

Robust Adaptive Posicast Controller [★]

Yildiray Yildiz ^{*} Anuradha Annaswamy ^{**}

^{*} *Mechanical Engineering Department, Bilkent University, 06800
Ankara, Turkey (e-mail: yyildiz@bilkent.edu.tr).*

^{**} *Department of Mechanical Engineering, Massachusetts Institute of
Technology, MA 02139 USA (e-mail: aanna@mit.edu)*

Abstract: Adaptive Posicast Controller that is robust to delay-mismatch is introduced in this paper. Inspired from a recent result on guaranteed delay margins in adaptive control, the original adaptive laws of the above mentioned controller are modified using projection to compensate the uncertainty in the input delay. It is conjectured and shown in simulations that even though the assumed upper bound for the delay value is exceeded, Adaptive Posicast Controller with projection algorithm keeps all the system signals bounded.

© 2015, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.

Keywords: Robust adaptive control, time delay systems, projection algorithm.

1. INTRODUCTION

The instability inducing effect of time-delays in adaptive systems and the solution of this problem have been a topic of a large body of research. Among them, the very recent ones are presented in Pepe and Jiang (2006); Mazenc et al. (2008); Jankovic (2009); Bekiaris-Liberis and Krstic (2010b); Bekiaris-Liberis et al. (2010); Bekiaris-Liberis and Krstic (2010a); Mazenc and Niculescu (2011); Mazenc et al. (2011); Bekiaris-Liberis and Krstic (2013).

Adaptive Posicast Controller (APC) is developed for linear time-invariant systems with input time delays (Niculescu and Annaswamy (2003)) and later extended for a larger class of systems (Yildiz et al. (2010a)). Due to its simple structure it has been one of the rare examples of adaptive controllers that is successfully implemented in an industrial setting (see Yildiz et al. (2010b, 2011)). Although APC is developed for known input delays, the robustness of the controller for delay mismatch, the case when the assumed and actual input delay values are different, is shown for specific cases in experimental tests. However, no theoretical result has been reported yet in this issue.

Recently it has been shown by Matsutani et al. (2013) that, a closed loop system with a linear time invariant plant whose states are measurable and a conventional model reference adaptive controller with projection algorithm, has a guaranteed delay (τ) margin. Inspired by this result, in this paper it is conjectured that the APC with projection algorithm is guaranteed to be robust to a delay-mismatch of $\delta\tau \leq \bar{\delta\tau}$, where $\delta\tau = \tau_{actual}/\tau_{assumed}$ and $\bar{\delta\tau}$ is an upper bound that can be calculated. In the simulation results it is shown that a) a conventional MRAC with projection algorithm can provide a certain delay margin, above which the signals grow in an unbounded fashion, b) when APC is used for the same plant with the input delay value that causes instability, all the signals remain bounded and

c) when the delay value exceeds the assumed delay value for APC, a projection modification in the adaptive laws provides robustness by stopping parameter drift due to delay mismatch. The proof of the conjecture introduced in this paper is currently under investigation. It is noted that the control of an uncertain ODE system with unknown input delay, in the case of full state measurements, is presented in Bresch-Pietri and Krstic (2009).

In Section 2, the result on the guaranteed delay margin for adaptive controllers is briefly presented for the scalar case. In Section 3, Robust Adaptive Posicast Controller (RAPC) is introduced for the scalar case. In Section 4, RAPC is introduced for the output measurement case. In Section 5, simulation results are provided and in Section 6, a summary is given.

2. DELAY MARGIN FOR ADAPTIVE CONTROL

In this section, the result obtained by the author Annaswamy and her colleagues in Matsutani et al. (2012) is briefly explained.

Consider the following scalar plant dynamics

$$\dot{x}(t) = ax(t) + u(t - \tau) \quad (1)$$

where a is unknown and its absolute value is assumed to be bounded with a known bound. The input time delay τ is also assumed to be unknown. It can be shown that there exists a τ^* such that $\forall \tau < \tau^*$, the closed loop system with plant (1) and a standard adaptive control input

$$u(t) = k(t)x(t) + r(t) \quad (2)$$

together with the following adaptive law has globally bounded solutions for any given initial conditions $x(\xi)$ and $k(\xi)$, for $\xi \in [-\tau, 0]$, satisfying $f(k(\xi)) < 1$, $\forall \xi$:

$$\dot{k} = \gamma \text{Proj}(k, -x(t)e(t)) \quad (3)$$

[★] Sponsor and financial support acknowledgment goes here. Paper titles should be written in uppercase and lowercase letters, not all uppercase.

where $\gamma > 0$ is the adaptation rate,

$$\text{Proj}(\theta, y) = \begin{cases} y - \frac{\nabla f(\theta)(\nabla f(\theta))^2}{\|\nabla f(\theta)\|^2} y f(\theta) & \text{if } f(\theta) > 0 \text{ and } y^T f(\theta) > 0 \\ y & \text{otherwise} \end{cases} \quad (4)$$

and the function $f(\cdot)$ is convex. An example of these functions can be given as

$$f(\theta) = \frac{\|\theta\|^2 - \theta_{\max}'^2}{\epsilon^2 + 2\epsilon\theta_{\max}'} \quad (5)$$

where θ_{\max}' and ϵ are arbitrary positive constants.

Consider the sets Ω_0 and Ω_1 that are defined as

$$\begin{aligned} \Omega_0 &= \{\theta \mid -\theta_{\max}' \leq \theta \leq \theta_{\max}'\} \\ \Omega_1 &= \{\theta \mid -\theta_{\max} \leq \theta \leq \theta_{\max}\} \end{aligned} \quad (6)$$

where $\theta_{\max} = \theta_{\max}' + \epsilon$. It is noted that when the function given in (5) is used and θ is a scalar, the projection algorithm (4) becomes

$$\text{Proj}(\theta, y) = \begin{cases} y \frac{\theta_{\max}^2 - \theta^2}{\theta_{\max}^2 - \theta_{\max}'^2} & \text{if } \theta \in \Omega_1 \setminus \Omega_0 \wedge y\theta > 0 \\ y & \text{otherwise} \end{cases} \quad (7)$$

The extension of this result for higher order systems with measurable states is provided in Matsutani et al. (2013).

3. BEYOND DELAY MARGIN: ROBUST ADAPTIVE POSICAST CONTROLLER

3.1 Adaptive Posicast Controller Design

The result presented in Section 2 shows that a conventional adaptive controller with adaptive laws modified using projection algorithm can ensure the boundedness of all signals up to certain input delay value τ^* . In addition, τ^* can be determined based on controller parameters and plant uncertainty bounds. When the delay value goes beyond the delay margin τ^* , there is possibility of instability. On the other hand, Adaptive Posicast Controller (APC) is developed for the cases of known input delay and if an upper bound $\bar{\tau}$ on possible system delays is known, APC can be designed based on this upper bound. Below, the design of APC is briefly explained.

Consider the plant given in (1) with a delay value of $\bar{\tau}$

$$\dot{x}(t) = ax(t) + u(t - \bar{\tau}). \quad (8)$$

If a is known, a fixed controller given as

$$u(t) = k^*x(t + \bar{\tau}) + r(t) \quad (9)$$

results in a closed loop dynamics as

$$\begin{aligned} \dot{x}(t) &= (a + k^*)x(t) + r(t - \bar{\tau}) \\ &= a_m x(t) + r(t - \bar{\tau}). \end{aligned} \quad (10)$$

If k is chosen such that $a_m < 0$, the state x converges to the reference r . To solve the problem of non-causality in the control input (9), $x(t + \bar{\tau})$ can be estimated using plant dynamics (1) and the control law can be modified as

$$u(t) = k_x^* x(t) + \int_{-\tau}^0 \beta^*(\sigma) u(t + \sigma) d\sigma + r(t) \quad (11)$$

where $k_x^* = ke^{a\tau}$ and $\beta^*(\sigma) = ke^{a\sigma}$.

When a is unknown, the controller parameters in (9) need to be adaptive. To do this, first, (10) is defined to be the ideal system response and rewritten as

$$\dot{x}_m(t) = a_m x_m(t) + r(t - \bar{\tau}). \quad (12)$$

Defining $e(t) = x(t) - x_m(t)$, it can be shown that there exists a $\bar{\tau}^*$ such that given initial conditions $x(\eta), k_x(\eta), \beta(\eta, \sigma), \eta \in [-\bar{\tau}, 0]$ and $u(\eta)$ for $\eta \in [-2\bar{\tau}, 0]$, the following controller

$$u(t) = k_x(t)x(t) + \int_{-\tau}^0 \beta(t, \sigma) u(t + \sigma) d\sigma + r(t) \quad (13)$$

with the adaptive laws

$$\begin{aligned} \dot{k}_x(t) &= -\gamma_x e(t)x(t - \bar{\tau}) \\ \frac{\partial \beta(t, \sigma)}{\partial t} &= -\gamma_\beta e(t)u(t + \sigma - \bar{\tau}) \end{aligned} \quad (14)$$

ensures that all the signals are bounded and $\lim_{t \rightarrow \infty} e(t) = 0, \forall \bar{\tau} \leq \bar{\tau}^*$.

The design of APC for state accessible case with higher order plants is very similar to the scalar case and therefore is not shown here.

3.2 Properties of the Projection Algorithm

Before stating the Robust Adaptive Posicast Controller conjecture, it is important to emphasize a useful property of the projection algorithm which is used in proving the robustness of the adaptive systems to time-delays. This property guarantees the boundedness of the adaptive parameters independent of the boundedness of other system signals.

Lemma 1: Consider the following dynamics

$$\begin{aligned} \theta(0) &\in \Omega_1 \\ \dot{\theta} &= \text{Proj}(\theta, y) \end{aligned} \quad (15)$$

where projection algorithm is as explained in (7). Then, assuming $t_0 = 0$, if $|\theta(0)| \leq \theta_{\max}$, then $|\theta(t)| \leq \theta_{\max}, \forall t \geq 0$. The proof of this lemma can be found in Lavretsky (2008).

3.3 Robust Adaptive Posicast Controller Design

Consider the following plant model

$$\dot{x}(t) = ax(t) + u(t - \delta_\tau \bar{\tau}) \quad (16)$$

where a is an unknown constant, $\bar{\tau}$ is the nominal time delay and $\delta_\tau = \bar{\tau}_{\text{actual}}/\bar{\tau}$. For simplicity the input gain is assumed to be 1. This model represents the case when the actual value of the input time delay is unknown. The following conjecture states that the APC together with the projection algorithm provides robustness to the adaptive system up to a certain value of δ_τ .

Conjecture 1: Given initial conditions $k_x(\eta), x(\eta), \beta(\eta, \sigma)$ for $\eta \in [-\delta_\tau \bar{\tau}, 0]$ and $u(\eta)$ for $\eta \in [-2\delta_\tau \bar{\tau}, 0]$, there exists a δ_τ^* such that for all $\delta_\tau \in [0, \delta_\tau^*]$ the closed loop system consisting of the plant (16), controller (13) and the adaptive laws given in (15) has bounded solutions for all $t \geq 0$.

It is noted that the projection algorithm (15) is applied to each adaptive parameter, $k_x(t)$ and $\beta(t, \sigma)$, separately, noting that the adaptive laws without projection modification is given in (14). For example, in the case of $k_x, \theta = k_x$ and $y = -e(t)x(t - \bar{\tau})$ in (15).

Remark 1: It is noted that the APC stabilizes the system for all delay values that are smaller than $\bar{\tau}$. Therefore, unlike conventional MRAC, APC does not need any modification (projection or other types) to its adaptive laws to provide delay robustness up to the delay value of $\bar{\tau}$.

Remark 2: The conjecture states that when the delay value exceeds $\bar{\tau}$, APC still ensures that all the solutions of the closed loop system are bounded, provided that the adaptive laws are modified using projection.

Remark 3: A strong evidence for the stated conjecture is the recent proofs provided in Matsutani et al. (2012) and Matsutani et al. (2013) where it is shown that projection algorithm guarantees a certain delay margin τ^* when conventional MRAC is used for linear time invariant systems in the scalar and state accessible cases. Encouraged by these results the proof of the stated conjecture is currently under investigation.

Remark 4: To the best of authors' knowledge, projection algorithm has not been implemented for a function $\beta(t, \sigma)$, given in (13), before. To achieve this, first, the norm of $\beta(t, \sigma)$ is defined as

$$\|\beta(t, \sigma)\| = \sup_{\sigma \in [-\bar{\tau}, 0]} |\beta(t, \sigma)|. \quad (17)$$

If (17) is substituted in (5), it is obtained that

$$f(\beta) = \frac{(\sup_{\sigma \in [-\bar{\tau}, 0]} |\beta(t, \sigma)|)^2 - \beta_{\max}'^2}{\epsilon^2 + 2\epsilon\beta_{\max}'}, \quad (18)$$

where β_{\max} and β_{\max}' are defined similar to before. It can be seen that substituting (18) in (4) results in the same projection operator for scalar parameters given in (7).

Remark 5: In real computer implementations, the finite integral term in (13) need to be discretized as

$$\int_{-\bar{\tau}}^0 \beta(t, \sigma) u(t + \sigma) d\sigma = \left(\beta_1 u(t - \bar{\tau}) + \beta_2 u(t - \bar{\tau} + \bar{\tau}/n) + \dots + \beta_n u(t - \bar{\tau}/n) \right) \times \frac{\bar{\tau}}{n}. \quad (19)$$

Therefore, the projection algorithm need to be applied to all of the adaptive control parameters $\beta_1, \beta_2, \dots, \beta_n$ resulting from this approximation.

4. ROBUST ADAPTIVE POSICAST CONTROLLER: OUTPUT FEEDBACK CASE

4.1 Case 1: Known Time Delay

Consider the following plant dynamics

$$y(t) = k \frac{N(s)}{D(s)} u(t - \tau) \quad (20)$$

where, s is the differential operator, $N(s)$ and $D(s)$ are monic polynomials that are coprime, k is a positive constant and the relative degree of this plant is smaller or equal to 2. It is also assumed that τ , input time delay, is known and the delay-free part of the plant is minimum phase. A reference model, representing ideal closed loop system behavior is represented using the following input-output description

$$y_m(t) = k_m \frac{N_m(s)}{D_m(s)} r(t - \tau). \quad (21)$$

It is assumed that the reference model (21) is stable and minimum phase, its relative degree is equal to or greater than that of the plant and the numerator and denominator polynomials are monic.

The plant dynamics (20) can be represented using state space description as

$$\begin{aligned} \dot{X} &= AX(t) + Bu(t - \tau) \\ y(t) &= h^T X(t). \end{aligned} \quad (22)$$

Consider the following state space descriptions of two signal generators:

$$\begin{aligned} \dot{v}_1(t) &= Mv_1(t) + \mu u(t - \tau) \\ \dot{v}_2(t) &= Mv_2(t) + \mu y(t). \end{aligned} \quad (23)$$

It can be shown that there exists α_1^* , α_2^* and k_r^* such that the control input

$$u(t) = \alpha_1^{*T} v_1(t + \tau) + \alpha_2^{*T} v_2(t + \tau) + k_r^* r(t) \quad (24)$$

ensures that the closed loop transfer function matches with the reference model (21). Since the control law (24) is non-causal, the following procedure is used to obtain a causal controller:

It is known that (Narendra and Annaswamy (2005)) there exists $a, b \in \mathbb{R}^n$ such that

$$y(t) = a^T v_1(t) + b^T v_2(t). \quad (25)$$

Using (25), the signal generator dynamics (23) can be rewritten as

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} M & 0 \\ \mu a^T & M + \mu b^T \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} + \begin{bmatrix} \mu \\ 0 \end{bmatrix} u(t - \tau). \quad (26)$$

With the following definition,

$$F = \begin{bmatrix} M & 0 \\ \mu a^T & M + \mu b^T \end{bmatrix} \quad (27)$$

it can be shown after some manipulations that the following causal control law is equivalent to (24):

$$u(t) = \zeta_1^{*T} v_1(t) + \zeta_2^{*T} v_2(t) + \int_{-\tau}^0 \psi^*(\rho) u(t + \rho) d\rho + k_r^* r(t) \quad (28)$$

where

$$\begin{aligned} [\zeta_1^{*T} \ \zeta_2^{*T}] &= [\alpha_1^{*T} \ \alpha_2^{*T}] e^{F\tau} \\ \psi^*(\rho) &= [\alpha_1^{*T} \ \alpha_2^{*T}] e^{-F\rho} \begin{bmatrix} \mu \\ 0 \end{bmatrix}. \end{aligned} \quad (29)$$

When the coefficients of the plant polynomials $N(s)$ and $D(s)$ and the positive high frequency gain k is unknown, the controller parameters in (28) need to adapt themselves continuously based on the tracking error and other system signals to achieve the boundedness of all signals and asymptotic stability of the tracking error.

Theorem 1: Given initial conditions $X(t_0), u(\rho), \rho \in [t_0 - 2\tau, t_0], \zeta_1(\sigma), \zeta_2(\sigma), k_r(\sigma), \psi(\sigma), \sigma \in [t_0 - \tau, t_0]$, the following control law

$$\begin{aligned} u(t) &= \zeta_1^T(t) v_1(t) + \zeta_2^T(t) v_2(t) + \int_{-\tau}^0 \psi(t, \rho) u(t + \rho) d\rho \\ &+ k_r(t) r(t) \end{aligned} \quad (30)$$

together with the adaptive laws

$$\begin{aligned} \dot{\zeta}_1^T &= -\gamma_{\zeta_1} e_1(t) v_1(t - \tau) \\ \dot{\zeta}_2^T &= -\gamma_{\zeta_2} e_1(t) v_2(t - \tau) \\ \dot{k}_r &= -\gamma_{k_r} e_1(t) r(t - \tau) \\ \frac{\partial \psi(t, \rho)}{\partial t} &= -\gamma_{\psi} e_1(t) u(t + \rho - \tau) \end{aligned} \quad (31)$$

ensures that all the closed loop system signals are bounded and $\lim_{t \rightarrow \infty} (y(t) - r(t)) = 0$. The proof of this theorem can be found in Yildiz et al. (2010a).

4.2 Case 2: Uncertain Time Delay

Consider the following plant dynamics:

$$y(t) = k \frac{N(s)}{D(s)} u(t - \delta_\tau \tau) \quad (32)$$

where k is an unknown positive high frequency gain, $N(s)$ and $D(s)$ are monic coprime polynomials with unknown coefficients, τ is a known input gain and δ_τ is an unknown constant. Here, $\delta_\tau = \tau_{\text{actual}}/\tau$ represents the uncertainty in the time delay.

Conjecture 2: Given initial conditions as in *Theorem 1*, there exists a δ_τ^* such that the plant dynamics (32) and the controller (30) with the adaptive laws given as

$$\begin{aligned} \dot{\zeta}_1^T &= \gamma_{\zeta_1} \text{Proj}(\zeta_1^T, -e_1(t) v_1(t - \tau)) \\ \dot{\zeta}_2^T &= \gamma_{\zeta_2} \text{Proj}(\zeta_2^T, -e_1(t) v_2(t - \tau)) \\ \dot{k}_r &= \gamma_{k_r} \text{Proj}(k, -e_1(t) r(t - \tau)) \\ \frac{\partial \psi(t, \rho)}{\partial t} &= \gamma_{\psi} \text{Proj}(\psi(t, \rho), -e_1(t) u(t + \rho - \tau)) \end{aligned} \quad (33)$$

have bounded solutions for all $\delta_\tau \in [0, \delta_\tau^*]$.

Remark 6: As mentioned in Section 3.3, where Robust Adaptive Posicast Controller design is explained for scalar plants, there exists a strong evidence for this conjecture in Matsutani et al. (2013) and Matsutani et al. (2013).

5. SIMULATION RESULTS

Consider the following plant dynamics

$$\dot{x}(t) = ax(t) + bu(t - \tau) \quad (34)$$

where the state constant $a = -0.8$, input constant $b = 1.6$ and the input delay $\tau = 0.66$. The reference model is given as

$$\dot{x}_m(t) = a_m x_m(t) + b_m r(t) \quad (35)$$

where $a_m = -2$ and $b_m = 2$. It can be shown that a fixed controller given as

$$u(t) = k_x x(t) + k_r r(t) \quad (36)$$

where $k_x = -0.75$ and $k_r = 1.25$ provides exact model matching between (35) and the delay free part of (34).

The state input constant a is changed to 0.3, which introduces a 62.5% uncertainty, and a conventional adaptive controller is designed which is given as

$$\begin{aligned} u(t) &= k_x(t) x(t) + k_r r(t) \\ \dot{k}_x &= -\gamma e(t) x(t) \end{aligned} \quad (37)$$

where $\gamma = 1$ and $e(t) = x(t) - x_m(t)$. The initial condition for the adaptive controller is set to $k_x(0) = -0.75$. Fig. 1 shows the tracking performance of the adaptive controller (37). When there is no delay in the system, the response is good. However, when an input delay $\tau = 0.66$ is introduced, the closed loop system gets very close to instability. Figure 2 shows how the controller parameter k_x evolves with and without delay cases. k_x drifts continuously for the nonzero delay case.

To prevent the parameter drift and thus instability, the adaptive control laws are modified using projection, where

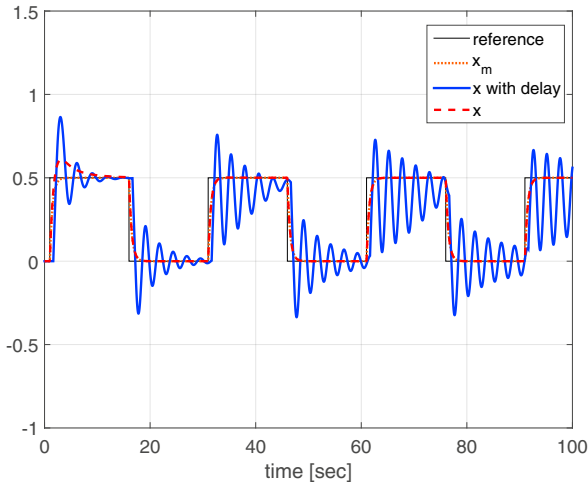


Fig. 1. Evolution of the reference, reference model output x_m and plant output x , when conventional MRAC is used without projection for delay values of $\tau = 0$ and $\tau = 0.66$.

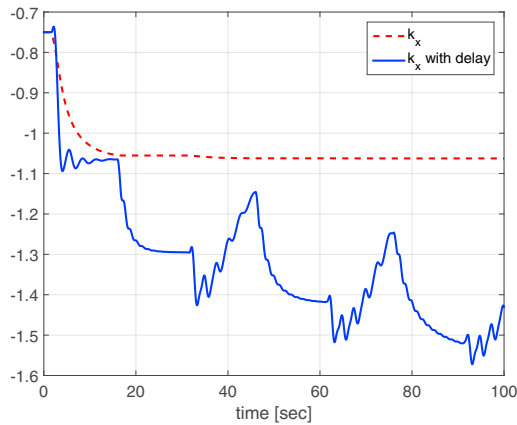


Fig. 2. Evolution of the controller parameter k_x , when conventional MRAC is used without projection for delay values of $\tau = 0$ and $\tau = 0.66$.

projection parameters are selected as $k_{\max} = 1$ and $\epsilon = 0.01$. Fig. 3 shows tracking performance comparison between the cases when the projection algorithm is used and not used. When projection is used, instability is eliminated however a small steady state error ($\approx 4.5\%$) is introduced, as expected. Fig. 4 shows how projection algorithm help stop parameter drift. It is noted that the steady state error can be made smaller by relaxing the projection algorithm parameters defining the bounds of k_x .

Fig. 5 shows the comparison between the tracking performances of MRAC with projection and APC without projection for $\tau = 0.66$. As expected, APC performs much better than MRAC even without any projection algorithm.

Fig. 6 - 7 shows that when a 55% delay uncertainty is introduced, which results in $\tau = 1.02$, the response of the APC becomes oscillatory and k_x , one of adaptive parameters of APC drifts continuously. However, when the adaptive laws are modified, i.e. when RAPC is used, the

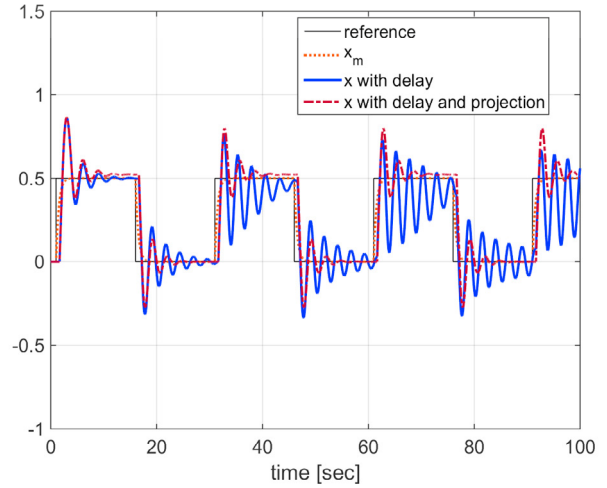


Fig. 3. Evolution of the reference, reference model output x_m and plant output x , when conventional MRAC is used with and without projection for $\tau = 0.66$.

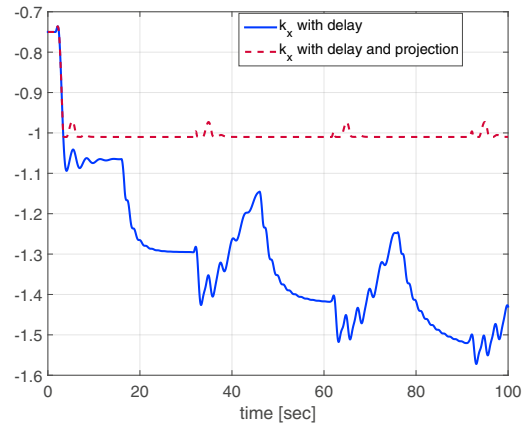


Fig. 4. Evolution of the controller parameter k_x , when conventional MRAC is used with and without projection for $\tau = 0.66$.

amplitude of the oscillations decreases considerably and the dangerous parameter drift stops.

6. SUMMARY

Robust Adaptive Posicast Controller (RAPC) is introduced in this paper. Two conjectures, one for the scalar case and the other for the output feedback case, about the boundedness of closed loop system signals are given. Simulation results are provided that verify the robustness properties of RAPC.

REFERENCES

- Bekiaris-Liberis, N., and Krstic, M. (2010). Stabilization of strict-feedback linear systems with delayed integrators. *Automatica*, 46(11), 1902–1910.
- Bekiaris-Liberis, N. and Krstic, M. (2010a). Delay-adaptive feedback for linear feedforward systems. *Systems and Control Letters*, 59, 277–283.

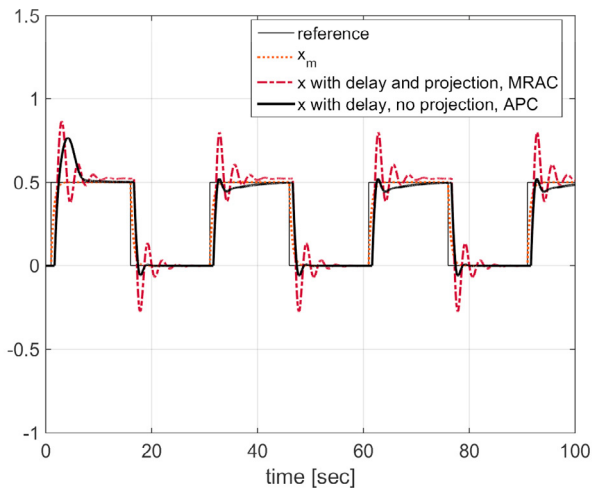


Fig. 5. Evolution of the reference, reference model output x_m and plant output x , when conventional MRAC is used with projection and APC is used without projection for $\tau = 0.66$.

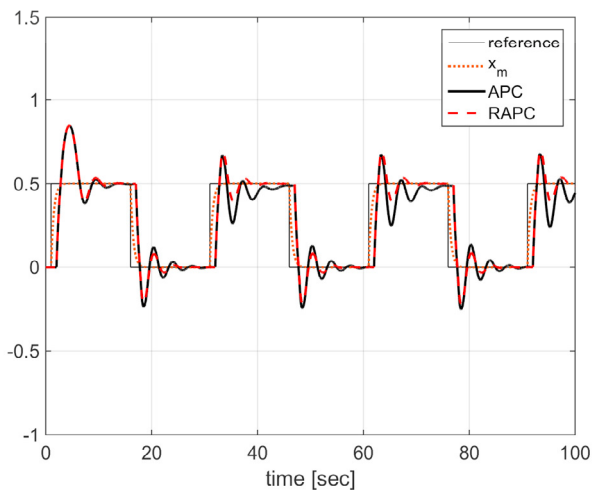


Fig. 6. Evolution of the reference, reference model output x_m and plant output x , when APC and RAPC is used, with 55% delay uncertainty.

- Bekiaris-Liberis, N. and Krstic, M. (2010b). Input delay compensation for forward complete and feedforward nonlinear systems. *IEEE Transactions on Automatic Control*, 55(2), 287–303.
- Bekiaris-Liberis, N. and Krstic, M. (2013). Compensation of state-dependent input delay for nonlinear systems. *IEEE Transactions on Automatic Control*, 58(2), 275–289.
- Bresch-Pietri, D. and Krstic, M. (2009). Adaptive trajectory tracking despite unknown input delay and plant parameters. *Automatica*, 45(9), 2074–2081.
- Jankovic, M. (2009). Cross-term forwarding for systems with time delay. *IEEE Transactions on Automatic Control*, 54(3), 498–511.
- Lavretsky, E. (2008). Adaptive control: Introduction, overview, and applications. In *Lecture notes from IEEE Robust and Adaptive Control Workshop*.
- Matsutani, M., Annaswamy, A., and Lavretsky, E. (2012). Guaranteed delay margins for adaptive control of scalar

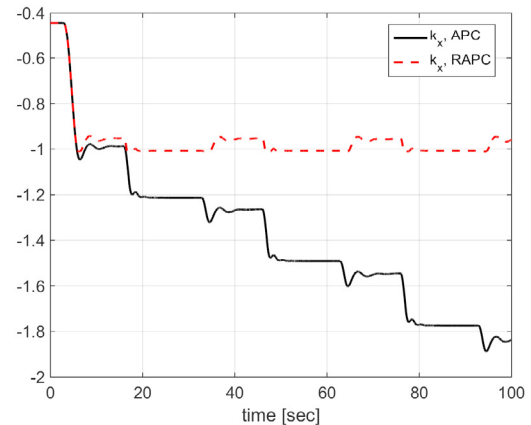


Fig. 7. Evolution of the controller parameter k_x , when APC and RAPC is used in the presence of 55% delay uncertainty.

- plants. In *Proc. Amer. Control Conf.*, 7297–7302.
- Matsutani, M., Annaswamy, A., and Lavretsky, E. (2013). Guaranteed delay margins for adaptive systems with state variables accessible. In *Proc. Amer. Control Conf.*, 3362–3369.
- Mazenc, F., Malisoff, M., and Lin, Z. (2008). Further results on input-to-state stability for nonlinear systems with delayed feedbacks. *Automatica*, 44(9), 2415–2421.
- Mazenc, F. and Niculescu, S.I. (2011). Generating positive and stable solutions through delayed state feedback. *Automatica*, 47(3), 525–533.
- Mazenc, F., Niculescu, S.I., and Bekaik, M. (2011). Backstepping for nonlinear systems with delay in the input revisited. *SIAM Journal on Control and Optimization*, 49, 2263–2278.
- Narendra, K.S. and Annaswamy, A.M. (2005). *Stable adaptive systems*. Dover Publications, New York.
- Niculescu, S.I. and Annaswamy, A.M. (2003). An adaptive smith-controller for time-delay systems with relative degree $n^* \leq 2$. *Systems and Control Letters*, 49, 347–358.
- Pepe, P. and Jiang, Z.P. (2006). A lyapunov-krasovskii methodology for iss and iiss of time-delay systems. *Systems and Control Letters*, 55, 1006–1014.
- Yildiz, Y., Annaswamy, A., Kolmanovsky, I., and Yanakiev, D. (2010a). Adaptive posicast controller for time-delay systems with relative degree $n^* \leq 2$. *Automatica*, 46(2), 279–289.
- Yildiz, Y., Annaswamy, A., Yanakiev, D., and Kolmanovsky, I. (2010b). Spark ignition engine fuel-to-air ratio control: An adaptive control approach. *Control Engineering Practice*, 18(12), 1369–1378.
- Yildiz, Y., Annaswamy, A., Yanakiev, D., and Kolmanovsky, I. (2011). Spark ignition engine idle speed control: An adaptive control approach. *IEEE Transactions On Control Systems Technology*, 19(5), 990–1002.