THE EMPIRICAL PERFORMANCE OF TRANSACTION PERIOD LENGTH IN CASH-IN-ADVANCE MODELS OF MONEY DEMAND

A Master's Thesis

By

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July 2001

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ABSTRACT

THE EMPIRICAL PERFORMANCE OF TRANSACTION PERIOD LENGTH IN CASH-IN-ADVANE MODELS OF MONEY DEMAND

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M. A. in Economics

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July 2001

Cash-in-advance models of money demand make strong predictions about stochastic properties of key endogenous macroeconomic variables. Hodrick, Kocherlakota, and Lucas (1991) showed that the models by Lucas (1984), Svensson (1985), and Stokey, and Lucas (1987) were not generating dataset consistent with the sample values. In this study we investigate whether Arnwine (2000) can generate consistent statistical values of key endogenous variables. We use the same methods as Hodrick, Kocherlakota, and Lucas (1991).

Keywords and phrases: Cash-in-advance models of money demand, transaction

ÖZET

PARA ÇEKME ARALIĞI EKLENMIŞ ÖN ÖDEME KISITLI PARA TALEBİ MODELLERİNİN EMPİRİK PERFORMANSI

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Temmuz 2001

Ön ödeme kısıtılı para talebi modelleri makroekonomik önemli değiskenler hakkında oldukça güçlü tahminler yapmaktalar. Yalnız, Hodrick, Kocherlakota ve Lucas (1991) gösterdiki, Lucas (1984), Svensson (1985) ve Stokey ve Lucas (1987) ön ödeme kısıtlı modelleri, makroekonomik datalarla tutarlı tahminler yapamiyorlar. Biz bu çalışmada, para çekme aralığı eklenmiş ön ödeme kısıtlı para talebi modellerinin empirik performansını inceliyoruz. Bunun için Hodrick, Kocherlakota ve Lucas (1991)'ın methodlarını kullaniyoruz.

Anahtar kelimeler ve ifadeler: Ön ödeme kısıtı, para çekme aralığı

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to Assist. Prof. Dr. Neil Arnwine for his excellent supervision. His patience and expert guidance bring my research up to this point. I am grateful to Erdem Basci, Levent Akdeniz and Sabit Khakimzhanov for showing keen interest to the subject and matter and accepting to rewiev this thesis.

I am grateful to my family for their love and encouragement.

I owe special thanks to my friends: Zafer Akin, Mehmet Germeyanoglu, Zeynal Karaca, Orhan Hasanaliyev, O. Faruk Baykal, Mustafa Kesir, Erkan Kose, Salih Ucar, Mustafa Gok, Ismail Tevrizoglu. They were all with me during my study.

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CHAPTER 1

INTRODUCTION

In this study we test whether transaction period length in cash-in-advance models can generate reasonable endogenous variables, consistent with the sample values. Qualitatively we find that making transaction period length endogenous enables cash-in-advance models generate substantial variation in most of the endogenous variables.

Cash-in-advance models of money demand are very strong in the sense that within these models explicit formulas for the key endogenous variables may be obtained. Lucas (1980), Svenson (1985), and Lucas and Stokey's (1987), cash-in-advance models make theoretical predictions about income velocity of money, inflation rate, nominal interest rate, and realized real interest rate. However, these CIA models have not been supported empirically. For instance studies by Giovannini, and Labadie (1991) and Hodrick, Kocherlakota and Lucas (1991) showed, by using calibrated discrete state space representation of the economy, that predictions of these models were not consistent with the realized data. They failed to generate reasonable endogenous variables given the consumption growth and money growth exogenous.

Cooley and Hansen (1989) study the welfare costs of an inflation tax using a cash-in-advance model of an economy. They find that when economy is simulated by using monthly data the welfare cost of inflation tax is much lower. This is because under inflation transacting more often is better. In their case, as the transaction period length is exogenously fixed, shorter transactions length, which is one month, will have less welfare cost.

Arnwine (2000) introduces the transaction period length concept into cash-in-advance models. This innovation is an attempt to improve upon the performance of CIA models. He focuses on the frequency with which consumers conduct financial transactions, and the role that this plays in the determination of the price level and the money velocity. The study of finance period length has been extensively considered with the major contributions being Baumol (1952), Tobin (1956), and Miller and Orr (1966). The innovation of the paper is that the transaction period length is determined in a general equilibrium, cash-in-advance setting. In the model, the consumer may obtain cash, in a costly transaction. The consumer selects cash balances to balance the opportunity cost of holding cash with the cost of incurring transaction. Since there is an opportunity cost of holding money, a consumer will only transact when cash balances have reached to zero. The inclusion of this innovation in an empirical CIA model should, at least enable the income velocity of money to be more variable compared to the other models mentioned above.

Section 1.1- Theoretical Cash-in-Advance Literature

The cash-in advance literature first started with Clower (1967). He assumes that, a consumer cannot spend more than the money he has on hand, and indicates this as a constraint for the consumers. This brought a new microfoundation for the demand for money.

Lucas (1980) is a theoretical study of determination of prices, interest rates and currency exchange rates, set in an infinitely lived two-country world. This economy is subject to both to stochastic endowment shocks and to monetary instability. The demand for money is derived by treating money symmetrically with other assets. Money has a value and is held because it gives return-liquidity services, whereas ordinary assets have direct value and are held because they give return-dividends. The price of money can be determined by an asset pricing equation as the price of the other assets, if the liquidity services were specified. He specified a stochastic steady state where all prices and interest rates are endogenous functions of exogenous stochastic processes of money supply and output. Also he derives the demand for money via cash-in-advance constraint instead of postulating that real balances give direct utility.

Lucas (1980) predicts unit velocity because of the information structure of the model. First, people learn exactly how much they will buy in the current period, and second they purchase using currency balances, which they accumulated for this purpose. This makes people to accumulate money just as much they will use in the period.

Svensson (1985) provides an innovation to Lucas (1980) of less complicated precautionary money demand, which allows variable money velocity. Svensson (1985) is a study of the demand for money, of the determination of the price level and nominal and real interest rates. Svennson finds that money is priced just like any of asset in a CAPM setting. Also he searches how the asset prices are affected by changes in money supply and in income.

Svensson (1985) model differs from Lucas's (1982), in that consumers must decide on their cash balances before they know the current state and hence before they know their consumption. This timing gives rise to combined transactions, precautionary, and store-of-value demand for money. Since he allows nonbinding liquidity constraints, the simple quantity equation does not hold, and he gets a more general, and more reasonable demand for money. He provides a microfoundation for money demand within a general equilibrium setting. Relative to Lucas (1982) he has a more reasonable demand for money with variable velocity.

Lucas, and Stokey (1987) introduce a credit good in order to allow for the velocity of money to fluctuate. Consumers can make some purchases with credit or cash. The setting of the model allows more variation in velocity. This is made by both information structure and the possibility of substitution of the money with credit.

Inserting transaction cost in the economy is not new in the literature. Brito, and Hartley (1995) compare the optimality of holding credit cards, and the high interest rate burden of credit cards, to not holding credit cards and transaction costs that comes from not holding credit cards debts.

Section 1.2- Empirical Performance of Cash-in Advance Literature

Hodrick, Kocherlakota, Lucas (1991) explores whether this class of models can produce realistic predictions about stochastic properties of the endogenous variables when the exogenous process is taken as the money growth and consumption growth. They develop new numerical methods to solve these models. They simultaneously examine for fifteen statistics including variation of velocity, correlations of velocity with money growth, consumption growth and inflation. They also search for expected values of inflation interest rate and money growth and their standard deviations, and correlations of inflation with money growth, nominal interest rate and realized real interest rate.

HKL (1991) find that in the cash-only model the predicted velocity of money is always constant. In the cash-credit model of Lucas, Stokey (1987) velocity does vary because agents have the chance of substitution between cash and the goods. But, the cash-credit model is unable to generate realistic predictions about the other endogenous variables when the parameters are chosen to generate reasonable variability in velocity. The cash only model has general poor performance; only 3 of 15 statistics fall in the range of the predictions of the model. The cash credit model seems to be more successful compared to cash only model, however only 5 of the 15 statistics fall in the range of model's predictions.

In a complementary study to HKL (1991), Giovannini, Labadie (1991), develop a method to solve and simulate cash-in-advance models of money and asset prices. The most important result of their study is the very high covariation between ex ante returns on stocks and nominal bonds. They also find that the real returns on stocks are only occasionally negatively related to inflation being in conflict with the data, and that ex ante real interest rates are uncorrelated with expected inflation. The mean velocity and the variation of velocity are smaller than the sample value. Actually, the velocity is unitary for Lucas (1982), and Svensson (1985), and variation zero.

In this study we use the methods of Hodrick, Kocherlakota and Lucas (1991) to test the value of the variable period length innovation. For easy comparison, we simulate an economy in the same way as HKL (1991) but with a variable number of transactions per period. We have taken the annual VAR and the solution concept directly from HKL (1991). To solve for the equilibrium we generated an algorithm (for cash only model).

We calculate velocity, real and nominal interest rates, inflation and real balances. We consider only the cash-only version tested by HKL (1991)

Our goal is to explain relationships between endogenous variables. The unconditional moments we examine are the same as HKL (1991). We also arrange the parameter ranges the same as HKL (1991). Following HKL (1991) to allow the model greatest chance of success, we calculate several first order and second order unconditional moments for some variables of interest over a large parameter range using a specific utility function.

We have written the program in Gauss, and it is in Appendix A.

The thesis is organized as follows. Chapter 2 describes the model and gives the solution. Chapter 3 gives the formulas that are used to calculate endogenous variables. Chapter 4 contains a description of resources of the data and estimation of the vector auto regression. Chapter 5 presents the predictions of the models. Chapter 6 gives the conclusions. In appendix A, I give the program that solves the model and calculates the predictions of the model. Appendix B presents tables. Bellman equation is in appendix C.

CHAPTER 2

THE MODEL, THE SOLUTION AND THE METHOD

Section 2.1- The Model

There is a representative agent exchange economy. The agent chooses the amount he will consume and the amount of the money would use to buy assets. The model differs from Lucas and Stokey (1987) and Svensson (1985) in the way that the agent may transact when he needs. Financial transaction is an exchange in which the consumer receives money balances. The single consumption good must be purchased with money, so the consumer is required carry a non-negative money balance. So the consumer also chooses the transaction period length. As holding cash has an opportunity cost the agent will not transact until the cash balances has reached to zero. The aggregate money supply in that period is X_t and the aggregate non-storable endowment at time is y_t. We denote $\frac{X_{t+1}}{X_t}$ as

ω_t and $\frac{y_t}{y_{t-1}}$ as γ_t. Let {γ t, w_t} stationary ergodic Markov chain with transition probability matrix Π, where the typical element Π_{is} gives the probability of moving from state i to state s.

A representative consumer maximizes the discounted utility of consumption over an infinite lifetime,

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U(c_t) \qquad \text{t=0, 1, 2,...} \tag{1}$$

Where β is the discount factor, c_t is the consumption in the period, and $U(.)$ is the utility function. The consumer's utility function is,

$$U(c_t) = \frac{c_t^{1-\alpha} - 1}{1-\alpha} \tag{2}$$

where $1-\alpha$ is the curvature of the utility function.

The consumer obtains cash in a costly financial transaction, and then uses the cash to purchase consumption goods until the cash is depleted. Then another financial transaction is undertaken. The CIA constraint for this problem is:

$$c_t \leq \frac{M_t}{P_t} \cdot n_t \qquad (3)$$

where M_t is the nominal money transacted, and P_t is the price level. The agent begins the period holding cash M_t, and conducts n_t transactions per period. The agent's budget constraint, in real terms is,

$$\frac{M_{t+1}}{P_t} + \frac{Q_t}{P_t} z_{t+1} + c_t + T_t n_t = (\frac{Q_t}{P_t} + y_t) z_t + (w_t - 1)\frac{X_t}{P_t} + \frac{M_t}{P_t}) \qquad (4)$$

Q_t is the nominal stock price. T_t is the cost of transaction, and consumer takes it as given. n_t is the number of transactions, therefore the velocity of money, and z_t is the share. The right side of the equilibrium constitutes the wealth of the agent. It consists of any unspent cash balances, receipts of nominal dividends $z_t y_t$, the resale value of stock holdings $\frac{Q_t z_t}{P_t}$, and the lump-sum monetary transfers $\frac{(\omega_t - 1)}{P_t} X_t$.

The consumer's problem is to select simultaneously the initial real money balance, the length of the time between financial transactions, and the investment share that he will trade in the security market, ant the consumption amount that will maximize his life time expected utility, given the prizes P_t and Q_t given. The timing is as follows: By the beginning of the period he learns the state, and chooses $c_t, n_t, M_{t+1}, z_{t+1}$ given

M_t, z_t. From the last period also given X_t, random endowment, the consumer takes P_t, T_t, Q_t as given from market equilibrium.

In equilibrium markets should clear, this implies,

$z_{t+1} = 1$

$y_t = c_t + n_t T_t$

$M_{t+1} = X_{t+1}$

Following HKL (1991) we analyze the stationary equilibria, in which the prices depend on the current state. In our setting we assume that the current state is the current level of money supply, the lagged level of real endowment, and the current rates of growth of money and endowment.

Below I give the the meanings of the abreviations.

c	consumption
y	output
M	money demand
X	money supply
T	transaction cost
Q	nominal stock price
z	share of capitol stock owned by the agent
γ	otuput growth
ω	money growth
λ, μ	lagrange multipliers of budget, and cash-in-advance constraints
t	time

Section 2.2-The Solution of the Model

The first order conditions for the solution are given below:

$$u_{c,t} = \lambda_t + \mu_t \quad (5)$$

$$\mu_t \frac{M_t}{P_t} = T_t \lambda_t \quad (6)$$

$$\frac{\lambda_t}{P_t} = \beta E\left[\frac{\lambda_{t+1}}{P_{t+1}} + \frac{\mu_{t+1} n_{t+1}}{P_{t+1}}\right] \quad (7)$$

$$\lambda_t \frac{Q_t}{P_t} = E_t\left[\left(\frac{Q_{t+1}}{P_{t+1}} + y_{t+1}\right)\lambda_{t+1}\right] \quad (8)$$

Equation 5 is the first order condition with respect to consumption good. The marginal utility of consumption is equal to the sum of the shadow prices of budget constraint and the CIA constraint. Equation 6 is the first order condition with respect to velocity of the money, or equivalently number of transactions in a period that is consumers hold money until the transactions value of real balances equals the utility value of transacting. Equation 7 is the first order condition with respect to M_{t+1}, which is obtained by using envelope theorem. Equation 7 differs from the usual cash-in-advance money demand condition by its inclusion of a velocity term. Varying the velocity of money allows the consumer to alter the equilibrium transaction value of money. Equation 8 is the first order condition with respect to z_{t+1}. It is standard capital asset pricing equation.

To simplify capital asset pricing equation assume that, in equilibrium,

$$T_t \equiv \Phi \frac{M_t}{P_t} \quad (9)$$

This implies that the real cost of transacting is proportional to the level of real balances transacted. T represents the physical cost to society of providing transacting technology in the form of real balances. We view equation 9 as an equilibrium condition only. Under the model the consumers are price takers with respect to T. Under this assumption, equation 6 becomes,

$$\mu_t = \Phi \lambda_t \quad (10)$$

Then equation 5 becomes,

$$u_{c,t} = (1+\Phi)\lambda_t \quad (11)$$

By using equation 8 and 11 and obtain equation 12,

$$u_{c,t} = \beta E_t \left[(1+\Phi n_{t+1}) u_{c,t+1} \left(\frac{P_t}{P_{t+1}}\right) \right] \quad (12)$$

To solve 12, we make some substitutions from equations (1) through (11).

$$c_t \equiv \frac{M_t}{P_t} n_t$$

$$y_t - T_t n_t = \frac{M_t}{P_t} n_t$$

$$y_t = (1+\Phi)\frac{M_t}{P_t} n_t$$

$$c_t = (1+\Phi)^{-1} y_t$$

The first equation comes from cash-in-advance constraint, second equation comes from market clearing condition, and the others follow. By the above equations we have that,

$$\frac{P_t}{P_{t+1}} = \frac{n_t}{n_{t+1}} \frac{y_{t+1}}{y_t} \frac{M_t}{M_{t+1}} \tag{13}$$

By substituting equation 13 in 12 we obtain, the following equation,

$$l_t = \beta E_t \left[(l_{t+1} + \Phi).\gamma_{t+1}^{1-\alpha} \right] \frac{1}{\omega_t} \tag{14}$$

where $l_t = \frac{1}{n_t}$, that is the length of the period of the transaction. Equation 14 is a contraction mapping therefore it has unique solution. We solve (14) computationally by using Tauchen and Hussey's (1991) technique.

We can write equation 14 in the form of states as follows,

$$l_i = \beta E_i \left[\left(l_j + \Phi \right) \gamma_j^{1-\alpha} \right] \frac{1}{\omega_i} \quad (15)$$

In equation 15, i is the current state and j is the other state that is probable to occur in the next period after the current state.

Section 2.3- The Method

We follow the solution method of Hodrick, Kocherlakota and Lucas (1991). They do not restrict CIA constraint in all states. They obtain exogenous process by estimating bivariate vector autoregressions (VAR) using quarterly and annual data on consumption growth and money growth data. They use Tauchen, and Hussey's (1991) method to approximate these VARs by a Markov chain. This method allows calculating formulas for endogenous variables.

The key to calculating the equilibrium values of endogenous variables is to approximate equation 15. As mentioned in section 2.2, i is the current state and j is the other state that is probable to occur in the next period after the current state. However j can take sixteen possible values because the Markov Chain we use in our study is 16 state one. As Tauchen, and Hussey (1991) gives transition probabilities from one state to another, we are able to calculate the expectation equation in 15.

CHAPTER 3

FORMULAS OF THE ENDOGENOUS VARIABLES

Formulas for velocity, realized real interest rate and nominal interest rate, inflation and the growth of real balances are calculated by Svensson(1985).

Our formulas are also similar, but different for velocity, inflation and money growth. The formulas are listed in Table1.

We calculate velocity of circulation of velocity as $v_i = n_i = \frac{1}{l_i}$. Formula for money growth is obtained as follows,

$$M_i = \frac{P_i}{c_i} n_i \qquad \text{(From equation 3)}$$

So, given the current state is i and next state is j the money growth rate will be

$$m_g = \gamma_j \frac{l_j}{l_i} - 1.$$

To calculate expected inflation we define,

$E\left(\frac{P_j}{P_i}\right) = \frac{\omega_j}{\gamma_j}\frac{l_i}{l_j}$. This is from equation 13.

.

CHAPTER 4

DATA AND ESTIMATION OF THE VECTOR AUTOREGRESSIONS

In our analysis we use the vector autoregression of money growth and consumption growth that is estimated by HKL (1991). This was for complete comparison of the models they tested and the model we test sources.

Tauchen (1987) describes quadrature procedure that constructs approximating Markov chains for VARs. This procedure chooses grid points and transition probabilities to match the conditional moments of the estimated (Gaussian) VAR. HKL (1991) estimates VARs by using this method using 16 states. Also they check the method's ability to estimate the VARs by estimating the VARs from the data generated from the Markov chain. The two VARs correspond very closely.

Hodrick, Kocherlakota, and Lucas (1991) obtain the Markov process from VARs by using the procedures developed by Tauchen (1991). The VAR is estimated with annual (1950-1986). The data series and sources are listed below.

Real per capita consumption is the sum of consumption in 1982 dollars of nondurables and services divided by total population. The price level is the sum of the current dollar series divided by consumption measured in 1982 dollars. The per capita money stock is M2 divided by total population.

They chose M2 as the monetary aggregate because a first order Markov process in the growth rates of money and endowment implies a stationary velocity in the models. Since M1 velocity appears to be nonstationary over the sample period the models would be rejected immediately. So M2 was more appropriate for calibrating the CIA models they used.

In their setting they assume the exogeneous process is first order Markov. They use Schwarz (1978) criterion and likelihood ratio tests to asses the appropriate order of the VAR. They alo report that a first order VAR is adequate. The Schwarz criterion always suggests the lower dimensional model.

DATA RESOURCES

Below I list the data sources that Hodrick, Kocherlakota and Lucas (1991) used. This part is taken from HKL (1991), Appendix C.

Annual data come from the 1987 *Economic Report of the President* unless otherwise indicated. Consumption of nondurables and services in 1982 (current) dollars: table B-2 (B-1); 1987 observations from the 1988 report. Population: table B-31; 1986-87 observations from the 1988 report. Money stock: M2; 1948-83 from Balke and Gordon (1986); 1984-87 from 1988 report. Nominal interest rate: Commercial paper rate for 4-6 month maturity; 1950087 from 1988 report, table B-71.

CHAPTER 5

PREDICTIONS OF THE MODEL

Using the algorithm obtained in chapter 2 and the Markov chains obtained from Tauchen, and Hussey (1991) we calculate the predictions of the model for the joint distribution of the endogenous and exogenous variables. We investigate whether the model can generate statistics consistent with sample moments computed from U.S. time-series data.

To compare the Arnwine (2000) with the Lucas, Stokey (1987) and Svensson (1985) we follow the same path as HKL (1991). Following HKL (1991) we calculate the several first and second order unconditional moments. Also we use the parameter range same as Hodrick, Kocherlakota and Lucas (1991). In our study, the parameter ranges are, $\beta \in \{.9,.92,....,1\}$, $\alpha \in \{0,.5,....,9.5\}$ and we add values for $\Phi \in \{.01,.003,...,.28\}$.

We investigate the same 15 statistics, HKL (1991). The statistics we investigate are; variation of velocity, correlations of velocity with money growth, output growth, and nominal interest rate, means and standard deviations of real and nominal interest rate, inflation, and real balance growth, correlations of inflation with money growth, consumption growth and nominal interest rate.

Table 2 lists the prediction of the cash-only model obtained by Hodrick, Kocherlakota and Lucas (1991). The predictions of the model, as they note, are poor. The most important failure is that, the model predicts no variation of the velocity. The other predictions are also not good. Only 3 of 15 sample statistics fall in the range attainable by the model predictions. The model also predicts constant expected inflation, and expected money growth in all states, which is not realistic.

Table 3 lists the predictions of our modified model. The model successfully generates variation of velocity. It also generates variation for all other statistics. 4 of 15 sample statistics fall in the range attainable by the model predictions. These three statistics are variation of velocity, expected inflation, expected real interest rate ,and correlation of inflation withrealized real interest rate. The variance in velocity mainly determined by the money shocks. This can be understood from the correlation between variance of velocity and money growth, which is very close to 1. While cash-only models of Lucas (1982) and Svensson (1985) predicts constant expected inflation and expected money growth regardless of the state, Arnwine (2000) predicts different values for different states. In our case expected nominal interest rate is independent of parameters α, β. It is determined by Φ.

CHAPTER 6

CONCLUSIONS

In this study we investigate whether Arnwine (2000) improves the capability of an empirical cash-in-advance model to generate deviations in velocity. For this, we use the same techniques as Hodrick, Kocherlakota, and Lucas (1991) use. We use the annual VAR of money growth and consumption growth obtained by them, by using US time series data, for complete comparison. We set the parameter range the same as theirs' for the same reason. We compute the same statistics predicted by the model again for comparison.

Lucas (1984) and Svensson (1985) show that adding about future cash needs can in principle allow velocity to vary. Hodrick, Kocherlakota, and Lucas (1991) show that such models are unsuccessful at generating any variation in velocity of money.

In this study, we find that the addition of a variable transactions period length allows for a great deal of variation in money velocity. Also adding such a variable to model improved the predictions of expected inflation and expected money growth.4 of 15 statistics fall in the range of the predictions of the model. 3 of 15 statistics fall in the range of predictions of cash-only model tested by HKL (1991).These 4 statistics are: variance of velocity, expected inflation, expected realized interest rate, and correlation between inflation and realized real interest rate.

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APPENDIX A

```
/*This program computes the equilibrium values of the endogenous variables*/

load pim[16,16] =c:\yavuz\transition.txt;
load t1[16,1]=c:\yavuz\sigma.txt;
load t2[16,1]=c:\yavuz\sigma1.txt;
load pist[16,1]=c:\yavuz\pisl.txt;

/*These are matrix from tauchen's program, annual case. First is transition probability
matrix*/
/* Second and third are money and endowment growth, and the last is the */
/*Stationary distribution matrix*/

a=1;                          /*initial alpha value */
fi=0.01;                      /*initial fi value*/
beta=0.9;                     /*initial beta value*/
b=a-1;
m=t2;

/*These loops are for calculating for different values of alpha, beta and fi*/

do while beta<1.02;   fi=0.01;
      do while fi<0.3;     a=1;
            do while a<10;

/* initial value for length of financial transaction*/

lengthk=ones(16,1);
length=ones(16,1);
label:
k=1;


          do while k<17;
                length[k,1]=(beta*(pim[k,.]*(t2.*t2^-a)*( lengthk[k,1]+fi)))*1/t1[k,1];
                k=k+1;
          endo;

m1=lengthk-length;
lengthk=length;
i=1;

/* this loop is for convergence */
```

```
        do while  i<17;
               if abs(m1[i,1])>0.001;
                    goto label;
               endif;
               i=i+1;
        endo;

mu=(fi/(1+fi)).*t2^-a;
m=t2.*length;
lambda=(1/(1+fi)).*t2^-a;

/*initial matrices for endogenous variables*/

infrate=zeros(16,16);
mg=zeros(16,16);
r=zeros(16,16);
i=zeros(16,1);
unity=ones(1,16);
unit=ones(16,1);
forexp=zeros(16,16);
forexpec=zeros(16,1);
forexpmg=zeros(16,16);
forexpr=zeros(16,16);
expinf=0;

/*this loop is for calculating the predictions of the endegenous variables*/

n=1;

do while n<17;

        y=1;
        do while y<17;
               infrate[n,y]=t2[y,1]*length[y,1]/(t1[y,1]*length[n,1]);
               forexp[n,y]=pim[n,y]*infrate[n,y];
               mg[n,y]=(t2[y,1]*length[y,1])/length[n,1]-1;
               forexpmg[n,y]=pim[n,y]*mg[n,y];
               y=y+1;
        endo;


i[n,1]=(pim[n,.]*(mu.*(1./(infrate[n,.])'))))./(pim[n,.]*(lambda.*(1./(infrate[n,.])')));
        s=1;

        do while s<17;
                r[n,s]=(1+i[n,1])/(infrate[n,s])-1;
```

```
                forexpr[n,s]=pim[n,s]*r[n,s];
                s=s+1;
        endo;
        n=n+1;

endo;

/*final formulas for endogenous formulas*/

expinfrate=unity*((forexp*unit).*pist)-1;
expr=unity*((forexpr*unit).*pist);
expmg=unity*((forexpmg*unit).*pist);
expi=unity*(pist.*i);

/* caculates correlation between velocity and consumption growth*/

v=1/length;
avrV=unity*(pist.*v);
avrgama=unity*(pist.*t2);
corrvgama=(unity*((v-avrV).*(t2-avrgama)));
varvgama=((unity*((v-avrV).*(v-avrV))))*((unity*((t2-avrgama).*(t2-avrgama))));


/*calculates correlation between velocity and money growth */

avrw=unity*(pist.*t1);
corrvw=(unity*((v-avrV).*(t1-avrw)));
varvw=((unity*((v-avrV).*(v-avrV))))*((unity*((t1-avrw).*(t1-avrw))));



/* calculates correlation between velocity and nominal interest rate*/

avri=expi;
corrvi=(unity*((v-avrV).*(i-avri)));
varvi=((unity*((v-avrV).*(v-avrV))))*((unity*((i-avri).*(i-avri))));



/* covariance of velocity */

avrV=unity*(pist.*v);
var=v-avrV;
variance=unity*(var.*var)/15;
```

/* corralation between inflation rate and money growth*/

```
avrpi=expinfrate;
avrw=unity*(pist.*t1);
corrpiw=(unity*(((forexp*unit)-avrpi).*(t1-avrw)));
varpiw=((unity*(((forexp*unit)-avrpi).*((forexp*unit)-avrpi))))*((unity*((t1-avrw).*(t1-
avrw))));
```

/* correlation between inflation and nominal interest rate*/

```
avrpi=expinfrate;
corrpii=(unity*(((forexp*unit)-avrpi).*(i-avri)));
varpii=((unity*((forexp*unit-avrpi).*(forexp*unit-avrpi))))*((unity*((i-avri).*(i-avri))));
```

/*correlation between inflation rate and realized real interest rate */

```
avrr=expr;
avrpi=expinfrate;
corrpir=(unity*(((forexp*unit)-avrpi).*((forexpr*unit)-avrr)));
varpir=((unity*((forexp*unit-avrpi).*(forexp*unit-avrpi))))*((unity*((forexpr*unit-
avrr).*(forexpr*unit-avrr))));
```

/* standart deviation of inflation rate*/

```
fark=ones(16,1);
avrpay=expinfrate;
pay=forexp*unit;
fark=pay-avrpay;
sigmapay=((unity*(fark.*fark))/15);
```

/*standart deviation of r*/

```
ismet=ones(16,1);
avrr=expr;
ismet=forexpr*unit-avrr;
sigmar=((unity*(ismet.*ismet))/15);
```

/*standart deviation of mg*/

```
musa=ones(16,1);
```

```
avrmg=expmg;
kazim=forexpmg*unit-avrmg;
sigmamg=((unity*(kazim.*kazim))/15);


/*standart deviation of i*/

memo=ones(16,1);
avri=expi;
germe=i-avri;
sigmai=((unity*(germe.*germe))/15);


print (variance~corrvgama~varvgama~corrvw~varvw~corrvi
~varvi~expinfrate~sigmapay~expi~sigmai~expr~sigmar~expmg~sigmamg~corrpiw~var
piw~corrpii~varpii~corrpir~varpir~a~beta~fi);

print unity*(pist.*v);

            a=a+0.5;
            endo;

      fi=fi+0.03;
      endo;

beta=beta+0.02;
endo;
```

APPENDIX B

Bellman equation for the problem.

$$V\left(\frac{M_t}{P_t},\frac{Q_t}{P_t} \mid \frac{X_t}{P_t},\omega_t,\gamma_t\right)=$$

$$U(c_t)+\beta\, V\left(\frac{M_{t+1}}{P_{t+1}},\frac{Q_{+1t}}{P_{t+1}} \mid \frac{X_{t+1}}{P_{t+1}},\omega_{t+1},\gamma_{t+1}\right)+$$

$$\lambda\left[\left(\frac{Q_t}{P_t}+y_t\right)z_t+(\omega_t-1)\frac{X_t}{P_t}+\frac{M_t}{P_t}-c_t-n_tT_t-\frac{M_{t+1}}{P_t}-\frac{Q_{t+1}}{P_{t+1}}z_{t+1}\right]+$$

$$\mu\left(\frac{M_t}{P_t}n_t-c_t\right)$$

subject to market clearing conditions

$$z_{t+1}=1$$

$$y_t=c_t+n_tT_t$$

$$M_{t+1}=X_{t+1}$$

APPENDIX C

TABLE 1

EXPRESSIONS FOR ENDEGENEOUS VARIABLES

Consumption velocity	$v(\gamma_i, \omega_i) = \dfrac{1}{l_i}$
Inflation rate (plus one)	$\pi(\gamma_i, \omega_i \mid \gamma_j, \omega_j) = \dfrac{\gamma_j}{\omega_j} \dfrac{l_j}{l_i}$
Nominal interest rate	$i(\gamma_i, \omega_i) = \dfrac{E\{\mu(\gamma_j, \omega_j)[1/\pi(\gamma_i, \omega_i \mid \gamma_j, \omega_j)] \mid (\gamma_i, \omega_i)\}}{E\{\lambda(\gamma_j, \omega_j)[1/\pi(\gamma_i, \omega_i \mid \gamma_j, \omega_j] \mid (\gamma_i, \omega_i)\}}$
Realized real interest rate	$r(\gamma_i, \omega_i \mid \gamma_j, \omega_j) = \dfrac{1 + i(\gamma_i, \omega_i)}{\pi(\gamma_i, \omega_i \mid \gamma_j, \omega_j)} - 1$
Growth rate of real balances	$m_g(\gamma_i, \omega_i \mid \gamma_j, \omega_j) = \gamma_j \dfrac{l_j}{l_i} - 1$

Note: The current state of the Markov chain is (γ_i, ω_i), and $(\gamma_i, \omega_i \mid \gamma_j, \omega_j)$ denotes transition from the current state to the state (γ_j, ω_j) in the next period.

TABLE 2
EASTIMATED VAR AND THEIR MARKOV COUNTERPARETS

	COEFFICIENTS ON			COVARIANCE MATRIX			TEST STATISTICS			LIKELIHOOD RATIO TEST	
DEPENDENT VARIABLE	Constant	ω_{t-1}	γ_{t-1}	R^2	σ	$\rho_{\omega\gamma}$	SC(I)	SC(2)	SC(3)	1 vs. 2	2 vs. 3
ω_t	.742	.685	-.400	.489	.02243	-.443				1.495	4.063
	(-.35)	(.116)	(-.321)				-16.439	-16.095	-15.84	[.827]	[.398]
γ_t	.68	.091	.239	.084	.01072						
	(-.163)	(-.054)	(-.150)								

Estimated Markov Counterpart

	Constant	ω_{t-1}	γ_{t-1}	R^2	σ	$\rho_{\omega\gamma}$
ω_t	.756	.635	-.362		.0222	.462
γ_t	.680	.091	.239		.01072	

SC(j) is the value of the Schwarz (1978) criterion for lag length j.
The likelihood ratio tests lag length j+1. The marginal level of significance of this test is in brackets. The statistics incorporate the degrees of freedom connection recommended.

TABLE 3

CASH MODEL SIMULATION RESULTS VS. SAMPLE VALUES (Annual Data 1950-87)

	Min	Min (α,β)	Max	Max (α,β)	Sample Value	Standard Error
$cv[v]$	.0000	(.95, 1.0)	.0009	(1.0, 1.0)	.0456	.0097
$corr[v,\gamma]$	-.1585	(1.5, 1.0)	.0000	(9.5, 1.0)	-.5000	.1447
$corr[v,\omega]$	.0000	(9.5, 1.0)	.0711	(0, .98)	-.0668	.2263
$corr[v,i]$	.0000	(2.5, 1.0)	.1555	(0, .98)	.5348	.2245
$E[\pi]$	.0389	All	.0389	All	.0434	.0079
$\sigma[\pi]$	.0297	All	.0297	All	.0283	.0061
$E[i]$	.0594	(0, .98)	.3901	(9.5, .90)	.0587	.0064
$\sigma[i]$	.0182	(0, .98)	.0537	(9.5, .90)	.0323	.0076
$E[r]$	.0201	(1.0, 1.0)	.3377	(9.5, .90)	.0148	.0053
$\sigma[r]$	.0116	(4.0, 1.0)	.0218	(9.5, .90)	.0200	.0046
$E[m_g]$	.0203	All	.0203	All	.0164	.0063
$\sigma[m_g]$	.0450	(9.5, 1.0)	.0474	(1.0, 1.0)	.0336	.0061
$corr[\pi,\omega]$	.9227	(1.0, 1.0)	.9254	(9.5, 1.0)	.3421	.1191
$corr[\pi,i]$	.9165	(0, .98)	.9274	(5.0, .98)	.7689	.0805
$corr[\pi,r]$	-.8812	(0, .90)	.4445	(9.5, .98)	-.1808	.1904

TABLE 4

ARNWINE (2000) SIMULATION RESULTS VS. SAMPLE VALUES (Annual Data 1950-87)

	Min	Min (α,β,Φ)	Max	Max (α,β,Φ)	Sample Value	Standard Error
$cv[v]$	.0039	1,.98,.28	94.1428	9.5,.90,.28	.0456	.0097
$corr[v,\gamma]$	.8913	1,.90,.01	.9762	9.5,.90,.01	-.5000	.1447
$corr[v,\omega]$	.7898	9.5,.96,.01	1.0000	1,1,.28	-.0668	.2263
$corr[v,i]$	-.1754	9.5,.96,.13	-.0415	1,.90,.01	.5348	.2245
$E[\pi]$	.0007	1,.90,.13	.1031	1,1,.28	.0434	.0079
$\sigma[\pi]$	1.0285	1,1,.04	1.316	9.5,1,.01	.0283	.0061
$E[i]$	.0100	All,All,.01	.2800	All,All,.28	.0587	.0064
$\sigma[i]$	.0000	All,All,.01	.0000	All,All,.28	.0323	.0076
$E[r]$	.0251	1,.90,.01	.4226	1,1,.28	.0148	.0053
$\sigma[r]$	.1128	1,.90,.04	.4636	1,1,.28	.0200	.0046
$E[m_g]$	.0258	1,.90,.13	.1274	1,1,.28	.0164	.0063
$\sigma[m_g]$	.1095	1,.90,.13	.3876	1,1,.28	.0336	.0061
$corr[\pi,\omega]$	-.2818	1,.90,.13	.0642	1,1,.04	.3421	.1191
$corr[\pi,i]$	-.9950	1,.90,.13	-.8459	1,1,.28	.7689	.0805
$corr[\pi,r]$	-.3987	1,1,.28	.0476	1,.90,.01	-.1808	.1904