AN EFFICIENT ALGORITHM TO EXTRACT COMPONENTS OF A COMPOSITE SIGNAL

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ABSTRACT

An efficient algorithm is proposed to extract components of a composite signal. The proposed approach has two stages of processing in which the time–frequency supports of the individual signal components are identified and then the individual components are estimated by performing a simple time–frequency domain incision on the identified support of the component. The use of a recently proposed time–frequency representation [1] significantly improves the performance of the proposed approach by providing very accurate description on the auto–Wigner terms of the composite signal. Then, simple fractional Fourier domain incision provides reliable estimates for each of the signal components in $O(N \log N)$ complexity for a composite signal of duration N.

1. INTRODUCTION

Analysis of multi-component signals have been an active research area since the introduction of the time–frequency concepts. The search for the signal components which have compact time–frequency supports typically starts with the careful examination of the time–frequency distribution of the composite signal. The Wigner distribution is the most commonly used time–frequency analysis tool which provides the highest resolution time–frequency characterization of the signal. However, because of its bilinear nature, the supports of the actual signal components may not be visible in the presence of cross–terms of the Wigner distribution. For instance, if the signal $s(t)$ is composed of m signal components, $x_i(t)$, $1 \leq i \leq m$, then the corresponding Wigner distribution [2] can be written as:

$$\begin{aligned} W_s(t,f) &= \int s(t+t'/2)s^*(t-t'/2)e^{-j2\pi f t'}\,\mathrm{d}t' \\ &= \int \sum_i x_i(t+t'/2)x_i^*(t-t'/2)e^{-j2\pi f t'}\,\mathrm{d}t' \\ &= \sum_i \int x_i(t+t'/2)x_i^*(t-t'/2)e^{-j2\pi f t'}\,\mathrm{d}t' \\ &\quad + \sum_{i\neq k} \int x_i(t+t'/2)x_k^*(t-t'/2)e^{-j2\pi f t'}\,\mathrm{d}t' \\ &= \sum_i W_{x_i}(t,f) + 2\sum_{i<k} \Re e\{W_{x_i x_k}(t,f)\}\,, \quad (1) \end{aligned}$$

where the high resolution auto–Wigner distributions corresponding to m individual signal components are accompanied by $m(m-1)/2$ cross–Wigner distributions [3]. As shown in Fig. 2(a), the cross–Wigner terms may partially or totally overlap with the auto–Wigner terms making it very difficult if not impossible to detect and identify the time–frequency supports of the individual signal components.

Since the cross–Wigner terms are oscillatory in nature [4], 2–D low pass filtering reduces the cross–term interference [5, 6]. However, the resolution of the auto–Wigner terms may degrade considerably resulting in identification of significantly larger supports for the signal components. This not only causes extraction of more noisy signal component estimates but also signal components with closely spaced time–frequency supports to be identified as only one signal component. Since the success of the component analysis is very much related to the accurate identification of the time–frequency supports of the signal components, smoothed Wigner distributions are not very suitable for the extraction of the signal components. The draw-backs of smoothed–Wigner distributions in the analysis and extraction of individual signal components can be partially overcome with the use of signal dependent sliding window time–frequency representations [7]. However, the high complexity of the computation of these representations, and more importantly, the use of the same time domain filtering of all signal components occurring at the same time but different frequencies limits the success of these approaches.

In this paper, the time–frequency supports of individual components are identified by using a recently developed time–frequency representation [1]. Since, in the new representation directional smoothing of arbitrarily chosen time–frequency regions is made possible, the interference of cross–Wigner terms can be greatly reduced with negligible distortion on the auto–Wigner terms. Therefore the reliable detection and high resolution identification can be performed very easily on the new time–frequency representation.

2. AN EFFICIENT ALGORITHM FOR THE IDENTIFICATION AND EXTRACTION OF SIGNAL COMPONENTS

Time–frequency based extraction of the individual signal components of a given multi-component signal can be con-

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ducted in two stages. In the first stage detection and identification of the individual signal components is performed on the time-frequency plane. Then, the signal components are estimated based on the obtained time-frequency information on them. As it is explained in the previous section, high resolution and accurate description of the time-frequency content of the individual signal components is essential in the over-all performance of the component extraction. Since, the currently used time-frequency representations do not provide such a description, the second stage of processing becomes significantly involved to provide reasonable results [8, 9]. In the following, we propose to use a recently introduced time-frequency representation in the first stage of the analysis. Since, this new representations provides the required time-frequency information very precisely, the signal components can be extracted very efficiently.

In order to demonstrate the efficiency of the new time-frequency representation, the five-component signal whose Wigner distribution is shown in Fig. 2(a) is analyzed as detailed in [1]. As shown in Fig. 2(c), signal components can be easily detected and their supports can be accurately described. The supports of the individual signal components can be identified either manually or automatically by using adaptive thresholding methods.

In the second stage of processing, the obtained information on the supports of the individual signal components is used to design proper time-frequency incision techniques to extract the components directly from the signal. To demonstrate the required processing for the signal component extraction, consider the supports of auto-terms of the Wigner distribution of a composite signal as shown in Fig. 3. In order to extract the signal component which is localized at the center of the time-frequency plane, a time-frequency incision around this component should be performed. Among many alternatives, the simplest incision can be performed by first applying a frequency domain mask $H_1(f)$ to $S(f)$ whose support is the same as the frequency axis projection of the signal component. Then, to the result a time-domain mask, whose support is the projection of the signal component on the time-axis, can be applied to approximate the signal component. This way, the estimated signal component will have its time-frequency support approximately limited into the dashed-box around the desired signal component. Formally, the component estimate is obtained by:

$$\hat{x}_i(t) = h_2(t)[h_1(t) * s(t)] \approx x_i(t) \quad . \tag{2}$$

In a more general case, if the supports of the auto-components in the time-frequency plane are as shown in Fig. 4, then it is not possible to extract $x_i(t)$ from $s(t)$, by successive masking in frequency and time domains. Because in this case there does not exist a rectangular region in the time-frequency plane, which contains only the the support of the i^{th} auto-component but not the others. However, a viable solution in this case is first to translate the origin of the time-frequency plane to approximate center (t_i, f_i) of the i^{th} auto-component as shown in Fig. 4. The required translation can be performed as:

$$\bar{s}(t) = s(t + t_i)e^{-j2\pi t f_i} \tag{3}$$

Note that the i^{th} component of the signal $\bar{s}(t)$ is $\bar{x}_i(t) = x_i(t + t_i)e^{-j2\pi t f_i}$. Then the fractional Fourier transform (FrFT) [10] of this signal is

$$\bar{s}_{a_i}(t) \equiv \{\mathcal{F}^{a_i}\bar{s}\}(t) \triangleq \int K_{a_i}(t, t')\bar{s}(t')\,\mathrm{d}t' \quad , \tag{4}$$

where $a_i = 2\phi_i/\pi$ is the order of the FrFT and $K_{a_i}(t, t')$ is the kernel of the transformation given in [10]. Since the WD of the a_i^{th} order FrFT of a signal is the same as the WD of the original signal rotated by angle of $a_i\pi/2$ in the clock-wise direction [10], the WD of $\bar{s}_{a_i}(t)$ is aligned with one of the axis as shown in Fig. 4. Thus after the elementary operations of translation and rotation in the time-frequency plane, the WD of $\bar{x}_{i,a_i}(t)$ fits into a compact rectangular region as shown in Fig. 4(c). Therefore, as it was the case for the WD in Fig. 3, the i^{th} component of $s(t)$ can be extracted in the transform domain by successive masking as:

$$\hat{\bar{x}}_{i,a_i}(t) = h_2(t)[h_1(t) * \bar{s}_{a_i}(t)] \quad , \tag{5}$$

where $h_2(t)$ is the dual of time-domain mask and $h_1(t)$ is the inverse Fourier transform of the dual of frequency domain mask $H_1(f)$. After obtaining an estimate for $\bar{x}_{i,a_i}(t)$, an estimate of $x_i(t)$ can be easily computed by reversing the operations of translation and rotation in the time-frequency plane:

$$\hat{\bar{x}}_i(t) = \mathcal{F}^{-a_i}[\bar{x}_{i,a_i}(t)] \tag{6}$$

$$\hat{x}_i(t) = \hat{\bar{x}}_i(t - t_i)e^{j2\pi f_i(t-t_i)} \quad . \tag{7}$$

In practice the required fractional Fourier transform can be directly carried on the given Nyquist rate samples of the composite signal $s(t)$ by using the algorithm given in [11]. As shown in [11], the complexity of the fractional Fourier transform is the same as FFT. Therefore, the overall complexity of the proposed signal component extraction algorithm is $O(N \log N)$ for a component whose time domain support is of approximately N samples in duration.

The required incision in the more general case shown in Fig. 4 can also be performed by using fractional Fourier domain filtering techniques given in [12, 13, 14]. However, the proposed techniques in [12, 13, 14] are for noise suppression. Therefore, there is a need for improvement in these techniques to suppress both the noise and the other signal components. We are currently working on these improvements and planning to report on the obtained results and their comparisons with the simple incision technique used in this work.

3. SIMULATIONS

In this section we investigate the performance of the proposed algorithm by conducting computer simulations on a complicated composite test signal which is composed of 5 chirp signals with Gaussian envelopes. As shown in Fig. 1, it is not possible to identify individual signal components of the composite signal. The corresponding Wigner distribution shown in Fig. 2(a) is very much cluttered with the cross-terms. Because of the significant overlaps between the cross-terms and the auto-terms, the auto-terms shown in Fig. 2(b) cannot be identified. As shown in Fig. 2(c), by using the first stage of the processing a significantly improved time-frequency representation of the composite signal can be obtained. As seen from this figure, as a result of the utilized directional filtering technique [1], the cross terms of the Wigner distribution are highly attenuated with little distortion on the auto-Wigner terms. As shown in Fig. 2(d), the error in the estimated auto-Wigner terms is negligible. Therefore, as a result of the first stage of processing, very accurate detection and support-identification of the signal components can be achieved.

To illustrate the performance of the second stage of processing, we present results on the extraction of two chirp components of the composite signal shown in Fig. 1. The estimated signal component corresponding to the chirp component of the original signal near the origin of the time-frequency plane is shown in Fig. 5(a). This result is obtained by performing time-frequency domain incision on a rotated time-frequency plane obtained by using fractional Fourier transformation of order 0.5 corresponding to $\pi/4$ radians of rotation. The error in the estimated signal component is shown in Fig. 5(b). As seen from this figure, the extracted signal component is a very close approximation of the original signal component with a normalized error of $E_i = 8.2 \times 10^{-4}$ which is defined as:

$$E_i = \frac{||\boldsymbol{x}_i - \hat{\boldsymbol{x}}_i||}{||\boldsymbol{x}_i||} \quad (8)$$

where $\boldsymbol{x}_i$ and $\hat{\boldsymbol{x}}_i$ are the actual and estimated signal components in vector notation.

The result of the estimated signal component corresponding to the shorter chirp component with a time center right below the origin is shown in Fig. 6(a). This result is obtained by first translating the origin of the time-frequency plane to the center of the chirp component. Then the time-frequency domain incision over the estimated support of the signal component is performed on a rotated time-frequency plane obtained by using fractional Fourier transformation of order 0.5 corresponding to $\pi/4$ radians of rotation. The difference plot of the estimated and actual signal component is shown in Fig. 6(b) to illustrate the accuracy of the algorithm. As seen from this figure, the extracted signal component is a very close approximation of the original signal component with a normalized error of $E_i = 4.8 \times 10^{-3}$.

4. CONCLUSIONS

A two-stage processing algorithm is proposed for the extraction of components of a composite signal. Based on a set of simulations, it is shown that the proposed two stage processing algorithm provides highly accurate estimates for the individual signal components. The use of a recently proposed time-frequency representation to detect and identify the time-frequency domain supports of the signal components play the key role in the success of the proposed approach. In the second stage, the use of fractional Fourier domain incision greatly increases the efficiency of the algorithm.

5. REFERENCES

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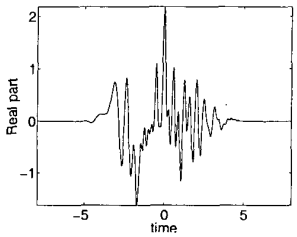

Figure 1: The time domain representation of a multi-component signal $s(t)$, which is composed of 5 linear-frequency modulated chirp signals.

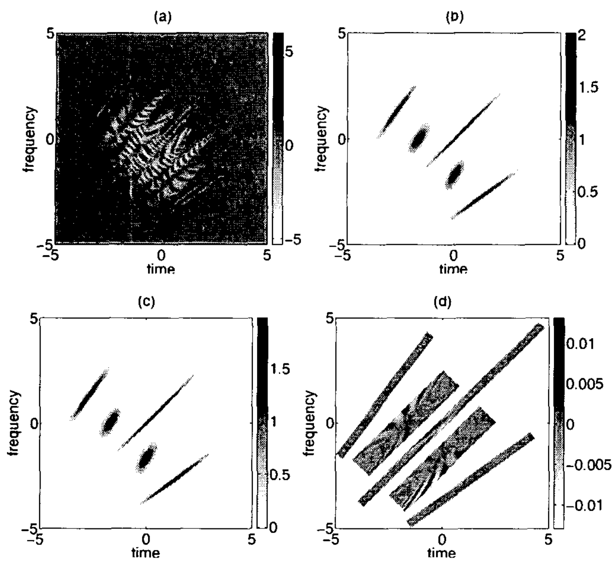

Figure 2: (a) The Wigner distribution of the signal $s(t)$ shown in Fig. 1, (b) the auto-components of the Wigner distribution, (c) the slices of the Wigner distribution smoothed by using the data-adaptive directional filtering algorithm in [1], (d) the difference of the smoothed slices from the auto-components of the Wigner distribution.

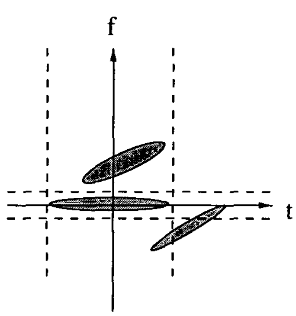

Figure 3: The extraction of the component centered at the origin of the time-frequency plane by using frequency and time domain masks.

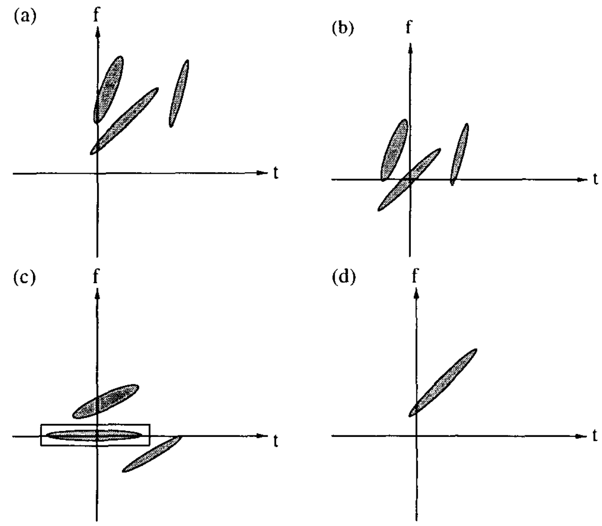

Figure 4: The supports of the Wigner distribution (only auto-terms) of $s(t)$ and its various transforms: (a) the WD of $s(t)$, (b) the WD of $\tilde{s}(t) = s(t+t_i)e^{-j2\pi f_i t}$, (c) the WD of $\tilde{s}_{a_i} = \mathcal{F}^{a_i}[\tilde{s}](t)$, (d) the WD of $\hat{x}_i(t) = \mathcal{F}^{-a_i}[h2(h_1 * \tilde{s}_{a_i})](t)$.

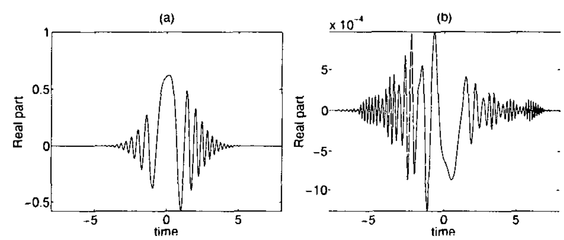

Figure 5: (a) The estimate of the long chirp component in Fig. 2(b) which is near the origin of the time-frequency plane, (b) the difference of the estimate from the actual signal component.

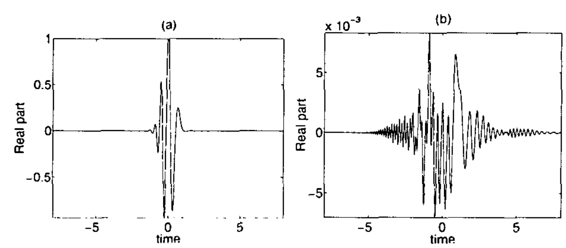

Figure 6: (a) The estimate of the short chirp component in Fig. 2(b) with the time center just below the origin, (b) the difference of the estimate from the actual signal component.