Fast Noniterative Steepest Descent Path Algorithm for Planar and Quasi-Planar Patch Geometries

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1 Introduction

Our motivation in developing fast direct (noniterative) algorithms for the solution of planar and quasi-planar geometries is based on the following observations:

- Fast iterative techniques [1–6], which have remarkable performances in solving large problems, are suffering from slow convergence or stagnation for resonant and near-resonant structures, examples of which are shown in Fig. 1.

- Conventional direct solvers, such as the Gaussian elimination, have $O(N^3)$ complexity, which is too high for the solution of large problems.
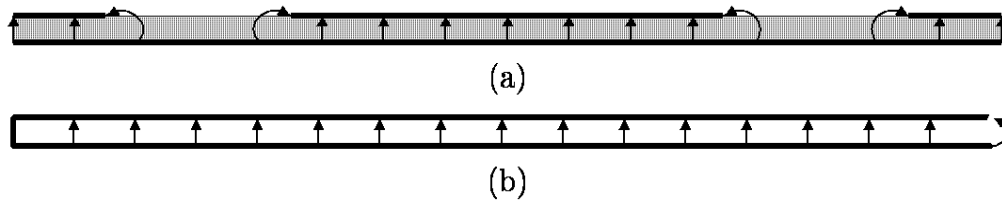

Figure 1: Examples of resonant structures: (a) planar microstrip geometry, (b) quasi-planar cavity geometry.

Our recent work on recursive T-matrix algorithms (RTMAs) resulted in noniterative solvers with reduced complexities [7–11]. However, the following problems remained open:

- Although RTMAs did reduce the complexity for "dense" geometries, similar reductions were not obtained for 2D and 3D planar geometries [11].

- Geometries involving layered media could not be solved using RTMAs.

Both of the above problems are overcome by combining RTMAs with another technique, namely, the steepest descent path (SDP) integration [12,13].

2 Steepest Descent Path (SDP) Formulation

The 3D Green's function (GF) for a homogeneous medium is given by

$$g(\mathbf{r},\mathbf{r}') = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} = \frac{i}{8\pi}\int_{-\infty}^{\infty} dk_\rho \frac{k_\rho}{k_z} H_0^{(1)}(k_\rho|\boldsymbol{\rho}-\boldsymbol{\rho}'|)e^{ik_z|z-z'|}. \tag{1}$$

The path of the above integral can be deformed to the SDP and the resulting integral can be discretized to obtain

$$g(\mathbf{r},\mathbf{r}') = \frac{k}{2\pi}\sum_m w_m \frac{1+is_m^2}{\sqrt{s_m^2 - i2}} H_0^{(1)}[k|\boldsymbol{\rho}-\boldsymbol{\rho}'|(1+is_m^2)]e^{ik_z(s_m)|z-z'|}. \tag{2}$$

Since the 3D GF can be expressed as a sum of a number of 2D GFs, the 3D problem can be solved by solving the same number of 2D problems. Note that the equivalent 2D problems involve "dense" geometries in 2D, making the recursive aggregate-T-matrix algorithm (RATMA) [7] a suitable candidate for a fast 2D solver.

For quasi-planar geometries, $|z - z'|$ remains small for all interactions between pairs of subscatterers, whereas $|z - z'| = 0$ for planar geometries. The above formulation can be extended to the layered geometries since the spectral-domain GF is readily available to be used in Eqs. (1) and (2). A symmetrized form of the spectral-domain dyadic GF derived by Chew [14] can be used for this purpose [15]. Special attention should be given to the branch-point and pole contributions when deforming the integration path to the SDP. A similar algorithm for the solution of 2D planar geometries was earlier developed by Chew and Lu [16,17].

RATMA was shown to have a computational complexity of $O(NP^2)$ [7,11] and P was shown to scale as $O(\sqrt{N}\log N)$ for the SDP formulation of 3D planar geometries [12,13]. Thus, the overall computational complexity of the fast direct algorithm based on the SDP (FDA/SDP) becomes $O(N^2 \log^2 N)$, which is significantly lower than $O(N^3)$.
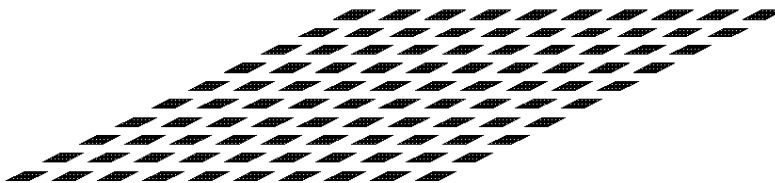

Figure 2: Geometry of a 10×10 array of 100 conducting square patches with dimensions $kw = 1.0$ and spacing $kd = 2.0$.

3 Results

Figure 2 shows a 10×10 array of 100 conducting square patches with dimensions $kw = 1.0$ and spacing $kd = 2.0$. Clearly, FDA/SDP is not limited to and does not

take advantage of periodic, planar, and canonical geometries involving identical patches. This canonical geometry is chosen merely because of the ease of reproducibility and comparison of the results. Figures 3(a) and (b) show the comparison of TE and TM scattered fields, respectively, as computed by the method of moments (MOM) and FDA/SDP of this paper. For both polarizations, the agreement between the two techniques is very good for the top 20-30 dB of the results. For lower values of RCS, occasional disagreements are observed. This problem is due to the fact that the geometry in Fig. 2 violates the addition theorems of harmonics even though the generalized RATMA [8] is used in this work. This situation can also be remedied by using windowed addition theorems [9]. A better remedy is to use the novel direct algorithms [18,19] that completely bypass this problem.
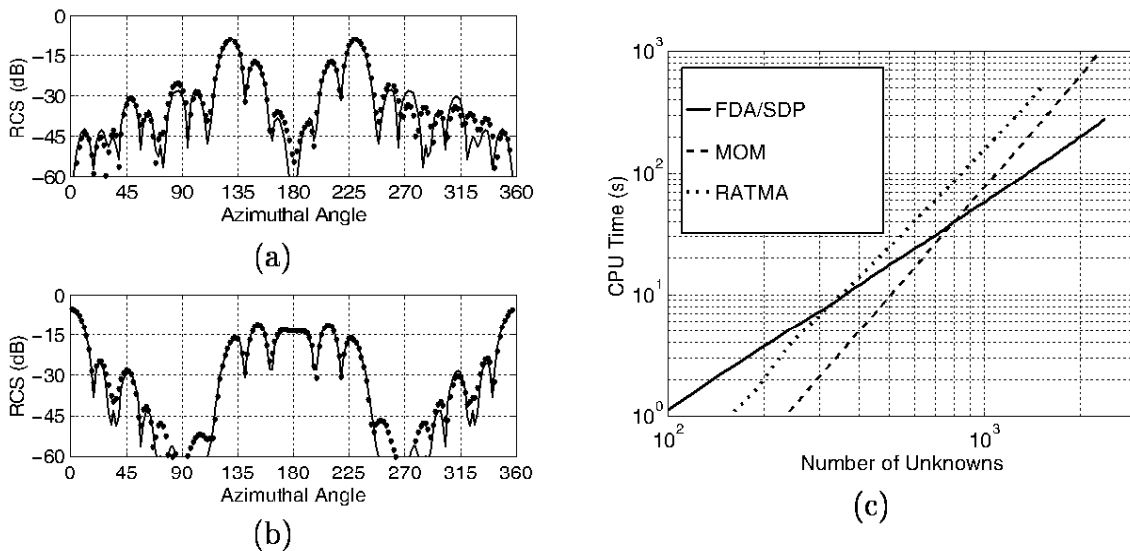

(a)

(b)

(c)

Figure 3: RCS of the two $\lambda \times \lambda$ conducting patches (as shown in Fig. 2) on the x-y plane for various patch separations: (a) TE and (b) TM polarized RCS curves for the array geometry shown in Fig. 2. Plane waves with TE and TM polarized electric fields are incident on the patches at $\theta = 45°$ and $\phi = 0°$. (c) Comparison of the CPU times required for various noniterative solution algorithms.

Finally, Fig. 3(c) shows a comparison of the solution times of FDA/SDP, MOM, and RATMA. FDA/SDP solution times are seen to increase with a smaller slope than those of MOM and RATMA, confirming the reduced computational complexity of FDA/SDP. The solution times presented in Fig. 3(c) are obtained by solving the scattering problems of increasingly larger planar arrays of patches, as shown in Fig. 2, without taking advantage of the periodicities and the symmetries of these arrays. RATMA was predicted and demonstrated to have $O(N^3)$ complexity for such 2D clustering of 3D scatterers [11]. On the other hand, FDA/SDP employs RATMA to solve 2D problems to take advantage of RATMA's reduced computational complexity for 2D clustering of 2D scatterers [11].

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