PARTICLE SWARM OPTIMIZATION BASED CHANNEL IDENTIFICATION IN CROSS-AMBIGUITY DOMAIN

Mehmet Burak Guldogan, Orhan Arikan

Bilkent University
Electrical and Electronics Engineering Department
Ankara, Turkey, e-mail:{guldogan,oarikan@ee.bilkent.edu.tr}

ABSTRACT

In this paper, a new array signal processing technique by using particle swarm optimization (PSO) is proposed to identify multipath channel parameters. The proposed technique provides estimates to the channel parameters by finding a global minimum of an optimization problem. Since the optimization problem is formulated in the cross-ambiguity function (CAF) domain of the transmitted signal and the received array outputs, the proposed technique is called as PSO-CAF. The performance of the PSO-CAF is compared with the space alternating generalized expectation maximization (SAGE) technique and with another recently proposed PSO based technique for various SNR values. Simulation results indicate the superior performance of the PSO-CAF technique over mentioned techniques for all SNR values.

Index Terms— direction of arrival (DOA), cross-ambiguity function (CAF), particle swarm optimization (PSO).

1. INTRODUCTION

To meet the ever increasing demand for more efficient utilization, the communication channels should be accurately modeled. To this end, antenna arrays and sophisticated signal processing techniques are used to estimate multipath channel parameters. There have been proposed many array signal processing techniques for reliable and accurate estimation of these channel parameters [1]. The maximum likelihood (ML) criterion based channel identification is a commonly used framework. In this framework, global maximum of the likelihood function over the channel parameter space should be found. Since the channel parameter space can be very large, the high dimensional search for the global maxima of the likelihood function creates issues in applications. Furthermore, the multimodal structure of the likelihood function complicates the search for the global maximum of the likelihood function. To reduce the computational complexity of the high dimensional search of the ML technique, the SAGE algorithm has been proposed. The SAGE algorithm has been successfully applied for joint channel parameter estimation and one of these efforts is reported in [2]. Unfortunately, the SAGE algorithm with gradient based search techniques are prone to converge to a local maximum of the likelihood function. To overcome this problem, various optimization techniques such as alternating projection method [3], and simulated annealing algorithms [4] have been proposed.

In this paper, a new array signal processing technique by using PSO is proposed to estimate multipath channel parameters. By finding a global minimum of an optimization problem, the proposed technique provides estimates to the channel parameters. Since the optimization problem is formulated in the CAF domain of the transmitted signal and the received array outputs, the proposed technique is called as PSO-CAF.

2. SIGNAL AND CHANNEL MODEL

In this section a commonly used parametric model for multipath channels is described. Consider that transmitted signals are written as a modulated train of pulses:

$$s(t) = \sum_{k=1}^{q} b_k p(t - (k-1)T) \ , \tag{1}$$

where $p(t)$ is the modulated pulse with time-bandwidth product larger than 1, and b_k's are ± 1. In a multipath environment, delayed, Doppler shifted and attenuated copies of the transmitted signal from a transmitter impinge on an M element receiver antenna array from different paths. The output of the antenna array can be written as:

$$\mathbf{x}(t) = \sum_{i=1}^{d} \mathbf{a}(\theta_i, \phi_i)\zeta_i s(t - \tau_i)e^{j2\pi\nu_i t} + \mathbf{n}(t) \tag{2}$$

where $\mathbf{x}(t) = [x_1(t), ..., x_M(t)]^T$ is the array output, d is the number of paths, $\mathbf{a}(\theta_i, \phi_i)$ is the steering vector, ϕ and θ are elevation and azimuth angles, respectively, ζ_i is the complex scaling factor of the ith path containing all the attenuation and phase terms, τ_i is the time delay of the ith path with respect to antenna origin, ν_i is the Doppler shift of the ith path and $\mathbf{n}(t) = [n_1(t), ..., n_M(t)]^T$ is spatially and temporally white circularly symmetric noise Gaussian distributed with covariance σ^2. For notational simplicity all unknown parameters are collected in vector, $\boldsymbol{\varphi} = [\boldsymbol{\varphi}_1, ..., \boldsymbol{\varphi}_d]$ where $\boldsymbol{\varphi}_i = [\tau_i, \nu_i, \theta_i, \phi_i]$.

3. MAXIMUM-LIKELIHOOD BASED PARAMETER ESTIMATION

ML estimation is a systematic approach, used in many parameter estimation problems. The likelihood function of the observed data is determined in order to use the ML method. Unknown channel parameters that maximize the likelihood function are considered to be ML estimates. Assuming that the noise on each pulse transmission are independent, the probability density function can be written as:

$$P[\mathbf{x}(t_1) \quad ... \quad \mathbf{x}(t_N)] = \prod_{k=1}^{N} \frac{1}{\mid \pi\sigma^2\mathbf{I}\mid} e^{-[\|\mathbf{e}(t_k)\|^2/\sigma^2]} \quad , \quad (3)$$

where $\mid \cdot \mid$ is for the determinant, $\|\cdot\|$ is for the norm, and

$$\mathbf{e}(t_k) = \mathbf{x}(t_k) - \sum_{i=1}^{d} \mathbf{a}(\theta_i, \phi_i)\zeta_i s(t_k - \tau_i)e^{j2\pi\nu_i t_k} \quad . \quad (4)$$

The ML estimates can be obtained as the maximum of the log-likelihood function:

$$\hat{\varphi} = \arg \max_{\varphi} \left\{ -\text{NMlog}\pi\sigma^2 - \frac{1}{\sigma^2} \sum_{k=1}^{N} \|\mathbf{e}(t_k)\|^2 \right\}. \quad (5)$$

Therefore, one needs to find the global maximum of this $4 \times d$ optimization problem to identify all 4 parameters of each d paths. SAGE algorithm, which has simpler maximization steps in lower dimensional spaces, has been proposed to reduce the computational complexity [5]. In SAGE, parameters are updated sequentially [2]. In Table 1, basic form of the SAGE algorithm is presented.

Table 1. Basic SAGE algorithm for reference

Initialize the algorithm.

for $j = 1$; $j \leq$ max. # iterations ; $j = j + 1$
  for $i = 1$; $i \leq d$; $i++$
   - Expectation step: estimate the complete (unobservable) data of ith signal path given measurements.
   - Maximization step: estimate each parameter of ith signal path sequentially by maximizing a properly chosen cost function.
   - Create a copy of the ith signal path with estimated parameters.
   - Subtract the copy signal from each antenna output.
  end
end

4. PARTICLE SWARM OPTIMIZATION

Particle swarm optimization (PSO) is a very powerful stochastic optimization algorithm, developed by Kennedy and Eberhart in 1995 [6]. It is inspired by animal social behaviors such as bird flocking. PSO has been successfully applied to many different global optimization applications [7], [8]. PSO

Table 2. PSO update steps

for each time step t, **do**
  for each particle l in the swarm **do**
   - $\boldsymbol{v}_l$ and $\mathbf{z}_l$ is updated using (6) and (7)
   - closeness of particle location to the solution is determined
   - $\mathbf{p}_l$ and $\mathbf{p}_g$ are updated
  end
end

algorithm operates on a set of solution candidates that are called as swarm of particles. The particles are flown through a multidimensional search space, where the position of each particle is adjusted according to its own memory and that of its neighbors. Each particle l consists of three vectors: its location in D dimensional search space $\mathbf{z}_l = [z_{l1}, z_{l2}, ..., z_{lD}]$, its historicaly best position $\mathbf{p}_l = [p_{l1}, p_{l2}, ..., p_{lD}]$ and its velocity $\boldsymbol{v}_l = [v_{l1}, v_{l2}, ..., v_{lD}]$. Best position is the position which has the best fitness to a predefined fitness function. Initially, the positions and velocities of each particle are randomly distributed over the search space. Then, in each time step, the velocity and location of each particle is updated by using following equations:

$$v_{lk} = \kappa \left(v_{lk} + c_1\epsilon_1 \ (p_{lk} - z_{lk}) + c_2\epsilon_2 \ (p_{gk} - z_{lk}) \right) \quad (6)$$
$$z_{lk} = z_{lk} + v_{lk} \quad , \quad (7)$$

where c_1 and c_2 are scaling factors that determine the relative pull of best position found particle and best position found by the swarm, ϵ_1 and ϵ_2 are random numbers, κ is the constriction factor and $\mathbf{p}_g$ is best position found by the swarm. Update procedure of the algorithm is summarized in Table 2.
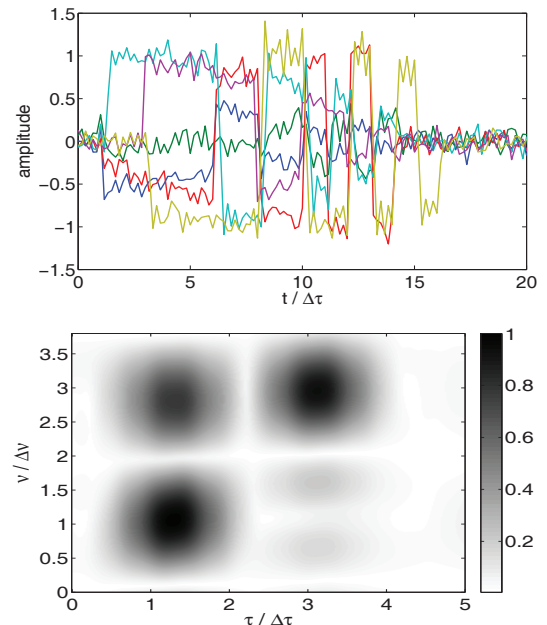

Fig. 1. Barker-13 coded 6 paths a-) in time domain, b-) in delay-Doppler domain.

5. PROPOSED PSO-CAF TECHNIQUE

When the number of paths increases, the ML approaches face significant challenges in finding the global maximum of the likelihood function. This is mainly because of the fact that likelihood maximization is performed in time domain, where there is a considerable overlap between the signals received from different paths. It is desirable to formulate an alternative optimization problem other than the time domain where the multipath signal components are localized with less of an overlapping problem. Since typical communication signals are phase or frequency modulated, their CAFs are significantly localized in the delay-Doppler domain. As used in radar signal processing applications, time delay of the Doppler shifted signals can be estimated by using CAF. The CAF between the received signal, $x_m(t)$, and the transmitted signal $s(t)$ is:

$$\boldsymbol{\chi}_{x_m,s}(\tau,\nu) = \int_{-\infty}^{\infty} x_m\left(t+\frac{\tau}{2}\right) s^*\left(t-\frac{\tau}{2}\right) e^{-j2\pi\nu t} dt. \quad (8)$$

Therefore, the transformation of the array signal outputs to the CAF domain localizes different multipath signals to their respective delay and Doppler cell. Peaks of these localized clusters can be detected by setting an adaptive threshold. To illustrate this phenomenon consider a synthetic multipath channel with 6 distinct paths. As shown in Fig. 1.a, the individual multipath signals overlap significantly in time at the output of an array element. However, as shown in Fig. 1.b, the CAF between the received signal and the transmitted signal localizes the contribution of different path components in delay-Doppler domain. This localization enables us to reformulate the channel identification problem as a set of loosely coupled optimization problems in lower dimensional parameter spaces. Effectiveness of the peak detection of each multipath cluster on CAF surface is also verified on ionospheric data [9]. In the following based on CAF, we provide the new optimization framework.

Assuming that, based on CAF processing of the received signal and the transmitted signal, we identified C clusters of paths in the CAF domain. Let the number of multipaths in the cth cluster be d_c. The path parameter optimization problem can be formulated for each cluster c, $1 \leq c \leq C$, as:

$$\hat{\boldsymbol{\varphi}}(S_c) = \arg\min_{\boldsymbol{\varphi}} \quad (9)$$

$$\sum_{m=1}^{M} \left\| \mathbf{vec}\left(\mathbf{W}_c\boldsymbol{\chi}_{x_m,s}(\tau,\nu) - \mathbf{W}_c\boldsymbol{\chi}_{\hat{x}_m(\boldsymbol{\varphi}(S_c)),s}(\tau,\nu)\right) \right\|^2,$$

where $\mathbf{vec}(.)$ is the vector operator stacking the columns of a matrix into a single column vector, $\mathbf{W}_c$ is a mask for the cth cluster, which selects the patch that will be used in the PSO optimization, S_c is the set containing path indexes of d_c multipath components in the cth cluster, and $\boldsymbol{\chi}_{\hat{x}_m(\boldsymbol{\varphi}(S_c)),s}(\tau,\nu)$ is the CAF between created c^{th} cluster multipath signal, $\hat{x}_m(t,\boldsymbol{\varphi}(S_c))$, and $s(t)$:

$$\boldsymbol{\chi}_{\hat{x}_m(\boldsymbol{\varphi}(S_c)),s}(\tau,\nu) = \sum_{i\in S_c} \zeta_i\hat{\mathbf{A}}_{m,i}(\tau,\nu) \quad . \quad (10)$$

In this equation, $\hat{\mathbf{A}}_{m,i}(\tau,\nu)$ is defined as:

$$\hat{\mathbf{A}}_{m,i}(\tau,\nu) = a_m(\theta_i,\phi_i)$$

$$\int_{-\infty}^{\infty} s\left(t-\tau_i+\frac{\tau}{2}\right) s^*\left(t-\frac{\tau}{2}\right) e^{-j2\pi(\nu-\nu_i)t} dt, \quad (11)$$

Using (10) and (11), (9) can be written in a compact form as:

$$\hat{\boldsymbol{\varphi}}(S_c) = \arg\min_{\boldsymbol{\varphi}} \sum_{m=1}^{M} \left\| \mathbf{vec}\left(\mathbf{W}_c\boldsymbol{\chi}_{x_m,s}(\tau,\nu)\right) - \mathbf{G}_{c,m}\boldsymbol{\zeta}_c \right\|^2$$
$$(12)$$

where
$$\mathbf{G}_{c,m} =$$

$$\left[\mathbf{vec}\left(\mathbf{W}_c\hat{\mathbf{A}}_{m,i}\right), ..., \mathbf{vec}\left(\mathbf{W}_c\hat{\mathbf{A}}_{m,i+d_c-1}\right)\right], \ i \in S_c, \quad (13)$$

and $\boldsymbol{\zeta}_c$ is the amplitude vector, which minimizes (12), given by

$$\hat{\boldsymbol{\zeta}}_c = \frac{1}{M} \sum_{m=1}^{M} \left(\mathbf{G}_{c,m}^H\mathbf{G}_{c,m}\right)^{-1} \mathbf{G}_{c,m}^H \mathbf{vec}\left(\mathbf{W}_c\boldsymbol{\chi}_{x_m,s}(\tau,\nu)\right) \ . $$
$$(14)$$

After substituting (14) into (12), channel parameter estimates for the cth cluster, $\boldsymbol{\varphi}(S_c)$, can be obtained as the minimum of the optimization problem over the remaining variables τ, ν, θ, ϕ using PSO.

6. SIMULATION RESULTS

In this section, performances of the PSO-CAF, SAGE and PSO-ML algorithms are compared on synthetic signals at different SNR values by using Monte Carlo simulations. PSO-ML is a recently developed PSO based algorithm, which searches parameter estimates using (5) [8]. For comparison reasons, the joint root-mean squared error (rMSE), is defined as:

$$\text{rMSE} = \sqrt{\frac{1}{dN_r} \sum_{\mu=1}^{N_r} \sum_{i=1}^{d} [\hat{\boldsymbol{\varphi}}_i^\mu - \boldsymbol{\varphi}_i^\mu]^2} \ , \quad (15)$$

where N_r is the number of Monte-Carlo simulations, $\hat{\boldsymbol{\varphi}}_i^\mu$ is the parameter estimates of the ith signal path found in the μth simulation and $\boldsymbol{\varphi}_i^\mu$ is the true parameter values of the ith path. A circular receiver array of M omnidirectional sensors at positions $[r\cos(m2\pi/M), r\sin(m2\pi/M)]$, $1 \leq \ldots \leq M$, is synthesized. The radius of the array $r = \lambda/4\sin(\pi/M)$ is chosen such that the distance between two neighboring sensors is $\lambda/2$, where λ is the carrier wavelength. The transmitted training signal consists of 6 Barker-13 coded pulses with a duration of $13\Delta\tau$ where $\Delta\tau$ is the chip duration. The pulse repetition interval is $30\Delta\tau$ resulting a total signal duration of $qT = 167\Delta\tau$. The SNR is defined at a single sensor relative to the noise variance.

In the experiment, there exists 10 equal power paths with parameter values $\boldsymbol{\theta} = [45, 50, 55, 60, 65, 70, 75, 57, 63, 68]^o$, $\boldsymbol{\phi} = [30, 35, 40, 45, 50, 55, 38, 47, 43, 33]^o$, $\boldsymbol{\tau} = \Delta\tau.[1, 1.25,$
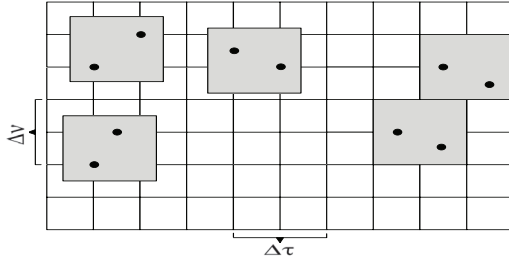
Fig. 2. 10 signal paths on delay-Doppler domain.

$1, 1.5, 2.5, 3, 4.25, 4.75, 4.75, 5.25]$, $\boldsymbol{\nu} = \Delta\nu.[1, 1.5, 2.5, 3, 2.75, 2.5, 1.5, 1.25, 2.5, 2.25]$. Position of each path on CAF surface is seen in Fig. 2. Notice that, each path has 4 parameters. Therefore, PSO-ML search for the path parameters in a 40-dimensional space and PSO-CAF sequentially searches five 8-dimensional spaces. Although the number of paths d is assumed to be known, it can also be estimated in our framework by adding extra dimensionality to PSO search and selecting the dominant scaling factors ζ. Moreover, there are excellent techniques to determine the number of paths [10], [11]. Same PSO settings are used for both PSO-CAF and PSO-ML as: *swarm size* $= 40$, $\kappa = 0.72984$, $c_1 = c_2 = 2.05$. Necessary number of PSO evaluations are conducted for both techniques to ensure the convergence. The joint-rMSE obtained from 100 Monte Carlo runs at each SNR is shown in Fig. 3. Obtained results show that PSO-CAF outperforms both SAGE and PSO-ML techniques significantly at all SNR values. Even at high SNR values, due to the existence of closely spaced clustered paths, SAGE and PSO-ML techniques fail to separate paths.

7. CONCLUSION

A new array signal processing technique operating in the CAF domain and using PSO is proposed for the estimation of multipath channel parameters. The PSO-CAF technique provides estimates to the channel parameters in a sequential search over lower dimensional spaces. Simulation results show that the PSO-CAF provides significantly better parameter estimates than the SAGE and recently proposed PSO-ML technique.

8. REFERENCES

[1] H. Krim and M. Viberg, "Two decades of array signal processing research: the parametric approach," *IEEE Signal Process. Mag.*, vol. 13, no. 4, pp. 67–94, Jul. 1996.

[2] B. H. Fleury, M. Tschudin, R. Heddergott, D. Dahlhaus, and K. I. Pedersen, "Channel parameter estimation in mobile radio environments using the SAGE algorithm," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 3, pp. 434–450, Mar. 1999.

[3] I. Ziskind and M. Wax, "Maximum likelihood localization of multiple sources by alternating projection," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 36, no. 10, pp. 1553–1560, Oct. 1988.

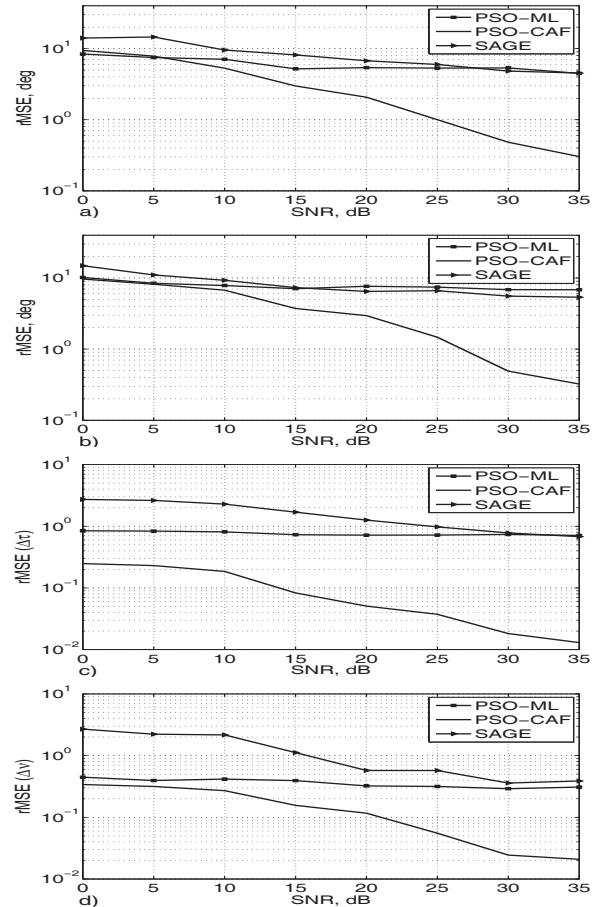[4] K. C. Sharman, "Maximum likelihood parameter estimation by

Fig. 3. Joint-rMSE, obtained with the PSO-CAF, the SAGE and the PSO-ML, of a-)θ, b-)ϕ, c-)τ, d-)ν of 10 signal paths.

simulated annealing," in *IEEE Int. Conf. Acoust. Speech Sign. Processing (ICASSP)*, 1988.

[5] J. A. Fessler and A. O. Hero, "Space-alternating generalized expectation-maximization algorithm," *IEEE Trans. Signal Process.*, vol. 42, no. 10, pp. 2664–2677, Oct. 1994.

[6] J. Kennedy and R. Eberhart, "Particle swarm optimization," in *IEEE Int. Conf. Neural Networks*, 1995.

[7] P. E. O. Yumbla, J. M. Ramirez, and C. A. C. Coello, "Optimal power flow subject to security constraints solved with a particle swarm optimizer," *IEEE Trans. Power Syst.*, vol. 23, no. 1, pp. 33–40, Feb. 2008.

[8] M. Li and Y. Lu, "Maximum likelihood DOA estimation in unknown colored noise fields," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 44, no. 3, pp. 1079–1090, Jul. 2008.

[9] M. B. Guldogan and O. Arikan, "A novel array signal processing technique for multipath channel parameter estimation," in *IEEE Signal Process. and Commun. Applications (SIU)*, 2007.

[10] M. Wax and T. Kailath, "Detection of signals by information theoretic criteria," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 33, no. 2, pp. 387–392, Apr. 1985.

[11] R. F. Brcich, A. M. Zoubir, and P. Pelin, "Detection of sources using bootstrap techniques," *IEEE Trans. Signal Process.*, vol. 50, no. 2, pp. 206–215, Feb. 2002.