PRIVATIZATION OF FORESTS: A REAL OPTIONS APPROACH

A Master's Thesis

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Department of Management Bilkent University Ankara February 2005

To my family

PRIVATIZATION OF FORESTS: A REAL OPTIONS APPROACH

The Institute of Economics and Social Sciences of Bilkent University

by

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In Partial Fulfilment of the Requirements for the Degree of MASTER OF SCIENCE

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February 2005

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science in Management.

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ABSTRACT

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This thesis analyzes the privatization of forests. It suggests that privatization can be regarded as a real option. The thesis develops a model that determines the optimal conditions for privatization of forests. Working in a real option framework, I show that delaying privatization could have a positive value attached to it and obtain the critical value at which the government privatizes the forest. I also show that the fundamental variables in the economy such as the interest rate and uncertainty can affect the decision to privatize.

Keywords: Real Options, Privatization, Economic Rent, Amenity

ÖZET

ORMANLARIN ÖZELLEŞTİRİLMESİ: REEL OPSİYON YAKLAŞIMI Siyahhan, Baran Master, İşletme Bölümü Tez Yöneticisi: Asst. Prof. Selçuk Caner

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Bu çalışma ormanların özelleştirilmesini incelemektedir. Çalışma, özelleştirmenin reel opsiyon olarak değerlendirilebileceğini anlatmaktadır. Tez, ormanların hangi koşullarda özelleştirilmesi gerektiğini gösteren bir model geliştirmektedir. Tez, reel opsiyon çerçevesinde, özelleştirmenin ertelenmesinin pozitif bir değeri olabileceğini kanıtlamakta ve devletin ormanı özelleştirmesi gereken eşik değerini bulmaktadır. Model, aynı zamanda, faiz oranı ve belirsizlik gibi temel ekonomik değişkenlerin özelleştirme kararı üzerinde etkili olduğunu göstermektedir.

Anahtar Kelimeler: Reel Opsiyon, Özelleştirme, Ekonomik Rant, Çevresel Fayda

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CHAPTER 1: INTRODUCTION

Consider a risk-neutral government that plans on privatizing a forest land which provides a certain social benefit (amenity value) to the community and potential economic opportunities from the sale of timber. Exactly when and under what conditions should the government privatize the forest? What are the factors that the government must take into account before going ahead with the plan? How should the government resolve the decision to privatize?

Although privatization is not a novel phenomenon, it is only in the past two decades that privatization movements have actually accelerated. Following the examples in the industrialized world, the developing countries soon adopted their own privatization programs. There are a number of motivating factors behind the privatization movements. The East European countries have mainly considered privatization as a tool to transform their countries into market economies whereas other developed and developing countries have privatized to achieve economic goals such as revenue generation and capital market development. Inefficiency of state owned enterprises (SOEs) is by far the most crucial and well-documented argument for privatization (see, for instance, Donahue, 1989; Lopez de Silanes, 1993 and Mueller, 1989). Privatization movements in Turkey also coincide with the trends observed in the last two decades across the world. The liberalization program announced in 1980 envisions the privatization of the SOEs with the objectives, among others, of stimulating the economy, strengthening the development of capital markets and increasing efficiency (Şımga-Muğan and Yüce, 2003).

Privatization of forests has generated lively debates in several countries. The privatization of forests dates back to the 18th century in Scandinavian countries. The former communist countries have recently brought up the issue although not all have gone on with the privatization plans. The question as to whether forests should be privatized has also spurred a debate in Turkey with environmentalists expressing their concern for the future stock of forest. Environmentalists are particularly concerned about the loss of endemic species and biodiversity. While the environmental concerns regarding the preservation and protection are well understood, economic benefits of preservation are understood less. It is particularly important to recognize the multidimensional nature of the problem. Forests are part of national wealth. As such, they bear a substantial value that entails both environmental and economic benefits. Economically, forests can be an important source of exports especially for developing countries. The Food and Agriculture Organization (FAO) estimates, for instance, that demand for wood and timber products will have grown by 2.2 billion m³ by 2010, corresponding to

a 2% increase per annum¹. Furthermore, forests are rich in plants that can be used for alimentary and medicinal purposes. Forest dwellers in many countries constitute yet another group who earn their livelihood from forests. Conversion of forests to agricultural use has economic implications and has been a controversial issue in many countries. Therefore, production of forests and forestry products is an economic process where optimal use of resources is dictated by economic theory. It is clear that the public at large should be interested in the optimum use of forests. However, the fact that multiple stakeholders are involved makes the management of forests a complex issue. In the event of a conflict between private and public interests, the economic use of forests should be regulated since no party has a license to use public resources for private purposes without paying appropriate prices which are to be determined based on the underlying value of the natural resources. Sustainable forest management, therefore, must formulate policies so that benefits derived from forests are not diminished. In other words, the challenge a government faces is not to prevent the harvesting of forests but to ensure via sustainable forest management that national wealth (rather than wealth of a single stakeholder) is maximized. Maximization of national wealth, in turn, requires an economic analysis that weighs the benefits of any policy against its costs.

One important aspect of sustainable forest management and economic analysis is property rights. The state must determine the ownership structure of forests. The recent trend is towards private ownership. However, if privatization

¹ Inderfor Ltd. "Forest Creation: Investment Opportunities in Argentina", Food and Agriculture

is perceived as depletion of resources, it would not be economically efficient. Efficient management of forestry resources including private use would increase the value of stock of forestry. By privatizing forests, governments aim to reduce bureaucracy, induce competition and promote more accountability and thereby increase the value of stock of forestry.² The Scandinavian experience shows that privatization of forests does not necessarily end in the destruction of forests so long as clear rules are set and abided by. The Swedish case is particularly exemplary. Although forests are privatized in Sweden, tree volume growth exceeds harvests by about 30% and the regeneration process works well.³ The Finnish experience in managing forests is equally striking. About 60% of forests in Finland are owned by private institutions or persons. There has not been, however, a decrease in the total area covered by forests. About three quarters of Finnish land is covered by forests which constitute a substantial export value for the country. Due to the efficient management of forests, the increase in stock has always exceeded harvesting volumes or drain⁴.

I develop a model where the government evaluates the decision to privatize forests. Specifically, I determine the optimal time and the critical value of the forest at which the government privatizes. The model explicitly takes into account the environmental and economic factors that affect the value of the forest to the community as a whole. The model explicitly determines the critical value that the government must take into account in its decision to privatize. I use the real

Organization, "Forestry Statistics-Today for Tomorrow"

² FAO, "The Challenge of Forest Management", Chapter 6, 1993

³ B. Hagglund "Sweden: Using the Forest as a Renewable Resource", Unasylva, No:167, 1991

⁴ Ministry of Agriculture and Forestry of Finland

options theory outlined by Dixit and Pindyck (1994). Valuation is key to natural resource management. NPV methods do not take into account uncertainties and the value of the options that are part of the value of natural resources. Thus, using real options, one can avoid subsidizing private interests or benefits in a land concession. The fundamental idea behind the model is that the government has an "option" to privatize forests. This option is similar to a financial put option where the buyer has the right (but not the obligation) to sell the underlying asset at a prespecified price. In determining the optimal time and conditions for privatization, the government solves an optimal stopping problem. At each point in time, the government has to decide whether to maintain public ownership (i.e. continue to delay privatization) or to privatize (i.e. stop and obtain the "termination payoff" or the privatization price). The fact that government evaluates its decision at each point in time ensures continuous revision and update of the forestry policy. Furthermore, the model allows for long-term policy evaluation. Long-term orientation is particularly important for forestry sector since short-run orientation may conceal the long-term benefits. Focusing merely on short-run benefits may cause excessive harvesting of the forest. This can be a threat to the community as the forest's capacity to provide goods and services is diminished over the longrun.

The thesis is structured as follows: Chapter 2 reviews the literature on real options and privatization. The model is developed in Chapter 3. Chapter 4 concludes and discusses some avenues for future research.

CHAPTER 2: RELATED WORK AND LITERATURE REVIEW

2.1. Literature on Real Options

Earliest studies on the application of options theory were done by Black and Scholes (1973). Merton (1973) generalized the options theory. This also coincides with the period during which researchers increasingly began to take note of the unrealistic features of the conventional discounted cash flow method (so often referred to as the NPV rule in standard finance textbooks) proposed by Dean (1951) and Bierman and Smidt (1960) based on the work of Fisher (1907). The conventional approach does not take into account, among others, the *irreversibility* of the investment (the fact that the initial investment cost is at least partially sunk), the *uncertainty* over the future cash flows of the investment and the flexibility in the *timing* of the investment (delaying the investment until factors that may act upon the investment ameliorate or abandoning the project altogether, rather than considering the investment only as a "now-or-never" decision). Among the early papers that study these issues are those of Cukierman (1980) and Bernanke (1983) who deal with the postponement of irreversible investment decisions and Tourinho (1979) and Brock, Rothschild and Stiglitz (1982) who consider the investment in natural resources. In the past two decades, real options theory was applied to a variety of areas of interest such as capital budgeting, strategic entry and capacity choices and exhaustible and inexhaustible natural resources. Majd and Pindyck (1987) investigate the implications of real options theory for capital budgeting decisions when investment is irreversible and not instantaneous (i.e. the project takes time to complete). McDonald and Siegel (1985) study project evaluation with the option to shut down while McDonald and Siegel (1986) and Myers and Majd (1983) dwell upon the valuation of investments and scrapping opportunities.

One essential feature of the real options is that it allows for the assessment of decisions on physical investments. For instance, Pindyck (1988), Williams (1991) and Trigeorgis (1996) consider the capacity choice and operational flexibility while Smith and McCardle (1996) look at the flexibilities in the exploration stage. More recently, Auguerrevere (2003) studies the effects of competitive interactions on investment decisions and on the dynamics of the price of a nonstorable commodity in a model of incremental investment with time to build and operating flexibility.

The real options are increasingly being used in modeling strategic choices in a competitive market. The basic question is whether a firm should invest in an industry characterized by competition. The firm facing competition has to assess whether it is worth using the preemptive advantage by entering the market (or increasing capacity) first or whether it is more apt to delay the investment (option value of postponement) and risk being the follower in the market. However, this strand of work takes off in the 1990s. Smets (1991) examines irreversible market entry for a duopoly with stochastic demand while Grenadier (1996) considers the strategic exercise of options applied to real estate markets and Weeds (2000) dwells upon the real options nature of investments when there is competition and preemption advantages.

The option-like characteristic of resource reserves is recognized in Tourinho (1979). He is the first to point out to the options in natural resources with applications to mining and oil industries. Paddock *et al.* (1986) extend the financial option theory to develop a model to price licenses in offshore drilling of petroleum and natural gas reserves. It is now common to value licenses for offshore oil reserves or mine fields as a compound American call option (see, among others, Paddock *et al.*, 1988; Bjerksund and Ekern, 1991; Pickles and Smith, 1993; Smit,1997; and Davis and Schantz, 2000). Brennan and Schwartz (1985) and Slade (2001) apply the option valuation technique to the mining industry which closely resembles the oil industry. The real options theory is also adapted to investment in forestry. The decision problem is to optimally harvest a given stock of forest for commercial use including timber and construction development (see, for instance, Morck *et al.*, 1989; Conrad, 1997 and Insley, 2002). In addition, the optimal timing for harvesting is addressed in these studies.

In all these studies, it is almost standard to consider the real options in investments from the perspective of the private firms. Although the specific context of the problem changes across studies (be it a firm investing in a resource reserve or a company that attempts to optimally enter a new market in a competitive environment) the fundamental problem solved remains to be that of a private firm and the public regulator is often neglected. Therefore, it comes as no surprise that the investments are usually considered as compound American calls.

2.2. Literature on Privatization

Although privatization is an option of the government which depends on proper valuation, the literature on privatization is mostly characterized by empirical studies. On the theoretical side, the effect of political considerations is of interest in the literature. The study of Boycko *et al.* (1996), which can be considered as one of the landmarks in the literature, sees privatization as a tool that can reduce inefficiency imposed on by politicians. It has been argued in the literature that privatization should "draw a line" between politicians and the firms to increase efficiency. Schmidt (1990) and Shapiro and Willig (1990) demonstrate this latter point through an information argument. More recently, Biais and Perotti (2002) and Alexeev and Kaganovich (2001) analyze the role of politically motivated agents in the privatization process and the firm efficiency. Another strand of studies has concentrated on the optimal auctions in privatization. Cornelli and Li (1997), for instance, inspect the tradeoff between revenue maximization and efficiency considerations when the government auctions a public enterprise in the presence of foreign buyers. Similarly, Maskin and Riley (1989) consider the optimal number of shares that should be offered to a potential buyer. Laffont and Tirole (1993) study the optimal scheme of privatization by introducing a policy tool that monitors the performance of the firm to balance between revenue maximization and economic efficiency after privatization. Boycko, Shleifer and Vishny (1994) and Katz and Owen (1997, 2002) focus attention on voucher privatizations that were particularly popular in Eastern Europe, notably in the Czech Republic and the Soviet Union.

Alternatively, Dharkadwar *et al.* (2000) argue that the principal-agent problem is at the heart of the ineffectiveness of several privatized firms and provide a model based on agency theory to increase post-privatization effectiveness. Errunza and Mazumdar (2001), to which this thesis is most related, analyze privatization of state owned enterprises (SOEs) as a put option for the government. The implicit put option formulation enables them to develop several policy options in the sale and sequencing of the sale of SOEs.

Empirical studies measuring the impact of privatization and comparing the performance of the public firm with that of the private firm have proliferated particularly after the spread of privatization in the early 1980s (see, among others, Donahue, 1989; Mueller, 1989; Megginson *et al.*, 1994 and Boubakri and Cossett,

1998). Other areas of interest are the macroeconomic effects of privatization (Boutchkova and Megginson, 2000 and Perotti and van Oijen, 2001) and the methods of or trends in privatization movements (Kikeri et al. 1992; Perotti, 1995; Jones et al. 1999 and Biais and Perotti, 2002). Megginson and Netter (2001) provide a good review of the privatization literature with emphasis on the empirical studies.

This thesis develops a model of privatization as a real option. It assesses the optimal conditions for the privatization of forests where the social benefit and the potential revenue the forest creates are the arguments of an implicit put option. In that sense, it establishes a link between the real option literature and the privatization literature. The model builds on Errunza and Mazumdar's (2001) notion that privatization is a put option. I show that Dixit and Pindyck's (1994) methodology can be used to solve the model suggested by Errunza and Mazumdar (2001). Use of real options to model divestment decisions is new to the literature. Although there are studies that consider entry and exit decisions, the emphasis is more on entry in a market in the presence of competition than on exit from the market. Therefore, the basic question in these studies is whether the firm should exercise its *call* option. The model developed in the subsequent pages changes the nature of the problem. The question is to optimally *divest* a renewable resource. Therefore, the government holds a put option rather than a call option from the start. Use of put options rather than call options changes the analysis of Dixit and Pindyck (1994) although the insights are the same (e.g. the government takes into account the opportunity cost of privatization). Furthermore, the model is closely

related to Brennan and Schwartz (1985) who develop a general model for natural resource investments and Morck *et al.* (1989) who study valuation of forestry resources. Like these studies, the model developed in the thesis also deals with a renewable resource. In other words, regeneration of forests must be accounted for in the model and this results in partial differential equations similar to those in Brennan and Schwartz (1985) and Morck *et al.* (1989). It is, nevertheless, possible to extend the model to nonrenewable resources and any other public asset that the government plans to privatize.

CHAPTER 3: MODELING PRIVATIZATION OF FORESTS

3.1. The Model

Before responding to questions raised in Introduction, I first formulate the problem the government faces and define the factors that it will take into account when executing the privatization plan. In the subsequent pages, I develop a model based on real options outlined by Dixit and Pindyck (1994). The model developed by Dixit and Pindyck (henceforth referred to as DP) incorporates irreversibility of initial investment, uncertainty affecting the variables of investment and flexibilities of waiting and abandoning investment. Through dynamic programming and contingent claims analysis, DP model investment as a call option and determine the value of delaying investment. The value of waiting reflects the price the firm would pay in order to delay investment. In other words, the value of waiting is an opportunity cost of investing and must be included in the overall cost of investment. The firm invests immediately only if the full cost of investment (i.e. initial investment and opportunity cost of waiting) is less than the expected cash flow from investment. When the expected cash flow falls short

of the full cost, the firm waits. The value of the project may thus be increased. The reasoning in investment decision is closely associated with the reasoning in financial markets. In the context of financial options, higher values of options imply postponement of exercise. Similarly, higher value of the opportunity cost (i.e. the value of waiting) implies postponement of investment. This basic notion can be incorporated into almost any setting such as investment in imperfect competition and sequential investment. A simple exercise is useful to illustrate that the opportunity cost of investment could be significant⁵. Suppose that a firm is considering an instantaneous investment opportunity that has an initial investment of \$1000. If the firm decides to undertake the project, it can sell the output for \$100 today. Assume, however, that the price of the product will be either \$50 or \$150 in the next period with equal probability. Assume also that the price will remain at the new level thereafter. According to the conventional NPV analysis, the firm should invest if the expected future cash flow from investment is positive. In the present context, assuming that the interest rate is 10%, the firm decides to invest since

$$NPV = -1000 + \sum_{t=0}^{\infty} \frac{100}{(1.1)^t} = -1000 + 1100 = 100$$

With the conventional NPV analysis, however, the firm ignores the opportunity cost of investing today. In other words, the firm has the option to wait another year and eliminate the price uncertainty. The firm would invest only if the price next period turned out to be \$150. The NPV of the project if the firm waits another year is

⁵ The example is based on Dixit and Pindyck (1994), Chapter 2

$$NPV = 0.5 \left[\frac{-1000}{1.1} + \sum_{t=1}^{\infty} \frac{150}{(1.1)^t} \right] = 295$$

The firm is clearly better off if it waits another year because the opportunity cost of investing today is large. Put another way, the full cost of the investment is not \$1000 but \$1295>\$1100. Since the full cost of the investment is greater than the expected future cash flow today, the firm should keep its option to delay investment and invest only if the price next period goes up.

In this thesis, I show that the same approach can be used to model divestment decisions such as privatization. Privatization (and divestment decisions for that matter) is similar to a financial *put* option where the owner of the option has the right to sell the underlying asset. In the current context, the government has flexibility with regard to the privatization of forests. This flexibility has a value. Specifically, the government has the option to sell (privatize) the forest. If the government decides to privatize, it receives the bid price (i.e. the strike price) and delivers the forest (i.e. the underlying asset). If the government decides to maintain its ownership, it does not exercise its option. Therefore, the payoff from privatization is either the bid price less the value of the forest or zero. This is the same pattern of payoff that one would derive from a put option. The counter party of the contract, the private firm, buys the forest in anticipation of future benefits such as exporting timber and developing theme parks.

Errunza and Mazumdar (2001) are the first to model the costs and benefits of privatization as part of a complex option. In their context, the European put option reflects the government guarantee on the debt of an SOE. That is,

$$D = F_T e^{-rT} - P(V, F_T, \sigma_V)$$

where F_T is the face value of the debt, V is the value of the SOE assets with volatility σ_V . The value of the European put, P, denotes the default risk premium of the debtholders. The face value of the debt, F_T , serves as the strike price of the put and the value of the assets of the firm, V, is similar to the value of the underlying stock. Errunza and Mazumdar then obtain the equity value of the private firm and the public firm to analyze the effects of privatization on such factors as efficiency and voting control.

The model developed in this thesis, as opposed to the model of Errunza and Mazumdar, indicates the optimal conditions under which the government should use its option to privatize forests. First, I solve the model assuming no uncertainty. When there is no uncertainty, it is straightforward to solve for the optimal time to privatize. Subsequently, I introduce uncertainty into the model. Instead of solving for the optimal time to privatize, I use the contingent claims analysis of Dixit and Pindyck (1994) to obtain the critical value of the forest that would induce the government to privatize the forest. This critical value determines whether or not the forest bears more value to the community if privatized.

Before going into the details of the model, I first introduce the variables of the model. The government owns, as an asset, a piece of forest whose value evolves stochastically. It is common in the literature to view the value of the forest, *V*, as following a stochastic process (see, for instance, Morck *et al.*, 1989 and Alvarez and Koskela, 2001). More formally, it evolves according to:

$$dV = a(V,t)dt + b(V,t)dz$$
⁽¹⁾

where dz is the increment of a Brownian motion, a(V,t) is the drift term which is the expected change in the value of the natural resource, E(dV) = a(V,t) and b(V,t)is the standard deviation or, rather, the variance rate such that $var(dV) = b^2(V,t)$.

The intrinsic value is a function of the potential rent, the social benefit and the growth rate of the forest over time. Specifically, the intrinsic value of the forest can be influenced by unforeseen ecological events such as the appearance of harmful insects or radical changes in the amount of rainfall, and by the economic factors, in particular, the economic rent. Economic rent is defined as the surplus value created during the production of a good or a service⁶. In other words, economic rent is the profit one earns above the opportunity cost. It is usually hard to create economic rent from many economic activities because other producers are likely to enter the market once economic rent is created. However, this need not be true for natural resource investments. Governments usually lease the right to extract resources to a certain firm. Hence, other companies are prohibited from entering the market. Furthermore, the fact that resources occur naturally considerably reduces the cost of production. Therefore, natural resources bear a great potential for creating economic rent. In the context of forestry resources, economic rent captures the changes in prices of forest products such as timber. In other words, it reflects the uncertain cash flows from harvesting the forest. The economic rent will be taken as stochastic after Samuelson (1965). The stochastic revenue process is commonly used in the literature (see, for example, Morck *et al.* 1989 and Insley, 2001) as well as studies that model the cost as the stochastic variable (Sarkar, 2003). The revenue (sales) stream is assumed to have a lognormal process as in equation $(2)^7$.

$$dR = \alpha_R(R, t)Rdt + \sigma_R(R, t)Rdz_R$$
(2)

where α_R is the drift term, σ_R denotes the uncertainty and dz_R is the increment of a Wiener process.

The second component of the intrinsic value of the forest is the social benefit or the amenity value of the forest. In the literature, it is standard to define amenity value of the forest as the non-timber benefits or functions derived from a standing forest (see, for instance, Scarpa *et al.*, 2000, Adamowicz *et al.*, 1996 and Sharma, 1992). The most recognized elements of the amenity value include climate, flood

⁶ Whiteman, "Economic Rent from Forest Operations in Suriname and a Proposal for Revising Suriname's Forest Revenue Sysytem" 1999, FAO, Rome.

⁷ Despite the studies that reveal the superiority of a mean-reverting process over the lognormal process (Lund, 1993; Hassett and Metcalf, 1995 and Schwartz, 1997), the use of the geometric Brownian motion is common in the literature (Brennan and Schwartz, 1985; McDonald and Siegel, 1986 and Dixit, 1989a).

control, recreational and hunting activities and biodiversity. Given the non-cash nature of some of the components of the amenity value, the amenity value of forests is usually assessed through the indices such as the naturalness index that reflect the amenities⁸. Scarpa *et al.* (2000) assess amenity value of the forests by subtracting the actual harvesting rate from the rate at which the forest would be cut had the owner of the forest maximized the timber revenues. Another common method used to measure amenity value is to monetarize the non-timber benefits by assessing how much people would be willing to pay for recreation (see, for instance, Willis, 1991; Kramer et al., 1992 and Elsasser, 1996). There are studies in the literature that take amenity as stochastic (see, for instance, Conrad, 1997; Forsyth, 2000 and Reed, 1993). Unforeseen ecological events such as the appearance of harmful insects or acid rains are likely to influence the value of the forest. Therefore, following Reed (1993), I will assume that the amenity value of the forest also follows a lognormal process:

$$dA = \alpha_A(A, t)Adt + \sigma_A(A, t)Adz_A$$
(3)

where α_A is the drift term, σ_A denotes the uncertainty and dz_A is the increment of a Wiener process.

The amenity value of the forest and the economic can be related. For instance, in response to an increase in the price of timber, the amenity value of the forest may decline since it would pay more to harvest and sell timber in the market. On

⁸ See, among others, Scarpa et al., 2000 and Raunikar and Buongiorno, 2001.

the other hand, both the amenity value and the economic rent may increase together due to scarcity in resources. Hence, the correlation between the amenity value and the economic rent, denoted by ρ_{AR} , may be positive or negative. I will assume, for ease of exposition, that amenity and economic rent are independent; hence, $\rho_{AR} = 0$. However, the model can incorporate non-zero correlation.

Another important factor that the government must consider is the forest volume. In other words, the government must realize and take into account that forests are a renewable resource. Tree growth determines the volume of available timber for sale. The rate of growth is also important for the amenities of the forest. As the tree volume increases, the community is expected to benefit more from the amenities of the forest. The forest. The change in growth is assumed to be deterministic based on Kilkki and Vaisanen, (1969) and Clark, (1976).

$$dX = g(t)dt \tag{4}$$

where g(t) is a positive, concave function of time. Equation (4) reflects the fact that tree growth is slow and takes time.

The ability of the government to decide on privatization creates an additional value to the intrinsic value of the resource. The total asset value of the forest must reflect this additional value. Therefore, the total asset value of the forest, E(t), is the sum of the intrinsic value of the forest outlined above and the value of government's option to privatize the forest. The option to privatize can be

PUT OPTION	PRIVATIZATION
Stock Price (V)	Value of the forest
Exercise price (K)	Privatization price
Time to expiration (t)	Infinity (due to perpetual option)
Volatility of stock price (σ)	Volatility of the value of the forest
Dividend on stock	Social benefit/Amenity value

Table 1. Correspondence between a Put Option and Privatization

regarded as a perpetual American put option. More specifically, the government holds the right to sell (privatize) the asset (the forest, whose value is V) at any time at a predetermined price, K, the exercise price. The correspondence between the privatization variables and put options is illustrated in Table 1. The total value of the forest can now be written as follows:

$$E(t) = V(R, A, X, t) + P(V, K, \sigma)$$
(5)

where E(t) denotes the total asset value of the forest over time, t, R is the implicit rent to be defined later, A stands for the amenity value of the forest, P denotes the value of the perpetual American put option and σ is the uncertainty coefficient. Other variables are V and K. The government is assumed to incur a certain maintenance cost, C, so long as it owns the forest. This paper assumes that K, the

privatization price, is the present value of all future costs; that is, $K = \frac{\sum_{i} C_{i}}{i}$, where *i* is the appropriate discount rate. Note that this formulation has a clear

analogy to capital-investment relationship. In particular, the privatization price is taken to be a stock variable whose value derives from the capitalized future cost, the flow variable. It is assumed that the government determines the privatization price. There is growing literature on the determination of privatization prices (see, for instance, Lopez-de-Silanes, 1997 and Aggarwal and Harper, 2000). However, one is also interested in knowing when the government should start the privatization program given the factors such as the amenity value and the potential economic rent. Assuming that there is no information asymmetry in the economy, the privatization price is known to private firms. In other words, the privatization price, K, is the reserve price of the government. The government would not accept a bid price below K. Note that the results do not change if the actual bid is higher than K. It is nevertheless possible to extend the model by relaxing the assumption of fixed K and introducing a quality variable such that the privatization price increases as the quality of timber ameliorates.

It is assumed that the government does not seek to profit from harvesting and the sale of timber. Its goal is rather to provide social benefit reflected in the amenity value of the forest. However, the government also takes into consideration the potential rent that might be extracted from the forest. If the amenity value is such that the economic rent becomes relatively significant then the government considers the privatization plan which also translates into revenue for itself. In brief, the government seeks to maximize the value of the asset it owns and attempts to solve the following optimization problem:

$$\max E\left\{ \int_{0}^{\infty} [V(t) + P(V, K, \sigma)] dt \right\}$$

subject to
$$dR = \alpha_{R}(R, t)Rdt + \sigma_{R}(R, t)Rdz_{R}$$

$$dA = \alpha_{A}(A, t)Adt + \sigma_{A}(A, t)Adz_{A}$$

$$dX = g(t)dt$$

(6)

where E designates the expectation operator.

In (6), the government maximizes the total value of the forest subject to the constraints on the control variables. The variables in the constraints are assumed to be uncorrelated.

Next, the motion that governs the stochastic value of the forest, *V*, must be specified. As argued above, the value of the forest is a function of economic rent, amenity, growth and time:

$$V = V(R, A, X, t)$$
⁽⁷⁾

The motion for the value of the forest can be obtained by expanding equation (7) using Ito's lemma.

$$dV = V_R dR + V_A dA + V_X dX + V_t dt + \frac{1}{2} V_{RR} (dR)^2 + \frac{1}{2} V_{AA} (dA)^2 + V_{RA} (dR) (dA)$$

Substituting for dR, dA and dX, we get

$$dV = V_R \left[\alpha_R R dt + \sigma_R R dz_R \right] + V_A \left[\alpha_A A dt + \sigma_A A dz_A \right] + g(t) V_X dt + V_t dt + \frac{1}{2} \sigma_R^2 R^2 V_{RR} dt + \frac{1}{2} \sigma_A^2 A^2 V_{AA} dt + \rho_{RA} \sigma_R \sigma_A R A V_{RA} dt$$

Rearranging the above,

$$dV = \left[\alpha_R R V_R + \alpha_A A V_A + g(t) V_X + V_t + \frac{1}{2}\sigma_R^2 R^2 V_{RR} + \frac{1}{2}\sigma_A^2 A^2 V_{AA}\right] dt + \sigma_R R V_R dz_R$$
$$+ \sigma_A A V_A dz_A$$

$$dV = \alpha_V V dt + \sigma_{VR} V dz_R + \sigma_{VA} V dz_A$$
(8)
where

$$\alpha_{V} = \frac{1}{V} \left[\alpha_{R} R V_{R} + \alpha_{A} A V_{A} + g(t) V_{X} + V_{t} + \frac{1}{2} \sigma_{R}^{2} R^{2} V_{RR} + \frac{1}{2} \sigma_{A}^{2} A^{2} V_{AA} \right]$$
(9)

$$\sigma_{VR} = \frac{1}{V} \left[\sigma_R R V_R \right] \tag{10}$$

and

$$\sigma_{VA} = \frac{1}{V} \left[\sigma_A A V_A \right] \tag{11}$$

Equation (8) shows that the change in the value of the forest is also stochastic. The uncertainty in the value of the forest stems from both amenity and economic rent. In equation (10), α_V is the expected rate of change in the value of the forest. σ_{VR} and σ_{VA} capture the uncertainty in the value of the forest related to economic rent and amenity, respectively. Equation (8) is fundamental to the solution of the problem. Next, I discuss the solution of the problem in the deterministic case.

3.2. The Deterministic Case

I start with the simple case where there is no uncertainty affecting the variables of the model. Although option valuation depends on the volatility of the underlying asset, the analysis in this section is useful for two reasons. First, it demonstrates that the flexibility regarding the timing of privatization has a value, which is ignored in the conventional NPV analysis. Second, it allows one to assess the role of uncertainty in the economy once uncertainty is introduced into the model.

When there is no uncertainty in the economy, the problem simplifies to a first order ordinary differential equation from which I obtain the value of option to wait. Since there is no uncertainty and hence $\sigma_{VR} = \sigma_{VA} = 0$, equation (8) reduces to:

$$dV = \alpha_v V dt \tag{12}$$

Integrating (12), I obtain the function for V(t):

$$V(t) = V_0 e_v^{\alpha t}$$
where $V(0) = V_0$
(13)

When the intrinsic value of the forest is given by equation (13), the value of the option at the privatization date can be written as:

$$P(V) = \max[(K - V_T)e^{-rT}, 0]$$
(14)

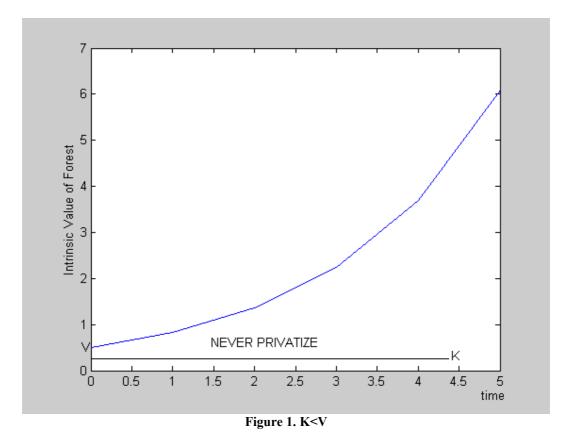
where r is the risk-free rate. Note that the appropriate discount rate is the risk-free rate since the government is assumed to be risk-neutral.

Equation (14) can also be seen as the payoff to the government of privatization at *T*. If the government privatizes the forest, it receives the privatization price *K* in return for the forest that has a time *T* value of V_T . If the government does not privatize, it does not realize the payoff.

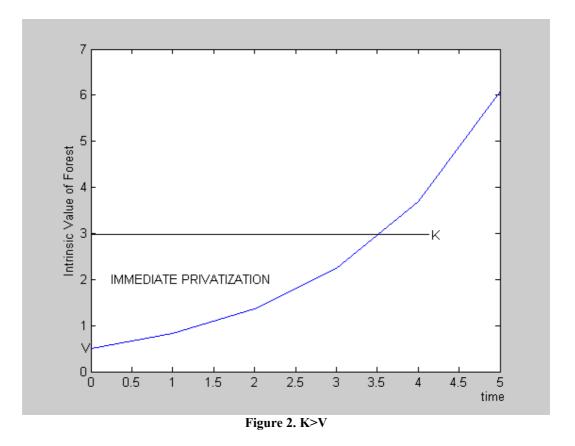
Substituting equation (13) into equation (14), we end up with equation (15):

$$P(V) = (K - Ve_v^{\alpha T})e^{-rT}$$
(15)

Equation (15) is the value today of privatization at some arbitrary time T given the current value of the forest, V. A closer inspection of equation (15) allows one to assess whether the government would privatize the forest today or would rather wait before privatization given the option to wait. In other words, there may be a value to waiting. The decision when to privatize depends on the sign of α_{ν} , the expected growth rate of the value of the forest.



Assume first that $\alpha_v > 0$. This implies that the value of the forest is an increasing function of time. The value of the forest can increase over time due to a rise in timber prices and/or an increase in the amenities of forest. The decision to privatize depends on the initial values of the privatization price, *K*, and *V*. If *K*<*V* initially, the government never privatizes the forest since the payoff from privatization is negative. Figure 1 illustrates this case. When *K*>*V* initially, it does not pay to postpone privatization since *V*(*t*) is an increasing function of time. In other words, the payoff from privatization decreases if the government waits. Figure 2 illustrates the case where the privatization price is greater than the value of the forest initially. In brief, when the value of the forest is expected to increase



over time, the government either privatizes immediately or never privatizes the forest. The payoff from privatization is hence $P(V) = \max [K-V,0]$.

Suppose now that $\alpha_v < 0$ implying that the expected change in the value of the forest in equation (8) is negative. This, in turn, implies that the value of the forest, V(t), is a decreasing function of time. The value of the forest can decline over time due to a decrease in timber prices or amenities of the forest. For instance, a road construction or the development of an industry near a forest may result in the cutting of trees and hence reduce the forest's amenity value. Even if K < V initially, P(V) > 0 since value of the forest eventually falls below the privatization price. To find the optimal time to privatize, the government maximizes the value

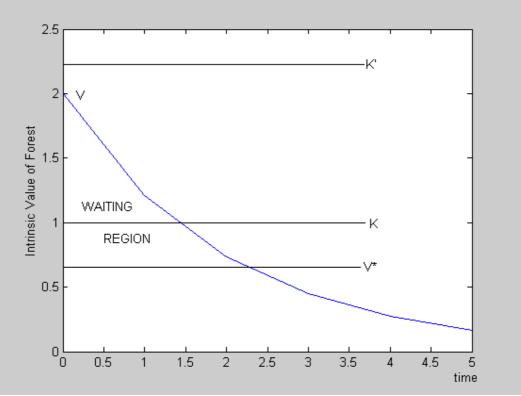


Figure 3. Privatization when the Forest Value Declines

of the put in equation (15) with respect to T. The first-order condition with respect to T is

$$\frac{dP(V)}{dT} = -rKe^{-rT} + (r - \alpha_V)Ve^{-(r - \alpha_V)T} = 0$$
(16)

Solving equation (16) yields⁹

$$T^* = \max\left\{\frac{-1}{\alpha_V} \ln \frac{(r - \alpha_V)V}{rK}, 0\right\}$$
(17)

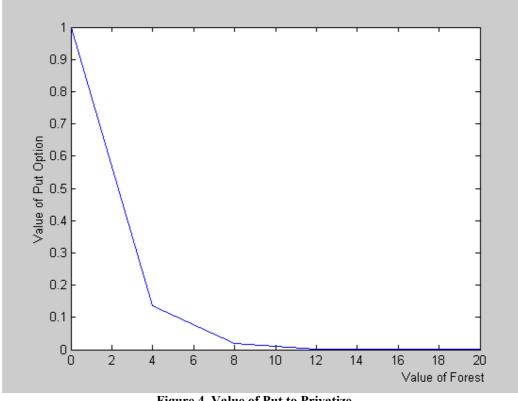


Figure 4. Value of Put to Privatize

In equation (17), if the privatization price, K, is not too much larger than the value of the forest, V, the government does not privatize immediately. In other words, $T^*>0$. This is shown in Figure 3. To find the critical value of forest at which the government privatizes the forest, we only need to set $T^* = 0$. Doing so yields

$$V^* = \frac{rK}{r - \alpha_V} < K \tag{18}$$

With the critical value of the forest, V^* , in hand, we only need to substitute the expression for T^* into equation (15) to obtain the value of option to privatize.

⁹ The second order condition is verified in Appendix A.

	K>V	K <v< th=""></v<>
α _V <0	Wait to Privatize	Wait to Privatize
α _V >0	Immediate Privatization	No Privatization

Table 2. Optimal Policies for Privatization of Forests

$$P(V) = \begin{cases} \left[\frac{(r-\alpha_{V})V}{rK}\right]^{r/\alpha_{V}} \left[\frac{-\alpha_{V}K}{r-\alpha_{V}}\right] \dots for \dots V \ge V * \\ K-V \dots for \dots V < V * \end{cases}$$
(19)

Equation (19) shows that the conventional NPV analysis does neglect the opportunity cost associated with the postponement of privatization. The government privatizes the forest when the value of the forest declines below the critical value, V^* . If, on the other hand, the value of the forest is greater than the critical value, it pays to delay privatization. In other words, the flexibility regarding the privatization has a positive value. Figure 4 shows the value of the option to privatize. As expected, the value of option to privatize is similar to a financial put option. The value of the option to privatize declines steadily as the value of the forest increases. When the value of the put reaches zero, it no longer pays to hold the option and the government privatizes. Table 2 summarizes the policy implications of the privatization program. In short, when the value of the forest is decreasing over time, the government waits until the payoff from privatization is maximized. When the value of the forest increases over time, the yalues of *K* and *V*.

3.3. The Stochastic Case

Next, I analyze the value of a put option when the value of the assets (i.e. the value of the forest) changes randomly through contingent claims analysis (CCA). I assume a nonzero variance term in equation (8), i.e. $\sigma_{VR} > 0$ and $\sigma_{VA} > 0$ throughout this section.

Before going into the details of the contingent claims analysis, it should be noted that the contingent claims analysis rests on the assumption of complete markets. Existence of complete markets implies that there are *pure securities* (also known as the Arrow-Debreu securities) in the market that pay \$1 in some state and nothing in the other states of the nature. In other words, existence of complete markets ensures that the security space is *spanned* by the pure securities. Spanning is a crucial assumption because it implies that either a single asset or a portfolio of assets in the market can replicate the risk and return characteristics of the value of the forest. In other words, spanning implies that the value of the hypothetical asset or portfolio is perfectly correlated with the value of the forest and that V can be regarded as the price of the asset (portfolio) in the market. Hence, the behavior of the value process of the forest is assumed to be similar to the behavior of stocks in financial markets. This is not an excessively restrictive assumption as it might at first seem. The assumption is realistic for commodities that are actually traded in the market such as oil, copper and timber. Furthermore, one need not search for a specific group of assets that would mimic the risk and return characteristics. It suffices that the value of the forest is replicated by a portfolio of assets (regardless of specific stocks) with a selffinancing strategy. Since markets are complete, one can obtain any risk and return characteristics with the pure securities. If markets are not complete so that such a replicating portfolio cannot be formed, the model can be solved through dynamic programming. The results, however, do not change if a solution is obtained through dynamic programming rather than contingent claims analysis. The advantage of using contingent claims analysis over dynamic programming is that it avoids the identification of a subjective discount rate that reflects the risk aversion of the decision-maker¹⁰.

Suppose that *Y* is a portfolio whose value is perfectly correlated with the value of the forest, *V*. According to the capital asset pricing model (CAPM), the return on the hypothetical portfolio is:

$$\mu = \mathbf{r} + \Phi \rho_{\rm YM} \sigma \tag{20}$$

where μ is the risk-adjusted return on the hypothetical portfolio, Φ is the market price of risk, ρ_{YM} is the correlation of the portfolio Y with the market portfolio and r is the exogenous risk-free discount rate. Note that since portfolio Y is perfectly correlated with the value of the forest, V, $\rho_{YM} = \rho_{VM}$. One crucial assumption to make at this point is that the expected change in the value of the forest, α_V , is

¹⁰ See Dixit and Pindyck (1994) for the comparison of dynamic programming and CCA.

equal to the risk-adjusted return, μ . To see why the equality should hold, consider the case where $\mu = \alpha_v + \delta$ without loss of generality. The latter equation implies that the risk-adjusted return on the stock is equal to the sum of the expected change in the value of the asset, α_v , and an indirect return, δ . δ can be thought of as the convenience yield on a real asset (e.g. a tree that grows more wood) or a dividend on the underlying stock. If δ were nonzero, the stock price would be expected to fall by the dividend amount. The holder of the put option would not exercise early because there is a positive probability that the option will go deeper in the money. In other words, probability of early exercise of an American put on a dividend paying stock decreases. Such a case implies that the government would hold on to its option to privatize rather than exercising it.

With these assumptions, it is now possible to obtain solution a using the no arbitrage condition. Following Dixit and Pindyck (1994), I first form a riskless portfolio composed of the option to privatize and an asset whose value is perfectly correlated with the value of the forest. Since the option to privatize is similar to a financial put option, the portfolio should hold the option to privatize and P_V units of the asset. Note that this is much like a portfolio held in the financial markets: holding a stock which provides positive return if the stock price increases and holding a put option on the same stock which yields a positive return if the price of the underlying asset falls. The total value of such a portfolio is then $\pi = P(V) + VP_V$.

The total return on this portfolio is influenced by two factors: first, the change in the value of the option to privatize, P(V) and second, the change in the value of the "stock" price, V. Hence the change in total return on the portfolio is:

$$d\pi = dP + P_V dV \tag{21}$$

Expanding dP using Ito's lemma and substituting in (21), we obtain:

$$d\pi = 2P_{V} \left[\alpha_{V} V dt + \sigma_{VR} V dz_{R} + \sigma_{VA} V dz_{A} \right] + \frac{1}{2} P_{VV} \left[\sigma_{VR}^{2} V^{2} + \sigma_{VA}^{2} V^{2} \right] dt$$
$$d\pi = \left[2\alpha_{V} V P_{V} + \frac{1}{2} \sigma_{VR}^{2} V^{2} P_{VV} + \frac{1}{2} \sigma_{VA}^{2} V^{2} P_{VV} \right] dt + 2\sigma_{VR} V P_{V} dz_{R} + 2\sigma_{VA} V P_{V} dz_{A} \quad (22)$$

Taking the expectation of equation (22) yields:

$$E(d\pi) = \left[2\alpha_{V}VP_{V} + \frac{1}{2}\sigma_{VR}^{2}V^{2}P_{VV} + \frac{1}{2}\sigma_{VA}^{2}V^{2}P_{VV}\right]dt$$
(23)

No arbitrage condition states that the expected change in the return on the portfolio should equal the riskless interest rate; that is, $E(d\pi) = r$. Then, the return on the portfolio whose value is $P(V) - VP_V$ should be equal to the riskless return. Hence, we have:

$$r[P - VP_V]dt = \left[2\alpha_V VP_V + \frac{1}{2}\sigma_{VR}^2 V^2 P_{VV} + \frac{1}{2}\sigma_{VA}^2 V^2 P_{VV}\right]dt$$
(24)

In equation (24), the left hand side is the riskless return on the hypothetical portfolio and the right hand side is the expected return on the portfolio. Rearranging and dividing both sides by dt, we end up with the following second order Cauchy differential equation:

$$\frac{1}{2}(\sigma_{VR}^2 + \sigma_{VA}^2)V^2 P_{VV} + (2\alpha_V + r)V P_V - rP = 0$$
(25)

As I have argued in Introduction, the government's problem can be seen as an *optimal stopping* problem. That is, at each point in time, the government assesses whether to maintain public ownership (i.e. continue to delay privatization) or to auction the forest (i.e. stop and realize the termination payoff K-V). Hence, the solution of equation (25) indicates the optimal stopping region for the government. However, boundary conditions must be specified to solve equation (25). The first boundary condition that can be imposed follows from the option theory:

$$\lim_{V \to \infty} P(V) = 0 \tag{26}$$

Boundary condition (26) states that as the value of the forest increases, the option value to wait tends to zero. Equation (26) has a clear analogy to stock options. One would expect the value of a put option to decrease as the price of the underlying stock tends to infinity.

However, equation (26) alone is not sufficient to solve the second order differential in equation (25). Equation (25) holds for the region where $V > V^*$. One is also interested in how the function behaves at the boundary $V=V^*$. We can impose continuity at the boundary:

$$P(V^*) = K - V^*$$
 (27)

Equation (27) is called the *value-matching* condition because it matches the value of $P(V^*)$ to the termination payoff. However, since the boundary V(t) is unknown, we need a third condition that specifies the behavior of the free boundary at V^* . Boundary condition (28), known as the *smooth-pasting* condition ensures the solution of the free boundary problem:

$$P_V(V^*) = -1$$
 (28)

Boundary condition (28) is called the smooth-pasting condition because it ensures that the slopes of the two functions coincide at the boundary.

To obtain an expression for P(V), equation (25) must be solved subject to the boundary conditions (26)-(28). Since there may be more than one function that solves equation (25), the easiest method is to conjecture a solution and then

substitute in the problem to see whether it works. One possible solution to the problem is given by:

$$P(V) = B_1 V^{\beta_1} + B_2 V^{\beta_2}$$
⁽²⁹⁾

where B_1 , B_2 , $\beta_1 > 0$ and $\beta_2 < 0$ are constants to be determined.

Boundary condition (26) implies that $B_1 = 0$ in equation (29). Hence we end up with:

$$P(V) = BV^{\beta} \tag{30}$$

where *B* and $\beta < 0$ are constants to be determined.

To obtain the threshold value V^* and B, one only needs to substitute equation (30) into value-matching and smooth-pasting conditions in equations (27) and (28). Solving these equations for V^* and B and substituting in P(V) yields:

$$V^* = \frac{\beta K}{\beta - 1} < K \tag{31}$$

$$B = -\frac{(\beta - 1)^{\beta - 1}}{\beta^{\beta} K^{\beta - 1}}$$
(32)

$$P(V) = \frac{-(\beta - 1)^{\beta - 1}}{\beta^{\beta} K^{\beta - 1}} V^{\beta}$$
(33)

where β is the negative root of a quadratic equation obtained by substituting equation (30) into equation (25):

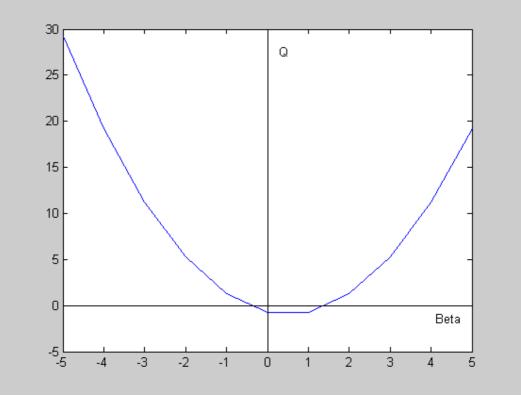


Figure 5. The Quadratic

$$Q = \frac{1}{2} (\sigma_{VR}^2 + \sigma_{VA}^2) \beta (\beta - 1) + (2\alpha_V + r) \beta - r = 0$$
(34)

When $\beta = 0$, Q = -r and when $\beta = 1$, $Q = 2 \alpha_v$. Hence, we are assured that β has one root greater than 0 and another less than 0. This is shown in Figure 5. Since β is negative by the boundary condition (26), β turns out to be:

$$\beta = \frac{1}{2} - \frac{(2\alpha_{V} + r)}{\sigma_{VR}^{2} + \sigma_{VA}^{2}} - \sqrt{\left(\frac{2\alpha_{V} + r}{\sigma_{VR}^{2} + \sigma_{VA}^{2}} - \frac{1}{2}\right)^{2} + \frac{2r}{\sigma_{VR}^{2} + \sigma_{VA}^{2}}}$$
(35)

Equation (31) gives the critical value of the forest at which government privatizes the forest. The government privatizes the forest when the threshold

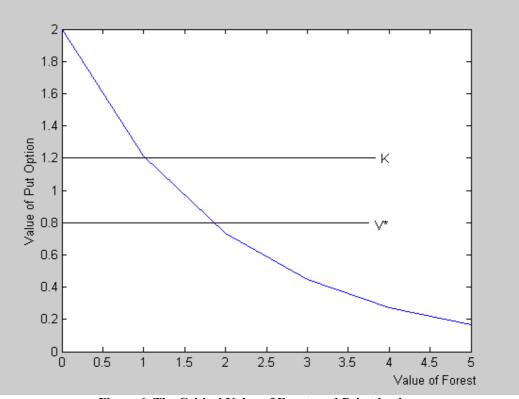


Figure 6. The Critical Value of Forest and Privatization

value, V^* , is hit for the first time from above. Figure 6 illustrates this point. Note that Figure 6 is similar to Figure 3 where the case for $\alpha_v < 0$ was explored. It is useful to compare the deterministic and stochastic cases and assess the role of uncertainty. Let V_d^* and V_s^* denote the critical values at which the government privatizes in the deterministic and stochastic cases, respectively. From equations (18) and (31), these values are

$$V_d^* = \frac{rK}{r - \alpha_V}$$
$$V_s^* = \frac{\beta K}{\beta - 1}$$

Of interest is the effect of uncertainty on the timing of privatization. Since V_d^* holds for $\alpha_v < 0$, assume for now that $\alpha_v < 0$. When $\alpha_v < 0$, $\frac{\beta}{\beta - 1} < \frac{r}{r - \alpha_v}$ implying that $V_s^* < V_d^*$. Hence, uncertainty causes the government to privatize at a later date than it would if there were no uncertainty in the economy.

A more formal way to assess the effect of uncertainty on privatization can be carried out through comparative statics. Note from equation (35) that β is a function of α_{v} , σ_{VR} , σ_{VA} and r. Without loss of generality, the analysis can be carried out on σ_{VR} . The same results apply to σ_{VA} . Taking the total differential of equation (34), we have

$$\frac{\partial Q}{\partial \beta} \frac{\partial \beta}{\partial \sigma_{_{VR}}} + \frac{\partial Q}{\partial \sigma_{_{VR}}} = 0 \tag{36}$$

where all derivatives are evaluated at $\beta < 0$ obtained in equation (35). Figure 6 shows that $\frac{\partial Q}{\partial \beta} < 0$ at β . In addition, differentiating Q with respect to σ_{VR} , we

have
$$\frac{\partial Q}{\partial \sigma_{VR}} = \sigma_{VR} \beta (\beta - 1) > 0$$
. Hence, $\frac{\partial \beta}{\partial \sigma_{VR}} > 0$. That is, as σ_{VR} increases, β

increases and $\beta/(\beta - 1)$ decreases since $\beta < 0$. This implies that the critical V^* at which the government privatizes the forest decreases, *ceteris paribus*. In other words, as uncertainty increases, the government delays privatization. This result is also in line with the observation that the government privatizes sooner when there is no uncertainty in the economy.

Similar comparative statics can be carried out to determine the effects of changes in α_v and *r* on privatization. Again, totally differentiating equation (34),

$$\frac{\partial Q}{\partial \beta} \frac{\partial \beta}{\partial \alpha_{\nu}} + \frac{\partial Q}{\partial \alpha_{\nu}} = 0$$
(37)

where $\frac{\partial Q}{\partial \alpha_V} = 2\beta \alpha_V$. Whether a change in α_V causes a higher or a lower critical

value, *V**, depends on the sign of α_v . If $\alpha_v < 0$, we have $\frac{\partial Q}{\partial \alpha_v} > 0$, which implies

that
$$\frac{\partial \beta}{\partial \alpha_V} > 0$$
. Hence, as α_v increases, β also increases, bringing down $\beta/(\beta - 1)$

and the critical value, V^* . This, in turn, causes the government to privatize later than it would otherwise, *ceteris paribus*. The case where $\alpha_v > 0$ produces the opposite effect. When $\alpha_v > 0$, α_v and V^* move in the same direction. Hence an increase in the expected rate of change of the forest implies that the government privatizes the forest sooner. This is also consistent with the results of the deterministic case where a positive expected rate of change implied immediate privatization of the forest provided that K > V initially.

The same analysis applies to capture the effect of interest rate on the critical value, V^* . Total differentiation yields,

$$\frac{\partial Q}{\partial \beta} \frac{\partial \beta}{\partial r} + \frac{\partial Q}{\partial r} = 0 \tag{38}$$

where all derivatives are again evaluated at β . Since the risk-free rate is assumed to be positive, we have $\frac{\partial Q}{\partial r} = \beta r - 1 < 0$. This implies that $\frac{\partial \beta}{\partial r} < 0$. Hence, an increase in the risk-free rate decreases β and increases $\beta/(\beta - 1)$ and the critical V^* . When V^* is higher, the government privatizes the forest sooner than it otherwise would.

As we have seen, certain parameters of the model must also be taken into account in the government's decision to privatize. The comparative statics analysis has implications for government policies regarding the privatization of forests. For instance, if uncertainty in the economy increases, the government delays privatization since the value of the forest is more volatile. The expected rate of change in the value of the forest is another important determinant of the privatization. The government should privatize those areas with a positive expected rate of change sooner than those areas that are expected to decrease in value. This is because when the expected rate of change is positive, the privatization payoff decreases faster than the privatization payoff when the rate is negative.

CHAPTER 4: CONCLUSION

Privatization is an important phenomenon. It is now regarded as one of the most essential tools to promote economic efficiency. However, privatization of natural resources and forests in particular is likely to meet resistance from a number of interest groups. The most salient conflict regarding the privatization of forests is between environmentalists and private firms. This thesis proposes a model that attempts to determine the optimal use of forests from the perspective of the whole society rather than a single interest group. The model, therefore, enables the government to decide on the optimal use of forests taking into account both economic rent and amenity value.

As argued above, the government privatizes the forest when the value of the forest falls below a certain threshold value. The government realizes a return that is equal to the difference between the privatization price and the value of the forest. The private firm buys the forest in anticipation of future benefits such as the sale of timber and/or development of theme parks. The government can thus reduce the inefficiency that may be associated with the management of the forest while the private firm increases the value of the forest by managing it efficiently.

We have also seen that the decision to privatize depends on both macro-level parameters such as the amount of uncertainty in the economy as well as more forest-specific parameters such as the expected rate of change in the value of the forest. The uncertainty stems from both the prices of timber and the amenities that are derived from forests.

The model developed in the thesis can be applied to other SOEs such as Tüpraş or Telekom since the fundamental intuition remains the same—that privatization is a put option of the government. Naturally, one should adapt the model to the setting where government privatizes a technology-producing firm rather than a renewable resource. In such a case, the government would be interested in the stochastic nature of innovation. That is, the control variable is the rate of innovation and the value of the assets of the SOE is determined by the stochastic evolution of technology over time. This, in turn, implies that the value of the firm's assets is stochastic. However, the model must be extended to include jump diffusion processes since innovation can cause jumps in the value of the firm assets.

There are, nevertheless, several limitations of the model, which, at the same time, are avenues for future research. First, the model dwells only upon the ownership structure of the forest. That is, the fundamental question is whether forests should be publicly or privately held. However, it does not address the conflicts of interest among private stakeholders. Private stakeholders do not constitute a homogeneous group. For instance, even if the government privatizes the forest, conflicts of interest are likely to arise between forest dwellers who earn their livelihood from forests and private firms that seek to profit from harvesting.

In addition, the model presents privatization as the sole policy tool. The model also applies to leasing contracts. However, leasing contracts might include restrictive covenants related to the use forest. For instance, the government might insist that harvests be lower than regeneration or that a certain degree of amenity as measured by some naturalness index be attained. It would be interesting to introduce the restrictive covenants and compare privatization and leasing as distinct policy tools.

One fundamental motivation behind almost all privatizations is to increase the efficiency. As argued in Introduction, by privatizing forests, governments aim to reduce bureaucracy that may be associated with the management of forests. An extension of the model could explicitly recognize the role for efficiency. Specifically, the optimal time to privatize and the critical value of the forest at which the government privatizes should reflect the potential gains in efficiency.

Finally, the model makes several assumptions that can be relaxed. For instance, the model assumes that growth of the forest is deterministic. This assumption can be relaxed since unforeseen ecological events such as the appearance of harmful insects or climactic changes are likely to alter the growth of the forest.

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APPENDIX A. SECOND ORDER CONDITION

To prove that the expression we obtained in equation (18) for T^* indeed maximizes the value of the put option, we need to check whether the second order condition is satisfied at T^* . The second order condition with respect to T is given by:

$$\frac{d^2 P(V)}{dT^2} = r^2 K e^{-rT} - (r - \alpha_V)^2 V e^{-(r - \alpha_V)T}$$
(A.1)

Recall that the optimal time to privatize is given by:

$$T^* = \max\left\{\frac{-1}{\alpha_V}\ln\frac{(r-\alpha_V)V}{rK}, 0\right\}$$
(A.2)

Substituting (A.2) into (A.1), we get

$$\frac{d^{2}P(V)}{dT^{2}} = r^{2}K \exp\left\{-r\left[\frac{-1}{\alpha_{V}}\ln\frac{(r-\alpha_{V})V}{rK}\right]\right\} - (r-\alpha_{V})^{2}V \exp\left\{-(r-\alpha_{V}\left[\frac{-1}{\alpha_{V}}\ln\frac{(r-\alpha_{V})V}{rK}\right]\right\}$$

$$\frac{d^{2}P(V)}{dT^{2}} = r^{2}K\left[\frac{(r-\alpha_{V})V}{rK}\right]^{r/\alpha_{V}} - (r-\alpha_{V})^{2}V\left[\frac{(r-\alpha_{V})V}{rK}\right]^{r/\alpha_{V}} \frac{rK}{(r-\alpha_{V})V}$$

$$\frac{d^{2}P(V)}{dT^{2}} = \left[\frac{(r-\alpha_{V})V}{rK}\right]^{r/\alpha_{V}} \left[r^{2}K - \frac{(r-\alpha_{V})^{2}VrK}{(r-\alpha_{V})V}\right]$$

$$\frac{d^{2}P(V)}{dT^{2}} = \left[\frac{(r-\alpha_{V})V}{rK}\right]^{r/\alpha_{V}} rK\alpha_{V} < 0$$
(A.3)

Equation (A.3) is negative since $\alpha_v < 0$. Hence, T^* maximizes P(V).

APPENDIX B. THE SOLUTION IN THE STOCHASTIC CASE

To check that the function P(V) we have come up with is indeed the solution to the differential equation (27) subject to the boundary conditions (28)-(30), I substitute the solution in (35) in equations (27)-(30). To reiterate, equation (35) stated that the value of the put to privatize is given by:

$$P(V) = \frac{-(\beta - 1)^{\beta - 1}}{\beta^{\beta} K^{\beta - 1}} V^{\beta}$$
(B.1)

The first boundary condition states that as the value of the forest increases, the option value of waiting decreases. That is,

$$\lim_{V \to \infty} P(V) = 0 \tag{B.2}$$

$$\lim_{V \to \infty} \frac{-(\beta - 1)^{\beta - 1}}{\beta^{\beta} K^{\beta - 1}} V^{\beta} = 0$$
(B.3)

Since $\beta < 0$, the first boundary condition is satisfied.

The second boundary condition is the value-matching condition which imposed continuity at the boundary:

$$P(V^*) = K - V^*$$
 (B.4)

Substituting for $P(V^*)$ and V^* in equation (B.4), we get

$$\frac{-(\beta-1)^{\beta-1}}{\beta^{\beta}K^{\beta-1}}\frac{\beta^{\beta}K^{\beta}}{(\beta-1)^{\beta}} = K - \frac{\beta K}{\beta-1}$$

$$\frac{-K}{\beta-1} = \frac{-K}{\beta-1}$$
(B.5)

From (B.5), it can be concluded that the value-matching condition is satisfied at the boundary.

Recall that the third boundary condition imposed that the slopes of the put function and the termination payoff coincide at the boundary:

$$P_V(V^*) = -1$$
 (B.6)

Substituting for $P_V(V^*)$, we end up with:

$$P_{V}(V^{*}) = \frac{-(\beta - 1)^{\beta - 1}}{\beta^{\beta - 1}K^{\beta - 1}} \frac{\beta^{\beta - 1}K^{\beta - 1}}{(\beta - 1)^{\beta - 1}} = -1$$
(B.7)

Equation (B.7) shows that the smooth-pasting condition is also satisfied by P(V).

Finally, we need to check whether P(V) satisfies the second order differential equation in (27). Recall that we had from equation (27):

$$\frac{1}{2}(\sigma_{VR}^{2} + \sigma_{VA}^{2})V^{2}P_{VV} + (2\alpha_{V} + r)VP_{V} - rP = 0$$
(B.8)

Substituting for P(V), we get

$$\frac{1}{2}(\sigma_{VR}^{2} + \sigma_{VA}^{2})V^{2}B\beta(\beta - 1)V^{\beta - 2} + (2\alpha_{V} + r)VB\beta V^{\beta - 1} - rBV^{\beta} = 0$$

$$\frac{1}{2}(\sigma_{VR}^{2} + \sigma_{VA}^{2})\beta(\beta - 1) + (2\alpha_{V} + r)\beta - r = 0$$
(B.9)

Note that equation (B.9) is the same as the quadratic equation (36) from which I obtained the root β . Therefore, equation (B.9) is satisfied by construction and P(V) satisfies the second order differential equation as well as the boundary conditions specified.