NETWORK DESIGN PROBLEMS IN WAVELENGTH DIVISION MULTIPLEXING OPTICAL NETWORKS

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By
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August 2001

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Master of Science.

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Abstract

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In this study, we analyze the network design problem arising in Wavelength Division Multiplexing (WDM) networks where traffic is static, wavelength interchanging is allowed and the location and number of the wavelength interchangers are to be determined. Given a topology and traffic data, we try to find the fiber and wavelength interchanger configuration with the minimum cost, that can establish all given connections. We present different formulations of the problem and some valid inequalities. Finally, we propose a heuristic method of generating feasible solutions, apply the method on three different topologies with varying traffic data, and present the results. The method is based on the idea of partitioning the problem into two; routing problem and wavelength assignment and interchanger location problem. Our results prove to be close to the lower bounds we generate, and indicate that the fiber cost performance of the case where all nodes are wavelength interchangers can be attained using a relatively small number of wavelength interchangers.

Keywords: Wavelength Division Multiplexing Networks, Capacitated Network Design, Multicommodity Network Flows, Routing and Wavelength Assignment

Özet

DALGABOYU BÖLÜŞÜMLÜ ÇOĞULLAMA KULLANILAN OPTİK İLETİŞİM AĞLARINDA AĞ TASARLAMA PROBLEMLERİ

Güneş Erdoğan
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Bu çalışmada, Dalga Bölüşümlü Çoğullama kullanılan ağlarda, trafiğin durağan olduğu, dalgaboyu dönüşümüne izin verildiği ve dalgaboyu değiştiricilerin sayısının ve yerlerinin belirlenmesinin sözkonusu olduğu ağ tasarım problemlerini inceledik. Ağ yapısı ve trafik bilgisi verildiği halde, en az maliyete sahip olan ve verilen baglantıları sağlayabilecek bir ağ tasarlamaya calıştık. Problemi ifade eden değişik formülasyonlar ve bazı geçerli eşitsizlikler sunduk. Sonuç olarak, olurlu çözümler üretmek icin bulgusal bir yöntem önerdik, yöntemi farklı trafik bilgileri ile üç farklı ağ yapısında uyguladık, ve sonuçları sunduk. Yöntem problemi iki probleme ayırmak fikri üzerine kuruludur: yol atama problemi ve dalgaboyu atama ve dalgaboyu değiştirici yeri saptama problemi. Sonuçlarımız, ürettiğimiz alt sınırlara yakındır, ve göstermektedir ki bütün düğümlerin dalgaboyu değiştirici olduğu durumdaki fiber maliyeti performansına, az miktarda dalgaboyu değiştiricisi kullanılarak da ulaşılabilir.

Anahtar sözcükler: Dalga Bölüşümlü Çoğullama Kullanılan Ağlar, Kapasiteli Ağ Tasarımı, Çok Ürünlü Ağ Akışı, Yol ve Dalgaboyu Atama

To my father and mother

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

Computer networking has been an important area of research for a long time. With the tremendous growth of the Internet, speed and capacity requirements for computer networks have increased considerably. Existing network technologies did not seem to satisfy this huge requirement. This was when all-optical networks came into the picture. All-optical networks offered higher speed, better reliability and more capacity than conventional networks. All-optical networks are networks where information is converted to light, transmitted as light, and reaches its final destination directly without being converted to electronic form in between. This method of transmission of messages is superior to the previous methods. All-optical networks promise data transmission rates several orders of magnitudes higher than the current networks. The key to high speeds in these networks is to maintain the signal in optical form so as to get rid of the conversion time from optical form to electronic form and vice versa. All-optical networks are considered as the transport networks of the future. The major applications for such networks are in video conferencing, scientific visualization,real-time medical imaging, high-speed super-computing and distributed computing [6],[13],[15]. To solve the capacity problem, Wavelength Division Multiplexing was developed. The most popular approach to utilize the high-capacity of all-optical networks is to divide optical spectrum into many different channels, each channel corresponding to a different wavelength.

This approach, called Wavelength-Division Multiplexing (WDM) allows multiple data streams to be transferred concurrently along the same fiber-optic cable, with different streams assigned separate wavelengths [13]. Although WDM increases the capacity of all-optical networks, it also increases the complexity of network management. Once a message is assigned a wavelength at its source node, this assignment cannot be changed at subsequent nodes. Networks which only encountered *capacity blocking* until now, are subject to a new type of blocking called *wavelength blocking*. In the former, a message cannot be delivered to its destination because all paths to destination are blocked by links that are used by other messages. In the latter, a message cannot be delivered to its destination because even if there exists a path to the destination, no wavelength that is unused on all links along the path can be found. To overcome this problem, devices that can change the wavelength assignment of a connection are used. These devices are referred as wavelength interchangers in this study. Much research has been done to investigate the effects of wavelength interchangers on routing, number of wavelengths required, blocking probability, throughput etc. Most of the approaches considered either no wavelength interchanging, called Wavelength Path Scheme (WP); or wavelength interchanging capability at each node, called Virtual Wavelength Path Scheme (VWP). The problem of determining the route and wavelength assignment of each connection in a WDM network is known as the Routing and Wavelength Assignment (RWA) problem. RWA problems have two main categories, *static* and *dynamic*. In the former, all connections are known a priori, whereas in the latter connection requests arrive randomly.

WDM networks received considerable interest from researchers. Raghavan and Upfal (1994) studied routing a set of requests (each of which is a pair of nodes to be connected by a path) using a limited number of wavelengths ensuring that different paths using the same wavelength never use the same physical link. They presented routing techniques and established connections between the expansion of a network and the number of wavelengths required for routing on it [6].

Ramaswami and Sivarajan (1995) studied maximizing the amount of dynamic traffic carried when there is a single fiber on each link and wavelength conversion is not allowed. They presented an IP formulation and proved upper bounds for both the IP and the LP that corresponds to its relaxation. They claimed that the upper bounds could be used as a metric to evaluate the performance of different RWA problems. Results of their experimentation showed that if all nodes have the capability of interchanging wavelength assignment, use of total capacity of the wavelength division multiplexing can be improved by 10-40% [9].

Wauters and Demeester (1996) presented formulations for maximizing the carried traffic for two cases; when wavelength interchanging is not allowed, and when every node is a wavelength interchanger. They used two kinds of formulations, namely *flow* and *path* formulations. While their flow formulations were quite close to the usual network formulations, path formulations were based on enumerating possible paths between source and destination pairs and choosing one among them. Finally, they presented an iterative heuristic RWA algorithm to minimize the number of wavelengths required to successfully deliver each message to its destination. The algorithm was based on performing local search on an initial routing and wavelength assignment. At each iteration, the path with the largest wavelength number (or all the paths that interfered with it) was tried to be rerouted on a smaller wavelength number. Results of their experimentation suggested that wavelength conversion did not make a significant reduction in the number of wavelengths required, and wavelength interchanging was not necessary at every node. They also concluded that wavelength interchanging capability at some specific nodes may be enough to overcome wavelength blocking [10].

Nagatsu, Okamoto and Sato (1996) proposed algorithms for RWA problem in a multi-fiber environment (more than one fiber can exist between two nodes) for both WP and WVP schemes. Their algorithm for the VWP scheme was aimed at minimizing the fiber requirement, whereas their algorithm for the WP scheme was aimed at minimizing the number of wavelengths required. They also proposed algorithms for failure restoration in VWP and WP schemes, in which they considered single-link-failures. They concluded that the difference between VWP and WP schemes increased as the number of wavelengths increased [11].

Banerjee and Mukherjee (1996) studied RWA for static and dynamic traffic in single-fiber WDM networks. They partitioned the RWA problem into two stages, first routing and then wavelength assignment. First problem was the well known multicommodity flow problem. They managed to obtain results close to the LP lower bound for the multicommodity flow problem, using a heuristic named "Randomized Rounding", which uses the solution of the LP relaxation to construct a feasible solution. For the wavelength assignment stage, they converted the problem into the graph coloring problem using the routing they obtained, and used *smallest-last* algorithm to minimize the number of wavelengths used. The algorithm basically starts by coloring the nodes with the maximum degree, and continues with coloring the smaller degree nodes. Their results were close to the LP lower bounds [12].

Bermond et al. (1996) presented upper and lower bounds for the number of wavelengths required to "gossip" (one-to-all communication) and "broadcast" (all-to-all communication) when each link has only one fiber, and wavelength interchanging is not possible, in networks with arbitrary topologies and particular networks of interest such as ring, torus, hypercube [13].

Armitage, Crochat and Le Boudec (1996) presented a tabu search algorithm for the WP scheme, namely Disjoint Alternate Path (DAP), that finds a routing minimizing the number of broken connections in case of a single-link failure [14].

Flammini and Scheideler (1997) studied routing a set of "dynamic" requests with a limited number of wavelengths, single fiber on each link, and without wavelength conversion. They suggested a protocol for routing, and applied their results to different topologies [15].

Qiao, Mei, Yoo and Zhang (1998) suggested slicing an optical network into several *Virtual Optical Networks* (VONs) and equipping each VON according to its traffic structure. They concluded that VONs supporting dynamic traffic require a small number of wavelengths and use of wavelength interchangers, but VONs supporting static traffic require a larger number of wavelengths and no wavelength interchangers [16].

Ramamurthy and Mukherjee (1998) presented a review/survey of the underlying technologies of WDM, WDM network design methods and analytical models used in wavelength-interchangeable networks. One of the questions they posed was: "An interesting question which has not been answered thoroughly is where (optimally) to place these few converters..." [20]. One of the outcomes of this thesis work is an algorithm to answer this question.

Zhang and Qiao (1998) studied wavelength assignment for "dynamic" traffic in multi-fiber WDM networks and presented an algorithm, namely Relative Capacity Loss algorithm, to minimize the probability of blocking. They claimed that wavelength blocking due to lack of wavelength interchangers can be dealt with using backup paths for rerouting and their intelligent wavelength assignment algorithm [21].

Alanyali and Ayanoglu (1998) presented two heuristics for routing and wavelength assignment of a set of static connection requests in WP scheme. First heuristic was aimed at minimizing the total weighted fiber length and did not consider fault tolerance, while second heuristic was an adaptation of the first heuristic for the fault tolerant case and considered several failure scenarios [22].

Yuan et al. (1998) assessed benefits of wavelength conversion and claimed that wavelength conversion could result in an increase of throughput in a environment under distributed control [23].

Qiao and Mei (1999) studied the minimum number of wavelengths required per link for a given network to be rearrangeably non-blocking in WP and VWP schemes. They claimed that WP and VWP performed equivalently in linear array topologies, while VWP performed slightly better in rings, meshes, tori and hypercubes [25].

Yates, Rumsewich and Lacey (1999) presented a review of performance improvements offered by wavelength interchanging. They also discuss the effects of the topology, number of wavelengths, and RWA algorithms on the performance improvements of wavelength interchanging. They concluded that in most networks, wavelength interchanging capability does result in a moderate improvement in performance. On the other hand, when path lengths are large and interference lengths are small, wavelength interchangers can result in a considerable increase of performance. They also concluded that wavelength interchanger capability at a limited number of nodes usually performs equivalent to the case where every node is a wavelength interchanger [26].

Subramaniam, Azizoglu and Somani (1999) studied the problem of finding the optimal placement of a given number of wavelength interchangers in the network, when the offered traffic is dynamic. They presented a dynamic programming algorithm to find the optimal placement of interchangers on a path, when link loads are nonuniform. Their results showed the importance of wavelength conversion. Optimally placed 4 interchangers on a 11-node, 10-edge path resulted in a reduction of blocking probability by more than two orders of magnitude [27].

Xiao and Leung studied (1999) algorithms for allocating a fixed number of wavelength interchangers in all-optical networks. They presented three IP formulations corresponding to three different objective functions. First was to maximize sum of (utilization of node i * number of wavelength interchangers at node i) over all nodes. Second was to maximize the product of (utilization of node i * number of wavelength interchangers at node i) over all nodes. The last objective was to maximize the minimum of (utilization of node i * number of wavelength interchangers at node i) over all nodes. They used dynamic programming to solve first two problems, and a greedy algorithm to solve the third problem which was proven to find the optimal [28].

Park, Shin and Lee (1999) proposed algorithms for routing and minimum wavelength requirement when routing is known. For wavelength interchanger location they simply recommended to allocate them to nodes in descending order of number of paths passing through number of nodes, until feasibility is attained. Finally, they gave an optical fiber dimensioning algorithm to determine the number of fibers on each edge required for feasibility of flow [29].

Xiao, Leung and Hung (2001) proposed an algorithm, namely the Two-stage Cut Saturation Algorithm, for designing an all-optical network with minimum cost. They concluded that their algorithm performed fairly well and if wavelength interchanging is allowed on all nodes, total cost of links may be reduced about 20% [30].

In the literature, WDM network design problem has many different metrics such as throughput, blocking probability, number of wavelengths required, total fiber length used, reliability, control complexity, etc. To the best of our knowledge, minimum number of wavelength interchangers and their optimal location is a problem that is virtually untouched. A few studies focus on optimally placing a limited number of wavelength interchangers on a network, in order to minimize blocking probability, or minimize the total fiber cost, but with given routing data. Actually, in a hybrid network composed of wavelength interchanging and non-interchanging nodes, RWA problem becomes harder, because wavelength assignment of a transmission may or may not be changed according to its route and the location of the wavelength interchangers. The actual overall problem is to design a minimum cost network while solving the corresponding RWA problem simultaneously, given the traffic data and the network topology.

In this thesis, a detailed analysis of Wavelength Division Multiplexing network design problem with minimum total fiber and wavelength interchanger costs, is carried out. Before stating the problem, facts about the structure of the communication network we analyze will be presented. We are given a connected graph with n nodes and m edges. Each node transmits and/or receives data. Each connection is assigned a wavelength at its source node. Each node is either a Wavelength Interchanger Cross-Connect (WIXC) or a Wavelength Selective Cross-Connect (WSXC), where the former has the capability of changing the wavelength assigned to a connection expressing through the node, and the latter does not have this capability. The former has an undetermined cost, since it is not commercially available at the time this thesis is submitted. The latter has a cost, but it is out of consideration, because each node requires one to transmit and receive messages. At least one fiber should be installed on an edge if the edge will be used. Fibers are unidirectional and each fiber can accommodate only one message of each distinct wavelength. So if two messages are assigned the same wavelength and flow between the same two nodes, then at least two fibers should be installed on that edge. We assume that each link has a variable cost per fiber installed but no fixed cost of installation. Throughout all formulations in this thesis, it is assumed that capacity just enough to accommodate the flows is necessary and sufficient. Providing extra capacity for reliability is out of consideration.

The problem can be stated as follows: Given a particular topology and traffic data, determine the configuration of fibers to be installed, number of wavelength interchanger devices and their locations, routing and wavelength assignment of each connection at each link it uses, such that the resulting network has the minimum cost.

In Chapter 2, various IP formulations of the problem are presented. First formulation is a binary formulation where connections are represented as binary variables. Second formulation is another binary formulation where "interchanger" constraints are stronger. Final formulation in Chapter 2 is an aggregated model where connections are consolidated according to their source nodes. Aggregation greatly reduces the number of variables, but some valid cuts which exploit the binary structure of the problem cannot be added. Next, valid cuts proposed for the models are presented. Finally, problems about the formulations are discussed.

In Chapter 3, subproblems of the main problem are identified. The problems are, 1) the integer multicommodity flow problem with variable upper bounds, 2) wavelength assignment problem, and 3) interchanger location problem. Hardness of the subproblems are discussed and IP formulations for each of the subproblems are presented. A worst case cost behaviour expression is derived for shortest path routing. A method of generating strong lower bounds for our problem using the multicommodity flow problem with variable upper bounds is also presented.

In Chapter 4, a solution method is proposed. The method can be summarized as follows: First, the problem is relaxed into an integer multicommodity flow problem with variable upper bounds. Second, a feasible solution for the case without any wavelength interchangers is generated using the solution from the first stage. More specifically, in stage two, routing found during the first stage is fixed and the wavelength assignment (without wavelength interchangers) problem is solved with the specified routing. If the cost of second stage is greater than the cost of the first stage, a third problem of wavelength interchanger placement is solved to determine how many wavelength interchangers are required and where they should be placed. Next, the results of the proposed method are presented and analyzed. The results are obtained by applying the method to three different topologies with randomly generated traffic data and varying levels of traffic density. The results are compared with the lower bounds generated using the method described in Chapter 3. The fact that the fiber cost performance when each node is a wavelength interchanger can be attained by a relatively small number of wavelength interchangers, is our most important contribution to the literature. The location of these wavelength interchangers depend both on the topology and the traffic, but most of the time, they happen to be located in the 'middle' of the graph or at the 'crossroads'.

The last chapter is the summary of the thesis. Results are summarized and possible areas of further research are highlighted.

Chapter 2

Formulating the problem

To have a better understanding of the structure of the problem, a precise mathematical expression of the problem is required. Although they are not very useful in solving the problem itself, the formulations of the problem offer many insights about ways of solving the problem. Three IP formulations of the problem with different strong and weak points are presented in this chapter. Before moving on to the formulations, it is necessary to state the notation to be used.

2.1 Notation

Let $G = (N, E)$ be the graph corresponding to the network topology where N is the set of nodes, $N = \{1, ..., |N|\}$, and E is the set of edges, $E = \{1, ..., |E|\}$.

Let K be the set of connection demands, with cardinality $|K|$. Each element of the set is a tuple (s_k, d_k), where s_k denotes the source and d_k denotes the destination of demand k.

Let W be the set of wavelengths available, $W = \{1, 2, ..., |W|\}$.

Let x_{ijkw} be the binary variable representing the flow of connection k from node i to node j, with the wavelength assignment w. In other words,

$$x_{ijkw} = \begin{cases} 1 & \text{if demand } k \text{ flows from node } i \text{ to node } j \text{ with} \\ & \text{the wavelength assignment } w \\ 0 & \text{otherwise} \end{cases}$$

Let f_{ij} be the number of fibers to be installed between nodes i and j.

Let a_i be the binary variable representing the existence of a WIXC. In other words,

$$a_i = \begin{cases} 1 & \text{if there is a WIXC at node } i \\ 0 & \text{if there is a WSXC at node } i \end{cases}$$

Let HC be the difference between the cost of a WIXC and the cost of a WSXC. Let c_{ij} be the cost of installing one fiber between nodes i and j.

Let y_{ik} be the parameter for the demand/supply of the network flow. In other words,

$$y_{ik} = \begin{cases} 1 & \text{if node } i \text{ is the source node of demand } k \\ -1 & \text{if node } i \text{ is the destination node of demand } k \\ 0 & \text{otherwise} \end{cases}$$

2.2 Minimal binary formulation

Based on the definitions above, the formulation is as follows:

(IP1)

Min $\sum_{(i,j)\in E} c_{ij} f_{ij} + HC(\sum_{i\in N} a_i)$
s.t.

$\sum_{j\in N}\sum_{w\in W} x_{ijkw} - \sum_{j\in N}\sum_{w\in W} x_{jikw}$	$=$	y_{ik}	$\forall i \in N, k \in K$	NC
$\sum_{k\in K}(x_{ijkw} + x_{jikw})$	$\leq$	f_{ij}	$\forall (i,j) \in E, w \in W$	BC
$\sum_{j\in N}\sum_{w\in W} w * (x_{jikw} - x_{ijkw})$	$\leq$	$(\lvert W\rvert - 1)^* a_i$	$\forall i \in N, k \in K$	
			where $y_{ik} = 0$	IC1
$\sum_{j\in N}\sum_{w\in W} w * (x_{ijkw} - x_{jikw})$	$\leq$	$(\lvert W\rvert - 1)^* a_i$	$\forall i \in N, k \in K$	
			where $y_{ik} = 0$	IC1

$x_{ijkw} \in B$
$f_{ij} \in I$
$a_i \in B$

Number of variables: $(2 * |E| * |K| * |W|) + |E| + |N|$
Number of constraints: $(3 * |N| * |K|) + (|E| * |W|)$

In IP1, Network Constraints (NC) provide the conservation of flow. Bundle constraints (BC) make sure that enough number of fibers are deployed on an edge to accommodate the demand through that edge. Interchanger constraints (IC1 & IC2) ensure that if a message changes its wavelength at node i, then node i must be a WIXC. Suppose a connection arrives at a node i with wavelength assignment w^1 and continues to neighbouring node j with wavelength assignment w^2. This results in a difference between the wavelength assignment of inflow and outflow, and IC1 and IC2 forces a_i to be at least $\frac{|w^1 - w^2|}{|W|-1}$. Since a_i values are constrained to be binary, this means that $a_i = 1$. Strong point of formulation IP1 is that it states the problem with minimum number of constraints that the author could. Another advantage is that IC1 and IC2 are composed of binary variables with non-binary coefficients. This property may be used for generating cover cuts while using commercial IP optimization packages, such as CPLEX.

2.3 Stronger binary formulation

IP1 can be further strengthened with stronger interchanger constraints. Assume that a message comes to node i with wavelength assignment w and leaves with wavelength assignment $w+1$. In this case, interchanger constraints of IP1 will force the interchanger assignment variable a_i to be at least $\frac{1}{|W|-1}$. The stronger interchanger constraints proposed are:

$$
\begin{array}{llllr}
\sum_{j\in N} x_{jikw} - \sum_{j\in N} x_{ijkw} & \leq & \mathrm{a}_i & \forall i\in N, k\in K, w\in W, where\ y_{ik}=0 & \text{IC'1} \\
\sum_{j\in N} x_{ijkw} - \sum_{j\in N} x_{jikw} & \leq & \mathrm{a}_i & \forall i\in N, k\in K, w\in W, where\ y_{ik}=0 & \text{IC'2}
\end{array}
$$

If this new set of interchanger constraints are used, the interchanger assignment variable a_i will be forced to be 1, which shows that this set of interchanger constraints are stronger. The price of strength is the increased number of constraints. Also, efficient cover cuts cannot be generated for this formulation because the coefficient matrix is composed of 0's and 1's. Second formulation is as follows:

(IP2)

Min $\sum_{(i,j)\in E} c_{ij} f_{ij} + HC(\sum_{i\in N} a_i)$
s.t.

$$
\begin{array}{llllr}
\sum_{j\in N}\sum_{w\in W} x_{ijkw} - \sum_{j\in N}\sum_{w\in W} x_{jikw} & = & \mathrm{y}_{ik} & \forall i\in N, k\in K & \text{NC} \\
\sum_{k\in K}(x_{ijkw} + x_{jikw}) & \leq & \mathrm{f}_{ij} & \forall (i,j)\in E, w\in W & \text{BC} \\
\sum_{j\in N} x_{jikw} - \sum_{j\in N} x_{ijkw} & \leq & \mathrm{a}_i & \forall w\in W, i\in N, k\in K & \\
 & & & \text{where}\quad \mathrm{y}_{ik}=0 & \text{IC'1} \\
\sum_{j\in N} x_{ijkw} - \sum_{j\in N} x_{jikw} & \leq & \mathrm{a}_i & \forall w\in W, i\in N, k\in K & \\
 & & & \text{where}\quad \mathrm{y}_{ik}=0 & \text{IC'2}
\end{array}
$$

$\mathrm{x}_{ijkw} \in B$

$\mathrm{f}_{ij} \in I$

$\mathrm{a}_i \in B$

Number of variables: $(2 * |E| * |K| * |W|) + |E| + |N|$
Number of constraints: $(|N| * |K|) + (|E| * |W|) + (2 * |K| * (|N| - 2) * |W|)$

2.4 Aggregated formulation

Two formulations presented have strong relaxations, which can be further strengthened with valid cuts. Unfortunately, for a small sized problem of 14 nodes, 21 edges and 60 connections, with 8 wavelengths available, the number of variables required is 20195 for both formulations presented. For larger problems, the number of variables becomes too large for commercially available MIP solver software. To overcome this problem, we used *aggregation* [18], [24], which is simply consolidating the connections according to their source nodes. Aggregated connection requests are referred as commodities for the rest of this study. Since consolidation is made using nodes, clearly, number of commodities becomes at most $|N|$. For the maximum reduction in the number of variables, we use a simple minimal cover formulation that ensures that the source or destination node of each connection is selected to be a commodity, and minimizes the number of commodities. Note that establishing a connection from node i to node j is no different than establishing a connection from node j to node i, which in turn means that source and destination nodes of all messages can be rearranged so that the source node of each connection is a commodity, without changing the problem. Once aggregation is done, connection set K becomes commodity set K' (which is a subset of the node set N), and flow parameter y_{ik} becomes the aggregated flow parameter Y_{ik}. Also, binary flow variables become general integer variables. The method is quite useful for reducing the number of variables, for example, number of variables required for the example above becomes at most 4739. But since we discard the binary structure of the problem, some valid cuts exploiting the binary structure of the original problem are no longer useful. Valid cuts will be discussed later in this chapter. Before presenting the aggregated formulation, a short description of the method of aggregation is presented:

Procedure *Aggregate*

Given node set N and connection set K.

1. Solve minimum set cover problem to select a set $K' = \{c_1, c_2, \ldots, c_{|K'|}\}$ of nodes with minimum cardinality suct that either the source or the destination of each connection in K is present in K'.

2. For $k := 1$ to $|K|$

   if $s_k \notin K'$ then interchange s_k and d_k

3. For $k := 1$ to $|K'|$

   For $i := 1$ to $|N|$

   if $i = c_k$, $Y_{ik} :=$ (number of elements of set K with source c_k)

   else, $Y_{ik} :=$ - (number of elements of set K with source c_k and destination i)

The aggregated formulation is as follows:

(IP3)

Min $\sum_{(i,j)\in E} c_{ij} f_{ij} + HC(\sum_{i\in N} a_i)$

s.t.

$$
\begin{array}{llll}
\sum_{j\in N}\sum_{w\in W} x_{ijkw} - \sum_{j\in N}\sum_{w\in W} x_{jikw} & = \mathrm{Y}_{ik} & \forall i \in N, k \in K' & \text{NC} \\
\sum_{k\in K}(x_{ijkw} + x_{jikw}) & \leq \mathrm{f}_{ij} & \forall (i,j) \in E, w \in W & \text{BC} \\
\sum_{j\in N} x_{ijkw} - \sum_{j\in N} x_{jikw} & \leq \mathrm{M}^*\mathrm{a}_i & \forall w \in W, i \in N, k \in K' & \\
 & & \text{where} \quad \mathrm{Y}_{ik} \leq 0 & \text{IC}
\end{array}
$$

$\mathrm{x}_{ijkw} \in I$

$\mathrm{f}_{ij} \in I$

$\mathrm{a}_i \in B$

Number of variables: $(2 * |E| * |K'| * |W|) + |E| + |N|$

Number of constraints: $(|N| * |K'|) + (|E| * |W|) + (|K'| * |N-1| * |W|)$

In the aggregated formulation, interchanger location constraints (IC) are modified to cope with the aggregated flow structure. What they state is simply: If, at a node other than the source node of the commodity, the outflow of the commodity for a wavelength assignment is more than the inflow of the commodity with the particular wavelength assignment, then the node is a wavelength interchanger. Clearly, this means that the flow was assigned a wavelength it was not assigned before. This time, we cannot state the second part, since we have aggregated the flows according to their source nodes, their destinations may be different. Hence, the outflow of a commodity with a wavelength assignment may be less than the inflow of the commodity with the particular wavelength assignment at a node other than the source node of the commodity. Structure of the network constraints and the bundle constraints are the same as the binary formulations presented before.

Aggregation has been used in the literature for a variety of multicommodity flow problems. Due to the fact that the performance of branch & bound relies on the speed of the simplex algorithm, smaller number of variables grant a considerable advantage to the aggregated formulations. Gendron, Crainic and Frangioni (1998) point out that ([19]), LP relaxations of the aggregated formulations for multicommodity flow problems are much easier to solve, but it is also more difficult to identify inequalities that tighten the lower bound. During the study, aggregation was used on multicommodity flow formulations for generating lower bounds due to the improvement of speed it offers. Even with aggregation, number of variables became too large to handle for a 32-node 50-link topology due to the large number of nodes. Although this formulation is not as degenerate as the previous two models, LP relaxation is too weak and the problem appears to have a symmetric structure which reduces the efficiency of the branch & bound. Thus, valid inequalities to tighten the lower bound and to shrink the search space are the next focus of the study.

2.5 Valid inequalities

In this section, valid inequalities from the literature and valid inequalities proposed by the author are presented. Before stating the first set of valid inequalities, some definitions are required. Let S and T be subsets of node set N. Furthermore, let $T = N \setminus S$. Let D_{ST} be the amount of traffic the network has to carry between partitions S and T. Let E_{ST} be the set of edges that connect partitions S and T. Then,

(VI1)

$$\sum_{(i,j)\in E_{ST}} f_{ij} \quad \geq \quad \lceil \frac{D_{ST}}{|W|} \rceil \quad \forall S, T \subset N, T = N \setminus S$$

This first set of valid inequalities is known as the cutset inequalities in the literature. T. Magnanti, P. Mirchandani and E. Vachani have shown that the cutset inequalities are facet defining for the two-facility capacitated network loading problem (TFLP) , when subgraphs defined by S and T are connected and $D_{ST} > 0$ [7]. TFLP problem is the problem of designing a capacitated network with zero flow costs, where facilities of fixed capacity can be installed on edges. Two types of facilities with different capacities and costs are considered. The problem is quite similar to a relaxation that will be used to generate lower bounds in Chapter 5. In fact, the only difference between the two problems is that the formulation we will present allows only one type of facility. Although we do not give a proof that the cutset inequalities are facet defining for our problems, they proved to be very strong during the experimentation. The problem about the cutset inequalities is that, they are exponential in number. Every subset S of N that satisfies $1 \leq |S| \leq \lfloor \frac{N}{2} \rfloor$ gives a probable cut (so that T will cover the subsets with greater number of elements). Total number of probable cutset inequalities is $2^{|N|-1} - 1$ if $|N|$ is odd and $2^{|N|-1} + \frac{\binom{|N|}{|N|/2}}{2} - 1$ if $|N|$ is even. Enumerating all probable cutsets (also checking each and every one of the probable cutsets for connectivity of S and T) is not feasible for networks with more than 20 nodes. Note that each node constitutes a connected S set. Likewise, every edge constitutes a connected S set with two elements. Valid inequalities corresponding to these S sets can be found easily without having to enumerate the possible subsets of N. Number of constraints that can be generated in this manner is $|N| + |E|$, and these cuts can be used to strengthen all three formulations.

Analysis of optimum solution of the LP relaxation of formulation IP2 by barrier method of CPLEX, motivated the author to find the second set of valid cuts that will be presented shortly. The output suggested that barrier algorithm divided the flow uniformly between all wavelengths and sent the divided flows through the shortest paths. Because of the structure of the bundle constraints, this kind of flow could only increase the fiber requirement by $\frac{1}{|W|}$. What the problem requires is simply: At least one fiber is required if one unit of flow passes through an edge. To state this in terms of the formulation, the following set of valid inequalities have been introduced to the model.

(VI2)

$$\sum_{w \in W}(x_{ijkw} + x_{ijkw}) \quad \leq \quad \mathrm{f}_{ij} \quad \forall (i,j) \in E, k \in K$$

Number of valid inequalities: $(|E| * |K|)$

Actually, this set of valid inequalities can be extended to cover a larger number of connections. The statement above can be restated as : At least $n+1$ fibers are required if $n * |W| + 1$ units of flow pass through an edge. Unfortunately, for a $|K|$ connection problem where $|W|$ wavelengths are available, the corresponding number of valid inequalities generated for values of n larger than 0 are: $\binom{|K|}{|W|+1} * |W|$ for $n = 1$, $\binom{|K|}{(2*|W|)+1} * |W|$ for $n = 2$, and so on. Adding this many constraints expands the problem too much beyond tractability. However, valid inequalities generated by considering single connection case provided considerable tightening of lower bound during experimentation. Another advantage is that they are polynomial in number. Unfortunately, they decrease the speed of the simplex algorithm dramatically. Moreover, they depend on the binary flow structure, thus, they cannot be used in the aggregated formulation.

Third set of valid inequalities are motivated by the claim that symmetric structure of an IP may cause branch-and-bound to perform poorly because the problem barely changes after branching [17]. All of the formulations presented up to now have a symmetric structure. In fact given an optimal solution, $|W|!-1$ optimal solutions can be generated. A simple proof of the previous statement is as follows:

Proposition 1 *Given an optimal solution,* $|W|! - 1$ *optimal solutions can be generated.*

Proof: Given an optimal solution x^* , each flow has a wavelength assignment for each edge it flows on. Notice that a wavelength assignment is only a *label*. Changing the name of a label will not result in any change in the optimal solution value. Given an optimal labeling and its corresponding partitioning of flows ($|W|$ sets of flows), one can interchange the names of the labels without disrupting the optimality. So given $|W|$ labels and $|W|$ partitions, total number of possible one-to-one matchings is $|W|!$. Since we are given one of the assignments, number of distinct assignments that can be generated is $|W|! - 1$. □

To decrease this level of symmetry, the following set of valid inequalities is proposed.

(VI3)

$$\sum_{(i,j)\in E} \sum_{k\in K} x_{ijkw^1} \quad \geq \quad \sum_{(i,j)\in E} \sum_{k\in K} x_{ijkw^2} \quad \forall w^1, w^2 \in W, w^1 = w^2 + 1$$

Number of valid inequalities: $|W| - 1$

VI3 simply states that the most crowded wavelength should be $|W|$, next most crowded wavelength should be $|W| - 1$, and so on. As opposed to the valid inequalities presented up to this point, this third set of valid inequalities do not tighten the lower bound, instead, they shrink the branch-and-bound tree. Although very small in number, these inequalities shrink the search space considerably, but during the experimentations, it was noticed that they decrease the speed of the simplex algorithm. For small problems, this set of valid inequalities has worked well, but for large problems, they decreased the speed of the simplex algorithm too much to be useful.

2.6 Problems about the formulations

The first major problem about the formulations is the huge number of variables. For a large problem of 32 nodes, 50 edges and 100 connections, with 8 wavelengths available, first two formulations require 80082 variables (80032 binary variables and 50 integer variables), whereas the third (aggregated) formulation requires 12882 variables on the average (12832 binary variables and 50 integer variables), assuming that on the average half of the nodes will be selected as commodities. This many variables result in longer solution times for the LP relaxation, longer dual simplex times at each node of the branch-and-bound tree and larger branch-and-bound trees, and consequently huge time and memory requirements.

Second problem about the formulations is the high degree of degeneracy, especially for the first two formulations. It is known that LP relaxations of capacitated network design problems are often highly degenerate [19]. It is observed that the objective value of the relaxation tends to stall for a few thousands of iterations at a time. A few hundred, and frequently a few thousand dual simplex iterations are required for re-optimization at each node of the branch-and-bound tree. The experimentations has been made using ARPA2, NFSNET and a 32 node topology representing long distance telephone network with nodes corresponding to major US cities. For the rest of this study, this 32 node topology will be referred as MESH32. Even for the smallest network structure (NFSNET) used for experimentation, no optimal result could be obtained for a reasonable number of connections using the three formulations above.

Third problem about the formulations is a general problem about the capacitated network design problems. As Magnanti, Mirchandani, and Vachani point out: "In general, linear programming lower bounds are weak for most capacitated network design problems..." [7], [19]. Even though branch & bound finds the optimal solution at the early stages, a large number of nodes are required to be inspected to prove optimality. Adding (VI1) for 1 and 2 node S sets, and (VI2) to formulations (IP1) and (IP2) resulted with an increase of about 5% in the lower bound, but after a week of computer time, the author had to give up because of the unsatisfiable memory requirement of the branch-and-bound tree.

2.7 Proof of NP-Hardness

S. Even, A. Itai and A. Shamir proved that the integral multicommodity flow problem is NP-Complete [1]. A simple transformation from integral multicommodity flow problem will be presented to prove that our problem is NP-Hard.

Theorem 1 *The problem of routing, wavelength assignment and wavelength interchanger location with minimum total cost is NP-Hard*

Proof: We know that the integral multicommodity flow problem is NP-Hard [1]. The following formulation describes the integral multicommodity flow problem. Note that u_{ij} denotes the upper bound on total flow on edge (i, j).

Min $\sum_{k\in K}\sum_{(i,j)\in E} c_{ijk}x_{ijk}$

s.t.

$$\begin{array}{lcllr} \sum_j x_{ijk} - \sum_j x_{jik} & = & \mathrm{y}_{ik} & \forall i \in N, k \in K & \mathrm{NC} \\ \sum_{i,j}(x_{ijkw} + x_{jikw}) & \leq & \mathrm{u}_{ij} & \forall (i,j) \in E, w \in W & \mathrm{BC} \\ \mathrm{x}_{ijk} \in B & & & & \end{array}$$

If the upper bounds u_{ij} are replaced by variables (f_{ij}) with corresponding costs (c_{ij}), the resulting problem is also NP-Hard since the integral multicommodity flow problem is a special case of the resulting problem where $f_{ij} = u_{ij}$. Now assume we replace f_{ij} with $|W| * f_{ij}$. We know that the problem is NP-Hard for $|W| = 1$, and the problem will be no easier for other (positive) values of $|W|$. Finally, notice that the special case of our problem where all nodes are set to be wavelength interchangers, is equivalent to the integral multicommodity flow problem where upper bounds are replaced with $|W| * f_{ij}$. Consider the following transformation: take the network for an integral multicommodity flow problem where upper bounds are replaced with $|W| * f_{ij}$ and construct the following instance of our problem. Set the number of bandwidths to be equal to the capacity of a single link ($|W|$). Set all nodes to be wavelength interchangers. Set the cost of an interchanger to 0. Set the fiber costs to be equal in both problems and the flow cost of a connection for each wavelength assignment on an edge to be equal to the flow cost of that particular connection on that edge. In this case an optimal solution to the former problem can be transformed into an optimal solution to the latter in the following manner. For each edge, construct an arbitrary list of flows on that edge (in both directions). Assign (i mod $|W|+1$)th wavelength to the i'th flow on the list. This corresponds to an optimal solution to the latter problem, since we do not have to care for wavelength continuity and the flow and fiber costs are equal. Similarly, taking an optimal solution of the latter problem and placing each flow (not caring for the wavelength assignment of the flow) to its corresponding edge results in an optimal solution to the former problem. Clearly, the transformation is polynomial. So, we can conclude that the latter problem is NP-Hard as well as the former. □

Chapter 3

Exploring the subproblems

Failure to optimize the whole problem, together with the proof of NP-Hardness, led the author to search for a "good" method of generating feasible solutions. The metric of being good is, of course, the distance to a lower bound which might be generated by a relaxation of the original problem. To present more than a simple greedy algorithm, a better understanding of the main problem was required. In order to understand the grand problem, subproblems were identified.

The subproblems one can identify are the routing problem, the wavelength assignment problem and the wavelength interchanger location problem. Routing of connections may be fixed to solve wavelength assignment and wavelength interchanger location problems simultaneously. Wavelength interchanger locations (hence, number of wavelength interchangers) may be fixed to solve the RWA problem of the resulting topology. However, wavelength assignment cannot be fixed, because it depends on both the routing and the wavelength converter locations. Thus, the author decided that the most appropriate way to partition the problems is to find a routing (without wavelength assignments) that minimizes the fiber cost, and then solve the wavelength assignment and wavelength interchanger location problems simultaneously. In the rest of this chapter, these subproblems are analyzed.

3.1 Integral Multicommodity Flow Problem

To simplify the problem, wavelength assignment obligation was dropped (which is equivalent to assuming that each node is a wavelength interchanger).

The problem is to determine the routing of each connection. But even then, the problem was equivalent to the integer multicommodity flow problem with variable upper bounds, which is a provably hard problem. Note that this problem is a relaxation of our problem. The formulation which is equivalent to integral multicommodity flow problem with variable upper bounds is as follows:

(IP4)

Min $\sum_{(i,j)\in E} c_{ij} f_{ij}$
s.t.

$$\begin{array}{llll} \sum_{j\in N} x_{ijk} - \sum_{j\in N} x_{jik} & = & \mathrm{y}_{ik} & \forall i \in N, k \in K & \mathrm{NC} \\ |W|^{*}\mathrm{f}_{ij} - \sum_{k\in K}(x_{ijk} + x_{jik}) & \geq & 0 & \forall (i,j) \in E & \mathrm{BC} \\ \mathrm{x}_{ijk} \in B \\ \mathrm{f}_{ij} \in I \end{array}$$

Number of variables: $(2 * |E| * |K|) + |E|$
Number of constraints: $(|N| * |K|) + |E|$

Different than the formulations presented before, subscript w for the flow variables have been eliminated. This formulation, too, may be aggregated using the algorithm described in Chapter 2. Detailed studies about solving multicommodity flow problems both heuristically and optimally have been done ([3],[5],[18],[24]). Using Lagrangian relaxation of the multicommodity flow problem is recommended (relaxing the bundle constraints so that the LP relaxation of the rest of the problem gives integral results) to produce a solution close to the LP lower bound ([4]). Both Lagrangian relaxation and Lagrangian Dualization has been applied to the formulations, without success. Unfortunately, neither lower bounds improved, nor good feasible solutions could be generated.

Banerjee and Mukherjee tackled this problem using "Randomized Rounding", which uses the optimum solution of the LP relaxation to construct a feasible solution ([12]). Instead of a probabilistic rounding method, a partial column generation approach is proposed by the author. Please note that a "flow" formulation can be replaced by a "path" formulation that enumerates the paths for each source-destination pair and forces the formulation to select only one alternative for each source-destination pair. Also note that total number of feasible paths available increases exponentially with the number of nodes in the network. Author's intuition suggested that in an optimal solution, shortest paths were used much more frequently than longer paths. Hence, enumerating the first A shortest paths for the alternatives, and selecting among these alternatives seemed to be an efficient way to handle this problem. Many algorithms exist for finding the k-shortest paths (KSP) in a network with nonnegative flow costs. For a comparative study of existing studies on KSP, see [8]. Yen's algorithm was implemented in C and used for finding the k-shortest paths, together with Dijkstra's well known shortest path algorithm. This method will be referred as the KSP method for the rest of this study. Let x_{ka} be the variable representing the selection of a'th alternative for the k'th source-destination pair. Let A be the number of alternatives. Let c_{ka} be the cost of selecting a'th alternative for k'th connection. Let S_{ij} be the set of tuples (k, a). A tuple (k, a) is an element of set S_{ij} if and only if a'th alternative for k'th message passes through edge (i, j). Following is the formulation for the KSP method.

(IP5)

Min $\sum_{k \in K} \sum_{1 \leq a \leq A} c_{ka} x_{ka} + \sum_{(i,j) \in E} c_{ij} f_{ij}$

$$
\begin{array}{lcll}
\sum_{1 \leq a \leq A} x_{ka} & = & 1 & \forall k \in K \\
\sum_{(k,a) \in S_{ij}} x_{ka} & \leq & |W|^{*}\mathrm{f}_{ij} & \forall (i,j) \in E \\
\mathrm{x}_{ka} \in B & & & \\
\mathrm{f}_{ij} \in I & & &
\end{array}
$$

Number of variables: $(|K| * A) + |E|$
Number of constraints: $|K| + |E|$

Notice that as A grows large, the formulation is equivalent to flow formulation, at the expense of more time and memory requirement for the branch & bound. It is well known that the performance of branch & bound can be improved if a good solution is used for pruning. Thus, KSP method can be further improved by increasing A iteratively and using the optimal solution of the previous iteration as a starting solution. Here is a formal description of the KSP method.

Procedure *KSP*

1. For all source-destination pairs, find the k shortest paths connecting these pairs;

2. Use shortest path routing and determine the fiber cost;

3. For $a := 2$ to A

   Solve IP5 for a alternatives using the optimum solution for $a - 1$ alternatives as the initial solution of the branch & bound tree.

Finding a worst case upper bound expression for KSP method has been attempted by the author. The following result is necessary to prove the worst case behavior of shortest path routing. Let $SP(k)$ be the fiber cost of the shortest path connecting the source and destination of k'th connection.

Proposition 2 *Optimal value of the LP relaxation of IP4 is* $\frac{\sum_k SP(k)}{|W|}$

Proof: First, notice that replacing the equality signs of network constraints with '$\geq$' will not disturb the structure of the problem. Furthermore, this change will be useful since we will deal with the dual in the following steps. Notice that the structure of the problem is as in Figure 3.1. Note that $N1, N2, \ldots, NK$ denotes the network flow constraint blocks for connections $1, 2, \ldots, |K|$ correspondingly, BC denotes the bundle constraint block, and the rest of the constraint coefficient matrix is composed of '0's.
Each square corresponds to the network constraints of a commodity. The rectangular block of constraints are the bundle constraints that bind the commodities. Each column of flow variables consists of a '1' and a '-1' in the square representing the network constraints of the commodity and a '-1' in the

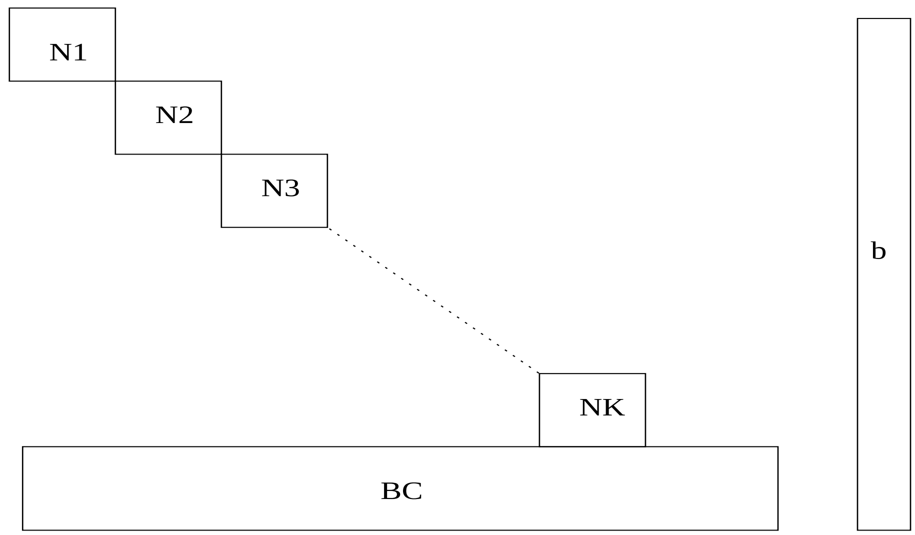


Figure 3.1: Structure of constraint matrix of IP4

rectangular block. Fiber variables are represented only in the rectangular block with a single $|W|$ value. Right hand side of the network flow constraints consists of one '1' and one '-1' for each commodity, and the rest of the right hand side values are zero. All constraints are '$\geq$' type. The structure of the dual of this formulation is as follows:

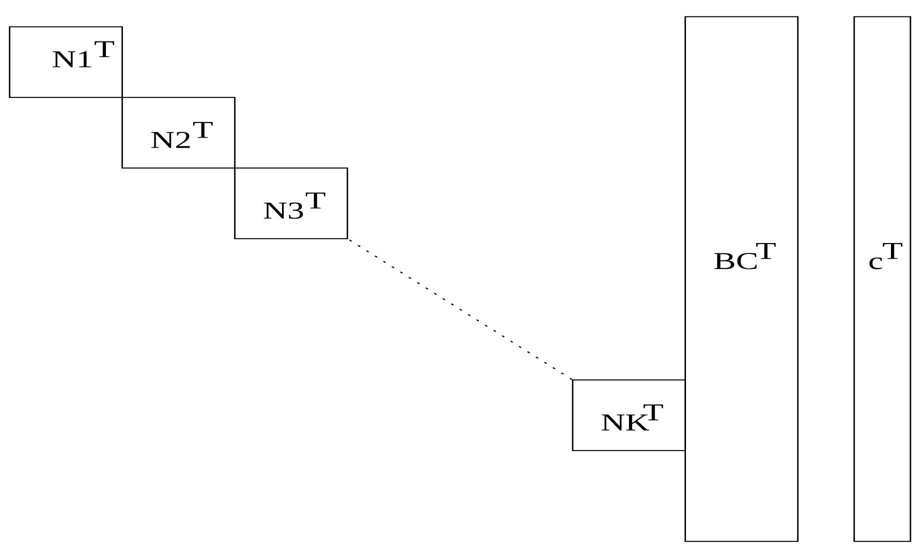


Figure 3.2: Structure of Dual of IP4

Note that every variable is nonnegative, each of the constraints is '$\leq$' type, and right hand side consists of '0's for constraints corresponding to flow variables and c_{ij}'s for the constraints corresponding to fiber variables. Notice that the lowermost part of the rectangular block only consists of variables corresponding to the bundle constraints of IP4, with right hand side values of c_{ij}. These constraints can be restated as:

$$w_i \leq \frac{c_{ij}}{|W|}$$

where w_i is the dual variable corresponding to the bundle constraint of primary variable f_{ij}. Notice that in every one of the rest of the dual constraints, one and only one nonzero coefficient exists for the dual variables representing the bundle constraints, which is '-1'. Applying Fourier-Motzkin elimination on dual variables representing the bundle constraints eliminates these variables and yields a right hand side vector consisting of '$\frac{c_{ij}}{|W|}$'s. In other words, assume that we restate each of the rest of dual constraints by taking the dual variable representing the bundle constraint to the right hand side. Notice that each dual variable corresponding to a bundle constraint constitutes an upper bound on a dual constraint, and the lowermost part of the rectangular block imposes upper bounds on the dual variables corresponding to bundle constraints. Thus, we can eliminate these variables by replacing them with their corresponding upper bounds. The structure of the resulting dual problem is as follows, where right hand side consists of $\frac{c_{ij}}{|W|}$ values:

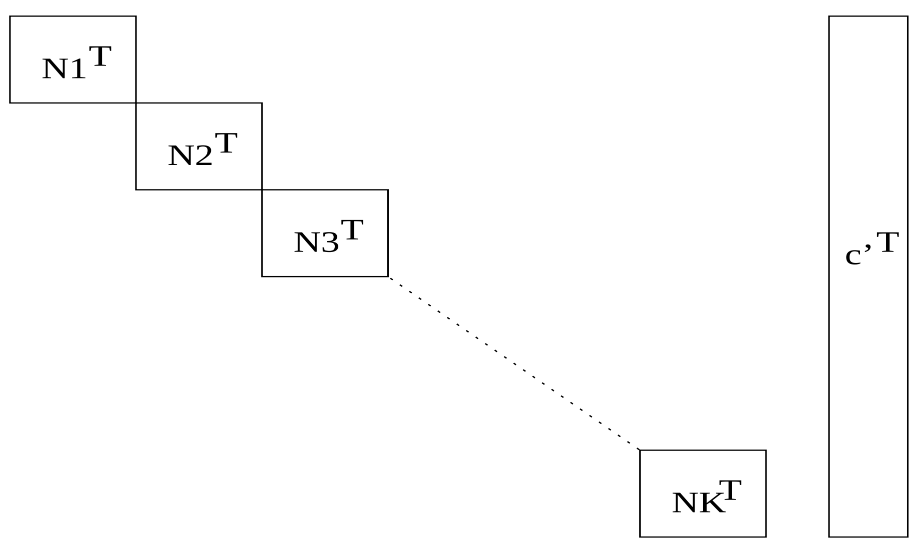


Figure 3.3: Structure of Modified Dual of IP4

Taking the dual of this problem results in the following primal problem:
Min $\sum_{k\in K}\sum_{(i,j)\in E}\frac{c_{ij}}{|W|}x_{ijk}$
s.t.

$$\begin{aligned}\textstyle\sum_{j\in N}x_{ijk}-\sum_{j\in N}x_{jik} &= \mathrm{y}_{ik} \quad \forall i\in N, k\in K \quad \mathrm{NC}\\ \mathrm{x}_{ijk}\geq 0\end{aligned}$$

Since the bundle constraints that bound the commodities together are eliminated, this problem is minimum cost flow for each commodity, and clearly, every commodity will follow the shortest path form its source to its destination, which has a cost of $\frac{SP(k)}{|W|}$ for each commodity k. So the overall cost of the optimum solution of this LP is: $\sum_{k\in K}\frac{SP(k)}{|W|}$. Notice that we did not disturb the problem, but just modified it. Thus, the optimal value of relaxation of IP4 is the same as the optimal value of this LP. □

Proposition 3 *Let Z_s denote the value of the shortest path routing and Z^* denote the value of the optimal solution. then,* $Z_s\leq Z^*(1+\frac{max\ c_{ij}}{min\ c_{ij}}*\frac{|W|-1}{|K|}*|E|)$

Proof: We are trying to construct a feasible solution to IP4, given the integral routing data. In this case, total cost of the fibers will be at least $\frac{\sum_k SP(k)}{|W|}$. Notice that number of fibers on an edge can be at most $\frac{|W|-1}{|W|}$ less than the value yielded by the bundle constraints of IP4. Considering the worst case, assume every edge is used in the shortest path routing. Then:

$$Z_s\leq\frac{\sum_k SP(k)}{|W|}+(\frac{|W|-1}{|W|}*\textstyle\sum_{(i,j)\in E}c_{ij})$$

Since $|E|*max\ c_{ij}\geq\sum_{(i,j)\in E}c_{ij}$,

$$Z_s\leq\frac{\sum_k SP(k)}{|W|}+(\frac{|W|-1}{|W|}*max\ c_{ij}*|E|)$$

Dividing and multiplying the last term by $min\ c_{ij}$,

$$Z_s\leq\frac{\sum_k SP(k)}{|W|}+(\frac{|W|-1}{|W|}*\frac{max\ c_{ij}}{min\ c_{ij}}*|E|*min\ c_{ij})$$

Since $min\ SP(k) \geq min\ c_{ij}$,

$$Z_s \leq \frac{\sum_k SP(k)}{|W|} + (\frac{|W|-1}{|W|} * \frac{max\ c_{ij}}{min\ c_{ij}} * |E| * min\ SP(k))$$

Dividing and multiplying the last term by $|K|$,

$$Z_s \leq \frac{\sum_k SP(k)}{|W|} + (\frac{|W|-1}{|W|} * \frac{max\ c_{ij}}{min\ c_{ij}} * |E| * min\ SP(k) * \frac{|K|}{|K|})$$

Since $\sum_k SP(k) \geq |K| * min\ SP(k)$,

$$Z_s \leq \frac{\sum_k SP(k)}{|W|} + (\frac{|W|-1}{|W|} * \frac{max\ c_{ij}}{min\ c_{ij}} * |E| * \frac{\sum_k SP(k)}{|K|})$$

$$Z_s \leq \frac{\sum_k SP(k)}{|W|}(1 + \frac{|W|-1}{|K|} * \frac{max\ c_{ij}}{min\ c_{ij}} * |E|)$$

Finally, by Proposition 2, $\frac{\sum_k SP(k)}{|W|} \leq Z^*$

$$Z_s \leq Z^*(1 + \frac{|W|-1}{|K|} * \frac{max\ c_{ij}}{min\ c_{ij}} * |E|)$$

□

The worst case expression offers insights about the cost behavior of shortest path routing. First, if $|W| = 1$, then routing all the connections on their shortest paths to their destination is the optimal solution, which is obvious. Next, it implies that as the size of the network grows, as the proportion of length of the longest link to the length of the shortest link increases, and as the number of wavelengths available increases the cost behavior may not be very good. Last and the most important implication is that, as $|K|$ grows large, the performance of shortest path routing improves. Obviously, for large connection request sets (150 or more), KSP method becomes harder to apply because of the increasing number of variables. But we know that, as the cardinality of the connection set increases, shortest-path routing tends to behave better. Thus, value of A (number of shortest path alternatives) may be decreased without a great loss of performance when $|K|$ is large.

Detailed analysis of the results of applying KSP method to NFSNET, ARPA2 and MESH32 topologies will be presented in Chapter 4.

3.2 Wavelength Assignment and Interchanger Location Problem

The name of this section suggests two subproblems, but actually, Wavelength Assignment and Interchanger Location problems require interacting decisions. A connection may (or may not) change its wavelength assignment in a node if the node is a WIXC (or not). If interchanger locations are fixed, then, each connection can be broken into several pieces at every WIXC on its route and assigned separate wavelengths for each piece. However, the aim of this study is to determine the number of WIXC's and where they should be placed. Thus, these problems are inseparable for our study.

Once routing is fixed, the problem becomes assigning wavelengths to each connection at each link it uses. Note that if each edge consists of a single fiber, and all nodes are WSXC's, the problem is equivalent to the graph k-colorability problem (with $k = |W|$), which is known to be NP-Complete ([2]).

Since three IP formulations involving wavelength assignment and interchanger location have been presented in Chapter 2, using the formulations to solve the wavelength assignment and interchanger location problem while fixing the routing, seemed appropriate. IP3 was selected for the reduction in the number of variables it offers. Following is the modified formulation for the case where formulation is fixed. Let r_{ijk} be the routing data, i.e., the amount of flow of commodity k from node i to node j.

(IP3')

Min $\sum_{(i,j)\in E} c_{ij} f_{ij} + HC(\sum_{i\in N} a_i)$
s.t.

$$
\begin{array}{lcll}
\sum_{w \in W} x_{ijkw} & = & \mathrm{r}_{ijk} & \forall (i,j) \in E, k \in K' \\
\sum_{k \in K} (x_{ijkw} + x_{jikw}) & \leq & \mathrm{f}_{ij} & \forall (i,j) \in E, w \in W \\
\sum_j x_{ijkw} - \sum_j x_{jikw} & \leq & \mathrm{M}^*\mathrm{a}_i & \forall w \in W, i \in N, k \in K, where\ Y_{ik} \leq 0 \\
\mathrm{x}_{ijkw} \in I & & & \\
\mathrm{f}_{ij} \in I & & & \\
\mathrm{a}_i \in B & & &
\end{array}
$$

Number of variables: $(2 * |E| * |K'| * |W|) + |E| + |N|$
Number of constraints: $(|E| * |K'|) + (|E| * |W|) + (|K'| * |N - 1| * |W|)$

Needless to say, many variables were eliminated from the model since edges which do not carry flows are discarded. This model is later used in Chapter 4 for determining the number and location of wavelength interchangers.

One other formulation to be presented is for wavelength assignment when routing is fixed and there are no wavelength converters. This formulation is used for the purpose of determining the maximum number of wavelength converters to be placed. Let c_1 be the optimal fiber cost of routing all the connections when all nodes are wavelength interchangers. Let c_2 be the optimal fiber cost when none of the nodes are wavelength interchangers. Clearly, $c_2 \geq c_1$. Let HC denote the cost of a single wavelength interchanger. If $c_2 - c_1 \leq HC$, then no wavelength interchangers are required and c_2 is the optimal value of the overall problem. So, if a quick way of finding a solution for the no interchanger case is found, it may be useful for assessing the value of a wavelength interchanger. Following is the formulation for wavelength assignment when routing is fixed. Let x_{kw} denote if k'th message is assigned wavelength w or not. Let R_{ij} denote the set of messages that pass through link (i, j).

(IP6)

Min $\sum_{(i,j) \in E} c_{ij} f_{ij}$
s.t.

$$\begin{array}{lll} \sum_{w \in W} x_{kw} & = & 1 \quad \text{k} \in K \\ \sum_{k \in R_{ij}} x_{kw} & \leq & \text{f}_{ij} \quad \forall (i,j) \in E, w \in W \\ \text{x}_{kw} \in B & & \\ \text{f}_{ij} \in I & & \end{array}$$

Number of variables: $(|K| * |W|) + |E|$
Number of constraints: $|K| + (|E| * |W|)$

3.3 Generating strong lower bounds

During the analysis of the integer multicommodity flow problem with variable upper bounds, it was noticed that even if the integrality constraints of the flow variables are dropped, the optimal value of the resulting problem is still close to that of the original problem. Hence, it seemed appropriate to use an aggregated formulation for multicommodity flow with variable upper bounds and force integrality constraints only on variables representing number of fibers, to obtain a strong lower bound. Furthermore, this formulation could be strengthened by adding cutset inequalities. For NFSNET topology, lower bounds generated proved to be equal to the value of the optimal solution of the integer multicommodity flow problem with variable upper bounds, 88.57% of the time (93 out of 105 instances). When the lower bound was not equal to the value of the optimal solution of the integer multicommodity flow problem with variable upper bounds, the difference between the objective values was not more that 5% of the lower bound. For larger topologies, solving the original problem to optimality was computationally expensive, so such data is not available for ARPA2 and MESH32 topologies.

In order to determine the cutset inequalities to be added the following methodology was used. Connected minimal subsets of nodes for non-planar topology (NFSNET), and minimal *faces* of planar network topologies (ARPA2 and MESH32) were used. A *face* is defined as the remaining connected components of a planar graph when edges and vertices of the graph is omitted.

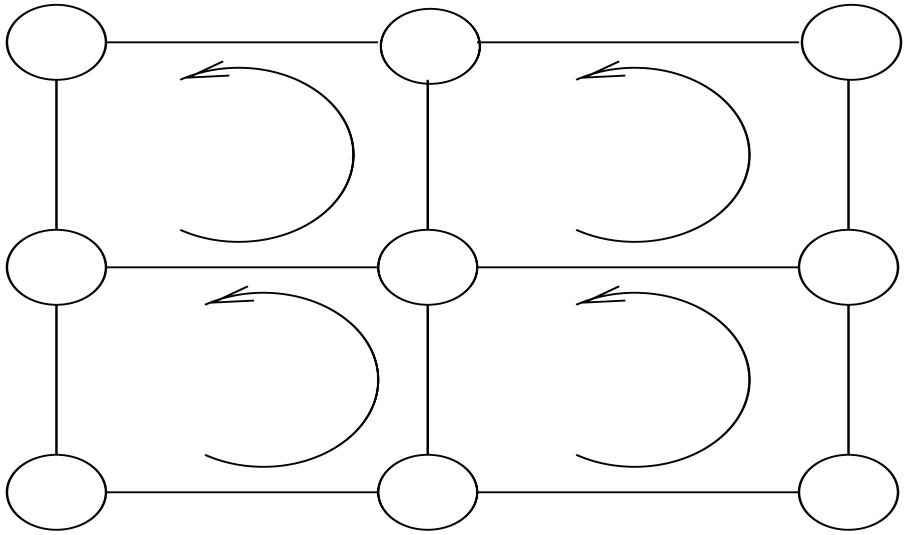


Figure 3.4: Faces of a simple mesh network

Large number of subsets of node set N, led the author to find a better way of searching for connected S-T partitions. Instead of taking subsets of the whole node set N, it seemed appropriate to aggregate some nodes to decrease the total number of subsets to be examined. Aggregated nodes were then connected with edges to denote if they are connected or not, and a condensed graph was constructed. This method reduced the 14 node NFSNET topology to a 7 node condensed graph, 21 element node ARPA2 topology to 6 node condensed graph, and 32 element node MESH32 topology to 19 node condensed graph. Cutset inequalities generated using the constructed graphs mentioned above proved to be effective for generating lower bounds during the experimentations.

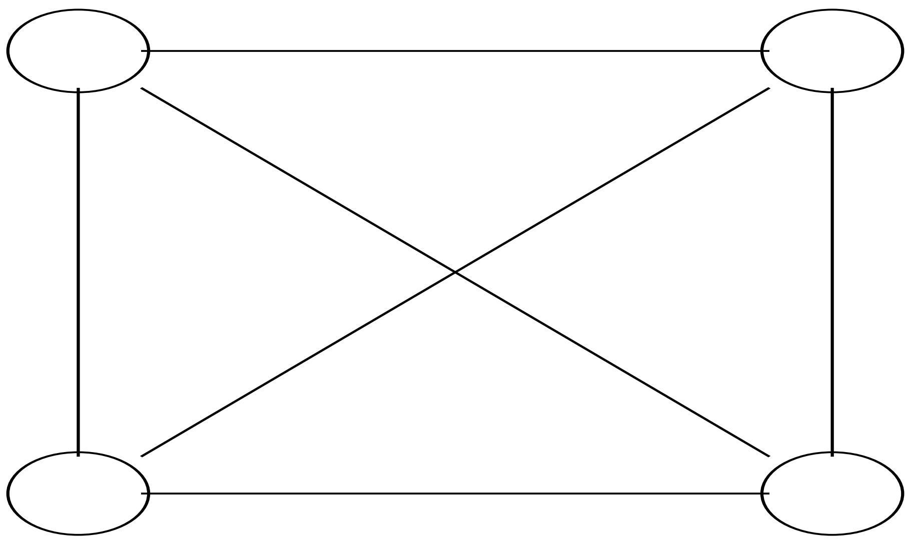


Figure 3.5: Condensed graph of the graph in Figure 3.4

Chapter 4

A solution method

4.1 Declaration of the overall procedure

Analysis of the subproblems declared in Chapter 3 led us to the idea of solving a subproblem, and then solving the rest of the problem while fixing the part solved before. Most appropriate choice seemed to first find the routing without wavelength assignment, due to the fact that KSP method proved to be a very efficient heuristic. When routing is fixed, the resulting problem is the wavelength assignment and interchanger location problem. This problem, too, involved a large number of variables. Instead of trying to solve the wavelength assignment and interchanger location problem, first, a solution for the case without any wavelength interchangers is generated. In case a wavelength assignment without any converters and with the same fiber cost as the routing can be found, the result of the wavelength assignment is optimal. If such a solution cannot be found, next step is to fix the number of fibers on every edge and to determine the minimum number of wavelength interchangers and their location that allows a feasible wavelength assignment. Following is the formal description of the procedure described above.

Procedure *Routing, Wavelength Assignment and Interchanger Location*

1. Use KSP method for a suitable value of A (number of alternatives), to find a feasible routing.

2. Solve IP6 with the routing data from step 1

   if the optimal value of IP6 is equal to that of KSP method, stop, existing solution is optimal

   else, go to step 3

3. Solve IP3 with the routing data from step 1, the f_{ij} values fixed to that of the best solution of KSP method and $HC = 1$.

4.2 Remarks about the procedure

Obviously, the complexity of the overall procedure is exponential. The procedure tries to solve large IP's for problems known to be NP-Hard, at all three stages. Fortunately, IP's used for the subproblems tend to behave well. Stages 2 and 3 seldom resulted in provably optimal solutions, but they produced good solutions in a relatively short time. Detailed results are provided at the end of this chapter. One important note is about the costing of the alternatives in stage 1. In terms of the problem, the path of a connection request does not matter since the only cost objects are fibers and wavelength interchangers. However, once routing is fixed, wavelength assignment and consequently final fiber quantities and wavelength interchanger locations are effected by the choice of path. It was noticed that many alternative optimal solutions exist for the first stage. The author's intuition suggested that as a connection uses more fibers, it interacts with more and more connections, making it harder to assign wavelengths. Thus, each alternative was assigned a small cost, namely $\frac{1}{1000}$'th of the total number of fibers it uses. This way, the formulation tried to minimize the total fiber cost and send the messages using the most direct routes simultaneously.

4.3 Analysis of Results

The method proposed in Section 4.1 was applied to three different topologies, namely, NFSNET, ARPA2, and MESH32. Randomly generated source-destination pairs were used during the experimentation. Cost of a fiber was assumed to be equal to its approximate length. 21 data sets for NFSNET, 21 data sets for ARPA2 and 9 data sets for MESH32 were tested. A total of 237 runs have been made to obtain the solutions. Same number of runs were required for obtaining the lower bounds. The computer used for the experimentation is a Sun Enterprise 4000 with a CPU clock of 248 Mhz and 1024 MB of real memory. The code was developed in C, and the callable library of CPLEX 5.0 was used for mixed integer optimization.

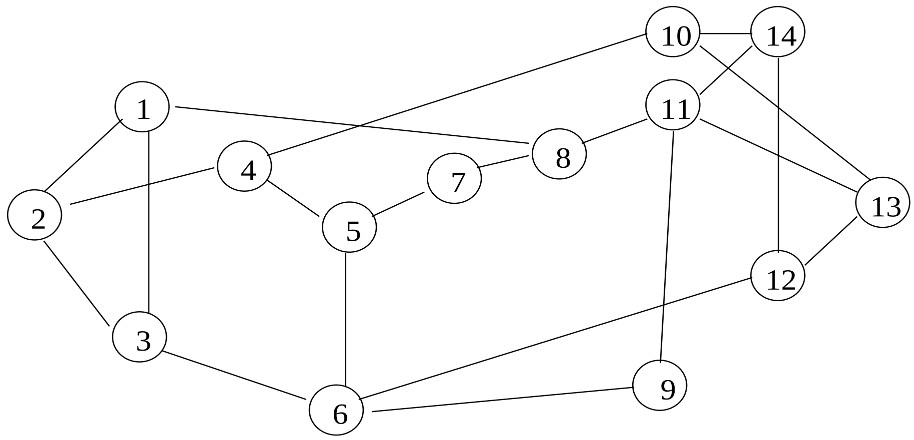
Rest of this chapter is composed of figures and detailed tables. Note that 'DS' is the column for the name of the data set, and 'NC' is the column that denotes the number of connections a data set involves. To better analyze the effects of traffic density and number of available wavelengths on the performance of the solution procedure, the procedure was applied to sets of connection requests for different number of wavelengths available. For example, the procedure was applied to data set 'ds7' for the NFSNET topology, involving 40 connections, for $|W| = 8, 10, 12, 14, 16$ where $|W|$ is the number of wavelengths available.

Tables do not include time data since the time for the overall procedure was limited to 12 hours of computer time (4 hours for each stage). Especially stages 2 and 3 tended to find the best solution in the early stages of branch & bound tree, but spent too much time to prove optimality. Since network design problems are not to be repeated daily in real life, length of the computation time can be afforded. Lower bound computations for NFSNET and ARPA2 topologies yielded the results in a few hours of computer time, whereas computations for the MESH32 topology took a few days.

The deviations do not exhibit any precise pattern. While they are quite close to the lower bounds, there seems no absolute guideline for predicting the behaviour of the results. Empirical evidence suggests that the type and the size of the topology have the greatest effects on the results. Next comes $|W|$ and $|K|$. It can be said that it is harder to obtain a solution with objective value close to the lower bound, as $|K|$ decreases and $|W|$ increases. This may be interpreted as a result of shortest path routing. Note that as proven in Proposition 3, the worst case behaviour of shortest path routing suggested that, with increasing $|W|$ and decreasing $|K|$, cost performance of shortest path routing decreases. The maximum of deviations for the KSP method for different topologies is 6.75%. The maximum of deviations for the wavelength assignment for different topologies is 25.36%. These results show that our heuristic performs quite well.

More importantly, a relatively small number of wavelength interchangers is sufficient for decreasing the fiber cost of WP scheme to the VWP scheme. Topology seems to have an effect also on the wavelength interchanger location, since the number of times a node is selected to be a wavelength interchanger seems to be concentrated at some specific nodes. For the NFSNET and MESH32 topologies, wavelength interchangers are more likely to be placed in the 'middle' of the graph, possibly due to the fact that KSP method selects the paths which use the minimum number of links. For the ARPA2 topology, wavelength interchangers are almost exclusively placed at 'crossroads', that is, the intersection points of the faces. To find a precise method for placing the wavelength interchangers just depending on the topology is not attempted, because during the experimentations it was observed that the location of the wavelength interchangers depended on the traffic data as well as the topology. Especially for the MESH32 topology, wavelength interchanger locations seemed to be scattered in a region, most probably because of the routing.

4.4 NFSNET



NFSNET Physical network with 14 nodes and 21 links

Figure 4.1: NFSNET topology

Lower Bounds for the NFSNET Topology

DS	NC	w:	8	10	12	14	16
ds1	20		2490	2315	2175	2050	2050
ds2	20		2450	2320	2190	2190	2190
ds3	20		2490	2440	2210	2190	2190
ds4	30		2890	2670	2490	2435	2190
ds5	30		2620	2490	2490	2460	2190
ds6	30		3200	2760	2670	2490	2370
ds7	40		3270	2840	2540	2490	2490
ds8	40		3190	2665	2540	2540	2370
ds9	40		3560	3010	2810	2690	2490
ds10	50		3775	3365	3080	2855	2490
ds11	50		3530	3040	2690	2640	2640
ds12	50		4065	3370	3230	2855	2730
ds13	60		4545	3730	3430	3240	3010
ds14	60		4455	3835	3510	2950	2790
ds15	60		4610	3890	3620	3260	3060
ds16	70		5160	4385	3860	3540	3100
ds17	70		4770	4100	3420	3290	3180
ds18	70		5045	4340	3770	3540	3080
ds19	80		5055	4320	3790	3310	3200
ds20	80		5515	4685	4130	3625	3350
ds21	80		5455	4570	4170	3685	3475

Table 4.1: Lower Bounds for the NFSNET Topology

Results for the NFSNET topology, KSP solved to 6 alternatives

			KSP	W.A.	KSP	W.A.	KSP	W.A.	KSP	W.A.	KSP	W.A.
DS	NC	w:	8	8	10	10	12	12	14	14	16	16
ds1	20		2490	2490	2365	2365	2365	2365	2240	2240	2240	2240
ds2	20		2470	2470	2370	2370	2240	2240	2240	2240	2240	2240
ds3	20		2590	2590	2500	2500	2220	2220	2200	2200	2200	2200
ds4	30		2900	3020	2780	2780	2560	2560	2510	2510	2200	2200
ds5	30		2740	2740	2540	2660	2540	2540	2470	2590	2240	2360
ds6	30		3200	3320	2790	2790	2690	2810	2500	2500	2370	2370
ds7	40		3280	3450	2860	2860	2730	2730	2540	2540	2540	2540
ds8	40		3250	3390	2865	2865	2540	2540	2540	2540	2370	2370
ds9	40		3560	3560	3010	3130	2810	2810	2690	2690	2540	2540
ds10	50		3775	3775	3375	3515	3145	3285	2905	3045	2620	2620
ds11	50		3660	3660	3230	3350	2690	2940	2640	2770	2640	2640
ds12	50		4070	4070	3475	3645	3230	3350	2970	3260	2840	2840
ds13	60		4570	4570	3900	3900	3430	3430	3350	3470	3080	3200
ds14	60		4455	4455	3910	4150	3570	3570	3150	3440	2935	3105
ds15	60		4735	4735	4100	4100	3620	3620	3310	3310	3150	3270
ds16	70		5295	5295	4405	4405	3955	3955	3590	3840	3315	3435
ds17	70		4865	4865	4125	4125	3510	3510	3330	3330	3285	3285
ds18	70		5085	5085	4435	4555	3770	3890	3540	3540	3080	3080
ds19	80		5215	5215	4380	4380	3820	3990	3350	3350	3200	3200
ds20	80		5625	5625	4685	4685	4220	4375	3625	3625	3450	3450
ds21	80		5530	5530	4635	4635	4170	4170	3685	3685	3475	3475

Table 4.2: Results for the NFSNET topology, KSP solved to 6 alternatives

Percent Deviations of KSP method
from lower bounds for the NFSNET topology

DS	NC	w: 8	10	12	14	16
ds1	20	%0,0000	%2,1598	%8,7356	%9,2683	%9,2683
ds2	20	%0,8163	%2,1552	%2,2831	%2,2831	%2,2831
ds3	20	%4,0161	%2,4590	%0,4525	%0,4566	%0,4566
ds4	30	%0,3460	%4,1199	%2,8112	%3,0801	%0,4566
ds5	30	%4,5802	%2,0080	%2,0080	%0,4065	%2,2831
ds6	30	%0,0000	%1,0870	%0,7491	%0,4016	%0,0000
ds7	40	%0,3058	%0,7042	%7,4803	%2,0080	%2,0080
ds8	40	%1,8809	%7,5047	%0,0000	%0,0000	%0,0000
ds9	40	%0,0000	%0,0000	%0,0000	%0,0000	%2,0080
ds10	50	%0,0000	%0,2972	%2,1104	%1,7513	%5,2209
ds11	50	%3,6827	%6,2500	%0,0000	%0,0000	%0,0000
ds12	50	%0,1230	%3,1157	%0,0000	%4,0280	%4,0293
ds13	60	%0,5501	%4,5576	%0,0000	%3,3951	%2,3256
ds14	60	%0,0000	%1,9557	%1,7094	%6,7797	%5,1971
ds15	60	%2,7115	%5,3985	%0,0000	%1,5337	%2,9412
ds16	70	%2,6163	%0,4561	%2,4611	%1,4124	%6,9355
ds17	70	%1,9916	%0,6098	%2,6316	%1,2158	%3,3019
ds18	70	%0,7929	%2,1889	%0,0000	%0,0000	%0,0000
ds19	80	%3,1652	%1,3889	%0,7916	%1,2085	%0,0000
ds20	80	%1,9946	%0,0000	%2,1792	%0,0000	%2,9851
ds21	80	%1,3749	%1,4223	%0,0000	%0,0000	%0,0000

Table 4.3: Percent Deviations of KSP method from the lower bounds for the NFSNET topology

Max Percent Deviation : %9.2683

Average Percent Deviation: %1.9821

KSP result was optimum %25.714 of the time

Percent Deviations of Wavelength Assignment
from the lower bound for the NFSNET topology

DS	NC	w: 8	10	12	14	16
ds1	20	%0,0000	%2,1598	%8,7356	%9,2683	%9,2683
ds2	20	%0,8163	%2,1552	%2,2831	%2,2831	%2,2831
ds3	20	%4,0161	%2,4590	%0,4525	%0,4566	%0,4566
ds4	30	%4,4983	%4,1199	%2,8112	%3,0801	%0,4566
ds5	30	%4,5802	%6,8273	%2,0080	%5,2846	%7,7626
ds6	30	%3,7500	%1,0870	%5,2434	%0,4016	%0,0000
ds7	40	%5,5046	%0,7042	%7,4803	%2,0080	%2,0080
ds8	40	%6,2696	%7,5047	%0,0000	%0,0000	%0,0000
ds9	40	%0,0000	%3,9867	%0,0000	%0,0000	%2,0080
ds10	50	%0,0000	%4,4577	%6,6558	%6,6550	%5,2209
ds11	50	%3,6827	%10,1974	%9,2937	%4,9242	%0,0000
ds12	50	%0,1230	%8,1602	%3,7152	%14,1856	%4,0293
ds13	60	%0,5501	%4,5576	%0,0000	%7,0988	%6,3123
ds14	60	%0,0000	%8,2138	%1,7094	%16,6102	%11,2903
ds15	60	%2,7115	%5,3985	%0,0000	%1,5337	%6,8627
ds16	70	%2,6163	%0,4561	%2,4611	%8,4746	%10,8065
ds17	70	%1,9916	%0,6098	%2,6316	%1,2158	%3,3019
ds18	70	%0,7929	%4,9539	%3,1830	%0,0000	%0,0000
ds19	80	%3,1652	%1,3889	%5,2770	%1,2085	%0,0000
ds20	80	%1,9946	%0,0000	%5,9322	%0,0000	%2,9851
ds21	80	%1,3749	%1,4223	%0,0000	%0,0000	%0,0000

Table 4.4: Percent Deviations of Wavelength Assignment from the lower bounds for the NFSNET topology

Max Percent Deviation : %16.6102

Average Percent Deviation: %3.4178

Wavelength Assignment result was optimum %20.000 of the time

WIXC requirements for the NFSNET topology

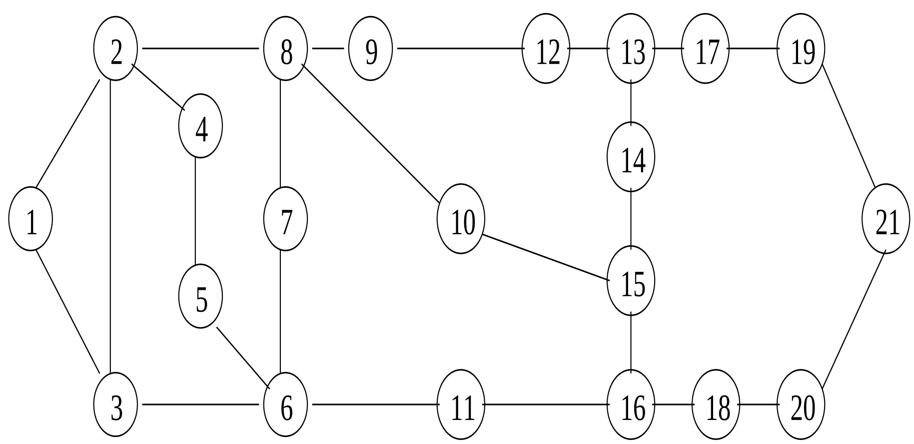
DS	NC	w:	8	10	12	14	16
ds1	20		none	none	none	none	none
ds2	20		none	none	none	none	none
ds3	20		none	none	none	none	none
ds4	30		none	none	4	none	4
ds5	30		none	none	none	none	none
ds6	30		none	none	6	none	none
ds7	40		5	8	8	none	none
ds8	40		none	none	none	none	none
ds9	40		5	6	none	13	none
ds10	50		none	none	10	10	none
ds11	50		none	none	none	none	none
ds12	50		none	none	8	5	6
ds13	60		none	2	none	4	none
ds14	60		none	none	5	5	8
ds15	60		none	5	none	6	none
ds16	70		none	none	none	6,11	6
ds17	70		none	none	none	none	none
ds18	70		none	6	6	none	none
ds19	80		none	none	5	none	none
ds20	80		none	none	13	5,11	13
ds21	80		none	none	none	5	none

Table 4.5: WIXC Requirements for the NFSNET topology

Node Frequencies for the NFSNET Topology	
Node	Frequency
5	29,0323%
6	25,8065%
8	12,9032%
13	9,6774%
4	6,4516%
10	6,4516%
11	6,4516%
2	3,2258%
1	0,0000%
3	0,0000%
7	0,0000%
9	0,0000%
12	0,0000%
14	0,0000%

Table 4.6: Node Frequencies for the NFSNET Topology

4.5 ARPA2



ARPA2 physical network with 21 nodes and 26 links

Figure 4.2: ARPA2 topology

Lower Bounds for the ARPA2 Topology

DS	NC	w:	8	10	12	14	16
ns1	40		7285	6285	5865	5590	5465
ns2	40		6525	5885	5590	5575	5535
ns3	40		5980	5815	5590	5475	5355
ns4	50		7575	6800	6155	5865	5625
ns5	50		7955	6840	6120	5625	5625
ns6	50		8010	7100	6230	6015	5590
ns7	60		8375	7245	6490	6240	5810
ns8	60		8575	7225	6485	5910	5660
ns9	60		9010	7650	6680	6260	5910
ns10	70		10335	8710	7790	7220	6465
ns11	70		10020	8485	7580	6485	6245
ns12	70		10340	8795	7935	6895	6530
ns13	80		10825	9035	8120	7010	6685
ns14	80		11635	9680	8500	7900	6720
ns15	80		10080	8560	7735	7090	6590
ns16	90		13110	10445	9135	8050	7560
ns17	90		13125	11010	9770	8250	7675
ns18	90		11645	9865	8665	7940	6950
ns19	100		12975	10430	9195	8050	7755
ns20	100		13115	10710	9145	8160	7760
ns21	100		13425	11355	9615	8310	7805

Table 4.7: Lower Bounds for the ARPA2 topology

Results for the ARPA2 topology, KSP solved to 6 alternatives

		KSP	W.A.	KSP	W.A.	KSP	W.A.	KSP	W.A.	KSP	W.A.
DS	NC	w: 8	8	10	10	12	12	14	14	16	16
ns1	40	7325	7325	6285	6285	5865	6035	5590	5730	5465	5465
ns2	40	6525	6705	6015	6015	5590	5770	5575	5745	5535	5535
ns3	40	6260	6790	5815	6140	5590	5770	5590	5770	5355	5525
ns4	50	7975	7975	6825	7035	6205	6535	5865	6215	5625	5795
ns5	50	7995	7995	6840	7020	6120	6640	5625	5975	5625	5805
ns6	50	8015	8015	7100	7270	6230	6810	6015	6515	5590	5980
ns7	60	8375	8375	7245	7245	6490	6660	6240	6410	5810	5810
ns8	60	8705	8705	7245	7245	6485	6935	5910	6655	5660	5830
ns9	60	9010	9270	7650	8000	6680	7300	6260	6985	5910	6410
ns10	70	10655	10655	8710	8710	7790	7960	7220	7710	6465	6905
ns11	70	10020	10020	8485	8485	7580	7750	6835	6835	6245	6595
ns12	70	10380	10560	8845	8845	7935	8315	6895	7245	6530	6710
ns13	80	11330	11330	9035	9035	8240	8410	7010	7180	6700	6840
ns14	80	11825	11825	9930	9930	8500	8830	7900	7900	7160	7330
ns15	80	10730	11050	8725	8895	7735	8085	7090	7090	6590	6730
ns16	90	13370	13370	10875	10875	9135	9135	8050	8385	7680	7680
ns17	90	13175	13175	11055	11195	9830	10000	8305	8305	7830	8000
ns18	90	11995	11995	9880	9880	8665	8805	7940	8110	6950	7090
ns19	100	12975	12975	10430	10430	9195	9195	8050	8400	7775	8205
ns20	100	13130	13130	11075	11075	9145	9145	8160	8160	7760	7760
ns21	100	13425	13425	11355	11355	9615	9615	8310	8310	7805	8155

Table 4.8: Results for the ARPA2 Topology, KSP solved to 6 alternatives

Percent Deviations of KSP Method
from the lower bounds for the ARPA2 topology

DS	NC	w: 8	10	12	14	16
ns1	40	%0,5491	%0,0000	%0,0000	%0,0000	%0,0000
ns2	40	%0,0000	%2,2090	%0,0000	%0,0000	%0,0000
ns3	40	%4,6823	%0,0000	%0,0000	%2,1005	%0,0000
ns4	50	%5,2805	%0,3676	%0,8123	%0,0000	%0,0000
ns5	50	%0,5028	%0,0000	%0,0000	%0,0000	%0,0000
ns6	50	%0,0624	%0,0000	%0,0000	%0,0000	%0,0000
ns7	60	%0,0000	%0,0000	%0,0000	%0,0000	%0,0000
ns8	60	%1,5160	%0,2768	%0,0000	%0,0000	%0,0000
ns9	60	%0,0000	%0,0000	%0,0000	%0,0000	%0,0000
ns10	70	%3,0963	%0,0000	%0,0000	%0,0000	%0,0000
ns11	70	%0,0000	%0,0000	%0,0000	%5,3971	%0,0000
ns12	70	%0,3868	%0,5685	%0,0000	%0,0000	%0,0000
ns13	80	%4,6651	%0,0000	%1,4778	%0,0000	%0,2244
ns14	80	%1,6330	%2,5826	%0,0000	%0,0000	%6,5476
ns15	80	%6,4484	%1,9276	%0,0000	%0,0000	%0,0000
ns16	90	%1,9832	%4,1168	%0,0000	%0,0000	%1,5873
ns17	90	%0,3810	%0,4087	%0,6141	%0,6667	%2,0195
ns18	90	%3,0056	%0,1521	%0,0000	%0,0000	%0,0000
ns19	100	%0,0000	%0,0000	%0,0000	%0,0000	%0,2579
ns20	100	%0,1144	%3,4080	%0,0000	%0,0000	%0,0000
ns21	100	%0,0000	%0,0000	%0,0000	%0,0000	%0,0000

Table 4.9: Percent Deviations of KSP Method from the lower bounds for the ARPA2 topology

Max Percent Deviation : %6.5476

Average Percent Deviation: %0.6860

KSP result was optimum %65.714 of the time

DS	NC	w: 8	10	12	14	16
ns1	40	%0,5491	%0,0000	%2,8986	%2,5045	%0,0000
ns2	40	%2,7586	%2,2090	%3,2200	%3,0493	%0,0000
ns3	40	%13,5452	%5,5890	%3,2200	%5,3881	%3,1746
ns4	50	%5,2805	%3,4559	%6,1738	%5,9676	%3,0222
ns5	50	%0,5028	%2,6316	%8,4967	%6,2222	%3,2000
ns6	50	%0,0624	%2,3944	%9,3098	%8,3126	%6,9767
ns7	60	%0,0000	%0,0000	%2,6194	%2,7244	%0,0000
ns8	60	%1,5160	%0,2768	%6,9391	%12,6058	%3,0035
ns9	60	%2,8857	%4,5752	%9,2814	%11,5815	%8,4602
ns10	70	%3,0963	%0,0000	%2,1823	%6,7867	%6,8059
ns11	70	%0,0000	%0,0000	%2,2427	%5,3971	%5,6045
ns12	70	%2,1277	%0,5685	%4,7889	%5,0761	%2,7565
ns13	80	%4,6651	%0,0000	%3,5714	%2,4251	%2,3186
ns14	80	%1,6330	%2,5826	%3,8824	%0,0000	%9,0774
ns15	80	%9,6230	%3,9136	%4,5249	%0,0000	%2,1244
ns16	90	%1,9832	%4,1168	%0,0000	%4,1615	%1,5873
ns17	90	%0,3810	%1,6803	%2,3541	%0,6667	%4,2345
ns18	90	%3,0056	%0,1521	%1,6157	%2,1411	%2,0144
ns19	100	%0,0000	%0,0000	%0,0000	%4,3478	%5,8027
ns20	100	%0,1144	%3,4080	%0,0000	%0,0000	%0,0000
ns21	100	%0,0000	%0,0000	%0,0000	%0,0000	%4,4843

Percent Deviations of Wavelength Assignment from the lower bounds for the ARPA2 topology

Table 4.10: Percent Deviations of Wavelength Assignment from the lower bounds for the ARPA2 topology

Max Percent Deviation : %13.5452

Average Percent Deviation: %3.1677

Wavelength Assignment result was optimum %21.905 of the time

WIXC Requirements for the ARPA2 Topology

DS	NC	w:	8	10	12	14	16
ns1	40		none	none	15,16	16	none
ns2	40		13	none	13	15	none
ns3	40		2,13,16	13,15	13	13	15
ns4	50		none	2	2,8	6	8,13,15
ns5	50		none	13	6,15	13,15	13
ns6	50		none	15,16	13,15,16	8,16	2,13,16
ns7	60		none	none	15	15,16	none
ns8	60		none	none	6,15	6,15,16	5,15,16
ns9	60		6,8	13,15	8,13,15	8,13,15	8,15,16
ns10	70		none	none	15	6,16	13
ns11	70		none	none	15,16	none	6
ns12	70		13	none	2,10,15	13,15	6,13
ns13	80		none	none	15	2,13,15	16
ns14	80		none	none	8	none	15,16
ns15	80		13,16	15,16	2,16	none	8,16
ns16	90		none	none	none	15,16	none
ns17	90		none	16	15,18	none	15
ns18	90		none	none	16	16	15,16
ns19	100		none	none	none	13,15,16	8,16
ns20	100		none	none	none	none	none
ns21	100		none	none	none	none	8,13,16

Table 4.11: WIXC Requirements for the ARPA2 Topology

Node Frequencies for the ARPA2 Topology

Node	Frequency
15	28,4404%
16	24,7706%
13	21,1009%
8	10,0917%
6	7,3394%
2	5,5046%
5	0,9174%
10	0,9174%
18	0,9174%
1	0,0000%
3	0,0000%
4	0,0000%
7	0,0000%
9	0,0000%
11	0,0000%
12	0,0000%
14	0,0000%
17	0,0000%
19	0,0000%
20	0,0000%
21	0,0000%

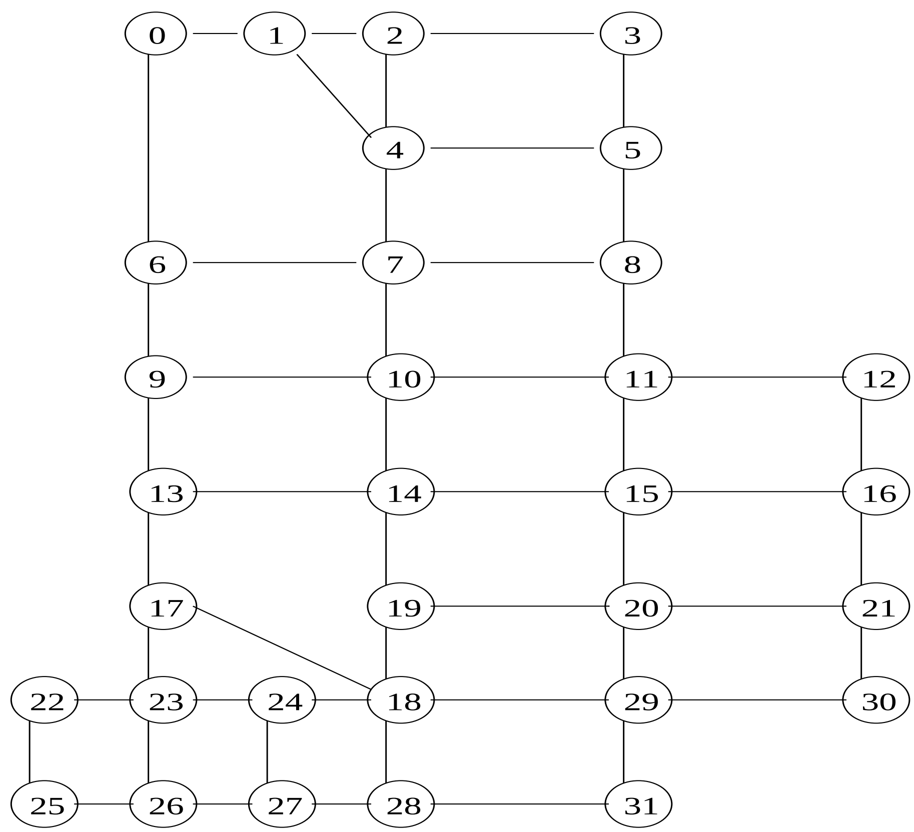
Table 4.12: Node Frequencies for the ARPA2 Topology

4.6 MESH32

Lower Bounds for the MESH32 Topology

DS	NC	w:	8	12	16
ls1	60		15516	12657	11300
ls2	60		16422	12984	11956
ls3	60		15441	12477	10815
ls4	80		19759	14877	12824
ls5	80		18278	14189	12504
ls6	80		17005	13565	11810
ls7	100		22850	17034	14187
ls8	100		23230	17293	14332
ls9	100		22129	16791	13871

Table 4.13: Lower Bounds for the MESH32 Topology



MESH32 physical network with 32 nodes and 50 links

Figure 4.3: MESH32 topology

Results for the MESH32 Topology, KSP solved to 6 alternatives

			KSP	W.A.	KSP	W.A.	KSP	W.A.
DS	NC	w:	8	8	12	12	16	16
ls1	60		15973	17279	13550	13854	13058	13058
ls2	60		17085	18914	14188	14940	13292	14225
ls3	60		15732	17013	13266	13272	12990	13187
ls4	80		20149	21060	15271	17948	13899	16047
ls5	80		19147	19640	15102	16302	13590	14165
ls6	80		17874	18498	14313	14953	12624	14214
ls7	100		25028	26861	17971	20911	14991	16903
ls8	100		24580	24936	18403	20562	14996	17399
ls9	100		23756	24605	17461	19593	14511	17389

Table 4.14: Results for the MESH32 Topology, KSP solved to 6 alternatives

Percent Deviations for the MESH32 Topology

			KSP	W.A.	KSP	W.A.	KSP	W.A.
DS	NC	w:	8	8	12	12	16	16
ls1	60		%2,9453	%11,3625	%7,0554	%9,4572	%15,5575	%15,5575
ls2	60		%4,0373	%15,1748	%9,2730	%15,0647	%11,1743	%18,9779
ls3	60		%1,8846	%10,1807	%6,3236	%6,3717	%20,1110	%21,9325
ls4	80		%1,9738	%6,5843	%2,6484	%20,6426	%8,3827	%25,1326
ls5	80		%4,7543	%7,4516	%6,4346	%14,8918	%8,6852	%13,2837
ls6	80		%5,1103	%8,7798	%5,5142	%10,2322	%6,8925	%20,3556
ls7	100		%9,5317	%17,5536	%5,5008	%22,7604	%5,6672	%19,1443
ls8	100		%5,8115	%7,3440	%6,4188	%18,9036	%4,6330	%21,3997
ls9	100		%7,3523	%11,1889	%3,9902	%16,6875	%4,6139	%25,3623

Table 4.15: Percent Deviations for the MESH32 Topology

Max Percent Deviation for KSP : %20.1110

Average Percent Deviation for KSP: %6.7510

Max Percent Deviation for WA : %25.3623

Average Percent Deviation for WA : %15.2510

WIXC Requirements for the MESH32 Topology

DS	NC	w:	8	12	16
ls1	60		11,12,14,15,20,29	14	none
ls2	60		12,14,15,16,19,20	7,14,16,20	11,14,15,16,19,20
ls3	60		11,12,14,15,16,18,21	11,12,14,18,19,20	20
ls4	80		8,11,14,15,16,18	7,8,11,14,16,29	12,14,15,16,18,29
ls5	80		11,15,17,18	7,8,14,28	5,14,15,29
ls6	80		8,15,18,19,29	11,15,19,20	15,20,21,25,28
ls7	100		15,16,18,29,30	10,11,14,16,21	12,14,15,16,21
ls8	100		9,11,12,15,16,25	8,11,14,25	7,9,14,15,16,25,28,29
ls9	100		5,9,11,14,15,30	9,12,14,15,24,29	10,11,12,18,19

Table 4.16: WIXC Requirements for the MESH32 Topology

Node Frequencies for the MESH32 Topology	
Node	Frequency
14	%13,7405
15	%12,9771
11	%9,9237
16	%9,1603
12	%6,8702
18	%6,1069
20	%6,1069
29	%6,1069
19	%4,5802
8	%3,8168
7	%3,0534
9	%3,0534
21	%3,0534
25	%3,0534
28	%2,2901
5	%1,5267
10	%1,5267
30	%1,5267
17	%0,7634
24	%0,7634
1	%0,0000
2	%0,0000
3	%0,0000
4	%0,0000
6	%0,0000
13	%0,0000
22	%0,0000
23	%0,0000
26	%0,0000
27	%0,0000
31	%0,0000
32	%0,0000

Table 4.17: Node Frequencies for the MESH32 Topology

Chapter 5

Conclusion

In this thesis, a detailed analysis of network design problems in wavelength division multiplexing optical networks has been carried out. The problem was proved to be NP-Hard. In spite of this fact, straightforward optimization was tried at first, which was unsuccessful because of the huge number of variables involved and the high degree of degeneracy of the models. Next, the problem was decomposed into two parts, to make the corresponding models more manageable. First problem was the routing problem and the second was the wavelength assignment and interchanger location problem. Appropriate methods for solving the subproblems were developed. A complete procedure for generating "good" feasible solutions was defined. The procedure was tested on three different real-world topologies for varying amounts of traffic load and the results seemed to be close to the lower bounds generated. The results suggested that the cost performance of the case where all nodes are wavelength interchangers can be attained with a relatively low number of wavelength interchangers. The location of the wavelength interchangers tends to be in the 'middle' of the graph, and at the nodes where two or more faces intersect. But since the performance of the wavelength interchangers heavily depend on the traffic, instead of placing the wavelength interchangers on some predetermined nodes,it is better to let the corresponding formulation to decide on the location of the wavelength interchangers.

It is mentioned in the literature that as the average number of links that the connections share, namely the interference length, decrease, it is harder to solve the wavelength assignment problem and the benefit of wavelength interchanging increases ([26]). During the experimental stage of this study, it was noticed that the second and third stages of the overall procedure heavily depend on the routing. KSP method was used for solving the routing problem and the alternatives were costed according to the number of links they use. This approach minimizes the total number of links used by connections while minimizing the total fiber cost. However, it is not entirely same with assuring a minimal interference length, or maximizing the interference length. Notice that the interference length depends on *all* choices of routing. Thus, maximizing the interference length while minimizing the total fiber cost is not an easy problem. Search for a better method of routing or the KSP method with a better costing for the alternatives that takes the interference length into account may be areas of further research.

Three different formulations with strong and weak points, together with three sets of valid inequalities are presented in Chapter 2. Unfortunately, these formulations were not very useful for generating good solutions. Another topic to be investigated may be the rounding techniques and heuristics that use the optimal solution of the LP relaxation of an MIP.

Yet another area of further research may be the local search techniques such as tabu search and simulated annealing. Shortest path routing can be obtained in polynomial time, and a local search technique may be used to further "polish" this initial solution. For example, fixing the routing of all but a few of the connections and rerouting those connections to improve the objective function value of the existing solution may be a way of generating solutions. How to choose the connections to be rerouted and which local search technique to use are questions to be considered.

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