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IFAC-PapersOnLine 48-12 (2015) 422–427

On the Discrete Adaptive Posicast Controller

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Abstract: In this paper, we present the discrete version of the Adaptive Posicast Controller (APC) that deals with parametric uncertainties in systems with input time-delays. The continuous-time APC is based on the Smith Predictor and Finite Spectrum Assignment with time-varying parameters adjusted online. Although the continuous-time APC showed dramatic performance improvements in experimental studies with internal combustion engines, the full benefits could not be realized since the finite integral term in the control law had to be approximated in computer implementation. It is shown in the literature that integral approximation in time-delay compensating controllers degrades the performance if care is not taken. In this work, we present a development of the APC in the discrete-time domain, eliminating the need for approximation. In essence, this paper attempts to present a unified development of the discrete-time APC for systems that are linear with known/unknown input time-delays. Performances of the continuous-time and discrete-time APC, as well as conventional Model Reference Adaptive Controller (MRAC) for linear systems with known time-delay are compared in simulation studies. It is shown that discrete-time APC outperforms its continuous-time counterpart and MRAC. Further simulations studies are also presented to show the performance of the design for systems with uncertain time-delay.

Keywords: discrete time systems, time-delay systems, adaptive control, robust control, Lyapunov stability.

1. INTRODUCTION

Adaptive Posicast Controller (APC) Yildiz et al. (2010) is a model reference adaptive controller for linear time invariant plants with known input time delays. Basic building blocks of this controller are the celebrated Smith Predictor Smith (1959) the finite spectrum assignment controller (FSA) Wang et al. (1999) and Manitius & Olbrot (1979) and the adaptive controller developed by Ortega & Lozano (1988) and Niculescu & Annaswamy (2003). APC has proved to be a powerful candidate for time-delay systems control both in simulation and experimental works. Successful experimental implementations include spark ignition engine idle speed control Yildiz et al. (2007) and fuel-to-air ratio control Yildiz et al. (2008) while simulation implementation on flight control is presented in Yildiz (2010). Recently, an extension of APC using combined/composite model reference adaptive control is presented Dydek et al. (2010). Although APC has successfully been implemented in various domains with considerable performance improvements, the premise of time-delay compensation using future output prediction, as proven by the theory, had to be approximately realized in these applications. The main reason behind this was that the APC had to be implemented using a microprocessor and therefore all the terms in the control laws had to be digitally approximated. This is a conventional approach in many control implementations and in most of the cases works perfectly well as long as the sampling is fast enough. One exception to this rule is the implementation of the finite spectrum assignment (FSA) controller. It is shown in Wang et al. (1999) that, as the sampling frequency increases, the phase margin of the FSA controller decreases. A remedy to this problem is provided in Mondie & Michiels (2003). Since APC is based on FSA controller, fast sampling to achieve good approximation of the continuous control laws may degrade the system performance.

To eliminate the need for approximation and, therefore, to exploit the full benefits of APC, a fully discrete time APC design is provided in this paper. A Lyapunov stability proof is given and the discrete APC is compared with its continuous counterpart in the simulation environment. A comparison with a conventional model reference adaptive controller is also provided. As expected, simulation results verify the advantage of developing the controller in the discrete domain over a continuous-time development followed by a discrete approximation.

There are already many successful methods proposed in the literature to compensate the effect of time-delays in continuous-time control systems. Among them, the very recent ones are presented in Mazenc & Niculescu (2011) and Krstic (2010). To see an analysis of robustness of nonlinear predictive laws to delay perturbations and a

10.1016/j.ifacol.2015.09.415

comprehensive list of delay-compensating controllers see Bekiaris-Liberis & Krstic (2013). Also, Krstic (2009) is a very recent important contribution to the field presenting predictive feedback in delay systems with extensions to nonlinear systems, delay-adaptive control and actuator dynamics modeled by PDEs.

In the discrete time domain, there are various solutions to model reference adaptive control problem with the natural inclusion of time delay Goodwin et al. (1980), Kokotovic (1991), and Akhtar & Bernstein (2004). The main contribution of the discrete time APC is that in the controller development, future state estimation, i.e. predictor feedback, is explicit, which helped the extension of the method to the control of uncertain input time-delay cases, in the discrete-time domain. It is noted that recently, uncertain input delay case is solved for the continuous time systems without approximating the delay in Bresch-Pietri & Krstic (2009). A preliminary result of this work is presented in Abidi & Yildiz (2011) without the extension to the uncertain time-delay problem. In Abidi & Xu (2015), an extension to nonlinear systems is presented along with more detailed proofs.

The organization of this paper is as follows: Section 2 gives the Problem Statement. Section 3 gives the Discrete-Time Adaptive Posicast Controller Design. Section 4 gives the Extension to Uncertain Upper Bounded Time-Delay. Section 5 gives the Simulation Examples. Section 6 gives the Conclusion.

2. PROBLEM STATEMENT

Consider a continuous-time plant given as

$$\begin{aligned}\dot{\mathbf{x}}(t) &= A\mathbf{x}(t) + B_{\mathrm{n}}\Lambda\mathbf{u}(t-\tau)\\ \mathbf{y}(t) &= C^T\mathbf{x}(t)\end{aligned} \tag{1}$$

where $\mathbf{x} \in \Re^n$ is the state vector, $A \in \Re^{n\times n}$ is a constant uncertain matrix, $B_{\mathrm{n}} \in \Re^{n\times m}$ is a constant known matrix, $\Lambda \in \Re^{m\times m}$ is a constant uncertain positive definite matrix, $\mathbf{u} \in \Re^m$ is the vector of the control inputs, $\tau \geq 0$ is the input time-delay, and $\mathbf{y} \in \Re^m$ is the plant output and $C \in \Re^{n\times m}$ is the output matrix. For the plant (1), the following assumptions are made:

Assumption 1. Input time-delay τ is known.

Assumption 2. Plant (1) is minimum-phase.

Suppose that the reference model is given as

$$\dot{\mathbf{x}}_{\mathrm{m}}(t) = A_{\mathrm{m}}\mathbf{x}_{\mathrm{m}} + B_{\mathrm{m}}\mathbf{r}(t-\tau) \tag{2}$$

where $A_{\mathrm{m}} \in \Re^{n\times n}$ is a constant Hurwitz matrix, $B_{\mathrm{m}} \in \Re^{n\times m}$ is a constant matrix and $\mathbf{r}$ is the desired reference command. The control problem is finding a bounded control input $\mathbf{u}$ such that $\lim_{t\to\infty}\|\mathbf{x}(t)-\mathbf{x}_{\mathrm{m}}(t)\| = 0$, while keeping all the system signals bounded.

3. DISCRETE-TIME ADAPTIVE POSICAST CONTROLLER DESIGN

In this section the discrete-time design of the APC will be presented. Consider the sampled-data form of (1) given by

$$\begin{aligned}\mathbf{x}_{k+1} &= \Phi\mathbf{x}_k + \Gamma\mathbf{u}_{k-p}\\ \mathbf{y}_k &= C^T\mathbf{x}_k\end{aligned} \tag{3}$$

where the matrices $\Phi \in \Re^{n\times n}$, $\Gamma \in \Re^{n\times m}$ are uncertain and p is selected such that $\tau = pT$ where T is the sampling interval.

Assumption 3. The time-delay p is known.

Assumption 4. The plant (3) is minimum-phase.

Assumption 5. The matrix $C^T\Gamma_{\mathrm{n}}$ is non-singular.

Consider the sampled-data form of the reference model (2)

$$\begin{aligned}\mathbf{x}_{\mathrm{m},k+1} &= \Phi_{\mathrm{m}}\mathbf{x}_{\mathrm{m},k} + \Gamma_{\mathrm{m}}\mathbf{r}_{k-p}\\ \mathbf{y}_{\mathrm{m},k} &= C^T\mathbf{x}_{\mathrm{m},k}.\end{aligned} \tag{4}$$

As in the continuous-time problem, the objective is to force the plant (3) to track the reference model (4) and thereby achieve $\lim_{k\to\infty}\mathbf{x}_k = \mathbf{x}_{\mathrm{m},k}$. The reference model (4) is designed by using the nominal values of the plant parameters. In other words, assuming that there exists a Φ_{n} and Γ_{n} that are equal to Φ and Γ without uncertainty.

Consider initially that Φ and Γ are known, in order to derive the controller, subtract (4) from (3) to obtain

$$\mathbf{x}_{k+1} - \mathbf{x}_{\mathrm{m},k+1} = \Phi\mathbf{x}_k - \Phi_{\mathrm{m}}\mathbf{x}_{\mathrm{m},k} + \Gamma\mathbf{u}_{k-p} - \Gamma_{\mathrm{m}}\mathbf{r}_{k-p}. \tag{5}$$

Further, the term $\Phi_{\mathrm{m}}\mathbf{x}_k$ is added and subtracted on the right hand side of (5) to obtain

$$\mathbf{e}_{k+1} = \Phi_{\mathrm{m}}\mathbf{e}_k + (\Phi - \Phi_{\mathrm{m}})\,\mathbf{x}_k + \Gamma\mathbf{u}_{k-p} - \Gamma_{\mathrm{m}}\mathbf{r}_{k-p}. \tag{6}$$

where $\mathbf{e}_k = \mathbf{x}_k - \mathbf{x}_{\mathrm{m},k}$. The goal is to have $\lim_{k\to\infty}\mathbf{x}_k = \mathbf{x}_{\mathrm{m},k}$ or in other words $\lim_{k\to\infty}\mathbf{e}_k = 0$, therefore, assuming that there exists a $\Theta \in \Re^{m\times n}$ and a positive-definite $\Theta_\gamma \in \Re^{m\times m}$ such that

$$\Phi - \Gamma_{\mathrm{n}}\Theta = \Phi_{\mathrm{m}} \;\&\; \Gamma = \Gamma_{\mathrm{n}}\Theta_\gamma \tag{7}$$

it is possible to construct a control law

$$\mathbf{u}_k = -\Theta_\gamma^{-1}\left(\Theta\mathbf{x}_{k+p} - \Theta_{\mathrm{r}}\mathbf{r}_k\right) \tag{8}$$

where the known matrix $\Theta_{\mathrm{r}} \in \Re^{m\times m}$ is selected such that $\Gamma_{\mathrm{m}} = \Gamma_{\mathrm{n}}\Theta_{\mathrm{r}}$. Since the controller (8) is non-causal, the future $\mathbf{x}_{k+p}$ is computed as

$$\begin{aligned}\mathbf{x}_{k+p} = \Phi^p\mathbf{x}_k + \big(&\Phi^{p-1}\Gamma\mathbf{u}_{k-p} + \Phi^{p-2}\Gamma\mathbf{u}_{k-p+1} + \cdots\\ &+ \Gamma\mathbf{u}_{k-1}\big).\end{aligned} \tag{9}$$

Substituting (9) in (8) leads to a controller of the form

$$\mathbf{u}_k = -\Theta_\gamma^{-1}\left(\Theta_{\mathrm{x}}\mathbf{x}_k + \Theta_{\mathrm{u}}\boldsymbol{\xi}_k - \Theta_{\mathrm{r}}\mathbf{r}_k\right) \tag{10}$$

where $\Theta_{\mathrm{x}} = \Theta\Phi^p \in \Re^{m\times n}$, $\Theta_{\mathrm{u}} = \Theta\left[\Gamma\ \Phi\Gamma\ \cdots\ \Phi^{p-1}\Gamma\right] \in \Re^{m\times pm}$ and $\boldsymbol{\xi}_k^T = \left[\mathbf{u}_{k-1}^T\ \cdots\ \mathbf{u}_{k-p}^T\right] \in \Re^{pm}$.

Consider (6), using (7) and (9) it is obtained that

$$\begin{aligned}\mathbf{e}_{k+1} = \Phi_{\mathrm{m}}\mathbf{e}_k &+ \Gamma_{\mathrm{n}}\left(\Theta_{\mathrm{x}}\mathbf{x}_{k-p} + \Theta_{\mathrm{u}}\boldsymbol{\xi}_{k-p}\right) + \Gamma_{\mathrm{n}}\Theta_\gamma\mathbf{u}_{k-p}\\ &- \Gamma_{\mathrm{n}}\Theta_{\mathrm{r}}\mathbf{r}_{k-p}.\end{aligned} \tag{11}$$

Substitution of the control law (10) in the tracking error (11) it is obtained that

$$\mathbf{e}_{k+1} = \Phi_{\mathrm{m}}\mathbf{e}_k \tag{12}$$

which is stable.

Proceeding now with uncertain Φ and Γ, the parameters Θ_{x}, Θ_{u} and Θ_γ become uncertain. The control law (10) is then modified to the form

$$\mathbf{u}_k = -\hat{\Theta}_{\gamma,k}^{-1}\left(\hat{\Theta}_{\mathrm{x},k}x_k + \hat{\Theta}_{\mathrm{u},k}\boldsymbol{\xi}_k - \Theta_r\mathbf{r}_k\right) \tag{13}$$

where $\hat{\Theta}_{\mathrm{x},k}$, $\hat{\Theta}_{\mathrm{u},k}$ and $\hat{\Theta}_{\gamma,k}$ are the estimates of Θ_{x}, Θ_{u} and Θ_γ respectively. To derive the estimation law for $\hat{\Theta}_{\mathrm{x},k}$, $\hat{\Theta}_{\mathrm{u},k}$ and $\hat{\Theta}_{\gamma,k}$ it is necessary to derive the closed-loop system.

Consider the system (11), adding and subtracting the term $\Gamma_n\hat{\Theta}_{\gamma,k-p}\mathbf{u}_{k-p}$ it is obtained that

$$\mathbf{e}_{k+1} = \Phi_m\mathbf{e}_k + \Gamma_n\left(\Theta_x\mathbf{x}_{k-p} + \Theta_u\boldsymbol{\xi}_{k-p} + \Theta_\gamma\mathbf{u}_{k-p}\right) \quad (14)$$
$$- \Gamma_n\hat{\Theta}_{\gamma,k-p}\mathbf{u}_{k-p} + \Gamma_n\hat{\Theta}_{\gamma,k-p}\mathbf{u}_{k-p} - \Gamma_n\Theta_r\mathbf{r}_{k-p}.$$

Define the estimation errors as $\tilde{\Theta}_{x,k} = \Theta_x - \hat{\Theta}_{x,k}$, $\tilde{\Theta}_{u,k} = \Theta_u - \hat{\Theta}_{u,k}$ and $\tilde{\Theta}_{\gamma,k} = \Theta_\gamma - \hat{\Theta}_{\gamma,k}$. Using these definitions the system (14) can be simplified to the form

$$\mathbf{e}_{k+1} = \Phi_m\mathbf{e}_k + \Gamma_n\left(\Theta_x\mathbf{x}_{k-p} + \Theta_u\boldsymbol{\xi}_{k-p}\right) \quad (15)$$
$$+\Gamma_n\tilde{\Theta}_{\gamma,k-p}\mathbf{u}_{k-p} + \Gamma_n\hat{\Theta}_{\gamma,k-p}\mathbf{u}_{k-p} - \Gamma_n\Theta_r\mathbf{r}_{k-p}.$$

Further, substitution of (13) into (16) it is obtained that

$$\mathbf{e}_{k+1} = \Phi_m\mathbf{e}_k + \Gamma_n\big(\tilde{\Theta}_{x,k-p}\mathbf{x}_{k-p} + \tilde{\Theta}_{u,k-p}\boldsymbol{\xi}_{k-p} + \tilde{\Theta}_{\gamma,k-p}\mathbf{u}_{k-p}\big) \quad (16)$$

which is the closed-loop dynamics of the system in terms of the parameter estimation errors. It is convenient to rewrite the error dynamics (16) in the augmented form

$$\mathbf{e}_{k+1} = \Phi_m\mathbf{e}_k + \Gamma_n\tilde{\Psi}^T_{k-p}\boldsymbol{\zeta}_{k-p} \quad (17)$$

where $\tilde{\Psi}^T_k = \left[\tilde{\Theta}_{x,k}\ \tilde{\Theta}_{u,k}\ \tilde{\Theta}_{\gamma,k}\right] \in \Re^{m\times(n+m(p+1))}$ and $\boldsymbol{\zeta}^T_k = \left[\mathbf{x}^T_k\ \boldsymbol{\xi}^T_k\ u^T_k\right] \in \Re^{n+m(p+1)}$. In order to proceed with the formulation of the adaptation law define $\mathbf{z}_{k+1} = C^T_\gamma(\mathbf{e}_{k+1} - \Phi_m\mathbf{e}_k) \in \Re^m$ where $C^T_\gamma = \left(C^T\Gamma_n\right)^{-1}C^T$ and substitute (17) to obtain

$$\mathbf{z}_{k+1} = \tilde{\Psi}^T_{k-p}\boldsymbol{\zeta}_{k-p}. \quad (18)$$

The adaptation laws must be formulated with the objective of minimizing $\mathbf{z}_{k+1}$ so that the tracking error would follow the dynamics $\mathbf{e}_{k+1} = \Phi_m\mathbf{e}_k$. Therefore, the adaptation laws are formulated as follows

$$\hat{\Psi}_{k+1} = \begin{cases} \hat{\Psi}_{k-p} + \epsilon_k P_{k+1}\boldsymbol{\zeta}_{k-p}\mathbf{z}^T_{k+1}, & k \in [p,\infty) \\ \hat{\Psi}_0, & k \in [0,p) \end{cases} \quad (19)$$

$$P_{k+1} = \begin{cases} P_{k-p} - \epsilon_k\dfrac{P_{k-p}\boldsymbol{\zeta}_{k-p}\boldsymbol{\zeta}^T_{k-p}P_{k-p}}{1+\epsilon_k\boldsymbol{\zeta}^T_{k-p}P_{k-p}\boldsymbol{\zeta}_{k-p}}, & k \in [p,\infty) \\ P_0 > 0, & k \in [0,p) \end{cases} \quad (20)$$

where $\epsilon_k \in \Re$ is a positive coefficient used to prevent a singular $\hat{\Theta}_{\gamma,k}$ and the matrix $P_k \in \Re^{(n+m(p+1))\times(n+m(p+1))}$ is a symmetric, positive-definite covariance matrix, Kokotovic (1991).

Remark 1. Note that, in order for $\hat{\Theta}_{\gamma,k}$ not to be singular then ϵ_k^{-1} must be selected such that it is not an eigenvalue of $-\hat{\Theta}^{-1}_{\gamma,k-p}SP_{k+1}\boldsymbol{\zeta}_{k-p}\mathbf{z}^T_{k+1}$ where $S = [\mathbf{0}\ \cdots\ \mathbf{0}\ I] \in \Re^{m\times(n+m(p+1))}$.

Theorem 1. The plant (3) and the adaptive laws (19) and (20) results in a closed-loop system with a bounded $\tilde{\Psi}_k$ and $\lim_{k\to\infty}\|\mathbf{e}_k\| = 0$ if $\epsilon_k > 0$.

Proof. To proceed with the proof, note that $\mathbf{z}^\top_k = [z_{1,k}\ z_{2,k}\ \cdots\ z_{m,k}]^\top$ and $\tilde{\Psi}^\top_k = \left[\tilde{\boldsymbol{\psi}}_{1,k}\ \tilde{\boldsymbol{\psi}}_{2,k}\ \cdots\ \tilde{\boldsymbol{\psi}}_{m,k}\right]^\top$, where $\tilde{\boldsymbol{\psi}}_{j,k} \in \Re^{(n+m(p+1))\times 1}$ and $j = 1,\ldots,m$. Now consider the following positive function

$$V_k = \sum_{j=1}^m\left(\sum_{i=0}^p\tilde{\boldsymbol{\psi}}^\top_{j,k-i}P^{-1}_{k-i}\tilde{\boldsymbol{\psi}}_{j,k-i}\right). \quad (21)$$

The forward difference of (21) is given by

$$\Delta V_k = V_{k+1} - V_k \quad (22)$$
$$= \sum_{j=1}^m\left[\tilde{\boldsymbol{\psi}}^\top_{j,k+1}P^{-1}_{k+1}\tilde{\boldsymbol{\psi}}_{j,k+1} - \tilde{\boldsymbol{\psi}}^\top_{j,k-p}P^{-1}_{k-p}\tilde{\boldsymbol{\psi}}_{j,k-p}\right].$$

Consider the update law (19), subtracting both sides from $\boldsymbol{\psi}_j$ it is possible to obtain

$$\boldsymbol{\psi}_j - \hat{\boldsymbol{\psi}}_{j,k+1} = \boldsymbol{\psi}_j - \hat{\boldsymbol{\psi}}_{j,k-p} - \epsilon_k P_{k+1}\boldsymbol{\zeta}_{k-p}z_{j,k+1} \quad (23)$$

and defining $\tilde{\boldsymbol{\psi}}_{j,k} = \boldsymbol{\psi}_j - \hat{\boldsymbol{\psi}}_{j,k}$ we obtain

$$\tilde{\boldsymbol{\psi}}_{j,k+1} = \tilde{\boldsymbol{\psi}}_{j,k-p} - \epsilon_k P_{k+1}\boldsymbol{\zeta}_{k-p}z_{j,k+1} \quad (24)$$

substitute (24) in (22) to obtain

$$\Delta V_k = \sum_{j=1}^m\left[\left(\tilde{\boldsymbol{\psi}}_{j,k-p} - \epsilon_k P_{k+1}\boldsymbol{\zeta}_{k-p}z_{j,k+1}\right)^\top P^{-1}_{k+1}\left(\tilde{\boldsymbol{\psi}}_{j,k-p} - \epsilon_k P_{k+1}\boldsymbol{\zeta}_{k-p}z_{j,k+1}\right) - \tilde{\boldsymbol{\psi}}^\top_{j,k-p}P^{-1}_{k-p}\tilde{\boldsymbol{\psi}}_{j,k-p}\right] \quad (25)$$

Grouping similar terms with each other leads to

$$\Delta V_k = \sum_{j=1}^m\Big[\tilde{\boldsymbol{\psi}}^\top_{j,k-p}\left(P^{-1}_{k+1} - P^{-1}_{k-p}\right)\tilde{\boldsymbol{\psi}}_{j,k-p} \quad (26)$$
$$-2\epsilon_k\tilde{\boldsymbol{\psi}}^\top_{k-p}\boldsymbol{\zeta}_{k-p}z_{j,k+1} + \epsilon_k^2\boldsymbol{\zeta}^\top_{k-p}P_{k+1}\boldsymbol{\zeta}_{k-p}z^2_{j,k+1}\Big].$$

Substituting $P^{-1}_{k+1} = P^{-1}_{k-p} + \epsilon_k\boldsymbol{\zeta}_{k-p}\boldsymbol{\zeta}^T_{k-p}$ into (27) and, since, $\epsilon_k > 0$ it is obtained

$$\Delta V_k \leq \sum_{j=1}^m\Big[\epsilon_k\tilde{\boldsymbol{\psi}}^\top_{j,k-p}\boldsymbol{\zeta}_{k-p}\boldsymbol{\zeta}^\top_{k-p}\tilde{\boldsymbol{\psi}}_{j,k-p} \quad (27)$$
$$-2\epsilon_k\tilde{\boldsymbol{\psi}}^\top_{j,k-p}\boldsymbol{\zeta}_{k-p}z_{j,k+1} + \epsilon_k^2\boldsymbol{\zeta}^\top_{k-p}P_{k+1}\boldsymbol{\zeta}_{k-p}z^2_{j,k+1}\Big].$$

Further, note that $z_{j,k+1} = \boldsymbol{\psi}^\top_{j,k-p}\boldsymbol{\zeta}_{k-p}$. Using this substitution in (71) results in

$$\Delta V_k \leq \sum_{j=1}^m\epsilon_k z^2_{j,k+1}\big[-1 + \epsilon_k\boldsymbol{\zeta}^\top_{k-p}P_{k+1}\boldsymbol{\zeta}_{k-p}\big]. \quad (28)$$

Using $\boldsymbol{\zeta}^T_{k-p}P_{k+1}\boldsymbol{\zeta}_{k-p} = \frac{\boldsymbol{\zeta}^T_{k-p}P_{k-p}\boldsymbol{\zeta}_{k-p}}{1+\epsilon_k\boldsymbol{\zeta}^T_{k-p}P_{k-p}\boldsymbol{\zeta}_{k-p}}$ in (28), ΔV_k becomes

$$\Delta V_k = -\sum_{j=1}^m\left[\frac{\epsilon_k z^2_{j,k+1}}{1+\epsilon_k\boldsymbol{\zeta}^\top_{k-p}P_{k-p}\boldsymbol{\zeta}_{k-p}}\right], \quad (29)$$

which can be rewritten in the form

$$\Delta V_k = -\frac{\epsilon_k\mathbf{z}^\top_{k+1}\mathbf{z}_{k+1}}{1+\epsilon_k\boldsymbol{\zeta}^\top_{k-p}P_{k-p}\boldsymbol{\zeta}_{k-p}}. \quad (30)$$

The result (30) implies that V_k is non-increasing and, thus, $\tilde{\Psi}_k$ is bounded. Consequently it is concluded that

$$\lim_{k\to\infty}\frac{\epsilon_k\mathbf{z}^\top_{k+1}\mathbf{z}_{k+1}}{1+\epsilon_k\boldsymbol{\zeta}^\top_{k-p}P_{k-p}\boldsymbol{\zeta}_{k-p}} = 0. \quad (31)$$

Following the steps in Abidi & Xu (2015), it is obtained that $\lim_{k\to\infty}\|\mathbf{e}_k\| = 0$.

4. EXTENSION TO UNCERTAIN UPPER-BOUNDED TIME-DELAY

Consider the system (3), but, with an uncertain input delay d such that

$$\begin{aligned}\mathbf{x}_{k+1} &= \Phi\mathbf{x}_k + \Gamma\mathbf{u}_{k-d}\\ \mathbf{y}_k &= C^T\mathbf{x}_k\end{aligned} \tag{32}$$

and the uncertain time-delay is assumed to have a known upper-bound such that $d \le p$ for a known p. Subtracting (4) from (32) and deriving the error dynamics as

$$\mathbf{e}_{k+1} = \Phi_{\mathrm{m}}\mathbf{e}_k + \Gamma_{\mathrm{n}}\Theta\mathbf{x}_k + \Gamma_{\mathrm{n}}\Theta_\gamma\mathbf{u}_{k-d} - \Gamma_{\mathrm{n}}\Theta_{\mathrm{r}}\mathbf{r}_{k-p} \tag{33}$$

where $\mathbf{e}_k = \mathbf{x}_k - \mathbf{x}_{\mathrm{m},k}$. Note that $\mathbf{x}_{k+p}$ can be written as

$$\begin{aligned}\mathbf{x}_{k+p} = \Phi^p\mathbf{x}_k + \big(&\Phi^{p-1}\Gamma\mathbf{u}_{k-d} + \Phi^{p-2}\Gamma\mathbf{u}_{k-d+1} + \cdots\\ &+ \Gamma\mathbf{u}_{k+p-d-1}\big)\end{aligned} \tag{34}$$

Substituting a p time steps delayed form of (34) into (33)

$$\begin{aligned}\mathbf{e}_{k+1} = \Phi_{\mathrm{m}}\mathbf{e}_k &+ \Gamma_{\mathrm{n}}\Theta\Phi^p\mathbf{x}_{k-p} + \Gamma_{\mathrm{n}}\Theta\left(\Phi^{p-1}\Gamma\mathbf{u}_{k-p-d}\right.\\ &+ \left.\Phi^{p-2}\Gamma\mathbf{u}_{k-p-d+1} + \cdots + \Gamma\mathbf{u}_{k-d-1}\right)\\ &+ \Gamma_{\mathrm{n}}\Theta_\gamma\mathbf{u}_{k-d} - \Gamma_{\mathrm{n}}\Theta_{\mathrm{r}}\mathbf{r}_{k-p}.\end{aligned} \tag{35}$$

Let $\boldsymbol{\xi}_k^T = \left[\mathbf{u}_{k-1}^T \ \cdots \ \mathbf{u}_{k-p}^T\right] \in \Re^{pm}$ and rewrite (35) as

$$\begin{aligned}\mathbf{e}_{k+1} = \Phi_{\mathrm{m}}\mathbf{e}_k &+ \Gamma_{\mathrm{n}}\Theta\Phi^p\mathbf{x}_{k-p} + \Gamma_{\mathrm{n}}(0\cdot\mathbf{u}_{k-2p} + \ \ldots\\ &+0\cdot\mathbf{u}_{k-p-d-1}) + \Gamma_{\mathrm{n}}\Theta\left(\Phi^{p-1}\Gamma\mathbf{u}_{k-p-d}\right.\\ &+ \left.\Phi^{p-2}\Gamma\mathbf{u}_{k-p-d+1} + \cdots + \Gamma\mathbf{u}_{k-d-1}\right) + \Gamma_{\mathrm{n}}\Theta_\gamma\mathbf{u}_{k-d}\\ &+ \Gamma_{\mathrm{n}}(0\cdot\mathbf{u}_{k-d+1} + \ldots + 0\cdot\mathbf{u}_{k-1}) - \Gamma_{\mathrm{n}}\Theta_{\mathrm{r}}\mathbf{r}_{k-p}.\end{aligned} \tag{36}$$

It is possible to simplify (36) further to the form

$$\begin{aligned}\mathbf{e}_{k+1} = \Phi_{\mathrm{m}}\mathbf{e}_k &+ \Gamma_{\mathrm{n}}\Theta_{\mathrm{x}}\mathbf{x}_{k-p} + \Gamma_{\mathrm{n}}\Theta_{\mathrm{u}}\boldsymbol{\xi}_{k-p} - \Gamma_{\mathrm{n}}\Theta_{\mathrm{r}}\mathbf{r}_{k-p}\\ &+ \Gamma_{\mathrm{n}}\Theta_{\mathrm{p}}\mathbf{u}_{k-p} + \Gamma_{\mathrm{n}}\Omega_{\mathrm{u}}\boldsymbol{\xi}_k\end{aligned} \tag{37}$$

where $\Theta_{\mathrm{x}} \in \Re^{m\times n}$, $\Theta_{\mathrm{p}} = \Theta\Phi^{p-d-1}\Gamma \in \Re^{m\times m}$, $\Theta_{\mathrm{u}} = \left[[0]_{m\times m(p-d)} \mid \Theta\Phi^{p-1}\Gamma \cdots \Theta\Phi^{p-d}\Gamma\right] \in \Re^{m\times m}$ and $\Omega_{\mathrm{u}} = \left[\Theta\Phi^{p-d-2}\Gamma \cdots \Theta\Gamma \mid [0]_{m\times md}\right] \in \Re^{m\times pm}$ are the matrices of uncertain parameters and note that some of the elements of Θ_{u} and Ω_{u} the matrices are zero as in (36).

The reason (35) is rewritten in the form (36) is to eliminate the dependency on the uncertain delay d. From (37) it seen that the system is written in terms of the known upper-bound p rather than the uncertain delay d. Proceeding further, assume a controller of the form

$$\mathbf{u}_k = -\hat{\Theta}_{\mathrm{p},k}^{-1}\left(\hat{\Theta}_{\mathrm{x},k}\mathbf{x}_k + \hat{\Theta}_{\mathrm{u},k}\boldsymbol{\xi}_k - \Theta_{\mathrm{r}}\mathbf{r}_k\right). \tag{38}$$

Substitution of (38) into (37) and after performing some simplifications it is obtained that

$$\begin{aligned}\mathbf{e}_{k+1} &= \Phi_{\mathrm{m}}\mathbf{e}_k + \Gamma_{\mathrm{n}}\tilde{\Theta}_{\mathrm{x},k-p}\mathbf{x}_{k-p} + \Gamma_{\mathrm{n}}\tilde{\Theta}_{\mathrm{u},k-p}\boldsymbol{\xi}_{k-p}\\ &\quad + \Gamma_{\mathrm{n}}\left(\Theta_{\mathrm{p}} - \hat{\Theta}_{\mathrm{p},k-p}\right)\mathbf{u}_{k-p} + \Gamma_{\mathrm{n}}\Omega_{\mathrm{u}}\boldsymbol{\xi}_k\\ &= \Phi_{\mathrm{m}}\mathbf{e}_k + \Gamma_{\mathrm{n}}\tilde{\Theta}_{\mathrm{x},k-p}\mathbf{x}_{k-p} + \Gamma_{\mathrm{n}}\tilde{\Theta}_{\mathrm{u},k-p}\boldsymbol{\xi}_{k-p}\\ &\quad + \Gamma_{\mathrm{n}}\tilde{\Theta}_{\mathrm{p},k-p}\mathbf{u}_{k-p} + \Gamma_{\mathrm{n}}\Omega_{\mathrm{u}}\boldsymbol{\xi}_k.\end{aligned} \tag{39}$$

Including the terms $\Gamma_{\mathrm{n}}\cdot 0\cdot\mathbf{x}_k + \Gamma_{\mathrm{n}}\cdot 0\cdot\mathbf{u}_k$ in (39) such that

$$\begin{aligned}\mathbf{e}_{k+1} = \Phi_{\mathrm{m}}\mathbf{e}_k &+ \Gamma_{\mathrm{n}}\tilde{\Theta}_{\mathrm{x},k-p}\mathbf{x}_{k-p} + \Gamma_{\mathrm{n}}\tilde{\Theta}_{\mathrm{u},k-p}\boldsymbol{\xi}_{k-p}\\ &+ \Gamma_{\mathrm{n}}\tilde{\Theta}_{\mathrm{p},k-p}\mathbf{u}_{k-p} + \Gamma_{\mathrm{n}}\cdot 0\cdot\mathbf{x}_k\\ &+ \Gamma_{\mathrm{n}}\cdot 0\cdot\mathbf{u}_k + \Gamma_{\mathrm{n}}\Omega_{\mathrm{u}}\boldsymbol{\xi}_k.\end{aligned} \tag{40}$$

and let $\boldsymbol{\zeta}_k^T = \left[\mathbf{x}_k^T \ \boldsymbol{\xi}_k^T \ \mathbf{u}_k^T\right] \in \Re^{n+m(p+1)}$, $\tilde{\Psi}_k^T = \left[\tilde{\Theta}_{\mathrm{x},k} \ \tilde{\Theta}_{\mathrm{u},k} \ \tilde{\Theta}_{\mathrm{p},k}\right] \in \Re^{m\times(n+m(p+1))}$ and $\Omega^T = [[0]\ \Omega_{\mathrm{u}}\ [0]] \in \Re^{m\times(n+m(p+1))}$ then it is possible to obtain the compact error dynamics of the form

$$\mathbf{e}_{k+1} = \Phi_{\mathrm{m}}\mathbf{e}_k + \Gamma_{\mathrm{n}}\tilde{\Psi}_{k-p}^T\boldsymbol{\zeta}_{k-p} + \Gamma_{\mathrm{n}}\Omega^T\boldsymbol{\zeta}_k. \tag{41}$$

Note that the error dynamics (41) is similar to (17) with the only difference being the extra term $\Gamma_{\mathrm{n}}\Omega^T\boldsymbol{\zeta}_k$ which exists due to the uncertainty in the delay. If the delay d is known and $d = p$ then Ω would be a null matrix. Using $\mathbf{z}_{k+1} = (C^T\Gamma_{\mathrm{n}})^{-1}C^T\left(\mathbf{e}_{k+1} - \Phi_{\mathrm{m}}\mathbf{e}_k\right)$ to obtain

$$\mathbf{z}_{k+1} = \tilde{\Psi}_{k-p}^T\boldsymbol{\zeta}_{k-p} + \Omega^T\boldsymbol{\zeta}_k, \tag{42}$$

where $\mathbf{z}_{k+1} \in \Re^m$. The adaptation law will be formulated in such a way as to be robust to the term $\Omega^T\boldsymbol{\zeta}_k$. Based on (42) and using an approach similar to Abidi (2014), the adaptation law is proposed as

$$\hat{\Psi}_{k+1} = \begin{cases}\hat{\Psi}_{k-p} + \epsilon_k\dfrac{\beta_k}{\varphi_k}Q\boldsymbol{\zeta}_{k-p}\mathbf{z}_{k+1}^T, & k\in[p,\infty)\\ \hat{\Psi}_0, & k\in[0,p)\end{cases} \tag{43}$$

where the scalar function $\varphi_k = 1 + \epsilon_k\boldsymbol{\zeta}_{k-p}^TQ\boldsymbol{\zeta}_{k-p} + \epsilon_k\gamma\lambda^2\|\boldsymbol{\zeta}_k\|^2$, the matrix Q is a constant positive definite matrix of dimension $n+m(p+1)$, γ,λ are positive tuning constants, β_k is a positive scalar weighing coefficient and $\epsilon_k > 0$ is a coefficient used to ensure a nonsingular $\hat{\Theta}_{\mathrm{p},k}$.

Consider the constant uncertainty Ω and assume that $\|\Omega\| = \lambda\rho$ where ρ is an uncertain positive constant, it is easy to see that $\|\Omega^T\boldsymbol{\zeta}_k\| \le \lambda\rho\|\boldsymbol{\zeta}_k\|$. Further, the weighing coefficient β_k can be defined as,

$$\beta_k = \begin{cases}1 - \dfrac{\lambda\hat{\rho}_k\|\boldsymbol{\zeta}_k\|}{\|\mathbf{z}_{k+1}\|}, & \text{if } \|\mathbf{z}_{k+1}\| \ge \lambda\hat{\rho}_k\|\boldsymbol{\zeta}_k\|\\ 0, & \text{if } \|\mathbf{z}_{k+1}\| < \lambda\hat{\rho}_k\|\boldsymbol{\zeta}_k\|\end{cases} \tag{44}$$

where $\hat{\rho}_k$ is the estimate of ρ and λ can be chosen as any constant as long as it satisfies $0 < \lambda < \lambda_{\max}$, with $\lambda_{\max}$ being defined later. The estimation law for ρ is given as

$$\hat{\rho}_{k+1} = \hat{\rho}_k + \epsilon_k\frac{\beta_k\lambda\gamma\|\boldsymbol{\zeta}_k\|\cdot\|\mathbf{z}_{k+1}\|}{\varphi_k}. \tag{45}$$

From (44) if $\|\mathbf{z}_{k+1}\| \ge \lambda\hat{\rho}_k\|\boldsymbol{\zeta}_k\|$ it is obtained that

$$\beta_k^2\mathbf{z}_{k+1}^T\mathbf{z}_{k+1} = \beta_k\mathbf{z}_{k+1}^T\mathbf{z}_{k+1} - \lambda\hat{\rho}_k\beta_k\|\boldsymbol{\zeta}_k\|\cdot\|\mathbf{z}_{k+1}\|. \tag{46}$$

The validity of the above adaption law is verified by the following theorem.

Theorem 2. Under the adaptation law (43) and the closed-loop dynamics (42) the tracking error $\mathbf{e}_k$ is bounded.

A procedure similar to that in *Theorem 1* can be used to verify the boundedness of $\|\mathbf{e}_k\|$.

5. SIMULATION EXAMPLES

To illustrate the advantages of the discrete-time APC, a flight control example with the longitudinal dynamics of a four-engine jet transport aircraft, Blakelock (1991) was used. The aircraft flies straight and level flight at 40,000 ft with a velocity of 600 ft/sec. Under these conditions, the nominal short period dynamics is given by

$$\begin{bmatrix}\dot{\alpha}(t)\\ \dot{q}(t)\end{bmatrix} = \begin{bmatrix}-0.323 & 1\\ -1.169 & -0.480\end{bmatrix}\begin{bmatrix}\alpha(t)\\ q(t)\end{bmatrix} + \begin{bmatrix}-0.018\\ -1.379\end{bmatrix}\sigma_e(t-\tau)$$

where α is the angle of attack in radians, q is the pitch rate in radians per second and σ_e is the elevator deflection also in radians. The time-delay value used in the simulation is given as $\tau = 0.4$s. Eigenvalues are $-0.4017 \pm 1.0785i$, giving a nominal short period natural frequency of $\omega_{\rm n} = 1.1423$ rad/s and a nominal damping ratio of $\zeta = 0.3517$.

To obtain a challanging scenerio, control effectiveness uncertainty was introduced resulting in a 30% decrease in elevator effectiveness. In addition, by adding further uncertainty to the state matrix, proximity of the open loop poles to the imaginary axis was halved and the damping ratio was reduced by 48%. The resulting plant becomes

$$\begin{bmatrix} \dot{\alpha}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} -0.323 & 1.005 \\ -1.176 & -0.077 \end{bmatrix} \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} -0.009 \\ -0.689 \end{bmatrix} \sigma_e(t-\tau). \quad (47)$$

In order to implement the controller (10) the reference model needs to be computed in discrete-time. To do this the nominal plant (47) will be sampled at $T_{\rm s} = 0.02$s resulting in the sampled-data plant

$$\begin{bmatrix} \alpha_{k+1} \\ q_{k+1} \end{bmatrix} = \begin{bmatrix} 0.993 & 0.0198 \\ -0.023 & 0.990 \end{bmatrix} \begin{bmatrix} \alpha_k \\ q_k \end{bmatrix} + \begin{bmatrix} -0.0006 \\ -0.027 \end{bmatrix} \sigma_{e,k-p} \quad (48)$$

where $p = \tau/T_{\rm s} = 20$. The reference model is designed using the LQR method ignoring the delay. The feedback matrix is calculated by selecting $Q_{\rm x} = diag(10, 10)$ and $R = 1$ resulting in a reference model of the form

$$\begin{bmatrix} \alpha_{{\rm m},k+1} \\ q_{{\rm m},k+1} \end{bmatrix} = \begin{bmatrix} 0.9924 & 0.0179 \\ -0.0622 & 0.9078 \end{bmatrix} \begin{bmatrix} \alpha_{{\rm m},k} \\ q_{{\rm m},k} \end{bmatrix} + \begin{bmatrix} 0.0021 \\ 0.0905 \end{bmatrix} r_{k-p}. \quad (49)$$

5.1 Discrete-Time Adaptive APC vs Discrete Approximation of Continuous-Time APC

The adaptive gains of the continuous-time APC are calculated as $\Psi_{\rm x} = diag(3.7, 8.3) \times 10^3$ and $\Psi_{\rm r} = 8.4 \times 10^3$. The gains used in the integral approximation are tuned to get the best performance. As for the discrete-time APC, the parameter values for $P_0 = diag(P_{{\rm x},0}, P_{{\rm u},0}\ P_{\gamma,0})$ where $P_{{\rm x},0} = diag(44, 105)$, $P_{{\rm u},0} = 5.1 I_{p\times p}$, $P_{\gamma,0} = 0.10$. The performance of the two controllers is shown in Fig.1. In Fig.1 it is seen that the continuous-time APC is oscillatory while the discrete-time APC has a short oscillatory period after which it is smooth throughout.

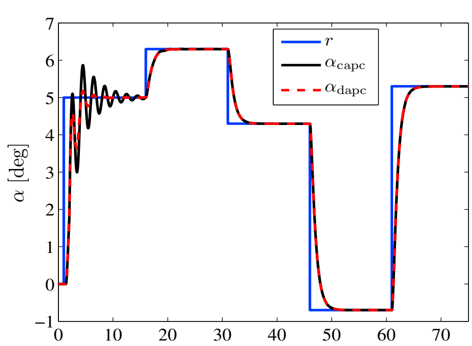

Fig. 1. Performance of the approximate continuous-time APC vs discrete-time APC with $\tau = 0.4$s

5.2 Discrete-Time Adaptive APC vs MRAC

The structure of the MRAC in discrete-time is similar to that of the discrete-time APC with the main difference being that the term $\hat{\Theta}_{{\rm u},k}\boldsymbol{\xi}_k$ is absent from the controller (13). Fig.2 shows that the MRAC is very oscillatory when an input-delay of 0.4s is introduced to the system. Even though it is well known that MRAC works well when there is no delay in the system its performance degrades considerably in the presence of delay. On the other hand the discrete-time APC is stable similar to the previous example. This example clearly presents the advantages discrete-time APC has over the conventional MRAC design.

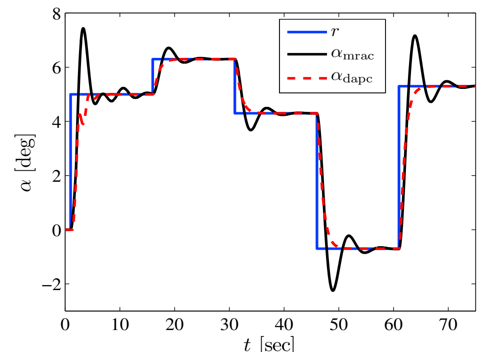

Fig. 2. Performance of the discrete-time APC vs MRAC with $\tau = 0.4$s

5.3 Uncertain upper-bounded time delay

Consider the system (47) where the time-dealy is assumed to be $\tau_{\rm p} = 0.4$s while the actual time-delay is $\tau_{\rm d} = 0.3$s. Selecting $\lambda = 0.015$, $\gamma = 100$ and $Q = diag(300, 150, 60, I_{p\times p})$. The system is simulated under these conditions and the results can be seen in Fig.3. The results show that the system converges within a reasonable error bound around the desired trajectory. Furthermore, the actual time is changed to $\tau_{\rm d} = 0.2$s while the remaining parameters remain unchanged and the system is simulated once more. The results from Fig.4 show that inspite of a 50% uncertainty in the time-delay, very good performance is still possible using this approach.

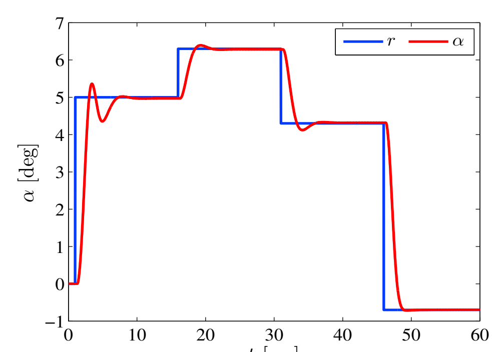

Fig. 3. Performance with $\tau_{\rm d} = 0.3$s and $\tau_{\rm p} = 0.4$s

6. CONCLUSION

In this paper, a discrete-time Adaptive Posicast Control (APC) method for time-delay systems has been derived. The method is extended to nonlinear systems and linear systems with uncertain upper bounded time-delay. The method is simulated and compared to a discrete-time approximation of the continuous-time APC and a MRAC by applying each method to a flight control problem,

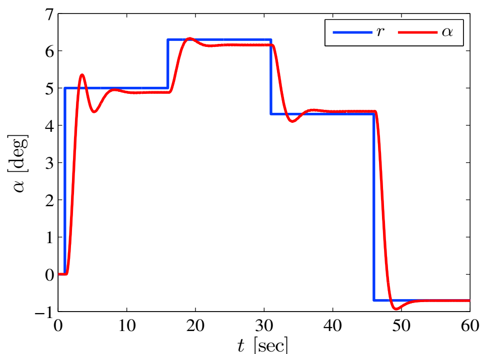

Fig. 4. Performance with $\tau_{\mathrm{d}} = 0.2$s and $\tau_{\mathrm{p}} = 0.4$s

where the short period dynamics of a jet transport aircraft were used as the plant model. Further simulation results are shown for nonlinear and unknown upper bounded time-delay cases. A potential for the discrete-time APC to outperform both the continuous-time APC and the conventional MRAC is highlighted. The stability of the closed loop system under different scenarios is discussed.

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