SCHEDULING WITH CONTROLLABLE PROCESSING TIMES IN A CNC ENVIRONMENT

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I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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Flexible manufacturing systems give a manufacturer some capabilities to consider and solve different manufacturing problems simultaneously instead of one by one in a sequential manner. Using those makes her more competitive in the market. One of those capabilities is controllable processing times. By using this capability, the due date requirements of customers can be satisfied much more effectively. Processing times of the jobs in a CNC machine can be easily controlled via machining conditions such that they can be increased or decreased at the expense of tooling cost. In this study, we consider the problem of scheduling a set of jobs by minimizing the sum of total weighted tardiness, tooling and machining costs on a single CNC machine. This problem is NP-hard since the total weighted tardiness problem is NP-hard alone. Moreover, the problem is non-linear because of the nature of the tooling cost. We proposed a DP-based heuristic to solve the problem for a given sequence and designed a local search algorithm that uses it as a base heuristic.

Keywords: Scheduling, Single Machine, Total Weighted Tardiness, Machining Conditions, Controllable Processing Times, Heuristics.

ÖZET

CNC ORTAMINDA KONTROL EDİLEBİLİR ÜRETİM ZAMANLARIYLA ÇİZELGELEME

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Esnek üretim sistemleri bir üreticiye farklı üretim problemlerini tek tek sırayla değil aynı anda çözebilmesini sağlayan kabiliyetler verir. Bu kabiliyetleri kullanmak onu pazarda çok daha rekabetçi bir hale getirir. Bu kabiliyetlerden birisi de bilgisayar sistemleri üzerinden kontrol edilebilir üretim zamanlarıdır. Bu kabiliyeti kullanarak müşterilerin teslim zamanı gereksinimleri daha efektif bir şekilde karşılanabilir. Bir CNC makinasında işlenecek olan işlerin üretim zamanları işleme koşulları üzerinden kontrol edilebilir, yani kesici uç maliyeti karşılığında arttırılabilir ya da azaltılabilir. Bu çalışmada, toplam ağırlıklı gecikme, kesici uç ve işleme maliyetlerini enazlayarak bir grup işin çizelgelenmesini gözönüne aldık. Tek başına toplam ağırlıklı gecikme problemi NP-zor olmasından dolayı, bizim incelediğimiz problem de NP-zor'dur. Ayrıca, kesici uç maliyetinin doğasından dolayı da problem doğrusal değildir. Bu çalışmamızda sırası verilmiş işlerin çizelgelenmesi için DP temelli bir yöntem ve bu yöntemi temel algoritma olarak kullanan bir genetik problem uzayı algoritması tasarladık.

Anahtar sözcükler: Çizelgeleme, Toplam Ağırlıklı Gecikme, İşleme Koşulları, Kontrol Edilebilir Üretim Zamanları, Yordamlama.

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Chapter 1

Introduction

Buyer-vendor relationship plays an important role in business. Usually, buyers desire reliable time delivery for meeting their schedules. From the perspective of the vendors, each buyer has a different priority. All of these require vendors to consider weighted tardiness problem in their scheduling decisions. If a manufacturer has a flexible manufacturing system, in addition to its other capabilities she also gains a capability to be more competitive in meeting customer due date requirements. This capability is to be able to control processing times, which is a readily available feature on modern CNC machines.

Processing times in a CNC machine are controlled by machining conditions, for example feed rate and cutting speed in a turning machine. We can increase or decrease the processing time of a job by changing the machining conditions. However, there is a cost which is incurred when we increase the processing speed. It is the tooling cost, to which a manufacturer should always pay attention to use CNC machines effectively. Gray *et al.* [22] and Veeramani *et al.* [53] give extensive surveys on the tool management issues of FMS, and emphasize that the lack of tooling considerations has resulted in the poor performance of these systems.

In the literature, the interaction between determining machining conditions and solving single machine weighted tardiness problem is never analyzed up to now. Moreover, in the industry, they are considered as decisions at two different levels of FMS hierarchy. Machining conditions are determined in the design level and scheduling problems are solved in the operational level. To convert the capability of controlling processing times to an advantage for a manufacturer, instead of solving these problems one by one in a sequential manner we propose a solution methodology for solving them together by considering the relations between them.

In this study, we consider the problem of scheduling a set of jobs by minimizing the sum of total weighted tardiness, tooling and machining costs on a single CNC machine. This problem is NP-hard since the total weighted tardiness problem is NP-hard alone. Moreover, the problem is non-linear because of the nature of the tooling cost. Processing times of the jobs are controllable because they depend on the machining conditions. Thus, finding an efficient algorithm that solves the problem exactly is almost impossible.

In our study, we first develop an efficient Dynamic Programming(DP) based heuristic algorithm that considers interactions among the jobs using a contributed cost function. The proposed algorithm minimizes the summation of total weighted tardiness, tooling and machining costs over a given sequence. We then employ a problem space heuristic based on a local search algorithm that uses the proposed algorithm as a base heuristic to determine the processing time of each job and the schedule of all jobs simultaneously to minimize the stated objective function. At each stage of the base algorithm, we generate a set of processing time alternatives for each job in a recursive equation (that corresponds to states in a DP format) in a backward DP algorithm. After finding the optimal processing time for the first job in the sequence, it generates the optimal processing times for the rest of the jobs by using the recursive equations and alternative states of each job in a forward DP algorithm.

We compared computationally our DP-based heuristic with mathematical formulation of the problem for a given sequence, and found that our heuristic decreases CPU times dramatically at the expense of losing little from solution quality. Also the computational results over the original problem show that our proposed local search algorithm produces solutions with much higher quality when compared to the sequential algorithm that determines machining conditions first and then solves the scheduling problem for the total weighted tardiness criterion. Although, CPU times of our algorithm are higher than the ones of the sequential algorithm, they are in acceptable ranges considering the improvement in the solution quality.

In the next chapter, we present a literature review on machining conditions optimization in tool management, controllable processing times and weighted tardiness concerns in scheduling literature. In Chapter 3, we define the scope of this study and give the mathematical formulations of the problem. In Chapter 4, we introduce our proposed heuristic approaches and in Chapter 5 we present an illustrative example for our proposed DP-based heuristic. Experimental design and computational results are given in Chapter 6. Finally, we conclude in Chapter 7 giving final remarks and future research directions.

Chapter 2

Literature Review

In literature, both tool management issues and scheduling problems have been extensively researched. However, they are considered separately. The interaction between them is ignored. In this section, we give a short literature review related to tool management, and scheduling with controllable processing times and with total weighted tardiness criterion, $1||\sum w_i T_i$.

In order to give the related literature in an organized manner, we will start with the tool management and machining condition issues in the following section. Then, we will give the literature on scheduling with controllable processing times and with weighted tardiness criterion. Finally, we will conclude by mentioning the drawbacks of the current literature that motivate us for this study.

2.1 Tool Management and Machining Conditions

Flexibility is a key requirement in manufacturing systems to cope with modern market environment which is characterized by diverse products, high quality and short lead time. Crama and Klundert [12] define the most vital component of flexibility as "the ability of machines to perform various operations on various products or parts". The term "flexible" is generally used to describe two aspects of the system [44]: (1) the ability to use alternative routings through the machines to perform a given set of operations, and (2) the ability to simultaneously machine different part types. This flexibility is achieved by the use of CNC machines which are capable of carrying multiple tools. Also, the versatility of an FMS is achieved by equipping each machine with a tool magazine. This magazine can hold a set of tools which the machine can use to perform a succession of operations while incurring low setup costs when switching from one tool to another. In reality, FMSs are only capable of processing a finite family of parts at any given time. The flexibility or randomness is limited by the allocation of supporting resources such as pallets, fixture, and tools. As FMSs expand into the low volume, high variety production environment, the number of pallets, fixture, and tools and the amount of handling of these resources are increased. The management of these resources, especially the tooling which accounts for a high percentage of the operating costs of an automated manufacturing environment, is an absolute must. Therefore the models including tool management improves the productivity for an FMS.

Due to its direct impact on system performance, its dynamic nature and the large amount of information involved, the tooling problem has been considered as one of the most important and complicated issues in automated manufacturing. Proper tool management ensures that the correct tools are on the appropriate machines at the right time so that the desired quantities of workpieces are manufactured and the machine utilizations are maintained. Tool inventory, maintenance and distribution issues determine the quantity of work produced and system utilizations.

Tool management is an important area of research which has been extensively studied for nearly a hundred years, since Taylor [50] first recognized in 1907 that the machining conditions should be optimized to minimize the machining cost. Malakooti and Deviprasas [35] list vital contributions on parameter selection in metal cutting from 1907 up to 1985 in their paper.

It is stated by Stecke [46] and Gray *et al.* [22] that approximately 50 percent of U.S. annual expenditures on manufacturing is in the metal working industry, and two thirds of metal working is metal cutting. Besides being a critical issue in factory integration, tool management has direct cost implications. Kouvelis [30] reports in his study that tooling accounts for 25 percent to 30 percent of both fixed costs and variable costs of production in an automated machining environment. The reason for such a high contribution of the tooling to the total manufacturing cost is related to the high material removal rate in metal cutting processes, and the consequent increased tool consumption rates and tool replacement frequencies.

Kaighobadi and Venkatesh [27] state that the lack of attention to cutting tool related issues is a main reason for making an FMS inflexible in practice. Gray *et al.* [22] and Veeramani *et al.* [53] give extensive surveys on the tool management issues in automated manufacturing systems, and emphasize that the lack of tool management considerations has resulted in the poor performance of these systems.

The optimization of the machining conditions for a single operation is a well known problem, where the decision variables are usually the cutting speed and the feed rate. These conditions are the key to economical machining operations. Knowledge of optimal cutting parameters for machining operations is required for process planning of metal cutting operations. Numerous models have been developed with the objective of determining optimal machining conditions.

Malakooti and Deviprasas [35] formulate a metal cutting operation, specifically for a turning operation, as a discrete multiple objective problem. The objectives are to minimize cost per part, production time per part, and roughness of the work surface, simultaneously. They discuss a heuristic gradient-based multiple criteria decision making approach which they apply to parameter selection in metal cutting. For the metal cutting problem, they show how efficient alternatives can be generated by a discrete variable approach and how the gradient-based multiple objective approach can be implemented to obtain the most preferred alternative. They also discuss their software package for microcomputers as a decision support system for parameter selection. They compare their computer aided machine parameter selection (CAMPS) package to some of the computer packages (used in 1987) in the market.

Duffuaa *et al.* [15] compare the results of a number of gradient based optimization algorithms with different machining models. Their approach is limited because of the use of gradient based methods which are not ideal for non-convex problems. They conclude that the generalized reduced gradient method is the most suitable for solving machining optimization models.

Petropoulos [39] has used geometric programming for optimization of machining parameters. Multi-pass turning optimization has been addressed by Ermer and Kromodihardjo [19]. They use a combination of linear and geometric programming.

Iwata *et al.* [26] use a stochastic approach to solve for optimal machining parameters. Eskicioglu and Eskicioglu [20] demonstrate the use of non-linear programming for machining parameter optimization. Hati and Rao [23] use sequential unconstrained minimization technique (SUMT) to solve a multi-pass turning operation.

Khan *et al.* [29] study machining condition optimization by genetic algorithms and simulated annealing. Although nonlinear and non-convex machining models developed with the objective of determining optimal cutting conditions are traditionally solved using gradient based algorithms, they study three non gradient based stochastic optimization algorithms and test their efficiency in solving several benchmark machining models which are complex because of non-linearities and non-convexity.

Stori *et al.* [47] integrate process simulation in machining parameter optimization and propose a methodology for incorporating simulation feedback to fine-tune analytic models during optimization process. They present a non-linear programming (NLP) optimization technique used to select process parameters based on closed-form analytical constraint equations relating to critical design requirements and execute simulation using these process parameters, providing predictions of the critical state variables. Then, they dynamically adapt constraint equation parameters using the feedback provided by the simulation predictions. They repeat this sequence until local convergence between simulation and constraint equation predictions has been achieved.

Thomas *et al.* [51] emphasize the importance of choice of optimized cutting tool parameters to control the required surface quality. Surface finish is an important requirement for many turned work pieces in machining operation. The authors dealt with the interactions between the cutting parameters and surface roughness. They investigated the effects of tool vibration on the resulting surface roughness in the dry turning operation of carbon steel. They chose a full factorial design that allowed to consider the three-level interactions between the cutting parameters (cutting speed, feed rate, tool nose radius, depth of cut, tool length, and workpiece length) on the two measured dependent variables (surface roughness and tool vibration). Their results show that the factors having the greatest influence on surface roughness are the second order interactions between cutting speed and tool nose radius, along with third-order interaction between feed rate, cutting speed and depth of cut. They had the best surface finish at a low feed rate, a large tool nose radius and a high cutting speed. They concluded that feed rate and tool nose radius produced the most important effects on surface roughness, followed by cutting speed.

Kyoung *et al.* [32] emphasized the importance of selecting tool size, tool path, cutting width at each tool path properly and calculating the machining time for optimal process planning. Since other factors depend on the tool size, it is the most important factor in their problem. They presented a method for selecting optimal tools for pocket machining for the components of injection mold. They applied the branch and bound method to select the optimal tools which minimize the machining time by using the range of feasible tools and the breadth-first search.

These models consider only the contribution of machining time and tooling cost to the total cost of operation, and they usually ignore the contribution of the non-machining time components to the operating cost, which could be significant for the multiple operation case. All of the time consuming events except the actual cutting operation are denoted as non-machining time components. Basic setup, tool interchanging, tool replacing, workpiece loading-unloading, tool tuning, tool approach and stabilization etc., are the typical examples of non-machining events. Machining conditions are the main determinants of these non-machining time components. These studies also exclude the tooling issues such as the tool availability and the tool life capacity limitations. Therefore, their results might lead to infeasibilities due to tool contention among operations for a limited number of tool types [37].

Aktürk and Avcı [4] proposed a solution procedure to make tool allocation and machining conditions selection decisions simultaneously. They also take into account the related tooling considerations of tool wear, tool availability, and tool replacing and loading times, since they affect both the machining and non-machining time components, hence the total cost of manufacturing. In their study, they extend single machining operation problem (SMOP) formulation by adding a new tool life constraint which enables them to include tooling issues like tool wear and tool availability. Furthermore, they propose a new cost measure to exploit the interaction between the number of tools required with the machining, tool replacing and loading times, and tool waste cost in conjunction with the optimum machining conditions for alternative operation-tool pairs. Consequently, they prevent any infeasibility that might occur for the tool allocation problem at the system level due to tool contention among tool life restrictions through a feedback mechanism.

Aktürk and Önen [6] proposed a new algorithm to solve lot sizing, tool allocation and machining conditions optimization problems simultaneously to minimize the total production cost in a CNC environment. They integrated the system, machine and tool level decisions for production of multiple parts consisting of multiple operations. This way, they avoid any infeasibility that may occur due to tool and machine hour availability limitations.

In a recent study, Aktürk [3] developed an exact approach to determine the optimum machining conditions and tool allocation decisions simultaneously to minimize the total production cost on a CNC turning machine where alternative tools can be used for each operation. He emphasized the tool management issues at the tool level such as the optimum machining conditions and tool selection-allocation decisions considering the tool life, machining operations and tool availability constraints. He presented a new mathematical model and proposed an efficient solution procedure to determine concurrently the optimal machining conditions of cutting speed and feed rate, the optimal operation-tool assignment and optimal allocation of tools.

2.2 Scheduling

Scheduling is concerned with determining the sequence in which available work should be processed to optimize system performance.

Standard formulations of the scheduling problem assume that job processing times are fixed and known in advance of scheduling. In practice, processing times are often a function of the amount and mix of resource inputs allocated to a job. These resources can vary depending on the system. For instance, in a production facility composed of CNC machines, machine cutting speed and feed rate are effective parameters changing the processing times and tool usage rates. In a relatively labor-intensive systems, processing time typically depend on the number and type of the workers allocated to the system (Daniels *et al.* [13]).

2.2.1 Controllable Processing Times

Processing time control and its impact on sequencing decisions and operational performance have received limited attention in the scheduling literature. Some models for single-processor systems have been developed and studied concerning controllable processing times. Extensions to parallel-machine environments are also addressed by researchers. A survey of the literature up to 1990 can be found in Nowicki and Zdrzalka [36].

Daniels and Sarin [14] consider the problem of joint sequencing and resource allocation when the scheduling criterion of interest is the number of tardy jobs and derive theoretical results that aid in developing the trade-off curve between the number of tardy jobs and the total amount of allocated resource.

Panwalker and Rajagopalan [38] consider the static single machine sequencing problem with a common due date for all jobs in which job processing times are controllable with linear costs. They develop a method to find optimal processing times and an optimal sequence to minimize a cost function containing earliness cost, tardiness cost and total processing cost.

Adiri and Yehudai [2] study the problem of scheduling identical parallel processors whose service rates can change between jobs. Trick [52] focuses on assigning single-operation jobs to identical machines while simultaneously controlling the processing speed of each machine.

Zdrzalka [57] deals with the problem of scheduling jobs on a single machine in which each job has a release date, a delivery time and a controllable processing time, having its own associated linearly varying cost and propose an approximation algorithm for minimizing the overall schedule cost.

Ishii et al. [25] consider the problem with parallel uniform machines in which the speed of a machine is a continuous nonnegative variable and the compression cost is a function of the speed of the machine.

Cheng et al. [10] consider a parallel machine scheduling problem with controllable processing times, where the job processing times can be compressed through incurring an additional cost, which is a convex function of the amount of compression. They formulate two problems, one to minimize the total compression cost plus the total flow time, and the other to minimize the total compression cost plus the sum of earliness and tardiness costs for the common due date schedule problem.

Daniels et al. [13] investigate the improvements in manufacturing performance that can be realized by broadening the scope of the production scheduling function to include both job sequencing and processing-time control through the deployment of a flexible resource. They consider an environment in which a set of jobs must be scheduled over a set of parallel manufacturing cells, each consisting of a single machine, where the processing time of each job depends on the amount of resource allocated to the associated cell.

Karabati and Kouvelis [28] solve the simultaneous scheduling and optimal processing-times selection problem in a flow line operated under a cyclic scheduling policy. They address the simultaneous scheduling and optimal-processing-times selection problem in a multi-product deterministic flow line operated under a cyclic scheduling approach. They provide a modeling framework for cyclic scheduling decisions that incorporate processing-times selection considerations. After presenting a linear program solving the optimal-processing-times selection problem for a given cyclic sequence, they demonstrate for large problems, how the use of a row generation scheme allows them to solve it more efficiently than standard linear programming codes. For the solution of the simultaneous scheduling and optimal-processing-times selection problem, they propose a simple procedure that iteratively solves cyclic scheduling and optimal-processing-times selection subproblems for given sequences.

Cheng and Shakhlevich [11] present two polynomial algorithms for the proportionate flow-shop problem with controllable processing times minimizing the makespan and compression cost. Sodhi *et al.* [45] develop models for determining economic processing speeds and tool loading to minimize the makespan required to produce a given set of parts in a flexible manufacturing system composed of several machines.

The concept of controllable processing times can also be observed in project management with controllable activity durations. In 1980, Vickson treats the problem of minimizing the total weighted flow cost plus job processing cost in a single machine sequencing problem for jobs having processing costs which are linear functions of processing times in his first study [55]. In his second study [56], he extends his initial study and presents simple methods for solving two single machine sequencing problems when job processing times are themselves decision variables having their own linearly varying costs. The objectives studied are minimizing the total processing cost plus either the average flow cost or the maximum tardiness cost. He treats only the problems with zero ready time and no precedence constraints.

Lee and Lei [34] present efficient algorithms for solving several special cases of multi-project scheduling problems with controllable project duration and hard resource constraints. Two types of problems are considered. In type I, the duration of each project includes a constant and a term that is inversely proportional to the amount of resource allocated. In type II, the duration of each individual project is a continuous decreasing function of the amount of resource allocated.

Erenguc *et al.* [18] give a formulation and an exact solution method for a nonpreemptive resource constrained project scheduling problem in which the duration/cost of an activity is determined by the mode selection and the duration reduction (crashing) within the mode.

2.2.2 The Total Weighted Tardiness

One of the first results in tardiness scheduling is the well known Elmaghraby lemma ([16]) which states that if a job's due date is greater than the total completion time of all jobs, then there is an optimal schedule in which that job is scheduled last.

Lawler [33] shows that total weighted tardiness problem, $1||\sum w_i T_i$, is strongly NP-hard and gives a pseudo polynomial algorithm for the total tardiness problem. Various enumerative solution methods have been proposed. In 1969, Emmons [17] derives several dominance rules for total tardiness problem that restrict the search for an optimal solution. Emmons' rules are used both branch and bound and dynamic programming algorithms (Fisher and Potts and Van Wassenhove [21, 40]). Rinnooy Kan *et al.* [43] extended these results to the weighted tardiness problem. Rachamadugu [41] identifies a condition characterizing adjacent jobs in an optimal sequence for $1||\sum w_i T_i$. Chambers *et al.* [9] develop new heuristic dominance rules and flexible decomposition heuristics. The exact approaches used in solving the total weighted tardiness problem are tested by Abdul-Razaq *et al.* [1] and they use Emmons' dominance rules to form a precedence graph for finding upper and lower bounds. They show that the most promising lower bound both in quality and time consumption is the linear lower bound method by Potts and Van Wassenhove [40], which obtained from Lagrangian relaxation of machine capacity constraints. Hoogeveen and Van de Velde [24] reformulate the problem by using slack variables and show that better Lagragian lower bounds can be obtained.

Szwarc [48] proves the existence of a special ordering for the single machine earliness-tardiness (E/T) problem with independent job penalties where the arrangement of two adjacent jobs in an optimal schedule depends on their start time. Szwarc and Liu [49] present a two-stage decomposition mechanism to $1||\sum w_i T_i$ when tardiness penalties are proportional to the processing times which proves to be powerful in solving the problem completely or reducing it to a smaller problem. Also, Aktürk and Yıldırım [7, 8] propose a new dominance rule and a lower bounding scheme that provides a sufficient condition for local optimality, which can be used in reducing the number of alternatives in any exact approach. Their proposed rule covers and extends the Emmons' results and generalizations of Rinnooy Kan *et al.* by considering the time dependent orderings between each pair of jobs.

Since the implicit enumerative algorithms may require considerable resources both in terms of computation times and memory, several heuristics and dispatching rules have been proposed. They are logical rules for choosing which available job to select for processing at a particular work center. In using dispatching rules, usually scheduling decisions are made sequentially rather than once. For static dispatching rules, the job priorities do not change over time while priorities might change over time for the dynamic dispatching rules. Vepsalainen and Morton [54] develop and test efficient dispatching rules for the weighted tardiness problem with specified due dates and delay penalties. They show that Apparent Tardiness Cost (ATC) rule outperforms many existing heuristic rules.

ATC is a composite dispatching rule that combines the weighted shortest processing time and minimum slack rules. Under the ATC rule jobs are scheduled one at a time; the job with highest ranking index is then selected to be processed next. The ranking index is a function of time t, processing times p_i, delay penalties w_i, and due dates dd_i of the remaining jobs. The ATC index can be defined as:

$$a_i = \frac{w_i}{p_i} exp(\frac{-max(0, dd_i - t - p_i)}{K\overline{p}})$$

where $\overline{p}$ is the average processing time of the remaining jobs at time t and K is the look-ahead parameter. It trades off job's urgency (slack) against machine utilization, but due to the more complex weighted criterion, an additional look ahead parameter is needed to assimilate the competing jobs which have different weights. In computational tests performed by Rachamadugu and Morton [42] an exponential function of the slack was found somewhat more efficient. Intuitively, the exponential look ahead works by ensuring timely completion of short jobs (steep increase of priority close to due date), and by extending the look ahead far enough to prevent long tardy jobs from overshadowing clusters of short jobs.

2.3 Summary

In the literature of scheduling with controllable processing times, to the best of our knowledge there is no study that considers the total weighted tardiness problem. Moreover, most of the studies assume that the processing times can be crashed in a range with linear compression cost. But, for our case, the processing times are closely related with tool and operation parameters.

In the literature related to the weighted tardiness problem, processing times are taken as constant, either deterministic or probabilistic. However, they are closely related with the machining conditions. Hence, the processing times of the jobs are controllable.

As a result, scheduling jobs which have controllable processing times under the total weighted tardiness criterion is an untouched topic in the literature. The objective of the research reported in this thesis is to show how closely machining conditions optimization and scheduling of the jobs in a CNC machine are related. These topics have been studied separately by many researches, however there is no study that integrates all of these and investigates the interactions among them.

In this chapter, we introduced a short review of the literature on tool management and scheduling issues which are related with our problem in some aspects, and stated the similarities and diversities of our problem between the problems studied in the literature.

In the next chapter, we give the definition and underlying assumptions of the problem, present the mathematical programming formulations of the original problem and its subproblem.

Chapter 3

Problem Statement and Modeling

In this chapter, we will first give the detailed definition of the problem, underlying assumptions and notation used throughout the study. Then, we will construct a mathematical formulation.

3.1 Problem Definition

We are given N jobs with specified depth of cut, length and diameter of the generated surface along the maximum allowable surface roughness attributes. Also, each job corresponds to one cutting operation. The problem is scheduling these jobs on a CNC machine in order to minimize the total weighted tardiness cost plus machining and tooling costs. Each job can be performed by a different tool type. When tool life is over, tool is changed. However, considering rapid tool change technologies, we assume that tool change times are negligible. The machining conditions of the CNC machine can be changed, and for each job it can be adjusted to different cutting speed and feed rate pair and this pair determines the processing time of the job. However, there are some constraints for these settings. The speed and feed rate have to satisfy the machining power, surface finish and tool life constraints. After detecting the feasible region of cutting speed and feed rate (consequently processing time), we have to make two decisions, what will the feasible machining setting for each job and what will be the processing sequence of the jobs.

3.1.1 Assumptions and Notations

The assumptions about the operating policy and characteristics of the system considered in this study are as follows:

- The CNC machine that is continuously available can process one job at a time.
- There are N jobs with no precedence relation, all ready at time zero.
- Each job has a different due date and tardiness penalty.
- Depth of cut, length and diameter of the surface, and maximum allowable surface roughness values for each job are given.
- Tool change times are negligible.
- Total usage of a tool cannot exceed 1.
- Each job corresponds to a single cutting operation.
- Each operation may require a different tool.
- There are unlimited amount of tools.
- Preemption is not allowed.
- Cutting speed and feed rate of the machine constitute the machining conditions and they can easily be adjusted to new settings.

The notation used throughout the paper is as follows :

α_i, β_i, γ_i : speed, feed, depth of cut exponents for operation-tool pair i

C_m,b,c,e : specific coefficient and exponents of the machine power constraint

C_s,g,h,l : specific coefficient and exponents of the surface roughness constraint

v_i : cutting speed for operation-tool pair i

f_i : feed rate for operation-tool pair i

D_i : diameter of the generated surface for the job i

d_i : depth of cut for job i

C_{t_i} : cost of tooling for operation-tool pair i

L_i : length of the generated surface for the job i

S_{r_i} : maximum allowable surface roughness for the operation i

C_i : Taylor s tool life constant for operation-tool pair i

C_0 : operating cost of the CNC machine ($/min)

H : maximum available horse power for all jobs

p_i : processing time of job i

p_i^u,p_i^l : upper and lower bounds for the processing time of job i

w_i : weight of job i

dd_i : due date of job i

s_i : starting time of job i

T_i : Tardiness of job i

Since each job has one operation, we use the same index for both operation and job. Now, we are ready to construct the mathematical model.

3.2 Mathematical Modeling

In the last part of this section we will give the mathematical formulation of the problem. However before presenting it, we will first give how lower and upper bounds of processing times, which are used in formulating the problem, can be found and second define the cost items depending on processing times of jobs in the objective function of the formulation.

3.2.1 SMOP and Lower & Upper Bounds for Processing Times

Single Machine Operation Problem (SMOP) is the problem of determining optimal machining conditions for an operation considering machining and tooling costs. This problem is formulated and solved optimally by Akturk and Avci [4]. The mathematical model of SMOP is given below:

$$\text{Minimize} \quad C_1 v_i^{-1} f_i^{-1} + C_2 v_i^{(\alpha_i - 1)} f_i^{(\beta_i - 1)}$$

subject to

$$C'_t v_i^{(\alpha_i-1)} f_i^{(\beta_i-1)} \leq 1 \quad i = 1 \ldots N \tag{3.1}$$

$$C'_m v_i^b f_i^c \leq 1 \quad i = 1 \ldots N \tag{3.2}$$

$$C'_s v_i^g f_i^h \leq 1 \quad i = 1 \ldots N \tag{3.3}$$

$$v_i, f_i > 0 \tag{3.4}$$

where

$$C_1 = \frac{\pi D_i L_i C_0}{12} \qquad C_2 = \frac{\pi D_i L_i d_i^{\gamma_i} C_{t_i}}{12 C_i}$$

$$C'_t = \frac{\pi D_i L_i d_i^{\gamma_i} \rho_i}{12 C_i} \qquad C'_m = \frac{C_m d_i^e}{H}$$

$$C'_s = \frac{C_s d_i^l}{S_{r_i}}$$

The first term in the objective function is machining cost and the second one is the tooling cost. The constraints are tool life, machine power and surface finishing constraints, respectively.

Koylu [31] showed that the surface roughness constraint in SMOP is always binding. This means that in SMOP v, f pairs on the surface roughness constraint are dominant over all other v, f points with respect to tooling and machining costs. Thus, there is only one optimal v, f pair for SMOP and there is a one to one relation between machining conditions and processing time.

As we stated before, our objective is minimizing the sum of total weighted tardiness, machining, and tooling costs. SMOP contains two terms of our objective: tooling and machining. For this reason, the processing time that corresponds to the optimal solution of SMOP gives an upper bound for our problem. If we increase the processing time above that value, the sum of tooling and machining will increase and we know that tardiness cost will not decrease. Thus, there is no gain to assign a processing time to a job which is greater than the optimal solution of SMOP.

Also there is a lower bound for the processing time of job due to the power of the machine or life of a tool. It is the intersection point of surface roughness (SR) and tool life (TL) or machine power (MP) constraints. This point can be found as follows:
If the v and f values at the intersection of TL and SR constraints

$$\begin{aligned}
\text{From SR constraint (inequality 3.3)} v_i &= (C'_s f_i^h)^{-\frac{1}{g}} \\
\text{put it into TL constraint (inequality 3.1)} \quad & C'_t (C'_s f_i^h)^{\frac{1-\alpha}{g}} f_i^{(\beta_i - 1)} = 1 \\
f_i &= (C'_t (C'_s)^{\frac{1-\alpha}{g}})^{-\frac{g}{h(1-\alpha)+g(\beta-1)}} \\
&= (C'_t)^{-\frac{g}{h(1-\alpha)+g(\beta-1)}} (C'_s)^{\frac{\alpha-1}{h(1-\alpha)+g(\beta-1)}} \\
v_i &= (C'_s (C'_t)^{-\frac{hg}{h(1-\alpha)+g(\beta-1)}} (C'_s)^{\frac{h(\alpha-1)}{h(1-\alpha)+g(\beta-1)}})^{-\frac{1}{g}} \\
&= (C'_t)^{\frac{h}{h(1-\alpha)+g(\beta-1)}} (C'_s)^{\frac{(1-\beta)}{h(1-\alpha)+g(\beta-1)}}
\end{aligned}$$

$$\begin{aligned}
(v_i f_i)^I &= (C'_t)^{\frac{h}{h(1-\alpha)+g(\beta-1)}} (C'_s)^{\frac{(1-\beta)}{h(1-\alpha)+g(\beta-1)}} (C'_t)^{-\frac{g}{h(1-\alpha)+g(\beta-1)}} (C'_s)^{\frac{\alpha-1}{h(1-\alpha)+g(\beta-1)}} \\
(v_i f_i)^I &= (C'_t)^{\frac{h-g}{h(1-\alpha)+g(\beta-1)}} (C'_s)^{\frac{(\alpha-\beta)}{h(1-\alpha)+g(\beta-1)}} \qquad (3.5)
\end{aligned}$$

If the v_i and f_i values at the intersection of TL and SR constraints

$$\begin{aligned}
\text{From SR constraint (inequality 3.3)} v_i &= (C'_s f_i^h)^{-\frac{1}{g}} \\
\text{put it into MP constraint (inequality 3.2)} \quad & C'_m (C'_s)^{-\frac{b}{g}} f_i^{\frac{cg-hb}{g}} = 1 \\
f_i &= (C'_m)^{\frac{g}{hb-cg}} (C'_s)^{\frac{b}{cg-hb}} \\
v_i &= (C'_s (C'_m)^{\frac{hg}{hb-cg}} (C'_s)^{\frac{hb}{cg-hb}})^{-\frac{1}{g}} \\
&= (C'_s)^{\frac{c}{hb-cg}} (C'_m)^{\frac{-h}{hb-cg}} \\
(v_i f_i)^{II} &= (C'_s)^{\frac{c}{hb-cg}} (C'_m)^{\frac{-h}{hb-cg}} (C'_m)^{\frac{g}{hb-cg}} (C'_s)^{\frac{1}{cg-hb}} \\
(v_i f_i)^{II} &= (C'_s)^{\frac{c-1}{hb-cg}} (C'_m)^{\frac{g-h}{hb-cg}} \qquad (3.6)
\end{aligned}$$

We choose the minimum of the $v_i f_i$ values calculated by equation 3.5 and 3.6 since the other one must be infeasible. By using the processing time equation below, we can calculate the lower bound for the processing time of a job as follows:

$$\begin{aligned} p_i &= \frac{\pi D_i L_i}{12 v_i f_i} \\ p_i^l &= \frac{\pi D_i L_i}{12 min\{(v_i f_i)^I, (v_i f_i)^{II}\}} \end{aligned} \tag{3.7}$$

3.2.2 Cost Items of the Problem

In this section, we give the cost items included in our objective function and show that all of them can be written depending on processing time. Actually, showing this for machining and tardiness costs is trivial. However, it requires more attention for tooling cost.

- Tooling cost of job i depending on v_i and f_i:

  $$Tool_i(v_i, f_i) = \frac{\pi D_i L_i d_i^{\gamma_i} C_{t_i}}{12 C_i} v_i^{(\alpha_i - 1)} f_i^{(\beta_i - 1)}$$

  Since, optimal machining conditions v_i and f_i are on surface roughness constraint we can write down the tooling cost depending on only processing time of job i.

  From Surface Roughness constraint, we can find v_i depending on f_i and using processing time equation 3.7 we can derive tooling cost that depends on processing time.

  $$C_s' v_i^g f_i^h = 1 \Longrightarrow v_i = (C_s'(v_i f_i)^h)^{\frac{1}{h-g}}$$

  $$\begin{aligned} Tool_i(v_i, f_i) &= \frac{\pi D_i L_i d_i^{\gamma_i} C_{t_i}}{12 C_i} v_i^{(\alpha_i - 1)} f_i^{(\beta_i - 1)} \\ &= \frac{\pi D_i L_i d_i^{\gamma_i} C_{t_i}}{12 C_i} v_i^{(\alpha_i - \beta_i)} (v_i f_i)^{(\beta_i - 1)} \\ &= \frac{\pi D_i L_i d_i^{\gamma_i} C_{t_i}}{12 C_i} ((C_s'(v_i f_i)^h)^{\frac{(\alpha_i - \beta_i)}{h-g}})(v_i f_i)^{(\beta_i - 1)} \\ &= \frac{\pi D_i L_i d_i^{\gamma_i} C_{t_i}}{12 C_i} (C_s')^{\frac{(\alpha_i - \beta_i)}{h-g}} (v_i f_i)^{\frac{h(\alpha_i - \beta_i)}{h-g}} (v_i f_i)^{(\beta_i - 1)} \end{aligned}$$

$$
\begin{aligned}
&= \frac{\pi D_i L_i d_i^{\gamma_i} C_{t_i}}{12 C_i} (C_s')^{\frac{(\alpha_i - \beta_i)}{h-g}} (v_i f_i)^{\frac{h(\alpha_i - 1) - g(\beta_i - 1)}{h-g}} \\
&= (\frac{\pi D_i L_i}{12 v_i f_i})^{\frac{h(1-\alpha_i) - g(1-\beta_i)}{h-g}} (\frac{\pi D_i L_i}{12})^{\frac{h\alpha_i - g\beta_i}{h-g}} d_i^{\gamma_i} C_{t_i} C_i^{-1} (C_s')^{\frac{(\alpha_i - \beta_i)}{h-g}} \\
Tool_i(p_i) &= \frac{c_{a_i}}{(p_i)^{c_{b_i}}}
\end{aligned}
$$

where

$$
\begin{aligned}
c_{a_i} &= \left(\frac{\pi D_i L_i}{12}\right)^{\frac{h\alpha_i - g\beta_i}{h-g}} d_i^{\gamma_i} C_{t_i} C_i^{-1} (C_s')^{\frac{(\alpha_i - \beta_i)}{h-g}} \\
c_{b_i} &= -\frac{h(1-\alpha_i) - g(1-\beta_i)}{h-g}
\end{aligned}
$$

The critical point here is that c_{a_i} and c_{b_i} are always positive because of possible values of the technical coefficients such that $\alpha > \beta > 1$, $h > 0$ and $g < 0$. Thus, the tooling cost is a convex function that is negatively proportional to the processing time.

- Machining Cost of job i depending on processing time:

$$Mach_i(p_i) = C_0 p_i$$

- The Weighted Tardiness Cost of job i depending on processing time:

$$Tard_i(s_i, p_i) = w_i Max\{0, s_i + p_i - dd_i\}$$

After showing that the objective function of our problem can be written depending on just processing times, we can formulate the problem.

3.2.3 Mathematical Formulation

Now, we will give the non-linear formulation of the problem. Actually, the constraints of the problem are not nonlinear. However the tooling cost item in the objective function adds the nonlinearity.

$$\text{Minimize Total Cost} = \sum_{i=1}^{N} (w_i T_i + Mach_i(p_i) + Tool_i(p_i))$$

subject to

$$s_i - s_j \geq p_i - (M + p_i)(1 - X_{ij}) \quad i, j = 1 \dots N \tag{3.8}$$

$$s_j - s_i \geq p_j - (M + p_i)X_{ij} \quad i, j = 1 \dots N \tag{3.9}$$

$$T_i \geq s_i + p_i - dd_i \quad i, j = 1 \dots N \tag{3.10}$$

$$p_i^l \leq p_i \leq p_i^u \quad i = 1 \dots N \tag{3.11}$$

$$X_{ij} = 0 \text{ or } 1 \quad i = 1 \dots N \tag{3.12}$$

$$T_i, s_i \geq 0 \tag{3.13}$$

$$p_i > 0 \tag{3.14}$$

The M in the formulation represents a big number. The decision variables in this formulation are p_i, s_i, T_i, X_{ij}. The objective function is the summation of three cost terms defined in the previous section. The first three constraint sets are the ones that comes from total weighted tardiness problem. The next constraint set defines the lower and upper bounds for the processing times which are calculated by using the SMOP formulation developed by Akturk and Avci [4].

As we stated in the previous chapter, $1||\sum w_i T_i$ is NP-hard. Actually it is a subproblem of our problem. It is the case where processing times are fixed. Thus, our problem is extremely hard to solve. To reduce the complexity in the problem and to develop efficient algorithms, as we have done in the next chapter, we formulate the problem for a given sequence. In this way, we are free from the sequencing problem, but there is still a scheduling problem. To convert the formulation above to one for a given sequence, it is enough to throw out the binary variable equation 3.12 and replace the sets of inequalities 3.8 and 3.9 with the one below:

$$s_i + p_i \leq s_{i+1} \quad i = 1 \dots (N - 1) \tag{3.15}$$

Solving the problem just for a sequence possibly will not produce good solutions, however it gives an initial point and an insight to deal with the problem.

3.3 Summary

In this chapter, we have given the definition and the underlying assumptions of the problem of finding machining conditions and scheduling jobs considering total weighted tardiness. We have presented that our objective is the sum of total weighted tardiness, tooling and machining costs. We showed that all of them can be formulated in the form of their own processing times of which upper and lower bounds can also be found by using the SMOP formulation. Lastly, we presented the mathematical formulations of the original problem and the problem for a given sequence.

In the following chapter, we will present the local search algorithm for the original problem and the DP-based algorithm for the subproblem.

Chapter 4

Proposed Heuristic Algorithms

In the previous chapter, we state our problem and give the underlying assumptions. We also present the mathematical programming formulations for the original problem and for the problem with a given sequence. Lawler [33] has showed that the single machine total weighted tardiness problem is strongly NP-hard. Actually, it is a subproblem of our problem. When we fixed the processing times in our problem, our problem reduces to $1||\sum w_i T_i$. Therefore, no algorithm is likely to be proposed to solve the problem optimally in polynomial time. Hence, it is justifiable to try heuristic approaches to solve our problem.

Weighted tardiness problem and determination of machining conditions are generally considered as separate problems and solved at different levels of factory management. Machining conditions are determined at the design and development level and the weighted tardiness problem is solved at the operational level. By considering this perspective, in this chapter, we will first present an algorithm for our problem that uses the usual sequential approach in the industry. Then, we will give our approach that solves the machining condition and the weighted tardiness problems simultaneously. In the last section, we describe in detail the DP-based heuristic that is used to solve the subproblem where the job sequence is given.

4.1 The Sequential Algorithm

The sequential algorithm consists of two levels. In the first level machining conditions, which are cutting speed and feed rate of the turning machine, are determined. Both cutting speed and feed rate specify the processing time of a job. By using the processing times determined in the first level, single machine total weighted tardiness problem is solved in the second level. As we design the algorithm, we use the best known procedures in each level and they are as follows:

1. The machining conditions considering tooling and machining costs are found by using the SMOP, whose mathematical model is given and explained in section 3.2.1. The procedure developed by Akturk and Avci [4] solves SMOP optimally.

2. To solve $1||\sum w_i T_i$ problem where the processing times are calculated in the first level, a Problem Space Genetic Algorithm (PSGA), designed by Avci *et al.* [5], is employed. This is the best published algorithm in the literature as stated by Avci *et al.* for the single machine weighted tardiness problem.

Although the sequential algorithm solves the subproblems in each level optimally or almost optimally, its drawback is obvious. The interaction between two levels is ignored. By using the flexibilities provided by a CNC machine, the due date requirement can be satisfied better or operation cost due to machining and tooling costs can be decreased. In the next section, we give an algorithm that exploits the interaction between two levels to generate better solutions.

4.2 The Proposed Simultaneous Algorithm

The proposed algorithm for our problem is a Problem Space Genetic Algorithm (PSGA) that determines the machining conditions and solve the total weighted tardiness problem simultaneously. PSGAs are basically local search algorithms.

To develop a PSGA, it is necessary to define an initial feasible solution, a base heuristic and a neighborhood structure.

The overall quality of the base heuristic affects the overall effectiveness of the problem space approach. Thus, it is very critical to use a relatively fast base heuristic which generates a relatively good solution.

The neighborhood is constructed through perturbation of the problem data and the search is performed in the space of these perturbations. A genetic algorithm (GA), which is based on a formalization of natural genetics, is used to search that space. Some characteristics of GAs are as follows:

- There is a *coding scheme* for possible solutions of the problem and these solutions are stored as strings. Each string is called a chromosome and each variable is referred to as gene.
- An *evaluation function* that estimates the quality of each solution (each string) in the set of solutions (called the population) is used.
- An *initial set of solutions* to the problem (the initial population) is randomly obtained or based on prior knowledge.
- A set of *genetic operations* that, using the information contained in a certain population (referred to as generation $G(t)$) and a set of genetic operators, creates a new population (the next generation, $G(t+1)$).
- A *termination condition* is defined at the end of the genetic process.

There are three main genetic operators: reproduction, crossover and mutation. The reproduction (or selection) operator creates a mating pool where strings are copied from $G(t)$ and await the action of crossover and mutation. Those strings from $G(t)$ with higher fitness values create a large number of copies in the mating pool. The crossover operator provides a mechanism for strings to mix attributes through a random process. The mutation operator produces the occasional alteration of a bit at a certain position in a string. Each bit is a candidate for mutation and will be selected according to the mutation probability.

In problem space search, the chromosomes represent the perturbation vectors and genes represent the perturbation amount for a single job. The original problem data is perturbed and the problem with perturbed data is solved by using the base heuristic to obtain a solution. Although the heuristic is applied on the perturbed values, the objective function is calculated with the original data values as expected. The new perturbation values are generated from the previous population by using the genetic algorithm instead of a random generation. Asexual and sexual reproduction and mutation operators are used in generations.

In this study, sequencing priorities of jobs, which are calculated with Apparent Tardiness Cost (ATC) dispatching rule, are perturbed.

In every generation, a population of chromosomes (perturbation vectors) is created. One of the important parameters of PSGA is the POPSIZE which gives the number of perturbation vectors in a population. The perturbation magnitude θ is the second parameter. The genes of the chromosomes can take values in a range of $(-\theta, \theta)$. The generation of initial population is done by taking random numbers in this range. Genetic algorithm operators are used in the forthcoming generations.

Fitness value shows the probability that the population member will be selected for breeding. The parameter ϕ determines the selectivity of the algorithm. The selection probability of better solutions increase as ϕ increases. The population loose diversity and converge to a population in which all members are identical in high values of ϕ. However, if ϕ is too small, the algorithm will converge very slowly using excessive computation time. We will use these fitness values in asexual and sexual reproduction.

In asexual reproduction, select a member from the current population randomly according to selectivity (fitness) values (a random number in (0,1) is taken, and if fitness of the member is greater than this number, it is selected). This member is directly passed to the next generation. In sexual reproduction, two parents are selected in the same selectivity logic, and combined through crossover to produce an offspring which is passed to the next generation. We work on a well known single point crossover operator. In single point crossover, a point is chosen randomly and the genes of the offspring up to that point is taken from the first parent and the following genes are copied from the second. %SEXUAL shows the percentage of sexual reproduction in the new generation. %SEXUAL·POPSIZE number of members are generated by sexual reproduction while the remaining are of asexual.

MUTPROB is the probability of mutation, and each gene has this probability to be mutated. If the gene is selected, then it is replaced by a newly generated random value taken in $(-\theta, \theta)$. After mutation operation, we get the new population. Now, go to step 3 as discussed below to perturb the data with the genes of this population and calculate the objective values of the POPSIZE number of solutions produced with this data.

The whole procedure of PSGA can be summarized as follows:

Step 1. Find the lower and upper bounds on processing times, p_i^l and p_i^u, for all jobs from their SMOP formulations as stated in §3.2.1.

Step 2. Create an initial population at random from a range of $(-\theta, \theta)$.

Step 3. For each member (chromosome) of the population do

Step 3.1. set the current time $t = 0$ and number of jobs scheduled $k = 0$ and calculate average of averages of processing times as follows:

$$\overline{p_{avg}} = \sum_{i=1}^{N} \frac{(p_i^u + p_i^l)/2}{N}$$

Step 3.2. For each job i at time t, calculate the ATC priorities as follows:

$$a_i = \frac{w_i}{p_{avg_i}} exp(-max(0, dd_i - t - p_{avg_i})/K\overline{p_{avg}})$$

Step 3.3. ATC priorities are normalized into interval [0,1] yielding $\eta_i(t)$ as follows, let $a_{min}(t) = \min_i a_i(t)$ and $a_{max}(t) = \max_i a_i(t)$:

$$\eta_i(t) = \frac{a_i(t) - a_{min}(t)}{a_{max}(t) - a_{min}(t)}$$

Step 3.4. Perturb the priorities of the jobs with this member by adding the perturbation vector δ_i to the priorities as follows:

$$\eta_i(t) = \eta_i(t) + \delta_i$$

Step 3.5. Select the job with the highest perturbed priority and schedule it next in the sequence. Set $t = t + p_{avg_i}$ and $k = k + 1$. If there are any unscheduled jobs, $k < N$, then go to step 2.1.

Step 3.6. For newly generated sequence, find the schedule by the base heuristic, which is the mathematical formulation given below or the DP-based heuristic explained in the next section, and calculate the objective function value for the given perturbation vector, denoted by $V(i)$.

$$\text{Minimize Total Cost} = \sum_{j=1}^{N}(w_j T_j + Mach_j(p_j) + Tool_j(p_j))$$

subject to

$$s_j + p_j \leq s_{j+1} \quad j = 1 \ldots (N-1) \tag{4.1}$$

$$p_j^l \leq p_j \leq p_j^u \quad j = 1 \ldots N \tag{4.2}$$

$$s_j \geq 0 \tag{4.3}$$

$$p_j > 0 \tag{4.4}$$

$$\text{where } T_j = max\{0, s_j + p_j - dd_j\}$$

Step 4. After finishing all members in the population, save the best and worst solutions. If the number of generations reaches the limit, then stop and report the best solution, else go to step 4 to generate a new population.

Step 5. Compute the *fitness* $f(i)$ of each member as follows, Let V_{max} be the maximum objective value in the population:

$$f_i = \frac{(V_{max} - V_i)^\phi}{\sum_i (V_{max} - V_i)^\phi}$$

Step 6. Apply crossover and mutation to get the next generation using the fitness distribution and update perturbation vectors, then go to step 2.

The algorithm above presents a single-start PSGA. It proceeds until the number of generations reaches MAXGEN. However, in multi-start, after generating that much populations, the algorithm restarts itself from the first step for NUMSTART times. The PSGA generates as many as the multiplication of POPSIZE, MAXGEN, and NUMSTART for each run.

4.3 The Proposed DP-based Heuristic

In this section, we present the proposed heuristic to solve the problem for a given sequence, for which we already gave the mathematical formulation in §3.2.3. This algorithm is to be used as an evaluation function (step 3.6) in the problem space genetic algorithm described in the previous section.

The motivation behind this algorithm was that if we can minimize the contribution of each job to the total cost, we can minimize the total cost. Thus we defined a contributed cost function for each job as follows:

$$ContCost_i(p_i) = Tool_i(p_i) + Mach(p_i) + \sum_{j=i}^{N} w_j \Delta Tard_j(p_i) \tag{4.5}$$

Each cost item in this function can be defined as the tooling cost, machining cost, and the deviation in the tardiness costs of itself and all jobs after itself depending on its own processing time. In constructing this function, we assumed that $p_k = p_k^l$ for $k = 1 \ldots i-1$, since otherwise we cannot define the contributed cost just depending on the processing time of job i because $\Delta Tard_j$, which is the deviation in the tardiness cost of job j, is also affected from the change in processing times of those jobs. By fixing them, we define $\Delta Tard_i$ over just p_i. In our algorithm, finding this contributed cost is named as *Graph Generation*, which corresponds to steps 2 and 3 below, since we generate a graph that shows how the contributed cost of job i changes.

For each job, we can easily construct the function above. However we cannot directly get the processing time that minimizes it and then use them to find the final schedule. The reason is that we assumed that processing times of the previous jobs are in their lower bounds as we constructed the contributed cost function. However their processing times may differ from the lower bounds. To deal with this problem we first define Δ_i, which is the total change deviation in the sum of processing times of jobs before job i:

$$\Delta_i = \sum_{k=1}^{i-1} p_k - p_k^l \tag{4.6}$$

Machining and tooling cost of a job is independent from Δ_i. However $\Delta Tard_i$ is dependent on it. What we do in the algorithm is that for all possible values of Δ_i we find the corresponding processing times of each job. Actually, this corresponds to the *State Generation* step in our algorithm. It is called so because we find the states, in other words ranges, of Δ_i and processing times for these states.

The proposed algorithm is similar to a backward DP. Beginning from the last job processed, for each job, generating *graphs* and *states* iteratively, it defines a function for each job that gives how the processing times of the jobs processed after that job depend on the deviation of processing time of that job. After finding the processing time of the first job, the algorithm finds the processing time of the second job, third job and so on. It is an approximation algorithm since the defined contributed cost considers the interaction between jobs on the basis of just tardiness costs. There is also an interaction through the tooling cost. However, due to the nonlinearity of tooling cost, developing a practical solution procedure is almost impossible by considering also that interaction. The step by step definition of the algorithm is as follows:

Step 1. set $i = N$.

Step 2.**(Graph Generation)** for $j = i$ to $j = N$ do:

> Step 2.1 Set $p_k = p_k^l$ for $k = 1, \ldots, i-1$ and construct the function $P_j(p_i)$ that shows how the processing time of job j depends on the processing time of job i.

Step 3.**(Graph Generation)** Construct the contributed cost for job i which is defined in equation 4.5.

Step 4. If $i > 1$ goto step 5, else goto step 7.

Step 5.**(State Generation)** Generate the function $P_i(\Delta_i)$, where Δ_i is the total deviation of processing times of jobs 1 to $i-1$ from their corresponding lower bounds as defined in equation 4.6.

Step 6. Generate the function $P_i(p_{i-1})$ from $P_i(\Delta_i)$ by replacing Δ_i with $p_{i-1} - p^l_{i-1}$, then set $i = i - 1$ and goto step 2.

Step 7. Find the minimum of the following total cost function for job 1:

$$ContCost_1(p_1) = \sum_{i=1}^{N} w_j \Delta Tard_i(p_i) + Tool_1(p_1) + Mach_1(p_1)$$

Step 8. Calculate $P_i(p^\star_1)$ for $i = 2 \ldots N$. This gives the processing times of all jobs in the sequence.

In the following two subsections, we will give further explanation about the steps 2, 3 and 5 of the proposed algorithm that are named as graph generation and state generation, respectively. These are the critical steps of the algorithm since most of the computational effort in the algorithm is spent in these steps.

4.3.1 Explanation of Graph Generation

In steps 2 and 3, our objective is to find the contributed cost of job i. For that purpose, we will first find how processing times of jobs processed after job i depend on the processing time of job i. Then, we will find how the tardiness costs of job i to job N deviate depending on the increase in the processing time of job i. By using this information, we will construct the contributed cost.

As we stated before, the machining and tooling costs of a job are only a function of its own processing time. However, the deviations in tardiness' of the other jobs are dependent on both starting time and their processing times. The problem in constructing the contributed cost of job i arises from calculating how we find the deviations of the tardiness costs of job j for $j = i \ldots N$ considering the changes in the processing times of job l for $l = i \ldots j$.

To find $\Delta Tard_j$ for $j = i \dots N$, we have to first construct the function $P_j(p_i)$ that shows how the processing time of job j depends on processing time of job i while $p_k = p_k^l$ for $k = 1 \dots i-1$. This is a piecewise linear function which can be discontinuous. For k^{th} state of p_i,which is defined as $r_{j,i}^k \leq p_i < r_{j,i}^{k+1}$, it returns a value by a function in the form of $c_{j,i}^k - x_{j,i}^k p_i$. In this function, $c_{j,i}^i$ symbolizes a parameter whose superscript k indicates which state it corresponds, and subscript j, i indicates which P function it corresponds. $x_{j,i}^k$ is 0 or 1 parameter which indicates p_j either decreases in the same magnitude as p_i increases or it remains the same in the $k^{th} state$. $r_{j,i}^k$ and $r_{j,i}^{k+1}$ are also parameters that show the boundaries of states. The number of states in all functions are also finite. The reason for that is explained in the next section.

For $i = N$, it is obvious that $P_N(p_N) = p_N \quad p_N^u \geq p_N \geq p_N^l$. For $i < N$, when we come to this step we have already known $P_j(p_{i+1})$ for $j = (i+2) \dots N$ and they are in the following form :

$$P_j(p_{i+1}) = \begin{cases} c_{j,i+1}^1 & r_{j,i+1}^1 \leq p_{i+1} < r_{j,i+1}^2 \\ c_{j,i+1}^2 - p_{i+1} & r_{j,i+1}^2 \leq p_{i+1} < r_{j,i+1}^3 \\ c_{j,i+1}^3 & r_{j,i+1}^3 \leq p_{i+1} < r_{j,i+1}^4 \\ c_{j,i+1}^4 - p_{i+1} & r_{j,i+1}^4 \leq p_{i+1} < r_{j,i+1}^5 \\ \vdots & \\ c_{j,i+1}^{Y^{j,i+1}} & r_{j,i+1}^{Y^{j,i+1}} \leq p_{i+1} < r_{j,i+1}^{Y^{j,i+1}+1} \end{cases} \tag{4.7}$$

This function is generated by setting $p_k = p_k^l$ for $k = 1 \dots i$ and making p_{i+1} variable. Thus, for

$$\Delta_{i+1} = \sum_{k=1}^{i+1} p_k - p_k^l \tag{4.8}$$

the function 4.7 above and the function 4.9 below are equivalent:

$$P_j(\Delta_{i+1}) = \begin{cases} c_{j,i+1}^1 & r_{j,i+1}^1 - p_{i+1}^l \leq \Delta_{i+1} < r_{j,i+1}^2 - p_{i+1}^l \\ c_{j,i+1}^2 - p_{i+1}^l - \Delta_{i+1} & r_{j,i+1}^2 - p_{i+1}^l \leq \Delta_{i+1} < r_{j,i+1}^3 - p_{i+1}^l \\ c_{j,i+1}^3 & r_{j,i+1}^3 - p_{i+1}^l \leq \Delta_{i+1} < r_{j,i+1}^4 - p_{i+1}^l \\ c_{j,i+1}^4 - p_{i+1}^l - \Delta_{i+1} & r_{j,i+1}^4 - p_{i+1}^l \leq \Delta_{i+1} < r_{j,i+1}^5 - p_{i+1}^l \\ \vdots & \\ c_{j,i+1}^{Y^{j,i+1}} & r_{j,i+1}^{Y^{j,i+1}} - p_{i+1}^l \leq \Delta_{i+1} < r_{j,i+1}^{Y^{j,i+1}+1} - p_{i+1}^l \end{cases} \tag{4.9}$$

From the previous iteration, we know how the processing time of job $i+1$ depends on p_i:

$$P_{i+1}(p_i) = \begin{cases} c^1_{i+1,i} & r^1_{i+1,i} \leq p_i < r^2_{i+1,i} \\ c^2_{i+1,i} - p_i & r^2_{i+1,i} \leq p_i < r^3_{i+1,i} \\ c^3_{i+1,i} & r^3_{i+1,i} \leq p_i < r^4_{i+1,i} \\ c^4_{i+1,i} - p_i & r^4_{i+1,i} \leq p_i < r^5_{i+1,i} \\ \vdots & \\ c^{Y^{i,i+1}}_{i+1,i} & r^{Y^{i,i+1}}_{i+1,i} \leq p_i < r^{Y^{i,i+1}+1}_{i+1,i} \end{cases} \tag{4.10}$$

Now, in equation 4.8, we put the $P_{i+1}(p_i)$ in place of p_{i+1}, make p_i a variable and set $p_k = p^l_k$ for $k = 1 \ldots i-1$. As a result we obtain following function for Δ_{i+1}:

$$\Delta_{i+1} = P_{i+1}(p_i) - p^l_{i+1} + p_i - p^l_i = \begin{cases} c^1_{i+1,i} - p^l_{i+1} + p_i - p^l_i & r^1_{i+1,i} \leq p_i < r^2_{i+1,i} \\ c^2_{i+1,i} - p^l_{i+1} - p^l_i & r^2_{i+1,i} \leq p_i < r^3_{i+1,i} \\ c^3_{i+1,i} - p^l_{i+1} + p_i - p^l_i & r^3_{i+1,i} \leq p_i < r^4_{i+1,i} \\ c^4_{i+1,i} - p^l_{i+1} - p^l_i & r^4_{i+1,i} \leq p_i < r^5_{i+1,i} \\ \vdots & \\ c^{Y^{i,i+1}}_{i+1,i} - p^l_{i+1} + p_i - p^l_i & r^{Y^{i,i+1}}_{i+1,i} \leq p_i < r^{Y^{i,i+1}+1}_{i+1,i} \end{cases} \tag{4.11}$$

When we put the Δ_{i+1}, which is defined with function 4.11, in its place in function 4.9 we find how the processing time of job j, $j = (i+2) \ldots N$, depends on p_i.

After finding $P_j(p_i)$ for $j = i \ldots N$, and knowing that $p_k = p^l_k$ for $k = 1 \ldots i-1$, we can construct $\sum_{j=i}^{N} w_j \Delta Tard_j(p_i)$. For this purpose, we use the following algorithm:

Step 1. From functions $P_j(p_i)$ for $j = i \ldots N$ collect all $r^k_{j,i}$ for $k = 1, \ldots, \omega_{j,i}+1$ where $\omega_{j,i}$ is the number of states of function $P_j(p_i)$. Call the set of them as Ω_i. Then eliminate duplicate entries in Ω_i and name remaining ones as $r^{[k]}_i$ for $k = 1 \ldots W_i$ such that $r^{[1]}_i < r^{[2]}_i < \ldots < r^{[W_i]}_i$.

Step 2. Set $p^0_j = P_j(p^l_i)$, where p^0_j is the processing time of job j when there is no deviation in p_i from p_i^l. We can formulate it as follows,

$$p_j^0 = c_{j,i}^{[0]} - x_{j,i}^{[0]} p_i \quad r_i^{[0]} \leq p_i \leq r_i^{[1]}$$

Step 3. For $k = 1$ to $W_i - 1$ do,

Step 3.1. Find the p_j for $j = i + 1 \dots N$ by using $P_j(p_i)$ over $r_i^{[k]} \leq p_i \leq r_i^{[k+1]}$, it is defined as follows:

$$p_j = c_{j,i}^{[k]} - x_{j,i}^{[k]} p_i \quad r_i^{[k]} \leq p_i \leq r_i^{[k+1]}$$

Step 3.2. Set $s_{i-1} = \sum_{t=1}^{i-2} p_t^l$ and for $j = i$ to N do

Step 3.2.1. Calculate starting time of job j

$$s_j = s_{j-1} + p_{j-1}$$

Step 3.2.2. Calculate the deviation in the tardiness cost of job j

$$\Delta Tard_j(p_i) = max\{0, s_j + (p_j - p_j^l) - dd_j\} \quad r_i^{[k]} \leq p_i \leq r_i^{[k+1]}$$

Step 3.3. Sum up all $w_j \Delta Tard_j(p_i)$ and find total weighted deviation in tardiness'

$$TotalTard_i(p_i) = \sum_{j=i}^{N} w_j \Delta Tard_j(p_i) = m_i^k p_i + n_i^k \quad r_i^{[k]} \leq p_i \leq r_i^{[k+1]}$$

m_i^k and n_i^k are just constants that come from the weighted summation of the constants in the $\Delta Tard_j$ functions. If we define Π as the set of jobs whose $Tard_j > 0$ for $j = i \dots N$:

$$m_i^k = \sum_{j \in \Pi} w_j (x_{j,i}^{[k]} - x_{j,i}^{[0]}) \quad \text{and} \quad n_i^k = \sum_{j \in \Pi} (s_j + (c_{j,i}^{[k]} - c_{j,i}^{[0]}) - dd_j)$$

At the end of this algorithm, we reach the function that shows the total deviation in tardiness costs of job i to job N depending on p_i. It is a piecewise linear function in the following form as in the Figure 4.1:

$$TotalTard_i(p_i) = \begin{cases} m_i^1 p_i + n_i^1 & r_i^{[1]} \leq p_i < r_i^{[2]} \\ m_i^2 p_i + n_i^2 & r_i^{[2]} \leq p_i < r_i^{[3]} \\ \vdots & \\ u_i^{W^i} p_i + n_i^{W^i} & r_i^{[W_i]} \leq p_i < r_i^{[W_i+1]} \end{cases} \tag{4.12}$$

When we sum up this function with $Tool_i(p_i)$ and $Mach_i(p_i)$, the *Graph Generation* steps finish, this means that we generated the contributed cost function. After checking whether we reached the first job, the algorithm either continues to the *State Generation* step or to the Step 7 of main algorithm where the processing time of the first job is found.

4.3.2 Explanation of State Generation

Our objective in this step is, by using the contributed cost function given in equation 4.5, to construct the function that gives the processing time of job i over the different states, or ranges, of Δ_i.

In steps 2 and 3 of the main algorithm, we generated the contributed cost for job i defined in the equation 4.5. As we stated before, while constructing this cost function, we set $p_k = p_k^l$ for $k = 1 \dots i-1$ and the processing time obtained by minimizing this function does not produce good solutions since most probably there will be jobs whose processing times are not in their lower bounds. We give the sum of the variations in the processing times jobs in equation 4.8 for job i. We have to include these variations in our total cost function and the cost item that we modify by including Δ_i is the total tardiness function which is stated in (4.12) above because the others are not dependent on Δ_i. The modified tardiness cost function is given below:

$$TotalTard_i'(p_i, \Delta_i) = \begin{cases} m_i^1 p_i + n_i^1 + m_i^1 \Delta_i & r_i^{[1]} \leq p_i + \Delta < r_i^{[2]} \\ m_i^2 p_i + n_i^2 + m_i^2 \Delta_i & r_i^{[2]} \leq p_i + \Delta < r_i^{[3]} \\ \vdots & \\ m_i^W p_i + n_i^W + m_i^W \Delta_i & r_i^{[W]} \leq p_i + \Delta < r_i^{[W+1]} \end{cases} \tag{4.13}$$

To find $P_i(\Delta_i)$, we have to solve the following minimization problem:

$$P_i(\Delta_i) = \{p_i \in argmin\{TotalTard_i'(p_i, \Delta_i) + Tool_i(p_i) + Mach_i(p_i) : p_i^l \leq p_i < p_i^u\}\}$$

For $\Delta_i = 0$, $TotalTard_i'(p_i, \Delta_i)$ is equivalent to $TotalTard_i(p_i)$ function and

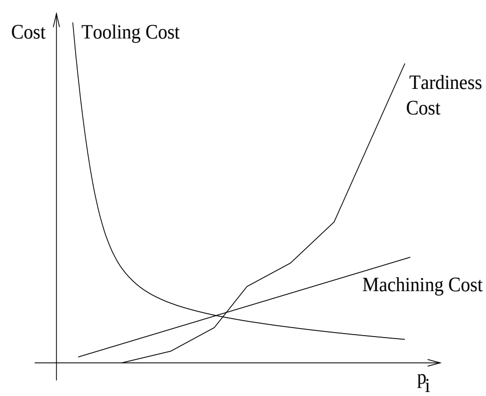

Figure 4.1: The cost items for each job

its example shape can be seen from the Figure 4.1. Obviously, as Δ_i increases, the total tardiness cost function moves towards to the y-axis. The reason for that, if a job has a tardiness value t for the state of $(\Delta_i = D, p_i = p)$ and we increase the Δ_i by δ, now that job has a tardiness value t for the state of $(\Delta_i = D+\delta, p_i = p-\delta)$. The other cost items, tooling and machining costs remain constant with respect to Δ_i since they are independent from the deviations in the processing times of the previous jobs in the sequence. For each Δ_i value, there is only one minimum value of the total cost function. Moreover there is a range of $x \leq \Delta_i \leq y$ such that that minimum point still remains as the minimum. Our purpose is to find the minimum values and corresponding "Δ-ranges" defined in the function $P_i(\Delta)$ and by this way to solve the minimization problem above.

To find the minimum of the total cost, we present the following algorithm:

Step 1. Set $P_i(\Delta_i) = 0$ for $0 \leq \Delta_i < \Delta_i^{max}$ where $\Delta_i^{max} = \sum_{k=1}^{i-1}(p_k^u - p_k^l)$.

Step 2. For $k = 1$ to $W - 1$ do

Step 2.1. Consider k^{th} and $(k+1)^{th}$ pieces in the $TotalTard'_j(p_i, \Delta_i)$ (line

k and $k+1$), which are:

$$L_i^k(p_i,\Delta_i) = \begin{cases} m_i^k p_i + n_i^k + m_i^k \Delta_i & r_i^{[k]} \le p_i + \Delta_i < r_i^{[k+1]} \\ \infty & o.w. \end{cases}$$

$$L_i^{k+1}(p_i,\Delta_i) = \begin{cases} m_i^{k+1} p_i + n_i^{k+1} + m_i^k \Delta_i & r_i^{[k+1]} \le p_i + \Delta_i < r_i^{[k+2]} \\ \infty & o.w. \end{cases}$$

Step 2.2. Generate the function $R_i(\Delta_i)$ which is:

$$R_i(\Delta_i) = \{p_i \in Argmin\{Min\{CL_i^k(\Delta_i,p_i), CL_i^{k+1}(\Delta_i,p_i)\} : p_i^l \le p_i < p_i^u\}\}$$

where

$$\begin{aligned} CL_i^k(\Delta_i,p_i) &= L_i^k(p_i,\Delta_i) + Tool(p_i) + Mach(p_i) \\ CL_i^{k+1}(\Delta_i,p_i) &= L_i^{k+1}(p_i,\Delta_i) + Tool(p_i) + Mach(p_i) \end{aligned}$$

Step 2.3. Find $P_i'(\Delta_i)$ by combining $R_i(\Delta_i)$ and $P_i(\Delta_i)$ as follows:

$$P_i'(\Delta_i) = \{p_i \in Argmin\{Min\{ContCost_i(R_i(\Delta_i)), ContCost_i(P_i(\Delta_i))\}\}\}$$

Step 2.4. Set $P_i(\Delta_i) = P_i'(\Delta_i)$

The algorithm is very simple. We take two adjacent pieces and analyze the contributed cost function that corresponds to these two pieces.

In step 2.2, if these two lines form a convex shape (as in Figure 4.2), to find $R_i(\Delta)$ we use *UseConvex* subroutine, else we use *UseConcave* subroutine. For readability, in the next two subsections, we will drop the subscript i from all parameters, functions and variables except for p_i and Δ_i.

4.3.2.1 *UseConvex* Subroutine

Two consecutive tardiness cost lines may form a convex shape as in the Figure 4.2. The reason for this is obvious, when a job becomes tardy at a point without affecting the other jobs, the slope of the function after that point increases.

$R(\Delta_i)$ has a direct formulation in this case which we call *UseConvex* subroutine. Consider L^k and L^{k+1}, $R(\Delta_i)$ is formulated as follows:

$$R(\Delta_i) = \begin{cases} r^{[k]} - \Delta_i & 0 \leq \Delta_i < r^{[k]} - min_k \\ min^k & r^{[k]} - min^k \leq \Delta_i < r^{[k+1]} - min^k \\ r^{[k+1]} - \Delta_i & r^{[k+1]} - min^k \leq \Delta_i < r^{[k+1]} - min^{k+1} \\ min^{k+1} & r^{[k+1]} - min^{k+1} \leq \Delta_i < r^{[k+2]} - min^{k+1} \\ r^{[k+2]} - \Delta_i & r^{[k+2]} - min^k \leq \Delta_i < \Delta_i^{max} \end{cases}$$

where

$$min^k = argmin\{CL^k(0, p_i) : p_i^l \leq p_i < p_i^u\}$$
$$min^{k+1} = argmin\{CL^{k+1}(0, p_i) : p_i^l \leq p < p_i^u\}$$

In this formulation $r^{[k]}$ is greater than min^k. In other cases, for example $r^{[k]} \leq min^k$, $r^{[k+1]} \leq min^{k+1}$ and so on, the formulation is modified by deleting ranges in which both sides of Δ_i are non-positive and by changing left side of the Δ_i,whose left side is negative but right size is positive,to zero.

The construction of this formula is based on graphical observations. Firstly, if we take the partial derivatives of CL^k and CL^{k+1} over their defined regions for Δ_i we see that $min^{k+1} \leq min^k$ since:

$$\frac{\partial}{\partial p_i} CL^k(\Delta_i, p_i) = m^k - \frac{c_a c_b}{p_i^{c_b - 1}} + C_0 = 0 \Longrightarrow p_i = (\frac{c_a c_b}{m^k C_0})^{\frac{1}{c_b - 1}} = min^k$$

$$\frac{\partial}{\partial p_i} CL^{k+1}(\Delta_i, p_i) = m^{k+1} - \frac{c_a c_b}{(p_i)^{c_b + 1}} + C_0 = 0 \Longrightarrow p_i = (\frac{c_a c_b}{m^{k+1} C_0})^{\frac{1}{c_b + 1}} = min^{k+1}$$

$$m^k < m^{k+1} \Longrightarrow min^{k+1} \leq min^k$$

Initially, let $r^{[k+2]} \geq r^{[k+1]} \geq r^{[k]} \geq min^k \geq min^{k+1}$ as in Figure 4.2. In this case, $R(\Delta_i) = r^{[k]} - \Delta_i$, and this case is valid for $0 \leq \Delta_i < r^{[k]} - min^k$.

When $r^{[k]} - min^k \leq \Delta_i < r^{[k+1]} - min^k$, the tardiness cost function becomes as in Figure 4.3. In this case, the minimum for $L1$ is the minimum of total cost function(considering only $L1$ and $L2$).

When $r^{k+1} - min^k \leq \Delta_i < r^{[k+1]} - min^{k+1}$, the tardiness cost function becomes as in Figure 4.4. In this case, the break point between two lines $bp_2 = r^{[k+1]} - \Delta_i$ is the minimum of total cost function.

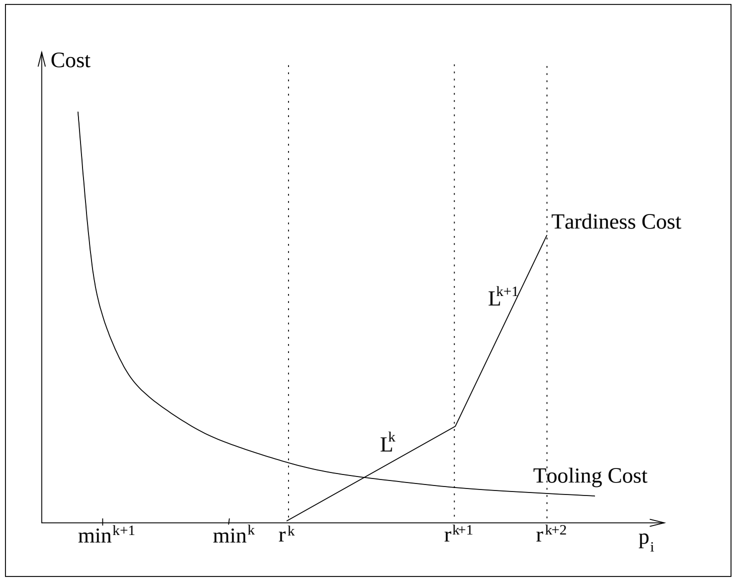

Figure 4.2: $0 \leq \Delta_i < r^{[k]} - min^k$

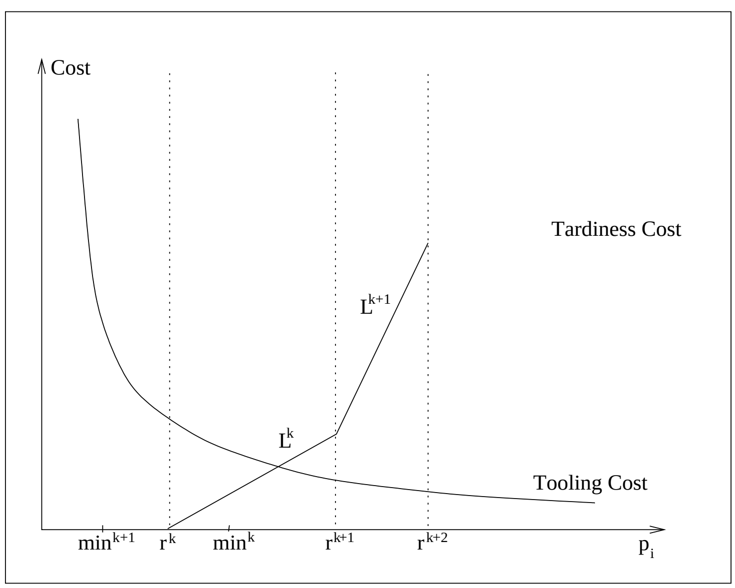

Figure 4.3: $r^{[k+1]} - min^k \leq \Delta_i < r^{[k+1]} - min^{k+1}$

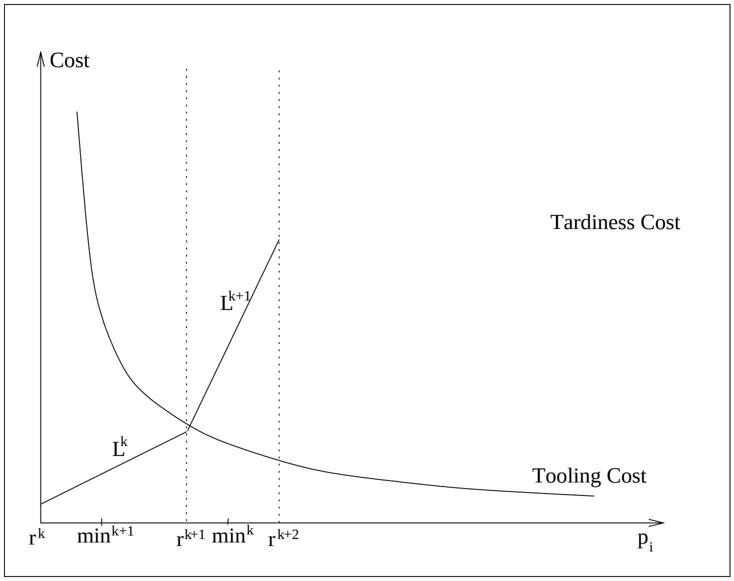

Figure 4.4: $r^{[k]} - min^k \leq \Delta_i < r^{[k+1]} - min^k$

The rest of the function $R(\Delta_i)$ can be derived in a similar way.

4.3.2.2 ***UseConcave*** Subroutine

Sometimes, two consecutive tardiness cost lines form a concave shape as in Figure 4.5. To explain the reason of this situation, let us consider 3 jobs i, j, and k whose processing order is job i, job j, and job k. Also let job k be tardy and have a constant processing time. While p_j is constant and as p_i increases tardiness of job k increases. However if at a point, say t, p_j may start to decrease as p_i increases, at that point, the tardiness of job k equals to a constant value and remains at that value as long as p_j decreases. This causes concavity because the slope of the total tardiness cost function falls down after point t. In such a case, $R(\Delta_i)$ requires a detailed algorithm. We developed an algorithm that can be used also for non-consecutive lines since we need it in step 2.3 of the *State Generation* algorithm (when two lines are not consecutive the formulation for convex case cannot be used and this algorithm is used again). Two tardiness lines for which

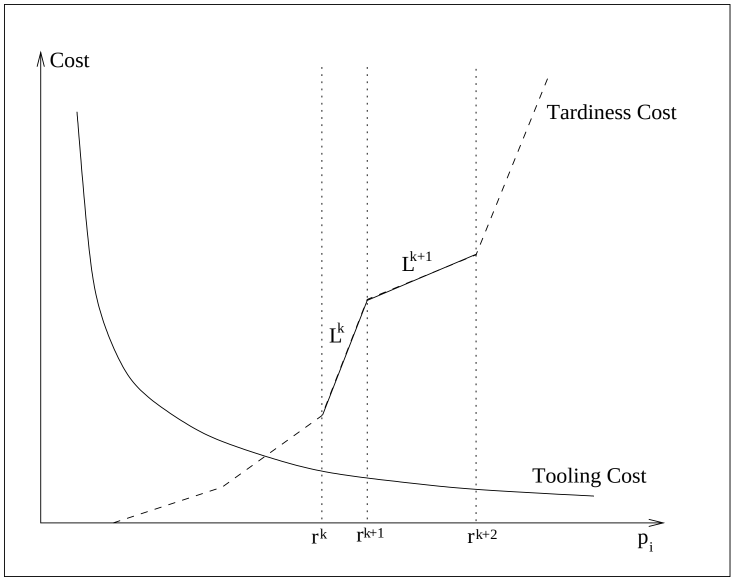

Figure 4.5: Two lines that form concave shape

we want to generate $R(\Delta_i)$ are the followings:

$$L^k(p_i, \Delta_i) = \begin{cases} m^k * p_i + n^k + m^k \Delta_i & r^{[k]} \leq p_i + \Delta_i < r^{[k+1]} \\ \infty & o.w. \end{cases}$$

$$L^s(p_i, \Delta_i) = \begin{cases} m^s * p_i + n^s + m^s \Delta_i & r^{[s]} \leq p_i + \Delta_i < r^{[s+1]} \\ \infty & o.w. \end{cases}$$

where $r^{[s]} \geq r^{[k+1]}$.

Now consider the following simplified versions of L_k and L_s:

$$L^k(p_i) = \begin{cases} m^k * p_i + (n^k)' & b_1 \leq p_i < b_2 \\ \infty & o.w. \end{cases}$$

$$L^s(p_i) = \begin{cases} m^s * p_i + (n^s)' & b_3 \leq p_i < b_4 \\ \infty & o.w. \end{cases}$$

where

$b_1 = r^{[k]} - \Delta_i$

$b_2 = r^{[k+1]} - \Delta_i$

$b_3 = r^{[s]} - \Delta_i$

$b_4 = r^{[s+1]} - \Delta_i$

$(n^k)' = n^k + m^k \Delta_i$

$(n^s)' = n^s + m^k \Delta_i$

Corresponding total cost functions CL^k and CL^s are similar to the ones in Figure 4.6. The possible locations of b_1, b_2, b_3, b_4, min^k, and min^s relative to each other change depending on the value of Δ_i and values of $r^{[k]}$, $r^{[s]}$, $r^{[k+1]}$, $r^{[s+1]}$. The possible ordering of them for each line as follows:

$$for\ L^k \left\langle \begin{array}{lll} (A) & min^k \leq b_1 < b2 & \text{for} \quad 0 \leq \Delta_i \leq r^{[k]} - min^k \\ (B) & b_1 \leq min^k \leq b2 & \text{for} \quad r^{[k]} - min^k \leq \Delta_i \leq r^{[k+1]} - min^k \\ (C) & b_1 < b2 \leq min^k & \text{for} \quad r^{[k+1]} - min^k \leq \Delta_i \leq \Delta_i^{max} \end{array} \right.$$

$$for\ L^s \left\langle \begin{array}{lll} (1) & min^s \leq b_3 < b4 & \text{for} \quad 0 \leq \Delta_i \leq r^{[s]} - min^s \\ (2) & b_3 \leq min^s \leq b4 & \text{for} \quad r^{[s]} - min^s \leq \Delta_i \leq r^{[s+1]} - min^s \\ (3) & b_3 < b4 \leq min^s & \text{for} \quad r^{[s+1]} - min^s \leq \Delta_i \leq \Delta_i^{max} \end{array} \right.$$

Graphically, (A), (B) and (C) indicate the position of the line L^k, as Δ_i increases, (1), (2) and (3) do the same thing for L^s. The positions of L^k and L^s are important for us. We can explain the reason of that with an example, consider the case that both of these lines position over minimums of contributed cost lines corresponding to them for a defined range of Δ_i. This provides us an important advantage: now we know that for these two pieces of contributed cost, the processing time of job j that minimizes the $ContCost'$ is either m^s or m^k over this range of Δ_i. All of the combinations of these cases give us this kind of advantages and a way for constructing $R(\Delta)$.

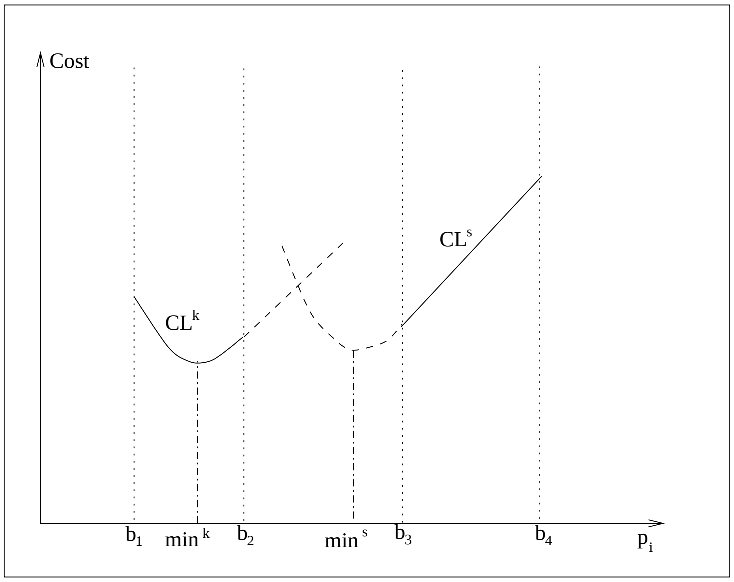

Figure 4.6: The illustration of possible total costs for two tardiness lines

In our algorithm we construct the $R(\Delta_i)$ for each possible combinations of A, B, C and 1, 2, 3 over defined ranges of Δ_i. The figure for possible combinations and the flow between these combinations and some information about cost items are given in Figure4.7. For example, $(A, 1)$ corresponds to $min^k \leq b_1 < b2$ and $min^s \leq b_3 < b4$; $(A, 2)$ corresponds to $min^k \leq b_1 < b2$ and $b_3 \leq min^s < b4$; and so on. The outgoing arcs from $(A, 1)$ means that from $(A, 1)$ we can go either $(A, 2)$ or $(B, 1)$. If $r^{[k]} - min^k > r^{[s]} - min^s$ then we go from $(A, 1)$ to $(A, 2)$, else we go from $(A, 1)$ to $(B, 1)$. We discuss how to construct $R(\Delta_i)$ from these combinations in appendix A.

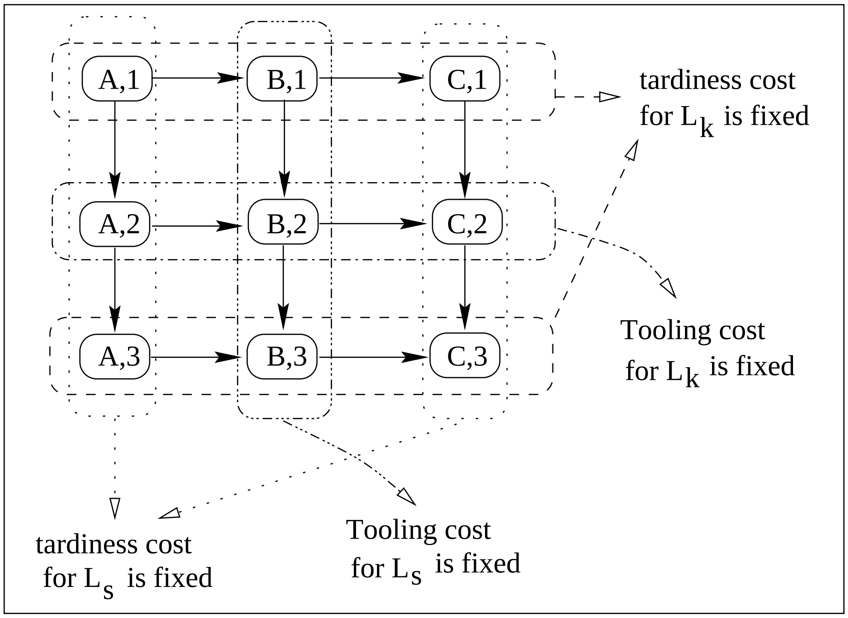

Figure 4.7: Possible combinations of locations of ranges for two tardiness lines and information about cost items corresponding to each line

4.4 Complexity Analysis

In the main algorithm, for each job, a number of operations are performed such as graph generation and state generation. Since the number of jobs is N, the complexity of the algorithm is determined by the complexity of these operations.

In the state generation step, we should determine the maximum number of break points on the contributed cost function for a job. If there is no concavity, which is explained previously, then the maximum number of break points is N. Each such point represents when a job becomes tardy. On the other hand, each job can cause concavity once on each line segment. Therefore, the total number of line segments is at most (N^2). Since we determine states using break points and the minimum of the line segments, the total number of states for any job can be at most $(2N^2)$. In state generation, for job i, we use the total number of line segments for $(N - i)$ jobs. As a result, W, in step 2 of the state generation algorithm, is at most $(2N^3 - 2N^2)$. In step 2.2, while comparing two line segments, we can get at most 6 states (4 end points and 2 minimums of lines) for $R_i(\Delta_i)$, which implies that the total number of comparisons in step 2.3 is at most $(6N^2)$. Therefore, state generation performs $O(N^5)$.

In the graph generation steps, we combine equation 4.11 and equation 4.9. Since the number of comparisons are at most $(N^2 * N^2)$ and since we perform this $(N - i)$ times, the step 2 of the main algorithm performs $O(N^5)$. In step 3 of the main algorithm, contributed cost performs $O(N^4)$ since W is at most $(2N^3 - 2N^2)$ and while determining total tardiness we sum up $(N - i)$ times $\Delta Tard$. As a result, the graph generation performs $O(N^5)$.

Since we use state and graph generation once for each job, the algorithm performs $O(N^6)$.

4.5 Summary

In this chapter, we first presented the sequential algorithm that reflects the general approach in the factory management. Then we gave our proposed simultaneous algorithm, which is a problem space genetic algorithm that uses the proposed DP-based heuristic or mathematical formulation of the problem. Lastly, we presented the DP-based heuristic in detail.

In the next chapter, we will give an illustrative example over a *toy* problem to clarify some of the steps of the proposed DP-based heuristic.

Chapter 5

An Illustrative Example

In this chapter, we will solve a small scheduling problem for a given sequence to show how the proposed DP-based heuristic works. We construct an example for four jobs with the sequence of 1, 2, 3, 4 and job 1 is processed first. Machining cost, C_0, is 1. The data for each job is given in table 5.1.

Job #	w	dd	p^l	p^u	c_a	c_b
1	3	2	0.1	6	4	1
2	1	3	0.2	5	4	2
3	2	1	0.1	6	4	0.5
4	3	6	0.3	7	4	1.5

Table 5.1: Problem data

Job 4:

We start with generating Total Tardiness Function of job 4. Since there is no job after job 4, its $TotalTard'$ function includes only its own tardiness cost:

$$s_4 = p_1^l + p_2^l + p_3^l = 0.4$$
$$\Delta_4^{max} = (6-0.1)+(5-0.2)+(6-0.1) = 16.6$$
$$P_4(p_4) = p_4$$
$$TotalTard_4(p_4) = \begin{cases} 0 & 0.3 \leq p_4 < 5.6 \\ 3(p_4 - 5.6) & 5.6 \leq p_4 < 7 \end{cases}$$
$$TotalTard'_4(p_4, \Delta_4) = \begin{cases} 0 & 0.3 \leq p_4 + \Delta_4 < 5.6 \\ 3(p_4 + \Delta_4 - 5.6) & 5.6 \leq p_4 + \Delta_4 < 7 + 16.6 = 23.6 \end{cases}$$

This function gives how tardiness cost value changes according to the change in the processing times of job 1 to job 4. Now we have to generate $P_4(\Delta_4)$. The total contributed cost lines that correspond to each tardiness cost range is as follows:

$$CL^1(\Delta_4, p_4) = \begin{cases} p_4 + 4/(p_4)^{1.5} & 0.3 \leq p_4 + \Delta_4 < 5.6 \\ \infty & o.w. \end{cases}$$
$$CL^2(\Delta_4, p_4) = \begin{cases} 4p_4 - 16.8 + 3\Delta_4 + 4/(p_4)^{1.5} & 5.6 \leq p_4 + \Delta_4 < 23.6 \\ \infty & o.w. \end{cases}$$

The points that give minimums of lines CL^1 and CL^2 are $min^1 = 2.05$ and $min^2 = 1.18$, respectively.

We construct $R(\Delta_4)$ by using *UseConvex* subroutine. Since there is no more line $P_4(\Delta_4) = R(\Delta_4)$:

$$P_4(\Delta_4) = R(\Delta_4) = \begin{cases} 2.05 & 0.00 \leq \Delta_4 < 3.55 \\ 5.60 - \Delta_4 & 3.55 \leq \Delta_4 < 4.42 \\ 1.18 & 4.42 \leq \Delta_4 < 23.60 \end{cases}$$

Then we reduce $P_4(\Delta_4)$ to $P_4(p_3)$ by setting $\Delta_4 = p_3 - 0.1$:

$$P_4(p_3) = \begin{cases} 2.05 & 0.10 \leq p_3 < 3.65 \\ 5.70 - p_3 & 3.65 \leq p_3 < 4.52 \\ 1.18 & 4.52 \leq p_3 < 23.60 \end{cases}$$

Job 3:

We have to generate $TotalTard$ function of job 3. The only job after job 3 is job 4 and we already know how its processing time changes with respect to the processing time of job 3 from $P_4(p_3)$. We calculate the tardiness costs for each job and sum them up:

$$s_4 = p_1^l + p_2^l + p_3 = 0.3 + p_3$$

$$Tard_4(p_3) = Max\{0, p_3 + P_4(p_3) - 5.7\} = \begin{cases} 0 & 0.1 \le p_3 < 4.52 \\ p_3 - 4.52 & 4.52 \le p_3 < 6 \end{cases}$$

$$s_3 = p_1^l + p_2^l = 0.3$$

$$\Delta_3^{max} = (5 - 0.2) + (6 - 0.1) = 10.7$$

$$Tard_3(p_3) = Max\{0, s_3 + p_3 - 1\} = \begin{cases} 0 & 0.1 \le p_3 < 0.7 \\ p_3 - 0.7 & 0.7 \le p_3 < 6 \end{cases}$$

$$TotalTard(p_3) = w_3 Tard_3(p_3) + w_4 Tard_4(p_3) = \begin{cases} 0 & 0.10 \le p_3 < 0.70 \\ 2(p - 0.70) & 0.70 \le p_3 < 4.52 \\ 5(p - 2.99) & 4.52 \le p_3 < 6 \end{cases}$$

$$TotalTard_3'(\Delta_3, p_3) = \begin{cases} 0 & 0.10 \le p_3 + \Delta_3 < 0.70 \\ 2(p + \Delta_3 - 0.70) & 0.70 \le p_3 + \Delta_3 < 4.52 \\ 5(p + \Delta_3 - 2.99) & 4.52 \le p_3 + \Delta_3 < 6 + 10.7 = 16.7 \end{cases}$$

Now we have to calculate $P_3(\Delta_3)$. We have 3 piecewise parts in total cost function of job 3:

$$CL^1(\Delta_3, p_3) = \begin{cases} p_3 + 4/(p_3)^{0.5} & 0.10 \le p_3 + \Delta_3 < 0.70 \\ \infty & o.w. \end{cases}$$

$$CL^2(\Delta_3, p_3) = \begin{cases} 3p_3 - 1.40 + 2\Delta_3 + 4/(p_3)^{0.5} & 0.70 \le p_3 + \Delta_3 < 4.52 \\ \infty & o.w. \end{cases}$$

$$CL^3(\Delta_3, p_3) = \begin{cases} 6p - 14.95 + 5\Delta_3 + 4/(p_3)^{0.5} & 4.52 \le p_3 + \Delta_3 < 16.7 \\ \infty & o.w. \end{cases}$$

The processing times that minimizes CL^1, CL^2 and CL^3 are $min_1 = 1.59$, $min_2 = 0.76$, and $min_3 = 0.48$, respectively.

As it is seen all consecutive lines form a convex shape. Thus we can use

UseConvex subroutine and find the $P_3(\Delta_3)$ easily:

$$P_3(\Delta_3) = \begin{cases} 0.76 & 0.00 \leq \Delta_3 < 3.76 \\ 4.52 - \Delta_3 & 3.76 \leq \Delta_3 < 4.04 \\ 0.48 & 4.04 \leq \Delta_3 < 10.7 \end{cases}$$

Then convert $P_3(\Delta_3)$ to $P_3(p_2)$:

$$P_3(p_2) = \begin{cases} 0.76 & 0.20 \leq p_2 < 3.96 \\ 4.72 - p_2 & 3.96 \leq p_2 < 4.24 \\ 0.48 & 4.24 \leq p_2 < 10.7 \end{cases}$$

<u>*Job 2:*</u>

We know $P_3(p_2)$ and $P_2(p_2)$ but do not know $P_4(p_2)$. We have to calculate it by using $P_4(p_3)$ and $P_3(p_2)$:

$$P_4(\Delta_3) = \begin{cases} 2.05 & 0.00 \leq \Delta_3 < 3.55 \\ 5.60 - \Delta_3 & 3.55 \leq \Delta_3 < 4.42 \\ 1.18 & 4.42 \leq \Delta_3 < 10.7 \end{cases}$$

$$\Delta_3 = P_3(p_2) + p_2 - 0.3 = \begin{cases} 0.46 + p_2 & 0.20 \leq p_2 < 3.96 \\ 4.42 & 3.96 \leq p_2 < 4.24 \\ 0.18 + p_2 & 4.24 \leq p_2 < 5 \end{cases}$$

When we inserted Δ_3 into $P_4(\Delta_3)$ we obtain $P_4(p_2)$:

$$P_4(p_2) = \begin{cases} 2.05 & 0.20 \leq p_2 < 3.09 \\ 5.14 - p_2 & 3.09 \leq p_2 < 3.96 \\ 1.18 & 3.96 \leq p_2 < 5 \end{cases}$$

Now, we can calculate the $TotalTard$ function of job 2:

$$s_4 = p_1^l + p_2 + P_3(p_2) = \begin{cases} 0.86 + p_2 & 0.20 \leq p_2 < 3.96 \\ 4.82 & 3.96 \leq p_2 < 4.24 \\ 0.58 + p_2 & 4.24 \leq p_2 < 6 \end{cases}$$

$$Tard_4(p_2) = Max\{0, s_4 + P_4(p_2) - 6\} = \begin{cases} 0 & 0.2 \leq p_2 < 4.24 \\ p_2 - 4.24 & 4.24 \leq p_2 < 5 \end{cases}$$

$s_3 = 0.1 + p_2$

$$Tard_3(p_2) = Max\{0, s_3 + P_3(p_2) - 1\} = \begin{cases} p_2 - 0.14 & 0.2 \le p_2 < 3.96 \\ 3.82 & 3.96 \le p_2 < 4.24 \\ p_2 - 0.42 & 4.24 \le p_2 < 5 \end{cases}$$

$s_2 = p_1^l = 0.1$

$\Delta_2^{max} = 6 - 0.1 = 5.9$

$$Tard_2(p_2) = Max\{0, s_2 + p_2 - 3\} = \begin{cases} 0 & 0.2 \le p_2 < 2.9 \\ p_2 - 2.9 & 2.9 \le p_2 < 5 \end{cases}$$

$$TotalTard_2(p_2) = \sum_{i=2}^{4} w_i Tard_i(p_2) = \begin{cases} 2(p_2 - 0.14) & 0.20 \le p_2 < 2.90 \\ 3(p_2 - 1.06) & 2.90 \le p_2 < 3.96 \\ 1(p_2 + 4.75) & 3.96 \le p_2 < 4.24 \\ 6(p_2 - 2.74) & 4.24 \le p_2 < 5 \end{cases}$$

$$TotalTard_2'(\Delta_2, p_2) = \begin{cases} 2(p_2 + \Delta_2 - 0.14) & 0.20 \le p_2 + \Delta_2 < 2.90 \\ 3(p_2 + \Delta_2 - 1.06) & 2.90 \le p_2 + \Delta_2 < 3.96 \\ 1(p_2 + \Delta_2 + 4.75) & 3.96 \le p_2 + \Delta_2 < 4.24 \\ 6(p_2 + \Delta_2 - 2.74) & 4.24 \le p_2 + \Delta_2 < 10.9 \end{cases}$$

As it is seen the first and the second pieces in Total Tardiness function form a concave shape. Now, we have to also use *UseConcave* function to calculate $P_2(\Delta)$.

$$CL^1(\Delta_2, p_2) = \begin{cases} 3p_2 + 2\Delta_2 - 0.28 + 4/(p_2)^2 & 0.20 \le p_2 + \Delta_2 < 2.90 \\ \infty & o.w. \end{cases}$$

$$CL^2(\Delta_2, p_2) = \begin{cases} 4p_2 + 3\Delta_2 - 3.18 + 4/(p_2)^2 & 2.90 \le p_2 + \Delta_2 < 3.96 \\ \infty & o.w. \end{cases}$$

$$CL^3(\Delta_2, p_2) = \begin{cases} 2p_2 + \Delta_2 + 4.75 + 4/(p_2)^2 & 3.96 \le p_2 + \Delta_2 < 4.24 \\ \infty & o.w. \end{cases}$$

$$CL^4(\Delta_2, p_2) = \begin{cases} 7p_2 + 6\Delta_2 - 16.44 + 4/(p_2)^2 & 4.24 \le p_2 + \Delta_2 < 10.9 \\ \infty & o.w. \end{cases}$$

The processing times that minimizes CL^1, CL^2, CL^3, and CL^4 are $min^1 = 1.39$, $min^2 = 1.26$, $min^3 = 1.59$, and $min^4 = 1.05$, respectively.

L^1 and L^2 form a convex shape. Thus $R(\Delta_2)$ for CL^1 and CL^2 is constructed by *UseConvex* subroutine:

$$R(\Delta_2) = \begin{cases} 1.39 & 0.00 \le p_2 < 1.51 \\ 2.90 - p_2 & 1.51 \le p_2 < 1.64 \\ 1.26 & 1.64 \le p_2 < 2.70 \\ 3.96 - p_2 & 2.70 \le p_2 < 5 \end{cases}$$

Initially $P_2(\Delta_2) = R(\Delta_2)$. L^1 and L^2 form a concave shape. Thus $R(\Delta_2)$ for CL^1 and CL^2 is constructed by *UseConcave* subroutine. The initial values of b_1, b_2, b_3, b_4 are for $\Delta_2 = 0$ as follows:

$$b_1 = 2.9 \quad b_2 = 3.96 \quad b_3 = 3.96 \quad b_4 = 4.24$$

The initial order of them with respect to min^2 and min^3 as follows:

$$min^2 < b_1 < b_2 \quad min^3 < b_3 < b_4$$

This order corresponds to state $(A, 1)$ in figure 4.7. Since $b_3 - min^3 = 2.37 > 1.64 = b_1 - min^2$, the next state is $(B, 1)$ and the current state is valid as long as Δ_2 is in the interval (0,1.64).

$$\begin{aligned} F(\Delta_2) &= CL^2(r^{[2]} - \Delta_2, \Delta_2) - CL^3(r^{[3]} - \Delta_2, \Delta_2) \\ &= 4(2.90 - \Delta_2) + 3\Delta_2 - 3.18 + \tfrac{4}{(2.90-\Delta_2)^2} \\ &\quad -(2(3.96 - \Delta_2) + \Delta_2 + 4.75 + \tfrac{4}{(3.96-\Delta_2)^2}) \\ &= -4.25 + \tfrac{4}{(2.90-\Delta_2)^2} - \tfrac{4}{(3.96-\Delta_2)^2} \end{aligned}$$

$\Delta_2^\star = 2.03$ such that $F(\Delta_2^\star) = 0$. Since $\Delta_2^\star$ is not in the interval (0, 1.64):

$$R(\Delta_2) = \ 2.90 - \Delta_2 \quad 0 \le \Delta_2 \le 1.64$$

We have to find the $R(\Delta_2)$ for other possible Δ_2 ranges. For $\Delta_2 = 1.64$, values of b_1, b_2, b_3, b_4 are as follows:

$$b_1 = 1.26 \quad b_2 = 2.32 \quad b_3 = 2.32 \quad b_4 = 2.0$$

We are now in state $(B, 1)$. Since $b_3 - min^3 = 0.73 < 1.06 = b_2 - min^2$, the next state is $(B, 2)$ and the current state is valid as long as Δ_2 is in the interval (1.64, 2.37).

$$\begin{aligned} F(\Delta_2) &= CL^2(min^2, \Delta_2) - CL^3(r^{[s]} - \Delta_2, \Delta_2) \\ &= 4(1.26) + 3\Delta_2 - 3.18 + \tfrac{4}{(1.26)^2} \\ &-(2(3.96 - \Delta_2) + \Delta_2 + 4.75 + \tfrac{4}{(3.96-\Delta_2)^2}) \\ &= 4\Delta_2 - \tfrac{4}{(3.96-\Delta_2)^2} - 8.29 \end{aligned}$$

Over the defined interval (1.64, 2.37), $F(\Delta_2)$ is always less than zero. This means that $CL^2 < CL^3$ and the minimum point of CL^2 is less than the minimum point of CL^3. It follows that

$$R(\Delta_2) = \begin{cases} 2.90 - \Delta_2 & 0 \leq \Delta_2 \leq 1.64 \\ 1.26 & 1.64 \leq \Delta_2 \leq 2.37 \end{cases}$$

For $\Delta_2 = 2.37$, values of b_1, b_2, b_3, b_4 are as follows:

$$b_1 = 0.53 \quad b_2 = 1.59 \quad b_3 = 1.59 \quad b_4 = 1.87$$

We are now in state $(B, 2)$. The next states are $(C, 2)$ and then $(C, 3)$. We make calculations similar to above and construct the $R(\Delta_2)$ fully as follows:

$$R(\Delta_2) = \begin{cases} 2.90 - \Delta_2 & 0 \leq \Delta_2 < 1.64 \\ 1.26 & 1.64 \leq \Delta_2 < 2.56 \\ 1.59 & 2.56 \leq \Delta_2 < 2.66 \\ 4.24 - \Delta_2 & 2.66 \leq \Delta_2 < 5.9 \end{cases}$$

Since the rest of graph is convex, we can again use *UseConvex* subroutine and combine it with the $R(\Delta_2)$ functions generated up to now. This gives $P_2(\Delta_2)$ function.

$$P_2(\Delta_2) = \begin{cases} 1.39 & 0.00 \leq \Delta_2 < 1.51 \\ 2.90 - \Delta_2 & 1.51 \leq \Delta_2 < 1.64 \\ 1.26 & 1.64 \leq \Delta_2 < 2.56 \\ 1.59 & 2.56 \leq \Delta_2 < 2.66 \\ 4.24 - \Delta_2 & 2.66 \leq \Delta_2 < 3.20 \\ 1.05 & 3.20 \leq \Delta_2 < 5.9 \end{cases}$$

Job 1:

The rest of the algorithm is trivial. We find $P_4(p_1)$, $P_3(p_1)$ and $P_2(p_1)$ by using

$P_2(\Delta_2)$. They are as follows:

$$P_4(p_1) = \begin{cases} 2.05 & 0.10 \leq p_1 < 1.62 \\ 3.67 - p_1 & 1.62 \leq p_1 < 2.49 \\ 1.18 & 2.49 \leq p_1 < 6 \end{cases}$$

$$P_3(p_1) = \begin{cases} 0.76 & 0.10 \leq p_1 < 3.24 \\ 0.48 & 3.24 \leq p_1 < 6 \end{cases}$$

$$P_2(p_1) = \begin{cases} 1.39 & 0.10 \leq p_1 < 1.53 \\ 1.57 & 1.53 \leq p_1 < 2.49 \\ 4.06 - p_1 & 2.49 \leq p_1 < 3.24 \\ 4.34 - p_1 & 3.24 \leq p_1 < 3.30 \\ 1.05 & 3.30 \leq p_1 < 6 \end{cases}$$

From them, we generate the Total Tardiness cost function for job 1. It is as follows:

$$TotalTard_1(p_1) = \begin{cases} 0 & 0.10 \leq p_1 < 1.95 \\ 3(p_1 - 0.30) & 1.95 \leq p_1 < 2.00 \\ 6(p_1 - 1.15) & 2.00 \leq p_1 < 3.30 \\ 9(p_1 - 1.87) & 3.30 \leq p_1 < 6 \end{cases}$$

Then, we minimize the total contributed cost function directly. Its minimum is 1.15 and this is the processing time of job 1. We can now obtain the processing times of the job 2,3 and 4 form functions P_2, P_3 and P_4, respectively. They are as follows:

$$p_2 = 1.39 \quad p_3 = 0.76 \text{ and } p_4 = 2.05$$

The objective function value is 21.45 and this is the optimal solution which is also found with mathematical formulation.

In the next chapter we describe and discuss the experimental design of our problem and the computational results.

Chapter 6

Experimental Design

In this chapter, the experimental factors of the problem are specified and the performance of both the proposed DP-based heuristic and local search algorithm are tested and compared with bench-mark algorithms. Both local search and DP-based heuristic are coded in C language and compiled with GNU compiler. The nonlinear model of the original problem for a given sequence is formulated in GAMS 2.25 and solved by MINOS 5.3. All problems are solved on a sparc station (Sun Enterprize 4000) under SunOS 5.7.

The experimental settings of the problem is explained in §6.1. The computational results of the proposed DP-based heuristic for the problem for a given sequence are presented in §6.2. The appropriate values for PSGA parameters and the two-stage benchmark algorithm are given and discussed in §6.3. Finally in §6.4, the results of the algorithms are evaluated.

6.1 Experimental Settings

We developed a four-factorial experimental design to test both proposed DP-based heuristic and PSGA. In Table 6.1 these factors are listed where UN stands for uniform distribution. Two factors, number of jobs and tooling cost, can take values in two levels and the remaining two, tardiness factor and range of due date, can take values in three levels . Thus the experimental design is a $2*2*3*3$ full factorial design.

Factors	Definition	Level 1	Level 2	Level 3
N	Number of jobs	40	80	-
C_t	Tooling cost	UN[0.5,1.5]	UN[3.5,4.5]	-
TF	Tardiness factor	0.2	0.5	0.8
RDD	Range of due date	0.2	0.5	0.8

Table 6.1: Experimental design factors

The experimental factors are explained briefly below:

- N : Number of jobs processed on the machine. Just considering the weighted tardiness cost item, the difficulty of the problem increases exponentially when we increase the number of jobs. Also, as the number of job increases, the decision on the processing times of the jobs becomes more critical since an increase in the processing time of a job has greater effect on the tardiness cost value.

- C_{t_i} : Cost of tool assigned to job i. As tooling cost increases, the tradeoff between tooling cost and the tardiness + machining cost increases. This increases the difficulty of the problem.

- TF and RDD : Tardiness factor and Range of Due Date. These two factors are used to assign due dates of jobs. Due dates of jobs are determined by using the following formula:

$$dd_i = UN[(1 - TF - RDD/2), (1 - TF + RDD/2)] * \Sigma\bar{p}_i$$

  where $\bar{p}_i$ is the average processing time of job i and $\Sigma\bar{p}_i$ is the sum of average processing times of all jobs. As it can be realized from the formula, as RDD increases the range that due dates of jobs are assigned from increases. As TF increases the mean of due dates decreases and this causes due dates to become tighter.

The technological coefficients of the tools are given in Table 6.2 below with the other parameters which are given appropriate values after some trial runs.

α	=	UN[3.60,4.30]
β	=	UN[1.20,1.60]
γ	=	UN[1.05,1.20]
C	=	UN[10000000,60000000]
b	=	UN[0.80,0.96]
c	=	UN[0.70,0.83]
e	=	UN[0.65,0.75]
C_m	=	UN[1.50,2.50]
g	=	-UN[1.45,1.69]
h	=	UN[1.005,1.104]
l	=	UN[0.18,0.40]
C_s	=	UN[20000,25000]*10000
D	=	UN[1.5,3.0]
d	=	UN[0.2,0.3]
S	=	UN[300,500]
C_0	=	0.5

Table 6.2: Technical coefficients and parameters

6.2 Results for the Problem for a Given Sequence

By using our full factorial design, we first tested our DP-based algorithm. For the benchmark, we used the mathematical formulation given in the §3.2.3 (p. 24). We coded this formulation in commercial package GAMS that uses MINOS solver and applied the same factorial design. For each factor combination, we took 25 replications by using 25 different seeds. Therefore, for each algorithm we took 900 runs. The results of these runs are given in Appendix B, which contain the sum and individual values of cost items and the run times. The summary of these runs are listed in the Table 6.3 showing the minimum, maximum and average results.

The averages of 25 replications for each factor combination are given in Table 6.4. "*% Deviations* " in this table shows what percent the value of the optimal solution is less than the value of the solution heuristic for "*Obj*" and what percent the CPU time (in seconds) of our heuristic is less than the one of the mathematical formulation for "*CPU*".

The results show that our proposed heuristic deviates from the optimal solution about only 2% on the average. However, we gain about 82% improvement in the CPU time on the average. The most difficult case of the problem is the factor combination of (1 1 0 2) where the number of jobs, tooling cost and the range of due date are at their highest level and the tardiness factor is at its lowest level. This can be observed from the CPU times for this combination in Table 6.4. Even in the worst case our algorithm deviates only 17% on the average.

	Objective			Runtimes (in CPU seconds)		
Algoritms	Min	Average	Max	Min	Average	Max
DP-Based Algorithm	25.54	134.41	410.16	0.03	0.14	0.82
Math. Model in GAMS	25.12	131.40	410.05	0.40	0.73	1.48

Table 6.3: Summary results for the problem with given sequence

Even if the average CPU times for the mathematical model in TableDpGams may seem too small. However, if we consider that it is a base algorithm called hundreds of times, its effect in the main algorithm is significant. The paired samples statistics, correlations, test results, and estimated marginal grand means by each factor for both objective values and CPU times of the compared algorithms are given in appendix D.1. In the next section, we present the results of proposed PSGAs that use these solution procedures as their base heuristics and compare them with a benchmark algorithm.

				DP-based Algo.		Math. Model		Deviations	
N	C_t	TF	RDD	Obj	CPU	Obj	CPU	Obj	CPU
0	0	0	0	31.68	0.09	30.90	0.54	0.02	0.83
0	0	0	1	32.03	0.11	31.15	0.57	0.03	0.81
0	0	0	2	35.23	0.13	33.74	0.59	0.04	0.79
0	0	1	0	46.91	0.06	46.78	0.52	0.00	0.88
0	0	1	1	47.18	0.06	47.03	0.52	0.00	0.88
0	0	1	2	49.65	0.07	49.41	0.53	0.00	0.87
0	0	2	0	72.05	0.06	72.05	0.46	0.00	0.88
0	0	2	1	72.40	0.06	72.39	0.47	0.00	0.88
0	0	2	2	71.35	0.06	71.34	0.49	0.00	0.89
0	1	0	0	77.34	0.14	73.51	0.62	0.05	0.77
0	1	0	1	77.86	0.15	73.61	0.63	0.05	0.77
0	1	0	2	82.31	0.16	75.53	0.65	0.08	0.75
0	1	1	0	94.39	0.08	93.71	0.56	0.01	0.86
0	1	1	1	95.10	0.08	93.92	0.58	0.01	0.86
0	1	1	2	98.87	0.10	95.91	0.58	0.03	0.83
0	1	2	0	121.10	0.06	121.07	0.53	0.00	0.89
0	1	2	1	121.30	0.06	121.25	0.53	0.00	0.89
0	1	2	2	120.31	0.06	119.97	0.54	0.00	0.88
1	0	0	0	68.59	0.18	67.50	0.82	0.02	0.78
1	0	0	1	72.04	0.26	70.16	0.91	0.03	0.71
1	0	0	2	83.20	0.43	78.78	1.02	0.05	0.58
1	0	1	0	119.83	0.13	119.74	0.68	0.00	0.80
1	0	1	1	121.96	0.14	121.87	0.73	0.00	0.81
1	0	1	2	131.53	0.15	131.43	0.79	0.00	0.82
1	0	2	0	210.87	0.10	210.87	0.61	0.00	0.83
1	0	2	1	212.40	0.11	212.40	0.60	0.00	0.82
1	0	2	2	210.29	0.11	210.28	0.61	0.00	0.81
1	1	0	0	177.69	0.31	166.90	1.15	0.06	0.73
1	1	0	1	189.32	0.35	167.90	1.27	0.11	0.72
1	1	0	2	211.56	0.46	176.29	1.31	0.17	0.65
1	1	1	0	230.80	0.15	229.31	1.01	0.01	0.85
1	1	1	1	234.51	0.15	231.20	1.05	0.01	0.85
1	1	1	2	244.08	0.17	239.93	1.09	0.02	0.84
1	1	2	0	324.23	0.11	324.15	0.81	0.00	0.86
1	1	2	1	325.58	0.11	325.48	0.81	0.00	0.86
1	1	2	2	323.13	0.12	322.96	0.87	0.00	0.87

Table 6.4: Comparison of DP-based heuristic with Math model in GAMS

6.3 Local Search Parameters and Results

In this section, we give, discuss and compare the results of the following three algorithms:

- **The Proposed Simultaneous Algorithms:** This is the proposed algorithm which is explained in § 4.2. We run this PSGA for two different base heuristics:
  - **PSGA[DP-based]:** It uses the DP-based algorithm explained in § 4.3.
  - **PSGA[Math Model]:** It uses the math formulation of the problem for given sequences which is modeled in GAMS.
- **Sequential Algorithm:** As explained in § 4.1, this is the algorithm that reflects the general approach in the industry towards our problem.

The parameters used in these PSGAs are given in Table 6.5. For the sequential PSGA, we assigned parameters to values which give the best results for the total weighted tardiness problem stated by Avci *et al.*[5]. As choosing the values of parameters for our proposed PSGAs, we used the same values for %SEXUAL, π, θ, MUTPROB, CROSSOVER. We chose the values of MAXGEN and POPSIZE by considering the time constraint. Since as we increase the MAXGEN the solution quality increases, we decided to employ a single start for our proposed PSGAs.

The same factorial design in the previous section is used for all PSGAs listed above. However, this time, we take 5 replications for each factor combination since CPU times are much higher for PSGAs when compared to the algorithms in the previous section.Therefore, for each algorithm we took 180 runs and in total 540 runs. The results of these runs are given in Appendix C, in which the first section contains the sum and individual values of cost items and the second section contains the CPU times.

Definitions of Parameters			Sequential	Simultaneous
POPSIZE	:	size of the population in a generation	50	20
MAXGEN	:	number of generations	100	30
% SEXUAL	:	probability of sexual reproduction	0.8	0.8
CROSSOVER	:	crossover type in sexual reproduction	single	single
MUTPROB	:	mutation probability for each gene	0.05	0.05
ϕ	:	selectivity of the algorithm	4	4
θ	:	perturbation magnitude	1	1
NUMSTART	:	number of restarts	5	1

Table 6.5: Definitions and levels of PSGA parameters for the sequential and the proposed simultaneous algorithms

The summary of the results are given in Table 6.6. On the average, the proposed PSGA[Dp-based] provides 62% improvement in solution quality over the sequential algorithm. The time-wise loss that corresponds to this improvement is only 86 CPU seconds. The results of PSGA[Dp-based] and PSGA[Math Model] are as expected. Since the only difference between these two PSGAs are their base heuristics, their time and solution quality comparison results are not much different from the comparison results of DP-based heuristic and Mathematical Formulation for a given sequence. On the average, the loss in solution quality in PSGA[Dp-based] is just 3% compared to PSGA[Math Model]. However, time-wise gain is about 361 seconds or 75%. Time-wise and cost-wise deviations of algorithms with respect to each other are given in Table 6.7, the ratios are calculated as (high-low)/(high).

Algorithms	Objective			Runtimes (CPU seconds)		
	Min	Average	Max	Min	Average	Max
Sequential	26.39	246.40	984.15	8.48	33.71	79.83
PSGA[DP-based]	20.75	93.36	282.33	10.15	119.29	476.29
PSGA[Math Model]	20.75	90.42	282.87	300.55	480.23	859.81

Table 6.6: Summary results of PSGA's for the problem

The averages of 5 replications for each factor combination can be seen in Table 6.8. For all factor combinations, there is an obvious cost-wise improvement. The runtime of PSGA[Dp-based] can decrease up to 10 seconds and increase up to 476 seconds, while runtime of PSGA[Math Model] is between 300 and 859 seconds and seem more stable. This means that the commercial solver MINOS cannot properly catch some characteristics of the problem while our proposed PSGA[Dp-based] manages to exploit them very well. When the factor of N,

				Time-wise Comparison		Cost-wise Comparison	
				Sequential vs PSGA[DP-based]	PSGA[DP-based] vs PSGA[Math Model]	sequential vs PSGA[DP-based]	PSGA[DP-based] vs PSGA[Math Model]
N	C_t	TF	RDD				
0	0	0	0	0.88	0.71	0.49	0.00
0	0	0	1	0.86	0.74	0.63	0.02
0	0	0	2	0.90	0.73	0.68	0.01
0	0	1	0	0.76	0.84	0.56	0.01
0	0	1	1	0.79	0.82	0.63	0.02
0	0	1	2	0.81	0.82	0.69	0.04
0	0	2	0	0.65	0.88	0.56	0.00
0	0	2	1	0.66	0.88	0.57	0.00
0	0	2	2	0.69	0.88	0.61	0.03
0	1	0	0	0.89	0.71	0.58	0.02
0	1	0	1	0.87	0.75	0.60	0.04
0	1	0	2	0.88	0.75	0.67	0.02
0	1	1	0	0.81	0.82	0.59	0.00
0	1	1	1	0.83	0.80	0.59	0.03
0	1	1	2	0.83	0.81	0.65	0.03
0	1	2	0	0.67	0.89	0.57	0.00
0	1	2	1	0.71	0.89	0.56	0.02
0	1	2	2	0.75	0.87	0.60	0.02
1	0	0	0	0.73	0.56	0.54	0.03
1	0	0	1	0.82	0.56	0.62	0.06
1	0	0	2	0.86	0.43	0.64	0.11
1	0	1	0	0.37	0.80	0.59	0.02
1	0	1	1	0.57	0.77	0.65	0.02
1	0	1	2	0.55	0.73	0.68	0.02
1	0	2	0	0.15	0.83	0.57	0.01
1	0	2	1	0.31	0.83	0.55	0.02
1	0	2	2	0.30	0.83	0.59	0.01
1	1	0	0	0.80	0.63	0.63	0.08
1	1	0	1	0.82	0.63	0.65	0.09
1	1	0	2	0.83	0.57	0.68	0.12
1	1	1	0	0.57	0.81	0.65	0.03
1	1	1	1	0.59	0.82	0.64	0.05
1	1	1	2	0.49	0.82	0.67	0.05
1	1	2	0	0.22	0.87	0.62	0.02
1	1	2	1	0.41	0.87	0.59	0.01
1	1	2	2	0.38	0.86	0.64	0.00
Averages				0.67	0.77	0.61	0.03

Table 6.7: Deviations of Algorithms for the original problem with respect to each other

number of jobs, is fixed, deviation in solution times of the sequential algorithm and PSGA[Math Model] are not much when compared to PSGA[Dp-based]. The reason for this is that the sequential algorithm and PSGA[Math Model] do not exploit the problem structure to improve the solution quality, and to improve the run time, respectively. However, our DP-based heuristic behaves according to the structure of the problem.

				Sequential		PSGA[DP-based]		PSGA[GAMS]	
N	C_t	TF	RDD	Obj	CPU	Obj	CPU	Obj	CPU
0	0	0	0	49.15	10.89	24.96	83.31	24.92	296.97
0	0	0	1	64.50	10.70	23.73	71.88	23.35	304.92
0	0	0	2	70.37	9.11	22.48	77.61	22.19	308.27
0	0	1	0	72.86	10.55	32.21	41.96	31.93	275.96
0	0	1	1	81.74	10.62	30.49	46.12	29.80	285.26
0	0	1	2	95.46	9.52	29.87	46.12	28.71	274.66
0	0	2	0	106.97	10.37	47.00	28.56	46.80	251.13
0	0	2	1	106.18	10.14	45.56	28.74	45.58	250.20
0	0	2	2	118.00	9.52	46.37	29.47	45.03	249.41
0	1	0	0	138.71	10.95	58.90	88.38	57.96	322.38
0	1	0	1	148.32	10.83	59.02	71.08	56.47	325.26
0	1	0	2	166.80	9.88	55.63	72.49	54.40	327.03
0	1	1	0	170.78	11.07	70.66	52.35	70.38	306.79
0	1	1	1	172.67	10.96	70.44	57.65	68.14	307.30
0	1	1	2	198.70	9.82	69.88	48.69	67.48	303.50
0	1	2	0	210.21	9.92	89.54	29.20	89.26	282.63
0	1	2	1	202.72	9.38	89.23	31.43	87.68	279.76
0	1	2	2	223.46	9.02	89.03	32.85	87.14	276.51
1	0	0	0	114.11	54.61	52.90	197.85	51.41	475.91
1	0	0	1	134.36	40.38	50.55	229.91	47.64	540.17
1	0	0	2	142.78	45.14	51.24	313.44	45.70	584.54
1	0	1	0	185.31	51.63	75.16	80.60	73.34	410.46
1	0	1	1	201.67	37.00	70.33	86.83	69.07	405.22
1	0	1	2	226.78	47.83	73.00	99.29	71.68	397.48
1	0	2	0	294.62	47.70	128.13	56.08	127.46	326.97
1	0	2	1	299.13	39.32	135.96	56.19	132.90	330.88
1	0	2	2	341.28	40.46	140.09	56.58	138.90	339.80
1	1	0	0	374.20	47.29	137.15	226.80	125.96	653.18
1	1	0	1	377.78	41.37	133.31	232.28	121.27	658.71
1	1	0	2	427.82	47.03	135.17	268.00	119.05	661.15
1	1	1	0	476.72	46.88	166.16	99.82	161.54	568.34
1	1	1	1	469.41	41.83	169.34	94.78	160.84	570.60
1	1	1	2	541.59	45.45	177.17	85.90	169.00	498.64
1	1	2	0	606.89	47.12	233.04	57.85	229.17	457.64
1	1	2	1	586.60	36.38	237.79	58.30	234.60	446.81
1	1	2	2	671.74	39.67	239.37	61.71	238.47	461.38

Table 6.8: Comparison of the sequential and the proposed simultaneous algorithms

One other interesting point is that the results of the problem for the different RDD factors, holding other factors fixed, do not change much. Both PSGA[Dp-based] and PSGA[Math Model] tend to solve problems in a way that the sum of three cost items do not change too much depending on RDD.

The most of the improvements in solutions comes from a decrease in the tardiness cost in the expense of tooling cost. This is expected since a decrease in processing time of a job at the beginning of the sequence decreases the tardiness of all tardy jobs, however it increases just its own tooling cost. These improvements can be clearly seen in Appendix C. Also, The paired samples statistics, correlations, test results and estimated marginal grand means by each factor for both objective cost values and CPU times of the compared algorithms are given in appendix D.2.

6.4 Summary

In this section, we presented the experimental results of our proposed heuristic and the PSGAs. We compared both with a benchmark solution method. Our proposed PSGA that uses the proposed DP-based base heuristic gives the best performance considering time/cost ratio.

In the next section, we will give our concluding remarks about this study and related future research areas on this subject.

Chapter 7

Conclusion

This chapter provides a brief summary of the contributions of this thesis and addresses some possible extensions of this study for future research. We considered the scheduling problem of jobs with controllable machining times on a single CNC machine. The objective is minimizing the summation of total weighted tardiness, machining and tooling costs. We defined and formulated the problem mathematically and proposed a PSGA for the original problem and a DP-based heuristic for the problem for a given sequence. In the next section, we will make a short summary of the contributions we made to this problem and in § 7.2 we will suggest some future research directions.

7.1 Contributions

The integration of scheduling and tool management literature produces more realistic problems. There is no study in the literature considering the machining condition optimization and total weighted tardiness problem simultaneously. From this perspective, our study is the first one that considers both problems simultaneously.

We first formulate the nonlinear mathematical model of the problem. For this purpose, we presented a formulation showing that the tooling cost of a job can be written depending on its processing time and moreover this function is convex.

Since the problem is NP-hard, we developed a problem space genetic algorithm that uses the mathematical model of the problem or proposed dynamic programming based heuristic as its base algorithm. Also, we presented an algorithm that reflects the current approach in the industry for the problem and used it as a benchmark.

The proposed DP-based heuristic is designed to solve the problem for a given sequence. It is backward DP-based since it starts from the last job processed and goes until the first job collecting the information required to determine the processing time of a job depending on the processing times of the jobs before that job. When it reaches to the beginning, since there is no job before the first job, its processing time is calculated easily and then the processing time of other jobs are determined according to its processing time.

The DP-based heuristic is a strong one since it embraces the problem characteristics that arise from the interaction between the machining conditions and weighted tardiness criterion. The defined contributed cost function for each job make generating very good solutions highly possible since it manages to reflect the tradeoff between the tooling cost and total weighted cost.

The computational results confirm the strong relation between the machining conditions and total weighted tardiness problem. They indicate that when the interaction between them is ignored and they are solved independently from each other, the solution quality can be quite poor even if each problem is solved alone optimally. Our proposed heuristics showed outstanding performance compared to the benchmark algorithms.

7.2 Future Research Directions

This thesis showed that the interaction between the decisions on the different levels of flexible manufacturing system hierarchy has a strong impact on its efficiency. From this perspective, the solution methods for the problems generated by combining the problems from the different levels of FMS hierarchy is a very important research direction.

Our approach can be used to find machining conditions and solve some other traditional scheduling criteria simultaneously, for example earliness or the sum of earliness and tardiness. The solution method of the DP-based algorithm can be applied to the problems that have similar characteristics with our problem.

Also, a method to find a lower bound for our problem can be developed. This way, we can gain more information about the solution quality of our proposed algorithms.

Lastly, our proposed algorithms can be improved, for example the sequence generating procedure ATC, which is currently use, can be improved according to the characteristics of the problem or the contributed cost function can be reconstructed to make it reflect the interaction between the cost items better.

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Appendix A

Construction of $R_i(\Delta_i)$

Here, we give a formulation for $R_i(\Delta_i)$, which is explained in section 4.3.2, depending on the possible locations of b_1, b_2, b_3, b_4, min^k, and min^s relative to each other, which are also defined in section 4.3.2.

A.1 State (A,1)

The corresponding combinations are as in the figure A.1. Let Δ_i remains in this combination while it is in the interval $(0, X)$ such that $X \leq min\{r^{[k]}, r^{[s]}\}$. The minimum points of L^k and L^s are b_1 and b_3, respectively. While $CL^k > CL^s$, $R(\Delta_i) = b_3$, else $R(\Delta_i) = b_1$. We find which one of b_3 and b_1 gives the minimum of the total cost for given values of Δ_i by finding the zero point of $F = (CL^k - CL^s)$:

$$\begin{aligned} CL^k(b_1, \Delta_i) &= L^k(b_1, \Delta_i) + Tool(b_1) + Mach(b_1) \\ &= L^k(r^{[k]} - \Delta_i, \Delta_i) + Tool(r^{[k]} - \Delta_i) + Mach(r^{[k]} - \Delta_i) \\ &= m^k(r^{[k]} - \Delta_i) + n^k + m^k\Delta_i + \tfrac{c_a}{(r^{[k]} - \Delta_i)^{c_b}} + C_0(r^{[k]} - \Delta_i) \\ &= m^k r^{[k]} + n^k + \tfrac{c_a}{(r^{[k]} - \Delta_i)^{c_b}} + C_0(r^{[k]} - \Delta_i) \end{aligned}$$

$$\begin{aligned} CL^s(b_2, \Delta_i) &= L^s(b_2, \Delta_i) + Tool(b_2) + Mach(b_2) \\ &= L^s(r^{[s]} - \Delta_i, \Delta_i) + Tool(r^{[s]} - \Delta_i) + Mach(r^{[s]} - \Delta_i) \\ &= m^s(r^{[s]} - \Delta_i) + n_s + m^s\Delta_i + \tfrac{c_a}{(r^{[s]} - \Delta_i)^{c_b}} + C_0(r^{[s]} - \Delta_i) \\ &= m^s r^{[s]} + n_s + \tfrac{c_a}{(r^{[s]} - \Delta_i)^{c_b}} + C_0(r^{[s]} - \Delta_i) \end{aligned}$$

$$\begin{aligned} F(\Delta_i) &= CL^k(r^{[k]} - \Delta_i, \Delta_i) - CL^s(r^{[s]} - \Delta_i, \Delta_i) \\ &= r^{[k]}(m^k + C_0) - r^{[s]}(m^s + C_0) + \tfrac{c_a}{(r^{[k]} - \Delta_i)^{c_b}} - \tfrac{c_a}{(r^{[s]} - \Delta_i)^{c_b}} \end{aligned}$$

F has at most one root. There are three possible cases and solutions related to $\Delta_i^\star$, the root of F, are as follows:

- If there is no $\Delta_i^\star > 0$ such that $F(\Delta_i^\star) = 0$, then CL^k is always greater than CL^s. Thus

$$R(\Delta_i) = \ b_3 = r^{[s]} - \Delta_i \quad 0 \leq \Delta_i \leq X$$

- If there is a $\Delta_i^\star$ but it is not defined on the interval $(0, X)$, then CL^k is less than CL^s. Thus

$$R(\Delta_i) = \ b_1 = r^{[k]} - \Delta_i \quad 0 \leq \Delta_i \leq X$$

- If there is a $\Delta_i^\star$ and it is in the interval $(0, X)$ for $\Delta_i > \Delta_i^\star$, $CL^k > CL^s$ and for $\Delta_i \leq \Delta_i^\star$, $CL^k \leq CL^s$. Thus

$$R(\Delta_i) = \begin{cases} b_1 = r^{[k]} - \Delta_i & 0 \leq \Delta_i \leq \Delta_i^\star \\ b_3 = r^{[s]} - \Delta_i & \Delta_i^\star \leq \Delta_i \leq X \end{cases}$$

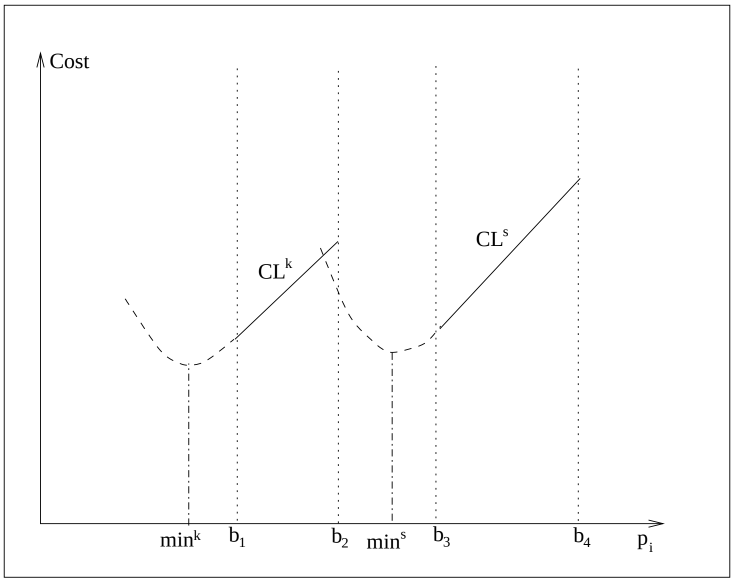

Figure A.1: The illustration of combination $(A, 1)$

The generation of $R(\Delta_i)$ for other combinations is similar to this one.

A.2 State (A,2)

Let Δ_i remains in this combination while it is in the interval (X, Y). The minimum point of L^k and L^s are b_1 and min^s, respectively. While $CL^k > CL^s$, $R(\Delta_i) = b_1$, else $R(\Delta_i) = min^s$. We find which one of b_1 and min_s gives the minimum of the total cost for given values of Δ_i by finding the zero point of $F = (CL^k - CL^s)$:

$$\begin{aligned} CL^k(b_1, \Delta_i) &= L^k(b_1, \Delta_i) + Tool(b_1) + Mach(b_1) \\ &= L^k(r^{[k]} - \Delta_i, \Delta_i) + Tool(r^{[k]} - \Delta_i) + Mach(r^{[k]} - \Delta_i) \\ &= m^k(r^{[k]} - \Delta_i) + n^k + m^k\Delta_i + \frac{c_a}{(r^{[k]} - \Delta_i)^{c_b}} + C_0(r^{[k]} - \Delta_i) \\ &= m^k r^{[k]} + n^k + \frac{c_a}{(r^{[k]} - \Delta_i)^{c_b}} + C_0(r^{[k]} - \Delta_i) \end{aligned}$$

$$\begin{aligned} CL^s(min^s,\Delta_i) &= L^s(min^s,\Delta_i) + Tool(min^s) + Mach(min^s) \\ &= m^s min^s + n^s + m^s\Delta_i + \tfrac{c_a}{(min^s)^{c_b}} + C_0 min^s \\ &= m^s min^s + n^s + \tfrac{c_a}{(min^s)^{c_b}} + C_0 min^s \end{aligned}$$

$$\begin{aligned} F(\Delta_i) &= CL^k(r^{[k]}-\Delta_i,\Delta_i) - CL^s(min^s,\Delta_i) \\ &= -C_0\Delta_i + \tfrac{c_a}{(r^{[k]}-\Delta_i)^{c_b}} - \tfrac{c_a}{(min^s)^{c_b}} + C_0(r^{[k]} - min^s) + m^k r^{[k]} - m^s min^s + n^k - n^s \end{aligned}$$

F may have at most two roots. There are three possible cases and solutions related to $\Delta_1^\star$ and $\Delta_2^\star$, the roots of F, are as follows:

- $F(\Delta_i)$ has no root, then CL^k is always greater than CL^s. Thus

$$R(\Delta_i) = \; min^s \quad X \le \Delta_i \le Y$$

- $F(\Delta_i)$ has roots $\Delta_1^\star$ and $\Delta_2^\star$ such that $\Delta_1^\star = \Delta_2^\star$, $R(\Delta_i)$ is the same with the case of no root.

- $F(\Delta_i)$ has roots $\Delta_1^\star$ and $\Delta_2^\star$ such that $\Delta_1^\star \neq \Delta_2^\star$. In this case for $\Delta_1^\star \le \Delta_i \le \Delta_2^\star$, $CL^k \le CL^s$ and for otherwise $CL^k > CL^s$. Thus we can write $R(\Delta_i)$ independently from the range (X,Y) as follows:

$$R(\Delta_i) = \begin{cases} min^s & \Delta_1^\star \le \Delta_i \le \Delta_2^\star \\ b_1 = r^{[k]} - \Delta_i & o.w. \end{cases}$$

  According to order of $\Delta_1^\star$, $\Delta_2^\star$,X and Y we can write $R(\Delta_i)$ easily. For example if $X < \Delta_1^\star < \Delta_2^\star < Y$ then:

$$R(\Delta_i) = \begin{cases} r^{[k]} - \Delta_i & X \le \Delta_i < \Delta_1^\star \\ min^s & \Delta_1^\star \le \Delta_i \le \Delta_2^\star \\ r^{[k]} - \Delta_i & \Delta_2^\star < \Delta_i < Y \end{cases}$$

A.3 State (B,2)

Let Δ_i remains in this combination while it is in the interval (X,Y). The minimum point of L^k and L^s are min^k and min^s, respectively. While $CL^k > CL^s$,

$R(\Delta_i) = min^k$, else $R(\Delta_i) = min^s$. We find which one of min_k and min_s gives the minimum of the total cost for given values of Δ_i by finding the zero point of $F = (CL^k - CL^s)$:

$$\begin{aligned} CL^k(min^k, \Delta_i) &= L^k(min^k, \Delta_i) + Tool(min^k) + Mach(min^k) \\ &= m^k min^k + n^k + m^k \Delta_i + \tfrac{c_a}{(min^k)^{c_b}} + C_0 min^k \end{aligned}$$

$$\begin{aligned} CL^s(min^s, \Delta_i) &= L^s(min^s, \Delta_i) + Tool(min^s) + Mach(min^s) \\ &= m^s min^s + n^s + m^s \Delta_i + \tfrac{c_a}{(min^s)^{c_b}} + C_0 min^s \end{aligned}$$

$$\begin{aligned} F(\Delta_i) &= CL^k(r^{[k]} - \Delta_i, \Delta_i) - CL^s(min^s, \Delta_i) \\ &= \Delta_i(min^k - min^s) + \tfrac{c_a}{(min^k)^{c_b}} - \tfrac{c_a}{(min^s)^{c_b}} + C_0(min^k - min^s) \\ &+ m^k min^k - m^s min^s + n^k - n^s \end{aligned}$$

The $\Delta_i^\star$ such that $F(\Delta_i^\star) = 0$ can be easily derived from the equation above:

$$\Delta_i^\star = \frac{\mathbf{C}}{min^k - min^s}$$

where

$$\mathbf{C} = \frac{c_a}{(min^k)^{c_b}} - \frac{c_a}{(min^s)^{c_b}} + C_0(min^k - min^s) + m^k min^k - m^s min^s + n^k - n^s$$

for $\Delta_i > \Delta_i^\star$, $CL^k > CL^s$ and for $\Delta_i \leq \Delta_i^\star$, $CL^k \leq CL^s$. Thus we can write $R(\Delta_i)$ independently from the range (X, Y) as follows:

$$R(\Delta_i) = \begin{cases} min^k & 0 \leq \Delta_i \leq \Delta_i^\star \\ min^s & \Delta_i^\star \leq \Delta_i \leq \infty \end{cases}$$

According to order of $\Delta_1^\star$, X and Y we can write $R(\Delta_i)$ easily as in state $(A, 2)$.

A.4 States (A,3), (C,1), (C,3)

The calculations for these states are very similar to the state $(A, 1)$. The rules for constructing $R(\Delta_i)$ for them and $(A, 1)$ are the same. $F(\Delta_i)$ function for each state as follows:

State $(A,3)$

$$\begin{aligned} F(\Delta_i) &= CL^k(r^{[k]}-\Delta_i,\Delta_i) - CL^s(r^{[s+1]}-\Delta_i,\Delta_i) \\ &= r^{[k]}(m^k+C_0) - r^{[s+1]}(m^s+C_0) + \frac{c_a}{(r^{[k]}-\Delta_i)^{c_b}} - \frac{c_a}{(r^{[s+1]}-\Delta_i)^{c_b}} \end{aligned}$$

State $(C,1)$

$$\begin{aligned} F(\Delta_i) &= CL^k(r^{[k+1]}-\Delta_i,\Delta_i) - CL^s(r^{[s]}-\Delta_i,\Delta_i) \\ &= r^{[k+1]}(m^k+C_0) - r^{[s]}(m^s+C_0) + \frac{c_a}{(r^{[k+1]}-\Delta_i)^{c_b}} - \frac{c_a}{(r^{[s]}-\Delta_i)^{c_b}} \end{aligned}$$

State $(C,3)$

$$\begin{aligned} F(\Delta_i) &= CL^k(r^{[k+1]}-\Delta_i,\Delta_i) - CL^s(r^{[s+1]}-\Delta_i,\Delta_i) \\ &= r^{[k+1]}(m^k+C_0) - r^{[s+1]}(m^s+C_0) + \frac{c_a}{(r^{[k+1]}-\Delta_i)^{c_b}} - \frac{c_a}{(r^{[s+1]}-\Delta_i)^{c_b}} \end{aligned}$$

A.5 States (B,1), (C,2), (B,3)

The calculations for these states are very similar to the state $(A,2)$. The corresponding $F(\Delta_i)$ for each state as follows :

State $(B,1)$

$$\begin{aligned} F(\Delta_i) &= CL^k(min^k,\Delta_i) - CL^s(r^{[s]}-\Delta_i,\Delta_i) \\ &= C_0\Delta_i - \frac{c_a}{(r^{[s]}-\Delta_i)^{c_b}} + \frac{c_a}{(min^k)^{c_b}} - C_0(r^{[s]}-min^k) \\ &-m^s r^{[s]} + m^k min^k - n^s + n^k \end{aligned}$$

State $(C,2)$

$$\begin{aligned} F(\Delta_i) &= CL^k(r^{[k+1]}-\Delta_i,\Delta_i) - CL^s(min^s,\Delta_i) \\ &= -C_0\Delta_i + \frac{c_a}{(r^{[k+1]}-\Delta_i)^{c_b}} - \frac{c_a}{(min^s)^{c_b}} + C_0(r^{[k+1]}-min^s) \\ &+m^k r^{[k+1]} - m^s min^s + n^k - n^s \end{aligned}$$

<u>State $(B,1)$</u>

$$
\begin{aligned}
F(\Delta_i) \quad & = CL^k(min^k, \Delta_i) - CL^s(r^{[s+1]} - \Delta_i, \Delta_i) \\
& = C_0\Delta_i - \frac{c_a}{(r^{[s+1]}-\Delta_i)^{c_b}} + \frac{c_a}{(min^k)^{c_b}} - C_0(r^{[s+1]} - min^k) \\
& -m^s r^{[s+1]} + m^k min^k - n^s + n^k
\end{aligned}
$$

All of these functions are in the same form and have at most one root. The rules to find $R(\Delta_i)$ are the same with $(A,2)$.

Appendix B

Results for The Problem with a Given Sequence

					DP-based Heuristic					Math. Model				
N	C_t	TF	RDD	$r\#$	Tard	Tool	Mach	Total	CPU	Tard	Tool	Mach	Total	CPU
0	0	0	0	1	4.33	15.75	5.45	25.54	0.13	4.294	15.691	5.448	25.432	0.63
0	0	0	0	2	10.88	18.72	6.85	36.45	0.08	7.395	19.692	6.395	33.481	0.61
0	0	0	0	3	10.51	23.67	7.53	41.71	0.09	11.504	22.163	7.661	41.327	0.55
0	0	0	0	4	3.84	19.07	5.18	28.08	0.11	4.324	18.02	5.241	27.585	0.5
0	0	0	0	5	5.93	20.42	5.9	32.26	0.1	5.604	19.271	5.853	30.728	0.52
0	0	0	0	6	5.44	20.64	7.19	33.27	0.08	5.561	20.458	7.201	33.22	0.5
0	0	0	0	7	6.36	18.47	7.15	31.98	0.09	6.142	18.02	7.111	31.273	0.54
0	0	0	0	8	5.1	18.77	5.57	29.44	0.13	4.704	18.06	5.506	28.27	0.6
0	0	0	0	9	5.62	20.75	5.22	31.6	0.08	6.405	19.713	5.309	31.427	0.54
0	0	0	0	10	6.52	18.32	7.09	31.93	0.13	4.936	18.262	6.818	30.017	0.55
0	0	0	0	11	4.36	15.63	5.73	25.72	0.14	4.042	15.404	5.673	25.119	0.5
0	0	0	0	12	9.74	23.8	9.28	42.83	0.07	10.942	21.914	9.45	42.306	0.5
0	0	0	0	13	6.44	18.97	5.5	30.91	0.08	6.945	18.192	5.566	30.702	0.46
0	0	0	0	14	5.88	17.74	6.81	30.43	0.13	6.074	16.745	6.848	29.667	0.59
0	0	0	0	15	5.4	18.17	6.7	30.27	0.15	5.009	17.579	6.631	29.218	0.64
0	0	0	0	16	6.55	20.81	6.95	34.31	0.08	5.579	20.237	6.848	32.664	0.48
0	0	0	0	17	4.15	18.86	5.57	28.59	0.09	4.562	17.617	5.626	27.805	0.61
0	0	0	0	18	5.89	19.18	5.65	30.72	0.1	6.149	18.844	5.679	30.672	0.57
0	0	0	0	19	6.29	17.01	4.28	27.58	0.08	6.471	16.757	4.304	27.531	0.5
0	0	0	0	20	7.45	17.51	5.15	30.11	0.06	6.893	16.769	5.071	28.733	0.48
0	0	0	0	21	6.53	19.16	4.22	29.91	0.05	7.639	17.693	4.329	29.661	0.5
0	0	0	0	22	6.25	19.86	5.3	31.41	0.09	6.249	19.01	5.299	30.558	0.45
0	0	0	0	23	5.43	24.27	5.81	35.51	0.07	6.86	22.083	5.962	34.904	0.59
0	0	0	0	24	6.8	19.77	5.43	32	0.09	7.002	19.431	5.458	31.891	0.55
0	0	0	0	25	4.36	19.28	5.69	29.32	0.06	4.754	17.787	5.749	28.29	0.56
0	0	0	1	1	6.61	16.86	5.32	28.79	0.08	7.036	16.109	5.388	28.533	0.53
0	0	0	1	2	8.49	19.44	6.46	34.39	0.09	8.534	19.121	6.466	34.122	0.61
0	0	0	1	3	11.74	23.77	7.63	43.15	0.14	11.63	22.521	7.621	41.772	0.54
0	0	0	1	4	5.28	17.56	5.47	28.3	0.15	4.75	16.972	5.41	27.132	0.57
0	0	0	1	5	5.3	19.91	5.8	31.01	0.11	5.565	19.387	5.838	30.79	0.59
0	0	0	1	6	8.7	18.8	7.6	35.11	0.13	7.239	18.963	7.404	33.606	0.6
0	0	0	1	7	7.31	19.83	6.88	34.03	0.07	7.839	18.98	6.96	33.779	0.56
0	0	0	1	8	4.69	18.42	5.53	28.64	0.14	4.242	18.374	5.472	28.087	0.55
0	0	0	1	9	6.09	20.06	5.28	31.43	0.08	6.523	19.538	5.327	31.387	0.58
0	0	0	1	10	5.59	18.32	6.91	30.82	0.13	4.914	18.668	6.768	30.351	0.48
0	0	0	1	11	6.93	17.22	5.72	29.88	0.1	6.111	15.87	5.621	27.602	0.5
0	0	0	1	12	10.16	22.9	9.48	42.55	0.09	10.176	21.63	9.487	41.292	0.59
0	0	0	1	13	6.39	18.58	5.54	30.51	0.08	6.791	17.953	5.599	30.344	0.56
0	0	0	1	14	5.29	18.65	6.63	30.56	0.15	5.839	17.608	6.718	30.165	0.57
0	0	0	1	15	6.45	18.21	6.76	31.43	0.13	5.742	17.59	6.635	29.967	0.58
0	0	0	1	16	5.92	20.83	6.94	33.69	0.08	5.357	19.885	6.895	32.137	0.52
0	0	0	1	17	5.33	18.01	5.62	28.97	0.1	5.101	17.924	5.595	28.62	0.53
0	0	0	1	18	7.89	17.75	6.24	31.88	0.13	7.124	16.23	6.075	29.43	0.66
0	0	0	1	19	4.88	16.5	4.34	25.71	0.12	5.278	16.002	4.393	25.672	0.53
0	0	0	1	20	7.1	18.07	4.96	30.12	0.08	8.259	16.25	5.138	29.647	0.54
0	0	0	1	21	6.78	17.09	4.39	28.27	0.1	6.996	16.835	4.424	28.254	0.58
0	0	0	1	22	7.22	18.63	5.56	31.4	0.14	5.795	18.076	5.414	29.285	0.54
0	0	0	1	23	9.16	23.46	5.87	38.49	0.08	10.119	21.989	5.97	38.079	0.62
0	0	0	1	24	6.8	20.12	5.6	32.52	0.09	5.658	19.4	5.466	30.524	0.63
0	0	0	1	25	5.85	17.35	5.96	29.16	0.1	5.003	17.367	5.82	28.19	0.58
0	0	0	2	1	6.94	14.59	5.94	27.47	0.15	5.995	14.182	5.746	25.923	0.62
0	0	0	2	2	8.26	18.63	6.61	33.5	0.12	8.382	17.942	6.643	32.967	0.59
0	0	0	2	3	12.34	21.5	8.22	42.05	0.18	11.387	19.435	8.077	38.9	0.65
0	0	0	2	4	11.09	18.38	5.21	34.68	0.13	11.094	18.377	5.21	34.682	0.6
0	0	0	2	5	9.59	19.68	5.84	35.11	0.09	9.221	19.866	5.793	34.88	0.52
0	0	0	2	6	8.44	17.71	7.83	33.98	0.17	7.381	17.73	7.625	32.735	0.59
0	0	0	2	7	11.21	18.69	7.02	36.92	0.1	11.196	18.677	7.022	36.896	0.51
0	0	0	2	8	9.73	19.81	5.53	35.07	0.09	8.746	18.511	5.531	32.789	0.6
0	0	0	2	9	10.62	18.91	5.46	34.98	0.09	10.028	18.814	5.417	34.26	0.58
0	0	0	2	10	10.75	19.52	6.79	37.06	0.1	9.743	19.35	6.671	35.764	0.53
0	0	0	2	11	9.15	15.92	5.76	30.84	0.1	7.828	15.847	5.632	29.307	0.63
0	0	0	2	12	16.62	22.27	9.45	48.34	0.1	16.977	21.762	9.502	48.241	0.6
0	0	0	2	13	10.19	17.61	5.73	33.53	0.11	9.929	17.142	5.72	32.792	0.6
0	0	0	2	14	6.61	23.07	6.38	36.06	0.17	8.383	17.394	6.752	32.529	0.65
0	0	0	2	15	9.48	18.42	6.73	34.64	0.16	8.705	17.826	6.59	33.121	0.6
0	0	0	2	16	14.86	18.57	7.29	40.73	0.16	12.756	19.228	7.034	39.018	0.6
0	0	0	2	17	8.9	17.04	5.89	31.82	0.11	7.268	17.052	5.745	30.066	0.57
0	0	0	2	18	7.53	20.78	5.75	34.06	0.14	8.195	17.959	5.801	31.955	0.6
0	0	0	2	19	8.94	16.1	4.58	29.62	0.14	6.924	16.179	4.399	27.502	0.55
0	0	0	2	20	8.9	18.61	4.92	32.43	0.08	10.117	16.772	5.079	31.967	0.64
0	0	0	2	21	8.23	19.16	4.22	31.61	0.07	9.226	17.74	4.33	31.295	0.55
0	0	0	2	22	13.33	19.83	5.31	38.47	0.11	12.965	19.253	5.275	37.493	0.64
0	0	0	2	23	10.53	19.87	6.31	36.71	0.18	9.7	19.994	6.201	35.895	0.6
0	0	0	2	24	10.55	22.21	5.71	38.47	0.15	9.292	18.498	5.613	33.403	0.56
0	0	0	2	25	10.25	15.97	6.44	32.66	0.16	6.486	16.535	6.059	29.08	0.56
0	0	1	0	1	12.59	17.98	5.2	35.77	0.04	12.94	17.475	5.232	35.646	0.45
0	0	1	0	2	20.41	21.9	6.2	48.52	0.07	21.18	20.866	6.276	48.322	0.61
0	0	1	0	3	26.75	25.33	7.39	59.47	0.05	27.701	24.076	7.471	59.248	0.59
0	0	1	0	4	18.05	20.66	5.01	43.73	0.07	18.093	20.614	5.017	43.724	0.47
0	0	1	0	5	17.07	22.82	5.51	45.4	0.07	17.584	22.18	5.549	45.313	0.5
0	0	1	0	6	22.16	22.09	7.06	51.31	0.06	22.653	21.456	7.104	51.213	0.47
0	0	1	0	7	25.15	20.06	6.86	52.07	0.07	25.286	19.893	6.865	52.044	0.49
0	0	1	0	8	15.49	21.01	5.22	41.72	0.06	16.063	20.254	5.272	41.589	0.54
0	0	1	0	9	22.23	20.89	5.21	48.33	0.07	22.376	20.712	5.22	48.308	0.47
0	0	1	0	10	20.23	21.19	6.48	47.91	0.04	20.264	21.154	6.487	47.904	0.45
0	0	1	0	11	14.95	18.7	5.34	38.99	0.07	15.141	18.468	5.351	38.96	0.48
0	0	1	0	12	31.5	23.01	9.34	63.85	0.08	31.555	22.952	9.342	63.849	0.59
0	0	1	0	13	21.06	19.2	5.48	45.74	0.06	21.271	18.921	5.495	45.687	0.54
0	0	1	0	14	15.43	20.22	6.46	42.12	0.04	16.121	19.276	6.529	41.925	0.56
0	0	1	0	15	13.46	20.83	6.33	40.62	0.1	13.509	19.625	6.335	39.468	0.54
0	0	1	0	16	30.07	21.74	6.71	58.51	0.06	30.085	21.721	6.707	58.513	0.57
0	0	1	0	17	18.12	19.36	5.48	42.96	0.07	18.46	18.908	5.502	42.87	0.49
0	0	1	0	18	20.92	21.29	5.49	47.7	0.07	21.78	20.106	5.562	47.449	0.55
0	0	1	0	19	15.79	19.16	4.11	39.05	0.05	16.361	18.412	4.151	38.924	0.47
0	0	1	0	20	18.01	18.61	4.92	41.54	0.05	18.569	17.861	4.963	41.392	0.53
0	0	1	0	21	21.71	19.05	4.23	44.98	0.06	21.747	18.991	4.228	44.965	0.53
0	0	1	0	22	23.66	21.05	5.13	49.85	0.05	23.671	21.047	5.129	49.847	0.5
0	0	1	0	23	24.94	24.27	5.81	55.02	0.08	25.808	23.139	5.875	54.822	0.56
0	0	1	0	24	20.13	21.36	5.29	46.78	0.05	20.56	20.818	5.324	46.702	0.51
0	0	1	0	25	14.68	20.73	5.47	40.88	0.07	15.361	19.841	5.531	40.733	0.52

					DP-based Heuristic					Math. Model				
N	C_t	TF	RDD	$r\#$	Tard	Tool	Mach	Total	CPU	Tard	Tool	Mach	Total	CPU
0	0	1	1	1	15.92	17.98	5.20	39.09	0.04	16.34	17.39	5.24	38.97	0.54
0	0	1	1	2	19.65	21.90	6.20	47.75	0.06	20.55	20.67	6.29	47.51	0.56
0	0	1	1	3	30.73	25.33	7.39	63.45	0.06	31.62	24.18	7.46	63.27	0.52
0	0	1	1	4	15.61	20.66	5.01	41.29	0.05	16.01	20.18	5.05	41.23	0.51
0	0	1	1	5	16.95	22.82	5.51	45.28	0.06	17.46	22.18	5.55	45.19	0.50
0	0	1	1	6	21.33	22.09	7.06	50.47	0.06	21.80	21.48	7.10	50.38	0.47
0	0	1	1	7	24.21	20.06	6.86	51.12	0.06	24.37	19.86	6.87	51.10	0.51
0	0	1	1	8	15.26	21.01	5.22	41.49	0.07	15.71	20.43	5.26	41.40	0.56
0	0	1	1	9	22.71	20.89	5.21	48.81	0.07	22.85	20.72	5.22	48.79	0.59
0	0	1	1	10	22.65	21.18	6.49	50.31	0.05	22.76	21.05	6.50	50.31	0.46
0	0	1	1	11	20.12	18.70	5.34	44.16	0.07	20.31	18.47	5.35	44.14	0.48
0	0	1	1	12	30.81	23.19	9.32	63.32	0.08	30.97	23.01	9.34	63.31	0.53
0	0	1	1	13	21.36	19.20	5.48	46.03	0.07	21.58	18.90	5.50	45.97	0.44
0	0	1	1	14	18.70	20.22	6.46	45.38	0.07	19.48	19.16	6.54	45.17	0.59
0	0	1	1	15	13.83	20.74	6.37	40.95	0.10	13.71	19.34	6.37	39.42	0.53
0	0	1	1	16	29.14	21.74	6.71	57.58	0.06	29.15	21.72	6.71	57.58	0.53
0	0	1	1	17	20.71	19.36	5.48	45.54	0.06	21.02	18.96	5.50	45.47	0.51
0	0	1	1	18	15.52	21.29	5.49	42.30	0.07	16.44	20.02	5.57	42.03	0.52
0	0	1	1	19	13.79	19.16	4.11	37.06	0.08	14.47	18.25	4.16	36.89	0.48
0	0	1	1	20	19.15	18.61	4.92	42.67	0.06	19.65	17.96	4.96	42.56	0.53
0	0	1	1	21	19.29	19.16	4.22	42.67	0.06	19.60	18.79	4.24	42.63	0.54
0	0	1	1	22	21.84	21.05	5.13	48.02	0.05	21.87	21.03	5.13	48.02	0.46
0	0	1	1	23	26.81	24.27	5.81	56.89	0.06	27.69	23.10	5.88	56.67	0.56
0	0	1	1	24	18.66	21.34	5.29	45.29	0.07	19.20	20.63	5.34	45.17	0.52
0	0	1	1	25	16.49	20.73	5.47	42.69	0.06	16.97	20.13	5.51	42.60	0.47
0	0	1	2	1	14.02	16.72	5.46	36.20	0.06	12.39	16.82	5.32	34.53	0.52
0	0	1	2	2	19.11	21.90	6.20	47.22	0.08	20.11	20.55	6.31	46.96	0.59
0	0	1	2	3	23.81	23.93	7.63	55.37	0.06	22.78	23.46	7.53	53.77	0.63
0	0	1	2	4	25.09	20.66	5.01	50.77	0.06	25.46	20.23	5.04	50.73	0.50
0	0	1	2	5	21.36	22.84	5.51	49.70	0.06	22.15	21.85	5.58	49.58	0.47
0	0	1	2	6	17.10	21.41	7.12	45.63	0.07	17.25	21.11	7.14	45.49	0.49
0	0	1	2	7	27.66	19.83	6.88	54.38	0.08	27.91	19.48	6.91	54.29	0.58
0	0	1	2	8	21.30	21.01	5.22	47.53	0.07	21.76	20.45	5.26	47.46	0.57
0	0	1	2	9	24.95	20.89	5.21	51.05	0.07	25.27	20.51	5.24	51.02	0.50
0	0	1	2	10	26.35	21.15	6.49	53.98	0.07	26.79	20.60	6.53	53.93	0.50
0	0	1	2	11	22.50	18.39	5.36	46.25	0.06	22.59	18.29	5.37	46.24	0.50
0	0	1	2	12	41.99	23.86	9.28	75.13	0.05	42.42	23.34	9.31	75.06	0.55
0	0	1	2	13	24.47	19.20	5.48	49.14	0.10	24.72	18.87	5.50	49.09	0.48
0	0	1	2	14	22.17	20.22	6.46	48.85	0.05	22.73	19.49	6.51	48.73	0.59
0	0	1	2	15	18.91	20.46	6.27	45.65	0.09	19.07	20.29	6.29	45.64	0.58
0	0	1	2	16	31.17	21.74	6.71	59.62	0.05	31.29	21.60	6.72	59.61	0.53
0	0	1	2	17	21.86	19.36	5.48	46.70	0.06	22.22	18.92	5.50	46.64	0.50
0	0	1	2	18	20.50	20.70	5.61	46.81	0.07	20.47	19.53	5.61	45.62	0.52
0	0	1	2	19	18.13	19.16	4.11	41.39	0.06	18.66	18.52	4.15	41.32	0.49
0	0	1	2	20	22.09	18.61	4.92	45.61	0.06	22.69	17.79	4.97	45.46	0.54
0	0	1	2	21	22.39	19.16	4.22	45.77	0.06	22.65	18.87	4.24	45.75	0.48
0	0	1	2	22	30.37	21.05	5.13	56.56	0.07	30.65	20.73	5.15	56.53	0.51
0	0	1	2	23	22.28	24.27	5.81	52.36	0.07	23.26	23.03	5.89	52.17	0.50
0	0	1	2	24	21.25	20.79	5.33	47.37	0.09	21.53	20.43	5.36	47.32	0.58
0	0	1	2	25	16.10	20.73	5.47	42.30	0.06	16.59	20.14	5.51	42.23	0.47
0	0	2	0	1	31.49	18.11	5.19	54.78	0.04	31.54	18.05	5.19	54.78	0.45
0	0	2	0	2	42.68	21.90	6.20	70.79	0.05	42.80	21.77	6.21	70.78	0.52
0	0	2	0	3	54.11	25.33	7.39	86.83	0.08	54.41	24.99	7.41	86.81	0.48
0	0	2	0	4	43.66	20.66	5.01	69.34	0.05	43.66	20.66	5.02	69.34	0.43
0	0	2	0	5	40.70	23.02	5.50	69.21	0.04	40.74	22.97	5.50	69.21	0.46
0	0	2	0	6	49.18	22.09	7.06	78.33	0.06	49.27	21.99	7.07	78.33	0.48
0	0	2	0	7	54.75	20.06	6.86	81.67	0.05	54.82	19.99	6.86	81.67	0.42
0	0	2	0	8	34.29	21.01	5.22	60.52	0.06	34.46	20.81	5.23	60.50	0.52
0	0	2	0	9	44.86	20.89	5.21	70.95	0.05	44.90	20.84	5.21	70.95	0.46
0	0	2	0	10	50.92	21.32	6.48	78.71	0.05	50.92	21.32	6.48	78.71	0.44
0	0	2	0	11	40.86	18.70	5.34	64.90	0.07	40.87	18.69	5.34	64.90	0.42
0	0	2	0	12	67.30	23.86	9.28	100.43	0.05	67.33	23.82	9.28	100.43	0.46
0	0	2	0	13	47.27	19.20	5.48	71.94	0.06	47.31	19.14	5.48	71.94	0.48
0	0	2	0	14	43.71	20.22	6.46	70.40	0.04	43.75	20.18	6.47	70.39	0.45
0	0	2	0	15	32.12	21.80	6.16	60.08	0.06	32.38	21.49	6.18	60.05	0.53
0	0	2	0	16	66.37	21.74	6.71	94.82	0.06	66.37	21.74	6.71	94.82	0.43
0	0	2	0	17	45.15	19.36	5.48	69.98	0.07	45.19	19.32	5.48	69.98	0.50
0	0	2	0	18	44.80	21.29	5.49	71.58	0.04	44.93	21.14	5.50	71.57	0.42
0	0	2	0	19	36.05	19.16	4.11	59.31	0.07	36.14	19.05	4.11	59.31	0.52
0	0	2	0	20	38.92	18.61	4.92	62.44	0.04	39.03	18.48	4.92	62.43	0.48
0	0	2	0	21	41.35	19.13	4.22	64.70	0.07	41.34	19.13	4.22	64.70	0.41
0	0	2	0	22	54.38	21.05	5.13	80.56	0.08	54.38	21.05	5.13	80.56	0.51
0	0	2	0	23	49.14	24.27	5.81	79.22	0.05	49.28	24.12	5.81	79.22	0.45
0	0	2	0	24	40.92	21.43	5.29	67.63	0.06	41.02	21.31	5.29	67.63	0.44
0	0	2	0	25	36.01	20.73	5.47	62.21	0.04	36.05	20.69	5.47	62.21	0.46
0	0	2	1	1	35.58	18.11	5.19	58.88	0.07	35.71	17.96	5.20	58.86	0.49
0	0	2	1	2	43.91	21.90	6.20	72.01	0.05	44.06	21.73	6.21	72.00	0.57
0	0	2	1	3	56.94	25.33	7.39	89.66	0.06	57.42	24.77	7.42	89.62	0.50
0	0	2	1	4	41.29	20.66	5.01	66.97	0.05	41.29	20.66	5.02	66.97	0.50
0	0	2	1	5	40.23	23.02	5.50	68.75	0.06	40.28	22.97	5.50	68.74	0.46
0	0	2	1	6	45.12	22.09	7.06	74.27	0.06	45.23	21.97	7.07	74.27	0.51
0	0	2	1	7	54.00	20.06	6.86	80.92	0.05	54.10	19.96	6.86	80.91	0.48
0	0	2	1	8	36.13	21.01	5.22	62.36	0.07	36.20	20.93	5.23	62.36	0.44
0	0	2	1	9	44.36	20.89	5.21	70.45	0.06	44.43	20.81	5.21	70.45	0.44
0	0	2	1	10	52.89	21.28	6.48	80.64	0.06	52.89	21.28	6.48	80.64	0.41
0	0	2	1	11	46.34	18.70	5.34	70.38	0.06	46.38	18.66	5.34	70.38	0.46
0	0	2	1	12	68.16	23.86	9.28	101.29	0.05	68.25	23.76	9.28	101.28	0.47
0	0	2	1	13	45.93	19.20	5.48	70.61	0.05	46.01	19.12	5.48	70.61	0.49
0	0	2	1	14	45.54	20.22	6.46	72.22	0.06	45.61	20.14	6.47	72.22	0.40
0	0	2	1	15	31.90	21.80	6.16	59.87	0.06	32.21	21.44	6.18	59.84	0.50
0	0	2	1	16	64.90	21.74	6.71	93.34	0.06	64.90	21.74	6.71	93.34	0.49
0	0	2	1	17	45.24	19.36	5.48	70.07	0.05	45.28	19.31	5.48	70.07	0.44
0	0	2	1	18	42.19	21.29	5.49	68.97	0.07	42.49	20.95	5.51	68.95	0.52
0	0	2	1	19	33.47	19.16	4.11	56.73	0.05	33.53	19.09	4.11	56.73	0.40
0	0	2	1	20	38.52	18.61	4.92	62.05	0.04	38.66	18.45	4.92	62.03	0.48
0	0	2	1	21	38.13	19.16	4.22	61.51	0.06	38.17	19.12	4.22	61.51	0.50
0	0	2	1	22	53.97	21.05	5.13	80.15	0.06	53.97	21.05	5.13	80.15	0.43
0	0	2	1	23	54.51	24.27	5.81	84.59	0.06	54.91	23.82	5.83	84.56	0.46
0	0	2	1	24	39.02	21.44	5.29	65.75	0.06	39.07	21.38	5.29	65.74	0.53
0	0	2	1	25	41.29	20.73	5.47	67.49	0.04	41.34	20.68	5.47	67.49	0.50

					DP-based Heuristic					Math. Model				
N	C_t	TF	RDD	$r\#$	Tard	Tool	Mach	Total	CPU	Tard	Tool	Mach	Total	CPU
0	0	2	2	1	26.25	18.11	5.19	49.55	0.05	26.42	17.91	5.20	49.52	0.45
0	0	2	2	2	39.59	21.90	6.20	67.69	0.05	39.77	21.70	6.22	67.68	0.55
0	0	2	2	3	44.42	25.33	7.39	77.14	0.05	44.86	24.82	7.42	77.10	0.51
0	0	2	2	4	46.83	20.66	5.01	72.51	0.05	46.85	20.64	5.02	72.51	0.41
0	0	2	2	5	42.36	23.08	5.49	70.93	0.06	42.39	23.05	5.49	70.93	0.48
0	0	2	2	6	40.38	22.09	7.06	69.53	0.05	40.50	21.95	7.07	69.53	0.48
0	0	2	2	7	49.70	19.85	6.87	76.41	0.06	49.62	19.92	6.86	76.41	0.50
0	0	2	2	8	38.53	21.01	5.22	64.76	0.06	38.75	20.75	5.24	64.73	0.46
0	0	2	2	9	43.83	20.89	5.21	69.93	0.06	43.90	20.81	5.21	69.92	0.50
0	0	2	2	10	50.96	21.31	6.48	78.74	0.06	50.96	21.31	6.48	78.74	0.44
0	0	2	2	11	44.88	18.70	5.34	68.92	0.04	44.88	18.70	5.34	68.92	0.43
0	0	2	2	12	76.79	23.86	9.28	109.93	0.06	76.87	23.77	9.28	109.92	0.45
0	0	2	2	13	45.79	19.20	5.48	70.47	0.06	45.85	19.13	5.48	70.46	0.54
0	0	2	2	14	46.27	20.22	6.46	72.95	0.06	46.45	20.01	6.48	72.94	0.56
0	0	2	2	15	39.06	21.80	6.16	67.02	0.07	39.25	21.58	6.18	67.01	0.54
0	0	2	2	16	60.85	21.74	6.71	89.30	0.07	60.86	21.73	6.71	89.30	0.44
0	0	2	2	17	44.82	19.36	5.48	69.65	0.05	44.90	19.27	5.48	69.65	0.51
0	0	2	2	18	39.00	21.29	5.49	65.78	0.06	39.48	20.70	5.52	65.71	0.54
0	0	2	2	19	34.83	19.16	4.11	58.09	0.06	35.05	18.90	4.12	58.07	0.55
0	0	2	2	20	39.86	18.61	4.92	63.38	0.07	40.09	18.34	4.93	63.36	0.44
0	0	2	2	21	39.81	19.16	4.22	63.19	0.06	39.88	19.09	4.22	63.19	0.53
0	0	2	2	22	57.83	21.05	5.13	84.01	0.03	57.83	21.05	5.13	84.01	0.49
0	0	2	2	23	47.30	24.27	5.81	77.38	0.04	47.60	23.92	5.82	77.35	0.54
0	0	2	2	24	38.39	21.41	5.29	65.08	0.06	38.67	21.08	5.31	65.06	0.50
0	0	2	2	25	35.29	20.73	5.47	61.49	0.05	35.46	20.54	5.48	61.48	0.41
0	1	0	0	1	20.23	35.85	8.46	64.55	0.18	16.06	36.28	8.06	60.41	0.62
0	1	0	0	2	19.61	50.96	8.62	79.19	0.16	23.51	42.65	9.05	75.20	0.62
0	1	0	0	3	27.63	56.24	10.38	94.24	0.13	26.26	49.62	10.24	86.12	0.69
0	1	0	0	4	18.44	49.01	7.62	75.07	0.10	18.13	46.31	7.60	72.04	0.64
0	1	0	0	5	20.88	56.32	8.43	85.62	0.15	21.57	47.79	8.49	77.85	0.65
0	1	0	0	6	21.94	58.96	10.20	91.10	0.15	21.56	48.79	10.17	80.52	0.66
0	1	0	0	7	16.07	48.77	9.08	73.92	0.13	16.14	48.45	9.09	73.67	0.60
0	1	0	0	8	23.74	42.55	8.30	74.59	0.15	18.02	44.54	7.82	70.37	0.65
0	1	0	0	9	20.11	48.32	7.17	75.60	0.08	21.49	46.48	7.28	75.26	0.59
0	1	0	0	10	16.30	48.69	8.97	73.97	0.19	15.70	49.25	8.92	73.87	0.57
0	1	0	0	11	19.69	39.93	8.50	68.12	0.15	13.80	38.94	8.02	60.77	0.54
0	1	0	0	12	33.66	61.77	12.89	108.32	0.19	29.33	54.62	12.35	96.30	0.61
0	1	0	0	13	22.10	44.79	7.88	74.77	0.14	21.23	44.25	7.81	73.28	0.64
0	1	0	0	14	20.34	41.17	9.90	71.41	0.19	16.38	41.42	9.44	67.23	0.69
0	1	0	0	15	18.73	41.43	9.90	70.06	0.22	18.71	41.42	9.90	70.03	0.72
0	1	0	0	16	21.06	55.06	8.94	85.05	0.15	17.31	53.50	8.66	79.47	0.60
0	1	0	0	17	19.54	44.21	8.02	71.76	0.12	17.35	43.29	7.81	68.45	0.60
0	1	0	0	18	19.57	45.05	7.87	72.49	0.20	19.57	45.05	7.87	72.49	0.60
0	1	0	0	19	23.90	37.23	6.53	67.66	0.10	20.23	36.85	6.21	63.29	0.60
0	1	0	0	20	25.68	39.70	7.58	72.96	0.14	21.11	39.66	7.16	67.93	0.67
0	1	0	0	21	20.96	47.44	5.80	74.21	0.07	22.13	43.94	5.89	71.96	0.57
0	1	0	0	22	20.91	45.80	7.26	73.96	0.08	18.38	45.92	7.08	71.39	0.59
0	1	0	0	23	21.52	55.71	7.61	84.84	0.09	24.50	51.64	7.84	83.98	0.66
0	1	0	0	24	24.28	46.59	8.14	79.00	0.13	22.19	46.94	7.96	77.09	0.55
0	1	0	0	25	18.88	43.90	8.21	70.99	0.15	18.26	42.40	8.15	68.81	0.59
0	1	0	1	1	19.24	38.67	8.09	66.00	0.17	19.02	36.24	8.07	63.33	0.62
0	1	0	1	2	24.05	47.79	9.18	81.01	0.17	23.35	42.16	9.10	74.61	0.61
0	1	0	1	3	25.54	54.10	9.87	89.52	0.10	26.98	52.22	10.00	89.20	0.64
0	1	0	1	4	17.79	48.71	7.71	74.20	0.11	17.46	45.21	7.68	70.35	0.65
0	1	0	1	5	19.67	49.61	8.35	77.63	0.12	20.34	48.80	8.41	77.55	0.70
0	1	0	1	6	24.64	55.58	10.52	90.74	0.15	21.06	48.79	10.17	80.02	0.63
0	1	0	1	7	24.74	47.27	9.84	81.84	0.17	21.36	44.21	9.49	75.06	0.59
0	1	0	1	8	14.24	59.00	7.52	80.76	0.17	17.62	44.54	7.82	69.98	0.63
0	1	0	1	9	22.80	47.11	7.42	77.32	0.11	20.79	46.79	7.26	74.84	0.60
0	1	0	1	10	17.46	50.13	8.95	76.54	0.15	17.38	49.00	8.94	75.33	0.65
0	1	0	1	11	22.60	39.65	8.37	70.62	0.13	16.26	40.39	7.91	64.56	0.57
0	1	0	1	12	33.32	65.02	12.98	111.33	0.25	26.88	55.52	12.25	94.65	0.64
0	1	0	1	13	24.85	43.66	8.12	76.64	0.16	22.45	43.41	7.88	73.74	0.62
0	1	0	1	14	17.93	42.68	9.37	69.97	0.22	17.05	43.05	9.28	69.38	0.61
0	1	0	1	15	26.73	41.74	10.81	79.28	0.18	19.93	39.82	10.09	69.83	0.68
0	1	0	1	16	19.44	54.99	8.84	83.27	0.13	17.14	53.34	8.67	79.15	0.60
0	1	0	1	17	20.08	45.11	7.89	73.08	0.10	19.22	43.19	7.82	70.23	0.57
0	1	0	1	18	18.14	44.09	8.20	70.43	0.17	16.69	42.91	8.06	67.67	0.69
0	1	0	1	19	17.03	39.72	6.13	62.89	0.11	17.38	37.33	6.17	60.88	0.56
0	1	0	1	20	20.43	42.03	7.06	69.53	0.15	20.36	40.91	7.06	68.33	0.64
0	1	0	1	21	19.89	43.91	5.90	69.70	0.11	20.59	43.02	5.95	69.57	0.61
0	1	0	1	22	15.85	47.17	7.03	70.05	0.13	16.05	46.52	7.04	69.61	0.61
0	1	0	1	23	32.20	51.97	8.31	92.48	0.14	28.55	49.70	7.99	86.25	0.66
0	1	0	1	24	19.87	48.68	7.85	76.39	0.12	21.65	46.47	8.00	76.12	0.64
0	1	0	1	25	22.39	44.61	8.37	75.38	0.15	17.37	44.81	7.93	70.10	0.62
0	1	0	2	1	21.64	34.99	8.80	65.43	0.23	14.46	35.54	8.14	58.14	0.74
0	1	0	2	2	23.19	58.02	9.24	90.45	0.17	21.23	42.63	9.06	72.92	0.61
0	1	0	2	3	28.55	47.94	10.96	87.45	0.16	22.94	48.59	10.35	81.87	0.71
0	1	0	2	4	22.91	48.62	7.47	79.00	0.11	24.28	46.83	7.57	78.68	0.62
0	1	0	2	5	25.41	52.59	8.53	86.54	0.16	26.65	45.91	8.65	81.21	0.63
0	1	0	2	6	20.65	56.70	10.56	87.91	0.16	21.19	44.32	10.63	76.14	0.66
0	1	0	2	7	26.28	45.19	9.60	81.07	0.21	24.18	45.33	9.39	78.89	0.70
0	1	0	2	8	22.67	47.22	7.78	77.68	0.16	22.90	44.69	7.81	75.40	0.61
0	1	0	2	9	24.36	62.94	7.31	94.61	0.15	23.64	45.87	7.35	76.86	0.56
0	1	0	2	10	26.50	46.85	9.35	82.70	0.18	25.61	45.40	9.28	80.28	0.62
0	1	0	2	11	25.31	38.94	8.46	72.71	0.15	19.28	39.32	7.99	66.59	0.54
0	1	0	2	12	33.80	74.55	12.02	120.38	0.19	33.91	57.52	12.05	103.47	0.66
0	1	0	2	13	19.97	55.79	7.46	83.22	0.16	23.82	43.58	7.87	75.26	0.65
0	1	0	2	14	25.32	45.06	9.86	80.25	0.21	19.37	42.51	9.33	71.20	0.68
0	1	0	2	15	26.72	42.62	10.23	79.58	0.18	21.65	42.94	9.74	74.32	0.72
0	1	0	2	16	31.37	60.16	9.48	101.01	0.18	24.99	48.60	9.12	82.71	0.62
0	1	0	2	17	22.82	44.01	8.17	75.01	0.12	20.19	41.52	7.99	69.70	0.67
0	1	0	2	18	21.42	46.27	8.15	75.84	0.21	21.74	41.59	8.19	71.52	0.76
0	1	0	2	19	20.33	39.33	6.15	65.80	0.10	19.82	37.95	6.12	63.89	0.60
0	1	0	2	20	27.13	40.44	7.54	75.11	0.15	23.69	38.74	7.25	69.68	0.68
0	1	0	2	21	27.23	41.55	6.26	75.04	0.12	23.66	42.64	5.98	72.27	0.61
0	1	0	2	22	28.38	44.59	7.31	80.27	0.11	25.94	45.41	7.13	78.48	0.54
0	1	0	2	23	29.60	49.01	8.51	87.13	0.16	23.64	49.63	8.00	81.27	0.63
0	1	0	2	24	26.44	45.01	8.26	79.71	0.14	24.85	45.24	8.11	78.20	0.71
0	1	0	2	25	21.22	44.40	8.35	73.97	0.18	17.94	43.31	8.07	69.31	0.74

					DP-based Heuristic					Math. Model				
N	C_t	TF	RDD	$r\#$	Tard	Tool	Mach	Total	CPU	Tard	Tool	Mach	Total	CPU
0	1	1	0	1	28.02	41.27	7.76	77.04	0.10	27.57	40.47	7.72	75.76	0.55
0	1	1	0	2	39.39	46.36	8.86	94.60	0.09	38.41	45.72	8.78	92.91	0.55
0	1	1	0	3	44.79	54.10	9.86	108.74	0.09	45.10	53.75	9.89	108.74	0.55
0	1	1	0	4	33.90	53.19	7.22	94.32	0.06	35.41	51.45	7.30	94.16	0.53
0	1	1	0	5	35.26	54.80	8.05	98.11	0.07	37.02	52.56	8.16	97.74	0.59
0	1	1	0	6	39.48	53.72	9.84	103.05	0.08	39.06	53.32	9.81	102.18	0.52
0	1	1	0	7	36.85	53.61	8.72	99.17	0.07	37.14	53.17	8.74	99.05	0.53
0	1	1	0	8	33.54	47.63	7.71	88.88	0.08	33.42	46.11	7.70	87.22	0.60
0	1	1	0	9	37.50	50.66	7.02	95.18	0.05	39.09	48.66	7.13	94.88	0.54
0	1	1	0	10	35.28	54.50	8.56	98.33	0.07	36.10	53.53	8.61	98.24	0.48
0	1	1	0	11	27.60	45.61	7.59	80.79	0.06	28.87	44.06	7.67	80.59	0.55
0	1	1	0	12	54.25	61.90	12.04	128.18	0.15	48.16	63.13	11.56	122.84	0.61
0	1	1	0	13	36.11	49.67	7.43	93.21	0.08	36.90	48.55	7.48	92.92	0.56
0	1	1	0	14	29.46	47.33	8.98	85.77	0.08	28.88	47.15	8.94	84.97	0.58
0	1	1	0	15	29.85	46.68	9.40	85.94	0.14	29.85	46.68	9.40	85.94	0.59
0	1	1	0	16	41.47	59.36	8.30	109.13	0.06	42.33	58.38	8.35	109.06	0.57
0	1	1	0	17	32.44	47.84	7.49	87.78	0.06	33.80	46.17	7.59	87.56	0.50
0	1	1	0	18	36.49	49.40	7.66	93.55	0.09	36.45	48.03	7.66	92.14	0.62
0	1	1	0	19	30.17	43.63	5.77	79.57	0.05	31.60	41.93	5.85	79.38	0.53
0	1	1	0	20	32.36	45.56	6.75	84.67	0.07	33.07	44.71	6.79	84.57	0.57
0	1	1	0	21	37.18	46.53	5.74	89.45	0.04	38.47	45.03	5.82	89.31	0.55
0	1	1	0	22	36.90	51.37	6.77	95.04	0.05	37.59	50.52	6.81	94.92	0.57
0	1	1	0	23	42.12	56.65	7.54	106.31	0.07	44.91	53.03	7.75	105.68	0.61
0	1	1	0	24	37.73	50.47	7.76	95.96	0.08	37.76	49.81	7.76	95.33	0.59
0	1	1	0	25	30.84	48.44	7.66	86.94	0.09	31.57	47.48	7.72	86.77	0.53
0	1	1	1	1	30.49	42.22	7.62	80.34	0.10	31.19	41.28	7.67	80.13	0.47
0	1	1	1	2	36.47	47.94	8.63	93.04	0.08	36.60	47.52	8.64	92.76	0.65
0	1	1	1	3	48.22	55.00	9.81	113.03	0.09	49.41	53.52	9.90	112.83	0.54
0	1	1	1	4	30.30	53.16	7.22	90.69	0.06	31.95	51.22	7.31	90.48	0.61
0	1	1	1	5	36.30	53.04	8.16	97.50	0.09	37.08	51.68	8.21	96.98	0.56
0	1	1	1	6	39.17	51.97	9.93	101.07	0.08	39.20	51.53	9.94	100.67	0.61
0	1	1	1	7	38.94	52.24	8.95	100.13	0.06	36.99	52.05	8.82	97.85	0.62
0	1	1	1	8	31.49	48.77	7.54	87.80	0.07	33.00	46.96	7.64	87.59	0.59
0	1	1	1	9	39.63	49.30	7.15	96.08	0.09	39.51	48.41	7.15	95.08	0.64
0	1	1	1	10	38.05	54.33	8.57	100.94	0.07	38.59	53.73	8.60	100.92	0.54
0	1	1	1	11	33.04	45.77	7.58	86.39	0.05	33.79	44.88	7.62	86.29	0.65
0	1	1	1	12	56.67	61.05	12.24	129.95	0.13	47.09	63.78	11.51	122.38	0.62
0	1	1	1	13	36.67	49.01	7.46	93.13	0.08	37.22	48.38	7.49	93.09	0.58
0	1	1	1	14	31.47	50.15	8.91	90.53	0.12	32.10	46.84	8.96	87.90	0.61
0	1	1	1	15	36.71	45.41	9.96	92.08	0.15	29.33	46.74	9.40	85.47	0.61
0	1	1	1	16	40.55	59.39	8.29	108.23	0.06	41.32	58.51	8.34	108.17	0.58
0	1	1	1	17	35.30	47.26	7.53	90.09	0.07	36.50	45.81	7.61	89.92	0.54
0	1	1	1	18	28.86	51.38	7.48	87.72	0.07	30.84	48.91	7.60	87.36	0.57
0	1	1	1	19	27.47	43.50	5.77	76.74	0.08	29.14	41.37	5.88	76.40	0.59
0	1	1	1	20	34.53	43.93	6.85	85.31	0.12	35.05	43.22	6.89	85.15	0.60
0	1	1	1	21	36.92	45.62	5.91	88.45	0.08	35.98	44.64	5.85	86.46	0.58
0	1	1	1	22	35.27	51.50	6.77	93.53	0.06	36.12	50.54	6.81	93.46	0.64
0	1	1	1	23	44.23	56.76	7.53	108.52	0.06	47.22	52.97	7.75	107.94	0.53
0	1	1	1	24	36.83	51.76	7.84	96.43	0.11	37.10	48.32	7.86	93.28	0.52
0	1	1	1	25	32.00	50.14	7.56	89.71	0.08	33.22	48.70	7.64	89.56	0.55
0	1	1	2	1	23.45	46.91	7.58	77.95	0.19	25.46	40.49	7.72	73.67	0.60
0	1	1	2	2	29.60	65.92	8.14	103.66	0.11	36.49	46.38	8.73	91.59	0.59
0	1	1	2	3	39.71	52.39	9.99	102.09	0.10	39.93	52.13	10.01	102.07	0.55
0	1	1	2	4	43.96	49.89	7.49	101.34	0.07	42.89	49.00	7.43	99.32	0.57
0	1	1	2	5	35.77	72.58	7.78	116.12	0.07	39.39	53.35	8.12	100.85	0.56
0	1	1	2	6	33.27	52.83	9.85	95.96	0.09	33.82	52.13	9.90	95.85	0.56
0	1	1	2	7	51.59	48.64	9.83	110.06	0.12	40.10	50.04	8.96	99.10	0.58
0	1	1	2	8	37.15	49.10	7.52	93.76	0.05	38.73	47.18	7.63	93.53	0.59
0	1	1	2	9	40.70	49.64	7.08	97.42	0.09	41.26	48.93	7.12	97.30	0.59
0	1	1	2	10	49.51	49.73	9.26	108.49	0.15	41.96	51.26	8.77	101.99	0.51
0	1	1	2	11	32.01	52.31	7.42	91.74	0.09	35.83	44.28	7.66	87.76	0.56
0	1	1	2	12	59.58	65.07	11.42	136.07	0.12	59.74	64.89	11.43	136.07	0.62
0	1	1	2	13	42.63	49.81	7.73	100.17	0.09	41.20	46.43	7.64	95.27	0.58
0	1	1	2	14	35.76	49.94	8.94	94.65	0.13	36.24	46.50	8.99	91.72	0.65
0	1	1	2	15	36.11	47.38	9.36	92.84	0.14	36.24	47.22	9.37	92.83	0.66
0	1	1	2	16	43.49	57.75	8.38	109.62	0.10	44.00	57.20	8.42	109.61	0.54
0	1	1	2	17	36.43	47.84	7.49	91.77	0.06	37.53	46.58	7.57	91.68	0.51
0	1	1	2	18	34.29	48.00	7.68	89.98	0.11	34.28	47.66	7.68	89.62	0.57
0	1	1	2	19	32.41	43.62	5.77	81.79	0.06	34.40	41.24	5.89	81.53	0.57
0	1	1	2	20	37.69	44.99	6.88	89.56	0.12	38.32	42.67	6.93	87.92	0.61
0	1	1	2	21	38.46	45.52	5.80	89.78	0.08	39.11	44.76	5.84	89.70	0.61
0	1	1	2	22	36.77	65.78	6.37	108.93	0.05	44.00	50.50	6.81	101.31	0.50
0	1	1	2	23	39.03	57.68	7.49	104.19	0.06	41.17	54.99	7.62	103.79	0.61
0	1	1	2	24	37.81	50.18	7.74	95.74	0.09	38.77	49.06	7.81	95.64	0.64
0	1	1	2	25	33.36	46.90	7.76	88.02	0.11	33.73	46.50	7.80	88.01	0.61
0	1	2	0	1	46.47	44.11	7.51	98.09	0.05	46.82	43.73	7.53	98.07	0.52
0	1	2	0	2	59.52	50.84	8.45	118.81	0.06	59.74	50.59	8.46	118.80	0.66
0	1	2	0	3	72.16	57.14	9.68	138.97	0.04	72.64	56.61	9.70	138.95	0.48
0	1	2	0	4	60.16	53.19	7.22	120.58	0.05	60.51	52.82	7.23	120.56	0.53
0	1	2	0	5	61.09	55.10	8.03	124.22	0.06	61.34	54.83	8.04	124.21	0.53
0	1	2	0	6	65.77	56.70	9.62	132.08	0.06	66.15	56.27	9.64	132.06	0.54
0	1	2	0	7	66.14	54.59	8.66	129.39	0.08	66.44	54.25	8.68	129.37	0.52
0	1	2	0	8	51.52	49.35	7.50	108.37	0.06	51.52	49.33	7.50	108.36	0.52
0	1	2	0	9	60.72	50.86	7.01	118.59	0.06	61.34	50.13	7.05	118.52	0.52
0	1	2	0	10	65.78	56.23	8.47	130.48	0.06	66.41	55.53	8.50	130.44	0.56
0	1	2	0	11	54.31	46.18	7.56	108.04	0.04	54.49	45.98	7.57	108.04	0.52
0	1	2	0	12	82.07	71.03	11.09	164.19	0.07	83.28	69.65	11.15	164.09	0.47
0	1	2	0	13	63.16	49.67	7.42	120.25	0.07	63.27	49.55	7.43	120.25	0.51
0	1	2	0	14	57.78	49.23	8.81	115.83	0.07	57.85	49.17	8.82	115.83	0.54
0	1	2	0	15	50.41	50.27	9.18	109.85	0.07	50.96	49.64	9.21	109.81	0.60
0	1	2	0	16	77.82	59.68	8.28	145.77	0.06	77.86	59.63	8.28	145.77	0.55
0	1	2	0	17	60.72	47.84	7.49	116.06	0.06	60.87	47.69	7.50	116.06	0.53
0	1	2	0	18	59.86	52.34	7.44	119.63	0.07	60.79	51.27	7.48	119.54	0.56
0	1	2	0	19	51.83	43.95	5.75	101.53	0.06	51.95	43.82	5.76	101.53	0.58
0	1	2	0	20	53.01	48.09	6.61	107.71	0.06	53.91	47.05	6.66	107.63	0.49
0	1	2	0	21	57.65	46.53	5.74	109.92	0.05	57.99	46.15	5.76	109.90	0.58
0	1	2	0	22	68.05	51.71	6.76	126.52	0.05	68.20	51.55	6.76	126.51	0.47
0	1	2	0	23	68.20	57.68	7.49	133.36	0.06	69.04	56.74	7.53	133.31	0.51
0	1	2	0	24	58.20	52.51	7.61	118.32	0.07	58.80	51.84	7.65	118.29	0.54
0	1	2	0	25	53.29	50.14	7.56	110.99	0.04	53.39	50.04	7.57	110.99	0.52

					DP-based Heuristic					Math. Model				
N	C_t	TF	RDD	$r\#$	Tard	Tool	Mach	Total	CPU	Tard	Tool	Mach	Total	CPU
0	1	2	1	1	50.69	43.57	7.54	101.80	0.07	50.63	43.47	7.54	101.64	0.52
0	1	2	1	2	60.37	51.14	8.44	119.95	0.05	61.47	49.71	8.51	119.68	0.65
0	1	2	1	3	74.66	57.22	9.67	141.55	0.05	75.28	56.53	9.71	141.52	0.61
0	1	2	1	4	57.90	53.19	7.22	118.32	0.06	58.08	53.00	7.23	118.31	0.49
0	1	2	1	5	60.70	55.06	8.03	123.79	0.07	61.18	54.53	8.06	123.76	0.59
0	1	2	1	6	62.34	55.69	9.67	127.70	0.06	62.75	55.23	9.69	127.67	0.53
0	1	2	1	7	65.66	53.99	8.69	128.35	0.09	65.89	53.74	8.71	128.34	0.49
0	1	2	1	8	53.45	49.68	7.49	110.62	0.06	53.77	49.33	7.50	110.61	0.56
0	1	2	1	9	59.92	50.86	7.01	117.79	0.06	60.74	49.90	7.06	117.70	0.54
0	1	2	1	10	67.53	56.16	8.48	132.17	0.07	68.71	54.80	8.54	132.05	0.52
0	1	2	1	11	59.70	46.15	7.56	113.41	0.05	59.97	45.86	7.57	113.40	0.48
0	1	2	1	12	82.66	70.97	11.09	164.72	0.06	84.33	69.05	11.18	164.56	0.48
0	1	2	1	13	61.71	49.67	7.42	118.80	0.06	61.92	49.44	7.43	118.79	0.54
0	1	2	1	14	59.57	49.23	8.81	117.61	0.07	59.64	49.15	8.82	117.61	0.50
0	1	2	1	15	50.25	50.27	9.18	109.70	0.05	50.61	49.87	9.20	109.67	0.58
0	1	2	1	16	76.31	59.68	8.28	144.27	0.06	76.38	59.61	8.28	144.27	0.47
0	1	2	1	17	60.81	47.84	7.49	116.14	0.05	60.96	47.68	7.50	116.14	0.53
0	1	2	1	18	56.72	52.34	7.44	116.49	0.06	58.25	50.56	7.51	116.32	0.54
0	1	2	1	19	49.12	43.87	5.76	98.74	0.06	49.72	43.21	5.79	98.71	0.53
0	1	2	1	20	52.58	48.09	6.61	107.29	0.06	53.36	47.21	6.65	107.22	0.53
0	1	2	1	21	54.28	46.53	5.74	106.55	0.06	54.80	45.95	5.77	106.52	0.59
0	1	2	1	22	67.59	51.61	6.76	125.97	0.05	68.00	51.16	6.78	125.94	0.52
0	1	2	1	23	72.89	57.68	7.49	138.05	0.07	73.66	56.83	7.53	138.02	0.55
0	1	2	1	24	56.51	52.51	7.61	116.63	0.06	57.11	51.84	7.65	116.60	0.51
0	1	2	1	25	58.45	50.14	7.56	116.16	0.06	58.64	49.94	7.57	116.15	0.52
0	1	2	2	1	41.32	43.58	7.57	92.47	0.08	41.64	42.50	7.59	91.73	0.51
0	1	2	2	2	56.91	49.98	8.49	115.39	0.06	57.12	49.75	8.51	115.38	0.61
0	1	2	2	3	61.88	56.91	9.69	128.48	0.07	63.31	55.23	9.79	128.32	0.59
0	1	2	2	4	63.33	53.19	7.22	123.75	0.06	63.56	52.95	7.23	123.74	0.52
0	1	2	2	5	62.99	55.10	8.03	126.13	0.05	63.35	54.70	8.05	126.10	0.56
0	1	2	2	6	57.03	56.15	9.64	122.82	0.08	58.08	54.97	9.71	122.76	0.57
0	1	2	2	7	62.33	53.64	8.78	124.74	0.07	61.72	53.18	8.74	123.65	0.58
0	1	2	2	8	55.92	48.75	7.53	112.21	0.09	56.16	48.50	7.55	112.20	0.52
0	1	2	2	9	59.63	51.04	7.00	117.67	0.05	60.12	50.48	7.03	117.63	0.48
0	1	2	2	10	66.52	54.97	8.53	130.03	0.05	66.56	54.93	8.54	130.03	0.48
0	1	2	2	11	58.31	46.30	7.55	112.16	0.05	58.37	46.23	7.56	112.16	0.49
0	1	2	2	12	91.35	71.03	11.09	173.46	0.06	92.85	69.32	11.17	173.34	0.50
0	1	2	2	13	61.61	49.67	7.42	118.70	0.06	61.86	49.40	7.43	118.69	0.51
0	1	2	2	14	61.02	48.74	8.88	118.64	0.08	60.41	48.50	8.85	117.76	0.51
0	1	2	2	15	57.28	50.27	9.18	116.73	0.05	57.88	49.61	9.21	116.69	0.63
0	1	2	2	16	72.18	59.68	8.28	140.14	0.05	72.48	59.35	8.30	140.13	0.53
0	1	2	2	17	60.21	47.84	7.49	115.55	0.05	60.55	47.47	7.51	115.53	0.51
0	1	2	2	18	54.68	50.03	7.55	112.26	0.10	55.27	49.29	7.59	112.15	0.56
0	1	2	2	19	50.42	43.88	5.75	100.05	0.05	50.98	43.27	5.78	100.03	0.51
0	1	2	2	20	60.03	45.01	6.95	111.98	0.06	54.59	46.75	6.68	108.02	0.60
0	1	2	2	21	55.91	46.53	5.74	108.18	0.05	56.45	45.93	5.77	108.14	0.48
0	1	2	2	22	71.40	51.64	6.76	129.81	0.06	71.83	51.17	6.78	129.79	0.52
0	1	2	2	23	65.74	57.68	7.49	130.90	0.04	67.27	55.88	7.57	130.72	0.55
0	1	2	2	24	56.48	51.85	7.69	116.02	0.09	56.38	51.03	7.69	115.10	0.53
0	1	2	2	25	51.86	50.14	7.56	109.57	0.05	52.66	49.25	7.61	109.52	0.57
1	0	0	0	1	21.14	50.31	9.41	80.86	0.16	22.68	47.98	9.52	80.18	0.80
1	0	0	0	2	18.67	46.74	10.22	75.62	0.18	21.36	42.37	10.45	74.18	0.85
1	0	0	0	3	19.73	49.57	12.27	81.57	0.20	21.38	46.35	12.42	80.15	0.88
1	0	0	0	4	8.54	40.13	8.65	57.31	0.16	10.43	37.21	8.81	56.45	0.72
1	0	0	0	5	14.67	48.53	9.28	72.47	0.18	17.26	44.74	9.49	71.49	0.91
1	0	0	0	6	11.07	50.54	11.49	73.09	0.19	14.42	44.20	11.86	70.47	0.94
1	0	0	0	7	15.08	41.42	10.01	66.52	0.14	16.71	38.93	10.16	65.80	0.91
1	0	0	0	8	11.30	44.46	9.12	64.88	0.15	13.59	41.32	9.31	64.22	0.70
1	0	0	0	9	20.83	49.49	13.37	83.69	0.17	21.98	46.91	13.46	82.35	0.86
1	0	0	0	10	11.65	36.63	10.29	58.57	0.22	11.50	35.08	10.28	56.85	0.72
1	0	0	0	11	15.58	42.62	9.38	67.58	0.16	16.82	40.78	9.46	67.06	0.79
1	0	0	0	12	12.80	46.72	9.94	69.46	0.16	14.74	44.10	10.10	68.94	0.81
1	0	0	0	13	14.57	48.75	11.93	75.25	0.18	17.02	45.19	12.18	74.38	0.86
1	0	0	0	14	11.99	38.73	9.94	60.65	0.17	13.58	36.49	10.07	60.14	0.70
1	0	0	0	15	14.75	45.93	10.00	70.68	0.15	16.75	43.00	10.15	69.89	0.74
1	0	0	0	16	13.88	43.78	10.40	68.06	0.15	15.72	40.98	10.59	67.29	0.75
1	0	0	0	17	16.10	39.74	11.96	67.80	0.31	16.13	38.62	11.96	66.71	0.91
1	0	0	0	18	14.47	35.92	7.46	57.85	0.18	16.31	33.29	7.60	57.19	0.74
1	0	0	0	19	10.32	39.77	8.88	58.97	0.18	11.89	37.20	9.01	58.10	0.74
1	0	0	0	20	21.03	43.39	9.49	73.92	0.15	22.45	41.51	9.58	73.54	0.81
1	0	0	0	21	14.22	46.67	9.91	70.80	0.17	15.94	44.22	10.04	70.19	0.84
1	0	0	0	22	15.61	46.56	10.24	72.41	0.18	18.39	42.33	10.53	71.25	0.85
1	0	0	0	23	13.50	39.79	11.54	64.83	0.33	10.62	39.12	11.20	60.94	0.92
1	0	0	0	24	12.70	46.20	9.94	68.84	0.17	15.13	42.58	10.15	67.86	0.83
1	0	0	0	25	8.91	34.25	10.01	53.16	0.16	10.81	31.00	10.21	52.02	0.83
1	0	0	1	1	24.21	47.94	9.75	81.91	0.23	23.00	45.81	9.68	78.49	0.95
1	0	0	1	2	21.72	44.37	10.53	76.61	0.28	20.40	42.57	10.44	73.41	1.00
1	0	0	1	3	23.69	48.78	12.27	84.74	0.26	25.19	46.72	12.40	84.30	0.95
1	0	0	1	4	12.70	40.13	8.65	61.48	0.17	15.05	36.56	8.86	60.48	0.91
1	0	0	1	5	14.76	48.62	9.27	72.66	0.17	16.82	45.83	9.41	72.06	0.93
1	0	0	1	6	20.00	45.53	12.12	77.66	0.53	20.27	41.41	12.16	73.84	1.03
1	0	0	1	7	15.31	41.42	10.01	66.75	0.16	17.61	38.03	10.24	65.87	0.93
1	0	0	1	8	19.79	41.86	9.42	71.07	0.36	17.98	41.76	9.30	69.03	0.83
1	0	0	1	9	23.92	46.55	13.60	84.08	0.33	23.80	45.26	13.60	82.66	0.89
1	0	0	1	10	22.93	35.55	10.63	69.11	0.30	17.54	35.52	10.28	63.34	0.83
1	0	0	1	11	21.92	40.47	9.85	72.25	0.27	19.21	38.35	9.67	67.23	0.93
1	0	0	1	12	21.58	45.61	10.13	77.32	0.21	21.89	43.26	10.19	75.34	0.94
1	0	0	1	13	15.96	48.75	11.93	76.64	0.18	18.95	44.03	12.29	75.26	0.97
1	0	0	1	14	19.41	38.73	9.94	68.07	0.16	20.89	36.62	10.06	67.57	0.82
1	0	0	1	15	23.28	45.93	10.00	79.21	0.16	25.38	42.94	10.16	78.48	0.87
1	0	0	1	16	13.68	44.30	10.70	68.69	0.34	12.96	40.22	10.68	63.86	0.83
1	0	0	1	17	16.30	40.12	11.84	68.26	0.38	17.25	38.69	11.95	67.90	0.94
1	0	0	1	18	14.23	36.42	7.43	58.08	0.15	16.44	33.08	7.61	57.14	0.83
1	0	0	1	19	15.89	36.09	9.14	61.12	0.41	16.92	34.77	9.26	60.95	0.96
1	0	0	1	20	25.40	43.39	9.49	78.28	0.17	28.24	39.41	9.72	77.36	0.84
1	0	0	1	21	18.45	43.56	10.09	72.11	0.26	18.63	43.35	10.12	72.09	0.89
1	0	0	1	22	16.49	46.01	10.33	72.83	0.17	18.37	42.45	10.52	71.34	0.91
1	0	0	1	23	13.80	40.61	11.07	65.48	0.37	14.60	39.49	11.16	65.25	1.00
1	0	0	1	24	20.05	43.15	10.53	73.73	0.27	17.66	40.13	10.42	68.21	0.97
1	0	0	1	25	20.01	32.70	10.12	62.83	0.29	20.66	31.77	10.17	62.59	0.90

					DP-based Heuristic					Math. Model				
N	C_t	TF	RDD	$r\#$	Tard	Tool	Mach	Total	CPU	Tard	Tool	Mach	Total	CPU
1	0	0	2	1	35.31	45.57	10.38	91.25	0.32	25.80	44.82	9.83	80.45	1.05
1	0	0	2	2	37.01	46.85	10.21	94.08	0.16	39.25	43.72	10.36	93.32	0.99
1	0	0	2	3	37.49	44.90	13.02	95.41	0.65	32.04	44.84	12.63	89.51	1.12
1	0	0	2	4	27.09	38.25	8.77	74.11	0.38	27.85	37.19	8.84	73.88	0.85
1	0	0	2	5	58.07	42.42	10.86	111.35	0.50	31.70	44.26	9.62	85.57	1.04
1	0	0	2	6	37.99	43.81	12.68	94.48	0.69	30.15	42.93	12.05	85.13	1.08
1	0	0	2	7	23.71	40.78	10.15	74.64	0.18	24.58	37.79	10.27	72.64	1.05
1	0	0	2	8	33.96	41.21	9.49	84.65	0.44	31.80	41.45	9.34	82.59	0.91
1	0	0	2	9	46.83	47.64	13.44	107.90	0.27	48.09	45.78	13.55	107.42	1.21
1	0	0	2	10	19.23	34.73	10.54	64.50	0.58	17.68	34.37	10.40	62.45	0.98
1	0	0	2	11	34.02	41.18	10.28	85.47	0.62	23.18	38.23	9.83	71.23	0.98
1	0	0	2	12	34.97	46.23	10.02	91.23	0.19	35.63	44.35	10.12	90.10	0.96
1	0	0	2	13	23.75	43.58	13.01	80.33	0.75	17.42	42.85	12.54	72.81	1.01
1	0	0	2	14	27.06	37.02	10.33	74.41	0.42	25.80	35.00	10.25	71.05	0.87
1	0	0	2	15	29.97	45.93	10.00	85.90	0.15	32.19	42.93	10.17	85.29	0.96
1	0	0	2	16	28.46	39.75	10.83	79.04	0.34	27.30	39.94	10.71	77.95	1.15
1	0	0	2	17	22.33	38.64	12.00	72.97	0.71	22.73	38.07	12.05	72.85	1.13
1	0	0	2	18	29.04	32.00	7.77	68.81	0.41	28.60	31.60	7.77	67.96	1.00
1	0	0	2	19	20.62	36.88	9.25	66.75	0.33	19.70	35.09	9.26	64.04	1.02
1	0	0	2	20	38.46	41.92	9.58	89.96	0.28	39.72	40.21	9.67	89.61	0.91
1	0	0	2	21	30.52	46.08	10.40	87.01	0.43	26.78	42.24	10.26	79.28	1.06
1	0	0	2	22	25.05	46.56	10.24	81.85	0.14	27.16	43.73	10.42	81.31	1.02
1	0	0	2	23	17.40	39.86	11.25	68.51	0.66	17.47	39.53	11.25	68.25	1.09
1	0	0	2	24	30.89	46.11	10.50	87.50	0.40	25.93	40.65	10.50	77.08	1.07
1	0	0	2	25	24.93	32.73	10.35	68.01	0.65	25.25	32.17	10.37	67.79	0.93
1	0	1	0	1	71.34	50.43	9.40	131.17	0.12	72.03	49.51	9.44	130.98	0.71
1	0	1	0	2	75.19	47.11	10.20	132.50	0.13	75.58	46.61	10.21	132.40	0.71
1	0	1	0	3	84.01	50.75	12.16	146.92	0.15	84.31	50.36	12.18	146.84	0.73
1	0	1	0	4	45.85	40.35	8.63	94.83	0.15	46.68	39.33	8.67	94.68	0.71
1	0	1	0	5	66.36	48.68	9.27	124.31	0.12	66.54	48.47	9.27	124.28	0.70
1	0	1	0	6	64.24	50.54	11.49	126.27	0.14	65.19	49.27	11.53	125.99	0.71
1	0	1	0	7	67.88	41.42	10.01	119.32	0.14	68.05	41.23	10.02	119.29	0.62
1	0	1	0	8	63.37	44.46	9.12	116.94	0.13	63.38	44.45	9.12	116.95	0.66
1	0	1	0	9	87.44	50.40	13.29	151.12	0.13	88.39	49.18	13.33	150.90	0.74
1	0	1	0	10	63.49	37.91	10.07	111.47	0.11	63.50	37.91	10.07	111.47	0.59
1	0	1	0	11	66.79	42.62	9.38	118.79	0.13	67.23	42.05	9.39	118.67	0.65
1	0	1	0	12	69.27	46.72	9.94	125.94	0.12	69.36	46.63	9.94	125.93	0.62
1	0	1	0	13	66.26	48.75	11.93	126.94	0.13	66.59	48.33	11.95	126.87	0.69
1	0	1	0	14	67.64	38.73	9.94	116.31	0.10	67.76	38.60	9.94	116.30	0.64
1	0	1	0	15	76.43	45.93	10.00	132.36	0.11	76.64	45.70	10.01	132.34	0.62
1	0	1	0	16	58.63	44.39	10.36	113.38	0.14	58.94	44.01	10.37	113.32	0.67
1	0	1	0	17	48.64	43.66	11.59	103.89	0.15	50.16	41.60	11.70	103.46	0.79
1	0	1	0	18	58.97	36.42	7.43	102.82	0.12	59.18	36.17	7.44	102.80	0.61
1	0	1	0	19	54.96	39.77	8.88	103.61	0.13	55.58	38.94	8.90	103.42	0.67
1	0	1	0	20	78.39	43.39	9.49	131.27	0.15	78.69	43.03	9.51	131.23	0.69
1	0	1	0	21	67.39	46.67	9.91	123.97	0.15	67.73	46.27	9.93	123.93	0.77
1	0	1	0	22	62.29	46.33	10.25	118.87	0.16	62.40	46.16	10.26	118.82	0.60
1	0	1	0	23	48.40	45.13	10.77	104.30	0.15	49.22	44.06	10.81	104.10	0.75
1	0	1	0	24	63.39	46.72	9.91	120.02	0.13	63.54	46.55	9.92	120.01	0.63
1	0	1	0	25	54.12	34.41	10.00	98.53	0.13	54.60	33.85	10.02	98.47	0.72
1	0	1	1	1	68.49	50.43	9.40	128.32	0.14	69.08	49.68	9.43	128.19	0.77
1	0	1	1	2	75.18	47.08	10.20	132.46	0.14	75.81	46.30	10.22	132.33	0.66
1	0	1	1	3	83.94	50.75	12.16	146.86	0.14	84.61	49.90	12.20	146.71	0.80
1	0	1	1	4	50.14	40.38	8.63	99.16	0.16	50.87	39.51	8.67	99.04	0.76
1	0	1	1	5	71.75	48.68	9.27	129.70	0.13	72.02	48.39	9.28	129.68	0.75
1	0	1	1	6	56.76	50.54	11.49	118.79	0.14	57.74	49.19	11.53	118.46	0.81
1	0	1	1	7	69.30	41.42	10.01	120.73	0.13	69.57	41.10	10.02	120.69	0.78
1	0	1	1	8	68.02	44.46	9.12	121.60	0.13	68.05	44.44	9.12	121.60	0.58
1	0	1	1	9	87.49	50.40	13.29	151.18	0.13	88.54	49.05	13.34	150.93	0.80
1	0	1	1	10	67.08	37.91	10.07	115.05	0.12	67.12	37.87	10.07	115.05	0.58
1	0	1	1	11	61.10	42.62	9.38	113.10	0.14	61.58	41.99	9.40	112.97	0.67
1	0	1	1	12	72.47	46.72	9.94	129.13	0.14	72.62	46.56	9.95	129.12	0.70
1	0	1	1	13	62.50	48.75	11.93	123.18	0.15	62.86	48.34	11.95	123.14	0.74
1	0	1	1	14	74.34	38.73	9.94	123.00	0.13	74.57	38.46	9.95	122.97	0.68
1	0	1	1	15	80.51	45.93	10.00	136.44	0.15	80.85	45.55	10.01	136.41	0.66
1	0	1	1	16	56.01	44.39	10.36	110.75	0.14	56.43	43.86	10.38	110.67	0.74
1	0	1	1	17	52.13	43.66	11.59	107.38	0.16	53.09	42.48	11.65	107.22	0.89
1	0	1	1	18	60.46	36.42	7.43	104.31	0.12	60.63	36.22	7.44	104.29	0.62
1	0	1	1	19	52.53	39.77	8.88	101.18	0.15	53.14	38.96	8.90	101.00	0.71
1	0	1	1	20	86.79	43.39	9.49	139.68	0.17	87.13	43.00	9.51	139.64	0.75
1	0	1	1	21	68.89	46.67	9.91	125.47	0.15	69.24	46.27	9.93	125.43	0.78
1	0	1	1	22	65.11	46.56	10.24	121.91	0.14	65.47	46.14	10.26	121.87	0.76
1	0	1	1	23	54.21	45.13	10.77	110.11	0.15	55.04	44.08	10.81	109.93	0.78
1	0	1	1	24	62.16	46.67	9.91	118.74	0.13	62.45	46.34	9.93	118.71	0.73
1	0	1	1	25	76.30	34.41	10.00	120.71	0.14	76.62	34.06	10.01	120.69	0.63
1	0	1	2	1	69.69	50.43	9.40	129.52	0.14	70.63	49.21	9.45	129.29	0.89
1	0	1	2	2	98.21	47.12	10.20	155.53	0.13	98.72	46.50	10.22	155.43	0.82
1	0	1	2	3	94.35	49.58	12.22	156.15	0.22	94.24	49.56	12.22	156.02	0.85
1	0	1	2	4	69.41	40.44	8.63	118.47	0.12	69.94	39.82	8.65	118.41	0.75
1	0	1	2	5	84.81	48.68	9.27	142.76	0.14	85.29	48.13	9.29	142.71	0.79
1	0	1	2	6	72.70	50.54	11.49	134.73	0.13	74.39	48.28	11.59	134.26	0.87
1	0	1	2	7	70.05	41.42	10.01	121.48	0.14	70.63	40.72	10.04	121.40	0.84
1	0	1	2	8	80.69	44.46	9.12	134.27	0.14	80.92	44.20	9.13	134.25	0.73
1	0	1	2	9	116.56	50.40	13.29	180.24	0.12	117.44	49.34	13.33	180.10	0.97
1	0	1	2	10	61.10	37.91	10.07	109.08	0.13	61.28	37.71	10.08	109.07	0.64
1	0	1	2	11	67.31	42.62	9.38	119.31	0.13	67.72	42.12	9.39	119.22	0.67
1	0	1	2	12	88.56	46.72	9.94	145.22	0.13	89.02	46.18	9.97	145.17	0.80
1	0	1	2	13	66.01	48.75	11.93	126.69	0.14	66.41	48.28	11.95	126.64	0.72
1	0	1	2	14	71.59	38.73	9.94	120.26	0.14	72.03	38.21	9.96	120.20	0.69
1	0	1	2	15	89.88	45.93	10.00	145.80	0.13	90.36	45.37	10.02	145.75	0.72
1	0	1	2	16	63.56	44.39	10.36	118.30	0.14	64.28	43.46	10.41	118.14	0.84
1	0	1	2	17	63.33	42.17	11.68	117.18	0.27	63.41	42.08	11.69	117.18	0.91
1	0	1	2	18	69.18	36.42	7.43	113.03	0.14	69.71	35.80	7.46	112.97	0.82
1	0	1	2	19	62.52	39.77	8.88	111.17	0.13	63.30	38.77	8.92	111.00	0.76
1	0	1	2	20	99.06	43.39	9.49	151.94	0.15	99.45	42.94	9.51	151.90	0.75
1	0	1	2	21	76.39	46.67	9.91	132.97	0.16	76.92	46.06	9.94	132.92	0.76
1	0	1	2	22	75.98	46.56	10.24	132.78	0.14	76.64	45.78	10.28	132.70	0.75
1	0	1	2	23	63.46	45.13	10.77	119.36	0.15	64.45	43.88	10.83	119.16	0.93
1	0	1	2	24	66.26	46.57	9.92	122.75	0.16	67.02	45.67	9.96	122.64	0.83
1	0	1	2	25	84.75	34.41	10.00	129.16	0.12	85.10	34.02	10.01	129.13	0.71

					DP-based Heuristic					Math. Model				
N	C_t	TF	RDD	$r\#$	Tard	Tool	Mach	Total	CPU	Tard	Tool	Mach	Total	CPU
1	0	2	0	1	147.57	50.43	9.40	207.40	0.11	147.72	50.27	9.41	207.39	0.60
1	0	2	0	2	178.89	47.31	10.19	236.40	0.10	179.02	47.17	10.20	236.38	0.69
1	0	2	0	3	199.41	50.75	12.16	262.33	0.11	199.46	50.71	12.17	262.33	0.64
1	0	2	0	4	115.69	40.45	8.63	164.77	0.12	115.69	40.45	8.63	164.77	0.64
1	0	2	0	5	161.36	48.68	9.27	219.30	0.08	161.36	48.68	9.27	219.31	0.54
1	0	2	0	6	157.61	50.54	11.49	219.64	0.10	157.85	50.28	11.49	219.63	0.59
1	0	2	0	7	161.98	41.42	10.01	213.41	0.09	161.98	41.42	10.01	213.41	0.63
1	0	2	0	8	143.34	44.46	9.12	196.92	0.11	143.34	44.46	9.12	196.92	0.71
1	0	2	0	9	210.44	50.40	13.29	274.12	0.11	210.44	50.40	13.29	274.12	0.59
1	0	2	0	10	158.38	37.91	10.07	206.36	0.09	158.38	37.91	10.07	206.36	0.56
1	0	2	0	11	145.75	42.62	9.38	197.74	0.10	145.85	42.51	9.38	197.74	0.58
1	0	2	0	12	166.26	46.72	9.94	222.92	0.10	166.26	46.72	9.94	222.92	0.54
1	0	2	0	13	161.21	48.75	11.93	221.88	0.11	161.21	48.75	11.93	221.88	0.68
1	0	2	0	14	155.28	38.73	9.94	203.94	0.11	155.28	38.73	9.94	203.95	0.59
1	0	2	0	15	169.89	45.93	10.00	225.82	0.10	169.89	45.93	10.00	225.82	0.57
1	0	2	0	16	146.89	44.39	10.36	201.63	0.10	146.89	44.39	10.36	201.64	0.57
1	0	2	0	17	123.24	43.66	11.59	178.49	0.11	123.25	43.65	11.59	178.49	0.61
1	0	2	0	18	137.15	36.42	7.43	181.01	0.09	137.15	36.42	7.44	181.01	0.63
1	0	2	0	19	139.50	39.77	8.88	188.15	0.11	139.63	39.62	8.88	188.13	0.63
1	0	2	0	20	175.25	43.39	9.49	228.14	0.12	175.25	43.39	9.49	228.14	0.56
1	0	2	0	21	162.12	46.67	9.91	218.70	0.09	162.12	46.67	9.91	218.70	0.62
1	0	2	0	22	148.72	46.56	10.24	205.52	0.11	148.72	46.56	10.24	205.52	0.59
1	0	2	0	23	136.37	45.13	10.77	192.27	0.12	136.40	45.10	10.77	192.27	0.58
1	0	2	0	24	150.33	46.83	9.91	207.06	0.11	150.33	46.83	9.91	207.06	0.62
1	0	2	0	25	153.48	34.41	10.00	197.89	0.11	153.48	34.41	10.00	197.89	0.63
1	0	2	1	1	146.28	50.43	9.40	206.11	0.13	146.45	50.25	9.41	206.11	0.71
1	0	2	1	2	172.71	47.34	10.19	230.24	0.09	172.83	47.21	10.19	230.23	0.59
1	0	2	1	3	204.07	50.75	12.16	266.99	0.10	204.13	50.69	12.17	266.99	0.47
1	0	2	1	4	123.78	40.45	8.63	172.86	0.11	123.81	40.42	8.63	172.85	0.57
1	0	2	1	5	164.44	48.68	9.27	222.39	0.10	164.44	48.68	9.27	222.39	0.61
1	0	2	1	6	147.04	50.54	11.49	209.07	0.12	147.32	50.24	11.50	209.05	0.64
1	0	2	1	7	163.57	41.42	10.01	215.00	0.10	163.59	41.40	10.01	215.00	0.68
1	0	2	1	8	146.79	44.46	9.12	200.37	0.11	146.79	44.46	9.12	200.37	0.64
1	0	2	1	9	211.72	50.40	13.29	275.40	0.09	211.75	50.37	13.29	275.40	0.64
1	0	2	1	10	161.14	37.91	10.07	209.11	0.10	161.14	37.91	10.07	209.11	0.54
1	0	2	1	11	139.36	42.62	9.38	191.36	0.09	139.45	42.53	9.38	191.36	0.57
1	0	2	1	12	165.88	46.72	9.94	222.54	0.10	165.88	46.72	9.94	222.54	0.58
1	0	2	1	13	159.31	48.75	11.93	219.98	0.11	159.33	48.72	11.93	219.98	0.57
1	0	2	1	14	159.92	38.73	9.94	208.58	0.11	159.92	38.73	9.94	208.58	0.57
1	0	2	1	15	172.39	45.93	10.00	228.31	0.12	172.43	45.88	10.00	228.31	0.58
1	0	2	1	16	142.76	44.39	10.36	197.51	0.10	142.80	44.35	10.36	197.51	0.54
1	0	2	1	17	129.09	43.66	11.59	184.34	0.11	129.16	43.59	11.59	184.34	0.69
1	0	2	1	18	134.06	36.42	7.43	177.91	0.10	134.06	36.42	7.44	177.91	0.50
1	0	2	1	19	137.45	39.77	8.88	186.10	0.09	137.55	39.66	8.88	186.09	0.61
1	0	2	1	20	190.22	43.39	9.49	243.10	0.11	190.24	43.37	9.49	243.10	0.64
1	0	2	1	21	159.11	46.67	9.91	215.69	0.10	159.11	46.67	9.91	215.69	0.60
1	0	2	1	22	150.93	46.56	10.24	207.74	0.12	150.96	46.53	10.24	207.74	0.68
1	0	2	1	23	146.51	45.13	10.77	202.41	0.10	146.57	45.07	10.77	202.41	0.58
1	0	2	1	24	148.03	46.83	9.91	204.76	0.11	148.03	46.83	9.91	204.76	0.55
1	0	2	1	25	167.78	34.41	10.00	212.19	0.11	167.78	34.41	10.00	212.19	0.57
1	0	2	2	1	140.41	50.43	9.40	200.24	0.12	140.64	50.18	9.41	200.23	0.67
1	0	2	2	2	185.34	47.32	10.19	242.85	0.11	185.48	47.16	10.20	242.84	0.62
1	0	2	2	3	190.02	50.75	12.16	252.94	0.11	190.16	50.60	12.17	252.93	0.66
1	0	2	2	4	125.95	40.45	8.63	175.03	0.11	126.06	40.33	8.63	175.02	0.60
1	0	2	2	5	159.19	48.68	9.27	217.14	0.10	159.26	48.61	9.27	217.14	0.61
1	0	2	2	6	156.72	50.54	11.49	218.75	0.11	157.03	50.20	11.50	218.72	0.64
1	0	2	2	7	154.47	41.42	10.01	205.90	0.13	154.54	41.35	10.01	205.90	0.56
1	0	2	2	8	153.95	44.46	9.12	207.52	0.13	153.95	44.46	9.12	207.52	0.59
1	0	2	2	9	225.92	50.40	13.29	289.60	0.11	226.06	50.24	13.29	289.60	0.67
1	0	2	2	10	135.33	37.91	10.07	183.30	0.10	135.33	37.91	10.07	183.30	0.56
1	0	2	2	11	137.80	42.62	9.38	189.80	0.12	137.94	42.46	9.38	189.79	0.58
1	0	2	2	12	169.74	46.72	9.94	226.40	0.12	169.74	46.72	9.94	226.40	0.63
1	0	2	2	13	155.27	48.75	11.93	215.95	0.10	155.34	48.67	11.93	215.95	0.54
1	0	2	2	14	145.41	38.73	9.94	194.08	0.11	145.45	38.69	9.94	194.08	0.57
1	0	2	2	15	175.59	45.93	10.00	231.52	0.10	175.68	45.83	10.00	231.52	0.69
1	0	2	2	16	134.49	44.39	10.36	189.24	0.11	134.53	44.35	10.36	189.24	0.55
1	0	2	2	17	129.43	43.66	11.59	184.68	0.12	129.53	43.56	11.59	184.68	0.70
1	0	2	2	18	133.15	36.42	7.43	177.01	0.12	133.17	36.40	7.44	177.01	0.57
1	0	2	2	19	143.44	39.77	8.88	192.09	0.12	143.53	39.67	8.88	192.08	0.60
1	0	2	2	20	179.85	43.39	9.49	232.74	0.12	179.96	43.29	9.50	232.74	0.62
1	0	2	2	21	153.62	46.67	9.91	210.21	0.12	153.67	46.62	9.91	210.20	0.65
1	0	2	2	22	151.59	46.56	10.24	208.39	0.13	151.64	46.50	10.25	208.39	0.60
1	0	2	2	23	143.89	45.13	10.77	199.79	0.10	143.97	45.04	10.77	199.78	0.62
1	0	2	2	24	140.53	46.83	9.91	197.27	0.12	140.57	46.78	9.91	197.27	0.64
1	0	2	2	25	170.41	34.41	10.00	214.82	0.11	170.45	34.37	10.00	214.82	0.61
1	1	0	0	1	47.78	135.34	11.82	194.93	0.16	58.00	120.43	12.38	190.81	1.11
1	1	0	0	2	50.22	128.84	13.00	192.05	0.32	46.84	115.39	12.82	175.05	1.21
1	1	0	0	3	47.37	141.36	15.31	204.04	0.32	49.69	128.90	15.45	194.04	1.12
1	1	0	0	4	47.64	109.90	12.44	169.98	0.35	40.85	95.84	12.03	148.71	1.12
1	1	0	0	5	36.92	136.36	11.64	184.92	0.23	49.52	117.97	12.30	179.79	1.17
1	1	0	0	6	52.33	130.38	15.51	198.21	0.39	43.24	120.94	15.01	179.18	1.21
1	1	0	0	7	38.31	116.86	12.79	167.97	0.30	42.74	106.13	13.06	161.93	1.12
1	1	0	0	8	29.17	127.29	11.28	167.74	0.24	41.41	108.39	11.96	161.76	1.07
1	1	0	0	9	43.59	138.58	16.10	198.27	0.23	52.31	120.92	16.63	189.86	1.28
1	1	0	0	10	30.17	107.96	12.65	150.78	0.24	31.30	99.78	12.72	143.80	1.07
1	1	0	0	11	45.40	119.93	12.34	177.67	0.24	46.02	109.65	12.38	168.05	1.26
1	1	0	0	12	32.07	143.13	12.28	187.47	0.24	44.60	125.61	12.92	183.14	1.15
1	1	0	0	13	60.63	141.68	16.44	218.74	0.42	46.81	128.81	15.66	191.28	1.27
1	1	0	0	14	28.93	114.70	11.85	155.48	0.20	38.14	101.55	12.38	152.06	1.06
1	1	0	0	15	33.62	131.68	12.10	177.41	0.18	42.84	119.27	12.55	174.66	1.06
1	1	0	0	16	44.02	106.38	13.60	163.99	0.49	44.29	105.34	13.62	163.24	1.06
1	1	0	0	17	64.86	99.11	17.34	181.31	0.58	43.25	96.47	15.68	155.40	1.34
1	1	0	0	18	35.82	97.13	9.40	142.35	0.25	38.51	87.35	9.54	135.40	1.11
1	1	0	0	19	38.25	108.62	11.88	158.74	0.33	38.39	94.15	11.88	144.43	1.07
1	1	0	0	20	40.95	127.42	10.99	179.36	0.19	51.64	110.43	11.55	173.62	1.15
1	1	0	0	21	41.92	131.08	12.72	185.72	0.27	48.29	115.23	13.06	176.58	1.21
1	1	0	0	22	45.41	123.08	12.94	181.43	0.32	43.93	114.87	12.85	171.66	1.15
1	1	0	0	23	51.65	113.26	15.02	179.92	0.44	35.11	105.21	13.95	154.26	1.18
1	1	0	0	24	38.69	126.10	12.81	177.60	0.28	42.32	112.50	13.01	167.83	1.08
1	1	0	0	25	40.90	91.84	13.37	146.11	0.44	31.06	92.08	12.79	135.93	1.09

					DP-based Heuristic					Math. Model				
N	C_t	TF	RDD	$r\#$	Tard	Tool	Mach	Total	CPU	Tard	Tool	Mach	Total	CPU
1	1	0	1	1	44.39	133.52	11.93	189.84	0.29	53.39	119.49	12.43	185.30	1.24
1	1	0	1	2	43.79	132.47	12.60	188.87	0.27	46.82	116.11	12.78	175.71	1.38
1	1	0	1	3	68.77	137.99	16.28	223.04	0.35	55.50	127.53	15.52	198.56	1.22
1	1	0	1	4	41.22	110.24	11.89	163.36	0.31	42.81	96.40	12.00	151.21	1.27
1	1	0	1	5	88.48	120.45	13.81	222.74	0.38	50.58	117.15	12.36	180.08	1.34
1	1	0	1	6	51.91	159.19	15.72	226.81	0.37	42.92	117.21	15.24	175.37	1.48
1	1	0	1	7	41.89	117.53	13.06	172.48	0.31	41.08	106.68	13.04	160.81	1.25
1	1	0	1	8	33.42	128.16	11.24	172.82	0.21	47.56	107.33	12.01	166.90	1.32
1	1	0	1	9	56.97	121.94	16.92	195.83	0.39	50.63	122.06	16.57	189.25	1.29
1	1	0	1	10	46.84	106.14	13.27	166.24	0.29	40.46	96.44	12.92	149.83	1.10
1	1	0	1	11	45.62	119.41	12.52	177.55	0.27	46.31	106.04	12.56	164.91	1.27
1	1	0	1	12	53.61	150.93	13.31	217.85	0.37	51.35	120.08	13.22	184.65	1.19
1	1	0	1	13	81.46	138.56	17.69	237.72	0.69	47.58	123.03	16.02	186.63	1.39
1	1	0	1	14	52.29	107.42	12.79	172.50	0.30	44.93	101.08	12.41	158.41	1.12
1	1	0	1	15	61.09	144.92	13.15	219.16	0.27	52.96	115.44	12.74	181.15	1.20
1	1	0	1	16	45.37	115.15	13.96	174.49	0.44	41.05	104.45	13.68	159.18	1.29
1	1	0	1	17	78.03	110.25	17.86	206.14	0.60	44.29	94.96	15.81	155.05	1.42
1	1	0	1	18	35.58	97.18	9.39	142.15	0.24	37.96	87.82	9.53	135.31	1.16
1	1	0	1	19	61.41	97.57	13.17	172.15	0.50	40.31	90.39	12.10	142.79	1.26
1	1	0	1	20	41.22	128.20	10.89	180.31	0.21	51.92	112.44	11.46	175.82	1.26
1	1	0	1	21	54.36	127.44	13.37	195.17	0.33	47.44	115.99	13.03	176.47	1.28
1	1	0	1	22	53.08	118.03	13.44	184.55	0.43	46.05	111.20	13.09	170.34	1.28
1	1	0	1	23	52.58	134.33	14.77	201.69	0.33	38.20	106.09	13.89	158.18	1.37
1	1	0	1	24	35.89	126.45	12.72	175.06	0.32	42.76	110.50	13.12	166.38	1.20
1	1	0	1	25	47.28	94.31	12.94	154.53	0.31	42.53	94.08	12.67	149.28	1.20
1	1	0	2	1	54.05	130.26	12.36	196.67	0.26	55.46	119.27	12.44	187.17	1.24
1	1	0	2	2	73.85	128.18	13.06	215.10	0.31	68.13	115.87	12.80	196.79	1.35
1	1	0	2	3	94.68	158.49	17.03	270.20	0.56	59.08	128.22	15.53	202.83	1.37
1	1	0	2	4	60.33	135.11	11.93	207.38	0.35	57.30	98.95	11.87	168.12	1.35
1	1	0	2	5	97.62	133.00	13.71	244.33	0.51	61.14	119.29	12.27	192.70	1.34
1	1	0	2	6	80.02	153.63	16.26	249.91	0.50	59.12	114.94	15.41	189.47	1.42
1	1	0	2	7	55.83	115.02	13.59	184.45	0.33	51.08	101.08	13.38	165.53	1.33
1	1	0	2	8	71.25	143.15	12.36	226.76	0.40	60.69	106.05	12.10	178.83	1.31
1	1	0	2	9	80.68	133.03	17.03	230.74	0.39	74.40	119.69	16.71	210.80	1.39
1	1	0	2	10	40.22	108.90	13.16	162.27	0.56	37.95	94.72	13.03	145.69	1.17
1	1	0	2	11	59.30	116.10	13.01	188.41	0.33	49.33	107.53	12.49	169.34	1.29
1	1	0	2	12	64.90	158.07	13.02	236.00	0.32	65.53	122.46	13.11	201.10	1.32
1	1	0	2	13	70.66	147.73	17.05	235.45	0.52	43.50	128.06	15.73	187.28	1.32
1	1	0	2	14	62.27	121.69	13.29	197.25	0.47	45.28	100.35	12.46	158.09	1.22
1	1	0	2	15	46.86	159.57	11.95	218.38	0.27	58.10	117.90	12.65	188.64	1.34
1	1	0	2	16	82.52	128.30	15.46	226.28	0.60	55.56	99.77	14.02	169.34	1.31
1	1	0	2	17	79.00	119.98	17.39	216.37	0.77	48.74	97.79	15.62	162.14	1.44
1	1	0	2	18	40.09	99.41	9.15	148.65	0.26	47.77	86.79	9.59	144.15	1.06
1	1	0	2	19	57.74	113.47	12.66	183.87	0.82	41.81	93.00	11.99	146.79	1.28
1	1	0	2	20	80.22	117.95	12.22	210.40	0.31	65.26	110.04	11.59	186.88	1.35
1	1	0	2	21	89.14	118.68	14.63	222.45	0.50	53.61	114.23	13.14	180.98	1.35
1	1	0	2	22	89.29	110.96	14.63	214.89	0.52	53.69	114.87	12.87	181.43	1.25
1	1	0	2	23	82.13	131.41	15.78	229.32	0.63	41.10	108.21	13.79	163.10	1.46
1	1	0	2	24	62.13	118.08	13.81	194.02	0.43	52.79	106.92	13.34	173.05	1.33
1	1	0	2	25	72.59	93.05	13.71	179.35	0.53	48.22	96.19	12.57	156.98	1.21
1	1	1	0	1	101.89	135.56	11.81	249.25	0.16	107.40	128.19	12.03	247.63	1.04
1	1	1	0	2	98.05	137.53	11.95	247.52	0.14	102.88	131.61	12.11	246.60	1.00
1	1	1	0	3	109.03	150.89	14.55	274.47	0.14	113.22	145.79	14.70	273.71	1.04
1	1	1	0	4	69.51	117.50	11.12	198.13	0.14	76.75	107.71	11.44	195.90	1.00
1	1	1	0	5	96.09	137.04	11.61	244.75	0.15	101.13	130.90	11.78	243.81	1.14
1	1	1	0	6	92.47	143.72	14.08	250.28	0.13	99.92	134.07	14.39	248.37	1.10
1	1	1	0	7	90.50	123.33	12.33	226.16	0.14	93.48	119.74	12.44	225.66	0.93
1	1	1	0	8	86.59	128.16	11.24	225.99	0.14	93.60	119.26	11.50	224.36	1.01
1	1	1	0	9	110.18	142.83	15.73	268.73	0.17	116.93	134.22	16.00	267.15	1.03
1	1	1	0	10	82.12	113.01	12.16	207.29	0.14	84.77	109.84	12.26	206.87	0.85
1	1	1	0	11	91.83	125.75	11.77	229.34	0.15	96.52	120.00	11.94	228.46	1.05
1	1	1	0	12	94.41	143.67	12.26	250.34	0.14	98.75	138.43	12.40	249.58	0.94
1	1	1	0	13	89.50	156.35	14.52	260.36	0.17	99.83	142.14	14.99	256.95	1.11
1	1	1	0	14	89.05	115.48	11.82	216.35	0.12	91.71	112.37	11.91	216.00	0.97
1	1	1	0	15	101.23	131.68	12.10	245.01	0.14	105.37	126.34	12.26	243.96	0.93
1	1	1	0	16	81.52	128.39	12.59	222.51	0.16	87.36	121.14	12.82	221.32	1.04
1	1	1	0	17	80.01	113.78	14.94	208.74	0.30	77.48	109.92	14.80	202.20	1.18
1	1	1	0	18	79.75	101.57	9.02	190.34	0.14	82.83	97.85	9.12	189.79	0.86
1	1	1	0	19	73.94	116.49	11.04	201.48	0.17	81.68	106.49	11.33	199.50	0.92
1	1	1	0	20	100.99	128.20	10.89	240.08	0.13	105.90	122.11	11.06	239.07	0.94
1	1	1	0	21	91.94	136.37	12.25	240.56	0.16	98.91	127.62	12.52	239.05	1.00
1	1	1	0	22	85.00	134.43	12.00	231.43	0.15	89.88	128.18	12.20	230.26	1.04
1	1	1	0	23	71.88	130.00	12.86	214.74	0.18	79.99	118.55	13.24	211.78	1.17
1	1	1	0	24	90.00	130.47	12.32	232.79	0.14	94.94	124.53	12.50	231.97	0.98
1	1	1	0	25	72.24	109.19	12.06	193.48	0.13	75.35	105.24	12.17	192.77	0.91
1	1	1	1	1	99.33	135.59	11.81	246.73	0.13	106.05	126.91	12.09	245.05	1.07
1	1	1	1	2	97.02	137.58	11.94	246.55	0.14	101.99	131.70	12.11	245.80	1.01
1	1	1	1	3	107.54	150.89	14.55	272.98	0.14	113.40	143.79	14.78	271.97	1.01
1	1	1	1	4	73.63	117.62	11.12	202.36	0.17	79.95	109.80	11.36	201.11	1.10
1	1	1	1	5	101.90	137.18	11.60	250.68	0.15	106.05	132.39	11.74	250.17	1.06
1	1	1	1	6	83.99	143.61	14.09	241.68	0.16	91.48	133.43	14.41	239.33	1.19
1	1	1	1	7	91.48	123.33	12.33	227.15	0.14	94.23	120.05	12.43	226.71	0.97
1	1	1	1	8	91.04	128.16	11.24	230.44	0.15	97.97	119.43	11.50	228.90	1.04
1	1	1	1	9	109.48	142.74	15.73	267.96	0.15	116.90	132.82	16.06	265.78	1.09
1	1	1	1	10	85.33	113.01	12.16	210.50	0.14	88.56	109.23	12.28	210.07	0.91
1	1	1	1	11	85.28	125.75	11.77	222.79	0.14	90.90	118.66	11.99	221.55	1.02
1	1	1	1	12	97.13	143.67	12.26	253.05	0.12	101.90	137.99	12.42	252.31	1.01
1	1	1	1	13	97.87	165.24	14.95	278.06	0.25	95.99	143.62	14.94	254.54	1.12
1	1	1	1	14	94.73	115.30	11.82	221.86	0.13	98.95	110.22	11.99	221.16	1.06
1	1	1	1	15	105.04	131.68	12.10	248.82	0.13	108.93	126.88	12.24	248.04	1.00
1	1	1	1	16	89.03	124.91	13.06	227.01	0.19	86.37	117.56	12.98	216.90	1.14
1	1	1	1	17	80.86	135.43	14.63	230.93	0.23	81.09	111.77	14.73	207.59	1.13
1	1	1	1	18	81.22	101.57	9.02	191.81	0.13	84.49	97.66	9.12	191.28	1.00
1	1	1	1	19	71.37	116.49	11.04	198.90	0.16	78.44	107.77	11.29	197.50	1.10
1	1	1	1	20	109.31	128.20	10.89	248.40	0.13	114.55	121.88	11.07	247.50	1.04
1	1	1	1	21	93.03	136.37	12.25	241.65	0.15	99.46	128.19	12.50	240.15	1.04
1	1	1	1	22	88.42	134.44	12.00	234.86	0.16	93.34	128.18	12.20	233.71	0.97
1	1	1	1	23	77.79	130.07	12.86	220.72	0.15	87.18	117.60	13.28	218.06	1.14
1	1	1	1	24	88.16	130.39	12.32	230.88	0.14	94.92	122.10	12.59	229.60	1.04
1	1	1	1	25	94.67	109.24	12.06	215.97	0.14	98.28	104.87	12.19	215.34	0.95

					DP-based Heuristic					Math. Model				
N	C_t	TF	RDD	$r\#$	Tard	Tool	Mach	Total	CPU	Tard	Tool	Mach	Total	CPU
1	1	1	2	1	99.67	135.55	11.81	247.03	0.15	106.05	127.23	12.08	245.36	1.11
1	1	1	2	2	120.54	137.55	11.94	270.03	0.15	126.20	130.81	12.14	269.15	1.07
1	1	1	2	3	116.06	150.91	14.55	281.52	0.16	125.05	139.95	14.93	279.94	1.12
1	1	1	2	4	93.08	117.62	11.12	221.81	0.16	100.43	108.49	11.42	220.33	1.12
1	1	1	2	5	113.53	137.19	11.60	262.32	0.14	119.04	130.73	11.79	261.56	1.05
1	1	1	2	6	98.29	143.71	14.08	256.08	0.17	106.98	132.50	14.47	253.95	1.08
1	1	1	2	7	90.89	123.33	12.33	226.56	0.16	96.46	116.67	12.57	225.70	1.09
1	1	1	2	8	102.71	128.16	11.24	242.11	0.15	110.13	118.92	11.52	240.57	1.05
1	1	1	2	9	139.01	142.99	15.72	297.73	0.15	145.49	135.05	15.98	296.52	1.19
1	1	1	2	10	78.06	113.01	12.16	203.23	0.13	83.83	105.61	12.43	201.87	1.00
1	1	1	2	11	91.74	125.75	11.77	229.25	0.12	97.91	118.30	12.00	228.21	1.08
1	1	1	2	12	111.62	143.67	12.26	267.54	0.14	118.58	135.08	12.53	266.18	1.08
1	1	1	2	13	90.02	156.35	14.52	260.89	0.15	98.14	146.04	14.84	259.01	1.10
1	1	1	2	14	90.92	115.13	11.83	217.88	0.16	95.84	108.90	12.04	216.79	1.12
1	1	1	2	15	113.74	131.68	12.10	257.52	0.14	117.83	126.66	12.25	256.73	1.00
1	1	1	2	16	92.32	125.11	12.94	230.37	0.22	93.78	116.17	13.04	222.99	1.05
1	1	1	2	17	96.83	112.57	14.98	224.38	0.40	90.55	112.42	14.70	217.67	1.26
1	1	1	2	18	88.71	101.57	9.02	199.29	0.13	93.32	96.06	9.18	198.56	0.95
1	1	1	2	19	109.11	115.96	11.91	236.99	0.31	88.30	107.25	11.32	206.87	1.01
1	1	1	2	20	121.16	128.20	10.89	260.25	0.15	127.09	121.09	11.10	259.28	1.12
1	1	1	2	21	100.16	136.37	12.25	248.78	0.15	108.59	126.12	12.59	247.29	1.09
1	1	1	2	22	105.82	131.04	12.36	249.21	0.24	105.29	124.55	12.36	242.20	1.13
1	1	1	2	23	82.13	158.92	12.60	253.65	0.23	93.99	118.82	13.23	226.04	1.19
1	1	1	2	24	90.55	130.35	12.32	233.22	0.15	97.09	122.24	12.58	231.92	1.05
1	1	1	2	25	102.93	109.27	12.05	224.26	0.15	106.82	104.62	12.20	223.64	1.05
1	1	2	0	1	180.92	135.89	11.80	328.61	0.12	183.13	133.34	11.86	328.33	0.88
1	1	2	0	2	203.81	137.87	11.94	353.62	0.12	204.51	137.13	11.95	353.59	0.82
1	1	2	0	3	226.30	151.46	14.53	392.29	0.11	226.67	151.05	14.54	392.27	0.81
1	1	2	0	4	143.22	118.03	11.11	272.36	0.11	143.89	117.30	11.12	272.32	0.81
1	1	2	0	5	193.07	137.81	11.59	342.47	0.10	193.42	137.45	11.60	342.46	0.75
1	1	2	0	6	189.46	144.00	14.07	347.54	0.12	190.57	142.80	14.10	347.47	0.91
1	1	2	0	7	185.77	123.13	12.34	321.23	0.12	185.81	123.07	12.34	321.21	0.80
1	1	2	0	8	168.24	128.16	11.24	307.64	0.11	170.27	125.89	11.30	307.46	0.81
1	1	2	0	9	236.14	143.57	15.71	395.42	0.10	236.73	142.93	15.72	395.38	0.83
1	1	2	0	10	177.72	113.01	12.16	302.89	0.11	177.98	112.73	12.17	302.88	0.69
1	1	2	0	11	172.68	125.75	11.77	310.19	0.12	173.58	124.76	11.79	310.14	0.89
1	1	2	0	12	192.96	143.67	12.26	348.88	0.09	193.35	143.25	12.27	348.87	0.86
1	1	2	0	13	187.70	156.35	14.52	358.57	0.11	190.02	153.77	14.58	358.36	0.85
1	1	2	0	14	177.18	116.07	11.80	305.04	0.10	177.36	115.87	11.81	305.04	0.76
1	1	2	0	15	196.23	131.68	12.10	340.01	0.10	196.90	130.95	12.12	339.96	0.75
1	1	2	0	16	172.15	128.39	12.59	313.14	0.10	172.63	127.88	12.60	313.11	0.81
1	1	2	0	17	146.89	124.51	14.23	285.63	0.12	149.91	120.90	14.34	285.15	0.87
1	1	2	0	18	159.55	101.57	9.02	270.14	0.11	160.10	100.98	9.03	270.10	0.87
1	1	2	0	19	160.51	116.49	11.04	288.04	0.11	161.67	115.23	11.07	287.97	0.84
1	1	2	0	20	200.14	128.20	10.89	339.23	0.11	200.27	128.06	10.89	339.23	0.74
1	1	2	0	21	189.17	136.37	12.25	337.79	0.10	189.64	135.88	12.26	337.77	0.76
1	1	2	0	22	173.17	134.90	11.98	320.04	0.12	174.33	133.57	12.01	319.92	0.83
1	1	2	0	23	163.85	130.58	12.84	307.28	0.11	165.50	128.76	12.89	307.14	0.85
1	1	2	0	24	179.37	130.88	12.31	322.56	0.11	180.28	129.91	12.33	322.52	0.76
1	1	2	0	25	173.41	109.68	12.04	295.14	0.11	173.57	109.51	12.05	295.13	0.67
1	1	2	1	1	179.33	135.97	11.79	327.10	0.12	181.30	133.75	11.85	326.90	0.83
1	1	2	1	2	197.60	137.88	11.94	347.41	0.10	198.75	136.64	11.97	347.35	0.84
1	1	2	1	3	230.83	151.46	14.53	396.82	0.10	231.45	150.80	14.55	396.80	0.90
1	1	2	1	4	151.05	118.04	11.11	280.19	0.11	152.14	116.87	11.14	280.14	0.83
1	1	2	1	5	195.97	137.70	11.59	345.26	0.10	196.91	136.69	11.61	345.22	0.81
1	1	2	1	6	178.53	144.00	14.07	336.61	0.11	180.49	141.82	14.13	336.44	0.92
1	1	2	1	7	187.09	123.33	12.33	322.76	0.11	187.50	122.90	12.34	322.74	0.84
1	1	2	1	8	171.24	128.16	11.24	310.64	0.11	174.22	124.76	11.33	310.30	0.87
1	1	2	1	9	237.21	143.58	15.71	396.50	0.13	238.10	142.62	15.73	396.45	0.80
1	1	2	1	10	180.44	113.01	12.16	305.61	0.10	180.82	112.60	12.17	305.59	0.75
1	1	2	1	11	166.42	125.75	11.77	303.93	0.12	167.14	124.99	11.79	303.91	0.81
1	1	2	1	12	192.45	143.67	12.26	348.37	0.10	193.25	142.81	12.28	348.33	0.77
1	1	2	1	13	185.80	156.35	14.52	356.67	0.11	188.02	153.88	14.58	356.48	0.81
1	1	2	1	14	181.56	116.10	11.80	309.46	0.12	182.15	115.47	11.82	309.44	0.70
1	1	2	1	15	198.31	131.68	12.10	342.09	0.09	200.14	129.62	12.15	341.91	0.86
1	1	2	1	16	167.70	128.39	12.59	308.69	0.11	169.01	126.97	12.63	308.61	0.85
1	1	2	1	17	152.52	124.51	14.23	291.26	0.13	156.19	120.17	14.36	290.73	0.91
1	1	2	1	18	156.41	101.57	9.02	267.00	0.09	156.97	100.97	9.03	266.97	0.73
1	1	2	1	19	158.51	116.49	11.04	286.05	0.10	159.52	115.43	11.07	286.01	0.81
1	1	2	1	20	214.79	128.20	10.89	353.89	0.11	215.76	127.17	10.91	353.85	0.82
1	1	2	1	21	186.00	136.37	12.25	334.62	0.11	186.91	135.38	12.27	334.56	0.73
1	1	2	1	22	174.97	134.90	11.98	321.85	0.12	176.96	132.64	12.04	321.64	0.75
1	1	2	1	23	173.94	130.58	12.84	317.36	0.14	175.75	128.59	12.89	317.23	0.80
1	1	2	1	24	176.89	130.95	12.31	320.15	0.10	177.86	129.92	12.33	320.11	0.83
1	1	2	1	25	187.60	109.59	12.05	309.24	0.11	188.15	109.00	12.06	309.20	0.79
1	1	2	2	1	173.37	135.92	11.80	321.08	0.12	175.79	133.15	11.87	320.81	0.93
1	1	2	2	2	209.95	137.90	11.94	359.78	0.11	211.26	136.49	11.97	359.72	0.82
1	1	2	2	3	216.25	151.39	14.54	382.18	0.12	217.85	149.61	14.58	382.04	0.89
1	1	2	2	4	151.92	117.90	11.11	280.93	0.14	156.11	113.13	11.25	280.49	1.05
1	1	2	2	5	190.03	137.63	11.59	339.26	0.10	192.16	135.31	11.65	339.12	0.91
1	1	2	2	6	187.85	144.00	14.07	345.93	0.11	189.85	141.73	14.13	345.72	0.98
1	1	2	2	7	177.73	123.33	12.33	313.39	0.12	178.75	122.20	12.36	313.31	0.82
1	1	2	2	8	178.41	128.16	11.24	317.81	0.12	181.59	124.54	11.34	317.46	0.96
1	1	2	2	9	250.95	143.50	15.71	410.16	0.12	252.53	141.76	15.75	410.05	0.91
1	1	2	2	10	154.35	113.01	12.16	279.52	0.10	155.70	111.53	12.20	279.43	0.80
1	1	2	2	11	164.58	125.75	11.77	302.09	0.11	165.94	124.27	11.80	302.01	0.90
1	1	2	2	12	196.02	143.67	12.26	351.95	0.11	197.51	142.03	12.30	351.84	0.83
1	1	2	2	13	181.10	156.35	14.52	351.97	0.12	184.99	151.96	14.64	351.59	0.89
1	1	2	2	14	167.13	115.86	11.81	294.80	0.11	167.90	115.03	11.83	294.76	0.84
1	1	2	2	15	201.37	131.68	12.10	345.15	0.13	203.38	129.44	12.16	344.97	0.76
1	1	2	2	16	159.19	128.39	12.59	300.17	0.11	161.07	126.34	12.65	300.06	0.85
1	1	2	2	17	152.96	124.51	14.23	291.71	0.12	156.09	120.90	14.34	291.33	0.98
1	1	2	2	18	155.23	101.57	9.02	265.82	0.13	156.28	100.43	9.04	265.75	0.77
1	1	2	2	19	164.56	116.49	11.04	292.09	0.12	165.48	115.50	11.06	292.05	0.80
1	1	2	2	20	203.90	128.20	10.89	342.99	0.11	205.44	126.54	10.93	342.92	0.81
1	1	2	2	21	180.08	136.37	12.25	328.70	0.13	181.92	134.32	12.30	328.55	0.84
1	1	2	2	22	175.43	134.90	11.98	322.31	0.12	177.47	132.55	12.04	322.06	0.84
1	1	2	2	23	170.95	130.58	12.84	314.37	0.13	173.75	127.44	12.93	314.12	1.02
1	1	2	2	24	169.18	130.88	12.31	312.37	0.11	171.02	128.89	12.36	312.28	0.86
1	1	2	2	25	190.05	109.63	12.05	311.72	0.11	191.01	108.60	12.07	311.68	0.76

Appendix C

Results for The Original Problem

C.1 Cost Values

N	C_t	TF	RDD	$r\#$	Sequential				PSGA[DP-based Algo.]				PSGA[Math. Model]			
					Tard	Tool	Mach	Total	Tard	Tool	Mach	Total	Tard	Tool	Mach	Total
0	0	0	0	1	10.16	7.62	11.4	29.18	2.79	13.91	7.82	24.52	2.52	14.24	7.72	24.49
0	0	0	0	2	39.01	6.84	10.07	55.92	2.13	13.64	6.55	22.32	2.14	13.54	6.58	22.25
0	0	0	0	3	38.97	8.14	11.99	59.1	4.11	16.22	7.93	28.25	4.2	15.98	7.98	28.16
0	0	0	0	4	30.98	7.08	10.6	48.66	3.05	17.33	6.23	26.61	3.1	17.36	6.21	26.66
0	0	0	0	5	36.63	6.62	9.62	52.87	3.39	13.67	6.05	23.11	3.29	13.67	6.06	23.03
0	0	0	1	1	7.37	7.62	11.4	26.39	1.34	14.05	7.78	23.16	1.24	13.66	7.89	22.79
0	0	0	1	2	14.07	6.84	10.07	30.97	1.92	12.88	6.75	21.55	1.77	12.73	6.8	21.3
0	0	0	1	3	24.53	8.14	11.99	44.66	2.42	15.87	7.97	26.26	2.12	16.1	7.88	26.09
0	0	0	1	4	22.78	7.08	10.6	40.46	3.22	16.55	6.34	26.11	2.77	15.78	6.53	25.08
0	0	0	1	5	163.78	6.62	9.62	180.02	2.15	13.26	6.15	21.56	2.36	12.87	6.25	21.48
0	0	0	2	1	43.2	7.62	11.4	62.22	1.08	12.97	8.12	22.18	0.72	12.88	8.15	21.75
0	0	0	2	2	13.62	6.84	10.07	30.53	0.77	13.51	6.48	20.75	0.77	13.5	6.48	20.75
0	0	0	2	3	17.9	8.14	11.99	38.03	1.53	14.96	8.26	24.75	1.14	14.93	8.26	24.33
0	0	0	2	4	21.17	7.08	10.6	38.85	1.97	14.01	6.93	22.9	2.22	13.57	7.04	22.83
0	0	0	2	5	165.99	6.62	9.62	182.23	0.84	15.38	5.62	21.84	0.48	15.12	5.67	21.27
0	0	1	0	1	37.87	7.62	11.4	56.89	6.83	17.43	7.06	31.33	6.79	17.31	7.05	31.15
0	0	1	0	2	65.16	6.84	10.07	82.07	5.92	16.31	6.02	28.26	5.57	16.68	5.97	28.23
0	0	1	0	3	66.01	8.14	11.99	86.14	9.31	20.06	7.25	36.61	9.27	19.56	7.33	36.15
0	0	1	0	4	55	7.08	10.6	72.68	10.47	19.18	5.92	35.58	9.59	19.23	5.97	34.79
0	0	1	0	5	50.28	6.62	9.62	66.52	7.71	15.92	5.63	29.27	7.29	16.47	5.56	29.32
0	0	1	1	1	18.49	7.62	11.4	37.51	5.77	16.3	7.25	29.32	5.11	16.95	7.14	29.19
0	0	1	1	2	27.03	6.84	10.07	43.94	4.69	16.38	6.01	27.09	4.04	16.1	6.05	26.19
0	0	1	1	3	43.9	8.14	11.99	64.02	7.03	19.86	7.22	34.1	6.15	20.03	7.12	33.29
0	0	1	1	4	42.22	7.08	10.6	59.9	8.66	19.56	5.98	34.19	8.36	18.96	6.03	33.35
0	0	1	1	5	187.11	6.62	9.62	203.35	4.57	17.83	5.34	27.74	6.01	15.24	5.73	26.98
0	0	1	2	1	94.59	7.62	11.4	113.61	3.42	16.98	7.14	27.54	3.39	16.32	7.24	26.94
0	0	1	2	2	26.67	6.84	10.07	43.57	5.11	16.78	5.86	27.75	4.35	16.39	5.98	26.72
0	0	1	2	3	34.01	8.14	11.99	54.14	4.74	20.23	7.22	32.19	4.67	19.86	7.23	31.77
0	0	1	2	4	42.55	7.08	10.6	60.23	5.58	18.91	6.1	30.59	5.67	18.32	6.19	30.18
0	0	1	2	5	189.53	6.62	9.62	205.77	6.74	19.37	5.18	31.28	6.4	15.9	5.66	27.96
0	0	2	0	1	84.78	7.62	11.4	103.8	19.81	18.89	6.91	45.6	18.85	19.32	6.88	45.05
0	0	2	0	2	98.42	6.84	10.07	115.33	16.31	18.38	5.77	40.46	17.05	18.19	5.78	41.02
0	0	2	0	3	100.45	8.14	11.99	120.57	24.11	22.22	7.07	53.4	24.04	22.16	7.1	53.29
0	0	2	0	4	92.38	7.08	10.6	110.06	25.84	21.37	5.77	52.99	24.56	21.85	5.76	52.17
0	0	2	0	5	68.87	6.62	9.62	85.11	18.41	18.75	5.37	42.53	18.01	19.18	5.3	42.49
0	0	2	1	1	42.7	7.62	11.4	61.72	18.28	19.36	6.8	44.44	17.23	19.84	6.73	43.8
0	0	2	1	2	45.63	6.84	10.07	62.54	13.88	18.96	5.71	38.55	14.19	19.25	5.68	39.11
0	0	2	1	3	71.66	8.14	11.99	91.79	19.65	22.36	6.9	48.91	22.7	22.06	7	51.75
0	0	2	1	4	68.74	7.08	10.6	86.42	24.39	23.35	5.56	53.3	24.72	21.53	5.79	52.05
0	0	2	1	5	212.18	6.62	9.62	228.42	18.68	18.65	5.26	42.59	17.86	17.96	5.39	41.2
0	0	2	2	1	127.1	7.62	11.4	146.12	17.94	19	6.95	43.89	16.26	19.49	6.81	42.56
0	0	2	2	2	47.69	6.84	10.07	64.59	16.92	18.67	5.73	41.33	16.43	18.56	5.77	40.76
0	0	2	2	3	58.49	8.14	11.99	78.61	20.05	22.45	6.91	49.41	16.87	22.31	6.99	46.17
0	0	2	2	4	71.29	7.08	10.6	88.97	25.6	20.59	5.89	52.09	24.87	20.36	5.95	51.18
0	0	2	2	5	195.46	6.62	9.62	211.7	21.28	18.52	5.35	45.15	20.76	18.35	5.35	44.47
0	1	0	0	1	81.87	13.75	20.06	115.68	11.12	32.73	11.78	55.63	11.68	31.97	11.9	55.55
0	1	0	0	2	129.95	12	17.63	159.58	10.99	30.6	10	51.59	10.6	30.78	9.98	51.35
0	1	0	0	3	134.66	14.62	21.47	170.75	16.24	39.24	11.6	67.08	15.08	38.29	11.75	65.13
0	1	0	0	4	93.6	12.92	18.95	125.47	15.46	39.78	9.59	64.83	15.58	37.86	9.86	63.3
0	1	0	0	5	92.55	12.2	17.31	122.07	12.46	33.59	9.3	55.35	13.53	31.39	9.56	54.47
0	1	0	1	1	41.25	13.75	20.06	75.05	9.07	33.62	11.55	54.24	11	30.37	12.16	53.53
0	1	0	1	2	54.91	12	17.63	84.54	8.56	31.39	9.85	49.8	8.19	31.3	9.86	49.35
0	1	0	1	3	95.13	14.62	21.47	131.22	14.99	40.67	11.15	66.81	12.88	39.76	11.35	63.99
0	1	0	1	4	73.65	12.92	18.95	105.52	16.07	45.47	8.71	70.25	14.67	38.13	9.82	62.62
0	1	0	1	5	315.74	12.2	17.31	345.26	11.15	33.7	9.17	54.02	11.89	31.48	9.48	52.86
0	1	0	2	1	146.72	13.75	20.06	180.53	9.41	29.79	12.3	51.49	8.64	30.25	12.17	51.06
0	1	0	2	2	61.65	12	17.63	91.28	5.43	33.74	9.4	48.58	7.12	31.72	9.62	48.46
0	1	0	2	3	78.34	14.62	21.47	114.44	8.89	40.14	11.41	60.44	9.14	39.21	11.56	59.91
0	1	0	2	4	77.01	12.92	18.95	108.88	11.24	37.5	9.94	58.68	11.08	36.61	10.05	57.74
0	1	0	2	5	309.34	12.2	17.31	338.86	9	41.86	8.12	58.97	12.14	33.56	9.16	54.85
0	1	1	0	1	127.16	13.75	20.06	160.96	19.48	35.95	11.34	66.77	19.6	35.53	11.4	66.53
0	1	1	0	2	162.42	12	17.63	192.05	18.39	34.01	9.58	61.98	18.37	33.98	9.59	61.95
0	1	1	0	3	169.22	14.62	21.47	205.31	23.51	44.83	11	79.34	23.94	44.52	11.02	79.48
0	1	1	0	4	123.85	12.92	18.95	155.72	26.56	43.32	9.29	79.18	27.49	41.98	9.4	78.87
0	1	1	0	5	110.36	12.2	17.31	139.88	20.46	36.58	9.01	66.05	20.43	35.5	9.15	65.08
0	1	1	1	1	66.03	13.75	20.06	99.84	18.15	36.2	11.18	65.53	16.78	36	11.27	64.05
0	1	1	1	2	73.78	12	17.63	103.41	15.19	34.84	9.47	59.5	15.54	33.75	9.58	58.87
0	1	1	1	3	122.51	14.62	21.47	158.6	21.83	48.09	10.36	80.28	21.95	44.95	10.75	77.65
0	1	1	1	4	99.67	12.92	18.95	131.54	26.09	43.61	9.19	78.89	26.22	41.7	9.49	77.41
0	1	1	1	5	340.44	12.2	17.31	369.96	21.33	38.07	8.6	68	18.32	35.27	9.14	62.73
0	1	1	2	1	206.93	13.75	20.06	240.74	13.3	38.06	10.99	62.35	14.49	34.86	11.52	60.87
0	1	1	2	2	83.17	12	17.63	112.8	15.5	39.59	8.86	63.95	14.84	37.72	9.02	61.57
0	1	1	2	3	103.53	14.62	21.47	139.62	19.64	45.12	10.8	75.57	18.72	42.86	11.09	72.67
0	1	1	2	4	104.62	12.92	18.95	136.49	22.26	44.17	9.19	75.63	20.7	45.03	9.24	74.97
0	1	1	2	5	334.32	12.2	17.31	363.84	23.29	40.31	8.28	71.88	21.38	37.11	8.85	67.34
0	1	2	0	1	182.73	13.75	20.06	216.54	34.74	39.37	11.06	85.16	34.57	39.65	11	85.22
0	1	2	0	2	199.04	12	17.63	228.67	31.34	37.52	9.31	78.17	31.84	36.97	9.36	78.17
0	1	2	0	3	207.75	14.62	21.47	243.84	43.74	47.71	10.74	102.19	43.64	47.88	10.76	102.28
0	1	2	0	4	167.62	12.92	18.95	199.49	44.2	45.99	9.19	99.37	43.53	45.2	9.35	98.08
0	1	2	0	5	132.98	12.2	17.31	162.5	35.51	38.36	8.92	82.79	35.94	37.63	8.99	82.55
0	1	2	1	1	100.22	13.75	20.06	134.03	31.97	42.45	10.76	85.18	32.51	40.55	10.94	84
0	1	2	1	2	98.35	12	17.63	127.99	29.52	37.78	9.28	76.58	29.65	37.44	9.3	76.4
0	1	2	1	3	156.03	14.62	21.47	192.12	39.1	48.91	10.58	98.59	39.87	47.84	10.61	98.31
0	1	2	1	4	130.59	12.92	18.95	162.46	47.06	45.86	8.99	101.91	45.19	43.79	9.38	98.35
0	1	2	1	5	367.5	12.2	17.31	397.02	35.43	39.81	8.66	83.9	34.35	38.19	8.81	81.34
0	1	2	2	1	240.03	13.75	20.06	273.84	32.13	42.8	10.59	85.53	32.45	39.69	10.98	83.12
0	1	2	2	2	109.48	12	17.63	139.11	29.96	39.29	9.11	78.37	28.83	38.66	9.16	76.64
0	1	2	2	3	136.19	14.62	21.47	172.28	36.46	48.73	10.54	95.72	36.01	46.8	10.74	93.54
0	1	2	2	4	136.54	12.92	18.95	168.41	44.7	44.72	9.28	98.7	43.77	44.07	9.41	97.26
0	1	2	2	5	334.16	12.2	17.31	363.68	37.67	40.52	8.66	86.85	37.2	39.13	8.82	85.15

					Sequential				PSGA[DP-based Algo.]				PSGA[Math. Model]			
N	C_t	TF	RDD	$r\#$	Tard	Tool	Mach	Total	Tard	Tool	Mach	Total	Tard	Tool	Mach	Total
1	0	0	0	1	49.86	12.83	18.56	81.25	6.2	28.21	11.54	45.95	5.51	28.51	11.42	45.44
1	0	0	0	2	95.21	16.14	24.39	135.75	7.39	36.83	14.95	59.18	6.32	36.41	15	57.74
1	0	0	0	3	98.49	14.74	22.2	135.44	7.36	36.84	12.92	57.12	7.34	35.84	13.17	56.35
1	0	0	0	4	90.39	13.31	19.59	123.29	6.27	40.25	10	56.52	7.57	35.68	10.6	53.85
1	0	0	0	5	65.55	11.95	17.32	94.82	4.98	31.2	9.53	45.72	5.14	28.52	10.02	43.68
1	0	0	1	1	37.08	12.83	18.56	68.47	2.1	30.05	11.01	43.17	2.53	28.37	11.37	42.27
1	0	0	1	2	54.71	16.14	24.39	95.24	1.7	38.43	14.64	54.78	2.74	35.29	15.23	53.26
1	0	0	1	3	106.38	14.74	22.2	143.33	2.6	35.3	13.04	50.94	4.43	31.42	13.86	49.71
1	0	0	1	4	205.18	13.31	19.59	238.08	7.73	39.84	9.94	57.51	3.77	36.37	10.54	50.69
1	0	0	1	5	97.39	11.95	17.32	126.67	4.03	32.95	9.36	46.33	4.17	27.99	10.12	42.28
1	0	0	2	1	37.97	12.83	18.56	69.36	1.03	28.25	11.52	40.8	1.82	25.3	12.13	39.25
1	0	0	2	2	122.31	16.14	24.39	162.84	4.05	38.82	14.56	57.43	1.77	33.26	15.72	50.75
1	0	0	2	3	203.83	14.74	22.2	240.78	2.68	36.78	13	52.47	1.2	34.82	13.29	49.31
1	0	0	2	4	78.05	13.31	19.59	110.95	6.11	40.26	10.03	56.4	2.84	33.74	10.98	47.55
1	0	0	2	5	100.71	11.95	17.32	129.98	7.19	32.49	9.44	49.12	3.42	28.1	10.11	41.63
1	0	1	0	1	121.03	12.83	18.56	152.43	22.75	34.75	10.7	68.2	20.69	33.04	10.87	64.6
1	0	1	0	2	213.69	16.14	24.39	254.22	22.84	45.58	13.71	82.12	22.31	45.1	13.78	81.19
1	0	1	0	3	177.27	14.74	22.2	214.22	25.98	44.52	12.2	82.7	26.29	44.45	12.14	82.88
1	0	1	0	4	130.92	13.31	19.59	163.82	23.63	45.23	9.73	78.59	22.65	43.59	9.77	76
1	0	1	0	5	112.58	11.95	17.32	141.86	17.48	37.84	8.88	64.2	16.8	36.21	9.03	62.05
1	0	1	1	1	107.76	12.83	18.56	139.15	17.28	35.48	10.41	63.17	15.66	36.33	10.33	62.31
1	0	1	1	2	114.19	16.14	24.39	154.72	17.13	45.76	13.72	76.61	20.3	47.21	13.76	81.27
1	0	1	1	3	188.56	14.74	22.2	225.51	12.32	43.87	11.92	68.11	11.7	42.84	12.03	66.57
1	0	1	1	4	267.22	13.31	19.59	300.12	23.23	45.97	9.63	78.83	21.4	46.57	9.55	77.52
1	0	1	1	5	159.56	11.95	17.32	188.84	17.48	38.54	8.9	64.92	12.63	36.04	9.02	57.69
1	0	1	2	1	86.06	12.83	18.56	117.45	10.8	34.97	10.51	56.28	7.53	34.79	10.49	52.81
1	0	1	2	2	254.44	16.14	24.39	294.97	22.91	49.23	13.45	85.59	23.79	48.72	13.52	86.03
1	0	1	2	3	321.67	14.74	22.2	358.62	16.1	47	12.07	75.17	16.42	47.3	12.02	75.75
1	0	1	2	4	136.31	13.31	19.59	169.21	23.81	44.65	9.7	78.16	22.47	44.42	9.81	76.7
1	0	1	2	5	164.4	11.95	17.32	193.67	22.03	39.15	8.63	69.81	19.61	38.8	8.68	67.1
1	0	2	0	1	242.41	12.83	18.56	273.8	65.39	37.54	10.46	113.4	65.23	36.83	10.42	112.48
1	0	2	0	2	386.35	16.14	24.39	426.88	83.66	51.53	13.4	148.59	82.33	50.31	13.55	146.2
1	0	2	0	3	284.98	14.74	22.2	321.93	76.92	49.55	11.96	138.43	76.92	49.55	11.96	138.43
1	0	2	0	4	207.79	13.31	19.59	240.69	74.53	47.36	9.65	131.54	74.39	47.5	9.62	131.52
1	0	2	0	5	180.52	11.95	17.32	209.79	59.78	40.24	8.69	108.71	60.03	39.92	8.73	108.69
1	0	2	1	1	229.44	12.83	18.56	260.83	63.95	39.29	10.07	113.31	64.18	37.55	10.27	112
1	0	2	1	2	217.97	16.14	24.39	258.51	98.48	50.3	13.56	162.34	89.25	51.04	13.5	153.79
1	0	2	1	3	299.03	14.74	22.2	335.97	79.51	51.28	11.46	142.24	77.9	50.76	11.48	140.14
1	0	2	1	4	338.38	13.31	19.59	371.28	81.71	47.44	9.53	138.68	81.93	48.25	9.52	139.71
1	0	2	1	5	239.8	11.95	17.32	269.08	75.32	39.05	8.86	123.22	70.76	39.22	8.88	118.86
1	0	2	2	1	173.66	12.83	18.56	205.05	62.38	37.42	10.37	110.17	59.56	37.61	10.33	107.5
1	0	2	2	2	431.57	16.14	24.39	472.1	110.49	51.4	13.31	175.21	105.18	50.73	13.37	169.29
1	0	2	2	3	463.88	14.74	22.2	500.83	88.94	49.45	11.87	150.27	91.4	50.62	11.76	153.77
1	0	2	2	4	215.83	13.31	19.59	248.73	81.87	47.17	9.55	138.59	85.27	47.45	9.58	142.29
1	0	2	2	5	250.44	11.95	17.32	279.71	77.19	40.4	8.59	126.19	72.29	40.83	8.54	121.66
1	1	0	0	1	273.52	22.92	32.81	329.24	24.87	71.29	15.88	112.04	23.96	65.51	16.91	106.38
1	1	0	0	2	427.41	28.12	41.51	497.03	26.83	102.01	18.92	147.77	27.53	87.9	20.75	136.18
1	1	0	0	3	366.22	26.18	38.93	431.33	32.15	103.98	17	153.13	33.34	86.16	18.8	138.3
1	1	0	0	4	276.33	23.65	34.55	334.54	34.63	98.04	14.39	147.06	31.4	89.98	15.09	136.47
1	1	0	0	5	225.94	21.7	31.23	278.88	26.91	85.33	13.5	125.73	25.33	72.31	14.82	112.47
1	1	0	1	1	241.82	22.92	32.81	297.54	19.99	77.57	15.09	112.65	18.43	70.1	16.05	104.58
1	1	0	1	2	227.6	28.12	41.51	297.23	20.94	101.96	18.87	141.77	20.4	94.5	19.76	134.66
1	1	0	1	3	391.84	26.18	38.93	456.95	18.45	100.32	17.16	135.93	21.3	85.72	18.74	125.76
1	1	0	1	4	455.05	23.65	34.55	513.26	34.61	99.36	14.08	148.04	27.16	90.06	14.86	132.08
1	1	0	1	5	270.99	21.7	31.23	323.92	28.97	86.02	13.19	128.18	21.98	72.9	14.4	109.29
1	1	0	2	1	209.36	22.92	32.81	265.08	11.41	78	15.05	104.45	8.77	75.21	15.33	99.31
1	1	0	2	2	452.51	28.12	41.51	522.14	12.83	102	18.87	133.7	9.87	94.79	19.84	124.49
1	1	0	2	3	622.85	26.18	38.93	687.96	20.88	109.63	16.11	146.63	17.01	93.41	17.96	128.39
1	1	0	2	4	267.38	23.65	34.55	325.58	41.29	99.7	13.87	154.85	23.46	88.93	14.86	127.25
1	1	0	2	5	285.41	21.7	31.23	338.34	30.27	93.84	12.13	136.24	15.61	87.36	12.83	115.81
1	1	1	0	1	396.72	22.92	32.81	452.45	42.3	79.9	15.46	137.66	42.02	75.65	15.86	133.53
1	1	1	0	2	595.57	28.12	41.51	665.2	52.49	106.79	19.12	178.39	53.84	105.18	19.24	178.27
1	1	1	0	3	472.71	26.18	38.93	537.81	60.41	105.85	17.19	183.44	57.32	105.48	17.31	180.11
1	1	1	0	4	324.9	23.65	34.55	383.11	64.01	100.14	14.28	178.43	58.89	98.56	14.5	171.95
1	1	1	0	5	292.11	21.7	31.23	345.05	48.86	90.56	13.44	152.86	45.27	84.55	14.03	143.85
1	1	1	1	1	351.31	22.92	32.81	407.03	41.25	86.85	14.6	142.69	35.19	86.3	15.04	136.52
1	1	1	1	2	324.41	28.12	41.51	394.03	51.83	115.91	18.35	186.1	49.95	105.96	19.06	174.98
1	1	1	1	3	500.8	26.18	38.93	565.91	39.47	116.36	16.34	172.16	42.27	110.57	16.83	169.68
1	1	1	1	4	522.17	23.65	34.55	580.37	57.49	115.98	13.2	186.67	53.09	105.83	13.99	172.92
1	1	1	1	5	346.76	21.7	31.23	399.69	47.95	98.61	12.5	159.06	42.19	94.73	13.15	150.08
1	1	1	2	1	305.73	22.92	32.81	361.46	21.81	103.85	13.32	138.98	25.31	94.44	13.95	133.7
1	1	1	2	2	627.86	28.12	41.51	697.49	56.07	131.61	16.61	204.29	40.42	131.26	17.34	189.03
1	1	1	2	3	763.2	26.18	38.93	828.31	43.55	126.07	15.87	185.49	44.36	123.04	16.19	183.6
1	1	1	2	4	345.74	23.65	34.55	403.95	56.64	117.15	13.17	186.97	49.08	111.93	13.35	174.36
1	1	1	2	5	363.8	21.7	31.23	416.74	59.36	98.57	12.18	170.1	45.84	106.73	11.73	164.3
1	1	2	0	1	545.87	22.92	32.81	601.59	86.48	87.32	15.29	189.08	81.67	89.43	15.13	186.23
1	1	2	0	2	794.16	28.12	41.51	863.79	118.35	120.82	18.43	257.61	122.69	122.35	18.22	263.26
1	1	2	0	3	601.74	26.18	38.93	666.85	121.38	120.25	16.65	258.29	122.41	117.52	16.85	256.77
1	1	2	0	4	418.39	23.65	34.55	476.59	118.7	114.37	13.74	246.81	114.91	111.53	13.79	240.23
1	1	2	0	5	372.68	21.7	31.23	425.61	98.22	102.41	12.77	213.4	90.83	95.07	13.47	199.37
1	1	2	1	1	505.1	22.92	32.81	560.83	85.68	86.05	14.86	186.58	87.07	88.66	14.78	190.51
1	1	2	1	2	458.99	28.12	41.51	528.62	131.11	125.09	17.94	274.14	126.49	124.57	17.85	268.91
1	1	2	1	3	634.13	26.18	38.93	699.23	112.63	126.37	16.12	255.12	110.15	125.93	16.25	252.32
1	1	2	1	4	597.27	23.65	34.55	655.48	125.72	112.62	13.61	251.96	120.99	112.88	13.78	247.66
1	1	2	1	5	435.88	21.7	31.23	488.82	107.45	101.34	12.35	221.14	102.88	97.93	12.77	213.58
1	1	2	2	1	422.26	22.92	32.81	477.98	81.28	94.92	14.06	190.26	78.73	92.76	14.34	185.82
1	1	2	2	2	823.98	28.12	41.51	893.6	141.4	122.89	18.04	282.33	138.61	126.57	17.7	282.87
1	1	2	2	3	919.04	26.18	38.93	984.15	124.79	117.3	16.5	258.59	121.65	124.91	16.06	262.61
1	1	2	2	4	437.63	23.65	34.55	495.83	121.72	114.85	13.49	250.06	123.87	108.81	13.66	246.34
1	1	2	2	5	454.2	21.7	31.23	507.14	100.27	103.39	11.94	215.59	104.93	97.21	12.55	214.69

C.2 CPU Time Values

N	C_t	TF	RDD	$r\#$	Sequential	PSGA[DP-based Algo.]	PSGA[Math. Model]
0	0	0	0	1	11.2	114.8	367.1
0	0	0	0	2	9.8	122.4	364.3
0	0	0	0	3	13.9	102.0	366.5
0	0	0	0	4	16.3	83.6	371.7
0	0	0	0	5	14.4	108.5	382.4
0	0	0	1	1	12.6	128.1	373.9
0	0	0	1	2	11.7	136.8	379.4
0	0	0	1	3	14.8	129.5	380.8
0	0	0	1	4	16.8	82.9	372.1
0	0	0	1	5	10.2	10.2	392.4
0	0	0	2	1	8.5	135.2	390.7
0	0	0	2	2	9.1	120.5	374.7
0	0	0	2	3	14.5	151.7	388.9
0	0	0	2	4	11.8	11.8	391.9
0	0	0	2	5	10.2	104.1	385.9
0	0	1	0	1	11.8	60.7	342.4
0	0	1	0	2	9.6	72.2	339.5
0	0	1	0	3	13.1	47.1	333.6
0	0	1	0	4	16.0	40.6	350.4
0	0	1	0	5	14.1	49.8	356.3
0	0	1	1	1	12.5	81.7	346.4
0	0	1	1	2	10.8	76.0	352.0
0	0	1	1	3	14.4	61.4	355.5
0	0	1	1	4	16.6	43.3	350.2
0	0	1	1	5	11.3	49.9	368.7
0	0	1	2	1	11.2	81.7	364.4
0	0	1	2	2	10.8	77.4	334.5
0	0	1	2	3	13.8	64.5	345.3
0	0	1	2	4	13.1	40.8	340.6
0	0	1	2	5	9.9	48.0	352.9
0	0	2	0	1	11.3	37.7	310.6
0	0	2	0	2	11.3	37.6	303.9
0	0	2	0	3	12.7	36.6	307.2
0	0	2	0	4	14.7	33.5	320.3
0	0	2	0	5	13.2	35.2	324.2
0	0	2	1	1	11.2	36.9	302.8
0	0	2	1	2	11.0	38.0	304.2
0	0	2	1	3	13.4	37.8	300.6
0	0	2	1	4	16.1	33.6	319.3
0	0	2	1	5	10.3	34.4	327.0
0	0	2	2	1	10.3	41.3	305.5
0	0	2	2	2	9.8	39.8	303.0
0	0	2	2	3	13.0	39.3	321.4
0	0	2	2	4	14.7	33.4	308.1
0	0	2	2	5	10.2	34.9	314.5
0	1	0	0	1	11.8	142.3	393.9
0	1	0	0	2	11.4	120.4	399.4
0	1	0	0	3	12.9	114.6	395.6
0	1	0	0	4	16.0	101.5	408.2
0	1	0	0	5	14.5	105.5	408.8
0	1	0	1	1	12.9	156.1	397.4
0	1	0	1	2	11.7	135.7	407.6
0	1	0	1	3	14.5	126.8	395.0
0	1	0	1	4	17.7	82.6	405.8
0	1	0	1	5	10.3	10.3	417.9
0	1	0	2	1	10.7	144.0	401.9
0	1	0	2	2	12.7	129.5	398.2
0	1	0	2	3	13.6	125.4	399.1
0	1	0	2	4	13.3	13.3	422.7
0	1	0	2	5	9.9	94.3	415.2
0	1	1	0	1	12.2	90.1	372.3
0	1	1	0	2	11.6	76.4	382.6
0	1	1	0	3	13.1	75.9	370.3
0	1	1	0	4	16.2	50.3	387.8
0	1	1	0	5	14.4	59.1	393.3
0	1	1	1	1	12.2	97.8	375.0
0	1	1	1	2	12.0	86.7	381.8
0	1	1	1	3	14.0	80.3	377.1
0	1	1	1	4	17.8	58.1	386.4
0	1	1	1	5	11.1	63.2	391.2
0	1	1	2	1	10.2	108.8	378.5
0	1	1	2	2	10.9	77.8	369.7
0	1	1	2	3	13.7	77.6	383.7
0	1	1	2	4	14.2	43.6	373.2
0	1	1	2	5	10.4	44.5	390.9
0	1	2	0	1	10.6	38.2	340.0
0	1	2	0	2	10.3	37.4	345.8
0	1	2	0	3	12.5	37.9	343.4
0	1	2	0	4	13.7	35.2	360.1
0	1	2	0	5	13.1	35.5	363.9
0	1	2	1	1	11.0	41.7	332.2
0	1	2	1	2	9.9	40.5	348.6
0	1	2	1	3	12.1	44.9	342.0
0	1	2	1	4	14.1	34.9	356.1
0	1	2	1	5	10.8	36.8	352.1
0	1	2	2	1	9.7	55.7	335.5
0	1	2	2	2	9.3	47.8	338.2
0	1	2	2	3	13.1	42.4	346.5
0	1	2	2	4	12.7	34.2	341.1
0	1	2	2	5	10.1	39.9	356.8

N	C_t	TF	RDD	$r\#$	Sequential	PSGA[DP-based Algo.]	PSGA[Math. Model]
1	0	0	0	1	73.2	275.2	508.8
1	0	0	0	2	57.3	329.9	609.1
1	0	0	0	3	63.7	229.4	586.6
1	0	0	0	4	77.1	186.3	605.1
1	0	0	0	5	75.0	243.8	578.8
1	0	0	1	1	56.2	281.4	573.0
1	0	0	1	2	57.2	290.5	687.4
1	0	0	1	3	59.2	347.5	721.3
1	0	0	1	4	36.8	274.6	674.8
1	0	0	1	5	48.8	237.0	617.3
1	0	0	2	1	57.9	456.1	638.1
1	0	0	2	2	52.9	476.3	748.1
1	0	0	2	3	47.5	406.0	753.1
1	0	0	2	4	79.8	358.7	719.7
1	0	0	2	5	45.4	326.2	701.8
1	0	1	0	1	48.7	87.0	432.0
1	0	1	0	2	55.2	121.6	536.1
1	0	1	0	3	60.6	96.0	507.4
1	0	1	0	4	69.1	92.6	525.8
1	0	1	0	5	73.3	92.8	483.1
1	0	1	1	1	57.5	125.7	442.4
1	0	1	1	2	43.9	141.7	492.6
1	0	1	1	3	47.6	123.3	548.0
1	0	1	1	4	39.5	85.7	509.4
1	0	1	1	5	54.1	83.4	476.1
1	0	1	2	1	60.4	166.8	483.9
1	0	1	2	2	57.6	201.5	507.9
1	0	1	2	3	55.9	114.2	497.8
1	0	1	2	4	79.1	94.8	483.7
1	0	1	2	5	46.5	86.0	498.1
1	0	2	0	1	58.0	70.1	376.8
1	0	2	0	2	47.6	69.3	401.9
1	0	2	0	3	56.9	70.5	401.9
1	0	2	0	4	64.3	70.0	412.8
1	0	2	0	5	69.6	70.6	418.4
1	0	2	1	1	47.6	72.0	385.4
1	0	2	1	2	50.6	73.7	430.6
1	0	2	1	3	52.6	67.9	390.2
1	0	2	1	4	41.3	71.2	422.7
1	0	2	1	5	52.2	68.2	410.9
1	0	2	2	1	50.2	77.5	395.8
1	0	2	2	2	43.7	71.2	418.7
1	0	2	2	3	47.2	73.3	425.2
1	0	2	2	4	68.1	71.3	436.3
1	0	2	2	5	43.3	67.2	418.8
1	1	0	0	1	50.5	332.5	735.4
1	1	0	0	2	44.2	347.5	820.0
1	1	0	0	3	49.1	305.7	824.9
1	1	0	0	4	74.0	225.7	816.7
1	1	0	0	5	69.2	255.1	804.4
1	1	0	1	1	56.1	331.6	737.4
1	1	0	1	2	46.2	325.3	811.2
1	1	0	1	3	63.9	333.2	859.8
1	1	0	1	4	41.0	237.5	818.2
1	1	0	1	5	55.8	265.4	804.3
1	1	0	2	1	66.7	428.1	794.1
1	1	0	2	2	55.4	465.0	819.3
1	1	0	2	3	54.2	299.2	850.4
1	1	0	2	4	75.3	264.7	837.3
1	1	0	2	5	50.3	311.2	798.8
1	1	1	0	1	47.6	156.0	639.1
1	1	1	0	2	41.6	179.4	710.4
1	1	1	0	3	60.5	109.1	713.4
1	1	1	0	4	65.4	108.6	705.5
1	1	1	0	5	66.9	102.0	712.5
1	1	1	1	1	43.0	146.9	616.5
1	1	1	1	2	54.3	148.8	715.3
1	1	1	1	3	56.4	142.7	734.6
1	1	1	1	4	41.9	92.1	717.4
1	1	1	1	5	56.6	90.4	685.7
1	1	1	2	1	61.1	133.8	581.1
1	1	1	2	2	51.8	107.2	599.0
1	1	1	2	3	50.5	98.6	627.5
1	1	1	2	4	73.4	104.3	663.7
1	1	1	2	5	51.6	119.5	603.0
1	1	2	0	1	46.2	73.3	516.1
1	1	2	0	2	48.9	73.9	547.4
1	1	2	0	3	56.0	72.7	576.5
1	1	2	0	4	61.4	70.5	593.4
1	1	2	0	5	69.2	72.2	570.9
1	1	2	1	1	38.0	79.5	518.7
1	1	2	1	2	45.2	74.6	557.7
1	1	2	1	3	43.7	71.4	541.7
1	1	2	1	4	41.4	74.0	573.0
1	1	2	1	5	51.6	71.5	561.7
1	1	2	2	1	46.3	85.5	556.7
1	1	2	2	2	40.9	79.9	545.0
1	1	2	2	3	43.6	79.3	573.1
1	1	2	2	4	64.8	73.5	592.0
1	1	2	2	5	49.1	75.9	596.9

Appendix D

Statistical Analysis of Results

D.1 Analysis of The Problem with a Given Sequence

	Cost Pairs	Mean	N	Std. Deviation	Std. Error Mean
1	DP-based Algo	134.41	900	86.52	2.88
	Math Model	131.40	900	85.47	2.84

Table D.1: Paired samples statistics for DP-based algorithm and Math Model solved in GAMS

	Cost Pairs	N	Correlation	Sig.
1	DP-based Algo & Math Model	900	0.995	0.000

Table D.2: Paired samples correlations for DP-based algorithm and Math Model solved in GAMS

	Cost Pairs	St Dev	S.E. Mean	95% CI of Difference		t	Sig.
				Lower	Upper		
1	DP-based Algo & Math Model	8.29	0.28	2.46	3.54	10.88	0.000

Table D.3: Paired samples test results for DP-based algorithm and Math Model solved in GAMS

Source	Dependent variables	Sum of squares	DF	Mean Square	F	Sig.
Corrected Model	DP-based Algo	6460870.17	35	184596.29	594.44	0.000
	Math Model	6322880.48	35	180653.73	639.73	0.000
Intercept	DP-based Algo	16259042.75	1	16259042.75	52357.56	0.000
	Math Model	15539585.80	1	15539585.80	55028.32	0.000
N	DP-based Algo	3193768.11	1	3193768.11	10284.61	0.000
	Math Model	3015682.77	1	3015682.77	10679.05	0.000
C_t	DP-based Algo	1480828.57	1	1480828.57	4768.58	0.000
	Math Model	1312562.50	1	1312562.50	4648.01	0.000
TF	DP-based Algo	1170138.30	2	585069.15	1884.05	0.000
	Math Model	1367883.87	2	683941.93	2421.95	0.000
RDD	DP-based Algo	8101.71	2	4050.85	13.04	0.00
	Math Model	2731.90	2	1365.95	4.84	0.008

Table D.4: Test of Between-Subjects Effects for the cost values of DP-based algorithm and Math Model solved in GAMS

			95% CI of Difference	
Dependent Variable	Mean	S.E. Mean	Lower	Upper
DP-based Algo	134.41	0.59	133.25	135.56
Math Model	131.40	0.56	130.30	132.50

Table D.5: Estimated Marginal Grand Mean for the cost values of two stage algorithm and proposed PSGAs

				95% CI of Difference	
Dependent Variable	N	Mean	S.E. Mean	Lower	Upper
DP-based Algo	0	74.84	0.83	73.21	76.47
	1	139.98	0.83	192.35	195.61
Math Model	0	73.51	0.79	71.96	75.07
	1	189.29	0.79	187.73	190.84

Table D.6: Estimated Marginal Mean by the factor N for the cost values of DP-based algorithm and Math Model solved in GAMS

				95% CI of Difference	
Dependent Variable	C_t	Mean	S.E. Mean	Lower	Upper
DP-based Algo	0	93.84	0.83	92.21	95.47
	1	174.97	0.83	173.34	176.60
Math Model	0	93.21	0.79	91.66	94.77
	1	169.59	0.79	168.03	171.14

Table D.7: Estimated Marginal Mean by the factor C_t for DP-based algorithm and Math Model solved in GAMS

Dependent Variable	TF	Mean	S.E. Mean	95% CI of Difference	
				Lower	Upper
DP-based Algo	0	94.90	1.02	92.91	96.90
	1	126.23	1.02	124.24	128.23
	2	182.09	1.02	180.09	184.08
Math Model	0	87.16	0.97	85.26	89.07
	1	125.02	0.97	123.12	126.92
	2	182.02	0.97	180.11	183.92

Table D.8: Estimated Marginal Mean by the factor TF for DP-based algorithm and Math Model solved in GAMS

Dependent Variable	RDD	Mean	S.E. Mean	95% CI of Difference	
				Lower	Upper
DP-based Algo	0	131.291	1.02	129.29	133.29
	1	133.47	1.02	131.48	135.47
	2	138.46	1.02	136.46	140.46
Math Model	0	129.71	0.97	127.80	131.61
	1	130.70	0.97	128.79	132.60
	2	133.79	0.97	131.89	135.70

Table D.9: Estimated Marginal Mean by the factor RDD for DP-based algorithm and Math Model solved in GAMS

	Time Pairs	Mean	N	Std. Deviation	Std. Error Mean
1	DP-based Algo	0.14	900	0.11	0.004
	Math Model	0.72	900	0.24	0.008

Table D.10: Paired samples statistics for CPU times of DP-based algorithm and Math Model solved in GAMS

Cost Pairs		N	Correlation	Sig.
1	DP-based Algo & Math Model	900	0.746	0.000

Table D.11: Paired samples correlations for CPU times of DP-based algorithm and Math Model solved in GAMS

Cost Pairs		St Dev	S.E. Mean	95% CI of Difference		t	Sig.
				Lower	Upper		
1	DP-based Algo & Math Model	0.17	0.005	-0.59	-0.57	-100.99	0.000

Table D.12: Paired samples test results for CPU times of DP-based algorithm and Math Model solved in GAMS

Source	Dependent variables	Sum of squares	DF	Mean Square	F	Sig.
Corrected Model	DP-based Algo	8.71	35	0.25	78.61	0.000
	Math Model	48.59	35	1.39	396.06	0.000
Intercept	DP-based Algo	18.27	1	18.27	5766.39	0.000
	Math Model	471.22	1	471.22	134428.82	0.000
N	DP-based Algo	2.68	1	2.68	845.58	0.000
	Math Model	26.86	1	26.86	7661.55	0.000
C_t	DP-based Algo	0.19	1	0.19	59.45	0.000
	Math Model	6.87	1	6.87	1959.75	0.000
TF	DP-based Algo	3.60	2	1.80	568.84	0.000
	Math Model	7.73	2	3.87	1104.61	0.000
RDD	DP-based Algo	0.30	2	0.15	48.05	0.000
	Math Model	0.61	2	0.30	86.70	0.000

Table D.13: Test of Between-Subjects Effects for the cost values of DP-based algorithm and Math Model solved in GAMS

			95% CI of Difference	
Dependent Variable	Mean	S.E. Mean	Lower	Upper
DP-based Algo	0.142	0.002	0.14	0.146
Math Model	0.724	0.002	0.72	0.727

Table D.14: Estimated Marginal Grand Mean for the CPU times of two stage algorithm and proposed PSGAs

				95% CI of Difference	
Dependent Variable	N	Mean	S.E. Mean	Lower	Upper
DP-based Algo	0	0.09	0.003	0.08	0.09
	1	0.20	0.003	0.19	0.20
Math Model	0	0.55	0.003	0.54	0.55
	1	0.87	0.003	0.89	0.90

Table D.15: Estimated Marginal Mean by the factor N for the CPU times of DP-based algorithm and Math Model solved in GAMS

				95% CI of Difference	
Dependent Variable	C_t	Mean	S.E. Mean	Lower	Upper
DP-based Algo	0	0.13	0.003	0.12	0.12
	1	0.16	0.003	0.15	0.16
Math Model	0	0.64	0.003	0.63	0.64
	1	0.81	0.003	0.80	0.81

Table D.16: Estimated Marginal Mean by the factor C_t for the CPU times of DP-based algorithm and Math Model solved in GAMS

Dependent Variable	TF	Mean	S.E. Mean	95% CI of Difference	
				Lower	Upper
DP-based Algo	0	0.23	0.003	0.22	0.24
	1	0.11	0.003	0.10	0.12
	2	0.08	0.003	0.08	0.09
Math Model	0	0.84	0.003	0.83	0.85
	1	0.72	0.003	0.71	0.73
	2	0.61	0.003	0.61	0.62

Table D.17: Estimated Marginal Mean by the factor TF for the CPU times of DP-based algorithm and Math Model solved in GAMS

Dependent Variable	RDD	Mean	S.E. Mean	95% CI of Difference	
				Lower	Upper
DP-based Algo	0	0.12	0.003	0.12	0.13
	1	0.14	0.003	0.13	0.14
	2	0.17	0.003	0.16	0.17
Math Model	0	0.69	0.003	0.69	0.70
	1	0.72	0.003	0.72	0.73
	2	0.76	0.003	0.75	0.76

Table D.18: Estimated Marginal Mean by the factor RDD for the CPU times of DP-based algorithm and Math Model solved in GAMS

D.2 Analysis of The Original Problem

	Cost Pairs	Mean	N	Std. Deviation	Std. Error Mean
1	Sequential	246.40	180	191.98	14.31
	PSGA[DP-Algo]	93.35	180	62.87	4.68
2	Sequential	246.40	180	191.98	14.31
	PSGA[Math Model]	90.42	180	61.32	4.57
3	PSGA[DP-Algo]	93.35	180	62.87	4.68
	PSGA[Math Model]	90.42	180	61.32	4.58

Table D.19: Paired samples statistics for the sequential algorithm and proposed PSGAs

	Cost Pairs	N	Correlation	Sig.
1	Sequential & PSGA[DP-Algo]	180	0.895	0.000
2	Sequential & PSGA[Math Model]	180	0.895	0.000
3	PSGA[DP-Algo] & PSGA[Math Model]	180	0.997	0.000

Table D.20: Paired samples correlations for the sequential algorithm and proposed PSGAs

	Cost Pairs	St Dev	S.E. Mean	95% CI of Difference		t	Sig.
				Lower	Upper		
1	Sequential & PSGA[DP-Algo]	138.54	10.32	132.66	173.42	14.82	0.000
2	Sequential & PSGA[Math Model]	139.89	10.43	135.40	176.55	14.95	0.000
3	PSGA[DP-Algo] & PSGA[Math Model]	4.68	0.35	2.24	3.62	8.39	0.000

Table D.21: Paired samples test results for PSGA parameter sets

Source	Dependent variables	Sum of squares	DF	Mean Square	F	Sig.
Corrected Model	Sequential	5136740.55	35	146764.01	14.46	0.000
	PSGA[DP-Algo]	675598.10	35	19302.80	86.61	0.000
	PSGA[Math Model]	642144.45	35	18346.98	84.99	0.000
Intercept	Sequential	10928342.66	1	10928342.65	1077.18	0.000
	PSGA[DP-Algo]	1568789.70	1	1568789.70	7039.3	0.000
	PSGA[Math Model]	1471746.45	1	1471746.45	6817.75	0.000
N	Sequential	2306545.09	1	2306545.09	227.35	0.000
	PSGA[DP-Algo]	292351.51	1	292351.51	1311.81	0.000
	PSGA[Math Model]	264796.78	1	264796.78	1226.65	0.000
C_t	Sequential	1662564.60	1	1662564.60	163.87	0.000
	PSGA[DP-Algo]	200263.42	1	200263.42	898.60	0.000
	PSGA[Math Model]	181260.16	1	181260.16	839.67	0.000
TF	Sequential	508777.60	2	254388.80	25.07	0.000
	PSGA[DP-Algo]	111396.73	2	55698.37	249.92	0.000
	PSGA[Math Model]	122346.78	2	61173.39	283.38	0.000
RDD	Sequential	45300.87	2	22650.44	2.23	0.111
	PSGA[DP-Algo]	50.92	2	25.46	0.11	0.892
	PSGA[Math Model]	38.61	2	19.31	0.09	0.914

Table D.22: Test of Between-Subjects Effects for the cost values of sequential algorithm and proposed PSGAs

			95% CI of Difference	
Dependent Variable	Mean	S.E. Mean	Lower	Upper
Sequential	246.40	7.51	231.56	261.23
PSGA[DP-Algo]	93.36	1.11	91.16	95.56
PSGA[Math Model]	90.42	1.09	88.26	92.59

Table D.23: Estimated Marginal Grand Mean for the cost values of the sequential algorithm and proposed PSGAs

				95% CI of Difference	
Dependent Variable	N	Mean	S.E. Mean	Lower	Upper
Sequential	0	133.20	10.62	112.21	154.19
	1	359.60	10.62	338.61	380.58
PSGA[DP-Algo]	0	53.06	1.57	49.94	56.17
	1	133.66	1.57	130.54	136.76
PSGA[Math Model]	0	52.07	1.55	49.01	55.13
	1	128.78	1.55	125.72	131.83

Table D.24: Estimated Marginal Mean by the factor N for the cost values of the sequential algorithm and proposed PSGAs

				95% CI of Difference	
Dependent Variable	C_t	Mean	S.E. Mean	Lower	Upper
Sequential	0	150.29	10.62	129.31	171.28
	1	342.51	10.62	321.52	363.49
PSGA[DP-Algo]	0	60.00	1.57	56.89	63.11
	1	126.71	1.57	123.60	129.82
PSGA[Math Model]	0	58.69	1.55	55.63	61.75
	1	122.16	1.55	119.10	125.22

Table D.25: Estimated Marginal Mean by the factor C_t for the cost values of the sequential algorithm and proposed PSGAs

Dependent Variable	TF	Mean	S.E. Mean	95% CI of Difference	
				Lower	Upper
Sequential	0	184.07	13.00	158.37	209.78
	1	241.14	13.00	215.44	266.84
	2	313.98	13.00	288.28	339.68
PSGA[DP-Algo]	0	67.09	1.93	63.27	70.89
	1	86.22	1.93	82.41	90.03
	2	126.76	1.93	122.95	130.57
PSGA[Math Model]	0	62.52	1.90	58.78	66.28
	1	83.49	1.90	79.74	87.24
	2	125.25	1.90	121.50	123.00

Table D.26: Estimated Marginal Mean by the factor TF for the cost values of the sequential algorithm and proposed PSGAs

Dependent Variable	RDD	Mean	S.E. Mean	95% CI of Difference	
				Lower	Upper
Sequential	0	233.38	13.00	207.67	259.08
	1	237.09	13.00	211.39	262.79
	2	268.73	13.00	243.03	243.43
PSGA[DP-Algo]	0	92.98	1.93	89.17	96.79
	1	92.98	1.93	89.17	96.79
	2	94.11	1.93	90.30	97.92
PSGA[Math Model]	0	90.84	1.90	87.10	94.60
	1	89.78	1.90	86.03	93.53
	2	90.65	1.90	86.90	94.40

Table D.27: Estimated Marginal Mean by the factor RDD for the cost values of the sequential algorithm and proposed PSGAs

Time Pairs		Mean	N	Std. Deviation	Std. Error Mean
1	Sequential	33.71	180	22.65	1.69
	PSGA[DP-Algo]	119.29	180	98.74	7.36
2	Sequential	33.71	180	22.65	1.69
	PSGA[Math Model]	480.23	180	154.16	11.49
3	PSGA[DP-Algo]	119.29	180	98.74	7.36
	PSGA[Math Model]	480.23	180	154.16	11.49

Table D.28: Paired samples statistics for CPU times of the sequential algorithm and proposed PSGAs

Cost Pairs		N	Correlation	Sig.
1	Sequential & PSGA[DP-Algo]	180	0.511	0.000
2	Sequential & PSGA[Math Model]	180	0.748	0.000
3	PSGA[DP-Algo] & PSGA[Math Model]	180	0.771	0.000

Table D.29: Paired samples correlations for the CPU times of the sequential algorithm and proposed PSGAs

	Cost Pairs	St Dev	S.E. Mean	95% CI of Difference		t	Sig.
				Lower	Upper		
1	Sequential - PSGA[DP-Algo]	89.31	6.66	-98.71	-72.44	-12.87	0.000
2	Sequential - PSGA[Math Model]	138.03	10.29	-466.82	-426.21	-43.40	0.000
3	PSGA[DP-Algo] - PSGA[Math Model]	100.25	7.47	-375.68	-346.19	-48.30	0.000

Table D.30: Paired samples test results for the CPU times of the sequential algorithm and proposed PSGAs

Source	Dependent variables	Sum of squares	DF	Mean Square	F	Sig.
Corrected Model	Sequential	84818.946	35	2423.40	49.55	0.000
	PSGA[DP-Algo]	1581396.43	35	45182.75	39.69	0.000
	PSGA[Math Model]	4162208.99	35	118920.26	186.62	0.000
Intercept	Sequential	204579.92	1	204579.92	4183.18	0.000
	PSGA[DP-Algo]	2561428.28	1	2561428.28	2250.29	0.000
	PSGA[Math Model]	41511244.48	1	41511244.48	65141.49	0.000
N	Sequential	81597.33	1	81597.33	1668.47	0.000
	PSGA[DP-Algo]	440827.227	1	440827.23	387.28	0.000
	PSGA[Math Model]	2520620.70	1	2520620.70	3955.48	0.000
C_t	Sequential	63.21	1	63.21	1.29	0.257
	PSGA[DP-Algo]	1430.42	1	1430.42	1.26	0.264
	PSGA[Math Model]	418413.63	1	418413.63	656.59	0.013
TF	Sequential	441.50	2	220.75	25.07	0.000
	PSGA[DP-Algo]	778085.33	2	389042.61	249.92	0.000
	PSGA[Math Model]	703989.99	2	351995.00	283.38	0.000
RDD	Sequential	868.79	2	434.40	8.88	0.000
	PSGA[DP-Algo]	13259.93	2	6629.97	5.82	0.004
	PSGA[Math Model]	2949.05	2	1474.52	2.31	0.103

Table D.31: Test of Between-Subjects Effects for the CPU times of the sequential algorithm and proposed PSGAs

Dependent Variable	Mean	S.E. Mean	95% CI of Difference	
			Lower	Upper
Sequential	33.71	0.521	32.68	34.74
PSGA[DP-Algo]	119.29	2.51	114.32	124.26
PSGA[Math Model]	480.23	1.88	476.51	483.95

Table D.32: Estimated Marginal Grand Mean for the CPU times of the sequential algorithm and proposed PSGAs

Dependent Variable	N	Mean	S.E. Mean	95% CI of Difference	
				Lower	Upper
Sequential	0	12.42	0.73	10.96	13.88
	1	55.00	0.73	53.55	56.46
PSGA[DP-Algo]	0	69.80	3.56	62.77	76.83
	1	168.78	3.56	161.75	175.81
PSGA[Math Model]	0	361.89	2.66	356.63	367.15
	1	598.56	2.66	593.30	603.82

Table D.33: Estimated Marginal Mean by the factor N for CPU times of the sequential algorithm and proposed PSGAs

Dependent Variable	C_t	Mean	S.E. Mean	95% CI of Difference	
				Lower	Upper
Sequential	0	34.30	0.74	32.85	35.76
	1	33.12	0.74	31.66	34.57
PSGA[DP-Algo]	0	116.47	3.56	109.44	123.50
	1	122.11	3.56	115.08	129.14
PSGA[Math Model]	0	432.01	2.66	426.75	437.27
	1	528.44	2.66	523.18	533.70

Table D.34: Estimated Marginal Mean by the factor C_t for the CPU times of the sequential algorithm and proposed PSGAs

Dependent Variable	TF	Mean	S.E. Mean	95% CI of Difference	
				Lower	Upper
Sequential	0	35.32	0.90	33.53	37.10
	1	34.23	0.90	32.45	36.02
	2	31.59	0.90	29.80	33.37
PSGA[DP-Algo]	0	209.84	4.36	201.23	218.45
	1	92.23	4.36	83.68	100.90
	2	55.74	4.36	47.13	64.34
PSGA[Math Model]	0	560.07	3.26	553.63	566.51
	1	473.25	3.26	466.81	479.69
	2	407.36	3.26	400.92	413.80

Table D.35: Estimated Marginal Mean by the factor TF for the CPU times of the sequential algorithm and proposed PSGAs

Dependent Variable	RDD	Mean	S.E. Mean	95% CI of Difference	
				Lower	Upper
Sequential	0	36.46	0.90	34.68	38.25
	1	31.09	0.90	29.30	32.87
	2	33.59	0.90	31.80	35.37
PSGA[DP-Algo]	0	111.53	4.36	102.92	120.138
	1	115.09	4.36	106.48	123.70
	2	131.25	4.36	122.64	139.86
PSGA[Math Model]	0	474.60	3.26	468.16	481.05
	1	482.11	3.26	475.67	488.55
	2	483.97	3.26	477.52	490.41

Table D.36: Estimated Marginal Mean by the factor RDD for the CPU times of the sequential algorithm and proposed PSGAs

VITA

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