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On Stable Controller Design For Robust Stabilization of Time Delay Systems

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Abstract: This paper studies the problem of robust stabilization of an infinite dimensional plant by a stable and possibly low order controller. The plant of interest is assumed to have only finitely many simple unstable zeros, however, may have infinitely many unstable poles. In the literature, it has been shown that the problem can be reduced to an interpolation problem and it is possible to obtain lower and upper bounds of the multiplicative uncertainty under which an infinite dimensional stable controller can be generated by a modified Nevanlinna-Pick formulation. We propose that the same interpolation problem can be solved approximately by a finite dimensional approach and present a finite dimensional interpolation function which can be used to find a stable controller. We illustrate this idea by a numerical example and additionally show the effects of the free design parameters of the rational interpolating outer function approach on the numerical example.

Keywords: Strong stabilization, robust stabilization, infinite dimensional systems, $\mathcal{H}^\infty$ control, low degree controller

1. INTRODUCTION

This paper is about strong and robust stabilization of SISO infinite dimensional plants, specifically time delay systems, by a finite dimensional controller. Strong stability requires a stable controller to be designed. A stable controller has two main advantages: it is robust to sensor failures (i.e. undetermined feedback input or saturation of control input) as described by Doyle et al. (1990), and Ünal and İftar (2012c) and it is testable stand-alone as mentioned by van de Wal et al. (2002). It is possible to test a stable controller by its input-output relationship practically by applying some test signals as an open-loop system before using it with the original plant to prevent undesired errors.

It is essential to mention that strong stability means achieving stabilization by a stable controller which is different than some other strong stability concepts used in the infinite dimensional system theory, e.g. Hale and Lunel (2002).

There is a rich literature for strong stabilization of finite dimensional plants, see e.g. Campos-Delgado and Zhou (2003), Cheng et al. (2007), Cheng et al. (2011), Gümüşsoy and Özbay (2009), Petersen (2009), and Gündeş and Özbay (2011) and see also Gümüşsoy et al. (2008) for sensitivity shaping of infinite dimensional systems by fixed order stable controllers. Nevertheless, robust stabilization by a stable controller remains to be an active open research area and to our knowledge; the most recent contribution, which has been made by Wakaiki et al. (2013), gives some sufficient conditions as discussed below in more detail. Most recently, the same idea has been extended to the mixed sensitivity minimization by stable controllers, Wakaiki and Yamamoto (2014); see also Gümüşsoy and Özbay (2009) for an alternative earlier approach. We refer to Ünal and İftar (2012b) and Ünal and İftar (2012c) for recent results on stable $\mathcal{H}^\infty$ controller design for plants with input-output delays. It should be noted that for time delay systems, parity interlacing property, p.i.p., (having even number of poles between any pair of extended right half plane zeros) is necessary (and sufficient with added restrictions) for the existence of a strongly stabilizing controller, Ünal and İftar (2012a).

In their paper, Wakaiki et al. (2013) have studied infinite dimensional plants having finitely many simple unstable zeros but possibly infinitely many unstable poles. The authors have tried to find a way to calculate upper and lower bounds for the largest multiplicative uncertainty under which a robustly stabilizing stable controller can be generated. They have used a method developed in Gümüşsoy and Özbay (2009) and Özbay (2010), where an extension to the well known Nevanlinna-Pick interpolation algorithm is used. After calculating the upper and lower bounds, Wakaiki et al. (2013) also proposed how to generate a stable controller for a given multiplicative uncertainty. In this approach, for each bound, a modified version of the Nevanlinna-Pick interpolation problem is solved and the resulting interpolating function is an infinite dimensional one. Since this interpolating function is a part of the designed controller, the resulting controller ends up to be infinite dimensional, independent of the other components.

In this paper, we present an application of an old method appearing in Vidyasagar (1985) and Doyle et al. (1990),

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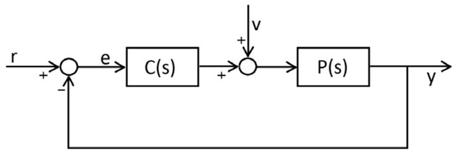

Fig. 1. The standard unity feedback system.

where a finite dimensional approach is used to find an interpolating outer function, without considering the $\mathcal{H}^\infty$ norm condition to be satisfied for robustness. We apply this method to solve the relaxed problems described by Wakaiki et al. (2013) to calculate "approximate" upper and lower bounds, then check the resulting $\mathcal{H}^\infty$ norm condition to verify that the stable controller designed also satisfies the robustness condition.

The paper is organized as follows: in Section 2 some details and brief results from Wakaiki et al. (2013) are presented. In Section 3 the method proposed in Vidyasagar (1985) and Doyle et al. (1990) is recalled and it is applied to the relaxed problems to calculate approximate maximum allowable uncertainty bounds. The effects of different free design parameters on the numerically calculated interpolation function are investigated in Section 5. Finally, Section 6 contains some concluding remarks on the proposed method.

2. PROBLEM DEFINITION

In this paper, we consider the feedback system shown in Figure 1 where C is required to be a stable controller, i.e. $C \in \mathcal{H}^\infty$, for a given infinite dimensional plant P. Recall that the feedback system (C, P) is stable if and only if $S = (1 + PC)^{-1}$, CS and PS are $\mathcal{H}^\infty$ functions. A controller $C \in \mathcal{H}^\infty$ leading to a stable feedback system (C, P), is said to be a strongly stabilizing controller for the given plant P.

In this paper we consider the same class of plants considered in Wakaiki et al. (2013):

$$P = \frac{N}{D} \tag{1}$$

such that $N \in \mathcal{H}^\infty$, $D \in \mathcal{H}^\infty$ and the pair (N, D) are strongly co-prime as in Smith (1989). A controller strongly stabilizing P is a robustly stabilizing controller for the set

$$\mathcal{P}_\rho := \{P_\Delta = (1 + W\Delta)P : \Delta \in \mathcal{H}^\infty, \|\Delta\|_\infty < 1/\rho\} \tag{2}$$

if and only if it satisfies

$$\|WT\|_\infty \leq \rho \tag{3}$$

where $T = PC(1 + PC)^{-1}$, and W characterizes the frequency distribution of the multiplicative plant uncertainty. In this work we assume that $W, W^{-1} \in \mathcal{H}^\infty$.

As in Wakaiki et al. (2013) we consider an inner-outer factorization of N and D so that

$$P = \frac{M_n}{M_d} N_0 \tag{4}$$

where M_n is inner and finite dimensional, M_d is inner and possibly infinite dimensional and $N_0, N_0^{-1} \in \mathcal{H}^\infty$.

Note that when M_d is infinite dimensional the plant contains infinitely many poles in $\mathbb{C}_+$ (e.g. a neutral time delay system with asymptotic pole chains in $\mathbb{C}_+$); however requiring $N_0^{-1} \in \mathcal{H}_\infty$ imposes a restriction that the plant is not strictly proper.

Example: A typical plant example satisfying the above conditions is the neutral time delay system

$$P(s) = \frac{(s - \alpha)(s - 4e^{-s} + 1)}{(s - 10)(s - 15)(2e^{-s} + 1)} \tag{5}$$

where the factorization is in the form

$$\begin{aligned} M_n(s) &:= \frac{(s - \alpha)(s - p)}{(s + \alpha)(s + p)} \\ M_d(s) &:= \frac{(s - 10)(s - 15)(2e^{-s} + 1)}{(s + 10)(s + 15)(e^{-s} + 2)} \\ N_o(s) &:= \frac{(s + \alpha)(s + p)(s - 4e^{-s} + 1)}{(s - p)(s + 10)(s + 15)(e^{-s} + 2)} \end{aligned} \tag{6}$$

with $p > 0$ being the only root of the quasi-polynomial $(s - 4e^{-s} + 1)$ in $\mathbb{C}_+$; p can be calculated numerically by using `qpmr` or `Yalta` packages, see Vyhlídal and Zítek (2014) and Avanessoff et al. (2013); for the above example, $p = 0.7990$.

In the rest of the paper, it is assumed that $W = KW_0$ where both W_0 and W_0^{-1} belong to $\mathcal{H}_\infty$ and $K > 0$. Thus we have robust stability if

$$\|W_0 T\|_\infty < \frac{1}{\rho\, K}. \tag{7}$$

Problem Definition: In summary, the problem at hand is to find a strongly stabilizing C for P satisfying (7) for the largest possible K. We call the largest $K > 0$ for which the above problem is solvable *the largest allowable uncertainty bound*. By Ünal and İftar (2012a), in order to have a feasible solution to the strong stabilization problem, P is assumed to satisfy the parity interlacing property. In Wakaiki et al. (2013) some upper and lower bounds are computed for the largest allowable uncertainty bound.

Brief Outline of the Proposed Solution: Wakaiki et al. (2013) showed that for a given $K > 0$ the strong robust stabilization problem is solvable if and only if there exists a function F satisfying all three conditions given below:

$$F, F^{-1} \in \mathcal{H}^\infty \tag{8}$$

$$\|W - M_d F\|_\infty \leq \rho \tag{9}$$

$$F(z_i) = \frac{W(z_i)}{M_d(z_i)}, i = 1, ..., n, \tag{10}$$

where $z_1, \ldots, z_n$ are the zeros of P in $\mathbb{C}_+$. Furthermore, once such a function F is constructed, a feasible controller is given by

$$C = \frac{W - M_d F}{M_n N_0 F} \tag{11}$$

Since the above problem is not straight forward to solve, Wakaiki et al. (2013) have proposed two relaxed problems each of which defines a necessary (respectively, a sufficient) condition to calculate upper and lower bounds for the largest possible multiplicative uncertainty K. In the relaxed problem, which is designed to solve for the lower bound, they have defined W_s to be $W_s, W_s^{-1} \in \mathcal{RH}^\infty$ such that $|W_s(j\omega)| \leq \rho - |W(j\omega)|$ for almost all $\omega \in \mathbb{R}$. If finitely many simple unstable zeros of the plant are called as z_i

then we define $\beta_i = W(z_i)/(M_d(z_i)W_s(z_i))$ for $i = 1, ..., n$. Wakaiki et al. (2013) proved that if G is a solution to the modified Nevanlinna-Pick problem with the interpolation data $(z_i, \beta_i)_{i=1}^n$ then

$$C = \frac{W - M_d W_s G}{M_n N_0 W_s G} \tag{12}$$

is a solution for the *strong and robust stabilization problem.* A similar definition is given by Wakaiki et al. (2013) for the upper bound calculations and it relies on a solution of another modified Nevanlinna-Pick problem.

Modified Version of Nevanlinna-Pick Problem: Suppose $z_1, ..., z_n \in \mathbb{C}_+$ are distinct and none of them coincide with $s = 1$; let $\beta_1, ..., \beta_n \in \mathbb{C} \setminus \{0\}$. Determine whether there exists a function G such that $G, G^{-1} \in \mathcal{H}^\infty$, $\|G\|_\infty \leq 1$ and $G(z_i) = \beta_i$ for $i = 1, ..., n$.

The condition $G^{-1} \in \mathcal{H}^\infty$ is the modification on the original Nevanlinna-Pick problem. The modified version of the problem is proved to be solvable by Gümüşsoy and Özbay (2009) and Özbay (2010) if and only if there exists an integer set $[k_1, ..., k_n]$ such that Pick matrix $\mathbf{P}([k_1, ..., k_n])$,

$$\mathbf{P}([k_1, ..., k_n]) := \left[\frac{-\log\beta_p - \log\overline{\beta_q} + j2\pi(k_q - k_p)}{1 - \phi(z_p)\overline{\phi(z_q)}} \right]_{p,q=1}^n \tag{13}$$

is positive semi definite where

$$\phi : \mathbb{C}_+ \to \mathbb{D} \quad : \quad \phi(s) = \frac{s-1}{s+1}. \tag{14}$$

Wakaiki et al. (2013) have given a solution to the modified Nevanlinna-Pick problem which ends up with an infinite dimensional G appearing in the designed controller, (12). As a result, the generated controller solves the *strong and robust stabilization problem* for an infinite dimensional plant with an infinite dimensional controller.

Remark. By the above design in (12), $F = W_s G$ is outer. In order to guarantee a bound on $\|F^{-1}\|_\infty$, Gümüşsoy and Özbay (2009) proposes a modified version of the Nevanlinna-Pick interpolation, where a parameter σ appears, so that $\|F^{-1}\|_\infty \leq e^\sigma$. In the numerical example considered below, we will use a relatively large σ and illustrate that this leads to a large controller gain at low frequencies.

3. AN INTERPOLATING RATIONAL OUTER FUNCTION

In Vidyasagar (1985) (see also Doyle et al. (1990)) a constructive method is outlined to generate a rational interpolating function, say U, which is guaranteed to be outer (i.e. both the function itself and its inverse are stable)[1] under some constraints related to parity interlacing property.

Let us assume that we have some $a_i \in \mathbb{C}_+$ and b_i as the interpolation data in the way $U(a_i) = b_i$ for $i = 1, ..., n$. Define $U_1(s) = b_1$, so that $U_1(a_1) = b_1$, clearly U_1 is outer and satisfies the first interpolation condition. Now suppose that an outer U_k is constructed in such a way that it satisfies the first k interpolation conditions for $1 \leq k < n$. Then, it is possible to define U_{k+1} as

$$U_{k+1}(s) = (1 + c_{k+1}H_{k+1}(s))^l U_k(s) \tag{15}$$

where $H_{k+1} \in \mathcal{H}^\infty$ and such that $H_{k+1}(a_i) = 0$ for $i = 1, ..., k$. This choice of H_{k+1} makes $U_{k+1}(a_i) = U_k(a_i)$ for $i = 1, ..., k$ independent of c and l. As a result, U_{k+1} is guaranteed to satisfy the interpolation data $(a_i, b_i)_{i=1}^k$. If it is possible to choose c and l such that $U_{k+1}(a_{k+1}) = b_{k+1}$ and $|c_{k+1}| < 1/\|H_{k+1}\|_\infty$ then U_{k+1} satisfies the interpolation data $(a_i, b_i)_{i=1}^{k+1}$ and it is outer. At the end of the algorithm, $U = U_n$ satisfies all the interpolation data and is outer.

By this method, it is possible to generate interpolating rational outer functions. It is also possible to tune the degree of the outer function by changing the value of parameter l, when c does not satisfy the norm condition. One additional important point is the choice of H_i functions. Due to the imposed requirements, zero location of the function is obvious (i.e. zeros of H_k have to be at a_i for $i = 1, ..., k-1$). However, the pole location is not constrained by the requirements. This means that the pole location can be a free design parameter to be determined to shape the resulting interpolation function. If we recall the requirements of the modified Nevanlinna-Pick problem from Section 2, we seek for an outer function G satisfying certain interpolation data, and a norm condition as $\|G\|_\infty < 1$. The method described in this section gives a function satisfying the first two conditions (i.e. interpolation data and being outer) but not necessarily the norm constraint.

4. A COMPUTATIONAL METHOD FOR THE PROBLEM SOLUTION

In this section, we try to make use of the method in Section 3 in order to find a norm constrained interpolating rational outer function to solve the modified Nevanlinna-Pick problem defined in Wakaiki et al. (2013).

Let us first recall the problem definition. The problem data: z_i for $i = 1, ..., n$ are the unstable zeros of the plant P (they are assumed to be distinct). The problem is to find a G such that

$$\begin{aligned} &G(z_i) = \beta_i \\ &G \in \mathcal{H}^\infty \\ &G^{-1} \in \mathcal{H}^\infty \\ &\|G\|_\infty < 1. \end{aligned}$$

Proposed Algorithm:

- Define $\mathbf{r} = (r_1, ..., r_n)$ to be the relative pole location for the corresponding z_i within each H_k which will be a part of the interpolation operation. (i.e. $\mathbf{r} = (1, ..., 1)$ is the case called as fixed pole location)
- Use the inner-outer factorization of the plant as given in (6), $P = M_d N_0 / M_n$
- Define l_{max} to be the maximum allowable relative degree for the interpolant G.
- At each step fix $\mathbf{r}$ to find maximum possible K as follows:

  while(bilinear search over K)

[1] An outer function need not be invertible in $\mathcal{H}^\infty$, in fact we seek an outer function whose inverse is also in $\mathcal{H}^\infty$, such functions are called unimodular in $\mathcal{H}^\infty$. But we will look for an outer function, then using a parameter $\sigma > 0$ as in the above remark, we will put a bound on the inverse of the interpolating outer function, using an idea from Gümüşsoy and Özbay (2009).

- Define $W = KW_0$ with current K
- Estimate W_s by using the MATLAB function `fitmagfrd` as explained in Wakaiki et al. (2013)
- Calculate $\beta_i = W(z_i)/M_d(z_i)W_s(z_i)$ for $i = 1, ...n$
- Let $U_1(s) = \beta_1$

  for $k = 2 : n$
  while $(l < l_{max})$
  $U_k = (1 + c_k H_k)^l U_{k-1}$
  Find c_k from $U_k(z_k) = \beta_k$
  if $((\|U_k\|_\infty < 1)$ and $(c_k < 1/\|H_k\|_\infty))$
  Conditions are satisfied, break while
  else
  $l = l + 1$
  end if
  end while
  end for
  $G = U_n$
  if a feasible G is constructed
  increase K
  else
  decrease K
  end if
  end while

The proposed algorithm is best explained with an example.

5. AN ILLUSTRATIVE EXAMPLE

Wakaiki et al. (2013) have studied a numerical example and derived upper and lower bounds for the multiplicative uncertainty under which a stable controller can be generated. The example is formed by the plant given in (5) and the factorization in (6) and the rest of the problem data is as follows:

$$\left.\begin{aligned} & W(s) = K\frac{s+1}{s+10} \\ & \rho = 1 \\ & 2 \le \alpha < 10 \\ & K > 0 \end{aligned}\right\} \quad (16)$$

See Figure 2 of Wakaiki et al. (2013) for the lower and upper bounds of the largest allowable K for which the robust strong stabilization problem is solvable with data given in (16) for the plant (5), with $\alpha \in (2\ ,10)$. In what follows we illustrate the application of the algorithm proposed in Section 4. Our objective is to find a finite dimensional G as an alternative to the infinite dimensional one constructed in Wakaiki et al. (2013).

5.1 Interpolating Rational Outer Function by an Inner $H(s)$

Having reached the original results from Wakaiki et al. (2013), we started to replace the interpolation function generating part with the method of Section 3. For the given numerical example, W_s and W_n are generated by MATLAB built-in function `fitmagfrd` and the interpolation data calculated as $z_1 = \alpha$, $\beta_1 = W(\alpha)/(M_d(\alpha)W_s(\alpha))$ and $z_2 = p = 0.7990$, $\beta_2 = W(p)/(M_d(p)W_s(p))$. Recall that p is the only unstable zero of the infinite dimensional part of the plant and α is the simple zero of the plant. The

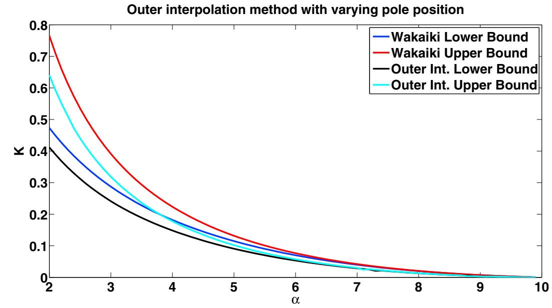

Fig. 2. Upper and lower bounds calculated for the maximum allowable multiplicative uncertainty by a finite dimensional interpolation function which is generated by the outer interpolation method of Section 3 using varying pole location in $H(s)$ as the interstage function, having degree $l < 5$.

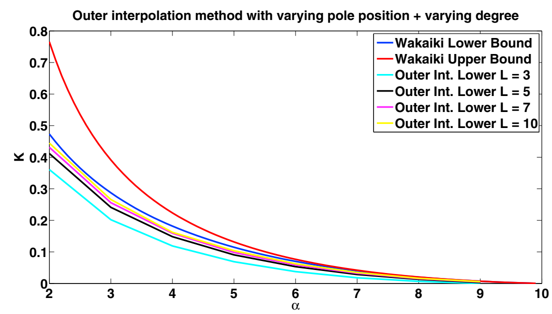

Fig. 3. Lower bounds calculated for the maximum allowable multiplicative uncertainty by a finite dimensional interpolation function which is generated by the outer interpolation method of Section 3 using varying pole location in $H(s)$ as the interstage function, having degree $l \in \{3, 5, 7, 10\}$.

objective is to calculate the maximum allowable multiplicative uncertainty bound under which a stable robust controller can be generated. W_s is replaced by W_n for upper bound calculations. Since we have two interpolation conditions, and both zeros are real, we just need to design a single H function for the second interpolation phase. The simplest and immediate choice is $H(s) = \frac{(s-\alpha)}{(s+\alpha)}$ for which H becomes inner. It is observed that the bounds calculated by this choice of H are far away from the bounds determined in Wakaiki et al. (2013). So, the next step is to investigate different choices of H and if necessary increase the order l.

5.2 Outer Interpolation by Varying Pole Location

After having the unsatisfactory results which are explained in Section 5.1, we now introduce a new parameter to the problem as the pole location of the interstage function H. Accordingly, set $H(s) = \frac{(s-\alpha)}{(s+r\alpha)}$ where $r > 0$ is the design parameter. We search for the optimum r which maximizes the upper and lower bounds for a fixed α. The results are shown in the Figure 2.

As it was clearly observed that letting pole location to vary, instead of fixing it to make H inner, significantly improves both the upper and lower bound approximations. It is also important to understand what upper and lower

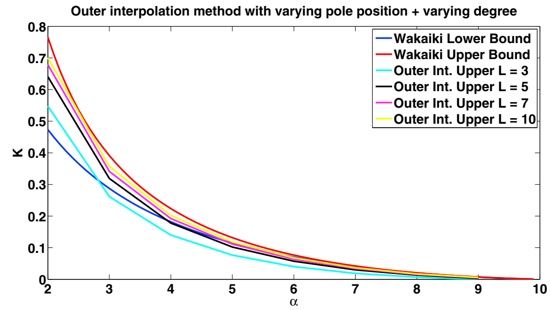

Fig. 4. Upper bounds calculated for the maximum allowable multiplicative uncertainty by a finite dimensional interpolation function which is generated by the outer interpolation method of Section 3 using varying pole location $F(s)$ as the interstage function, having degree $l \in \{3, 5, 7, 10\}$.

bounds mean in terms of the solution of the modified Nevanlinna-Pick problem. In the formulation derived by Wakaiki et al. (2013), lower bound is the bound under which the defined problem is certainly solvable by the given solution method (i.e. by calculating an infinite dimensional G). Similarly, upper bound is the bound above which the defined problem is certainly not solvable by the given solution method. In other words, for a fixed α in Figure 2 of Wakaiki et al. (2013), the optimum multiplicative uncertainty K under which a stable but infinite dimensional controller can be generated is between the defined upper and lower bounds. When this explanation is considered, the expected behavior of the approximated upper and lower bounds is to approach from above and from below, respectively. Figure 2 suggests that approximate lower bound behaves as expected whereas the approximate upper bound also approaches from below. To be sure about the approaching direction of the bounds an extra computation is done. In this computation, the varying pole location technique described in this section is used for some different values of $l \in \{3, 5, 7, 10\}$ to visualize the approaching direction of the approximate bounds calculated by the technique of Section 3. Figure 3 clearly shows that the approximate lower bound approaches from below to the original lower bound calculated by Wakaiki et al. (2013) as expected. However, as Figure 4 depicts the approximate upper bound also approaches to the original upper bound calculated by Wakaiki et al. (2013) from below. This unexpected behavior seems to grant a better upper bound than the original bound as a first impression, however, since the problems which are solved to generate the bounds are relaxed versions of the original problem, an approximate solution for the upper bound calculation is not meaningful. On the contrary, any approximate solution for the lower bound which stays strictly below the original bound is a suboptimal solution to the original problem. We make use of this fact to generate a low order interpolation function G for the case when $\alpha = 2$, $l = 5$, $K = 0.4117$, $r = 0.2046$ as given in (17) and the controller is given by (18). The comparison of the controller and G function designed by Wakaiki et al. (2013) and designed by the newly proposed method are given in Figures 5, 6, and 7. The resulting finite dimensional G is outer, i.e. both $G, G^{-1} \in \mathcal{H}^\infty$. The interpolation data that is required for the given value of $K = 0.4117$ is calculated by $\beta_{1,2} = W(z_{1,2})/M_d(z_{1,2})W_s(z_{1,2})$

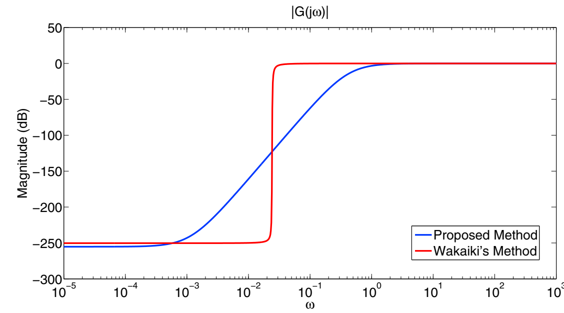

Fig. 5. The magnitude plots of $G(j\omega)$ obtained by Wakaiki et al. (2013) and by the proposed algorithm, using $l = 5$, when $\alpha = 2$.

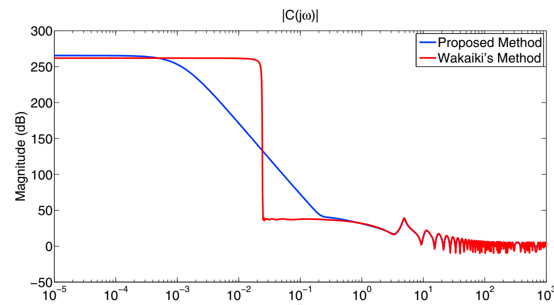

Fig. 6. The magnitude plot of the calculated $C(s)$ functions by both methods with $\alpha = 2$, and $l = 5$ for the proposed method.

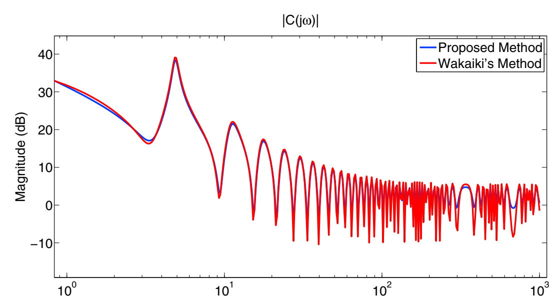

Fig. 7. Magnitude plot of the controller in the high frequency region.

for $z_1 = \alpha = 2$ and $z_2 = p = 0.7990$ that turn out to be $(z_i, \beta_i) = \{(0.7990, 0.1275), (2, 0.3955)\}$. For the constraint $\|G^{-1}\|_\infty \le e^\sigma$, we take $\sigma = 30$. It is a simple exercise to check that $G(0.7990) = 0.1275$ and $G(2) = 0.3955$ by using (17). As the last remark, Figure 5 shows that $\|G\|_\infty \le 1$; moreover $\|G^{-1}\|_\infty \le 5.7 \times 10^{12} < 1.0686 \times 10^{13} = e^{30}$. Thus, the function G constructed here is an admissible solution to the modified Nevanlinna-Pick problem.

$$G(s) = \frac{(s + 0.001147)^5}{(s + 0.4091)^5} \quad (17)$$

$$C = \frac{W - M_d W_s G}{M_n N_0 W_s G} \quad (18)$$

Note that robust stabilization problem requires N_0^{-1} to be a factor of the controller which means exact cancelation of the minimum phase part of the plant. However, robust stabilization problem considered here deals with uncertain plants of the form (2), characterized by the weight W. If it is dangerous to invert some portions of N_0 then, the uncertainty weight W must be chosen accordingly. In particular, as in (5) if there are infinitely many poles in $\mathbb{C}_+$, any parametric uncertainty in the time delay in M_d of (6) puts the problem outside the framework of uncertainty description of (2).

6. CONCLUSIONS

We have obtained some preliminary results towards robust stabilization of an infinite dimensional system with a possibly low order and stable controller. The plant of interest can only have finitely many unstable zeros but may posses infinitely many unstable poles. The strong and robust stabilization of such a plant was studied by Wakaiki et al. (2013) where the authors have proposed two relaxed problems to calculate lower and upper bounds for the multiplicative uncertainty under which a stable but infinite dimensional controller can be generated.

This paper is a first step to solve these relaxed problems by a finite dimensional controller using the rational interpolating outer function method of Section 3: the interpolant G is finite dimensional. We used the same numerical example as Wakaiki et al. (2013) and developed a finite dimensional outer G to achieve robust stabilization. We examined the effects of pole location of the interpolating function and relative degree of G on lower and upper bounds and concluded that the approximations get better as the degree increases for the lower bound. We also explained why it is not a good idea to solve the relaxed problem for the upper bound by an approximate approach.

In future studies we will extend this method to obtain finite dimensional strongly and robustly stabilizing controllers by using approximations of M_d and N_0 together with the finite dimensional G obtained here. Extension of the method to MIMO plants is also an interesting open problem.

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