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ABSTRACT

The influence of the Rashba spin-orbit coupling on the two-dimensional (2D) electrons and holes in a strong perpendicular magnetic field leads to different results of the Landau quantization in different spin projections. In Landau gauge the unidimensional wave vector describing the free motion in one in-plane direction is the same for both spin projections, whereas the numbers of the Landau quantization levels are different. For electron in *s*-type conduction band they differ by one, as was established earlier by Rashba[1], whereas for heavy holes in *p*-type valence band influenced by the 2D symmetry of the layer they differ by three. There are two lowest spin-splitted Landau levels for electrons as well as two lowest for holes. They give rise to four lowest energy levels of the 2D magnetoexcitons. It is shown that two of them are dipole-active in band-to-band quantum transitions, one is quadrupole-active and the fourth is forbidden. The optical orientation under the influence of the circularly polarized light leads to optical alignment of the magnetoexcitons with different orbital momentum projections on the direction of the external magnetic field.

Keywords: magnetoexcitons, Rashba spin-orbit splitting, Landau quantization.

1. INTRODUCTION

The influence of the spin-orbit coupling (SOC) on the two-dimensional (2D) Wannier-Mott excitons in double quantum well (DQW) structures, as well as the possibilities of the nonconventional electron-hole (e-h) pairing in these conditions were discussed in Ref.[2,3]. The main results are the breaking of the spin degeneracy of the electrons and holes, the changes of the exciton structure, and new properties of the Bose-Einstein condensed excitons. There are two types of SOC. One of them described by Dresselhaus[4] is known to be intrinsically present in zinc-blende structure. The Rashba spin-orbit coupling (RSOC)[1,5] depends on the electric field strength E_z perpendicular to the layer surface.

As was mentioned in Ref.[6,7] the Rashba model can be described by purely group theoretical means. For electron in *s*-like conduction band the total angular momentum with spin-orbit interaction equals to $j = 1/2$. Both wave vectors $\vec{k}$ and electric strength $\vec{E}$ are polar vectors, whereas their cross product $\left[\vec{k}\times\vec{E}\right]$ is an axial vector. Its point product with the spin axial vector $\vec{\sigma}$ gives rise to the triple scalar product $\left[\vec{k}\times\vec{E}\right]\cdot\vec{\sigma}$. This expression is an invariant under the action of the group symmetry elements forming identity representation Γ_1. In the first quantization representation the wave vector $\vec{k}$ is substituted by $-i\vec{\nabla}$. In the Γ_6-type conduction band the triple scalar product is the only term of the first order on $\vec{\nabla}$ and $\vec{E}$ compatible with the symmetry of the band.

The band structure described by the Hamiltonian with RSOC

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$$H_e = -\frac{\hbar^2 \Delta_{\parallel}}{2m_e}\hat{I} - i\tilde{\alpha}\left(\hat{\sigma}_x \frac{\partial}{\partial y} - \hat{\sigma}_y \frac{\partial}{\partial x}\right); \Delta_{\parallel} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}; \ \tilde{\alpha} = \alpha_e E_z \tag{1}$$

has the dispersion laws

$$E_e^{\pm} = \frac{\hbar^2 k_{\parallel}^2}{2m_e} \pm |\tilde{\alpha}| k_{\parallel}; k_{\parallel} = \sqrt{k_x^2 + k_y^2}. \tag{2}$$

One of them contains the loop of minima[1, 5]. The topmost valence band in our case is *p*-like with orbital quantum number $l = 1$ and with the total angular momentum equal to $j = 3/2$. The four-fold band states give rise to heavy and light holes forming in cubic crystals the irreducible representation Γ_8 in the point $k = 0$.

For the LH the effective Rashba Hamiltonian has the lowest order in $\vec{k}$ term and is the same as for the conduction electrons. For the HH the effective Rashba Hamiltonian happens to be the third order in $\vec{k}$ and remains the first order in spin operators $\vec{\sigma}$ as follows[6, 7]

$$H_h^{SOC} = \beta_h E_z \left(\hat{\sigma}_+ k_-^3 + \hat{\sigma}_- k_+^3\right), \tag{3}$$

where

$$\sigma_{\pm} = \frac{1}{2}\left(\hat{\sigma}_x \pm i\hat{\sigma}_y\right); k_{\pm} = \left(k_x \pm ik_y\right);$$
$$\hat{\sigma}_x = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}; \hat{\sigma}_y = \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix}; \hat{\sigma}_z = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}; \hat{I} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}. \tag{4}$$

The electric field strength E_z depends on the density of charges in the system[6, 7]. The interaction constants were evaluated in[2, 3] for different values of E_z, arriving to the conclusion that at $E_z = 100 \div 200 \frac{kV}{cm}$ the RSOC is a dominant mechanism for the energy band spin splitting.

2. THE LANDAU QUANTIZATION OF 2D ELECTRONS AND HOLES IN THE PRESENCE OF RSOC

Following the papers[1–7] the full Hamiltonians describing the Landau quantization of the 2D electrons and holes in a strong perpendicular magnetic field taking into account the RSOC consist from two parts. First of them are the zero order Hamiltonians for electrons and holes in a strong perpendicular magnetic field

$$H_e^0 = \frac{\hbar^2}{2m_e}\left(\hat{P}_e + \frac{|e|}{c}\vec{A}(\vec{r}_e)\right)^2; H_h^0 = \frac{\hbar^2}{2m_h}\left(\hat{P}_h - \frac{|e|}{c}\vec{A}(\vec{r}_h)\right)^2, \tag{5}$$

where $\hat{P}_e$ and $\hat{P}_h$ are 2D momenta equal to $-ih\vec{\nabla}_e$ and $-ih\vec{\nabla}_h$ correspondingly and the vector potential $\vec{A}(\vec{r})$ is written in Landau gauge, i.e. $\vec{A}(\vec{r}) = (A_x = -Hy; A_y = 0; A_z = 0)$. The second parts of the full Hamiltonians are the RSOC Hamiltonians (1) and (3), in which instead of usual momenta must be introduced the kinematic momenta

$$\hat{P}_e + \frac{|e|}{c}\vec{A}(\vec{r}_e) \text{ and } \hat{P}_h - \frac{|e|}{c}\vec{A}(\vec{r}_h), \tag{6}$$

what is equivalent to write instead of k_x^e, k_y^e, k_x^h and k_y^h the new expressions

$$K_x^e = \left(-i\frac{\partial}{\partial x_e} - \frac{y_e}{l^2}\right); K_y^e = -i\frac{\partial}{\partial y_e};$$
$$K_x^h = \left(-i\frac{\partial}{\partial x_h} + \frac{y_h}{l^2}\right); K_y^h = -i\frac{\partial}{\partial y_h}; l^2 = \frac{\hbar c}{|e| H} \tag{7}$$

correspondingly. l is the magnetic length. The new operators K_x and K_y do not commute

$$\left[K_x^e, K_y^e\right] = -\frac{i}{l^2}; \left[K_x^h, K_y^h\right] = \frac{i}{l^2}. \tag{8}$$

The RSOC Hamiltonians for 2D electrons and holes in a strong perpendicular magnetic field have the forms

$$H_e^{SOC} = \alpha_e E_z \left[\hat{\sigma}_x K_y^e - \hat{\sigma}_y K_x^e \right]; H_h^{SOC} = -\beta_h E_z$$
$$\times \{ \hat{\sigma}_x \left[K_y^{h^3} - \left(K_x^{h^2} K_y^h + K_y^h K_x^{h^2} + K_x^h K_y^h K_x^h \right) \right]$$
$$+ \hat{\sigma}_y \left[K_x^{h^3} - \left(K_y^{h^2} K_x^h + K_x^h K_y^{h^2} + K_y^h K_x^h K_y^h \right) \right] \}. \tag{9}$$

The full Hamiltonians are

$$H_e = H_e^0 + H_e^{SOC}; H_h = H_h^0 + H_h^{SOC}. \tag{10}$$

The solutions of the Landau quantization task for electrons and holes are chosen in the forms[1, 5]

$$\left| \Psi_e(R, p; x, y) \right\rangle = \frac{e^{ipx}}{\sqrt{L_x}} \begin{vmatrix} \Phi_1(y,p) \\ \Phi_2(y,p) \end{vmatrix};$$

$$\langle \Psi_e(R, p; x, y) | = \frac{e^{-ipx}}{\sqrt{L_x}} \begin{vmatrix} \Phi_1^*(y,p) & \Phi_2^*(y,p) \end{vmatrix};$$

$$\left| \Psi_h(R, q; x, y) \right\rangle = \frac{e^{iqx}}{\sqrt{L_x}} \begin{vmatrix} f_1(y,q) \\ f_2(y,q) \end{vmatrix};$$

$$\langle \Psi_h(R, q; x, y) | = \frac{e^{-iqx}}{\sqrt{L_x}} \begin{vmatrix} f_1^*(y,q) & f_2^*(y,q) \end{vmatrix}. \tag{11}$$

The actions of the operators K_x^e on the function e^{ipx_e} and of the operator K_x^h on the function e^{iqx_h} are

$$K_x^e \exp(ipx_e) = \left(p - \frac{y_e}{l^2} \right) e^{ipx_e}; K_x^h \exp(iqx_h) = \left(q + \frac{y_h}{l^2} \right) e^{iqx_h}. \tag{12}$$

Instead of variables y_e and y_h we will introduce the dimensionless variables

$$\eta_e = \frac{y_e}{l} - pl; \eta_h = \frac{y_h}{l} + ql. \tag{13}$$

The Schrödinger equations on the base of the full Hamiltonians (10) with the solutions in the forms (11) depend on two variables x and y. But taking into account the translational symmetry in direction x and the relations (12) these equations can be transformed in Schrödinger equations depending only on one variable. The cyclotron energies for electron and hole are $\hbar\omega_{ci} = \frac{\hbar|e|H}{m_i c}$; $i = e, h$. The Schrödinger equation describing the Landau quantization for electron looks as

$$\frac{1}{2}\hbar\omega_{ce}\left(\eta_e^2 - \frac{\partial^2}{\partial\eta_e^2}\right)\begin{vmatrix} \Phi_1(\eta_e) \\ \Phi_2(\eta_e) \end{vmatrix} - \frac{\alpha_e E_z}{l}\left(i\hat{\sigma}_x \frac{\partial}{\partial\eta_e} - \hat{\sigma}_y \eta_e \right)\begin{vmatrix} \Phi_1(\eta_e) \\ \Phi_2(\eta_e) \end{vmatrix} = \varepsilon_e \begin{vmatrix} \Phi_1(\eta_e) \\ \Phi_2(\eta_e) \end{vmatrix}. \tag{14}$$

Introducing the dimensionless energy and SOC constant α

$$\varepsilon_e = \frac{\varepsilon_e}{\hbar\omega_{ce}}; \alpha = \frac{\alpha_e E_z}{l\hbar\omega_{ce}} \tag{15}$$

we can transcribe the two-component equation (14) as follows

$$\frac{1}{2}\left(\eta_e^2 - \frac{\partial^2}{\partial\eta_e^2}\right)\Phi_1(\eta_e) - i\alpha\left(\eta_e + \frac{\partial}{\partial\eta_e}\right)\Phi_2(\eta_e) = \varepsilon_e \Phi_1(\eta_e);$$
$$\frac{1}{2}\left(\eta_e^2 - \frac{\partial^2}{\partial\eta_e^2}\right)\Phi_2(\eta_e) + i\alpha\left(\eta_e - \frac{\partial}{\partial\eta_e}\right)\Phi_1(\eta_e) = \varepsilon_e \Phi_2(\eta_e). \tag{16}$$

Acting in the same way we will obtain the one variable hole Hamiltonian

$$H_h(\eta_h) = \frac{1}{2}\hbar\omega_{ch}\left(\eta_h^2 - \frac{\partial^2}{\partial\eta_h^2}\right)\hat{I}$$

$$-\frac{\beta_h E_z}{l^3}\left\{i\hat{\sigma}_x\left(\frac{\partial^3}{\partial\eta_h^3}+3\eta_h^2\frac{\partial}{\partial\eta_h}+3\eta_h\right)+\hat{\sigma}_y\left(\eta_h^3+3\eta_h\frac{\partial^2}{\partial\eta_h^2}+3\frac{\partial}{\partial\eta_h}\right)\right\}. \quad (17)$$

The notations $\varepsilon_h=\frac{\varepsilon_h}{\hbar\omega_{ch}}$, $\beta=\frac{\beta_h E_z}{l^3\hbar\omega_{ch}}$ and the relation

$$\left(\eta\pm\frac{\partial}{\partial\eta}\right)^3=\eta^3\pm\frac{\partial^3}{\partial\eta^3}+3\left(\eta\frac{\partial^2}{\partial\eta^2}\pm\eta^2\frac{\partial}{\partial\eta}\right)+3\left(\eta\pm\frac{\partial}{\partial\eta}\right) \quad (18)$$

permits to simplify essentially the two component Schrödinger equation

$$\frac{1}{2}\left(\eta_h^2-\frac{\partial^2}{\partial\eta_h^2}\right)f_1(\eta_h)+i\beta\left(\eta_h-\frac{\partial}{\partial\eta_h}\right)^3 f_2(\eta_h)=\varepsilon_h f_1(\eta_h);$$

$$\frac{1}{2}\left(\eta_h^2-\frac{\partial^2}{\partial\eta_h^2}\right)f_2(\eta_h)-i\beta\left(\eta_h+\frac{\partial}{\partial\eta_h}\right)^3 f_1(\eta_h)=\varepsilon_h f_2(\eta_h). \quad (19)$$

Rashba[1] proposed the solution of equations (16) using the series expansions for the functions Φ_1 and Φ_2. We will use the same representations as follows

$$\Phi_1(\eta)=\sum_{n=0}^{\infty}a_n\varphi_n(\eta);\Phi_2(\eta)=\sum_{n=0}^{\infty}b_n\varphi_n(\eta);$$

$$f_1(\eta)=\sum_{n=0}^{\infty}c_n\varphi_n(\eta);f_2(\eta)=\sum_{n=0}^{\infty}d_n\varphi_n(\eta), \quad (20)$$

where $\varphi_n(\eta)$ are the eigenfunctions of the Landau quantization in Landau gauge with the orthogonality and normalization conditions

$$l\int_{-\infty}^{\infty}d\eta\varphi_n^*(\eta)\varphi_m(\eta)=\delta_{nm};\sum_n|a_n|^2+\sum_n|n_n|^2=1;\sum_n|c_n|^2+\sum_n|d_n|^2=1. \quad (21)$$

They obey to the differential equations

$$\left(\eta^2-\frac{\partial^2}{\partial\eta^2}\right)\varphi_n(\eta)=(2n+1)\varphi_n(\eta),$$

$$\frac{1}{\sqrt{2}}\left(\eta-\frac{\partial}{\partial\eta}\right)\varphi_n(\eta)=\sqrt{(n+1)}\varphi_{n+1}(\eta),$$

$$\frac{1}{\sqrt{2}}\left(\eta+\frac{\partial}{\partial\eta}\right)\varphi_n(\eta)=\sqrt{n}\varphi_{n-1}(\eta),$$

$$\left(\eta-\frac{\partial}{\partial\eta}\right)^3\varphi_n(\eta)=2\sqrt{2}\sqrt{(n+1)(n+2)(n+3)}\varphi_{n+3}(\eta),$$

$$\left(\eta+\frac{\partial}{\partial\eta}\right)^3\varphi_n(\eta)=2\sqrt{2}\sqrt{n(n-1)(n-2)}\varphi_{n-3}(\eta). \quad (22)$$

As one can see the expressions $\frac{1}{\sqrt{2}}\left(\eta-\frac{\partial}{\partial\eta}\right)$ and $\frac{1}{\sqrt{2}}\left(\eta+\frac{\partial}{\partial\eta}\right)$ play the role of increasing and decreasing differential operators. The equalities (22) transform the Schrödinger equations (16) and (19) into the linear relations between the Landau quantization functions. They are

$$\sum_{n=0}^{\infty}a_n\left(n+\frac{1}{2}-\varepsilon_e\right)\varphi_n(\eta)-i\alpha\sum_{n=0}^{\infty}b_n\sqrt{2}\sqrt{n}\varphi_{n-1}(\eta)=0;$$

$$\sum_{n=0}^{\infty}b_n\left(n+\frac{1}{2}-\varepsilon_e\right)\varphi_n(\eta)+i\alpha\sum_{n=0}^{\infty}a_n\sqrt{2}\sqrt{n+1}\varphi_{n+1}(\eta)=0 \quad (23)$$

for 2D conduction electron, and

$$\sum_{n=0}^{\infty}c_n\left(n+\frac{1}{2}-\varepsilon_h\right)\varphi_n(\eta)+i\beta 2\sqrt{2}\sum_{n=0}^{\infty}d_n\sqrt{(n+1)(n+2)(n+3)}\varphi_{n+3}(\eta)=0;$$

$$\sum_{n=0}^{\infty} d_n\left(n+\frac{1}{2}-\varepsilon_h\right)\varphi_n(\eta)-i\beta 2\sqrt{2}\sum_{n=0}^{\infty} c_n\sqrt{n(n-1)(n-2)}\varphi_{n-3}(\eta)=0 \tag{24}$$

for 2D heavy holes.

Multiplying these equations by $\varphi_s^*(\eta)$, where $s=0,1,2,...$, after the integration on the variable η in accordance with the condition (21), we will obtain the linear algebraic equations.

In the case of the 2D heavy holes the algebraic linear equations are

$$\begin{gathered}
c_0\left(\frac{1}{2}-\varepsilon_h\right)=0; c_1\left(\frac{3}{2}-\varepsilon_h\right)=0; c_2\left(\frac{5}{2}-\varepsilon_h\right)=0; \\
c_3\left(\frac{7}{2}-\varepsilon_h\right)=-i\beta 4\sqrt{3}d_0; d_0\left(\frac{1}{2}-\varepsilon_h\right)=i\beta 4\sqrt{3}c_3; \\
c_4\left(\frac{9}{2}-\varepsilon_h\right)=-i\beta 8\sqrt{3}d_1; d_1\left(\frac{3}{2}-\varepsilon_h\right)=i\beta 8\sqrt{3}c_4; \\
c_5\left(\frac{11}{2}-\varepsilon_h\right)=-i\beta 4\sqrt{30}d_2; d_2\left(\frac{5}{2}-\varepsilon_h\right)=i\beta 4\sqrt{30}c_5; \\
c_6\left(\frac{13}{2}-\varepsilon_h\right)=-i\beta 8\sqrt{15}d_3; d_3\left(\frac{7}{2}-\varepsilon_h\right)=i\beta 8\sqrt{15}c_6,...
\end{gathered} \tag{25}$$

and so on. As in the case of conduction electron the solution $\varepsilon_h=\frac{1}{2}$ is accompanied by the coefficients $c_0=1$ and by all another coefficients c_n and d_n equal to zero. The second spin splitted lowest Landau level for a heavy hole has a value $\varepsilon_h\neq\frac{1}{2}$, what leads to the solutions $c_0=c_1=c_2=0$. The fourth and the fifth equations lead to the dispersion equation

$$\left(\frac{7}{2}-\varepsilon_h\right)\left(\frac{1}{2}-\varepsilon_h\right)-48\beta^2=0; \varepsilon_h=2\pm\sqrt{\frac{9}{4}+48\beta^2}; c_3=\frac{-i\beta 4\sqrt{3}d_0}{\frac{3}{2}+\sqrt{\frac{9}{4}+48\beta^2}}. \tag{26}$$

All coefficients except d_0 and c_3 are equal to zero, what leads to the equalities

$$|c_3|^2+|d_0|^2=1; |d_0|^2=\frac{1}{1+\frac{48\beta^2}{\left(\frac{3}{2}+\sqrt{\frac{9}{4}+48\beta^2}\right)^2}}. \tag{27}$$

In the limiting case $\beta^2<\frac{1}{64}$ the second solution is

$$\varepsilon_h \approx \frac{1}{2}-16\beta^2; |d_0|^2\approx 1-\frac{16}{3}\beta^2; |c_3|^2\approx\frac{16}{3}\beta^2. \tag{28}$$

Two spin splitted LLLs for hole are

$$\begin{gathered}
\left|\Psi_h\left(R_3,q;x,y\right)\right\rangle=\frac{e^{iqx}}{\sqrt{L_x}}\begin{vmatrix}c_3\varphi_3(\eta)\\ d_0\varphi_0(\eta)\end{vmatrix}; \varepsilon_h=2-\sqrt{\frac{9}{4}+48\beta^2}; \\
\left|\Psi_h\left(R_4,q;x,y\right)\right\rangle=\frac{e^{iqx}}{\sqrt{L_x}}\begin{vmatrix}\varphi_0(\eta)\\ 0\end{vmatrix}; \varepsilon_h=\frac{1}{2}.
\end{gathered} \tag{29}$$

In difference on the electron SOC parameter $\alpha=\frac{\alpha_e E_z}{l\hbar\omega_{ce}}$, which decreases with increasing magnetic field strength H , the hole SOC parameter $\beta=\frac{\beta_h E_z}{l^3\hbar\omega_{ch}}$ has an inverse dependence, i.e. it increases with the increasing H . Only at small values of E_z and at not so high values of H the parameter β can be considered to be small, i.e. $\beta^2<\frac{1}{64}$.

There are four combinations of the electron and hole energies in the frame of these levels. They are represented in the Fig. 1 and are enumerated below:

$$f_1=(e,R_1;h,R_3);E(f_1)=E_g+\left(\frac{1}{2}-2\alpha^2\right)\hbar\omega_{ce}+\left(\frac{1}{2}-16\beta^2\right)\hbar\omega_{ch};$$

$$f_2=(e,R_1;h,R_4);E(f_2)=E_g+\left(\frac{1}{2}-2\alpha^2\right)\hbar\omega_{ce}+\frac{1}{2}\hbar\omega_{ch};$$

$$f_3=(e,R_2;h,R_3);E(f_3)=E_g+\frac{1}{2}\hbar\omega_{ce}+\left(\frac{1}{2}-16\beta^2\right)\hbar\omega_{ch};$$

$$f_4=(e,R_2;h,R_4);E(f_4)=E_g+\frac{1}{2}\hbar\omega_{ce}+\frac{1}{2}\hbar\omega_{ch}; \tag{30}$$

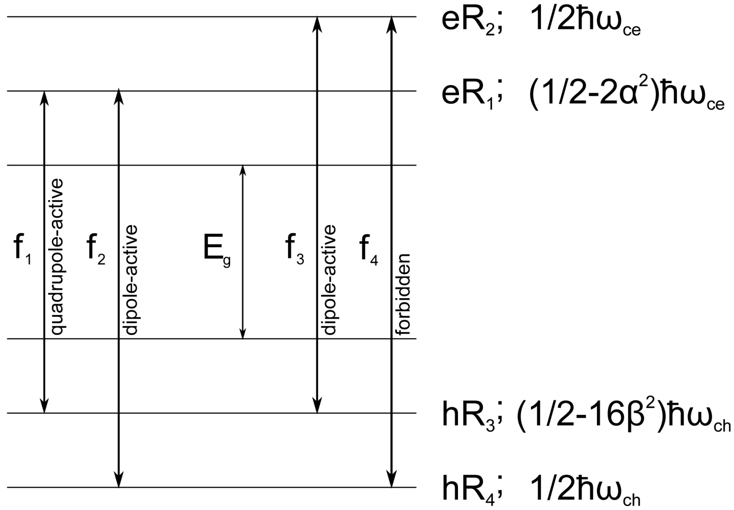

Fig. 1. Energy spectrum of the LLLs for electron and hole taking into account the RSOC. Four different combinations of the electron-hole pair states are represented.

In the next section the matrix elements of the Coulomb e-h interaction will be calculated.

3. THE COULOMB ELECTRON-HOLE INTERACTION, THE ENERGY SPECTRUM OF 2D MAGNETOEXCITONS AND THE BAND-TO-BAND QUANTUM TRANSITIONS

The exciton wave functions in the f_s compositions represented on the Fig. 1 are denoted as

$$\Psi_{ex}(\vec{k},f_s)=\frac{1}{\sqrt{N}}\sum_t e^{-ik_y tl^2}a^{\dagger}_{R_i,\frac{k_x}{2}+t}b^{\dagger}_{R_j,\frac{k_x}{2}-t}\left|0\right\rangle;\quad s=1,2,3,4;\, i=1,2;\; j=3,4, \tag{31}$$

where $a^{\dagger}$ and $b^{\dagger}$ are the creation electron and hole operators.

The average values of the electron-hole Coulomb interaction Hamiltonian H^{e-h}_{Coul} equals to

$$\langle\Psi_{ex}(\vec{k},f_s)|H^{e-h}_{Coul}\left|\Psi_{ex}(\vec{k},f_s)\right\rangle=-I_{ex}(e,R_i;h,R_j;\vec{k});\quad i=1,2;\; j=3,4. \tag{32}$$

The obtained values are represented on the figure 2.

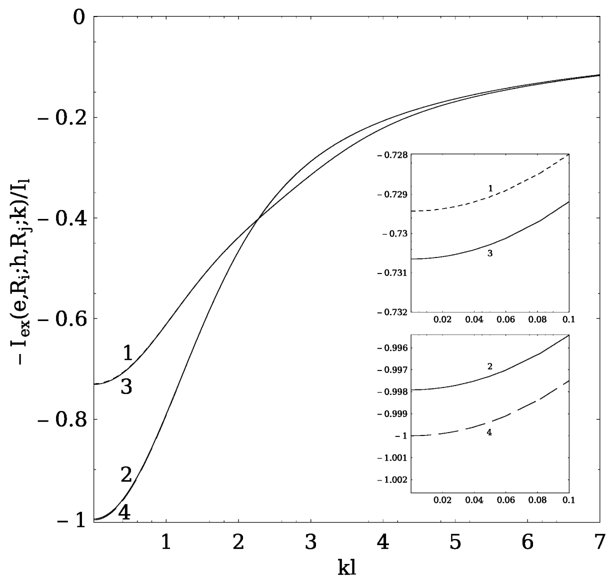

Fig. 2. The ionization potentials $I_{ex}(e,R_i;h,R_j;k)/I_l, i=1,2; j=3,4$ taken with the sign minus for spin-splitted LLLs for electrons and holes at E_z = 24 kV/cm and H = 15 T. The insets demonstrate the datailed dependences of these magnetoexciton states 1) $(e,R_1;h,R_3)$; 2) $(e,R_1;h,R_4)$; 3) $(e,R_2;h,R_3)$; 4) $(e,R_2;h,R_4)$.

The probability of the quantum transition in the exciton state f_1 is different from zero only if the small corrections proportional to $(Q_x+iQ_y)l$ are taken into account. It means that the transition to the exciton state f_1 is proportional to $(Q_x^2+Q_y^2)l^2 = |\vec{Q}_{2D}|^2 l^2$ and the corresponding quantum transition is quadrupole-active, being proportional to the square of the projection $\vec{Q}_{2D}$ on the layer surface of the light wave vector $\vec{Q}$ arbitrary oriented in the 3D space. In the Faraday geometry, when the wave vector $\vec{Q}$ is parallel to the direction of the external perpendicular magnetic field, the projection $\vec{Q}_{2D}=0$ and the quadrupole transition is forbidden. The quantum transitions in the states f_2 and f_3 are dipole active.

3. CONCLUSIONS

The influence of the Rashba spin-orbit coupling (RSOC) on the properties of the 2D magnetoexcitons was determined. The interdependence between the Landau quantization of the electron and hole orbital motions and their spin projections was revealed in the frame of Landau gauge. The spinor-type wave functions of the 2D conduction and valence electrons in the presence of the RSOC have different numbers of the Landau quantization functions for different spin projections. For example, they are $\varphi_0(y)$ and $\varphi_1(y)$ in one case, and $\varphi_3(y)$ and $\varphi_0(y)$ in another one. For conduction electron, if the number of Landau level is n for the up spin projection, it is equal to $n+1$ for the down spin projection. For the valence electron and for the heavy hole (HH) the number n of the Landau level for the down spin projection is accompanied by the number $n+3$ for the up spin projection. It is determined completely by the fact that the RSOC Hamiltonian for conduction electron is linear in the projections $k_\pm = k_x \pm ik_y$ of the in-plane wave vector $\vec{k}_\parallel$, whereas in the case of valent electron and heavy hole the corresponding Hamiltonian contains the third order of these projections, i.e. the expressions $(k_\pm)^3$. Two lowest Landau levels R_1 and R_2 for conduction electron and two lowest hole states R_3 and R_4 were considered. The wave functions were used to calculate the matrix elements of the Coulomb direct and exchange electron-hole (e-h) interactions corresponding to the combinations $f_1=(e,R_1;h,R_3)$; $f_2=(e,R_1;h,R_4)$; $f_3=(e,R_2;h,R_3)$ and $f_4=(e,R_2;h,R_4)$. The corresponding ionization potentials were expressed through the ionization potentials of the bare magnetoexciton states $I_{ex}^{(n,m)}(k)$ calculated earlier in[10]. With their help as well as with the knowledge of the coefficients

d_0, c_3 (28) it is possible to determine the dispersion laws (32) of the four new magnetoexciton bands taking into account the RSOC. The new dispersion laws could lead to new collective properties of the spinor-type 2D magnetoexcitons.

The optical quantum transitions from the ground state of the crystal to four magnetoexciton states were determined on the base of exciton wave functions (31) and electron-radiation interaction. It was shown that the quantum transitions in the states f_2 and f_3 corresponding to combinations $f_2 = (e, R_1; h, R_4)$ and $f_3 = (e, R_2; h, R_3)$ are dipole-active, the exciton state $f_1 = (e, R_1; h, R_3)$ is quadrupole-active, whereas the fourth combination $f_4 = (e, R_2; h, R_4)$ is forbidden. In the Faraday geometry, when the light wave vector $\vec{Q}$ is oriented along the magnetic field direction, the circular polarizations $\vec{\sigma}_{\vec{Q}}^{\pm}$ coincide with the exciton circular polarization $\vec{\sigma}_{\pm 1}$. The light circular polarization $\vec{\sigma}_{\vec{Q}}^{\mp}$ excites the exciton states $\vec{\sigma}_{\mp 1}$ because $\left(\vec{\sigma}_{\vec{Q}}^{\mp *} \cdot \vec{\sigma}_{\mp 1}\right) = \left(\vec{\sigma}_{\vec{Q}}^{\pm} \cdot \vec{\sigma}_{\mp 1}\right) = 1$, whereas $\left(\vec{\sigma}_{\vec{Q}}^{\pm *} \cdot \vec{\sigma}_{\mp 1}\right) = 0$. Such optical orientation of the exciton states under the influence of the circularly polarized light is named as optical alignment[13]. In $\vec{\sigma}_{\vec{Q}}^{-}$ polarization only the dipole-active quantum transition f_2 is allowed, whereas the dipole-active quantum transition f_3 and the quadrupole-active quantum transition f_1 are forbidden. In Faraday geometry and circular polarization $\vec{\sigma}_{\vec{Q}}^{+}$ the exciton states f_1 and f_3 having the circular polarization $\vec{\sigma}_1$ are allowed. One is dipole-active and another one is quadrupole-active.

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