PLACEMENT OF EXPRESS LINKS IN A DWDM OPTICAL NETWORK

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ABSTRACT

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M.S. in Industrial Engineering

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With the introduction of DWDM technology in telecommunication network systems, important advancements have been achieved in the problem of routing the increasing signal traffic between demand-supply nodes. The choice of the links to open, the number of links and routing of current traffic on these links in such an optical network system are important in terms of decreasing the complexity of the network and cost savings. The study in this thesis firstly introduces the use of express links, which enables those objectives, and then determines the appropriate network structure and routing. The study introduces two mathematical models as well as a lagrangian based heuristic for the solution of the problem.

Keywords: network loading problem, express links

ÖZET

DWDM OPTİK AĞLARDA EKSPRES LİNK YERLEŞTİRİLMESİ

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Telekomünikasyon ağ sistemlerinde DWDM teknolojisinin kullanımıyla beraber, artan sinyal trafiğinin arz-talep noktaları arasında rotalanması probleminde önemli ilerlemeler sağlanmıştır. Böyle bir optik ağ sisteminde açılacak linklerin seçimi, sayısı ve mevcut trafiğin bu linkler üzerinde rotalanması, hem maliyet kazancı, hem de ağ karmaşıklığının azaltılması açısından önemlidir. Yapılan çalışma, böyle bir ilerlemeye olanak veren ekspres link kullanımını tanıtmakta, daha sonra da önerilen iki matematiksel model ve lagrangian temelli sezgisel yaklaşım ile uygun link altyapısını ve trafiğin rotasını belirlemektedir.

Anahtar Sözcükler: ağ yükleme problemi, ekspres linkler

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To my country

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Chapter 1

INTRODUCTION

Design of telecommunication network systems is one of the interesting research areas that have been introduced to the study of many researchers. At first, the main aim of the design is to satisfy the desired supply-demand balance of customers or nodes in the network in some way. After the supply-demand balance of the network is provided, better designs of a telecommunication network can be looked for in order to give service under the conditions of less cost, less complexity, reasonable service time etc. In our research, we study a specific telecommunication network design problem, which utilizes new technologies developed by electric- electronics industry.

We present the specific telecommunication network design problem in Chapter 2 along with some technical information for telecommunications network systems, source of the problem and problem definition. This chapter also states the importance of the new technology for the telecommunication network we study.

In Chapter 3, the related literature work is given under two main titles. First part of the literature research presents the problems which show some similarities to our specific telecommunication network design problem. Those related works are mostly from the electrical engineering literature and design issues in the proper transmission of signals in a telecommunication network are mainly discussed. In the second part of Chapter 3, we give some examples from the literature of capacitated network design problems, which can be used to analyze our problem in a better way.

The analysis of our problem points out that we solve a kind of network loading problem for the design of a telecommunication network. We give two integer formulations for our problem in Chapter 4. First formulation resembles to the classic formulation of network flow problems, with some problem specific additions. The second formulation of the problem is less sized in terms of constraints and variables.

The models proposed are not capable of solving reasonably sized problems exactly in 72 hours. For this reason, we relax a set of constraints in our formulation to yield an easily solved model. Lagrangian relaxation of capacity constraints for the second formulation of the problem is given in Chapter 5. The resulting solution alone is not feasible for the original problem and moreover the lower bound that we obtain from the relaxed problem is too weak. In order to improve the lower bound of the lagrangian relaxation and to get a feasible solution for the original problem, we add a series of cuts; namely s-t cuts and some logical cuts. Those cuts are also added to original formulations of the problem to reach better lower bounds.

The lagrangian relaxed problem of second formulation with added cuts results in a feasible solution which is quite far away from the optimal solution. In Chapter 6, we present a heuristic which improves this feasible solution. We state the parameters of the heuristic and describe how the heuristic proceeds to find a good solution.

The computational results of our study are presented in Chapter 7. Firstly, we compare the performance of the two formulations when solved using CPLEX 9.0. Secondly, the performance comparison of second formulation and the heuristic is given in terms of time, gap and quality of lower and upper bounds. Lastly the cost comparison of networks with and without express links is analyzed under two measures that we define to evaluate the computational results of second formulation and the heuristic. Different network structures have been created and the features of those networks are also stated in the chapter.

In Chapter 8, we present the conclusions of our research. We state the main results and possible extensions of the problem for further research directions.

Chapter 2

PROBLEM DEFINITION

Matching demand with supply is a main concern in many fields of industry. Meeting customer demand on time with reasonable cost creates challenging problems, especially presented in the interest of operations researchers. One of such problems in real life is providing service on network systems, which are used by computer systems, telecommunication systems, delivery systems etc. A network system consists of two basic sets of elements; a set of nodes N, which become supply or demand points, and a set of links E connecting those points. The nodes of a network may represent customers, operation centers or service providers. The links of the network provide the connection of those nodes, for achieving the transfer of commodities between nodes.

In the telecommunication system we study, data transfer is achieved by the transmission of signals in the network. The links in the set E of network are the fiber optic cables through which the signal flows and those fiber optic cables connect the nodes of set N. The nodes in the set N of network are the operation centers in which the routing-switching decisions of signals are made. The nodes may become demand or supply points according to the origin-destination of the signal transmitted. Moreover, a node may become an intermediate point for a signal at which the routing decision of the signal is made to reach destination node of the signal. For this reason, an operation center needs to recognize the destination node of the signal. If the destination point of the signal is the node that the signal has just arrived, then the signal is processed there. If the destination node of the signal is a different node, then the operation center, which has become an intermediate point for that signal, has to find an appropriate connection to send the signal. For each of the two cases, the signal arriving at a node has to be processed for the correct action. As the number of signals processed at a node grows, the complexity of the telecommunication network increases. We need more operations to provide service, which make the networks more complex. Moreover the devices used for those operations create an important cost factor in the network.

2.1 Features of the Problem

A signal is the flow unit of telecommunication network traffic, which is sent through fiber optic cables on the links. A number of signals has to be sent from every node i to every node j, which is the traffic with origin i and destination j. A wavelength is assigned to each of the signals that are transmitted in an optical fiber. DWDM (Dense Wavelength Division Multiplex) is the name of technology for transmitting data by light waves via optical fibers. This technology allows us to send many signals together within a fiber optic cable, as long as the wavelength of each of the signals, which are carried through the same fiber, are different. That is, two signals with the same wavelength cannot be transmitted in the same fiber cable. One fiber optic cable can carry up to a number of signals with different wavelengths. The number of wavelengths available for a fiber cable of the telecommunication network is a capacity constraint for the transmission of signals, since a second fiber should be activated on a link if the number of signals to be transmitted on that link is more than the number of wavelengths available for one fiber cable.

A signal, which has to be transmitted from node ***i*** to node ***j***, is assigned a wavelength l_k as it departs from its source node ***i*** in a fiber cable. Along within that fiber, no change in the wavelength of the signal occurs. Its wavelength is kept until the signal reaches at an intermediate node on its route.

As soon as the signal with wavelength l_k reaches at an intermediate node, the signal may continue its route with the same wavelength l_k as it started from node *i*. The wavelength of the signal may also be converted to another wavelength l_m, which is not being used through the fiber that the signal is routed through. Then the signal departs from this intermediate node with a wavelength l_m, which is different than the wavelength l_k. Such wavelength conversion process is generally needed if two signals, which have the same wavelength, have to leave an intermediate node on their routes in the same fiber. The wavelength of one of the two signals has to be converted to another wavelength, which is free in that fiber. Wavelength conversion of a signal can only be done at nodes.

At the nodes of an optical network, there are devices, named OEO (optic-electronic-optic) converters, which convert the optical form of a signal to electronic form as soon as the signal arrives at a node. After the signal in electronic form is processed at the node, the form is converted to optical form by an OEO converter, before the signal leaves the node through a fiber. In our study, we assume that all OEO converters at the nodes have the feature of converting wavelength of a signal to another wavelength. The network systems that have the ability to provide full conversion opportunities at the nodes are called circuit-switched networks since any coincidence of same type of wavelength is prevented. In our work, we assume that full wavelength conversion is available at all nodes of the network, which allows us to convert wavelength of any signal to another wavelength at every node of the network.

2.2 Threshold of Signal Quality

One important issue in transmitting data in a telecommunication network is about the threshold of the signal quality. As the signal moves along a fiber cable, the quality of the signal decreases. After some distance, the decrease in the quality of the signal causes the signal to degrade beyond recovery. In our study we name the distance, after which the signal is useless, as the signal quality drop distance (SQDD). This SQDD value gives a threshold for maximum length of fiber cables constructed. The signals should be regenerated at certain distances between demand and supply points to keep the original data structure, before the signal quality drops below a certain threshold. Regeneration of signals guarantees the signal quality to be as live as if it was generated at its origin location. We assume that regeneration of signals is provided by the devices that are located at the nodes of the system and regeneration along links is not possible. The regeneration process of signals at nodes increases complexity of the network.

2.3 Main Cost Drivers of the Network

Regeneration of signals is one of the processes that create cost factors at nodes of the network. While a signal moves along in a fiber cable, it is amplified in order to distinguish the features of the signal from the distorting effect of noises, which arise along the travel distances. Amplification at certain points of the links is needed in order to reach the destination node of the signal or to pass through a node without losing quality and structure of the signal.

Amplification of signals along within the links is another cost driver for telecommunication network systems.

Main cost drivers of such a telecommunication network are the opening cost of links, the cost of devices which are used at the nodes for switching or routing the signals or converting the wavelength of a signal to another wavelength. Other than the cost issues of design of a telecommunication network, complexity of the network is another point that has to be carefully investigated for operating the network. As the number of signals processed at a node increase, the complexity of the system increases. This causes more time to be spent at the nodes for switching signals and converting wavelengths.

2.4 Problem Definition

The technological developments in electric-electronics industry present many new devices which increase efficiency of systems, create alternative systems or change the existing structure to compete with running time. In recent years, such new electric-electronic introduction, which is called ultra long-haul (ULH) DWDM technology, has been developed. The ULH technology enables us to bypass some nodes on the route of a signal. Direct links are created between two nodes on the route of a signal and that signal does not stop at the node(s) between the two nodes which are connected by direct links. These direct links are called "express links" in our study.

The ULH technology enables us to transmit signals over long distances without regenerating them, by using optical fiber links. Regeneration process, which can be done at intermediate nodes of the route of a signal, needs devices that increase the cost of network. By the use of ULH technology, less number of regeneration operations is needed in the network. The cost of the network will decrease since less number of optical devices will be used for the regeneration operation.

Other than the cost benefits of the ULH technology, advantages of direct links in reducing the complexity of the network cannot be neglected. Each one of regeneration processes, the decisions of routing and switching a signal is an extra operation, which increases the operating complexity of the network. The routing-switching decisions of a signal that uses direct links are not made at the node that the signal bypasses with those direct links. Also with the less number of regeneration processes, the complexity of the network is decreased a lot.

Arijs, Willems and Parys (2004) examined the use of ULH ultra long haul technology in a telecommunication network. They stated that the cost of electrical processing could be decreased by the introduction of ULH technology, without using optical switches at some nodes. Moreover, complexity of an all-optical network could be lessened. The authors studied an example of pan-European network for three following scenarios:

- Opaque network (Regeneration at every node for all channels).
- Transparent network with selective regeneration: regenerate channels in a node only when needed.
- Opaque network with express links.

The network contained 26 nodes and 34 links. They selected 8 nodes to act as the head or tail of the express links manually and constructed 13 express links with those nodes. They performed the case study by using WDM Guru, which is a commercial network planning solution that enables service providers and network equipment manufacturers to design resilient, cost-effective optical and SONET networks. The results of network, node and link costs of the solution were presented. They stated that express link design was 20% cheaper than the opaque design, since less number of OXC ports and transponders were used. Moreover the number of DWDM systems used in the express layer design were less than for the opaque and transparent design. On the other hand, express design needed more number of optical amplifiers, which are placed more frequently along the express links, in order not to be affected from noise since express links bypass some nodes.

As a result of their analysis, they stated the cost savings can be around 20% for the total network and it can be improved by optimally chosen express link placement. For this reason we look for a possible optimal solution for such systems, a mathematical model which includes as many of the real life cost drivers as possible while maintaining the signal transmission requirements.

In the specific problem that we study, we look for how one can provide service on an optical telecommunication network designed with reasonable cost values. Any node of the optical network is a supply and a demand point at the same time. We are given a network (N, E) with already operating links. The express link definition allows us to open new links which bypass some nodes of the network and connect two nodes which were not adjacent to each other before the express link was opened. We try to decide which existing links and the new express links will be operating, how many fibers will be activated on an operating link and the routes of signals in the telecommunication network.

Chapter 3

RELATED WORK FROM LITERATURE

At first glance, the specific problem that we have stated in Chapter 2 seems to be a research area presented to the interest of electric-electronic engineers. However, when we try to formulate the problem under several assumptions, we see that the problem is a kind of capacitated network design problem, which is also one of the research areas of network designers. The fiber cables can carry a limited number of signals with different wavelengths and this capacity restriction resembles to some of the studies in OR literature that are about capacitated network design.

The related topics for our problem can be classified under two main titles: the related work from IEEE literature, which study the signal transmission systems in telecommunication networks, and the related work from OR literature, which study similar network design problems in terms of formulation and problem modeling.

3.1 Related Work from IEEE Literature

In order to have a clear understanding of telecommunication network systems, we aim to provide detailed information for some of the problems from IEEE literature, which are related to our work. Other than giving the assumptions of the problems, we state the basic solution techniques of those studies. Although most of the studies related to our problem in this literature focus on routing signals or appropriate wavelength assignment problem, we summarize some of them to give an idea about the problem.

The study of Mukherjee, Banerjee and Ramamurthy (1996) presents principles for designing an optical wide-area WDM network with wavelength multiplexers and optical switchers. Packet forwarding is performed from one node to another by electronic switching and wavelength conversion is not possible. Once a wavelength is assigned to a lightpath, the wavelength stays the same during the transmission of signal.

The nonlinear model they present considers wavelength assignment to paths, capacity constraints about the fiber links and finding the path of an i-j node pair. Two different nonlinear objective functions are presented; one for minimizing the delay and the other one for maximizing the offered load. This optimization problem is NP-hard since several sub-problems of their problem are NP-hard. The solution approach concentrates on two of subproblems. A kind of simulated annealing approach, which utilizes node-exchange operations on a given initial virtual design, is used to find a good virtual topology. Secondly, they develop a flow deviation algorithm for minimizing the network-wide average packet delay. As a result they study the overall design, analysis, upgradability and optimization of a nation-wide WDM network, by considering the device capabilities.

Another study on routing and wavelength assignment problem (RWA) is given by Özdaglar and Bertsekas (2003). They propose an integer-linear programming formulation with a cost minimizing objective function under the assumption of no wavelength conversion. In this formulation, a wavelength is assigned to a lightpath. The model can also be modified for a system where sparse wavelength conversion is possible. Their experiments resulted in integral solutions most of the time and optimal or nearly optimal solutions for RWA problem can be obtained, even under the relaxation of integrality constraints.

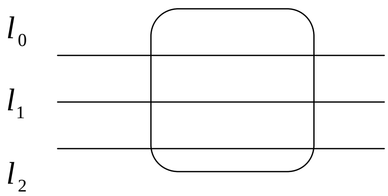
Ramaswami and Sivarajan (1995) inspect the problem of routing traffic between node pairs of an optical network. They try to find a path for each communicating i-j node pair and send the i-j traffic through that path. The traffic of each node pair is assigned a wavelength l. They emphasize the similarity between circuit-switched telephone networks and telecommunication networks. For a telephone call between i-j pair, circuit-switched telephone networks have to assign a circuit on each link of the i-j path. On the other hand, their optical network model has to assign the same wavelength to the i-j call (or traffic) on each link of the path. If the system had assumed that dynamic wavelength assignment converters are used at the intermediate nodes, then the optical network problem would become equivalent to circuit-switched telephone network problem.

In the paper, they solve the routing problem for a fixed set of connections and give an integer program. The objective is to maximize the number of connections that are successfully routed. Linear programming relaxation of the model gives an upper bound for the possible successfully routed connections with the assumption of no wavelength conversion. Later they derive a similar upper bound for a system where wavelength conversion is available and compare two cases; with and without wavelength conversion. They show that the upper bound found for the case with no wavelength conversion is a better bound on the carried traffic than the upper bound they found for the case with wavelength conversion.

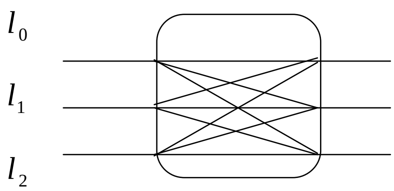
Ramaswami and Sivarajan (1995) state two main results for RWA problem of all-optical networks that they study. Firstly, large all-optical networks without wavelength conversion can be built and a number of successful connections per node can be guaranteed with a reasonable number of wavelengths available in the system. Secondly, their computations show that networks with wavelength converters offer a 10-40% increase in the amount of reuse achievable for the sample networks they have studied. The contribution of wavelength converters is more for larger networks than smaller ones, especially when the number of wavelengths available in the network is limited.

The lost traffic in a telecommunication network is another concern of researchers in telecommunication networks. Sanso, Soumis and Gendreu (1991) give a formulation to minimize the lost traffic in the network. The model basically consists of flow conservation constraints, capacity constraints and nonnegativity constraints. Capacity constraints assume that each arc has one type of capacity, in case the arc is used for the traffic flow. Flow conservation constraints have additional variables, which state the amount of lost traffic. The concentration of the study is mainly on the reliability problem in circuit-switched telecommunication networks. They present a new type of reliability measure which considers location of failure, capacity of the failed link and importance of lost calls. The measure depends on the evaluation of routing and rerouting policies in case of link failures in the network and considers the flexibility of the telecommunication network for rerouting flow after failure.

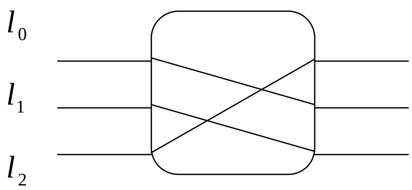
When the total number of wavelengths available in the network is not enough to route the traffic, changing wavelength of a signal at an intermediate node of its route enables to provide desired flow balance. Wavelength conversion capability at the intermediate nodes is classified in the study of Ramaswami and Sasaki (1998). Four cases for the wavelength conversion can be stated for ring, star and tree networks:



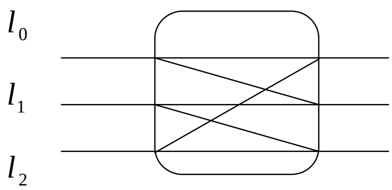
No conversion



Full conversion



Fixed conversion



Limited conversion

No conversion case corresponds to networks where wavelength conversion is not possible at the nodes of the system. In fixed conversion case, wavelength of a signal is converted to a different wavelength which is fixed for the initial wavelength. For the networks with limited conversion capability, wavelength of a signal has a limited number of wavelength alternatives to be converted. The full conversion case allows us to convert wavelength of a signal to any other wavelength that is available in the network.

The main focus of study of Ramaswami and Sasaki (1998) is on WDM networks, where the wavelength conversion capacity at the nodes is limited. They do not give a linear model to find how conversion will take place at the nodes; but theorems for ring and star networks are provided to have minimal wavelength conversion. The results show that ring and star networks can be constructed with minimal wavelength conversion capability, which can perform off-line channel assignment as good as networks with full wavelength conversion.

Wauters and Demester (1996) consider the blocking probabilities of two systems WP (wavelength path) and VWP (virtual wavelength path). WP case only routes the incoming wavelength to outgoing links appropriately and no wavelength conversion is allowed at the intermediate nodes. VWP, on the other hand, can convert wavelength of traffic to another wavelength at a cross-connect, which occurs at the intermediate nodes. WP is a more restricted case; that is blocking in WP occurs if no wavelength corresponding to the specific traffic can be found on the links of the route of the traffic. For a VWP, blocking occurs only if there is no wavelength to assign on the route to the specific traffic. Wauters and Demester show that when the number of wavelengths available on a fiber (in terms of fiber capacity) is more, the performance difference between WP and VWP is less. One conclusion about their study is that as the number of wavelengths that can be used on a fiber increases, shorter routes are possible for WP and the performance of WP approaches VWP; but never catches. Moreover, higher traffic load in the system means more blocking probability for both WP and VWP systems, especially significant for WP.

The studies from the IEEE literature show that the problem of routing and wavelength assignment was considered many times under different assumptions. The models mainly use the multicommodity flow constraints and the system is examined either from the very beginning with no constructed links or the possibility of rerouting traffic with only available links. Opening a new set of links over an existing system has not been examined with a model in the studies we examined. Moreover, most of the work has been towards networks with no wavelength conversion or limited wavelength conversion. The objectives proposed in these studies mainly aimed to minimize delay, number of wavelengths used or maximize total traffic that is successfully routed. Less attention has been paid to minimizing the cost of the network, depending on the number of system devices or link opening costs.

3.2 Related Work from OR Literature

In the previous section, the problem was generally introduced as a routing and wavelength assignment (RWA) problem. Considering the cost factors in the design of a network; the set of links chosen to open, the number of fibers operating on the links and the traffic route of each node pair in the system play a critical role in the expenses. The problem of designing a network where the links do not have capacities that limit the amount of flow has been studied many times in OR literature under the name of uncapacitated network design problem (UCNDP). However, the situation where the links have capacities, known as capacitated network design problem (CNDP), did not attract many researchers as UCNDP did [14]. Relaxations of CNDP generally yield weak lower bounds which are far from optimal solution. Especially, linear programming lower bounds are weak for most capacitated network design problems [18] and the gap between linear programming relaxation and the optimal solution is large.

The capacitated network design problem questions which links of the network should operate in order to provide conservation of flow between nodes. There is one type of link with a known capacity, which can be installed between two nodes of the network. In our problem, other than deciding which links will operate, we will decide the number of fiber cables to open on the selected links of the network. This means one more decision is to be made for each selected link. Then our specific problem turns out to be a kind of network-loading problem. In a classic network-loading problem, there are a number of different types of links with different capacities to open on the connections of the network. The designer chooses one of the link types to open on the selected arc. In our problem, we can think of different capacities as multiples of the capacity of one fiber cable. If we choose to open k number of fiber cables on the arc of network; then this means we have opened the link type with capacity $k.c$, where c is the capacity of one fiber cable. The network loading problem and the capacitated network design problem have some studies in the OR literature which will be introduced in following sections. Magnanti and Wong (1984) give a survey of network design problems, in which they consider general formulations of those problems. Their formulations mainly aim to solve transportation problems rather than the problems that appear in telecommunication and computer networks.

The capacitated network design problem [14] and the network loading problem [16], [17] are both NP-hard problems. Although our problem is a kind of network loading problem with routing costs in the objective function, we also examine the literature for CNDP to have an idea of the approaches for a capacitated case, in section 3.2.1. The work that has more relevance to our study is given in section 3.2.2 under the name of network loading problem.

3.2.1 Capacitated Network Design Problem (CNDP)

Among the studies about CNDP in literature,we mainly present the ones which have important contributions to solution techniques of CNDP or provide significant improved results compared to previous work.

Lagrangian relaxation has been widely used by many researchers that study CNDP and it became the starting point of many heuristics approaches [9], [10]. Holmberg and Yuan (2000) provide a lagrangian heuristic based branch and bound algorithm approach and state the features of two different lagrangian relaxations of a CNDP model. They compare the performances of CPLEX and their lagrangian-based branch and bound method for different network scenarios and conclude that their method is better in most of the cases by either finding the optimal solution or providing better solutions in one hour time. Crainic, Frangioni and Gendron (2001) also examine the results of bundle and subgradient methods for two lagrangian relaxations (shortest path relaxation and knapsack relaxation) of the problem. They compare the bounds obtained from different bundle-based relaxation methods and state that those methods are superior to subgradient approach since bundle-based methods converge faster and they are more robust to problems with different characteristics.

Sridhar and Park (2000) use a Benders-and-cut algorithm for a fixed-charge CNDP where the objective function is to minimize the installation cost of links. Problems on a complete graph with node numbers 6, 10, 15 and 20 are considered. They conclude that when the demand traffic is low, it is easier to solve the problem, however as traffic demand is higher, Benders-and-cut algorithm is quite effective to get better solutions.

Crainic, Gendreau and Farvolden (2000) consider a fixed charge capacitated multicommodity network design problem (CMND) for realistically sized problem instances, with node numbers changing from 20 to 100 and arc numbers from 100 to 700. They provide a simplex-based tabu search metaheuristic which gives good feasible solutions within reasonable computing efforts. The metaheuristic utilizes simplex pivot-type moves with column generation to find the space of continuous path flow variables. The technique also considers the actual mixed integer objective of capacitated multicommodity network design problem and the technique is robust with respect to type of problem; capacity of links, size of network and fixed costs.

Among the studies about CNDP, the most significant results so far have been obtained by Ghamlouche, Crainic and Gendreu. In 2003, the authors propose a new class of neighborhoods for metaheuristics to improve the range of moves by which the flow deviations are not restricted to paths that connect origin and destination. In this sense, their new tabu search algorithm, which utilizes cycle-based neighborhoods, provide better solutions and gaps compared to the study of Crainic, Gendreau and Farvolden 2000. A year later, Ghamlouche, Crainic and Gendreu (2004) add a path relinking procedure to their cycle-based neighborhood approach and obtain even better results than what they did in 2003.

3.2.2 Network Loading Problem

In a network design problem, when there is one type of link in terms of structure and capacity, the problem of how many links to open on an arc i-j becomes a kind of network loading problem. If the maximum number of links that can be opened on an arc i-j is m, then the number of actually opened links k, $0 \leq k \leq m$ determines the amount of flow that we can carry on the arc i-j. We can reconsider the problem as if there were m type of links available to open on an arc i-j with capacitites 1.c, 2.c, ... , m.c where c is the capacity of one link. Then the problem that we define in Chapter 2 is NP-hard since it generalizes the network loading problem with routing costs [16].

Like the capacitated network design problem, the network loading problem did not attract much attention in the literature. Among the studies about network loading problem, we can give two classes as single facility and multiple facility network loading problem. In single capacity case, a link type with capacity c can be installed an integer number of times on a link. For multiple facility case, a number of types of links are available with different capacities and a number of one link type can be installed on a arc of the network.

Gong (1995) study a network design problem for telecommunication problems. There are several types of links with different discrete sizes to be placed between appropriate nodes, in order to satisfy supply-demand balance with minimum cost. The traffic of a specific source-terminal pair travels on any single path without flow splitting across multiple links which have a common node. The complexity of their problem is due to discrete choice of link size and the single path requirement for each origin-destination pair.

Two models, a link based formulation and a path based formulation, are given to formulate the problem. The authors develop important facet defining inequalities for the link based formulation and show that these are also needed for the path based formulation. The branch and bound algorithm suggested for the path based formulation is computationally more effective than the link based formulation and they can solve problems with 15 nodes optimally.

Gabrel, Knippel and Minoux (1999) describe a solution procedure for capacitated network design problems with general step cost functions. They give a cost function which generalizes the single and multiple facility network loading problems. An implementation of constraint generation techniques have been given to get optimal solutions up to 20 nodes - 37 links and cost functions with an average six steps per link.

Magnanti and Mirchandani (1995), Mirchandani (2000), focus on the case where two types of facilities are available to choose for the arcs of the network. Magnanti and Mirchandani (1995) study a two-facility capacitated network design problem (TFLP) from the telecommunincation industry with no variable flow cost. The point-to-point communication demand of a network is to be met by using two types of links; link type-1 with one-unit capacity and link type-2 with *C*-unit capacity. The model assumes that the link type with *C*-unit capacity utilizes economies of scale and installing C number of one-unit capacity link is more expensive than constructing a single *C*-unit capacity link. Two approaches, lagrangian relaxation and a cutting plane technique with three classes of valid inequalities, are presented for the solution of the mixed integer program of the problem. They aim to improve lower bound of the problem by using stronger formulations than its linear programming relaxation and later seek for more efficient solution techniques. The lagrangian relaxation of capacity constraints of the problem results a network flow problem which satisfies integrality property. In this case, the lagrangian dual problem gives the same lower bound as the linear programming relaxation of the TFLP

(Geoffrion 1974). However, lagrangian relaxation of the flow conservation constraints and addition of a set of upper bound constraints to the relaxed problem yields a formulation P(LAG) which does not have integrality property. The lower bound of P(LAG) is better than the linear programming relaxation of the TFLP. Secondly, the valid inequalities found for improving the polyhedron decrease the integrality gap effectively under the conditions stated, while the size of linear program does not increase much.

Mirchandani (2000) used a projection based procedure to solve the same network loading problem of two types of facilities with capacities of one unit and C units. He suggests a mixed integer programming formulation that includes flow conservation and capacity constraints and additinoally cutset constraints which define facets under specific conditions. The projection of the model into a lower dimension is defined for the single commodity and multicommodity versions of this network loading problem with two link types. The polyhedral features of the projections is studied and several sets of facet defining inequalities are presented.

Agarwal (2002) presents a simple and effective heuristic algorithm for a multiple facility network design problem. In the scenarios studied, at most four types of links with different capacities are available. They study a complete graph; any node pair can be connected with any type of facility defined. Cost of installing a facility on an arc is considered in the model, however there is no cost related to flow variables. They provide gaps around %5 for problems up to 20 nodes and only feasible solutions are given for problems up to 99 nodes without an attempt to compute lower bound.

CHAPTER 3 RELATED WORK FROM LITERATURE

The network loading problem, as well as the capacitated network design problem are challenging problems to solve since both problems are NP-hard and their relaxations give weak lower bounds. We study a specific telecommunication network design problem whose formulation turns out to be a kind of network loading problem with additional traffic routing costs.

Chapter 4

TWO MODELS PROPOSED FOR THE PROBLEM

In order to model the problem that we define in the Chapter 2, we need to state several assumptions. First of all, we assume that full wavelength conversion is possible for the all nodes of the telecommunication network. Without this assumption, we need to prevent the use of a fiber by the paths of signals with same wavelength. Many studies [24], [21], [22] in the literature assign one appropriate wavelength to a signal and do not change the wavelength at intermediate nodes along the route. By this assumption, we can continue to send signals through a fiber cable until no more free wavelength is available in the fiber. A new fiber on a link will be needed if the fibers already opened on link are fully utilized.

Secondly, we assume that a fiber cable of express links that we are going to open has the same capacity with a fiber cable of normal link; that is both normal links and express links are assumed to consist of fibers which can carry CL=20 signals with different wavelengths in our study. This assumption also corresponds to the availability of 20 wavelengths in a fiber. Having a larger CL value means more capacity is available for one fiber cable. With a larger CL, we can decrease the necessary number of fibers that will be opened on an arc, however this also means that more number of wavelengths will be available in the network and cost of the network may increase due to increased number of wavelength availability in the network. On the other hand, smaller CL value means that a fiber can carry less number of signals with different wavelengths, which corresponds to less number of wavelength availability in the network. In this case, we need to open more number of fibers to transmit same number of signals in the network. With a smaller CL, our problem becomes a network loading problem where there are more number of link types available for an arc. If we have a quite large CL, opening only one fiber on selected arcs of the network may be enough to provide flow balance, which means we solve CNDP. In our study, we fix $CL = 20$ and create both of the scenarios by changing the demand density of the network. For a scenario with low dense demand pattern, we solve a CNDP with $CL=20$. However, if the demand density is high, we need to allow opening a number of fibers on each of the selected arcs and we solve a network loading problem.

The express links are chosen among the set of shortest paths of all node pairs in the network. If the distance of shortest path between any i-j node pair in the network is less than the SQDD (signal quality drop distance); then the shortest path is included into the set of express links that can be opened in the network. In other words, any i-j node pair can be connected by an express link if the shortest path between i-j is less than SQDD. An express link is constructed by connecting the normal links on the shortest path of i-j node pair. The nodes on the shortest path are bypassed by the express link. Moreover, the devices for amplification of signals need to be placed more frequently along the express links, since express links are assumed to be longer. For this reason, we assume that the unit-meter cost of constructing an express link is more expensive than a normal link. In the model, different unit meter costs are used for normal and express links.

For the routing cost of a signal, we assume if a signal uses a fiber of normal link or express link, the same OEO device cost is paid for every fiber used, independent of the length. If there are two paths available for a signal of i-j traffic with same number of arcs on the paths, we pay the same cost for sending a signal along any of these paths.

We assume that economies of scale do not exist for the cost of fibers opened on one arc. The cost of opening fibers on one arc is linearly proportional to the number of fibers operating on the link. As an example, cost of opening one fiber on an arc (k,m) has *c* cost, whereas three fibers on the same arc will have *3c* cost. This cost decision is more applicable to systems which work under the principle of leasing agreements.

For the demand pattern, we assume that almost all i-j node pairs of the network can have traffic. The demand of *k* units to be sent from node i to node j means that, we have *k* different signals to originate at node i with destination j. Those signals can be sent to node j separately through different paths which are arc disjoint, as well as through paths with common fibers as long as their wavelengths are different if they share the same fiber. This allows us to split traffic of an i-j node pair in the network.

After we state the assumptions, we give two formulations for the problem. First one “M-4” is an adapted version of the classic formulation that we observe for most of the network flow problems in the literature. The variables and parameters of the formulation M-4 is as follows:

Sets of arcs defined in the network (N,E):

A_X : set of normal links that can be opened.

$A_X = \{ (k,m) \cup (m,k) : \{k,m\} \in E \}$

A_{EL} : set of express links that can be opened.

$A_{EL} = \{ (k,m) : SP_{km} \leq SQDD, (k,m) \notin E \}$

where SP_{km} states the shortest path length between nodes *k* and *m*. We do not open an express link between i-j node pair, if there already exists a normal link between this pair and for this reason we have $A_X \cap A_{EL} = \varnothing$.

Set of Demand Pair:

K : set of origin-destination ordered pairs. $K = \{ (i,j) : d_{ij} > 0 \}$

where d_{ij} : number of signals to be sent from i to j.

Variables of model M-4:

X_{km}^{ij} : number of signals with origin-i and destination-j, using a normal fiber on arc (k,m) $\forall (k,m) \in A_X$, $\forall (i,j) \in K$

EL_{km}^{ij} : number of signals with origin-i and destination-j, using an express fiber on arc (k,m) $\forall (k,m) \in A_{EL}$, $\forall (i,j) \in K$

SLX_{km} : number of normal fibers to open on arc (k,m) $\forall (k,m) \in A_X$

SLE_{km} : number of express fibers to open on arc (k,m) $\forall (k,m) \in A_{EL}$

Parameters of model M-4:

FC_{km}: fixed cost of opening one fiber on arc (k,m).

where $FC_{km} = a + dist_{km}.C_X \quad \forall (k,m) \in A_X$, and $FC_{km} = a + dist_{km}.C_{EL}$ $\forall (k,m) \in A_{EL}$

The fixed cost of opening one fiber is a which corresponds to the cost of devices used at the head and tail nodes of an arc. $dist_{km}$ is the distance between nodes k and m. Unit meter costs of opening a fiber on normal and express link are C_X and C_{EL} respectively, with $C_X \le C_{EL}$.

C_{km} : fixed cost of one signal using a fiber on (k,m). We assume that this cost is same for all (k,m) arcs and index (k,m) is for the generalized formulation.

CL: maximum number of signals that can be carried by one fiber. Throughtout this study, CL is restricted to 20.

$MaxOnArcX_{km}$: maximum number of normal fibers that can be opened on arc (k,m). $\forall (k,m) \in A_X$

$MaxOnArcEL_{km}$: maximum number of express fibers that can be opened on arc (k,m). $\forall (k,m) \in A_{EL}$

Formulation of model M-4:

$$\text{Min} \sum_{(k,m)\in A_X} FC_{km}.SLX_{km} + \sum_{(k,m)\in A_{EL}} FC_{km}.SLE_{km} +$$

$$\sum_{(i,j)\in K} \sum_{(k,m)\in A_X} C_{km}.X_{km}^{ij} + \sum_{(i,j)\in K} \sum_{(k,m)\in A_{EL}} C_{km}.EL_{km}^{ij}$$

s.t.

$$\left(\sum_{k:(k,m)\in A_X} X_{km}^{ij} + \sum_{k:(k,m)\in A_{EL}} EL_{km}^{ij}\right) - \left(\sum_{n:(m,n)\in A_X} X_{mn}^{ij} + \sum_{n:(m,n)\in A_{EL}} EL_{mn}^{ij}\right)$$

$$= \begin{cases} d_{ij} & \text{if } j = m \\ -d_{ij} & \text{if } i = m \\ 0 & \text{if } i \neq m, j \neq m \end{cases} \qquad \forall (i,j)\in K,\ \forall m\in N \quad (1.1)$$

$$\sum_{(i,j)\in K} X_{km}^{ij} \leq CL\,.\,SLX_{km} \qquad \forall (k,m)\in A_X \quad (1.2)$$

$$\sum_{(i,j)\in K} EL_{km}^{ij} \leq CL\,.\,SLE_{km} \qquad \forall (k,m)\in A_{EL} \quad (1.3)$$

$$SLX_{km} \leq MaxOnArcX_{km} \qquad \forall (k,m)\in A_X \quad (1.4)$$

$$SLE_{km} \leq MaxOnArcEL_{km} \qquad \forall (k,m)\in A_{EL} \quad (1.5)$$

$$SLE_{km} \geq 0 \text{ integer } \forall (k,m)\in A_X,\ SLX_{km} \geq 0 \text{ integer, } \forall (k,m)\in A_{EL},$$

$$X_{km}^{ij} \geq 0 \text{ integer } \forall (k,m)\in A_X \quad \forall (i,j)\in K,$$

$$EL_{km}^{ij} \geq 0 \text{ integer } \forall (k,m)\in A_{EL},\ \forall (i,j)\in K.$$

We minimize the cost of fibers that are opened in the network and the routing cost of signals. C_{km} values are assumed to be same since they correspond to the cost of OEO devices, which are same for normal and express links, independent of length. FC_{km} values include a fixed cost of opening a fiber and a variable cost linearly proportional to the length of the fiber opened. First constraint is the flow balance constraint that gives which arcs are used for the traffic of specific i-j node pair. Constraints (1.2) and (1.3) find the total flow on an arc (k,m) and force the model to open enough number of fibers to let this traffic flow on the arc. The maximum number of fibers that are opened on an arc (k,m) is limited by constraints (1.4) and (1.5).

Other than the M-4 formulation of the problem, we can aggregate the flow on an arc (k,m) such that the signals with origin i is denoted by one variable as $X^{i}_{km} = \sum_{j:(i,j)\in K} X^{ij}_{km}$ and $EL^{i}_{km} = \sum_{j:(i,j)\in K} EL^{ij}_{km}$. By using the new variable with 3 indices, we give a second formulation "M-3" that is specific to our problem and we also use the assumption of full-wavelength conversion availability at the nodes of the network. Only the following new variables are introduced for "M-3".

Variables of model M-3:

X^{i}_{km} : number of signals with origin-i, using a normal fiber on arc (k,m) $\forall (k,m)\in A_X$, $\forall (i,j)\in K$

EL^{i}_{km}: number of signals with origin-i, using an express fiber on arc (k,m) $\forall (k,m)\in A_{EL}$, $\forall (i,j)\in K$

Formulation of model M-3:

$$\text{Min} \quad \sum_{(k,m)\in A_X} FC_{km}.SLX_{km} + \sum_{(k,m)\in A_{EL}} FC_{km}.SLE_{km} +$$

$$\sum_{\substack{i\in N\\ i\neq m}} \sum_{(k,m)\in A_X} C_{km}.X^{i}_{km} + \sum_{\substack{i\in N\\ i\neq m}} \sum_{(k,m)\in A_{EL}} C_{km}.EL^{i}_{km}$$

s.t.

$$\sum_{m:(i,m)\in A_X} X^i_{im} + \sum_{m:(i,m)\in A_{EL}} EL^i_{im} = \sum_{j:(i,j)\in K} d_{ij} \qquad \forall i \in N \quad (2.1)$$

$$\Big(\sum_{k:(k,j)\in A_X} X^i_{kj} + \sum_{k:(k,j)\in A_{EL}} EL^i_{kj}\Big) - \Big(\sum_{\substack{m:(j,m)\in A_X \\ m\neq i}} X^i_{jm} + \sum_{\substack{m:(j,m)\in A_{EL} \\ m\neq i}} EL^i_{jm}\Big) = d_{ij} \quad \forall i, j \in N, i \neq j \quad (2.2)$$

$$\sum_{\substack{i\in N \\ i\neq m}} X^i_{km} \leq CL \,.\, SLX_{km} \qquad \forall (k,m) \in A_X \quad (2.3)$$

$$\sum_{\substack{i\in N \\ i\neq m}} EL^i_{km} \leq CL \,.\, SLE_{km} \qquad \forall (k,m) \in A_{EL} \quad (2.4)$$

$$SLX_{km} \leq MaxOnArcX_{km} \qquad \forall (k,m) \in A_X \quad (2.5)$$

$$SLE_{km} \leq MaxOnArcEL_{km} \qquad \forall (k,m) \in A_{EL} \quad (2.6)$$

$$SLE_{km} \geq 0 \text{ integer } \forall (k,m) \in A_X, \qquad SLX_{km} \geq 0 \text{ integer } \forall (k,m) \in A_{EL}$$

$$X^i_{km} \geq 0 \quad \forall (k,m) \in A_X, \ \forall i \in N, i \neq m$$

$$EL^i_{km} \geq 0 \quad \forall (k,m) \in A_{EL}, \ \forall i \in N, i \neq m$$

Our objective function is almost the same as we state for M-4 formulation. The only difference is in the calculation of the routing cost. M-3 formulation finds the same cost since we still multiply the number of signals on a fiber with unit signal cost C_{km}. Our first constraint (2.1) guarantees that all demand with origin i has left the node i by using a normal or express link (i,j). Second constraint states that the flow that has originated at node i, should leave the demand d_{ij} at node j, after the flow with origin i has left node j. Note that (2.1) and (2.2) together make n^2 constraints, where n = |N| is the cardinality of node set N. (2.3) and (2.4) are capacity constraints of network, which determine the number of fibers to open. The constraints (2.5) and (2.6) are the limitations on the number of normal or express fibers that can be opened on an arc.

In order to compare two formulations in terms of number of constraints and number of variables, we give the following networks with appropriate changing terms in table 4.1. The demand in network number 2 is denser than first network. The third and fourth networks have 35 nodes and same demand pattern. We route the same number of signals in networks 3 and 4.

Table 4.1				variable number		constraint number	
Network number	Number of nodes	Number of arcs	Max. number of fibers	M-3	M-4	M-3	M-4
1	26	146	2	3952	36731	1114	7728
2	26	146	4	3952	77663	1114	15909
3	35	108	4	3888	85548	1585	30456
4	35	198	4	7128	156796	1853	30726

If we compare two formulations M-3 and M-4 in terms of number of constraints and variables, we see that the size of M-4 is much more than M-3. First of all, the number of constraints in (1.1) of M-4 formulation depends on the number of commodities |K|.|N|, whereas (2.1) and (2.2) are exactly $|N|^2$, which is less than |K|.|N| in the networks with many traffic pairs. For the same network (N, E), as the demand pattern is denser, the size of the model for M-4 is quite much affected whereas the size of M-3 is not affected. However the number of constraints of M-4 is less affected if the number of arcs is increased for the same node number.

After the computations in CPLEX 9.0, we see that an optimum solution cannot be found for the networks we study with 26 and 35 nodes, after 72 hours. We consider to relax the capacity constraints in a lagrangian way. The resulting relaxed problem P_{lag} is solved in seconds and yields a lower bound which is quite far away from the optimal solution of the problem. In most capacitated network design problems linear programming bounds are weak and our P_{lag} is provides lower bounds less than the LP lower bound. In order to improve lower bound of P_{lag} we add a set of s-t cuts and a logical cut to P_{lag} of M-3 formulation.

Chapter 5

LAGRANGIAN RELAXATION OF M-3 AND ADDED CUTS

In this chapter, we give the lagrangian relaxation of model M-3. Lagrangian relaxation of M-3 is solved quicker than the lagrangian relaxation of M-4. The relaxed model of M-3 is easily solved which enables us to use the relaxation in a subgradient algorithm described in section 5.2. Although the solution times for P_{lag} is around seconds, the lower bound we get from the relaxation is quite weak. In order to improve the lower bound, we add two sets of cuts to the formulation P_{lag} in sections 5.3 and 5.4.

5.1 Lagrangian Relaxation P_{lag}

We relax the original formulation P to yield P_{lag}. Among the constraints of formulation M-3, we move (2.3) and (2.4), which connect the flow variables and link opening variables, to the objective function with lagrangian multipliers l_{km}, as follows:

$$\text{Min} \quad \sum_{(k,m)\in A_X}(FC_{km} - l_{km}.CL)SLX_{km} \;+\; \sum_{(k,m)\in A_{EL}}(FC_{km} - l_{km}.CL)SLE_{km} +$$

$$\sum_{\substack{i\in N\\ i\neq m}}\sum_{(k,m)\in A_X}(C_{km}+l_{km})X^i_{km} \quad + \quad \sum_{\substack{i\in N\\ i\neq m}}\sum_{(k,m)\in A_{EL}}(C_{km}+l_{km})EL^i_{km}$$

s.t.

$$\sum_{m:(i,m)\in A_X} X^i_{im} + \sum_{m:(i,m)\in A_{EL}} EL^i_{im} = \sum_{j:(i,j)\in K} d_{ij} \qquad \forall i\in N \qquad (2.1)$$

$$\Big(\sum_{k:(k,j)\in A_X} X^i_{kj} + \sum_{k:(k,j)\in A_{EL}} EL^i_{kj}\Big) - \Big(\sum_{\substack{m:(j,m)\in A_X\\ m\neq i}} X^i_{jm} + \sum_{\substack{m:(j,m)\in A_{EL}\\ m\neq i}} EL^i_{jm}\Big) = d_{ij} \quad \forall i,j\in N, i\neq j \quad (2.2)$$

$$SLX_{km} \le MaxOnArcX_{km} \qquad \forall (k,m)\in A_X \quad (2.5)$$

$$SLE_{km} \le MaxOnArcEL_{km} \qquad \forall (k,m)\in A_{EL} \quad (2.6)$$

$$\sum_{i\in N, i\neq m} X^i_{km} \le CL.MaxOnArcX_{km} \qquad \forall (k,m)\in A_X \quad (2.33)$$

$$\sum_{i\in N, i\neq m} EL^i_{km} \le CL.MaxOnArcEL_{km} \qquad \forall (k,m)\in A_{EL} \quad (2.44)$$

$$SLE_{km} \ge 0 \text{ integer } \forall (k,m)\in A_X, \qquad SLX_{km} \ge 0 \text{ integer } \forall (k,m)\in A_{EL}$$

$$X^i_{km} \ge 0 \text{ integer } \forall (k,m)\in A_X,\ \forall i\in N,\ i\neq m$$

$$EL^i_{km} \ge 0 \text{ integer } \forall (k,m)\in A_{EL},\ \forall i\in N,\ i\neq m$$

In order to obtain a feasible flow from the solution of P_{lag}, for our original problem P, we add (2.33) and (2.44). These constraints limit the maximum flow on (k,m) arc with constant numbers $CL.MaxOnArcX_{km}$ and $CL.MaxOnArcEL_{km}$.

Resulting model is easily solved and the solution found from P$_{lag}$ provides a feasible flow balance solution for the problem P. However, SLE_{km} and SLX_{km} values are not given in the solution of P$_{lag}$ since the relaxed constraints would open necessary number of fibers in the solution. In order to find the values of SLE_{km} and SLX_{km} that will allow the feasible flow found by P$_{lag}$, the total traffic on every arc (k,m) is calculated with $\sum_{i \in N, i \neq m} X^i_{km}$ and $\sum_{i \in N, i \neq m} EL^i_{km}$. The necessary number of fibers to open on the arc (k,m) of the network is found by $SLX_{km} = \left\lceil \frac{\sum_{i \in N, i \neq m} X^i_{km}}{CL} \right\rceil$ and $SLE_{km} = \left\lceil \frac{\sum_{i \in N, i \neq m} EL^i_{km}}{CL} \right\rceil$. After the SLE_{km} and SLX_{km} values are obtained, the cost of the feasible flow for the network can be calculated.

5.2 Subgradient Algorithm

The problem P$_{lag}$ can be solved in seconds and this situation allows us to use the relaxed formulation in a subgradient algorithm iteratively. We use the algorithm that is described by Ghiani, Laporte and Musmanno (2003) and we give the main steps of this algorithm for updating the lagrangian multipliers of P$_{lag}$ problem as follows:

Initial Values of the algorithm:

t = 0, $l^0 = \mathbf{0}$, $LB = 0$, $L\_UB = 100000000$,

Iteration t of subgradient search algorithm:

1.1 Solve problem P$_{lag}$(l^t). If $LB(l^t) > LB$, update $LB = LB(l^t)$.

1.2 Find the feasible solution for original problem P by using the solution of $P_{lag}(l^t)$. Update L_UB, if $UB(l^t) < L\_UB$.

1.3 Determine e_{km}, $\forall(k,m)\in A_X \cup A_{EL}$. Calculate b^t for step size to update lagrangian multipliers.

1.4 Set $l_{km}^{t+1} = \max\{\ 0, (\ l_{km}^t + b^t \cdot e_{km})\ \}$ $\qquad \forall(k,m)\in A_X \cup A_{EL}$

1.5 Set t = t+1. If t < max_num_of iteration, go to step 1.1.

Otherwise terminate the algorithm.

LB is the objective function value of lagrangian relaxation solution, P_{lag} model. L_UB is the upper bound found by using the solution of lagrangian relaxed model. We get the feasible flow that we obtain from lagrangian solution and then open necessary number of optical fibers on the arcs (Step 1.2).

After a feasible solution is obtained from t^th iteration of subgradient method, we find the values of relaxed constraints (Step 1.3) by using the solution vector obtained from relaxed problem in Step 1.1.

$$e_{km}^t = \sum_{\substack{i\in N \\ i\neq m}} X_{km}^i - CL \cdot SLX_{km} \quad \forall(k,m)\in A_X$$

$$e_{km}^t = \sum_{\substack{i\in N \\ i\neq m}} EL_{km}^i - CL \cdot SLE_{km} \quad \forall(k,m)\in A_{EL} \ , \quad sum\_of\_(e^t)^2 = \sum_{(k,m)\in A_X \cup A_{EL}} (e_{km}^t)^2$$

and $\qquad b^t = a \cdot \dfrac{(L\_UB - LB)}{sum\_of\_(e^t)^2} \qquad$ for a =0.005

In step 1.4, lagrangian multipliers are updated according to the stated equation for all arcs defined in the network.

After updating the lagrangian multipliers at each subgradient iteration, each signal, which is using the (k,m) arc, pays a cost of $C_{km} + l_{km}$. With the added (2.33) and (2.44) feasibility constraints, we solve a kind of minimal cost multicommodity network flow problem. Since the resulting lagrangian relaxation has the integrality property, the best lagrangian lower bound obtained from subgradient algorithm will not be better than LP relaxation of the original problem (Geoffrion 1974). For this reason, we use subgradient algorithm to produce seeds for our heuristic iteratively, in Chapter 6.

5.3 S-T Cuts

The lower bound of lagrangian relaxation problem is too weak to use in the evaluation of a feasible solution that can be found for original problem P by any heuristic. This is not surprising since in most of the capacitated network design problems, lower bounds obtained from lagrangian relaxation alone are not good enough to provide nice gaps [9], [10]. We consider adding a set of cuts to the lagrangian relaxation, which can improve the lower bound. First of these set of cuts is the well-known S-T cuts [14], [17].

We choose a node subset S from the network N and name remaining nodes as the set T= N/S. The demand traffic (i,j) where i belongs to set S and j belongs to set T will use a number of fibers to leave set S (figure 5.1). In the same way, the traffic (i,j) where i belongs to set T and j belongs to set S has to enter set S (figure 5.2).

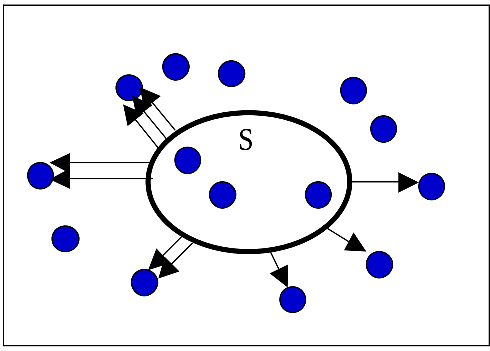


Figure 5.1

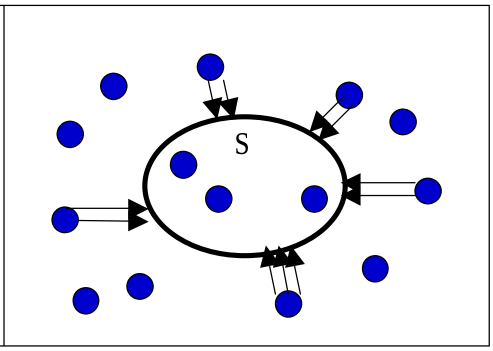


Figure 5.2

According to the statements above, we define the following for a selected $S \subset N$:

Let $\sum_{i \in S, j \in T} d_{ij} = out\_of\_[S]$ and $\sum_{j \in T, i \in S} d_{ji} = into\_[S]$. Then the following two inequalities are valid for the original problem.

$$\left\lceil \frac{out\_of\_[S]}{CL} \right\rceil \le \sum_{\substack{m:(i,m)\in A_X \\ m \in T, i \in S}} SLX_{im} + \sum_{\substack{m:(i,m)\in A_{EL} \\ m \in T, i \in S}} SLE_{im} \qquad (2.7)$$

$$\left\lceil \frac{into\_[S]}{CL} \right\rceil \le \sum_{\substack{m:(m,i)\in A_X \\ m \in T, i \in S}} SLX_{mi} + \sum_{\substack{m:(m,i)\in A_{EL} \\ m \in T, i \in S}} SLE_{mi} \qquad (2.8)$$

(2.7) and (2.8) provide improvement in the solution of P_{lag}. First one guarantees that enough number of fibers that leave the set S is opened in order to satisfy the flow of $\sum_{i \in S, j \in T} d_{ij}$ number of signals from S into T. In the same way, (2.8) guarantees that enough number of fibers that enter into set S from T is opened in order to satisfy the flow of $\sum_{j\in T, i\in S} d_{ji}$ number of signals from T into S. In most cases, some part of demand d_{ij} with $i, j \in T$ can enter set S, which means (2.7) and (2.8) will hold without equality. However these two inequalities force the lagrangian relaxation to open at least a number of fibers, which improves the lower bound of lagrangian solution.

We name the valid inequalities (2.7) and (2.8) as first level cut-set if |S|=1, and n^{th} level cut-set if |S|=n. First level cut-set strengthens the lower bound of P_{lag}, quite much as well as the second level cut-set does. However the contribution of n^{th} level cut-set to the improvement of lower bound decreases as n increases, if we add n^{th} level just after the 1^{st}, 2^{nd} …, $(n-1)^{th}$ levels. Moreover the computation time increases when the number of levels increases. Experimental results show that after the addition of 1^{st} 2^{nd} 3^{rd} and 4^{th} level cut-sets, addition of the 5^{th} and more levels do not provide much improvement in lower bound and the computation time increases tremendously. For this reason, cut-sets with level greater than 4 are not used in our formulations.

5.4 A Logical Cut

Other than the cut-set inequalities defined in previous section, we develop a logical cut, which may improve the lower bound of P_{lag} quite much, for networks with special characteristics related with network demand pattern.

We assume that for any node of the network, the total demand that has to leave node $i \in N$, $\sum_{j\in N} d_{ij}$, can be partitioned into two, with names $o\_close[i]$ and $o\_far[i]$ as follows:

$$\sum_{j:(i,j)\in A_X \cup A_{EL}} d_{ij} = o\_close[i] \quad \text{and} \quad \sum_{j:(i,j)\notin A_X \cup A_{EL}} d_{ij} = o\_far[i] \qquad \forall i \in N$$

since $$out\_of\_[i] = \sum_{j\in N} d_{ij} = \sum_{j:(i,j)\in A_X \cup A_{EL}} d_{ij} + \sum_{j:(i,j)\notin A_X \cup A_{EL}} d_{ij} \qquad \forall i \in N$$

By definitions above, $o\_close[i]$ is the total number of signals that has to be sent from node *i* to adjacent nodes of node i and $o\_far[i]$ is the total number of signals that has to be sent from node *i* to the nodes which are not adjacent to node *i*. It is obvious to see that any signal, which has to be sent from node *i* to a node which is not adjacent to node *i*, has to use at least two different arcs in the network. This means each signal of flow $o\_far[i]$ will spend at least two units capacity of fibers in the network. For the flow amount $o\_close[i]$, we can only say that at least one unit capacity of fibers in the network will be used, which is the trivial conclusion of using a fiber. If we formulize what we have said previously, we have the following:

$$\text{min\_tot\_links} = \left\lceil \frac{\sum_{i\in N}\left(o\_close[i] + 2.o\_far[i]\right)}{CL} \right\rceil$$

where $\lceil k \rceil$ gives the smallest integer $k*$ which satisfies $k \le k*$. The value "min_tot_links" provides a lower bound for the total number of fibers that has to be opened in the network, in order to satisfy flow of signals.

$$\text{min\_}tot\_links \le \sum_{(k,m)\in A_X} SLX_{km} + \sum_{(k,m)\in A_{EL}} SLE_{km} \qquad (2.9)$$

Although the logical inequality (2.9) alone improves the lower bound of P_{lag}, it does not contribute to the improvement of lower bound, when added to P_{lag}

after other 1st, 2nd, 3rd and 4th level cut-sets have been included in the formulation for our network examples. The contribution of (2.9) to the lower bound mainly depends on the demand pattern that is studied for the network. If we consider a demand pattern where each one of the nodes of the network has more traffic to be sent to the nodes, which are not adjacent to itself, than the traffic that will be sent to its adjacent nodes, we will have $o\_close[i]$ $< o\_far[i]$. As long as more percentage of $out\_of\_[i]$ consists of $o\_far[i]$, "min_tot_links" value is more likely to increase, since we multiply $o\_far[i]$ by two and increase the numerator in the function more than $o\_close[i]$ will do. The greater "min_tot_links" value strengthens the inequality (2.9).

The same partitioning of demand can be done for the number of signals that has to enter a node *i*.

$$\sum_{j:(j,i)\in A_X \cup A_{EL}} d_{ji} = in\_close[i] \qquad \text{and} \qquad \sum_{j:(j,i)\notin A_X \cup A_{EL}} d_{ji} = in\_far[i] \qquad \forall i \in N$$

$$\text{since} \quad \text{int}o\_[i] = \sum_{j\in N} d_{ji} = \sum_{j:(j,i)\in A_X \cup A_{EL}} d_{ji} + \sum_{j:(j,i)\notin A_X \cup A_{EL}} d_{ji} \qquad \forall i \in N$$

$$\text{min\_tot\_links2} = \left\lceil \frac{\sum_{i\in N}\left(in\_close[i] + 2.in\_far[i]\right)}{CL} \right\rceil$$

The value "min_tot_links2" provides a lower bound for the total number of fibers that has to be opened in the network, in order to satisfy flow of demand.

$$\min\_tot\_links2 \le \sum_{(k,m)\in A_X} SLX_{km} + \sum_{(k,m)\in A_{EL}} SLE_{km} \qquad (2.10)$$

We note here that:

$$\sum_{i\in N}(in\_far[i]) = \sum_{i\in N}(o\_far[i]) \quad \text{and} \quad \sum_{i\in N}(in\_close[i]) = \sum_{i\in N}(o\_close[i])$$

First equation is valid since either of $\sum_{i\in N}(in\_far[i])$ or $\sum_{i\in N}(o\_far[i])$ states the total traffic which is between nodes that are not adjacent to each other. The second equation is valid too, since $\sum_{i\in N}(in\_close[i])$ or $\sum_{i\in N}(o\_close[i])$ corresponds to the traffic between nodes which are connected by an arc. Then we conclude that (2.9) and (2.10) actually provide the same lower bound for the total number of fibers that has to be opened in the network and they give the same inequality.

We observe that after the addition of the 1-4th level s-t cuts to M-3 and M-4, both formulations give the same linear programming relaxation lower bound for the problem. However, the computation time for the bounds after the added s-t cuts is always better for M-3. Better lower bounds can be obtained from the formulations with s-t cuts, after a few minutes waiting time in CPLEX 9.0. We give detailed analysis of bounds with cuts in Chapter 7.

Chapter 6

THE HEURISTIC

Both M-3 and M-4 formulations of the problem result a difficult model which cannot be solved in reasonable time. The result of the lagrangian relaxation of the M-3 formulation can be made feasible when capacity constrains (2.33) and (2.44) are added to the relaxed problem. The final model finds a feasible flow for the original problem, as we state in Chapter 5. However, the quality of that solution is not so good and it needs to be improved to yield better objective values for the original problem P.

We analyze the features of the lagrangian solution in order to use it as a starting point for a better solution method. In the solution of P_{lag}, signals use a fiber without creating the cost of opening fibers since capacity constraints are relaxed. A signal with origin i and destination j uses a fiber on arc (k,m) as long as arc (k,m) is on the shortest path between i and j, and the constraints (2.33) and (2.44) are not violated. Even for a few number of signals, a fiber can be opened in the solution although these few units of flow can be routed to destination node through fibers that are already opened for larger number of signals on other arcs. By this way, the costs of fibers that are opened for small number of signals will not arise. Those fibers that are already opened for large number of signals, whose utilizations are close to one full-fiber capacity (CL), will not pay an additional fiber cost until the CL units is exceeded. For the reasons stated above, the solution of P_{lag}, which is feasible for P, does not provide a good upper bound. However we improve this upper bound by a heuristic that uses that feature of the lagrangian solution (Figure 6.1).

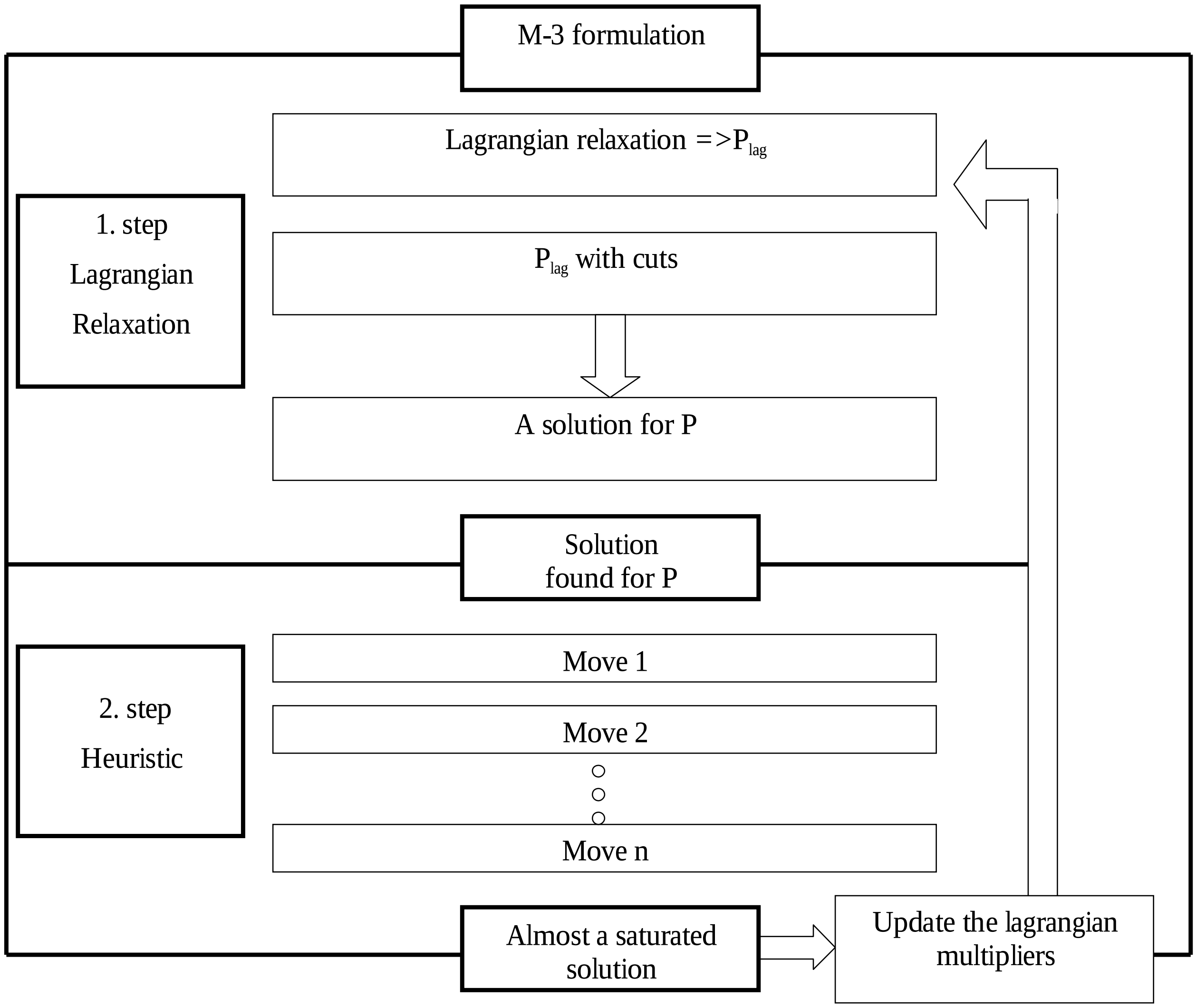


Figure 6.1 Overall solution approach

The main idea of one move of our heuristic is based on closing a number of fibers that carry small number of signals in the solution of P_{lag}, and routing that small number of signals through fibers, which are already opened for larger number of signals on other arcs. In order to give the details of our heuristic, we first give the definition EFF_{km} that we use in our procedure and later describe one move of the heuristic.

Excess-a-fiber-flow (EFF_{km}): We define the excess-a-fiber-flow as the number of signals on an arc (k,m) that does not fully use one fiber capacity. If the number of signals transmitted on arc (k,m) is shown by f_{km}, we calculate excess-a-fiber-flow on arc (k,m), EFF_{km}, as follows:

$$EFF_{km} = f_{km} - \left(\left\lceil \frac{f_{km}}{CL} \right\rceil - 1 \right) CL$$

$\left\lceil \frac{f_{km}}{CL} \right\rceil$ gives the number of fibers that are opened on arc (k,m) and $\left\lceil \frac{f_{km}}{CL} \right\rceil$-1 of those fibers are fully utilized by the signals. The number of signals that are transmitted on the remaining fiber gives the EFF_{km} value.

One move of the heuristic iteration:

After we get the solution of P_{lag}, we order the arcs according to their EFF_{km} values, from smallest to greatest. We assume that the arc with smallest EFF_{km} value is the most inefficient arc, on which we open a fiber for a few number of signals compared to other arcs whose EFF_{km} values are greater.

We select the first k number of arcs whose EFF_{km} values are smallest. One fiber of each selected arc is closed to yield problem P_{lag}' and we solve this problem. If the problem is feasible, then the signals, which have been using the fibers that we have just closed, can be routed through other links. If the problem is not feasible, we open the fibers on selected arcs and mark the k^{th} arc, which was the least inefficient arc among the selected ones, as the infeasibility arc. There can be only one infeasibility arc during the process. We decrease the number of fibers that we will close in the next move by one and set k=k-1.

A successful move of one heuristic iteration occurs if the problem is feasible after we close ***k*** number of fibers in the network. After a successful move, we set ***k*** = ***default***, where ***default*** equals 10 in our study. ***Default*** is the number of arcs that we select to check at the first move of the heuristic. In other words, we look for whether we can close 10 by 10 at each iteration at first. As long as we obtain a successful move, we continue to close 10 fibers in the network. However, after closing 10 fibers in the network a number of times, the problem may become infeasible and we decrease the number of fibers closed at each move. An unsuccessful move with k=1 means, only one fiber has been closed and the problem is infeasible. This indicates that the network will be more sensitive to the number of fibers we close in following moves. As long as the number of unsuccessful moves with *k*=1 increase, we do not lose time to check whether we can choose 10 or 8 arcs to close their fibers. After a number (***limit_default***) of unsuccessful moves with k=1 is met, we decrease the ***default*** value to 3 (***new_default***) in our computations since we are more likely to meet an unsuccessful move with greater ***k***. In the remaining moves of the heuristic, we can now close at most 3 (***new_default***) fibers. The use of this parameter set fastens our heuristic quite much. We proceed very fast at the beginning of an iteration by closing 10 fibers. As we make more moves in an iteration of heuristic, closing big batches of links is more likely to create infeasibility. We give the main steps of one move of the heuristic in figure 6.2.

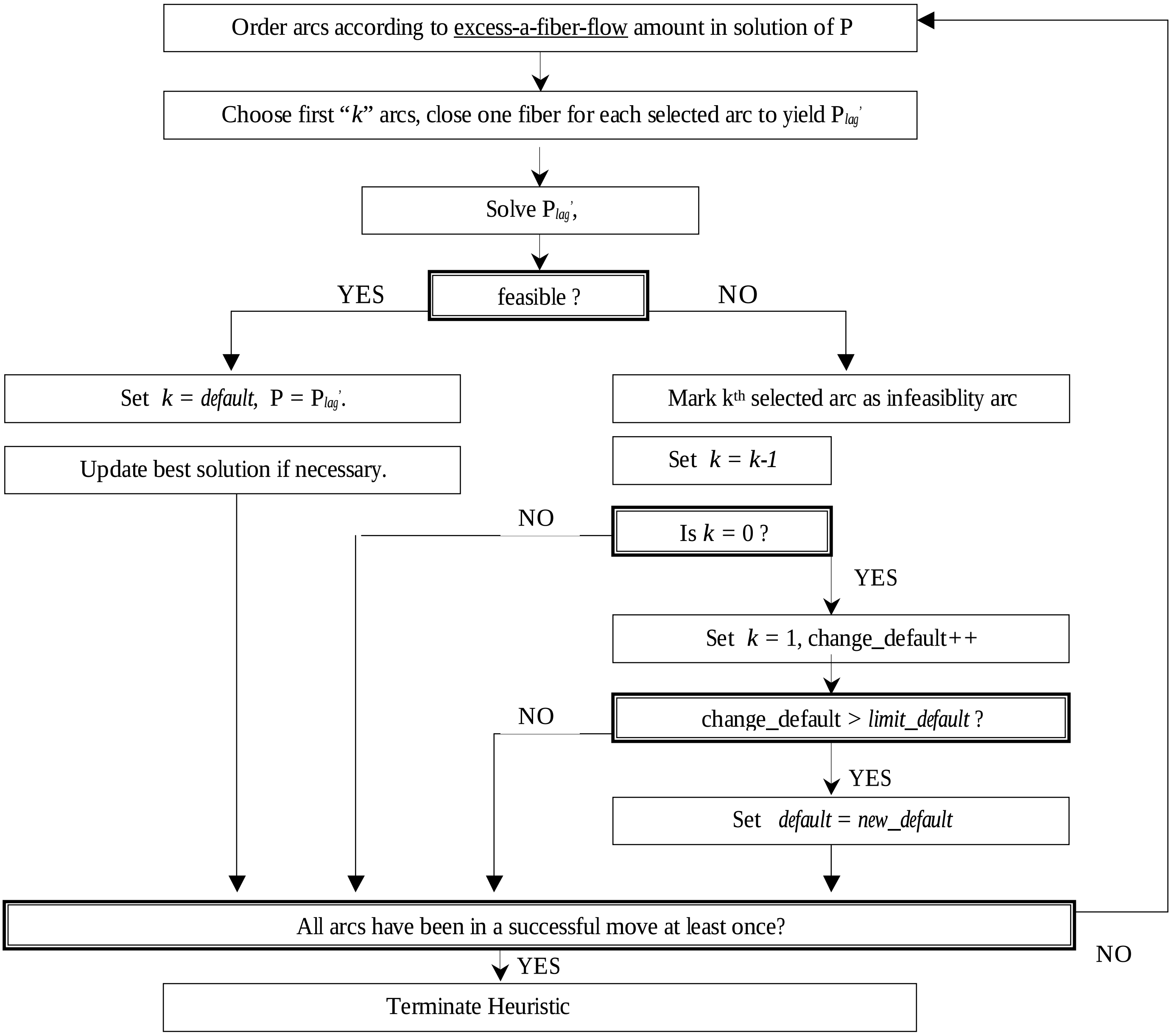


Figure 6.2: One move of heuristic iteration

According to experimental results, we have chosen heuristic parameters as *default*=10, *new_default*=3, and *limit_default*=6 in our computations.

After a successful move is obtained, we do not consider closing the arc, which is marked as infeasibility arc at that instant, until the end of current heuristic iteration. Closing a fiber on that arc will also lead to infeasibility in further moves of the same heuristic iteration and we do not have to check that arc again.

At the end of a heuristic iteration, we have a solution whose flow values use fibers with almost full utilization. Less number of fibers is opened in the network and the upper bound that we obtain from the lagrangian solution is improved quite much. We compare the results of heuristic with the results of formulation M-3 in CPLEX 9.0 in the following chapter.

Chapter 7

COMPUTATIONAL RESULTS

We have given two formulations for our problem with a slight difference in the definition of variables and flow balance equations. The computational results of those models are tested in CPLEX 9.0 and we see that our problem is not solved optimally with either M-3 or M-4. However, we compare computational performance of two models, M-3 and M-4 in terms of processing times and the quality of lower bounds and upper bounds provided for the problem in section 7.1. Other than the comparison of M-3 and M-4 formulations, we also examine the performance of our heuristic with formulation M-3, which is solved in CPLEX 9.0, to see which one of those can be preferred for different network structures in section 7.2.

In section 7.3, the costs of two different cases for the network system under consideration are inspected. The first case studies a network that operates with the availability of normal links; only SLX_{km} variables and the corresponding flow variables are defined. The second case is the same network and demand pattern with the availability of express links after the ULH technology is introduced. We have created networks and appropriate demand patterns for application in order to see whether the use of express links really provides cost savings for the scenarios we created.

The results were obtained for two networks, which are 26 nodes-146 arcs and 35 nodes-198 arcs. Table 7.0.1 gives the distance values for normal links and related SQDD used for determining express links. The maximum number of fibers that can be constructed on an arc was assumed to take values from the set L={1,2,4,6,8} for networks with express links. However, after removing the availability of express links, some of scenarios needed larger L values in order to provide a feasible flow balance. Table 7.0.2 shows the networks created and arrows with (i) state that there has been an increase in L value of the scenario for the network without express links.

Three different demand patterns were used for all networks 26-146-L and 35-198-L; D1 low dense, D2 medium dense and D3 high dense. High dense demand patterns were created in such a way that removing several links from the network would cause infeasibility of the problem. Other medium and low dense patterns were prepared according to D3 and we scaled D3 down to form D2. A number of communicating i-j node pairs was deleted randomly from the commodity set and total number of signals that have to be transmitted in the network with D2 has been decreased. We formed D1 from

Table 7.0.1	minimum distance (km)	average distance (km)	maximum distance (km)
network with 26 nodes (SQDD = 700)	157	427	1145
network with 35 nodes (SQDD = 110)	31	57	81

D2 in the same way we did to create D2 from D3. Second column shows the total number of signals that have to be transmitted in the network, for the corresponding demand pattern stated in first column. The ratio of unit meter cost of normal and express links was changed according to the set R={$\frac{2}{2}, \frac{2}{2{,}25}, \frac{2}{2{,}5}, \ldots, \frac{2}{4}$}. The scenario with R=$\frac{2}{3}$ means unit meter cost of normal fiber is 2 whereas it is 3 for express fiber.

Demand type	# of signals	with express links		without express links
D1	64	26-146-1	=>	26-70-1
D2	73	26-146-1	=>	26-70-1
D3	110	26-146-1	=>	26-70-1
D1	185	26-146-2	=>	26-70-2
D2	260	26-146-2	=>	26-70-2
D3	472	26-146-2	(i) =>	26-70-4
D1	630	26-146-4	=>	26-70-4
D2	1000	26-146-4	(i) =>	26-70-6
D3	1567	26-146-4	(i) =>	26-70-7
D1	1072	26-146-6	=>	26-70-6
D2	1429	26-146-6	(i) =>	26-70-10
D3	1976	26-146-6	(i) =>	26-70-10
D1	1264	26-146-8	=>	26-70-8
D2	1665	26-146-8	=>	26-70-8
D3	2256	26-146-8	(i) =>	26-70-12
D1	188	35-198-1	=>	35-108-1
D2	531	35-198-1	(i) =>	35-108-2
D3	589	35-198-1	(i) =>	35-108-2
D1	491	35-198-2	=>	35-108-2
D2	727	35-198-2	(i) =>	35-108-3
D3	1175	35-198-2	(i) =>	35-108-4
D1	982	35-198-4	=>	35-108-4
D2	1316	35-198-4	(i) =>	35-108-7
D3	1818	35-198-4	(i) =>	35-108-7
D1	1520	35-198-6	=>	35-108-6
D2	2221	35-198-6	(i) =>	35-108-10
D3	2999	35-198-6	(i) =>	35-108-10
D1	1585	35-198-8	=>	35-108-8
D2	2245	35-198-8	(i) =>	35-108-12
D3	3626	35-198-8	(i) =>	35-108-12

Table 7.0.2 Different network scenarios

The networks in table 7.0.2 will be named as "node # - arc # - *MaxOnArc*", where we assume $MaxOnArc = MaxOnArcX_{km} = MaxOnArcEL_{km}$ $\forall (k,m) \in A_X \cup A_{EL}$

A 35-198-2 network with D2 means that we study a network with 35 nodes, 198 arcs (total of normal and express arcs) with the availability of maximum 2 fibers that can be opened on an arc, for providing service to D2 type demand pattern. In other words, having $MaxOnArc$=2 means that we have two types of links to choose for opening on an arc with capacities CL and $2.CL$. Computations are performed on a 12 x 400 MHz UltraSPARC machine with 240 MB RAM. The heuristic and subgradient algorithm are coded in C 5.0 / CPLEX 9.0.

The gaps given our study were calculated as gap % = $\frac{(UB-LB)}{UB}$. Since the lower bounds that we obtain from the lagrangian relaxation of the problem, which is integral, cannot exceed the LP relaxation lower bound (Geoffrion, 1974), we use the lower bound of M-3 formulation for determining how close the solution of heuristic is to the optimal value in our computations. Our computations show that the gaps we find for the problem do not improve much after 10 minutes (Table 7.0.3). We do not get significant improvement in the gap; on the average no more than 1%. For this reason, we consider at most 10 minutes computation time for our calculations in sections 7.1 to 7.3.

7.1 Performance Comparisons of M-3 and M-4

Our preliminary computations have shown that the s-t cuts, which we define in Chapter 5, help to improve the lower bound of M-3 and M-4 formulations without relaxing the capacity constraints. However as the level of s-t cuts added to M-3 or M-4 increases, the computation time to reach the same upper bound, which we obtain without cuts, increases too. The addition of first-level s-t cuts to the formulation M-3 does not affect the time performance to reach the same upper bound, besides provides a better gap since the lower bound of M-3 is improved. In order to get the best performance from M-3 in terms of time, upper bound and the gap, we use the formulation M-3 with first level s-t cut set and the logical cut (2.9) in our computations in section 7.2 and 7.3. Table 7.1 shows the gap performances of M-3 and M-4 with first level s-t cuts, and the heuristic for D3 demand pattern and R=2/3.

D3, R = 2 / 3,00	M-3		H	
	Time	Gap	Time	Gap
26-146-1	1 min.	65,6	1 min.	32,8
	10 min.	35,1	10 min.	25,8
	25 min.	27,1	25 min.	24,8
	15,5 hrs.	23,0	1,4 hrs.	24,8
26-146-2	1 min.	37,5	1 min.	20,6
	10 min.	20,5	10 min.	19,0
	8,5 hrs.	17,0	55 min.	18,4
	22,6 hrs.	16,5	2,7 hrs.	18,4
26-146-4	1 min.	6,2	1 min.	6,5
	10 min.	4,6	10 min.	5,2
	25 min.	4,5	15 min.	5,0
	21 hrs.	4,0	30 min.	4,9
26-146-6	1 min.	5,6	1 min.	5,5
	10 min.	4,2	10 min.	4,8
	25 min.	3,8	30 min.	4,0
	1,7 hrs.	3,5	6 hrs.	3,8
26-146-8	1 min.	5,0	1 min.	5,6
	10 min.	3,5	10 min.	4,0
	24 min.	3,4	1,5 hrs.	3,7
	15 hrs.	3,4	4 hrs.	3,4

D3, R = 2 / 3,00	M-3		H	
	Time	Gap	Time	Gap
35-198-1	1 min.	36,4	1 min.	**
	10 min.	22,5	10 min.	16,5
	15 min.	18,6	35 min.	16,1
	10 hrs.	17,3	3,3 hrs.	15,1
35-198-2	1 min.	21,1	1 min.	**
	10 min.	9,0	10 min.	9,0
	1,4 hrs.	8,6	18 min.	8,2
	13,5 hrs.	8,1	6,5 hrs.	7,2
35-198-4	1 min.	13,0	1 min.	**
	10 min.	5,8	10 min.	5,4
	30 min.	5,6	23 min.	4,8
	15 hrs.	5,4	28 hrs.	4,7
35-198-6	1 min.	6,7	1 min.	**
	10 min.	3,2	10 min.	3,3
	1,6 hrs.	2,9	5,7 hrs.	2,7
	16 hrs.	2,6	26,6 hrs.	2,4
35-198-8	1 min.	5,5	1 min.	**
	10 min.	2,4	10 min.	2,6
	2 hrs.	1,9	2,8 hrs.	2,1
	13,5 hrs.	1,4	18 hrs.	1,6

Table 7.0.3 Solution gaps in long term

** Heuristic could not finish one iteration

D3, R= 2 / 3,00		M-3 Gap	M-4 Gap	H Gap
26-146-1	1 min.	65,6	68,8	32,8
	3 min.	58,0	66,9	30,7
	5 min.	39,6	66,8	30,7
	7 min.	35,2	66,8	29,0
	10 min.	35,1	40,9	25,8
26-146-1	1 min.	37,5	49,0	20,6
	3 min.	21,2	42,1	20,4
	5 min.	21,0	41,4	19,4
	7 min.	20,8	40,8	19,3
	10 min.	20,5	29,6	19,0
26-146-1	1 min.	6,2	17,7	6,5
	3 min.	4,6	14,9	5,2
	5 min.	4,6	14,0	5,2
	7 min.	4,6	13,6	5,2
	10 min.	4,6	13,6	5,2
26-146-1	1 min.	5,6	13,4	5,5
	3 min.	4,2	10,3	5,1
	5 min.	4,2	10,3	4,8
	7 min.	4,2	10,3	4,8
	10 min.	4,2	7,1	4,8
26-146-1	1 min.	5,0	12,4	5,6
	3 min.	3,6	9,7	4,0
	5 min.	3,5	9,7	4,0
	7 min.	3,5	9,7	4,0
	10 min.	3,5	7,7	4,0

D3, R= 2 / 3,00		M-3 Gap	M-4 Gap	H Gap
35-198-1	1 min.	36,4	*	**
	3 min.	36,2	47,4	16,5
	5 min.	36,2	47,4	16,5
	7 min.	32,3	47,4	16,5
	10 min.	22,5	47,4	16,5
35-198-2	1 min.	21,1	*	**
	3 min.	12,7	*	10,4
	5 min.	9,1	29,4	9,8
	7 min.	9,0	29,4	9,8
	10 min.	9,0	28,7	9,0
35-198-4	1 min.	13,0	*	**
	3 min.	6,6	*	5,4
	5 min.	6,4	23,7	5,4
	7 min.	6,0	23,5	5,4
	10 min.	5,8	23,5	5,4
35-198-6	1 min.	6,7	*	**
	3 min.	4,0	*	3,7
	5 min.	3,2	10,8	3,7
	7 min.	3,2	10,7	3,7
	10 min.	3,2	9,3	3,3
35-198-8	1 min.	5,5	*	**
	3 min.	3,8	*	3,2
	5 min.	2,4	12,2	3,2
	7 min.	2,4	8,8	2,6
	10 min.	2,4	8,1	2,6

Table 7.1

Performance comparisons of M-3, M-4 and the heuristic.
*   No integer solution found
** Heuristic could not finish one iteration

The upper bound that we obtain from M-3 is much tighter than the upper bound of M-4. M-3 formulation also improves the lower bound faster. For this reason we say that M-3 dominates M-4 formulation in terms of upper bound quality and time performance. The LP relaxations of M-3 and M-4 give the same lower bound for our problem with or without s-t cuts. Long time computational observations show that M-4 formulation may provide better lower bounds than M-3, however the gap that is provided by M-3 in minutes is much better than M-4 formulation. For those reasons, we can say that M-3 is a better formulation than M-4 and we will use M-3 in for our computations.

7.2 Performance Comparisons of M-3 and the Heuristic

The performance of heuristic was compared with M-3 formulation of the problem in CPLEX for the scenarios we created (Tables from 7.2.1a to 7.2.3a in Appendix). The computational results for different R values show that when L=1, the heuristic is quite fast enough to find a good solution in seconds, especially for 26 nodes network (Table 7.2.4). The upper bound that we obtain at particular time points by heuristic is much better than the performance of M-3 in CPLEX 9.0. However the gap for L=1 is quite large, although we can find solutions quickly.

D3		2 / 2,00		2 / 2,25		2 / 2,50		2 / 2,75		2 / 3,00		2 / 3,25		2 / 3,50		2 / 3,75		2 / 4,00	
		M-3 Gap	H Gap	M-3 Gap	H Gap	M-3 Gap	H Gap	M-3 Gap	H Gap	M-3 Gap	H Gap	M-3 Gap	H Gap	M-3 Gap	H Gap	M-3 Gap	H Gap	M-3 Gap	H Gap
35-198-1	1 min.	35,9	**	34,5		36,4	**	35,9	**	36,4	**	36,6	**	36,7	**	36,0		37,0	**
	3 min.	35,9	16,6	34,5	16,4	36,3	16,7	35,8	16,5	36,2	16,5	36,5	16,6	36,6	16,5	36,0	16,7	37,0	16,7
	5 min.	35,9	16,7	34,5	16,4	36,3	16,7	35,8	16,5	36,2	16,5	34,1	16,6	36,6	16,5	36,0	16,7	37,0	16,7
	7 min.	35,9	16,6	31,6	16,4	29,6	16,7	32,1	16,5	32,3	16,5	30,8	16,6	30,5	16,5	29,0	16,7	30,4	16,5
	10 min.	27,6	16,6	23,2	16,4	20,9	16,7	19,9	16,5	22,5	16,5	20,9	16,6	20,1	16,4	19,8	16,7	19,2	16,5
35-198-2	1 min.	20,6	**	24,2	**	24,4	**	24,4	**	21,1	**	25,2	**	23,1	**	22,9	**	25,4	**
	3 min.	10,9	10,4	13,4	10,5	14,4	10,6	14,6	10,5	12,7	10,4	13,8	10,6	12,5	10,7	14,6	10,5	14,1	10,6
	5 min.	9,9	10,3	9,8	10,5	9,5	10,6	10,1	10,5	9,1	9,8	9,6	10,6	9,5	10,0	9,8	9,9	9,0	10,5
	7 min.	9,1	10,2	9,7	9,8	9,5	9,9	9,4	10,4	9,0	9,8	9,5	9,9	9,4	9,9	8,6	9,7	9,0	10,5
	10 min.	9,1	10,0	9,5	9,4	8,8	9,4	9,4	10,4	9,0	9,0	9,5	9,9	9,3	9,8	8,6	9,0	9,0	9,1
35-198-4	1 min.	14,8	**	15,3	**	15,2	**	14,1	**	13,0	**	14,1	**	14,1	**	14,6	**	15,1	**
	3 min.	7,0	5,4	8,0	5,3	7,5	5,3	8,1	5,3	6,6	5,4	6,5	5,4	6,7	5,4	6,9	5,4	7,9	5,5
	5 min.	5,9	5,3	5,6	5,3	5,6	5,3	6,1	5,3	6,4	5,4	5,4	5,4	4,8	5,4	5,7	5,4	6,0	5,4
	7 min.	5,9	5,3	5,5	5,3	5,6	5,3	6,1	5,3	6,0	5,4	5,4	5,4	4,8	5,4	5,7	5,4	6,0	5,4
	10 min.	5,5	5,3	5,5	5,3	5,6	5,3	6,1	5,3	5,8	5,4	5,4	5,4	4,8	5,4	5,7	5,4	6,0	5,4
35-198-6	1 min.	6,6	**	7,5	**	6,3	**	6,9	**	6,7	**	7,8	**	6,9	**	7,2	**	6,9	**
	3 min.	4,3	3,7	4,8	3,7	4,7	3,7	4,5	3,7	4,0	3,7	3,3	3,8	4,2	3,8	4,0	3,8	4,3	3,8
	5 min.	3,7	3,7	3,1	3,7	3,0	3,7	3,1	3,6	3,2	3,7	2,5	3,7	3,4	3,8	3,2	3,8	3,4	3,8
	7 min.	3,5	3,5	3,1	3,6	2,9	3,7	3,1	3,6	3,2	3,7	2,5	3,6	3,4	3,5	2,8	3,5	2,9	3,3
	10 min.	3,0	3,5	2,7	3,2	2,9	3,7	3,1	3,6	3,2	3,3	2,5	3,6	3,0	3,5	2,8	3,5	2,9	3,3
35-198-8	1 min.	4,4	**	5,2	**	6,3	**	5,5	**	5,5	**	6,0	**	5,8	**	5,0	**	5,9	**
	3 min.	2,6	3,1	2,8	3,1	3,2	3,1	2,8	3,1	3,8	3,2	4,0	3,2	3,4	3,2	3,5	3,2	3,3	3,2
	5 min.	2,5	3,1	1,8	3,1	1,8	3,1	2,7	2,9	2,4	3,2	2,9	3,2	2,7	3,2	2,3	3,2	2,3	3,2
	7 min.	2,4	3,1	1,8	3,1	1,8	3,1	1,8	2,9	2,4	2,6	2,8	3,2	2,7	3,1	2,3	3,2	2,3	3,2
	10 min.	2,4	3,0	1,8	3,1	1,8	2,7	1,8	2,9	2,4	2,6	2,2	3,2	2,7	3,1	2,0	3,2	2,3	2,9

Table 7.2.3.b Comparison of gap performances of M-3 and heuristic for D3, 35-198-L.
** Heuristic could not finish one iteration

In table 7.2.3b, a 35-198-L network with D3 demand pattern was considered for the heuristic (H) and M-3 in CPLEX (M-3). The very first row of the table shows different R ratios.

The upper bound of heuristic is better than M-3 when L=2, however in some cases, they give almost the same upper bound. As the number L grows, the upper bound of M-3 approaches to the upper bound of heuristic. In most of the

	D1	D2	D3
26-146-1	3 sec.	3 sec.	4 sec.
26-146-2	5 sec.	10 sec.	12 sec.
26-146-4	18 sec.	34 sec.	38 sec.
26-146-6	16 sec.	14 sec.	42 sec.
26-146-8	26 sec.	20 sec.	18 sec.
35-198-1	10 sec.	47 sec.	1,2 min.
35-198-2	31 sec.	1,3 min.	2,1 min.
35-198-4	2,0 min.	2,0 min.	2,3 min.
35-198-6	2,0 min.	2,3 min.	1,5 min.
35-198-8	2,0 min.	2,3 min.	2,0 min.

Table 7.2.4 Time to complete first iteration of the heuristic

cases for L=6 and L=8, the upper bound of M-3 formulation is slightly better than the heuristic.

Other than the comparison of M-3 and heuristic, we comment on the overall performance of our solution approaches for different L values (Figures from 7.2.1a to 7.2.3a in Appendix). For example in figure 7.2.3b, we see that the gaps obtained for L=1 and L=2 are large, although quick solutions can be obtained, especially for L=1, by the heuristic. On the other hand, for L=4, 6 and 8 the gaps are around %5. According to the results, we can say that our approach can yield better gaps for problems whose L parameter is greater.

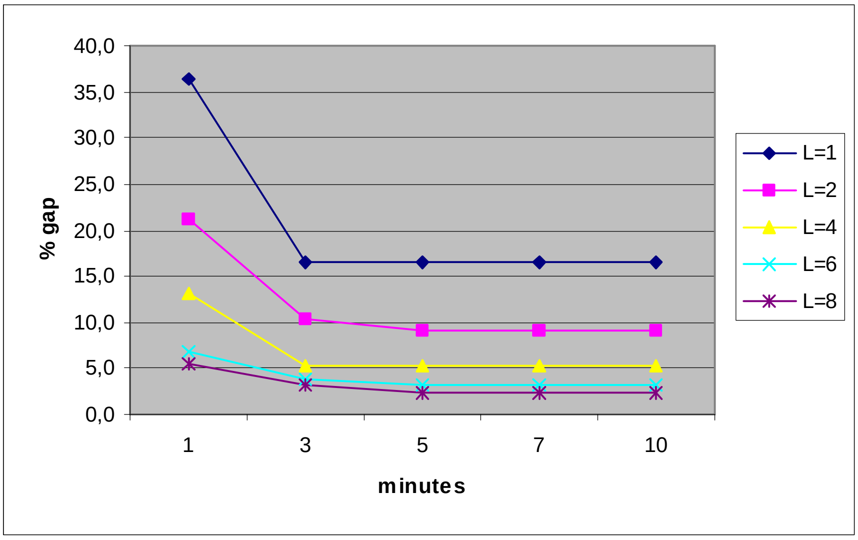


Figure 7.2.3.b Best gap obtained for scenario D3, 35-198-L, R=2/3

Another observation about the gap performances of our approach is about the number of arcs in the network. We can obtain different gap performances for two networks which differ only by number of their arcs (Table 7.2.5). The shaded cells in columns give the best gap obtained for the corresponding problem. If we compare two gaps that are in the same row we see that the gaps for networks with less number of arcs (26-70-L, 35-108-L) are almost half of the gaps that are obtained for networks with more number of arcs (26-146-L, 35-198-L). We can say that as the number of arcs available in the network increases, we provide larger gaps for the solution. For this reason, if we choose less number of shortest paths with distance less than SQDD as candidates of express links in the network, then the total number of arcs in the problem will decrease. This means that problems with less number of candidates for express links will have better gaps with our solution approach. We also observe that, the gaps are better for the networks with denser traffic than they are for the same networks with less dense traffic.

Demand type	# of signals	with express links	10 min. % gap			without express links	10 min. % gap	
			M-3	H			M-3	H
D1	64	26-146-1	30,1	19,7	=>	26-70-1	9,0	10,5
D2	73	26-146-1	31,2	23,9	=>	26-70-1	13,9	22,9
D3	110	26-146-1	35,1	25,8	=>	26-70-1	20,2	19,7
D1	185	26-146-2	28,8	30,0	=>	26-70-2	5,2	6,4
D2	260	26-146-2	24,7	26,0	=>	26-70-2	4,3	4,4
D3	472	26-146-2	20,5	19,0	**(i)=>**	26-70-4	6,6	6,6
D1	630	26-146-4	14,2	12,9	=>	26-70-4	5,6	5,2
D2	1000	26-146-4	8,4	8,2	**(i)=>**	26-70-6	3,8	4,2
D3	1567	26-146-4	4,6	5,2	**(i)=>**	26-70-7	2,2	2,1
D1	1072	26-146-6	8,1	8,0	=>	26-70-6	4,3	4,2
D2	1429	26-146-6	6,4	6,1	**(i)=>**	26-70-10	2,8	3,0
D3	1976	26-146-6	4,2	4,8	**(i)=>**	26-70-10	2,3	2,4
D1	1264	26-146-8	6,5	6,8	=>	26-70-8	2,9	3,3
D2	1665	26-146-8	5,1	5,1	=>	26-70-8	2,0	1,9
D3	2256	26-146-8	3,5	4,0	**(i)=>**	26-70-12	2,0	1,8
D1	188	35-198-1	58,4	33,4	=>	35-108-1	26,9	22,3
D2	531	35-198-1	25,6	20,7	**(i)=>**	35-108-2	11,1	10,9
D3	589	35-198-1	22,5	16,5	**(i)=>**	35-108-2	9,3	8,9
D1	491	35-198-2	29,5	23,8	=>	35-108-2	13,0	12,2
D2	727	35-198-2	15,5	16,3	**(i)=>**	35-108-3	8,7	8,8
D3	1175	35-198-2	9,0	9,0	**(i)=>**	35-108-4	4,9	5,3
D1	982	35-198-4	10,1	11,6	=>	35-108-4	6,0	6,3
D2	1316	35-198-4	8,9	7,8	**(i)=>**	35-108-7	5,5	4,3
D3	1818	35-198-4	5,8	5,4	**(i)=>**	35-108-7	3,4	3,1
D1	1520	35-198-6	7,3	7,7	=>	35-108-6	3,8	4,0
D2	2221	35-198-6	4,0	4,3	**(i)=>**	35-108-10	2,6	2,6
D3	2999	35-198-6	3,2	3,3	**(i)=>**	35-108-10	1,9	1,8
D1	1585	35-198-8	6,4	7,0	=>	35-108-8	4,2	3,3
D2	2245	35-198-8	3,9	4,5	**(i)=>**	35-108-12	3,0	2,6
D3	3626	35-198-8	2,4	2,6	**(i)=>**	35-108-12	1,3	1,2

Table 7.2.5 Comparing % gaps of two networks with same number of nodes, R=2/3

7.3 Comparison of Networks with and without Express Links

The introduction of express links in a network was one of the main aims of our problem and we tried to figure out the costs for a network with and without the availability of express links. We remove the availability of express links from the networks we study and this situation leads to decrease in the capacity of the network to provide flow balance. Removing express links from the networks with D1 does not cause infeasibility of network to achieve flow balance, since the demand is less dense. However networks with D2 and D3 type demand pattern cannot provide the required flow balance with given L values and only normal arcs. For those networks, which are infeasible without express links, we increase the maximum number of fibers that are allowed on a normal arc until the network becomes feasible with the given normal links (Table 7.0.2).

The optimal cost values for the networks with and without express links cannot be obtained and in order to compare cost expenses of two cases, we define two measures which use the lower and upper bounds of results to yield the approximate percentages of cost savings. First measure (M1) is the least cost savings measure, which utilizes the difference between lower bound of (NX) network without express links and the upper bound of (NXE) network with express links:

$$\%\ \mathrm{M1} = \frac{(\mathrm{LB\ of\ NX}) - (\mathrm{UB\ of\ NXE})}{\mathrm{LB\ of\ NX}}$$

The M1 is the guaranteed cost savings, since the best cost of NX is compared with the worst cost of NXE.

The second measure (M2) is the average expected savings, which is found by

$$\% \text{ M2} = \frac{[(\text{UB of NX}) + (\text{LB of NX})] - [(\text{UB of NXE}) + (\text{LB of NXE})]}{[(\text{UB of NX}) + (\text{LB of NX})]},$$

simplified from $$\frac{\frac{(\text{UB of NX}) + (\text{LB of NX})}{2} - \frac{(\text{UB of NXE}) + (\text{LB of NXE})}{2}}{\frac{(\text{UB of NX}) + (\text{LB of NX})}{2}}$$

This is a more optimistic and straightforward measure, which assumes that the optimal values are most probably in the middle of the bounds. This measure can give an idea for the accuracy of M1.

	D1		D2		D3	
	M1	M2	M1	M2	M1	M2
26-146-1	-7,5	7,6	-14,2	7,0	-12,7	12,6
26-146-2	-1,5	15,5	4,1	17,8	15,8	26,3
26-146-4	21,9	28,9	26,3	30,7	28,4	30,8
26-146-6	26,6	31,0	28,4	31,6	29,2	31,5
26-146-8	27,5	30,8	28,7	31,2	29,6	31,5
35-198-1	-18,1	14,0	7,7	22,0	14,4	25,1
35-198-2	5,9	22,5	14,6	24,8	22,1	27,5
35-198-4	21,0	27,3	22,1	26,7	24,9	28,1
35-198-6	23,7	27,9	25,1	27,6	26,9	28,7
35-198-8	22,9	26,6	24,8	27,3	27,8	29,0

Table 7.3 The percentages of cost savings with M1 and M2

The percentages of cost savings according to M1 and M2 are given in table 7.3. The graphical comparison for 35-198-L with D3 is given in figure 7.3.1.

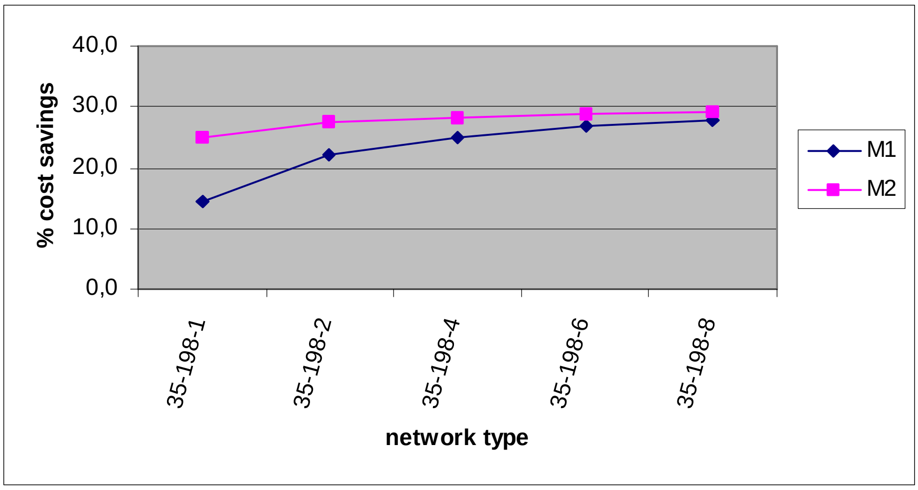


Figure 7.3.1 Cost savings for D3, 35-198-L, R=2/3

We also give the graphical comparisons of other networks in figures 7.3.2 and 7.3.3, in Appendix. The results for the given networks show that when the demand amount that we try to send in the same network (26-146-L or 35-198-L) increases, cost savings are greater. This observation is justified when we look at three scenarios of a network with different demand patterns. M1 measure guarantees more % cost savings for D3 than D1 in a 26 or 35 node network.

As the number of signals that we transmit in the network increases, we need to allow more number of fibers in the network and the cost savings of network system is largest for the L=8 case. Moreover, D3 pattern provides greater cost savings than D2 and D1 for the same L parameter, although the percentage savings are about the same.

Moreover the cost of the network with 35 nodes is more than the network with 26 nodes since more signals are to be sent and more number of links is going to be opened. Cost savings of same percentage mean more amounts of savings are possible for the larger network.

Chapter 8

CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

The technological introductions of electronics industry present new alternatives for the design of telecommunication networks. Placement of express links is one of those introductions, which has been examined in our study in terms of capacitated links, implementation cost of those links, the routes of traffic and network complexity.

We propose two models, M-3 and M-4, for providing the network flow balance with capacitated links. These models decide which links to open, how many fibers to operate on those links and the routes of signals that will be transmitted in the telecommunication network. The problem that we have considered so far is NP-hard and optimal solution of the formulations cannot be found for reasonably sized networks that we study. For this reason, we relax a set of constraints for M-3 formulation in a lagrangian way. Since the lower bound of the relaxation is quite weak, we add two sets of cuts to relaxed formulation: S-T cuts and a logical cut. In order to have quicker solutions for the networks we study, a heuristic has been developed for the lagrangian solution of the problem. We have compared the performance of our heuristic with performance of M-3 formulation in CPLEX 9.0 for different scenarios of networks with different demand patterns and network parameters.

The analysis of our computational results have several conclusions. First result is; as the parameter L grows, the gap that we can find from our heuristic and M-3 formulation improves. Our approach can not provide good bounds for L=1 in overall, however the heuristic is fast enough to find better upper bounds than M-3 formulation in short time. Second main result is about the cost savings that can be obtained by the introduction of express links in a telecommunication network. High complexity created by increasing number of traffic flow, high device cost for each process of signals especially at intermediate nodes lead us to consider the idea of bypassing some of nodes on signal routes. The worst case measure (M1) for the use of express links states that cost savings can be around 25% for a network with express links, especially for L=4, 6 and 8.

The problem that we have investigated decides which links to open, how many fibers to operate on those links and the routes of the signals. A further research can use the solution of our problem as a starting point for RWA algorithms. Routing decisions of signals will not be given in a RWA algorithm and only appropriate wavelength assignments to signals can be considered. The objectives of those algorithms can aim to maximize the number of signals that are routed successfully in the network, without a need of wavelength conversion. For the rest of the traffic that cannot be routed with a single wavelength, appropriate wavelength conversion points can be chosen in the telecommunication network.

We selected the candidates for express links from the set of shortest paths of all node pairs in the network. This selection criterion led us to a network with more number of arcs where the total number of arcs is almost doubled.

A further research can decrease the number of shortest paths that are selected as candidates of express links and study a network with less number of arcs. In this case, selection criterion for candidates of express links is vital. One may choose the shortest paths that connect two sets of nodes between which we have to send high number of signals. Another criterion may limit the degree of each node so that less number of shortest paths can be a candidate of express link for a node. Selecting a limited number of shortest paths as candidates of express links may provide better gaps for the solution, however this solution may not be good enough since all of the express links are not under consideration and this case may miss network designs that will provide more cost savings.

Besides the applicability of our study in a telecommunication network, we propose a different approach that can be used for network loading problems. Given a starting feasible solution, closing a number of capacitated links according to their flow/capacity utilization can also be considered for network loading problems where the link types do not have multiple capacities of unit CL capacity.

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APPENDIX

APPENDIX

D1		2 / 2,00		2 / 2,25		2 / 2,50		2 / 2,75		2 / 3,00		2 / 3,25		2 / 3,50		2 / 3,75		2 / 4,00	
		M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H
		Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap
26-146-1	1 min.	46,8	22,7	40,5	22,9	46,9	22,9	47,8	23,1	47,9	22,4	48,1	22,5	49,3	23,6	45,4	25,8	51,0	25,7
	3 min.	45,2	22,6	40,5	22,9	46,9	22,2	44,2	23,1	47,8	21,2	45,1	21,3	45,9	23,4	45,2	23,8	39,9	21,1
	5 min.	35,5	21,1	40,5	21,2	34,5	22,1	34,5	20,9	31,1	21,2	30,0	21,2	33,1	22,7	37,0	23,7	32,9	21,0
	7 min.	29,3	21,0	31,8	21,2	27,3	21,9	28,0	20,8	31,0	21,0	28,1	21,0	30,4	22,5	32,6	22,3	31,1	20,9
	10 min.	25,4	20,8	29,8	21,0	26,0	21,7	27,8	20,5	30,1	19,7	25,4	20,7	24,7	22,3	30,3	22,0	25,3	20,9
26-146-2	1 min.	49,7	32,7	52,3	30,6	49,8	32,5	51,7	33,1	50,7	32,8	52,0	34,3	52,5	34,0	53,8	34,8	55,4	33,4
	3 min.	30,4	30,7	30,3	29,9	31,6	32,0	30,2	31,5	30,6	31,3	31,5	32,7	32,5	32,2	31,7	33,0	32,4	31,7
	5 min.	29,1	30,2	29,6	29,4	30,9	31,4	29,8	31,1	29,7	30,6	30,6	31,6	30,8	31,8	30,9	32,4	30,6	31,2
	7 min.	28,3	29,8	28,7	28,9	30,7	31,2	29,4	30,7	29,0	30,2	30,3	31,3	30,2	31,4	30,0	32,1	30,0	30,8
	10 min.	27,9	29,5	28,5	28,7	30,1	30,6	29,1	30,5	28,8	30,0	30,0	31,0	29,1	31,0	29,7	31,7	29,6	30,4
26-146-4	1 min.	30,0	17,3	33,0	17,5	34,9	17,8	34,5	17,5	35,8	16,4	36,3	16,5	34,7	17,7	36,2	16,5	34,0	16,6
	3 min.	16,4	15,9	14,1	15,2	15,7	15,4	13,1	14,4	14,3	14,9	14,4	15,9	14,4	14,9	14,9	14,5	14,8	16,1
	5 min.	16,4	14,0	14,0	13,5	14,7	14,1	13,0	13,4	14,2	14,8	14,3	13,4	14,3	14,9	14,9	14,4	14,7	15,8
	7 min.	16,3	13,9	13,9	13,4	14,7	14,1	12,9	13,4	14,2	12,9	14,2	13,4	14,3	14,8	14,9	14,4	14,7	15,0
	10 min.	16,1	13,9	13,8	13,3	14,6	14,0	12,9	13,3	14,2	12,9	14,2	13,3	14,2	12,7	14,8	13,3	14,3	13,9
26-146-6	1 min.	10,2	9,3	12,6	10,0	12,3	9,9	12,7	9,8	13,7	10,3	14,5	10,4	11,4	9,5	11,4	10,4	16,0	9,5
	3 min.	7,9	9,3	8,8	9,0	8,5	8,5	8,4	9,1	8,1	9,5	9,1	9,4	8,1	9,3	8,7	9,9	9,6	9,3
	5 min.	7,9	8,5	8,6	8,5	8,5	8,5	8,3	9,1	8,1	9,3	9,1	8,1	8,0	9,2	8,6	8,9	9,6	9,3
	7 min.	7,9	8,5	8,6	8,4	8,4	8,5	8,0	8,7	8,1	8,0	9,1	8,1	8,0	8,7	8,0	8,9	9,6	9,3
	10 min.	7,9	8,5	8,5	8,4	8,4	7,5	7,9	7,8	8,1	8,0	9,0	8,1	7,8	8,1	8,0	8,5	9,6	8,6
26-146-8	1 min.	7,4	8,5	10,0	8,4	9,5	8,5	10,4	8,6	10,7	8,6	9,2	8,7	9,6	8,7	10,4	8,8	9,8	8,9
	3 min.	6,5	7,3	6,9	7,4	7,6	7,4	7,3	6,7	6,7	7,6	6,9	7,0	6,9	6,9	6,8	6,8	6,5	7,9
	5 min.	6,5	6,5	6,9	6,8	7,5	7,2	7,2	6,7	6,7	6,9	6,9	6,9	6,9	6,9	6,8	6,8	6,5	6,8
	7 min.	6,5	6,5	6,9	6,8	7,5	7,2	7,2	6,7	6,7	6,8	6,9	6,6	6,9	6,9	6,8	6,8	6,4	6,8
	10 min.	6,5	6,5	6,9	6,8	7,1	6,7	6,7	6,7	6,5	6,8	6,9	6,6	6,9	6,8	6,8	6,8	6,4	6,8

Table 7.2.1.a Comparison of gap performances of M-3 and heuristic for D1, 26-146-L.

D1		2 / 2,00		2 / 2,25		2 / 2,50		2 / 2,75		2 / 3,00		2 / 3,25		2 / 3,50		2 / 3,75		2 / 4,00	
		M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H
		Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap
35-198-1	1 min.	66,7	33,8	61,8	35,3	67,3	36,0	66,7	33,9	66,4	35,2	67,0	34,2	66,6	33,9	66,3	34,9	66,4	34,3
	3 min.	61,6	33,8	59,6	33,4	59,0	33,7	57,7	33,9	58,7	33,9	61,9	33,5	58,8	33,9	58,1	32,2	61,8	33,9
	5 min.	61,4	32,3	59,6	33,4	58,8	33,3	57,7	33,2	58,7	33,8	61,9	33,4	58,6	33,6	57,9	31,8	61,6	32,9
	7 min.	61,3	32,1	59,6	33,4	58,8	32,9	57,4	32,8	58,4	33,4	61,9	33,4	58,6	33,6	57,9	31,8	61,6	32,9
	10 min.	61,3	32,1	59,6	33,4	58,8	32,9	57,4	32,8	58,4	33,4	61,9	33,3	58,6	33,6	57,9	31,8	61,6	32,9
35-198-2	1 min.	49,9	27,0	50,6	27,0	52,1	27,2	50,0	26,7	50,2	26,7	51,9	26,9	52,1	27,0	51,4	26,9	50,8	26,8
	3 min.	49,9	26,4	49,5	26,5	51,1	27,0	50,0	24,8	50,2	26,5	50,9	25,9	51,0	24,5	50,2	25,8	48,8	24,5
	5 min.	40,1	25,8	39,4	26,0	37,2	24,2	44,8	24,7	44,9	25,0	43,6	25,2	44,0	24,5	42,9	25,2	40,7	24,5
	7 min.	30,3	25,5	28,2	25,7	25,9	24,2	38,0	23,5	29,5	23,8	29,9	24,5	28,4	24,5	27,1	24,5	26,1	24,1
	10 min.	28,6	24,6	26,7	24,0	25,8	24,1	27,6	23,5	29,5	23,8	27,2	24,3	26,2	24,5	26,9	23,9	26,1	24,0
35-198-4	1 min.	26,0	**	28,4	13,7	28,9	13,6	29,0	13,8	29,7	13,7	28,7	13,7	30,0	13,8	28,9	13,7	30,4	13,8
	3 min.	12,9	13,6	15,0	13,7	17,1	13,6	13,6	13,7	14,8	13,7	17,7	13,7	17,0	13,8	13,4	13,7	14,5	13,8
	5 min.	10,6	13,6	11,6	12,6	13,2	13,6	11,6	13,1	10,1	13,4	12,5	11,5	10,8	12,3	10,2	13,0	12,0	11,7
	7 min.	10,6	13,6	11,5	11,4	11,2	12,7	10,9	13,1	10,1	11,9	12,5	11,5	10,8	11,8	10,1	11,5	10,9	11,4
	10 min.	10,6	10,7	11,5	11,1	10,0	11,7	10,9	12,0	10,1	11,6	12,4	11,5	10,7	11,7	10,1	10,9	10,8	10,5
35-198-6	1 min.	16,8	**	18,1	**	16,9	**	17,6	**	19,4	**	18,0	**	19,3	**	19,0	**	17,9	**
	3 min.	7,9	8,6	10,0	8,7	9,5	8,7	8,1	8,7	9,0	8,7	8,9	8,7	9,6	8,8	8,8	8,8	8,9	8,8
	5 min.	7,1	8,0	7,8	8,7	6,1	8,7	6,8	8,7	7,4	8,7	7,3	8,3	7,6	8,6	7,2	8,7	7,2	8,7
	7 min.	7,1	8,0	7,7	8,6	6,1	8,6	6,7	8,3	7,4	7,7	7,1	8,3	6,9	8,6	7,2	8,7	7,0	8,2
	10 min.	7,1	8,0	7,7	8,5	6,1	7,6	6,7	8,3	7,3	7,7	7,1	8,3	6,9	7,2	6,8	8,3	7,0	6,8
35-198-8	1 min.	16,1	**	16,2	**	14,3	**	17,2	**	17,6	**	18,2	**	16,9	**	17,4	**	15,7	**
	3 min.	8,4	7,1	8,6	7,0	7,5	7,1	7,7	7,1	8,2	7,1	9,2	7,1	7,1	7,1	9,7	7,1	7,8	7,1
	5 min.	6,9	7,1	7,4	7,0	6,8	7,0	5,8	7,0	6,4	7,1	6,5	7,1	5,4	7,1	5,9	7,1	5,5	7,1
	7 min.	6,8	7,1	5,9	7,0	6,7	7,0	5,8	6,8	6,4	7,1	6,5	7,1	5,4	7,1	5,9	7,1	5,5	6,8
	10 min.	6,6	7,1	5,9	6,7	6,7	5,7	5,8	6,8	6,4	7,0	6,4	6,4	5,4	6,8	5,9	7,1	5,5	6,7

Table 7.2.1.b Comparison of gap performances of M-3 and heuristic for D1, 35-198-L.
** Heuristic could not finish one iteration

APPENDIX

D2		2 / 2,00		2 / 2,25		2 / 2,50		2 / 2,75		2 / 3,00		2 / 3,25		2 / 3,50		2 / 3,75		2 / 4,00	
		M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H
		Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap
26-146-1	1 min.	47,3	26,3	52,4	25,3	42,2	26,2	47,8	25,5	44,5	25,5	48,6	25,8	52,3	25,9	47,1	26,1	46,9	26,2
	3 min.	46,9	24,6	52,4	23,9	42,2	24,8	47,8	25,5	44,5	24,5	48,6	24,7	52,2	24,8	47,1	25,8	46,9	26,2
	5 min.	26,4	24,6	42,2	23,9	32,6	24,6	40,8	25,4	37,7	24,4	27,3	24,5	36,3	24,7	32,0	25,6	39,6	22,1
	7 min.	26,3	24,5	29,8	23,8	27,5	24,5	34,0	25,3	31,9	24,4	27,1	24,3	25,3	21,1	31,6	22,5	29,6	22,0
	10 min.	25,4	24,3	29,7	23,6	27,3	24,3	27,4	25,0	31,2	23,9	25,0	24,1	24,9	20,9	26,2	22,0	25,5	21,4
26-146-2	1 min.	45,1	28,1	42,6	27,1	45,2	27,8	45,2	28,8	43,0	29,5	46,0	29,5	47,3	29,7	47,3	29,5	48,0	29,7
	3 min.	25,2	26,2	26,2	26,6	27,7	27,1	25,9	27,0	27,5	28,1	27,1	27,6	25,9	28,7	26,4	28,3	25,4	28,0
	5 min.	24,6	25,6	24,8	26,1	25,5	26,7	25,5	26,6	26,0	26,5	25,4	27,0	25,3	25,6	24,9	26,8	25,2	27,6
	7 min.	24,4	24,8	24,5	25,8	24,8	26,4	25,2	26,3	25,2	26,2	24,8	26,7	24,2	25,2	24,6	26,5	24,9	27,4
	10 min.	23,9	24,4	24,2	25,6	24,6	26,2	24,9	25,4	24,7	26,0	24,2	26,1	23,3	25,0	24,3	26,3	24,4	27,2
26-146-4	1 min.	12,9	10,7	12,6	10,3	12,5	10,4	15,9	10,5	12,3	10,4	13,7	10,4	15,8	10,6	12,1	10,6	15,8	10,7
	3 min.	9,3	9,0	9,4	8,8	8,1	10,2	10,2	9,7	8,5	9,3	8,6	10,2	9,1	9,8	8,8	9,2	8,7	9,9
	5 min.	9,3	9,0	8,8	8,4	8,1	9,7	9,6	9,4	8,5	8,2	8,6	9,5	9,0	9,5	8,8	8,8	8,6	8,0
	7 min.	9,3	9,0	8,8	8,4	8,1	9,0	9,6	9,4	8,5	8,2	8,1	9,5	9,0	9,1	8,8	8,8	8,6	8,0
	10 min.	9,3	9,0	8,8	8,0	8,0	9,0	9,5	9,3	8,4	8,2	8,1	7,6	9,0	8,8	8,8	7,8	8,6	8,0
26-146-6	1 min.	8,5	7,8	7,2	7,6	7,1	7,6	8,8	7,7	8,1	7,7	9,1	7,8	11,3	7,9	9,3	8,0	8,6	8,1
	3 min.	5,6	7,7	6,1	7,5	6,0	7,4	6,0	7,6	6,5	7,5	6,8	7,3	6,7	7,2	5,8	7,6	7,0	7,9
	5 min.	5,6	7,7	6,1	7,5	6,0	7,3	6,0	7,0	6,5	7,4	6,1	6,3	6,7	7,2	5,8	7,5	6,9	7,1
	7 min.	5,6	6,6	6,1	7,3	5,9	7,3	6,0	6,9	6,4	7,4	6,1	6,3	6,7	6,8	5,8	6,9	6,9	6,8
	10 min.	5,6	6,6	6,1	7,3	5,9	7,0	5,9	6,8	6,4	6,1	6,1	6,3	6,7	6,4	5,7	6,1	6,6	6,8
26-146-8	1 min.	7,1	6,6	7,1	6,7	6,6	6,8	6,1	6,9	6,4	7,0	5,1	7,1	5,9	7,1	5,7	7,3	6,8	7,4
	3 min.	5,3	6,0	5,9	6,7	5,2	6,3	5,1	6,0	5,2	6,3	5,1	5,8	5,9	6,1	4,5	5,9	5,2	6,1
	5 min.	5,3	5,4	5,1	6,0	5,2	6,3	5,1	5,5	5,1	5,1	5,1	5,8	5,7	5,3	4,5	5,9	5,2	6,1
	7 min.	5,3	5,4	5,1	5,7	5,1	5,9	5,1	5,0	5,1	5,1	5,1	5,8	5,6	5,3	4,5	5,5	5,1	5,5
	10 min.	5,3	5,4	5,1	5,2	5,1	5,9	5,1	5,0	5,1	5,1	4,7	5,3	5,6	5,3	4,5	5,5	5,1	5,5

Table 7.2.2.a Comparison of gap performances of M-3 and heuristic for D2, 26-146-L.

D2		2 / 2,00		2 / 2,25		2 / 2,50		2 / 2,75		2 / 3,00		2 / 3,25		2 / 3,50		2 / 3,75		2 / 4,00	
		M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H
		Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap
35-198-1	1 min.	39,6	20,6	38,9	20,6	39,1	20,4	39,6	20,3	39,5	20,7	40,4	20,5	40,8	20,9	40,7	20,5	40,5	20,2
	3 min.	39,5	20,5	38,9	19,9	39,1	20,4	39,5	20,1	39,5	20,7	40,3	19,3	40,5	20,5	40,7	20,5	40,4	20,0
	5 min.	39,5	20,5	38,9	20,0	39,1	20,4	39,6	20,2	39,5	20,7	40,3	19,3	40,5	20,4	40,7	20,5	40,4	20,0
	7 min.	38,2	20,5	38,9	19,9	39,1	20,4	39,5	20,1	37,3	20,7	40,3	19,3	37,0	20,4	36,4	20,5	39,6	20,0
	10 min.	25,9	20,5	29,8	19,9	35,7	19,9	33,4	19,1	25,6	20,7	32,5	19,3	26,0	20,4	22,3	20,5	25,7	20,0
35-198-2	1 min.	37,6	**	38,2	**	37,9	**	37,8	**	35,5	**	34,9	**	37,3	**	37,9	**	38,1	**
	3 min.	27,6	16,1	31,3	16,1	34,2	16,4	34,5	16,4	28,1	16,3	34,9	16,4	32,5	16,4	32,7	16,5	34,2	16,4
	5 min.	18,2	15,5	20,5	16,1	22,2	15,4	23,8	15,7	17,8	16,3	23,7	16,3	15,2	16,3	20,7	16,5	23,6	16,4
	7 min.	18,2	15,5	15,5	16,1	16,9	15,4	17,8	15,6	15,9	16,3	17,0	16,3	15,2	16,3	17,0	15,8	17,3	16,3
	10 min.	16,9	15,5	15,5	16,1	16,2	15,4	17,3	15,5	15,5	16,3	15,9	15,7	15,1	16,2	16,2	15,7	17,3	16,3
35-198-4	1 min.	20,5	**	18,6	**	20,6	**	21,8	**	22,5	**	21,0	**	20,2	**	21,1	**	21,4	**
	3 min.	11,1	9,0	10,5	9,0	10,8	9,0	12,9	9,0	12,5	9,1	10,4	9,1	10,6	9,1	10,9	9,0	10,2	9,1
	5 min.	8,8	9,0	7,8	9,0	8,3	8,9	7,8	9,0	9,0	9,0	8,2	9,1	8,8	9,1	8,1	9,0	8,5	9,0
	7 min.	8,1	8,9	7,7	9,0	8,3	8,9	7,8	9,0	8,9	8,9	7,8	9,0	7,6	9,0	7,8	9,0	8,4	9,0
	10 min.	7,9	8,7	7,6	8,9	8,2	8,9	7,8	8,3	8,9	7,8	7,8	9,0	7,6	9,0	7,7	8,8	8,4	9,0
35-198-6	1 min.	11,1	**	10,3	**	12,4	**	12,6	**	11,2	**	10,2	**	12,4	**	13,2	**	10,5	**
	3 min.	6,0	4,3	5,5	4,3	6,6	4,3	5,2	4,3	4,7	4,3	6,1	4,3	5,1	4,4	7,3	4,4	5,2	4,4
	5 min.	4,6	4,2	4,0	4,3	4,9	4,3	4,7	4,3	4,0	4,3	4,8	4,3	3,9	4,3	4,7	4,4	4,2	4,4
	7 min.	4,6	4,2	4,0	4,2	4,9	4,3	4,4	4,3	4,0	4,3	4,4	4,3	3,9	4,3	4,7	4,4	4,2	4,4
	10 min.	4,6	4,2	4,0	4,2	4,9	4,3	4,4	4,3	4,0	4,3	4,2	4,3	3,9	4,3	4,1	4,3	4,2	4,4
35-198-8	1 min.	9,5	**	10,0	**	11,0	**	11,1	**	11,7	**	11,1	**	10,9	**	10,9	**	11,3	**
	3 min.	6,2	6,0	5,6	5,9	5,8	6,0	6,6	6,0	6,2	6,0	5,4	6,0	5,3	6,0	6,0	6,1	4,8	6,1
	5 min.	4,7	6,0	4,2	5,9	4,1	5,2	3,8	6,0	4,5	6,0	3,7	6,0	4,6	5,4	4,6	6,0	4,6	6,1
	7 min.	4,2	4,7	4,2	5,0	4,1	5,2	3,8	4,7	4,5	6,0	3,7	5,3	4,6	5,4	4,6	5,3	4,0	4,7
	10 min.	4,2	4,7	4,2	5,0	3,7	5,2	3,8	4,7	3,9	4,5	3,7	5,3	4,2	5,4	4,2	5,3	4,0	4,7

Table 7.2.2.b Comparison of gap performances of M-3 and heuristic for D2, 35-198-L.
** Heuristic could not finish one iteration

D3		2 / 2,00		2 / 2,25		2 / 2,50		2 / 2,75		2 / 3,00		2 / 3,25		2 / 3,50		2 / 3,75		2 / 4,00	
		M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H	M-3	H
		Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap	Gap
26-146-1	1 min.	65,8	29,9	65,4	29,0	64,2	29,3	66,0	30,1	65,6	32,8	64,7	29,9	64,3	31,6	64,2	32,6	64,7	32,1
	3 min.	61,1	29,9	55,4	29,0	59,6	29,2	59,0	29,6	58,0	30,7	60,9	29,9	58,4	31,5	53,3	31,9	48,9	32,0
	5 min.	47,7	29,4	39,3	29,0	37,8	29,2	40,5	29,6	39,6	30,7	39,2	29,8	41,1	31,1	38,7	31,7	48,9	31,6
	7 min.	38,3	29,3	32,3	28,7	30,0	28,2	36,9	28,7	35,2	29,0	37,3	29,8	34,4	31,0	36,1	31,6	40,2	31,2
	10 min.	34,9	29,0	31,1	27,9	30,0	28,2	36,2	28,4	35,1	25,8	33,3	28,7	32,3	29,6	34,4	30,8	32,5	30,7
26-146-2	1 min.	34,4	21,1	32,8	21,8	36,6	21,5	40,8	21,1	37,5	20,6	40,8	22,1	39,2	22,1	37,1	20,7	39,3	21,6
	3 min.	20,9	20,7	20,0	21,3	22,3	21,2	21,9	20,9	21,2	20,4	23,9	21,8	23,9	21,8	23,0	19,7	22,2	20,4
	5 min.	20,7	20,4	19,8	20,7	22,0	20,9	21,6	18,3	21,0	19,4	23,8	20,2	23,8	20,2	23,0	19,1	22,0	20,0
	7 min.	20,4	20,0	19,7	20,6	21,9	20,8	21,4	18,2	20,8	19,3	23,5	19,9	23,5	19,9	21,9	17,6	21,8	19,7
	10 min.	20,0	18,9	19,5	19,2	21,7	20,4	21,1	18,1	20,5	19,0	22,6	19,6	22,6	19,6	21,7	17,5	21,6	19,6
26-146-4	1 min.	6,2	6,5	7,8	6,3	7,9	5,1	6,0	6,5	6,2	6,5	6,5	6,6	7,1	6,6	7,0	6,7	6,7	6,8
	3 min.	5,4	5,5	5,3	5,4	5,6	4,9	4,3	5,0	4,6	5,2	5,5	5,1	5,7	5,2	5,2	5,7	5,6	5,1
	5 min.	5,4	5,5	5,3	5,4	5,2	4,9	4,3	5,0	4,6	5,2	5,3	5,0	5,7	5,2	4,7	5,1	5,6	5,1
	7 min.	5,2	5,4	5,3	5,1	5,2	4,9	4,3	4,9	4,6	5,2	5,3	5,0	5,6	5,1	4,7	5,1	5,6	5,1
	10 min.	5,2	5,4	5,2	5,1	5,2	4,9	4,3	4,9	4,6	5,2	5,3	5,0	5,6	5,1	4,7	5,1	5,6	5,1
26-146-6	1 min.	4,7	6,4	4,8	5,2	5,2	5,4	5,3	5,4	5,6	5,5	5,7	5,6	5,4	5,7	5,3	5,8	4,9	5,9
	3 min.	4,7	5,2	4,8	5,0	5,0	5,2	4,2	5,2	4,2	5,1	3,8	5,3	4,4	4,7	4,2	5,1	4,1	4,8
	5 min.	4,2	3,9	4,8	4,6	5,0	4,5	4,2	5,0	4,2	4,8	3,8	4,3	4,4	4,6	4,1	4,9	4,1	4,6
	7 min.	4,2	3,9	4,1	4,6	5,0	4,5	4,0	4,8	4,2	4,8	3,8	4,3	4,4	4,6	4,1	4,9	4,0	4,6
	10 min.	4,2	3,9	4,1	4,5	4,3	4,5	4,0	4,8	4,2	4,8	3,8	4,3	4,2	4,6	4,1	4,9	4,0	4,6
26-146-8	1 min.	5,1	5,8	4,3	5,7	4,8	5,5	3,4	5,8	5,0	5,6	4,4	6,3	4,9	5,4	4,3	5,9	4,4	6,2
	3 min.	3,9	4,5	3,8	4,2	4,3	4,6	3,4	4,3	3,6	4,0	3,9	5,0	3,8	4,9	3,7	4,8	4,4	4,7
	5 min.	3,9	4,3	3,8	4,1	4,3	4,2	3,4	4,3	3,5	4,0	3,8	4,9	3,8	4,0	3,4	4,2	4,3	4,0
	7 min.	3,9	4,3	3,8	4,1	4,1	4,0	3,4	4,3	3,5	4,0	3,8	4,5	3,8	3,8	3,4	4,2	4,2	4,0
	10 min.	3,9	3,8	3,4	3,8	4,1	4,0	3,4	4,3	3,5	4,0	3,8	4,5	3,8	3,8	3,4	3,9	4,2	3,6

Table 7.2.3.a Comparison of gap performances of M-3 and heuristic for D3, 26-146-L.

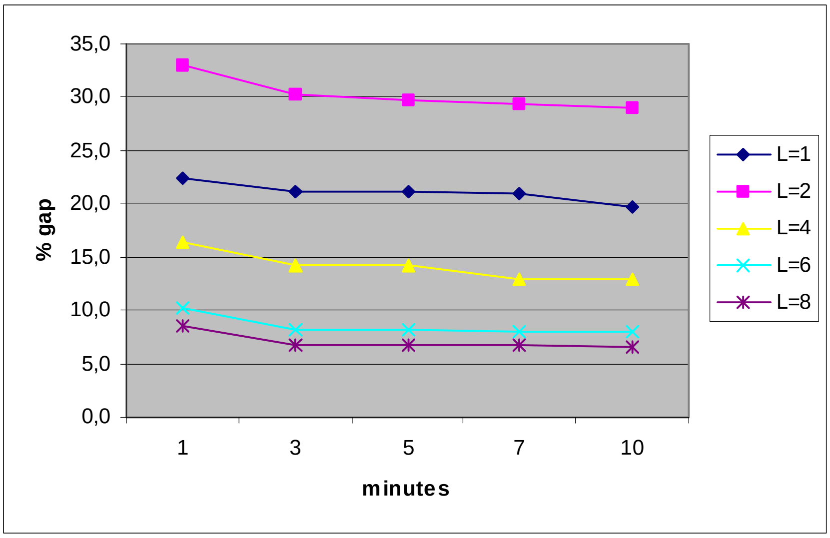

Figure 7.2.1.a Best gap obtained for scenario D1, 26-146-L, R=2/3

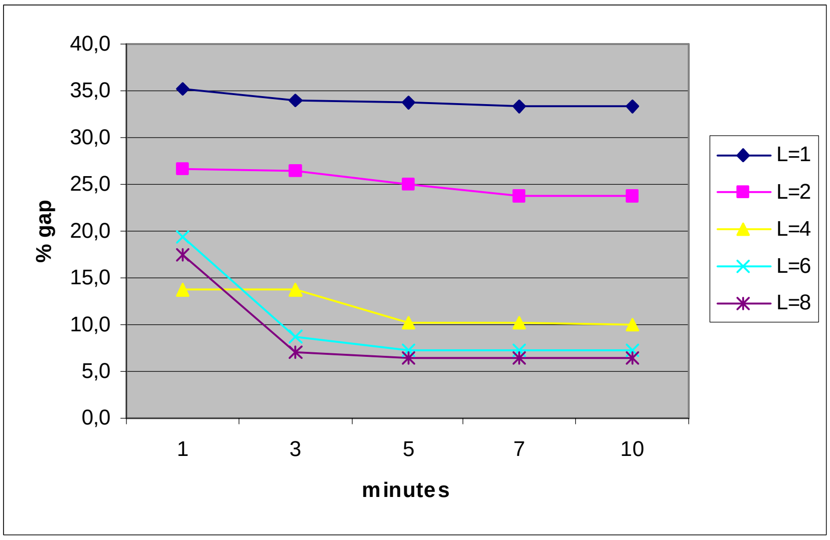

Figure 7.2.1.b Best gap obtained for scenario D1, 35-198-L, R=2/3

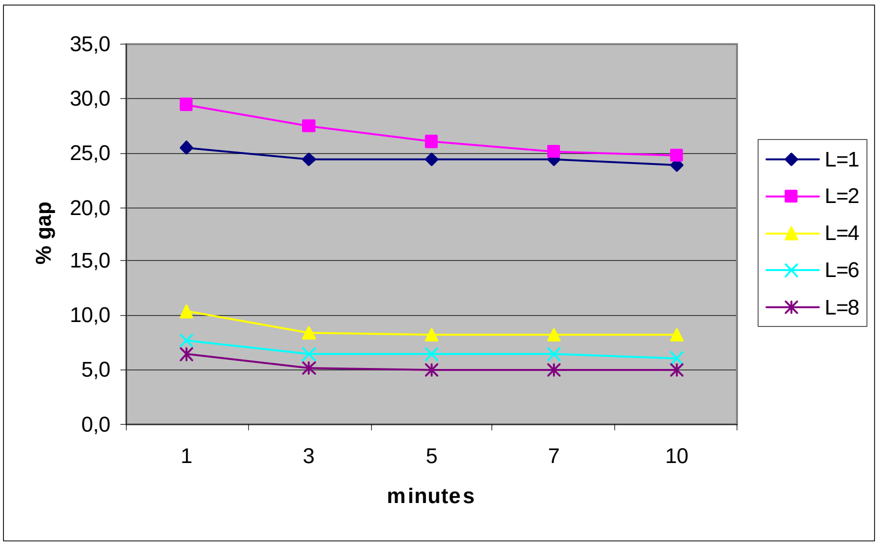

Figure 7.2.2.a Best gap obtained for scenario D2, 26-146-L, R=2/3

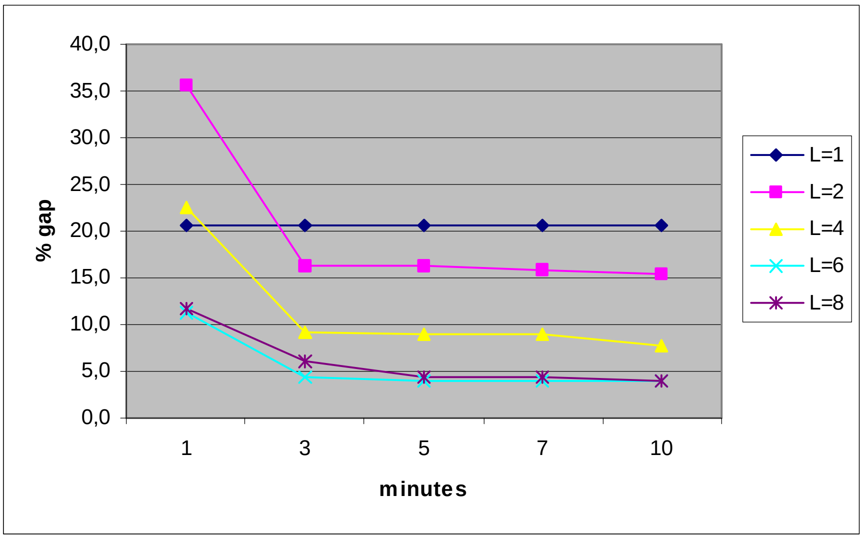

Figure 7.2.2.b Best gap obtained for scenario D2, 35-198-L, R=2/3

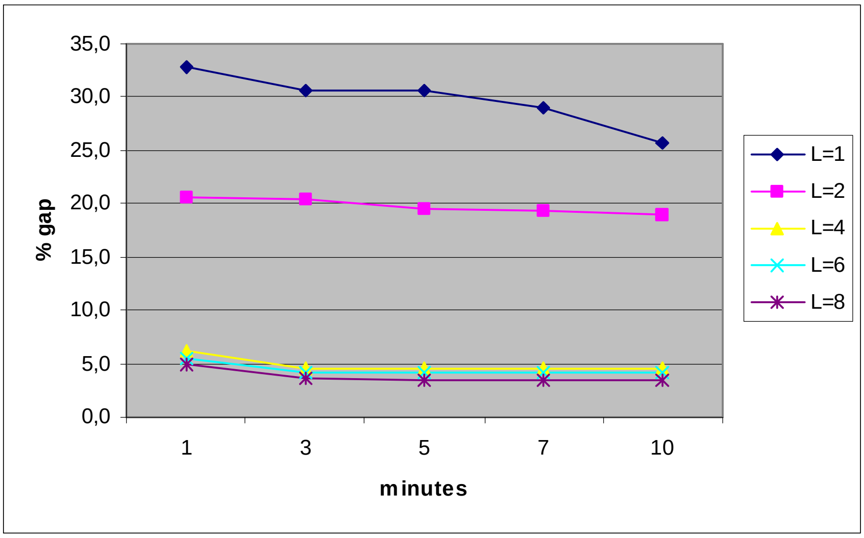

Figure 7.2.3.a Best gap obtained for scenario D3, 26-146-L, R=2/3

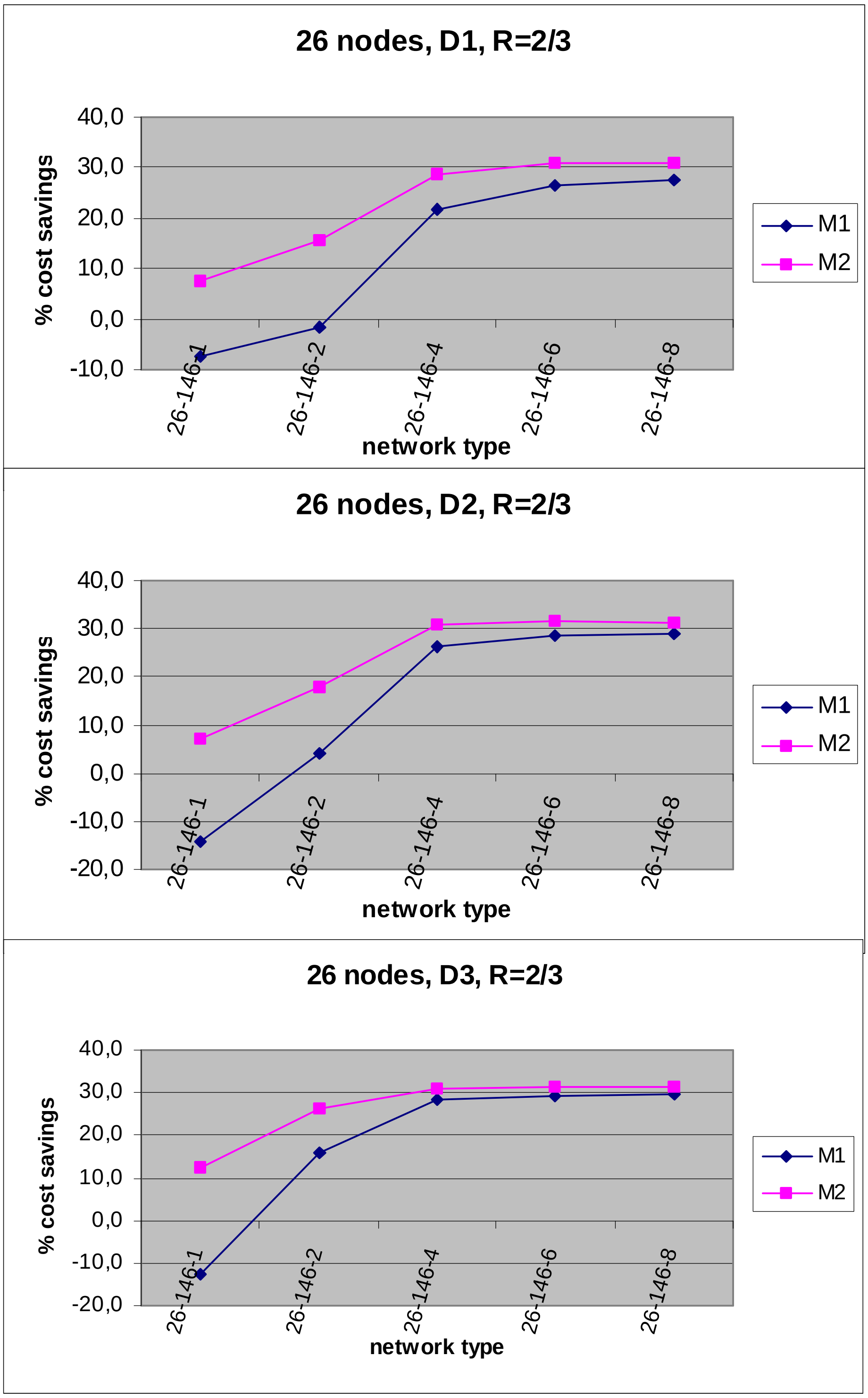

Figure 7.3.2 The percentages of cost savings for 26-146-L

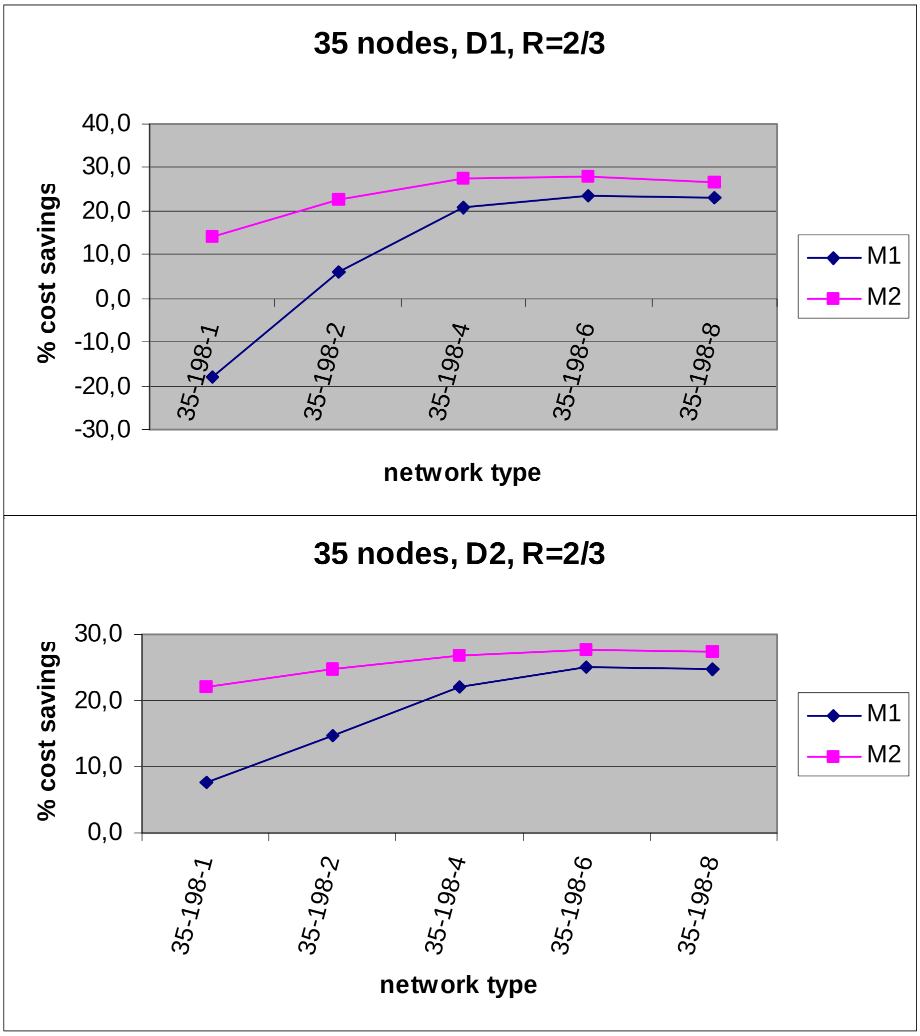

Figure 7.3.3 The percentages of cost savings for 35-198-L