.*

- $X_b(Q_c^*) = Q_c^* - Q_b \leqslant 0, \quad X_v(Q_c^*) = Q_c^* - Q_v \geqslant 0$
- $p_c \times \min\{-X_b(Q_c^*), X_v(Q_c^*)\} < J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b)$

Then if $\min\{-X_b(Q_c^*), X_v(Q_c^*)\}$ *amount of carbon credit and a unit discount* $d = (J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b) - p_c \times \min\{-X_b(Q_c^*), X_v(Q_c^*)\})/Q_c^*$ *are given by the vendor to the buyer for the order sizes greater than or equal to* Q_c^**, the buyer stays in a no worse situation by ordering* Q_c^* *units.*

Proof: See Appendix B.2.4 for the proof. ■

Theorem 34 and Theorem 35 have the following implication. If the buyer buys and the vendor sells carbon credits, the vendor gives free carbon credits to the buyer. However, if the monetary amount of credits the vendor sells or buyer buys is not sufficient to compensate the buyer's loss from ordering the centralized optimal quantity, the remaining loss of the buyer is compensated by quantity discounts or quantity discounts and fixed payments made by one party to the other. Here, giving more carbon credit to the buyer than he buys is not desired as it may not be practical in practice.

5.3.3 Coordination Mechanisms Proposed under Stochastic Demand and Carbon Cap Mechanism

Under stochastic demand and carbon cap mechanism, channel coordination is achieved either by quantity discounts given by the vendor to the buyer and a fixed payment made by one party to the other (Theorem 36) or only quantity discounts given by the vendor to the buyer (Theorems 37 and 38).

Let us define $Q_{cap} = \min\{Q_b, Q_v\}$.

Theorem 36 *Suppose $Q_d^0 < Q_c^0 \leqslant Q_{cap}$ holds. Then if a unit discount $d = c_b - c_v$ is given by the vendor to the buyer and a fixed payment of $J_b(Q_c^0, c_b - d) - J_b(Q_d^0, c_b)$ made by the buyer to the vendor for the order sizes greater than or equal to Q_c^0, the buyer stays in a no worse situation by ordering Q_c^0 units.*

Proof: See Appendix B.3.1 for the proof. ■

Theorem 37 *Suppose $Q_d^0 < Q_c^0 \leqslant Q_{cap}$ holds. Then if a unit discount $d = [J_b(Q_d^0, c_b) - J_b(Q_c^0, c_b)]/Q_c^0$ is given by the vendor to the buyer for the order sizes greater than or equal to Q_c^0, the buyer stays in a no worse situation by ordering Q_c^0 units.*

Proof: See Appendix B.3.2 for the proof. ■

Theorem 38 *Suppose the following conditions hold.*

- $Q_d^0 < Q_{cap}$
- $Q_c^0 > Q_{cap}$

Then if a unit discount $d = [J_b(Q_d^0, c_b) - J_b(Q_{cap}, c_b)]/Q_{cap}$ *is given by the vendor to the buyer for the order sizes greater than or equal to* Q_{cap}*, the buyer stays in a no worse situation by ordering* Q_{cap} *units.*

Proof: See Appendix B.3.3 for the proof. ■

Theorem 39 *Suppose* $Q_d^0 > Q_{cap}$ *holds. Then the channel is already coordinated.*

Proof: Since $Q_d^0 > Q_{cap}$, $Q_d^* = Q_{cap}$ from Theorem 26. Since $Q_d^0 < Q_c^0$ always holds from Proposition 11, $Q_c^0 > Q_{cap}$. Then $Q_c^0 = Q_{cap}$. Hence, the channel is already coordinated. ■

5.4 Numerical Analysis under Stochastic Demand

In this section numerical examples are illustrated for each emission regulating mechanism described in Section 5.1. In each example the optimal order sizes of coordinated and uncoordinated models are calculated and the coordination mechanisms proposed in Section 5.3 are applied.

5.4.1 Numerical Analysis under Stochastic Demand and Carbon Tax Mechanism

In this section two numerical examples are presented for the carbon tax mechanism. Examples 49 and 50 correspond to the applications of Theorem 28 and Theorem 29, respectively. The values of parameters and the probability distributions of demand are presented in Table 5.4. The results of the decentralized and the centralized solutions for each example are presented in Tables 5.5 and 5.6, respectively. The application of coordination mechanisms proposed in Section 5.3.1 to each example is summarized in Appendix B.4.

Table 5.4: Parameter Values of Examples 49 and 50 $p = 18$, $c_b = 13$, $c_v = 6$ and $t_v = 3$

Example	v	s	t_b	t_t	X
49	5	10	2	5	$U(160, 300)$
50	4	8	1.8	4.8	$U(120, 280)$

Table 5.5: Solutions of the Decentralized Model of Examples 49 and 50

Example	Q_d^*	$J_b(Q_d^*)$	$J_v(Q_d^*)$	$J_t(Q_d^*)$
49	239.1304	294.3478	956.5217	1250.8696
50	201.4545	200.1455	805.8182	1005.9636

Table 5.6: Solutions of the Centralized Model of Examples 49 and 50

Example	Q_c^*	$J_b(Q_c^*)$	$J_v(Q_c^*)$	$J_t(Q_c^*)$
49	263.4783	245.6522	1053.913	1299.5652
50	230.5455	141.9636	922.1818	1064.1455

5.4.2 Numerical Analysis under Stochastic Demand and Cap-and-Trade Mechanism

In this section six numerical examples are presented for the cap-and-trade mechanism. Examples 51-56 correspond to the applications of Theorems 30-35, respectively. The values of parameters and the probability distributions of demand are presented in Table 5.7. The results of the decentralized and the centralized solutions for each example are presented in Tables 5.8 and 5.9, respectively. The application of coordination mechanisms proposed in Section 5.3.2 to each example is summarized in Appendix B.5

Table 5.7: Parameter Values of Examples 51-56

Example	p	c_b	c_v	v	s	p_c	Q_b	Q_v	X
51	18	13	6	5	10	3	2800	3000	$U(1600, 3000)$
52	20	14	8	6	10	3	150	180	$U(45, 265)$
53	18	13	6	5	10	3	200	260	$U(150, 280)$
54	20	15	9	6	10	4	1200	1500	$U(900, 1600)$
55	15	10	4	3	6	2.5	220	270	$U(150, 320)$
56	15	11	6	4	5.5	2.5	600	650	$U(350, 800)$

Table 5.8: Solutions of the Decentralized Model of Examples 51-56

Example	Q_d^*	$X_b(Q_d^*)$	$X_v(Q_d^*)$	$J_b(Q_d^*)$	$J_v(Q_d^*)$	$J_t(Q_d^*)$
51	2330.4348	469.5652	669.5652	8982.6087	9321.7391	18304.3478
52	164.1667	-14.1667	15.8333	259.5833	492.5	752.0833
53	217.826	-17.8261	42.1739	656.9565	871.3043	1528.2609
54	1220.8333	-20.8333	279.1667	3964.5833	2441.6667	6406.25
55	230.2778	-10.2778	39.7222	756.1801	805.9722	1562.1528
56	540.9091	59.0909	109.0909	1455.6818	1352.2727	2807.9545

Table 5.9: Solutions of the Centralized Model of Examples 51-56

Example	Q_d^*	$X_b(Q_d^*)$	$X_v(Q_d^*)$	$J_b(Q_d^*)$	$J_v(Q_d^*)$	$J_t(Q_d^*)$
51	2573.913	226.087	426.087	8495.6522	10295.6522	18791.3044
52	191.6667	−41.6667	−11.6667	218.3333	575	793.3333
53	240.4348	−40.4348	19.5652	611.7391	961.7391	1573.4783
54	1279.1667	−79.1667	220.8333	3906.25	2558.3333	6464.5833
55	263.3333	−43.3333	6.6667	698.3333	921.6667	1620
56	609.0909	−9.0909	40.9091	1370.4545	1522.7273	2893.1818

5.4.3 Numerical Analysis under Stochastic Demand and Carbon Cap Mechanism

In this section six numerical examples are presented for the carbon cap mechanism. Each example corresponds to a specific case in Table 5.10. The values of parameters and the probability distributions of demand are presented in Table 5.11. The results of the decentralized and the centralized solutions for each example are presented in Tables 5.12 and 5.13, respectively. The application of coordination mechanisms proposed in Section 5.3.3 to each example is summarized in Appendix B.6.

Table 5.10: Classification of Examples for Carbon Cap Mechanism under Stochastic Demand

Example Index	Q_{cap}	Q_d^*	Q_c^*
57	Q_b	Q_d^0	Q_c^0
58	Q_v	Q_d^0	Q_c^0
59	Q_b	Q_d^0	Q_b
60	Q_v	Q_d^0	Q_v
61	Q_b	Q_b	Q_b
62	Q_v	Q_v	Q_v

Table 5.11: Parameter Values of Examples 57-62

Example	p	c_b	c_v	v	s	Q_b	Q_v	X
57	10	8	5	4	6	200	180	$U(120, 180)$
58	18	12	8	3	8	160	180	$U(60, 180)$
59	20	15	9	6	12	170	200	$U(120, 180)$
60	20	15	9	6	12	1600	1400	$U(900, 1500)$
61	15	7.5	5	3	7	105	180	$U(45, 135)$
62	25	18.5	15	9	14	1600	1150	$U(750, 1600)$

Table 5.12: Solutions of the Decentralized Model of Examples 57-62

Example	Q_d^0	Q_{cap}	Q_d^*	$J_b(Q_d^*)$	$J_v(Q_d^*)$	$J_t(Q_d^*)$
57	160	180	160	220	480	700
58	133.0435	160	133.0435	391.3043	532.1739	923.4783
59	159.2308	170	159.2308	573.4615	955.3846	1528.8462
60	1292.3077	1400	1292.3077	4234.6154	7753.8462	11988.4615
61	113.6842	105	105	512.5	262.5	775
62	1330.8333	1150	1150	4301.4706	4025	8326.4706

Table 5.13: Solutions of the Centralized Model of Examples 57-62

Example	Q_c^0	Q_{cap}	Q_c^*	$J_b(Q_c^*)$	$J_v(Q_c^*)$	$J_t(Q_c^*)$
57	175	180	175	197.5	525	722.5
58	153.913	160	153.913	349.5652	615.6522	965.2174
59	173.0769	170	170	548.3333	1020	1568.3333
60	1430.7692	1400	1400	3983.3333	8400	12383.3333
61	125.5263	105	105	512.5	262.5	775
62	1430	1150	1150	4301.4706	4025	8326.4706

Chapter 6

Conclusion

In this thesis, we study the channel coordination problem in a supply chain with two echelons under emission regulations. It assumed that the supply chain consists of a buyer (retailer) and a vendor (manufacturer). We study three emission regulating mechanisms, namely, cap-and-trade system, carbon tax and carbon cap. In Chapters 3 and 4, we assume that the demand faced by the buyer is deterministic. The buyer and the vendor operate in an infinite horizon under a lot-for-lot policy. Similarly, in Chapter 5, we assume that the demand faced by the buyer is stochastic and he/she operates under the conditions of the classical newsvendor problem.

For each demand setting and emission regulating policy, we propose two models, namely, the decentralized and the centralized models. In the decentralized model, we find the order quantity that minimizes (or maximizes) the average annual costs (or expected profits) of the buyer. Similarly, in the centralized model, we find the order quantity that minimizes (or maximizes) the average annual costs (or expected profits) of the buyer-vendor system. Under cap-and-trade mechanism, we also find the traded amount of carbon credits under the decentralized and centralized models. We further propose some coordination strategies including quantity discounts, carbon-credit sharing, carbon credit price discounts and fixed payments that compensate the buyer's loss due to ordering the centralized optimal order quantity. Additionally, we examine the impact of channel coordination on the optimal order quantities and on the cost (or expected profit) of the buyer, vendor, and the system by numerical analyses for each demand setting and emission regulating policy.

In Chapter 3, we extend the EOQ model to account for the two-level supply chain under the cap-and-trade mechanism. We examine the buyer's (or vendor's) decisions related to replenishment (or production) and inventory holding. We model the carbon emissions of the buyer and the vendor using a similar structure we used while modeling the costs. It is assumed that carbon credit buying price is at least as much as its selling price (i.e. $p_c^b \geq p_c^s$). In addition to the decentralized and centralized models we described in the previous paragraph, we further developed the "centralized model with carbon credit sharing" as a benchmark for coordination in order to achieve the maximum supply chain profitability. In this model, it is assumed that if one party sells carbon credits, he/she makes it available for the other party if he/she buys carbon credits. Thus, we show that the cost of the system under the centralized model with carbon credit sharing is less than its cost under the centralized model. The decrease in the system's cost is quantified in Equation (3.13). The optimal order quantities for the decentralized model and centralized model with carbon credit sharing under the cap-and-trade system are presented in Theorem 1 and Theorem 2, respectively.

In Chapter 3, we further examine how replenishment decisions can be coordinated under the cap-and-trade mechanism so as to compensate the buyer's loss under centralized optimal solution. We show that if both parties buy/sell carbon credits, channel coordination can be achieved by quantity discounts given by the vendor to the buyer (see Theorems 3 and 4). Similarly, if one party sells and the other buys carbon credits, channel coordination can be achieved with a combination of quantity discounts or fixed payments and carbon credit sharing (see Theorems 5 to 10).

We further analyze the impact of coordination on average annual emissions of the system in Chapter 3. We define R as the ratio of average annual emissions of the system under the centralized model with carbon credit sharing and the decentralized model. We study the effect of each parameter on R under different parameter settings. We observe that if a parameter is changed from its base value, R can increase above and/or decrease below 1 (see Figures 3.1-3.51). Therefore, we conclude that *channel coordination may not be good for the environment in terms of emission-related performance measures.*

Similar to Chapter 3, we use the EOQ model as a benchmark to account for a two-echelon setting under the carbon tax and carbon cap mechanisms in Chapter 4. Under both mechanisms, the carbon emissions of the buyer and the vendor follow similar structures with their average annual costs. Under the carbon tax mechanism, a monetary amount is paid to the regulatory agencies (i.e., government) for each unit of emission. The optimal order quantities resulting from the decentralized and the centralized model under the carbon tax mechanism are presented in Theorem 2 and Theorem 11, respectively. We further characterize the conditions under which the government collects more taxes under the decentralized and centralized optimal solutions (see Propositions 6 and 7). Similarly, we present the conditions under which the buyer and the vendor pay more taxes under the decentralized and centralized optimal solutions (see Propositions 4 and 5). If the buyer's average annual taxes decrease and the vendor's average annual taxes increase under the centralized solution, coordination is achieved by quantity discounts given by the vendor to the buyer. Similarly, if the buyer's average annual taxes increase and the vendor's average annual taxes decrease under the centralized solution, coordination is achieved when the vendor pays some amount of the buyer's taxes and gives him/her additional quantity discounts (see Theorems 12 to 17).

Also, we propose quantity discounts given by the vendor to the buyer, paying some amount of the buyer's taxes or combinations of these as the coordination mechanisms to compensate the buyer's loss under the centralized optimal solution (see Theorems 12 to 17).

Under the carbon cap mechanism, the buyer and the vendor cannot exceed the emission quotas allocated to them. The optimal order quantities resulting from the decentralized and the centralized models under deterministic demand are presented in Theorem 18 and Theorem 19, respectively. Since the emissions are not quantified with a monetary value, we propose quantity discounts as the coordination strategy to compensate the buyer's loss due to ordering the centralized optimal order quantity (see Theorem 20 and Theorem 21).

In Chapter 5, we extend our analyses in Chapters 3 and 4 for a perishable product with stochastic demand. We extend the single period problem (i.e., the newsvendor problem) for a two-level supply chain under carbon tax, cap-and-trade and carbon cap mechanisms. The order quantities that maximize the expected profit of the buyer (system) under carbon tax, cap-and-trade and carbon cap mechanisms are presented in Theorems 22, 24 and 26 (23, 25 and 27), respectively. In addition to quantity discounts and carbon credit sharing, we propose carbon price discounts so as to coordinate the channel (see Theorems 28 to 39).

Under the deterministic demand setting of our study, we consider the procurement decisions of an item with a cost-minimizing objective. This can be extended under the assumption that the demand is a function of the retail price. For instance, demand can be formulated as a linear or iso-elastic function of the retail price. Additionally, we investigate a single-period replenishment problem under stochastic demand. The emissions related considerations can also be integrated into different periodic or continuous review inventory models under stochastic demand. Also, we study the procurement decisions of a single item. An extension to our model would be to consider the joint replenishment problem of multiple items under environmental considerations.

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Appendix A

Proofs and Applications of Coordination Theorems under Deterministic Demand

A.1 Proofs of Coordination Mechanisms under Deterministic Demand and Cap-and-Trade Mechanism

A.1.1 Proof of Theorem 3

It suffices to show that

i) The buyer orders Q_s^*.

ii) $\overline{BC}(Q_s^*) \leqslant BC(Q_s^*, X_b(Q_s^*))$ (i.e., the buyer is not worse off.)

iii) $\overline{VC}(Q_s^*) \leqslant VC(Q_s^*, X_v(Q_s^*))$ (i.e., the vendor is not worse off.)

Let $Q < Q_s^*$. Then $\overline{BC}(Q) = BC(Q, X_b(Q))$. Since the global optimum of $BC(Q, X_b(Q))$, namely Q_d^*, satisfies $Q_d^* < Q_s^*$, Q_d^* is the optimal solution of $\overline{BC}(Q)$ when $Q < Q_s^*$.

Let $Q \geqslant Q_s^*$. Then

$$\begin{aligned}\overline{BC}(Q) &= BC(Q, X_b(Q)) - d \times D \\ &= BC(Q, X_b(Q)) - \tfrac{BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))}{D} \times D \\ &= BC(Q, X_b(Q)) - [BC(Q_c^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))].\end{aligned}$$

Since $BC(Q, X_b(Q))$ is a convex function of Q from Proposition 1 and the term $-[BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*), X_b(Q_d^*)]$ does not depend on Q, $\overline{BC}(Q)$ is also a convex function of Q. Then the global optimum of $\overline{BC}(Q)$ is Q_d^*. However, $Q_d^* \ngeqslant Q_s^*$, implying Q_d^* is not feasible when $Q \geqslant Q_s^*$. Since $\overline{BC}(Q)$ is a convex function in Q, we check the boundary condition, i.e., the buyer orders Q_s^*.

The buyer's cost at Q_s^* is

$$\begin{aligned}\overline{BC}(Q_s^*) &= BC(Q_s^*, X_b(Q_s^*)) - [BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))] \\ &= BC(Q_d^*, X_b(Q_d^*)).\end{aligned}$$

Hence, the buyer is not worse off.

The vendor's cost after coordination is

$\overline{VC}(Q_s^*) = VC(Q_s^*, X_v(Q_s^*)) + BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))$, which gives

$$\begin{aligned}VC(Q_d^*, X_v(Q_d^*)) - \overline{VC}(Q_s^*) = {}& [VC(Q_d^*, X_v(Q_d^*)) - VC(Q_s^*, X_v(Q_s^*))] \\ & -[BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))].\end{aligned}$$

Since

$BC(Q_d^*, X_b(Q_d^*)) + VC(Q_d^*, X_b(Q_d^*)) \geqslant BC(Q_s^*, X_b(Q_s^*)) + VC(Q_s^*, X_v(Q_s^*))$

or equivalently,

$VC(Q_d^*, X_v(Q_d^*)) - VC(Q_s^*, X_v(Q_s^*)) \geq BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))$

due to channel coordination principles, $VC(Q_d^*, X_v(Q_d^*)) - \overline{VC}(Q_s^*) \geqslant 0$. Hence, the vendor is not worse off.

The buyer is indifferent between ordering Q_d^* and Q_s^* since they result in the same cost. Hence, Q_s^* is the channel coordinating order quantity. ■

A.1.2 Proof of Theorem 5

It suffices to show that

i) The buyer orders Q_s^*.

ii) $\overline{BC}(Q_s^*) \leqslant BC(Q_d^*, X_b(Q_d^*))$ (i.e., the buyer is not worse off.)

iii) $\overline{BC}(Q_s^*) + \overline{VC}(Q_s^*) = SC(Q_s^*, X_s(Q_s^*))$
$= BC(Q_s^*, X_b(Q_s^*)) + VC(Q_s^*, X_v(Q_s^*)) - (p_c^b - p_c^s) \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\}$

iv) $\overline{VC}(Q_s^*) \leqslant VC(Q_d^*, X_v(Q_d^*))$ (i.e., the vendor is not worse off.)

Let $Q < Q_s^*$. Then $\overline{BC}(Q) = BC(Q, X_b(Q))$. Since the global optimum of $BC(Q, X_b(Q))$, namely Q_d^*, satisfies $Q_d^* < Q_s^*$, Q_d^* is the optimal solution of $\overline{BC}(Q, X_b(Q))$ when $Q < Q_s^*$.

Let $Q \geqslant Q_s^*$. Then

$$\overline{BC}(Q) = BC(Q, X_b(Q)) - p_c^b \times Y + [BC(Q_d^*, X_b(Q_d^*)) - BC(Q_s^*, X_b(Q_s^*)) + p_c^b \times Y]$$

$$= BC(Q, X_b(Q)) - p_c^b \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\} + BC(Q_d^*, X_b(Q_d^*))$$

$$- BC(Q_s^*, X_b(Q_s^*)) + p_c^b \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\}$$

$$= BC(Q, X_b(Q)) - [BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))].$$

Since $BC(Q, X_b(Q))$ is a convex function of Q from Proposition 1 and the term $-[BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*), X_b(Q_d^*)]$ does not depend on Q, $\overline{BC}(Q)$ is also a convex function of Q. Then the global optimum of $\overline{BC}(Q)$ is Q_d^*. However,

$Q_d^* \ngeqslant Q_s^*$, implying Q_d^* is not feasible when $Q \geqslant Q_s^*$. Since $\overline{BC}(Q)$ is a convex function in Q, we check the boundary condition, i.e., the buyer orders Q_s^*.

The buyer's cost at Q_s^* is

$$\begin{aligned}\overline{BC}(Q_s^*) &= BC(Q_s^*, X_b(Q_s^*)) - [BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))]. \\ &= BC(Q_d^*, X_b(Q_d^*)).\end{aligned}$$

Hence, the buyer is not worse off.

The vendor's cost after coordination is

$$\begin{aligned}\overline{VC}(Q_s^*) = {} & VC(Q_s^*, X_v(Q_s^*)) + p_c^s \times Y - [BC(Q_d^*, X_b(Q_d^*)) - BC(Q_s^*, X_b(Q_s^*)) \\ & +p_c^b \times Y], \text{ which gives}\end{aligned}$$

$$\begin{aligned}\overline{VC}(Q_s^*) &= VC(Q_s^*, X_v(Q_s^*)) - (p_c^b - p_c^s) \times Y + [BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))] \\ &= VC(Q_s^*, X_v(Q_s^*)) - (p_c^b - p_c^s) \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\} + BC(Q_s^*, X_b(Q_s^*)) \\ &\quad -BC(Q_d^*, X_b(Q_d^*)).\end{aligned}$$

The total cost of the buyer and the vendor after coordination is equal to

$$\begin{aligned}\overline{BC}(Q_s^*)) + \overline{VC}(Q_s^*) &= BC(Q_d^*, X_b(Q_d^*)) + VC(Q_s^*, X_v(Q_s^*)) + BC(Q_s^*, X_b(Q_s^*)) \\ &\quad -BC(Q_d^*, X_v(Q_d^*)) - (p_c^b - p_c^s) \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\} \\ &= BC(Q_s^*, X_b(Q_s^*)) + VC(Q_s^*, X_v(Q_s^*)) \\ &\quad -(p_c^b - p_c^s) \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\} \\ &= SC(Q_s^*, X_s(Q_s^*)).\end{aligned}$$

Recall from Equation (3.13), $SC(Q_s^*, X_s(Q_s^*)) = BC(Q_s^*, X_b(Q_s^*)) + VC(Q_s^*, X_v(Q_s^*)) - (p_c^b - p_c^s) \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\}$. Since the total cost of the system under centralized model with carbon credit sharing is at least as good as the total cost of the buyer and the vendor under decentralized model, we have

$BC(Q_s^*, X_b(Q_s^*)) + VC(Q_s^*, X_v(Q_s^*)) - (p_c^b - p_c^s) \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\} \leqslant BC(Q_d^*, X_b(Q_d^*)) + VC(Q_d^*, X_v(Q_d^*))$, which results in

$VC(Q_d^*, X_v(Q_d^*)) - [VC(Q_s^*, X_v(Q_s^*)) - (p_c^b - p_c^s) \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\}$
$+ BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))] \geqslant 0$. Hence,

$VC(Q_d^*, X_v(Q_d^*)) - \overline{VC}(Q_s^*) \geqslant 0$, implying the vendor is not worse off.

The buyer is indifferent between ordering Q_d^* and Q_s^* since they result in the same cost. Hence, Q_s^* is the channel coordinating order quantity. ■

A.1.3 Proof of Theorem 7

It suffices to show that

i) The buyer orders Q_s^*.

ii) $\overline{BC}(Q_s^*) \leqslant BC(Q_d^*, X_b(Q_d^*))$ (i.e., the buyer is not worse off.)

iii) $\overline{BC}(Q_s^*) + \overline{VC}(Q_c^*) = SC(Q_s^*, X_s(Q_s^*))$
$= BC(Q_s^*, X_b(Q_s^*)) + VC(Q_s^*, X_v(Q_s^*)) - (p_c^b - p_c^s) \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\}$

iv) $\overline{VC}(Q_s^*) \leqslant VC(Q_d^*, X_v(Q_d^*))$ (i.e., the vendor is not worse off.)

Let $Q < Q_s^*$. Then $\overline{BC}(Q) = BC(Q, X_b(Q))$. Since the global optimum of $BC(Q, X_b(Q))$, namely Q_d^*, satisfies $Q_d^* < Q_s^*$, Q_d^* is the optimal solution of $\overline{BC}(Q)$ when $Q < Q_s^*$.

Let $Q \geqslant Q_s^*$. Then

$$\overline{BC}(Q) = BC(Q, X_b(Q)) - d \times D - p_c^b \times min\{-X_b(Q_s^*), X_v(Q_s^*)\}$$

$$= BC(Q, X_b(Q)) - p_c^b \times min\{-X_b(Q_s^*), X_v(Q_s^*)\}$$

$$-\frac{[BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*)) - p_c^b \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\}]}{D} \times D$$

$$= BC(Q, X_b(Q)) - [BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))].$$

Since $BC(Q, X_b(Q))$ is a convex function of Q from Proposition 1 and the term $-[BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*), X_b(Q_d^*)]$ does not depend on Q, $\overline{BC}(Q)$ is also a convex function of Q. Then the global optimum of $\overline{BC}(Q)$ is Q_d^*. However, $Q_d^* \not\geqslant Q_s^*$, implying Q_d^* is not feasible when $Q \geqslant Q_s^*$. Since $\overline{BC}(Q)$ is a convex function in Q, we check the boundary condition, i.e., the buyer orders Q_s^*.

The buyer's cost at Q_s^* is

$$\overline{BC}(Q_s^*) = BC(Q_s^*, X_b(Q_s^*)) - [BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))]$$

$$= BC(Q_d^*, X_b(Q_d^*)).$$

Hence, the buyer is not worse off.

The vendor's cost after coordination is

$\overline{VC}(Q_s^*) = VC(Q_s^*, X_v(Q_s^*)) + p_c^s \times Y + d \times D$, which gives

$$\overline{VC}(Q_s^*) = VC(Q_s^*, X_v(Q_s^*)) + p_c^s \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\}$$

$$+ \frac{[BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*)) - p_c^b \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\}]}{D} \times D$$

$$= VC(Q_s^*, X_v(Q_s^*)) - (p_c^b - p_c^s) \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\}$$

$$+ [BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))].$$

The total cost of the buyer and the vendor after coordination is equal to

$$\overline{BC}(Q_s^*) + \overline{VC}(Q_s^*) = BC(Q_d^*, X_b(Q_d^*)) + VC(Q_s^*, X_v(Q_s^*)) + BC(Q_s^*, X_b(Q_s^*))$$

$$- BC(Q_d^*, X_b(Q_d^*)) - (p_c^b - p_c^s) \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\}$$

$$= BC(Q_s^*, X_b(Q_s^*)) + VC(Q_s^*, X_v(Q_s^*))$$

$$- (p_c^b - p_c^s) \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\}$$

$$= SC(Q_s^*, X_s(Q_s^*)).$$

Recall from Equation (3.13), $SC(Q_s^*, X_s(Q_s^*)) = BC(Q_s^*, X_b(Q_s^*)) + VC(Q_s^*, X_v(Q_s^*)) - (p_c^b - p_c^s) \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\}$. Since the total cost of the system under centralized model with carbon credit sharing is at least as good as the total cost of the buyer and the vendor under decentralized model, we have

$BC(Q_s^*, X_b(Q_s^*)) + VC(Q_s^*, X_v(Q_s^*)) - (p_c^b - p_c^s) \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\} \leq BC(Q_d^*, X_b(Q_d^*)) + VC(Q_d^*, X_v(Q_d^*))$, which results in

$$VC(Q_d^*, X_v(Q_d^*)) - [VC(Q_s^*, X_v(Q_s^*)) - (p_c^b - p_c^s) \times \min\{-X_b(Q_s^*), X_v(Q_s^*)\} + BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))] \geqslant 0.$$

Hence, $VC(Q_d^*, X_v(Q_d^*)) - \overline{VC}(Q_s^*) \geqslant 0$, implying the vendor is not worse off.

The buyer is indifferent between ordering Q_d^* and Q_s^* since they result in the same cost. Hence, Q_s^* is the channel coordinating order quantity. ■

A.1.4 Proof of Theorem 9

It suffices to show that

i) The buyer orders Q_s^*.

ii) $\overline{BC}(Q_s^*) \leqslant BC(Q_d^*, X_b(Q_d^*))$ (i.e., the buyer is not worse off.)

iii) $\overline{BC}(Q_s^*) + \overline{VC}(Q_s^*) = SC(Q_s^*, X_s(Q_s^*))$
$= BC(Q_s^*, X_b(Q_s^*)) + VC(Q_s^*, X_v(Q_s^*)) - (p_c^b - p_c^s) \times \min\{X_b(Q_s^*), -X_v(Q_s^*)\}$

iv) $\overline{VC}(Q_s^*) \leqslant VC(Q_d^*, X_v(Q_d^*))$ (i.e., the vendor is not worse off.)

Let $Q < Q_s^*$. Then $\overline{BC}(Q) = BC(Q, X_b(Q))$. Since the global optimum of $BC(Q, X_b(Q))$, namely Q_d^*, satisfies $Q_d^* < Q_s^*$, Q_d^* is the optimal solution of $\overline{BC}(Q)$ when $Q < Q_s^*$.

Let $Q \geqslant Q_s^*$. Then

$$\overline{BC}(Q) = BC(Q, X_b(Q)) - \bar{d} \times D + p_c^s \times min\{X_b(Q_s^*), -X_v(Q_s^*)\}$$

$$= BC(Q, X_b(Q)) + p_c^s \times min\{X_b(Q_s^*), -X_v(Q_s^*)\}$$

$$- \frac{BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*)) + p_c^s \times \min\{X_b(Q_s^*), -X_v(Q_s^*)\}}{D} \times D$$

$$= BC(Q, X_b(Q)) - [BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))].$$

Since $BC(Q, X_b(Q))$ is a convex function of Q from Proposition 1 and the term $-[BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*), X_b(Q_d^*)]$ does not depend on Q, $\overline{BC}(Q)$ is also a convex function of Q. Then the global optimum of $\overline{BC}(Q)$ is Q_d^*. However, $Q_d^* \ngeqslant Q_s^*$, implying Q_d^* is not feasible when $Q \geqslant Q_s^*$. Since $\overline{BC}(Q)$ is a convex function in Q, we check the boundary condition, i.e., the buyer orders Q_s^*.

The buyer's cost at Q_s^* is

$$\overline{BC}(Q_s^*) = BC(Q_s^*, X_b(Q_s^*)) - [BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))].$$

$$= BC(Q_d^*, X_b(Q_d^*)).$$

Hence, the buyer is not worse off.

The vendor's cost after coordination is

$\overline{VC}(Q_s^*) = VC(Q_s^*, X_v(Q_s^*)) + \bar{d} \times D - p_c^b \times Y$, which gives

$$\overline{VC}(Q_s^*) = VC(Q_s^*, X_v(Q_s^*)) - p_c^b \times \min\{X_b(Q_s^*), -X_v(Q_s^*)\}$$

$$+ \frac{BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*)) + p_c^s \times \min\{X_b(Q_s^*), -X_v(Q_s^*)\}}{D} \times D$$

$$= VC(Q_s^*, X_v(Q_s^*)) - (p_c^b - p_c^s) \times \min\{X_b(Q_s^*), -X_v(Q_s^*)\}$$

$$+ [BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))].$$

The total cost of the buyer and the vendor after coordination is equal to

$$
\begin{aligned}
\overline{BC}(Q_s^*) + \overline{VC}(Q_s^*) &= BC(Q_d^*, X_b(Q_d^*)) + VC(Q_s^*, X_v(Q_s^*)) \\
&\quad - (p_c^b - p_c^s) \times \min\{X_b(Q_s^*), -X_v(Q_s^*)\} + BC(Q_s^*, X_b(Q_s^*)) \\
&\quad - BC(Q_d^*, X_b(Q_d^*)) \\
&= BC(Q_s^*, X_b(Q_s^*)) + VC(Q_s^*, X_v(Q_s^*)) \\
&\quad - (p_c^b - p_c^s) \times \min\{X_b(Q_s^*), -X_v(Q_s^*)\} \\
&= SC(Q_s^*, X_s(Q_s^*)).
\end{aligned}
$$

Recall from Equation (3.13), $SC(Q_s^*, X_s(Q_s^*)) = BC(Q_s^*, X_b(Q_s^*)) + VC(Q_s^*, X_v(Q_s^*)) - (p_c^b - p_c^s) \times \min\{X_b(Q_s^*), -X_v(Q_s^*)\}$. Since the total cost of the system under centralized model with carbon credit sharing is at least as good as the total cost of the buyer and the vendor under decentralized model, we have

$BC(Q_s^*, X_b(Q_s^*)) + VC(Q_s^*, X_v(Q_s^*)) - (p_c^b - p_c^s) \times \min\{X_b(Q_s^*), -X_v(Q_s^*)\} \leqslant BC(Q_d^*, X_b(Q_d^*)) + VC(Q_d^*, X_v(Q_d^*))$, which results in

$$
\begin{aligned}
&VC(Q_d^*, X_v(Q_d^*)) - [VC(Q_s^*, X_v(Q_s^*)) - (p_c^b - p_c^s) \times \min\{X_b(Q_s^*), -X_v(Q_s^*)\} \\
&+BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))] \geqslant 0.
\end{aligned}
$$

Hence, $VC(Q_d^*) - \overline{VC}(Q_s^*) \geqslant 0$, implying the vendor is not worse off.

The buyer is indifferent between ordering Q_d^* and Q_s^* since they result in the same cost. Hence, Q_s^* is the channel coordinating order quantity. ■

A.2 Application of Coordination Mechanisms under Deterministic Demand and Cap-and-Trade Mechanism

Table A.1: Application of Coordination Mechanisms under Cap-and-Trade System

Example	$X_b(Q_s^*)$	$X_v(Q_s^*)$	**Applied Theorem**	**Coordination Strategy**
8	$X_b(Q_s^*) < 0$	$X_v(Q_s^*) > 0$	Theorem 5	Vendor gives 20.811 carbon credits to the buyer and the buyer makes a fixed payment of 75.291 to the vendor for $Q \geqslant Q_s^*$
13	$X_b(Q_s^*) > 0$	$X_v(Q_s^*) < 0$	Theorem 9	Buyer gives 5.929 carbon credits to the vendor and vendor gives a quantity discount of 0.354 per unit to the buyer for $Q \geqslant Q_s^*$
14	$X_b(Q_s^*) > 0$	$X_v(Q_s^*) > 0$	Theorem 3	Vendor gives a quantity discount of 0.078 per unit to the buyer for $Q \geqslant Q_s^*$
15	$X_b(Q_s^*) < 0$	$X_v(Q_s^*) > 0$	Theorem 7	Vendor gives 1.351 carbon credits and a quantity discount of 0.259 per unit to the buyer for $Q \geqslant Q_s^*$
16	$X_b(Q_s^*) > 0$	$X_v(Q_s^*) < 0$	Theorem 10	Buyer gives 4.945 carbon credits to the vendor and vendor gives a quantity discount of 0.673 per unit to the buyer for $Q \leqslant Q_s^*$
17	$X_b(Q_c^*) > 0$	$X_v(Q_c^*) > 0$	Theorem 4	Vendor gives a quantity discount of 0.209 per unit to the buyer for $Q \leqslant Q_s^*$
18	$X_b(Q_s^*) > 0$	$X_v(Q_s^*) > 0$	Theorem 4	Vendor gives a quantity discount of 0.229 per unit to the buyer for $Q \leqslant Q_s^*$
19	$X_b(Q_s^*) > 0$	$X_v(Q_s^*) < 0$	Theorem 10	Buyer gives 6.243 carbon credits to the vendor and vendor gives a quantity discount of 0.253 per unit to the buyer for $Q \leqslant Q_s^*$

Table A.2: The Costs of the Buyer and the Vendor after Coordination for Examples 8 and $13-19$

Example	$\overline{BC}(Q_s^*)$	$\overline{VC}(Q_s^*)$
8	979.983	422.365
13	746.107	614.521
14	627.826	552.493
15	739.996	848.166
16	644.981	818.75
17	1153.989	637.706
18	538.516	525.353
19	912.817	606.116

A.3 Proofs of Coordination Mechanisms under Deterministic Demand and Carbon Tax Mechanism

A.3.1 Proof of Theorem 12

It suffices to show that

i) The buyer orders Q_c^*.

ii) Giving quantity discount to the vendor is the convenient coordination mechanism at Q_c^*.

iii) $\overline{BC}(Q_c^*) \leqslant BC(Q_d^*)$ (i.e., the buyer is not worse off.)

iv) $\overline{VC}(Q_c^*) \leqslant VC(Q_d^*)$ (i.e., the vendor is not worse off.)

Let $Q < Q_c^*$. Then $\overline{BC}(Q) = BC(Q)$. Since $BC(Q)$ is a strictly convex function of Q, $\frac{\partial BC(Q)}{\partial Q} = 0$ gives Q_d^* as the optimal order quantity.

Let $Q \geqslant Q_c^*$. Then

$$\begin{aligned}\overline{BC}(Q) &= BC(Q) - d \times D = BC(Q) - \frac{BC(Q_c^*) - BC(Q_d^*)}{D} \times D \\ &= BC(Q) - [BC(Q_c^*) - BC(Q_d^*)].\end{aligned}$$

Since the term $-[BC(Q_c^*) - BC(Q_d^*)]$ does not depend on Q, $\overline{BC}(Q)$ is a strictly convex function of Q and $\frac{\partial \overline{BC}(Q)}{\partial Q} = 0$ gives Q_d^* as the optimal order quantity. However, $Q_d^* \ngeqslant Q_c^*$. Since $\overline{BC}(Q)$ is a strictly convex function of Q, we check the boundary condition, i.e., the buyer orders Q_c^*.

Suppose $2f_bD - g_bQ_c^*Q_d^* \geqslant 0$. Since $Q_d^* < Q_c^*$, $BT(Q_c^*) \leqslant BT(Q_d^*)$ from Proposition 4. Thus, the buyer's average annual tax does not increase under centralized solution and he/she does not need to be compensated by paying his/her taxes. Suppose $2f_vP - g_vQ_c^*Q_d^* \leqslant 0$. Since $Q_d^* < Q_c^*$, $VT(Q_c^*) \geqslant VT(Q_d^*)$ from from Proposition 5. Thus, the vendor's average annual tax does not decrease under centralized solution and it is not plausible for the vendor to pay the buyer's taxes to compensate his/her loss. Therefore, quantity discount should be given to the vendor at Q_c^*.

The buyer's cost at Q_c^* is

$$\overline{BC}(Q_c^*) = BC(Q_c^*) - [BC(Q_c^*) - BC(Q_d^*)] = BC(Q_d^*).$$

Hence, the buyer is not worse off.

The vendor's cost after coordination is

$\overline{VC}(Q_c^*) = VC(Q_c^*) + BC(Q_c^*) - BC(Q_d^*)$, which gives

$$VC(Q_d^*) - \overline{VC}(Q_c^*) = [VC(Q_d^*) - VC(Q_c^*)] - [BC(Q_c^*) - BC(Q_d^*)].$$

Since $VC(Q_d^*) - VC(Q_c^*) \geqslant BC(Q_c^*) - BC(Q_d^*)$ (i.e., the vendor's gain from the centralized solution is no less than the buyer's loss from the decentralized solution) due to channel coordination principles, $VC(Q_d^*) - \overline{VC}(Q_c^*) \geqslant 0$. Hence, the vendor is not worse off.

The buyer is indifferent between ordering Q_d^* and Q_c^* since they result in the same cost. Hence, Q_c^* is the channel coordinating order quantity. ■

A.3.2 Proof of Theorem 14

It suffices to show that

i) The buyer orders Q_c^*.

ii) Paying $BC(Q_c^*) - BC(Q_d^*)$ amount of buyer's average annual tax is the convenient coordination mechanism at Q_c^*.

iii) $\overline{BC}(Q_c^*) \leqslant BC(Q_d^*)$ (i.e., the buyer is not worse off.)

iv) $\overline{VC}(Q_c^*) \leqslant VC(Q_d^*)$ (i.e., the vendor is not worse off.)

Let $Q < Q_c^*$. Then $\overline{BC}(Q) = BC(Q)$. Since $BC(Q)$ is a strictly convex function in Q, $\frac{\partial BC(Q)}{\partial Q} = 0$ gives Q_d^* as the optimal order quantity.

Let $Q \geqslant Q_c^*$. Then

$$\overline{BC}(Q) = BC(Q) - [BC(Q_c^*) - BC(Q_d^*)] = BC(Q) + BC(Q_d^*) - BC(Q_c^*).$$

Since the term $BC(Q_d^*) - BC(Q_c^*)$ does not depend on Q, $\overline{BC}(Q)$ is a strictly convex function of Q and $\frac{\partial \overline{BC}(Q)}{\partial Q} = 0$ gives Q_d^* as the optimal order quantity. However, $Q_d^* \ngeqslant Q_c^*$. Since $\overline{BC}(Q)$ is a strictly convex function of Q, we check the boundary condition, i.e., the buyer orders Q_c^*.

Since $\min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\} \geqslant BC(Q_c^*) - BC(Q_d^*)$, $BT(Q_c^*) - BT(Q_d^*) \geqslant BC(Q_c^*) - BC(Q_d^*)$ and $VT(Q_d^*) - VT(Q_c^*) \geqslant BC(Q_c^*) - BC(Q_d^*)$. That is, buyer's average annual tax increases and vendor's average annual tax decreases by an amount at least as large as the buyer's loss from the centralized solution. Then, the vendor can compensate the buyer by paying $BC(Q_c^*) - BC(Q_d^*)$ amount of his/her average annual tax if he/she orders Q_c^*.

The buyer's cost at Q_c^* is

$$\overline{BC}(Q_c^*) = BC(Q_c^*) - [BC(Q_c^*) - BC(Q_d^*)] = BC(Q_d^*).$$

Hence, the buyer is not worse off.

The vendor's cost after coordination is

$\overline{VC}(Q_c^*) = VC(Q_c^*) + BC(Q_c^*) - BC(Q_d^*)$, which gives

$VC(Q_d^*) - \overline{VC}(Q_c^*) = [VC(Q_d^*) - VC(Q_c^*)] - [BC(Q_c^*) - BC(Q_d^*)]$.

Since $VC(Q_d^*) - VC(Q_c^*) \geqslant BC(Q_c^*) - BC(Q_d^*)$ (i.e., the vendor's gain from the centralized solution is no less than the buyer's loss from the decentralized solution) due to channel coordination principles, $VC(Q_d^*) - \overline{VC}(Q_c^*) \geqslant 0$. Hence, the vendor is not worse off.

The buyer is indifferent between ordering Q_d^* and Q_c^* since they result in the same cost. Hence, Q_c^* is the channel coordinating order quantity. ■

A.3.3 Proof of Theorem 16

It suffices to show that

i) The buyer orders Q_c^*.

ii) The vendor pays $\min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\}$ amount of the buyer's taxes and gives additional quantity discount at Q_c^*.

iii) $\overline{BC}(Q_c^*) \leqslant BC(Q_d^*)$ (i.e., the buyer is not worse off.)

iv) $\overline{VC}(Q_c^*) \leqslant VC(Q_d^*)$ (i.e., the vendor is not worse off.)

Let $Q < Q_c^*$. Then $\overline{BC}(Q) = BC(Q)$. Since $BC(Q)$ is a strictly convex function of Q, $\frac{\partial BC(Q)}{\partial Q} = 0$ gives Q_d^* as the optimal order quantity.

Let $Q \geqslant Q_c^*$. Then

$$\begin{aligned}
\overline{BC}(Q) &= BC(Q) - \bar{d} \times D - \min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\} \\
&= BC(Q) - \min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\} \\
-D \times & \frac{[BC(Q_c^*) - BC(Q_d^*) - \min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\}]}{D} \\
&= BC(Q) - [BC(Q_c^*) - BC(Q_d^*)].
\end{aligned}$$

Since the term $-[BC(Q_c^*) - BC(Q_d^*)]$ does not depend on Q, $\overline{BC}(Q)$ is a strictly convex function of Q and $\frac{\partial \overline{BC}(Q)}{\partial Q} = 0$ gives Q_d^* as the optimal order quantity. However, $Q_d^* \ngeqslant Q_c^*$. Since $\overline{BC}(Q)$ is a strictly convex function of Q, we check the boundary condition, i.e., the buyer orders Q_c^*.

Since $2f_bD - g_bQ_c^*Q_d^* \leqslant 0$ and $Q_d^* < Q_c^*$, $BT(Q_c^*) \geqslant BT(Q_d^*)$ from Proposition 4. Also, since $2f_vP - g_vQ_c^*Q_d^* \geqslant 0$ and $Q_d^* < Q_c^*$, $VT(Q_c^*) \leqslant VT(Q_d^*)$ from Proposition 5. Since $\min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\} \leqslant BC(Q_c^*) - BC(Q_d^*)$, either the vendor's gain in average annual taxes is not sufficient to compensate the buyer's loss in centralized solution or the buyer's loss is greater than his/her loss in average annual taxes in centralized solution. Thus, the buyer should pay $\min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\}$ amount of the vendor's taxes and give additional quantity discount if the buyer orders Q_c^*.

The buyer's cost at Q_c^* is

$$\overline{BC}(Q_c^*) = BC(Q_c^*) - [BC(Q_c^*) - BC(Q_d^*)] = BC(Q_d^*).$$

Hence, the buyer is not worse off.

The vendor's cost after coordination is

$$\overline{VC}(Q_c^*) = VC(Q_c^*) + BC(Q_c^*) - BC(Q_d^*), \text{ which gives}$$

$$VC(Q_d^*) - \overline{VC}(Q_c^*) = [VC(Q_d^*) - VC(Q_c^*)] - [BC(Q_c^*) - BC(Q_d^*)].$$

Since $VC(Q_d^*) - VC(Q_c^*) \geqslant BC(Q_c^*) - BC(Q_d^*)$ (i.e., the vendor's gain from the centralized solution is no less than the buyer's loss from the decentralized solution) due to channel coordination principles, $VC(Q_d^*) - \overline{VC}(Q_c^*) \geqslant 0$. Hence, the vendor is not worse off.

The buyer is indifferent between ordering Q_d^* and Q_c^* since they result in the same cost. Hence, Q_c^* is the channel coordinating order quantity. ■

A.4 Application of Coordination Mechanisms under Deterministic Demand and Carbon Tax Mechanism

Example 20: As seen in Table 4.3, $Q_d^* < Q_c^*$. Also, using Table 4.8, $2f_bD - g_bQ_c^*Q_d^* > 0$ and $2f_vP - g_vQ_c^*Q_d^* < 0$, i.e., $BT(Q_c^*) < BT(Q_d^*)$ and $VT(Q_c^*) > VT(Q_d^*)$. That is, the buyer's average annual tax decreases and the vendor's average annual tax increases under the centralized solution. The buyer's loss from ordering the centralized optimal quantity is $BC(Q_c^*) - BC(Q_d^*) = 10.8789$. Since $BT(Q_c^*) < BT(Q_d^*)$, the buyer does not need to be compensated by paying his/her taxes. Then according to Theorem 12, loss of the buyer should be compensated by giving quantity discount. The corresponding quantity discount to the loss of the buyer is $10.8789/D = 10.8789/90 = 0.1209$ per unit. Hence, the corresponding coordination strategy of the vendor is to give a quantity discount of 0.1209 per unit for order sizes greater than or equal to $Q_c^* = 180.0433$ to the buyer. The costs of the buyer and the vendor after coordination are $\overline{BC}(Q_c^*) = BC(Q_d^*) = 2045.1418$ and $\overline{VC}(Q_c^*) = 2864.6082$.

Example 21: As seen in Table 4.3, $Q_d^* < Q_c^*$. Also, using Table 4.8, $2f_bD - g_bQ_c^*Q_d^* < 0$ and $2f_vP - g_vQ_c^*Q_d^* < 0$, i.e., $BT(Q_c^*) > BT(Q_d^*)$ and $VT(Q_c^*) > VT(Q_d^*)$. That is, both the buyer and the vendor's average annual taxes increase under the centralized solution. The buyer's loss from ordering the centralized optimal quantity is $BC(Q_c^*) - BC(Q_d^*) = 8.2355$. Since $VT(Q_c^*) > VT(Q_d^*)$, it is not plausible for the vendor to pay the buyer's loss in average annual taxes. Then according to Theorem 12, loss of the buyer should be compensated by giving quantity discount. The corresponding quantity discount to the loss of the buyer is $8.2355/D = 8.2355/50 = 0.1647$ per unit. Hence, the corresponding coordination strategy of the vendor is to give a quantity discount of 0.1647 per unit for order sizes greater than or equal to $Q_c^* = 169.6053$ to the buyer. The costs of the buyer and the vendor after coordination are $\overline{BC}(Q_c^*) = BC(Q_d^*) = 1522.7128$ and $\overline{VC}(Q_c^*) = 1623.7183$.

Example 23: As seen in Table 4.3, $Q_d^* < Q_c^*$. Also, using Table 4.8, $2f_bD - g_bQ_c^*Q_d^* > 0$ and $2f_vP - g_vQ_c^*Q_d^* > 0$, i.e., $BT(Q_c^*) < BT(Q_d^*)$ and $VT(Q_c^*) < VT(Q_d^*)$. That is, both the buyer and the vendor's average annual taxes decrease under the centralized solution. The buyer's loss from ordering the centralized optimal quantity is $BC(Q_c^*) - BC(Q_d^*) = 14.6105$. Since $BT(Q_c^*) < BT(Q_d^*)$, the buyer does not need to be compensated by paying his/her taxes. Then according to Theorem 12, loss of the buyer should be compensated by giving quantity discount. The corresponding quantity discount to the loss of the buyer is $14.6105/D = 14.6105/50 = 0.2922$ per unit. Hence, the corresponding coordination strategy of the vendor is to give a quantity discount of 0.2922 per unit for order sizes greater than or equal to $Q_c^* = 93.1711$ to the buyer. The costs of the buyer and the vendor after coordination are $\overline{BC}(Q_c^*) = BC(Q_d^*) = 1218.3282$ and $\overline{VC}(Q_c^*) = 1479.052$.

Example 26: As seen in Table 4.3, $Q_d^* > Q_c^*$. Also, using Table 4.8, $2f_bD - g_bQ_c^*Q_d^* > 0$ and $2f_vP - g_vQ_c^*Q_d^* > 0$, i.e., $BT(Q_c^*) > BT(Q_d^*)$ and $VT(Q_c^*) > VT(Q_d^*)$. That is, both the buyer and the vendor's average annual taxes increase under the centralized solution. The buyer's loss from ordering the centralized optimal quantity is $BC(Q_c^*) - BC(Q_d^*) = 2.0111$. Since $VT(Q_c^*) > VT(Q_d^*)$, it is not plausible for the vendor to pay the buyer's loss in average annual taxes. Then according to Theorem 13, loss of the buyer should be compensated by giving quantity discount. The corresponding quantity discount to the loss of the buyer is $2.0111/D = 2.0111/40 = 0.0503$ per unit. Hence, the corresponding coordination strategy of the vendor is to give a quantity discount of 0.0503 per unit for order sizes less than or equal to $Q_c^* = 158.5226$ to the buyer. The costs of the buyer and the vendor after coordination are $\overline{BC}(Q_c^*) = BC(Q_d^*) = 1910.4665$ and $\overline{VC}(Q_c^*) = 147.8282$.

Example 29: As seen in Table 4.3, $Q_d^* > Q_c^*$. Also, using Table 4.8, $2f_bD - g_bQ_c^*Q_d^* < 0$ and $2f_vP - g_vQ_c^*Q_d^* < 0$, i.e., $BT(Q_c^*) < BT(Q_d^*)$ and $VT(Q_c^*) < VT(Q_d^*)$. That is, both the buyer and the vendor's average annual taxes decrease under the centralized solution. The buyer's loss from ordering the centralized optimal quantity is $BC(Q_c^*) - BC(Q_d^*) = 8.4815$. Since $BT(Q_c^*) < BT(Q_d^*)$, the buyer does not need to be compensated by paying his/her taxes. Then according to Theorem 13, loss of the buyer should be compensated by giving quantity discount. The corresponding quantity discount to the loss of the buyer is $8.4815/D = 14.6105/50 = 0.1696$ per unit. Hence, the corresponding coordination strategy of the vendor is to give a quantity discount of 0.1696 per unit for order sizes less than or equal to $Q_c^* = 140.6422$ to the buyer. The costs of the buyer and the vendor after coordination are $\overline{BC}(Q_c^*) = BC(Q_d^*) = 1564.3289$ and $\overline{VC}(Q_c^*) = 1545.5539$.

Example 30: As seen in Table 4.3, $Q_d^* > Q_c^*$. Also, using Table 4.8, $2f_bD - g_bQ_c^*Q_d^* < 0$ and $2f_vP - g_vQ_c^*Q_d^* > 0$, i.e., $BT(Q_c^*) < BT(Q_d^*)$ and $VT(Q_c^*) > VT(Q_d^*)$. That is, the buyer's average annual tax decreases and the vendor's average annual tax increases under the centralized solution. The buyer's loss from ordering the centralized optimal quantity is $BC(Q_c^*) - BC(Q_d^*) = 0.1822$. Since $BT(Q_c^*) < BT(Q_d^*)$, the buyer does not need to be compensated by paying his/her taxes. Then according to Theorem 13, loss of the buyer should be compensated by giving quantity discount. The corresponding quantity discount to the loss of the buyer is $0.1822/D = 0.1822/40 = 0.0046$ per unit. Hence, the corresponding coordination strategy of the vendor is to give a quantity discount of 0.0046 per unit for order sizes less than or equal to $Q_c^* = 137.3606$ to the buyer. The costs of the buyer and the vendor after coordination are $\overline{BC}(Q_c^*) = BC(Q_d^*) = 1281.8429$ and $\overline{VC}(Q_c^*) = 1166.1681$.

Example 31: As seen in Table 4.3, $Q_d^* > Q_c^*$. Also, using Table 4.8, $2f_bD - g_bQ_c^*Q_d^* > 0$ and $2f_vP - g_vQ_c^*Q_d^* < 0$, i.e., $BT(Q_c^*) > BT(Q_d^*)$ and $VT(Q_c^*) < VT(Q_d^*)$. That is, the buyer's average annual tax increases and the vendor's average annual tax decreases under the centralized solution. The buyer's loss from ordering the centralized optimal quantity is $BC(Q_c^*) - BC(Q_d^*) = 55.0751$. However, $\min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\} = BT(Q_c^*) - BT(Q_d^*) = 18.1009 < BC(Q_c^*) - BC(Q_d^*) = 55.0751$. That is, paying the buyer's loss in average annual taxes is not sufficient to compensate his loss from ordering centralized optimal quantity. Then according to Theorem 17, the remaining loss of the buyer should be compensated by giving quantity discount. The remaining loss of the buyer is $55.0751 - 18.1009 = 36.9742$ and the corresponding quantity discount is $36.9742/D = 36.9742/500 = 0.0739$ per unit. Hence, the corresponding coordination strategy of the vendor is to pay 18.1009 amount of the buyer's taxes and to give him/her a quantity discount of 0.0739 per unit for order sizes less than or equal to $Q_c^* = 531.7743$. The costs of the buyer and the vendor after coordination are $\overline{BC}(Q_c^*) = BC(Q_d^*) = 11933.105$ and $\overline{VC}(Q_c^*) = 13196.2544$.

Example 32: As seen in Table 4.6, $Q_d^* < Q_c^*$. Also, using Table 4.8, $2f_bD - g_bQ_c^*Q_d^* < 0$ and $2f_vP - g_vQ_c^*Q_d^* > 0$, i.e., $BT(Q_c^*) > BT(Q_d^*)$ and $VT(Q_c^*) < VT(Q_d^*)$. That is, the buyer's average annual tax increases and the vendor's average annual tax decreases under the centralized solution. The buyer's loss from ordering the centralized optimal quantity is $BC(Q_c^*) - BC(Q_d^*) = 25.2767$. Also, $\min\{BT(Q_c^*) - BT(Q_d^*), VT(Q_d^*) - VT(Q_c^*)\} = BT(Q_c^*) - BT(Q_d^*) = 26.2282 > BC(Q_c^*) - BC(Q_d^*) = 25.2767$. That is, the decrease in the vendor's average annual taxes is greater than the increase in buyer's average annual taxes. Then according to Theorem 14, loss of the buyer is compensated when the vendor pays $BC(Q_c^*) - BC(Q_d^*) = 25.2767$ amount of the buyer's taxes for order sizes greater than or equal to $Q_c^* = 53.7924$. The costs of the buyer and the vendor after coordination are $\overline{BC}(Q_c^*) = BC(Q_d^*) = 1232.8427$ and $\overline{VC}(Q_c^*) = 1549.2174$.

Table A.3: Application of Coordination Mechanisms under Carbon Tax Mechanism

Example Index	$BC(Q_c^*) - BC(Q_d^*)$	**Applied Theorem**	**Coordination Strategy**	$\overline{BC}(Q_c^*)$	$\overline{VC}(Q_c^*)$
20	10.8789	Theorem 12	Give a quantity discount of 0.1209 per unit for $Q \geqslant Q_c^*$	2045.1418	2864.6082
21	8.2355	Theorem 12	Give a quantity discount of 0.1647 per unit for $Q \geqslant Q_c^*$	1522.7128	1623.7183
23	14.6105	Theorem 12	Give a quantity discount of 0.2922 per unit for $Q \geqslant Q_c^*$	1218.3282	1479.052
26	2.0111	Theorem 13	Give a quantity discount of 0.0503 per unit for $Q \leqslant Q_c^*$	1910.4665	147.8282
29	8.4815	Theorem 13	Give a quantity discount of 0.1696 per unit for $Q \leqslant Q_c^*$	1564.3289	1545.5539

Table A.4: Application of Coordination Mechanisms Proposed in Section 4.1.3 (Continued)

Example Index	$BC(Q_c^*)$ $-BC(Q_d^*)$	**Applied Theorem**	**Coordination Strategy**	$\overline{BC}(Q_c^*)$	$\overline{VC}(Q_c^*)$
30	0.1822	Theorem 13	Give a quantity discount of 0.0046 per unit for $Q \leqslant Q_c^*$	1281.8429	1166.1681
31	55.0751	Theorem 17	Pay 18.1009 amount of the buyer's taxes and give a quantity discount of 0.0739 per unit for $Q \leqslant Q_c^*$	11933.105	13196.2544
32	25.2767	Theorem 14	Pay 25.2767 amount of the buyer's taxes for $Q \geqslant Q_c^*$	1232.8427	1549.2174

A.5 Proofs of Coordination Mechanisms under Deterministic Demand and Carbon Cap Mechanism

A.5.1 Proof of Theorem 20

It suffices to show that

i) The buyer orders Q_c^*.

ii) $\overline{BC}(Q_c^*) \leqslant BC(Q_d^*)$ (i.e., the buyer is not worse off.)

iii) $\overline{VC}(Q_c^*) \leqslant VC(Q_d^*)$ (i.e., the vendor is not worse off.)

Let $Q < Q_c^*$. Then $\overline{BC}(Q) = BC(Q)$. Since $BC(Q)$ is a strictly convex function of Q, $\frac{\partial BC(Q)}{\partial Q} = 0$ gives Q_d^* as the optimal order quantity.

Let $Q \geqslant Q_c^*$. Then

$$\begin{aligned}\overline{BC}(Q) &= BC(Q) - d \times D = BC(Q) - \frac{BC(Q_c^*) - BC(Q_d^*)}{D} \times D \\ &= BC(Q) - [BC(Q_c^*) - BC(Q_d^*)].\end{aligned}$$

Since the term $-[BC(Q_c^*) - BC(Q_d^*)]$ does not depend on Q, $\overline{BC}(Q)$ is a strictly convex function of Q and $\frac{\partial \overline{BC}(Q)}{\partial Q} = 0$ gives Q_d^* as the optimal order quantity. However, $Q_d^* \ngeqslant Q_c^*$. Since $\overline{BC}(Q)$ is a strictly convex function of Q, we check the boundary condition, i.e., the buyer orders Q_c^*.

The buyer's cost at Q_c^* is

$$\overline{BC}(Q_c^*) = BC(Q_c^*) - [BC(Q_c^*) - BC(Q_d^*)] = BC(Q_d^*).$$

Hence, the buyer is not worse off.

The vendor's cost after coordination is

$\overline{VC}(Q_c^*) = VC(Q_c^*) + BC(Q_c^*) - BC(Q_d^*)$, which gives

$$VC(Q_d^*) - \overline{VC}(Q_c^*) = [VC(Q_d^*) - VC(Q_c^*)] - [BC(Q_c^*) - BC(Q_d^*)].$$

Since $VC(Q_d^*) - VC(Q_c^*) \geqslant BC(Q_c^*) - BC(Q_d^*)$ (i.e., the vendor's gain from the centralized solution is no less than the buyer's loss from the decentralized solution) due to channel coordination principles, $VC(Q_d^*) - \overline{VC}(Q_c^*) \geqslant 0$. Hence, the vendor is not worse off.

The buyer is indifferent between ordering Q_d^* and Q_c^* since they result in the same cost. Hence, Q_c^* is the channel coordinating order quantity. ■

A.6 Application of Coordination Mechanisms under Deterministic Demand and Carbon Cap Mechanism

Example 33: As seen in Tables 4.14 and 4.15, $Q_d^* = Q_c^*$. Thus, the channel is already coordinated and there is no need to apply a coordination mechanism.

Example 34: As seen in Tables 4.14 and 4.15, $Q_d^* < Q_c^*$. The buyer's loss from ordering the centralized optimal quantity is $BC(Q_c^*) - BC(Q_d^*) = 4.8683$.

According to Theorem 20, loss of the buyer should be compensated by giving quantity discount. Hence, the corresponding coordination strategy of the vendor is to give a quantity discount of $4.8683/D = 4.8683/50 = 0.0974$ per unit for order sizes greater than or equal to $Q_c^* = 63.2456$ to the buyer. The costs of the buyer and the vendor after coordination are $\overline{BC}(Q_c^*) = BC(Q_d^*) = 690$ and $\overline{VC}(Q_c^*) = 468.113883$.

Example 35: As seen in Tables 4.14 and 4.15, $Q_d^* = Q_c^*$. Thus, the channel is already coordinated and there is no need to apply a coordination mechanism.

Example 36: As seen in Tables 4.14 and 4.15 in, $Q_d^* < Q_c^*$. The buyer's loss from ordering the centralized optimal quantity is $BC(Q_c^*) - BC(Q_d^*) = 7.1704$. According to Theorem 20, loss of the buyer should be compensated by giving quantity discount. Hence, the corresponding coordination strategy of the vendor is to give a quantity discount of $7.1704/D = 7.1704/50 = 0.1434$ per unit for order sizes greater than or equal to $Q_c^* = 100$ to the buyer. The costs of the buyer and the vendor after coordination are $\overline{BC}(Q_c^*) = BC(Q_d^*) = 715.3296$ and $\overline{VC}(Q_c^*) = 607.1704$.

Example 37: As seen in Tables 4.14 and 4.15, $Q_d^* = Q_c^*$. Thus, the channel is already coordinated and there is no need to apply a coordination mechanism.

Example 38: As seen in Tables 4.14 and 4.15, $Q_d^* = Q_c^*$. Thus, the channel is already coordinated and there is no need to apply a coordination mechanism.

Example 39: As seen in Tables 4.14 and 4.15, $Q_d^* > Q_c^*$. The buyer's loss from ordering the centralized optimal quantity is $BC(Q_c^*) - BC(Q_d^*) = 0.1345$. According to Theorem 21, loss of the buyer should be compensated by giving quantity discount. Hence, the corresponding coordination strategy of the vendor is to give a quantity discount of $0.1345/D = 0.1345/50 = 0.0027$ per unit for order sizes less than or equal to $Q_c^* = 138.7596$ to the buyer. The costs of the buyer and the vendor after coordination are $\overline{BC}(Q_c^*) = BC(Q_d^*) = 773.2051$ and $\overline{VC}(Q_c^*) = 477.6205$.

Example 40: As seen in Tables 4.14 and 4.15, $Q_d^* < Q_c^*$. The buyer's loss from ordering the centralized optimal quantity is $BC(Q_c^*) - BC(Q_d^*) = 4.9341$. According to Theorem 20, loss of the buyer should be compensated by giving quantity discount. Hence, the corresponding coordination strategy of the vendor is to give a quantity discount of $4.9341/D = 4.9341/50 = 0.0987$ per unit for order sizes greater than or equal to $Q_c^* = 95.9166$ to the buyer. The costs of the buyer and the vendor after coordination are $\overline{BC}(Q_c^*) = BC(Q_d^*) = 748.324$ and $\overline{VC}(Q_c^*) = 491.4676$.

Example 41: As seen in Tables 4.14 and 4.15, $Q_d^* > Q_c^*$. The buyer's loss from ordering the centralized optimal quantity is $BC(Q_c^*) - BC(Q_d^*) = 4.2618$. According to Theorem 21, loss of the buyer should be compensated by giving quantity discount. Hence, the corresponding coordination strategy of the vendor is to give a quantity discount of $4.2618/D = 4.2618/50 = 0.0852$ per unit for order sizes less than or equal to $Q_c^* = 161.4083$ to the buyer. The costs of the buyer and the vendor after coordination are $\overline{BC}(Q_c^*) = BC(Q_d^*) = 800.3517$ and $\overline{VC}(Q_c^*) = 478.4444$.

Example 42: As seen in Tables 4.14 and 4.15, $Q_d^* < Q_c^*$. The buyer's loss from ordering the centralized optimal quantity is $BC(Q_c^*) - BC(Q_d^*) = 1.3932$. According to Theorem 20, loss of the buyer should be compensated by giving quantity discount. Hence, the corresponding coordination strategy of the vendor is to give a quantity discount of $1.3932/D = 1.3932/50 = 0.0852$ per unit for order sizes greater than or equal to $Q_c^* = 100$ to the buyer. The costs of the buyer and the vendor after coordination are $\overline{BC}(Q_c^*) = BC(Q_d^*) = 823.6068$ and $\overline{VC}(Q_c^*) = 739.7265$.

Example 43: As seen in Tables 4.14 and 4.15, $Q_d^* = Q_c^*$. Thus, the channel is already coordinated and there is no need to apply a coordination mechanism.

Example 44: As seen in Tables 4.14 and 4.15, $Q_d^* = Q_c^*$. Thus, the channel is already coordinated and there is no need to apply a coordination mechanism.

Example 45: As seen in Tables 4.14 and 4.15, $Q_d^* > Q_c^*$. The buyer's loss from ordering the centralized optimal quantity is $BC(Q_c^*) - BC(Q_d^*) = 0.5103$. According to Theorem 21, loss of the buyer should be compensated by giving quantity discount. Hence, the corresponding coordination strategy of the vendor is to give a quantity discount of $0.5103/D = 0.5103/50 = 0.0852$ per unit for order sizes less than or equal to $Q_c^* = 77.556$ to the buyer. The costs of the buyer and the vendor after coordination are $\overline{BC}(Q_c^*) = BC(Q_d^*) = 773.75$ and $\overline{VC}(Q_c^*) = 448.3044$.

Example 46: As seen in Tables 4.14 and 4.15, $Q_d^* = Q_c^*$. Thus, the channel is already coordinated and there is no need to apply a coordination mechanism.

Example 47: As seen in Tables 4.14 and 4.15, $Q_d^* > Q_c^*$. The buyer's loss from ordering the centralized optimal quantity is $BC(Q_c^*) - BC(Q_d^*) = 6.2844$. According to Theorem 21, loss of the buyer should be compensated by giving quantity discount. Hence, the corresponding coordination strategy of the vendor is to give a quantity discount of $6.2844/D = 6.2844/50 = 0.1257$ per unit for order sizes less than or equal to $Q_c^* = 143.0194$ to the buyer. The costs of the buyer and the vendor after coordination are $\overline{BC}(Q_c^*) = BC(Q_d^*) = 826.3605$ and $\overline{VC}(Q_c^*) = 488.2822$.

Example 48: As seen in Tables 4.14 and 4.15, $Q_d^* = Q_c^*$. Thus, the channel is already coordinated and there is no need to apply a coordination mechanism.

Table A.5: Application of Coordination Mechanisms under Carbon Cap Mechanism

Example Index	$BC(Q_c^*)$ $-BC(Q_d^*)$	**Applied Theorem**	**Quantity Discount**	$\overline{BC}(Q_c^*)$	$\overline{VC}(Q_c^*)$
33	-	-	-	700.7165	459.1886
34	4.8683	Theorem 20	0.0974 for $Q \geqslant Q_c^*$	690	468.1139
35	-	-	-	671.25	453.75
36	7.1704	Theorem 20	0.1434 for $Q \geqslant Q_c^*$	715.3296	607.1704
37	-	-	-	717.2259	611.3446
38	-	-	-	766.5115	1477.486
39	0.1345	Theorem 21	0.0027 for $Q \leqslant Q_c^*$	773.2051	477.6205
40	4.9341	Theorem 20	0.0987 for $Q \geqslant Q_c^*$	748.324	491.4676
41	4.2618	Theorem 21	0.0852 for $Q \leqslant Q_c^*$	800.3517	478.4444
42	1.3932	Theorem 20	0.0279 for $Q \geqslant Q_c^*$	823.6068	739.7265
43	-	-	-	850	738.3333
44	-	-	-	773.75	450
45	0.5103	Theorem 21	0.0102 for $Q \leqslant Q_c^*$	773.75	448.3044
46	-	-	-	848.4415	513.4473
47	6.2844	Theorem 21	0.1257 for $Q \leqslant Q_c^*$	826.3605	488.2822
48	-	-	-	850.4478	539.9243

Appendix B

Proofs and Applications of Coordination Theorems under Stochastic Demand

B.1 Proofs of Coordination Theorems under Stochastic Demand and Carbon Tax Mechanism

B.1.1 Proof of Theorem 28

It suffices to show that

i) The buyer orders Q_c^*.

ii) $\overline{J}_b(Q_c^*) \geqslant J_b(Q_d^*, c_b)$

Let $Q < Q_c^*$. Then $\overline{J}_b(Q) = J_b(Q, c_b)$. Since $J_b(Q, c_b)$ is a strictly concave function in Q, $\partial J_b(Q, c_b)/\partial Q = 0$ gives Q_d^* as the optimal order quantity.

Let $Q \geqslant Q_c^*$. Then the expected profit function of the buyer is

$$\overline{J}_b(Q) = -(t_t + c_v)Q + \int_0^Q [px + (Q-x)v] f(x)\,\mathrm{d}x + \int_Q^\infty [pQ - (x-Q)s] f(x)\,\mathrm{d}x \\ - J_b(Q_c^*, c_b - d) + J_b(Q_d^*, c_b). \tag{B.1}$$

Since $\overline{J}_b(.)$ is a strictly concave function, the maximizer of Expression (B.1) can be found by checking the first order condition. That is,

$$\overline{J}_b^{'}(Q) = -(t_t + c_v) + pQf(Q) + \int_0^Q vf(x)\,\mathrm{d}x + \int_Q^\infty (p+s)f(x)\,\mathrm{d}x - pQf(Q) = 0.$$

This gives

$$F(Q_{crd}) = \frac{p + s - c_v - t_t}{p + s + v}.$$

This results in

$$Q_{crd} = F^{-1}\left(\frac{p + s - c_v - t_t}{p + s + v}\right) = Q_c^*.$$

Hence, the buyer orders Q_c^*. The buyer's expected profit after coordination is

$$\overline{J}_b(Q_c^*) = J_b(Q_c^*, c_b - d) - [J_b(Q_c^*, c_b - d) - J_b(Q_d^*, c_b)] = J_b(Q_d^*, c_b).$$

This implies that the buyer stays in a no worse situation by ordering Q_c^* units. ■

B.1.2 Proof of Theorem 29

It suffices to show that

i) $\overline{J}_b(Q_c^*) \geqslant J_b(Q_d^*, c_b)$

ii) The buyer orders Q_c^*.

The expected profit function after coordination is given by

$$\overline{J}_b(Q) = -(c_b + t_b - d)Q + \int_0^Q [px + (Q-x)v]f(x)\,\mathrm{d}x + \int_Q^\infty [pQ - (x-Q)s]f(x)\,\mathrm{d}x.$$

This is equivalent to

$$\begin{aligned}\overline{J}_b(Q) =&(v - c_b - t_b + \frac{J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b)}{Q_c^*})Q + (p-v)\mu \\ &- (p+s-v)\int_Q^\infty (x-Q)f(x)\,\mathrm{d}x.\end{aligned}$$

where $\mu = \int_0^\infty xf(x)\,\mathrm{d}x$ is the expected value of the demand.

Similarly, the expected profit at Q_c^* after coordination is given by

$$\begin{aligned}\overline{J}_b(Q_c^*) =&(v - c_b - t_b)Q_c^* + (p-v)\mu - (p+s-v)\int_{Q_c^*}^\infty (x-Q_c^*)f(x)\,\mathrm{d}x \\ &+ J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b).\end{aligned}$$

This is equivalent to

$$\overline{J}_b(Q_c^*) = J_b(Q_c^*, c_b) + J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b) = J_b(Q_d^*, c_b). \tag{B.2}$$

This implies that the buyer stays in a no worse situation by ordering Q_c^* units.

In order to show the buyer orders Q_c^* units, we need to show $Q_d^* < Q_d^*(c_b - d) < Q_c^*$.

Since $c_b - d < c_b$, we have $F(Q_d^*(c_b - d)) > F(Q_d^*)$. This implies $Q_d^* < Q_d^*(c_b - d)$. We also have $J_b(Q_d^*, c_b - d) > J_b(Q_d^*, c_b)$ since $J_b(Q, c_b)$ is a decreasing function of c_b for fixed values of Q. Since $J_b(Q_c^*, c_b - d) = J_b(Q_d^*, c_b)$ from Equation (B.2), then we have $J_b(Q_d^*, c_b - d) > J_b(Q_c^*, c_b - d)$. If $Q_c^* > Q_d^*$ and $Q_d^* < Q_d^*(c_b - d)$, $J_b(Q_d^*, c_b - d) > J_b(Q_c^*, c_b - d)$ holds if and only if $Q_d^* < Q_d^*(c_b - d) < Q_c^*$ due to the strict concavity of $J_b(Q, c_b)$ with respect to Q.

Hence, $J_b(Q, c_b - d) < \overline{J}_b(Q_c^*) = J_b(Q_c^*, c_b - d) \quad \forall Q \geqslant Q_c^*$. Since Q_d^* is the maximizer of $J_b(Q, c_b)$, $J_b(Q, c_b) < J_b(Q_d^*, c_b) = J_b(Q_c^*, c_b - d) \quad \forall Q < Q_c^*$ and

$Q \neq Q_d^*$.

Thus, Q_c^* is the maximizer of the new pricing schedule. ■

B.2 Proofs of Coordination Theorems under Stochastic Demand and Cap-and-Trade Mechanism

B.2.1 Proof of Theorem 32

It suffices to show that

i) The buyer orders Q_c^*.

ii) $\overline{J}_b(Q_c^*) \geqslant J_b(Q_d^*, c_b)$

Let $Q < Q_c^*$. Then $\overline{J}_b(Q) = J_b(Q, c_b)$. Since $J_b(Q, c_b)$ is a strictly concave function in Q, $\frac{\partial J_b(Q,c_b)}{\partial Q} = 0$ gives Q_d^* as the optimal order quantity.

Let $Q \geqslant Q_c^*$. Then the expected profit function of the buyer is

$$\overline{J}_b(Q) = J_b(Q, c_b) + p_c \times \frac{J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b)}{p_c} = J_b(Q, c_b) + J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b).$$

Since the term $J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b)$ does not depend on Q and $J_b(Q, c_b)$ is a strictly concave function, the expected profit maximizing value of Q is Q_d^* from the first order condition. However, $Q_d^* \ngeqslant Q_c^*$. Since $\overline{J}_b(Q)$ is a strictly concave function, we check the boundary condition. Thus, the buyer orders Q_c^*.

Since $X_b(Q_c^*) = Q_c^* - Q_b \leqslant 0$ and $X_v(Q_c^*) = Q_c^* - Q_v \geqslant 0$, the buyer buys and the vendor sells carbon credit at Q_c^*. Also, $p_c \times \min\{-X_b(Q_c^*), X_v(Q_c^*)\} \geqslant J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b)$. Then, the minimum of the amount of credit buyer buys and vendor sells is sufficient to compensate the buyer's loss from ordering the centralized optimal quantity. Thus, the vendor can compensate the buyer's loss by giving him/her carbon credits for free.

The buyer's expected profit at Q_c^* is

$$\overline{J}_b(Q_c^*) = J_b(Q_c^*, c_b) + J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b) = J_b(Q_d^*, c_b).$$

This implies that the buyer stays in a no worse situation by ordering Q_c^* units.
■

B.2.2 Proof of Theorem 33

It suffices to show that

i) The buyer orders Q_c^*.

ii) $\overline{J}_b(Q_c^*) \geqslant J_b(Q_d^*, c_b)$

Let $Q < Q_c^*$. Then $\overline{J}_b(Q) = J_b(Q, c_b)$. Since $J_b(Q, c_b)$ is a strictly concave function in Q, $\partial J_b(Q, c_b)/\partial Q = 0$ gives Q_d^* as the optimal order quantity.

Let $Q \geqslant Q_c^*$. Then the expected profit function of the buyer is

$$\overline{J}_b(Q) = J_b(Q, c_b) + \min\{-X_b(Q_c^*), X_v(Q_c^*)\} \times \frac{J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b)}{\min\{-X_b(Q_c^*), X_v(Q_c^*)\}}.$$

This is equivalent to

$$\overline{J}_b(Q) = J_b(Q, c_b) + J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b).$$

Since the term $J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b)$ does not depend on Q and $J_b(Q, c_b)$ is a strictly concave function, the expected profit maximizing value of Q is Q_d^* from the first order condition. However, $Q_d^* \ngeqslant Q_c^*$. Since $\overline{J}_b(Q)$ is a strictly concave function, we check the boundary condition. Thus, the buyer orders Q_c^*.

Since $X_b(Q_c^*) = Q_c^* - Q_b \leqslant 0$ and $X_v(Q_c^*) = Q_c^* - Q_v \geqslant 0$, the buyer buys and the vendor sells carbon credit at Q_c^*. Also, $p_c \times \min\{-X_b(Q_c^*), X_v(Q_c^*)\} \geqslant J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b)$. Then, the minimum of the amount of credit buyer buys and vendor sells is sufficient to compensate the buyer's loss from ordering the centralized optimal quantity. Thus, the vendor can compensate the buyer's loss by giving him carbon credits at a discounted price.

The buyer's expected profit at Q_c^* is

$$\overline{J}_b(Q_c^*) = J_b(Q_c^*, c_b) + J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b) = J_b(Q_d^*, c_b).$$

This implies that the buyer stays in a no worse situation by ordering Q_c^* units. ■

B.2.3 Proof of Theorem 34

It suffices to show that

i) The buyer orders Q_c^*.

ii) $\overline{J}_b(Q_c^*) \geqslant J_b(Q_d^*, c_b)$

Let $Q < Q_c^*$. Then $\overline{J}_b(Q) = J_b(Q, c_b)$. Since $J_b(Q, c_b)$ is a strictly concave function in Q, $\partial J_b(Q, c_b)/\partial Q = 0$ gives Q_d^* as the optimal order quantity.

Let $Q \geqslant Q_c^*$. Then the expected profit function of the buyer is

$$\begin{aligned}\overline{J}_b(Q) = &-(c_v + 2p_c)Q + p_c Q_b + \int_0^Q [px + (Q-x)v] f(x)\,\mathrm{d}x \\ &+ \int_Q^\infty [pQ - (x-Q)s] f(x)\,\mathrm{d}x + p_c \times \min\{-X_b(Q_c^*), X_v(Q_c^*)\} \\ &- J_b(Q_c^*, c_b - d) - p_c \times \min\{-X_b(Q_c^*), X_v(Q_c^*)\} + J_b(Q_d^*, c_b).\end{aligned}$$

This is equivalent to

$$\begin{aligned}\overline{J}_b(Q) = &- (c_v + 2p_c)Q + p_c Q_b + \int_0^Q [px + (Q-x)v] f(x)\,\mathrm{d}x \\ &+ \int_Q^\infty [pQ - (x-Q)s] f(x)\,\mathrm{d}x - J_b(Q_c^*, c_b - d) + J_b(Q_d^*, c_b).\end{aligned} \tag{B.3}$$

Since $\overline{J}_b(.)$ is a strictly concave function, the maximizer of Expression (B.3) can be found by checking the first order condition. That is,

$$\overline{J}_b^{'}(Q) = -(c_v + 2p_c) + pQf(Q) + \int_0^Q v f(x)\,\mathrm{d}x + \int_Q^\infty (p+s) f(x)\,\mathrm{d}x - pQf(Q) = 0.$$

This gives

$$F(Q_{crd}) = \frac{p + s - c_v - 2p_c}{p + s + v}.$$

This results in

$$Q_{crd} = F^{-1}(\frac{p + s - c_v - 2p_c}{p + s + v}) = Q_c^*.$$

Hence, the buyer orders Q_c^*. The buyer's expected profit after coordination is

$$\overline{J}_b(Q_c^*) = J_b(Q_c^*, c_b - d) - [J_b(Q_c^*, c_b - d) - J_b(Q_d^*, c_b)] = J_b(Q_d^*, c_b).$$

This implies that the buyer stays in a no worse situation by ordering Q_c^* units. ■

B.2.4 Proof of Theorem 35

It suffices to show that

i) $\overline{J}_b(Q_c^*) \geqslant J_b(Q_d^*, c_b)$

ii) The buyer orders Q_c^*.

The expected profit function after coordination is given by

$$\overline{J}_b(Q) = -(c_b + p_c - d)Q + \int_0^Q [px + (Q-x)v] f(x)\,\mathrm{d}x + \int_Q^\infty [pQ - (x-Q)s] f(x)\,\mathrm{d}x + p_c \times \min\{-X_b(Q_c^*), X_v(Q_c^*)\}.$$

This is equivalent to

$$\begin{aligned}\overline{J}_b(Q) =& (v - c_b - p_c + \frac{J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b) - p_c \times \min\{-X_b(Q_c^*), X_v(Q_c^*)\}}{Q_c^*})Q \\ &+ (p-v)\mu - (p+s-v)\int_Q^\infty (x-Q) f(x)\,\mathrm{d}x + p_c Q_b \\ &+ p_c \times \min\{-X_b(Q_c^*), X_v(Q_c^*)\}.\end{aligned}$$

where $\mu = \int_0^\infty x f(x)\,\mathrm{d}x$ is the expected value of the demand.

Similarly, the expected profit at Q_c^* after coordination is given by

$$\begin{aligned}\overline{J}_b(Q_c^*) =& (v - c_b - p_c)Q_c^* + (p-v)\mu - (p+s-v)\int_{Q_c^*}^\infty (x - Q_c^*) f(x)\,\mathrm{d}x + p_c Q_b \\ &+ J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b).\end{aligned}$$

This is equivalent to

$$\overline{J}_b(Q_c^*) = J_b(Q_c^*, c_b) + J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b) = J_b(Q_d^*, c_b). \tag{B.4}$$

This implies that the buyer stays in a no worse situation by ordering Q_c^* units.

Since the amount of carbon credit $\min\{-X_b(Q_c^*), X_v(Q_c^*)\}$ given by the vendor to the buyer does not depend on Q, the optimal order quantities of $J_b(Q, c_b - d)$ and $\overline{J}_b(Q)$ are the same. This implies that in order to show the buyer orders Q_c^* units, we need to show $Q_d^* < Q_d^*(c_b - d) < Q_c^*$.

Since $c_b - d < c_b$, we have $F(Q_d^*(c_b - d)) > F(Q_d^*)$. This implies $Q_d^* < Q_d^*(c_b - d)$. We also have $J_b(Q_d^*, c_b - d) > J_b(Q_d^*, c_b)$ since $J_b(Q, c_b)$ is a decreasing function of c_b for fixed values of Q. From Equation (B.4), we have $\overline{J}_b(Q_c^*) =$

$J_b(Q_c^*, c_b - d) + p_c \times \min\{-X_b(Q_c^*), X_v(Q_c^*)\} = J_b(Q_d^*, c_b)$. Then $J_b(Q_d^*, c_b - d) > J_b(Q_c^*, c_b - d) + p_c \times \min\{-X_b(Q_c^*), X_v(Q_c^*)\}$, which implies $J_b(Q_d^*, c_b - d) > J_b(Q_c^*, c_b - d)$. If $Q_c^* > Q_d^*$ and $Q_d^* < Q_d^*(c_b - d)$, $J_b(Q_d^*, c_b - d) > J_b(Q_c^*, c_b - d)$ holds if and only if $Q_d^* < Q_d^*(c_b - d) < Q_c^*$ due to the strict concavity of $J_b(Q, c_b)$ with respect to Q.

Thus, $J_b(Q, c_b - d) < J_b(Q_c^*, c_b - d) \quad \forall Q \geqslant Q_c^*$, which implies $\overline{J}_b(Q) < \overline{J}_b(Q_c^*) \quad \forall Q \geqslant Q_c^*$. Since Q_d^* is the maximizer of $J_b(Q, c_b)$, $J_b(Q, c_b) < J_b(Q_d^*, c_b) = \overline{J}_b(Q_c^*) \quad \forall Q < Q_c^*$ and $Q \neq Q_d^*$.

Hence, Q_c^* is the maximizer of the new pricing schedule. ■

B.3 Proofs of Coordination Theorems under Stochastic Demand and Carbon Cap Mechanism

B.3.1 Proof of Theorem 36

Since $Q_d^0 \leqslant Q_{cap}$, $Q_d^* = Q_d^0$ from Theorem 26. Similarly, since $Q_c^0 \leqslant Q_{cap}$, $Q_c^* = Q_c^0$ from Theorem 27. Then it suffices to show that

i) The buyer orders Q_c^0.

ii) $\overline{J}_b(Q_c^0) \geqslant J_b(Q_d^0, c_b)$

Let $Q < Q_c^0$. Then $\overline{J}_b(Q) = J_b(Q, c_b)$. Since $J_b(Q, c_b)$ is a strictly concave function in Q, $\partial J_b(Q, c_b)/\partial Q = 0$ gives Q_d^0 as the optimal order quantity.

Let $Q \geqslant Q_c^0$. Then the expected profit function of the buyer is

$$\overline{J}_b(Q) = -c_v Q + \int_0^Q [px + (Q-x)v] f(x)\,\mathrm{d}x + \int_Q^\infty [pQ - (x-Q)s] f(x)\,\mathrm{d}x \\ - J_b(Q_c^0, c_b - d) + J_b(Q_d^0, c_b). \tag{B.5}$$

Since $\overline{J}_b(.)$ is a strictly concave function, the maximizer of Expression (B.5) can be found by checking the first order condition. That is,

$$\overline{J}_b^{'}(Q) = -c_v + pQf(Q) + \int_0^Q vf(x)\,\mathrm{d}x + \int_Q^\infty (p+s)f(x)\,\mathrm{d}x - pQf(Q) = 0.$$

This gives

$$F(Q_{crd}) = \frac{p+s-c_v}{p+s+v}.$$

This results in

$$Q_{crd} = F^{-1}(\frac{p+s-c_v}{p+s+v}) = Q_c^0.$$

Since $Q_c^0 \leqslant Q_{cap}$, the buyer orders Q_c^0. The buyer's expected profit after coordination is

$$\overline{J}_b(Q_c^0) = J_b(Q_c^0, c_b - d) - [J_b(Q_c^0, c_b - d) - J_b(Q_d^0, c_b)] = J_b(Q_d^0, c_b).$$

This implies that the buyer stays in a no worse situation by ordering $Q_c^* = Q_c^0$ units. ■

B.3.2 Proof of Theorem 37

Since $Q_d^0 \leqslant Q_{cap}$, $Q_d^* = Q_d^0$ from Theorem 26. Similarly, since $Q_c^0 \leqslant Q_{cap}$, $Q_c^* = Q_c^0$ from Theorem 27. Then it suffices to show that

i) $\overline{J}_b(Q_c^0) \geqslant J_b(Q_d^0, c_b)$

ii) The buyer orders Q_c^0.

The expected profit function after coordination is given by

$$\overline{J}_b(Q) = -(c_b - d)Q + \int_0^Q [px + (Q - x)v]f(x)\,\mathrm{d}x + \int_Q^\infty [pQ - (x - Q)s]f(x)\,\mathrm{d}x.$$

This is equivalent to

$$\overline{J}_b(Q) = (v - c_b + \frac{J_b(Q_d^0, c_b) - J_b(Q_c^0, c_b)}{Q_c^0})Q + (p - v)\mu - (p + s - v)\int_Q^\infty (x - Q)f(x)\,\mathrm{d}x.$$

where $\mu = \int_0^\infty xf(x)\,\mathrm{d}x$ is the expected value of the demand.

Similarly, the expected profit at Q_c^0 after coordination is given by

$$\begin{aligned}\overline{J}_b(Q_c^0) =& (v - c_b)Q_c^0 + (p - v)\mu - (p + s - v)\int_{Q_c^0}^\infty (x - Q_c^0)f(x)\,\mathrm{d}x + J_b(Q_d^0, c_b) \\ & - J_b(Q_c^0, c_b).\end{aligned}$$

This is equivalent to

$$\overline{J}_b(Q_c^0) = J_b(Q_c^0, c_b) + J_b(Q_d^0, c_b) - J_b(Q_c^0, c_b) = J_b(Q_d^0, c_b). \tag{B.6}$$

This implies that the buyer stays in a no worse situation by ordering Q_c^0 units.

In order to show the buyer orders Q_c^0 units, we need to show $Q_d^0 < Q_d^0(c_b - d) < Q_c^0$.

Since $c_b - d < c_b$, we have $F(Q_d^0(c_b - d)) > F(Q_d^0)$. This implies $Q_d^0 < Q_d^0(c_b - d)$. We also have $J_b(Q_d^0, c_b - d) > J_b(Q_d^0, c_b)$ since $J_b(Q, c_b)$ is a decreasing function of c_b for fixed values of Q. Since $J_b(Q_c^0, c_b - d) = J_b(Q_d^0, c_b)$ from Equation (B.6), then we have $J_b(Q_d^0, c_b - d) > J_b(Q_c^0, c_b - d)$. If $Q_c^0 > Q_d^0$ and $Q_d^0 < Q_d^0(c_b - d)$, $J_b(Q_d^0, c_b - d) > J_b(Q_c^0, c_b - d)$ holds if and only if $Q_d^0 < Q_d^0(c_b - d) < Q_c^0$ due to the strict concavity of $J_b(Q, c_b)$ with respect to Q.

Hence, $J_b(Q, c_b - d) < \overline{J}_b(Q_c^0) = J_b(Q_c^0, c_b - d) \quad \forall Q \geqslant Q_c^0$. Since Q_d^0 is the maximizer of $J_b(Q, c_b)$, $J_b(Q, c_b) < J_b(Q_d^0, c_b) = J_b(Q_c^0, c_b - d) \quad \forall Q < Q_c^0$ and $Q \neq Q_d^0$.

Thus, $Q_c^* = Q_c^0$ is the maximizer of the new pricing schedule. ■

B.3.3 Proof of Theorem 38

Since $Q_d^0 \leqslant Q_{cap}$, $Q_d^* = Q_d^0$ from Theorem 26. Similarly, since $Q_c^0 > Q_{cap}$, $Q_c^* = Q_{cap}$ from Theorem 27. Then it suffices to show that

i) $\overline{J}_b(Q_{cap}) \geqslant J_b(Q_d^0, c_b)$

ii) The buyer orders Q_{cap}.

The expected profit function after coordination is given by

$$\overline{J}_b(Q) = -(c_b - d)Q + \int_0^Q [px + (Q - x)v] f(x)\,\mathrm{d}x + \int_Q^\infty [pQ - (x - Q)s] f(x)\,\mathrm{d}x.$$

This is equivalent to

$$\begin{aligned}\overline{J}_b(Q) =& (v - c_b + \frac{J_b(Q_d^0, c_b) - J_b(Q_{cap}, c_b)}{Q_{cap}})Q + (p - v)\mu \\ & - (p + s - v)\int_Q^\infty (x - Q) f(x)\,\mathrm{d}x.\end{aligned}$$

where $\mu = \int_0^\infty x f(x)\,\mathrm{d}x$ is the expected value of the demand.

Similarly, the expected profit at Q_{cap} after coordination is given by

$$\begin{aligned}\overline{J}_b(Q_{cap}) =& (v - c_b)Q_{cap} + (p - v)\mu - (p + s - v)\int_{Q_{cap}}^\infty (x - Q_{cap}) f(x)\,\mathrm{d}x \\ & + J_b(Q_d^0, c_b) - J_b(Q_{cap}, c_b).\end{aligned}$$

This is equivalent to

$$\overline{J}_b(Q_{cap}) = J_b(Q_{cap}, c_b) + J_b(Q_d^0, c_b) - J_b(Q_{cap}, c_b) = J_b(Q_d^0, c_b). \tag{B.7}$$

This implies that the buyer stays in a no worse situation by ordering Q_{cap} units.

In order to show the buyer orders Q_{cap} units, we need to show $Q_d^0 < Q_d^0(c_b - d) < Q_{cap}$.

Since $c_b - d < c_b$, we have $F(Q_d^0(c_b - d)) > F(Q_d^0)$. This implies $Q_d^0 < Q_d^0(c_b - d)$. We also have $J_b(Q_d^0, c_b - d) > J_b(Q_d^0, c_b)$ since $J_b(Q, c_b)$ is a decreasing function of c_b for fixed values of Q. Since $J_b(Q_{cap}, c_b - d) = J_b(Q_d^0, c_b)$ from Equation (B.7), then we have $J_b(Q_d^0, c_b - d) > J_b(Q_{cap}, c_b - d)$. If $Q_{cap} > Q_d^0$ and $Q_d^0 < Q_d^0(c_b - d)$, $J_b(Q_d^0, c_b - d) > J_b(Q_{cap}, c_b - d)$ holds if and only if $Q_d^0 < Q_d^0(c_b - d) < Q_{cap}$ due to the strict concavity of $J_b(Q, c_b)$ with respect to Q.

Hence, $J_b(Q, c_b - d) < \overline{J}_b(Q_c^*) = \overline{J}_b(Q_{cap}) = J_b(Q_{cap}, c_b - d) \quad \forall Q \geqslant Q_{cap}$. Since $Q_d^* = Q_d^0$ is the maximizer of $J_b(Q, c_b)$, $J_b(Q, c_b) < J_b(Q_d^0, c_b) = J_b(Q_{cap}, c_b - d) \quad \forall Q < Q_{cap}$ and $Q \neq Q_d^0$.

Thus, $Q_c^* = Q_{cap}$ is the maximizer of the new pricing schedule. ■

B.4 Application of Coordination Mechanisms under Stochastic Demand and Carbon Tax Mechanism

Example 49: According to Theorem 28, channel coordination can be achieved by a quantity discount of $d = c_b - c_v - t_v$ given by the vendor to the buyer and a fixed payment of amount $J_b(Q_c^*, c_b - d) - J_b(Q_d^*, c_b)$ made by the buyer to the vendor for the order sizes greater than or equal to Q_c^*. After receiving a discount of $d = c_b - c_v - t_v = 4$, the expected profit of the buyer becomes $J_b(Q_c^*, c_b - d) = 1299.565217$. The buyer should make a fixed payment of amount $J_b(Q_c^*, c_b - d) - J_b(Q_d^*, c_b) = 1299.565217 - 294.3478261 = 1005.217391$. Thus, the expected profit of the buyer after coordination is $J_b(Q_c^*, c_b - d) - [J_b(Q_c^*, c_b - d) - J_b(Q_d^*, c_b)] = J_b(Q_d^*, c_b) = 294.3478261$. The expected profit of the vendor after coordination is 1005.217391.

Example 50: According to Theorem 29, channel coordination can be achieved by a quantity discount of $[J_b(Q_d^*) - J_b(Q_c^*)]/Q_c^*$ given by the vendor to the buyer for the order sizes greater than or equal to Q_c^*. Then the discount given by the vendor to the buyer is $d = [J_b(Q_d^*) - J_b(Q_c^*)]/Q_c^* = [200.1454545 - 141.9636364]/230.5454545 = 0.252365931$. Thus, the expected profit of the buyer after coordination is $J_b(Q_c^*, c_b - d) = J_b(Q_d^*, c_b) = 200.1454545$. The expected profit of the vendor after coordination is 864.

B.5 Application of Coordination Mechanisms under Stochastic Demand and Cap-and-Trade Mechanism

Example 51: From Table 5.9, $X_b(Q_c^*) = Q_b - Q_c^* > 0$ and $X_v(Q_c^*) = Q_v - Q_c^* > 0$, i.e., the buyer and the vendor both sell carbon credit under the centralized solution. Thus, the buyer does not need extra carbon credit. According to Theorem 30, channel coordination can be achieved by quantity discount of $d = c_b - c_v - p_c$ given by the vendor to the buyer and a fixed payment of amount $J_b(Q_c^*, c_b - d) - J_b(Q_d^*, c_b)$ made by the buyer to the vendor for the order sizes greater than or equal to Q_c^*. After receiving a discount of $d = c_b - c_v - p_c = 4$, the expected profit of the buyer becomes $J_b(Q_c^*, c_b - d) = 18791.30435$. The buyer should make a fixed payment of amount $J_b(Q_c^*, c_b - d) - J_b(Q_d^*, c_b) = 18791.30435 - 8982.608696 = 9808.695652$ Thus, the expected profit of the buyer after coordination is $J_b(Q_c^*, c_b - d) - [J_b(Q_c^*, c_b - d) - J_b(Q_d^*, c_b)] = J_b(Q_d^*, c_b) = 8982.608696$. The expected profit of the vendor after coordination is 9808.695652.

Example 52: From Table 5.9, $X_b(Q_c^*) = Q_b - Q_c^* < 0$ and $X_v(Q_c^*) = Q_v - Q_c^* < 0$, i.e., the buyer and the vendor both buy carbon credit under the centralized solution. Since $X_b(Q_c^*) < 0$ and $X_v(Q_c^*) < 0$, the buyer needs carbon credit but the vendor does not have any carbon credit to give to the buyer. According to Theorem 31, channel coordination can be achieved by a quantity discount of $[J_b(Q_d^*) - J_b(Q_c^*)]/Q_c^*$ given by the vendor to the buyer for the order sizes greater than or equal to Q_c^*. Then the discount given by the vendor to the buyer is $d = [J_b(Q_d^*) - J_b(Q_c^*)]/Q_c^* = [259.5833333 - 218.3333333]/191.6666667 = 0.215217391$. Thus, the expected profit of the buyer after coordination is $J_b(Q_c^*, c_b - d) = J_b(Q_d^*, c_b) = 259.5833333$. The expected profit of the vendor after coordination is 533.75.

Example 53: From Table 5.9, $X_b(Q_c^*) = Q_b - Q_c^* < 0$ and $X_v(Q_c^*) = Q_v - Q_c^* > 0$, i.e., the buyer buys and the vendor sells carbon credit under the centralized solution. The buyer's loss from ordering the centralized optimal quantity is $J_b(Q_d^*) - J_b(Q_c^*) = 656.9565217 - 611.7391304 = 45.2173913$. $\min\{-X_b(Q_c^*), X_v(Q_c^*)\} = X_v(Q_c^*) = 19.56521739$ and $p_c \times X_v(Q_c^*) = 3 \times 19.56521739 = 58.69565217$. Since $p_c \times X_v(Q_c^*) = 58.69565217 > J_b(Q_d^*) - J_b(Q_c^*) = 45.2173913$, giving carbon credit for free to the buyer is sufficient to compensate his/her loss from ordering the centralized optimal quantity. Then according to Theorem 32, channel coordination can be achieved if the vendor gives $[J_b(Q_d^*) - J_b(Q_c^*)]/p_c$ amount of carbon credits to the buyer for free for the order sizes greater than or equal to Q_c^*. The amount of carbon credit corresponding to the loss of the buyer is $45.2173913/p_c = 45.2173913/3 = 15.07246377$. Thus, the expected profit of the buyer after coordination is $J_b(Q_c^*) + p_c \times [J_b(Q_d^*) - J_b(Q_c^*)]/p_c = J_b(Q_d^*) = 656.9565217$. The expected profit of the vendor after coordination is 916.5217391.

Example 54: From Table 5.9, $X_b(Q_c^*) = Q_b - Q_c^* < 0$ and $X_v(Q_c^*) = Q_v - Q_c^* > 0$, i.e., the buyer buys and the vendor sells carbon credit under the centralized solution. The buyer's loss from ordering the centralized optimal quantity is $J_b(Q_d^*) - J_b(Q_c^*) = 3964.583333 - 3906.25 = 58.33333333$. $\min\{-X_b(Q_c^*), X_v(Q_c^*)\} = X_v(Q_c^*) = 79.16666667$ and $p_c \times X_v(Q_c^*) = 4 \times 79.16666667 = 316.6666667$. Since $p_c \times X_v(Q_c^*) = 316.6666667 > J_b(Q_d^*) - J_b(Q_c^*) = 58.33333333$, giving carbon credits at a discounted price is sufficient to compensate his/her loss from ordering the centralized optimal quantity. Then according to Theorem 33, channel coordination can be achieved if the vendor gives $X_b(Q_c^*)$ amount of carbon credits to the buyer with a carbon price discount $d_c = [J_b(Q_d^*) - J_b(Q_c^*)]/\min\{-X_b(Q_c^*), X_v(Q_c^*)\}$ for the order sizes greater than or equal to Q_c^*. The corresponding carbon price discount is $[3964.583333 - 3906.25]/79.16666667 = 0.736842105$. Thus, the expected profit of the buyer after coordination is $J_b(Q_c^*) + \min\{-X_b(Q_c^*), X_v(Q_c^*)\} \times [J_b(Q_d^*) - J_b(Q_c^*)]/\min\{-X_b(Q_c^*), X_v(Q_c^*)\} = J_b(Q_d^*) = 3964.583333$. The expected profit of the vendor after coordination is 2500.

Example 55: From Table 5.9, $X_b(Q_c^*) = Q_b - Q_c^* < 0$ and $X_v(Q_c^*) = Q_v - Q_c^* > 0$, i.e., the buyer buys and the vendor sells carbon credit under the centralized solution. The buyer's loss from ordering the centralized optimal quantity is $J_b(Q_d^*) - J_b(Q_c^*) = 756.1805556 - 698.3333333 = 57.84722222$. $\min\{-X_b(Q_c^*), X_v(Q_c^*)\} = X_v(Q_c^*) = 6.666666667$ and $p_c \times X_v(Q_c^*) = 2.5 \times 6.666666667 = 16.66666667$. Since $p_c \times X_b(Q_c^*) = 16.66666667 < J_b(Q_d^*) - J_b(Q_c^*) = 57.84722222$, giving carbon credits for free is not sufficient to compensate his/her loss from ordering the centralized optimal quantity. Then according to Theorem 34, channel coordination can be achieved if the vendor gives $\min\{-X_b(Q_c^*), X_v(Q_c^*)\}$ amount of carbon credits for free and a quantity discount of $d = c_b - c_v - p_c$ to the buyer and the buyer makes a fixed payment of amount $J_b(Q_c^*, c_b - d) + p_c \times \min\{-X_b(Q_c^*), X_v(Q_c^*)\} - J_b(Q_d^*, c_b)$ for the order sizes greater than or equal to Q_c^*. Then the amount of carbon credits given by the vendor to the buyer for free is $X_v(Q_c^*) = 6.666666667$ and the quantity discount offered is $d = 3.5$. After receiving the carbon credits and the quantity discount, the expected profit of the buyer becomes $J_b(Q_c^*, c_b - d) + p_c \times X_v(Q_c^*) = 1620 + 16.66666667 = 1636.666667$ The fixed payment made by the buyer to the vendor is $1636.666667 - 756.1805556 = 880.4861111$. Thus, the expected profit of the buyer after coordination is $J_b(Q_c^*, c_b - d) - [J_b(Q_c^*, c_b - d) - J_b(Q_d^*, c_b)] = J_b(Q_d^*, c_b) = 756.1805556$. The expected profit of the vendor after coordination is 863.8194444.

Example 56: From Table 5.9, $X_b(Q_c^*) = Q_b - Q_c^* < 0$ and $X_v(Q_c^*) = Q_v - Q_c^* > 0$, i.e., the buyer buys and the vendor sells carbon credit under the centralized solution. The buyer's loss from ordering the centralized optimal quantity is $J_b(Q_d^*) - J_b(Q_c^*) = 1455.681818 - 1370.454545 = 85.22727273$. $\min\{-X_b(Q_c^*), X_v(Q_c^*)\} = -X_b(Q_c^*) = 9.090909091$ and $-p_c \times X_b(Q_c^*) = 2.5 \times 9.090909091 = 22.72727273$.

Since $-p_c \times X_b(Q_c^*) = 22.72727273 < J_b(Q_d^*) - J_b(Q_c^*) = 85.22727273$, giving carbon credits for free is not sufficient to compensate his/her loss from ordering the centralized optimal quantity. Then according to Theorem 35, channel coordination can be achieved if the vendor gives $\min\{-X_b(Q_c^*), X_v(Q_c^*)\}$ amount of carbon credits for free and a quantity discount of $d = [J_b(Q_d^*) - J_b(Q_c^*) - p_c \times \min\{-X_b(Q_c^*), X_v(Q_c^*)\}]/Q_c^*$ to the buyer for the order sizes greater than or equal to Q_c^*. Then the amount of carbon credits given by the vendor to the buyer for free is $-X_b(Q_c^*) = 9.090909091$ and the quantity discount offered is $d = (85.22727273 - 22.72727273)/609.0909091 = 0.10261194$. Thus, the expected profit of the buyer after coordination is $J_b(Q_c^*, c_b) + J_b(Q_d^*, c_b) - J_b(Q_c^*, c_b) = J_b(Q_d^*, c_b) = 1455.681818$. The expected profit of the vendor after coordination is 1437.5.

B.6 Application of Coordination Mechanisms under Stochastic Demand and Carbon Cap Mechanism

Example 57: From Table 5.11, $Q_{cap} = \min\{Q_b, Q_v\} = Q_v = 180$. From Tables 5.12 and 5.13, $Q_d^0 = 160$ and $Q_c^0 = 175$. Since $Q_d^0 = 160 < Q_{cap} = 180$, $Q_d^* = Q_d^0 = 160$ from Theorem 26. Similarly, since $Q_c^0 = 175 < Q_{cap} = 180$, $Q_c^* = Q_c^0 = 175$ from Theorem 27. Then according to Theorem 36, channel coordination can be achieved by a quantity discount of $d = c_b - c_v$ given by the vendor to the buyer and a fixed payment of amount $J_b(Q_c^*, c_b - d) - J_b(Q_d^*, c_b)$ made by the buyer to the vendor for the order sizes greater than or equal to Q_c^*. After receiving a discount of $d = c_b - c_v = 3$, the expected profit of the buyer becomes $J_b(Q_c^*, c_b - d) = 722.5$. The buyer should make a fixed payment of amount $J_b(Q_c^*, c_b - d) - J_b(Q_d^*, c_b) = 722.5 - 220 = 502.5$. Thus, the expected profit of the buyer after coordination is $J_b(Q_c^*, c_b - d) - [J_b(Q_c^*, c_b - d) - J_b(Q_d^*, c_b)] = J_b(Q_d^*, c_b) = 220$. The expected profit of the vendor after coordination is 502.5.

Example 58: From Table 5.11, $Q_{cap} = \min\{Q_b, Q_v\} = Q_b = 160$. From Tables 5.12 and 5.13, $Q_d^0 = 133.0434783$ and $Q_c^0 = 153.9130435$. Since $Q_d^0 = 133.0434783 < Q_{cap} = 160$, $Q_d^* = Q_d^0 = 133.0434783$ from Theorem 26. Similarly, since $Q_c^0 = 153.9130435 < Q_{cap} = 160$, $Q_c^* = Q_c^0 = 153.9130435$ from Theorem 27. Then according to Theorem 37, channel coordination can be achieved by a quantity discount of $[J_b(Q_d^0) - J_b(Q_c^0)]/Q_c^0$ given by the vendor to the buyer for the order sizes greater than or equal to Q_c^0. Then the discount given by the vendor to the buyer is $d = [J_b(Q_d^0) - J_b(Q_c^0)]/Q_c^0 = [391.3043478 - 349.5652174]/153.9130435 = 0.271186441$. Thus, the expected profit of the buyer after coordination is $J_b(Q_c^*, c_b - d) = J_b(Q_d^*, c_b) = 391.3043478$. The expected profit of the vendor after coordination is 573.9130435.

Example 59: From Table 5.11, $Q_{cap} = \min\{Q_b, Q_v\} = Q_b = 170$. From Tables 5.12 and 5.13, $Q_d^0 = 159.2307692$ and $Q_c^0 = 173.0769231$. Since $Q_d^0 = 159.2307692 < Q_{cap} = 170$, $Q_d^* = Q_d^0 = 159.2307692$ from Theorem 26. Similarly, since $Q_c^0 = 173.0769231 > Q_{cap} = 170$, $Q_c^* = Q_{cap} = 170$ from Theorem 27. Then according to Theorem 38, channel coordination can be achieved by a quantity discount of $[J_b(Q_d^0) - J_b(Q_{cap})]/Q_{cap}$ given by the vendor to the buyer for the order sizes greater than or equal to Q_{cap}. Then the discount given by the vendor to the buyer is $d = [J_b(Q_d^0) - J_b(Q_{cap})]/Q_{cap} = [573.4615385 - 548.3333333]/170 = 0.147812971$. Thus, the expected profit of the buyer after coordination is $J_b(Q_{cap}, c_b - d) = J_b(Q_d^*, c_b) = 573.4615385$. The expected profit of the vendor after coordination is 994.8717949.

Example 60: From Table 5.11, $Q_{cap} = \min\{Q_b, Q_v\} = Q_v = 1400$. From Tables 5.12 and 5.13, $Q_d^0 = 1292.307692$ and $Q_c^0 = 1430.769231$. Since $Q_d^0 = 1292.307692 < Q_{cap} = 1400$, $Q_d^* = Q_d^0 = 1292.307692$ from Theorem 26. Similarly, since $Q_c^0 = 1430.769231 > Q_{cap} = 1400$, $Q_c^* = Q_{cap} = 1400$ from Theorem 27. Then according to Theorem 38, channel coordination can be achieved by a quantity discount of $[J_b(Q_d^0) - J_b(Q_{cap})]/Q_{cap}$ given by the vendor to the buyer for the order sizes greater than or equal to Q_{cap}. Then the discount given by the vendor to the buyer is $d = [J_b(Q_d^0) - J_b(Q_{cap})]/Q_{cap} = [4234.615385 - 3983.333333]/1400 = 0.179487179$. Thus, the expected profit of the buyer after coordination is $J_b(Q_{cap}, c_b - d) = J_b(Q_d^*, c_b) = 4234.615385$. The expected profit of the vendor after coordination is 8148.717949.

Example 61: From Table 5.11, $Q_{cap} = \min\{Q_b, Q_v\} = Q_b = 105$. From Tables 5.12 and 5.13, $Q_d^0 = 113.6842105$ and $Q_c^0 = 125.5263158$. Since $Q_d^0 = 113.6842105 > Q_{cap} = 105$, $Q_d^* = Q_{cap} = 105$ from Theorem 26. Similarly, since $Q_c^0 = 125.5263158 > Q_{cap} = 105$, $Q_c^* = Q_{cap} = 105$ from Theorem 27. Thus, the channel is already coordinated from Theorem 39. The expected profit of the buyer is 512.5 and the expected profit of the vendor is 262.5.

Example 62: From Table 5.11, $Q_{cap} = \min\{Q_b, Q_v\} = Q_v = 1150$. From Tables 5.12 and 5.13, $Q_d^0 = 1330.833333$ and $Q_c^0 = 1430$. Since $Q_d^0 = 1330.833333 > Q_{cap} = 1150$, $Q_d^* = Q_{cap} = 1150$ from Theorem 26. Similarly, since $Q_c^0 = 1430 > Q_{cap} = 1150$, $Q_c^* = Q_{cap} = 1150$ from Theorem 27. Thus, the channel is already coordinated from Theorem 39. The expected profit of the buyer is 4301.470588 and the expected profit of the vendor is 4025.