Fast Acceleration Algorithm based on DFT Expansion for the Iterative MoM Analysis of Electromagnetic Radiation/Scattering from two-dimensional Large Phased Arrays

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Introduction: Analysis of the electromagnetic radiation/scattering from phased array problems using Method of Moments (MoM) in conjunction with iterative schemes is popular because it provides accurate and reliable results. However, it suffers greatly from storage requirements and computing time if the analyzed arrays are electrically very large. Fast Multiple Method based efforts [1]-[2], as well as the Conjugate Gradient-Fast Fourier Transform [3], are some typical attempts to reduce the computational complexity and hence to improve the overall computational efficiency. In this paper, an acceleration algorithm based on Discrete Fourier Transform (DFT) [4] is developed to reduce the computational complexity and memory storages of iterative MoM (IMoM) solution to $O(N_{tot})$, where N_{tot} is the total number of elements in the array. The iterative scheme used in this paper is the Biconjugate gradient method (BiCGM), which is a version of the conjugate gradient method (CGM) [5] and is very reliable in terms of convergence. The $O(N_{tot})$ complexity of the solution is due to this DFT based acceleration algorithm which divides the contributing elements into "strong" and "weak" interaction groups for a receiving element in the IMoM. The contributions from the strong group are obtained by conventional element-by-element computation to assure the fundamental accuracy. On the other hand, the entire induced current distribution is expressed in terms of a global domain DFT representation. Significant DFT terms are identified based on [6] and employed to obtain the interactions from the weak group. In general only a few significant DFT terms are sufficient to provide accurate results due to the fact that they provide minor corrections to the solution in contrast to the dominating strong group.

Formulation: Consider a rectangular finite planar periodic array of $(2N+1) \times (2M+1)$. It may consist of identical short and thin perfectly conducting wire dipoles oriented in the $\hat{y}$ direction at $z = 0$ plane in air (free-standing dipoles), as illustrated in Figure 1(a), or identical printed dipoles, again oriented in the $\hat{y}$ direction, on a grounded dielectric substrate as depicted in Figure 1(b). An electric field integral equation (EFIE) is formed by enforcing the boundary condition that the total E_y field must vanish on the dipole surfaces. This EFIE is solved using a Galerkin MoM solution, leading to the following set of linear equations:

$$\overline{\mathbf{Z}} \cdot \mathbf{I} = \mathbf{V} \tag{1}$$

where $\overline{\mathbf{Z}} = [Z_{nm,pq}], \mathbf{I} = [A_{nm}]$ and $\mathbf{V} = [V_{pq} e^{-j\beta_x p d_x} e^{-j\beta_y q d_y}]$ and A_{nm} is the unknown coefficient to be determined via IMoM-DFT approach. In (1), the right-hand side is related to the excitation at the pq^{th} dipole to radiate a direction beam according to (β_x, β_y), where β_x and β_y are the impressed (or excitation) phases, and $Z_{nm,pq}$ on the left-hand side is the mutual impedance between the nm^{th} and pq^{th} dipoles.

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Following the CGM [5] method, let $\mathbf{I}_i$ be the value of $\mathbf{I}$ at the i^{th} iteration with $\mathbf{I}_0$ being the initial guess. Then

$$\mathbf{R}_0 = \mathbf{V} - \overline{\mathbf{Z}} \cdot \mathbf{I}_0 \quad , \quad \mathbf{S}_0 = \mathbf{R}_0 \, . \tag{2}$$

At the i^{th} iteration, the BiCGM updates each vector in the following way:

$$\alpha_i = \frac{< \mathbf{R}_i, \mathbf{R}_i >}{< \overline{\mathbf{Z}} \cdot \mathbf{S}_i, \mathbf{S}_i >}, \ \mathbf{I}_{i+1} = \mathbf{I}_i + \alpha_i \mathbf{S}_i, \ \mathbf{R}_{i+1} = \mathbf{R}_i - \alpha_i \overline{\mathbf{Z}} \cdot \mathbf{S}_i \tag{3}$$

and

$$c_i = \frac{< \mathbf{R}_{i+1}, \mathbf{R}_{i+1} >}{< \mathbf{R}_i, \mathbf{R}_i >}, \ \mathbf{S}_{i+1} = \mathbf{R}_{i+1} + c_i \mathbf{S}_i \tag{4}$$

where $<,>$ denotes the inner product without complex conjugate. This procedure continues until a converged result is obtained. The repeated and time-consuming computations of $\overline{\mathbf{Z}} \cdot \mathbf{I}$ ($\mathbf{I} = \mathbf{I}_0$ in (2) and $\mathbf{I} = \mathbf{S}_i$ in (3)) type matrix-vector multiplications in IMoM are accelerated using this DFT based acceleration algorithm. In this algorithm, the contributing elements are divided into "strong" and "weak" regions such that

$$\overline{\mathbf{Z}} \cdot \mathbf{I} = \sum_{nm \in strong} A_{nm} Z_{nm,pq} + \sum_{nm \in weak} A_{nm} Z_{nm,pq} . \tag{5}$$

The contributions from the strong interaction group for each receiving element are generally the same and obtained via element-by-element computation. The size of the strong region is fixed and very small compared to the entire array. The computations of weak group interactions are based on a DFT representation of $\mathbf{I}$, which can be expressed by

$$A_{nm} = e^{-j\beta_x n d_x} e^{-j\beta_y m d_y} \sum_{k=-N}^{N} \sum_{l=-M}^{M} B_{kl} e^{-j2\pi \frac{kn}{2N+1}} e^{-j2\pi \frac{lm}{2M+1}} \tag{6}$$

where B_{kl} is the coefficient of the kl^{th} DFT term and can be found from the inverse DFT. For a rectangular array, the significant DFT terms can be determined based on the criterion presented in [6] where the DFT terms of two orthogonal column ($k = 0$) and row ($l = 0$) dominate. It should be noted that for an infinite array with uniform excitation, only B_{00} exists, and as a result, significant DFT coefficients will distribute by spreading out from ($k = 0, l = 0$) term with behaviors similar to sinc functions. Substituting (6) into (5) results in

$$\overline{\mathbf{Z}} \cdot \mathbf{I} = \sum_{nm \in strong} A_{nm} Z_{nm,pq} + \sum_{kl \in Q} B_{kl} C_{kl,pq} \tag{7}$$

where Q denotes the selected DFT terms, and

$$C_{kl,pq} = \sum_{nm \in weak} Z_{nm,pq} e^{-j\beta_x n d_x} e^{-j\beta_y m d_y} e^{-j2\pi \frac{kn}{2N+1}} e^{-j2\pi \frac{lm}{2M+1}} \, . \tag{8}$$

With this expression only Q DFT terms (Q is a small number and it is fixed), which can be computed and stored before the CGM proceeds, need to be computed for a given receiving element in addition to the interactions from the strong group. Consequently, $O(N_{tot})$ in computational complexity can be obtained.

$C_{kl,pq}$ in (8) denotes the contribution of the kl^{th} DFT term to the pq^{th} receiving element, where each DFT term represents a linear phase impression. In this paper, $C_{kl,pq}$ expression is calculated by dividing the weak group into "forward" and "backward" subgroups where

the "forward" group consists of elements in the front of the pq^{th} receiving element, and the rest of the weak group elements are included in the "backward" group. It should be mentioned that the number of elements in these two groups varies according to the location of the pq^{th} element, but the total number remains roughly the same for each receiving element. The details of this method will be explained during the presentation.

Results and Conclusion: Numerical results for free-standing dipoles obtained using IMoM-DFT approach are presented and compared with the conventional MoM solution. In the given examples a 31×31 thin (radius=$10^{-3}\lambda_0$, where λ_0 is the free-space wavelength), short (length=$0.4\lambda_0$) $\hat{y}$-directed dipole array elements with $d_x = 0.2\lambda_0$ and $d_y = 0.6\lambda_0$ is considered. The scan angle for this example is ($\theta = 20°$, $\phi = 80°$). The corresponding normalized array currents for the 4^{th} and 16^{th} (middle) rows are shown in Figure 2(a) and 2(b), respectively. Similarly, the currents for the 2^{nd} and 16^{th} (middle) columns are illustrated in Figures 3(a) and 3(b), respectively for the same array. The agreement between the conventional MoM and IMoM-DFT results are good. The algorithm uses 5×5 elements as the strong region (with receiving element located at the center) and 37 DFT terms. Consequently, the number of interaction terms with each receiving element is 62 (25 elements from the strong group and 37 DFT terms) that is independent of the array size. More numerical results including the printed dipole case will be presented.

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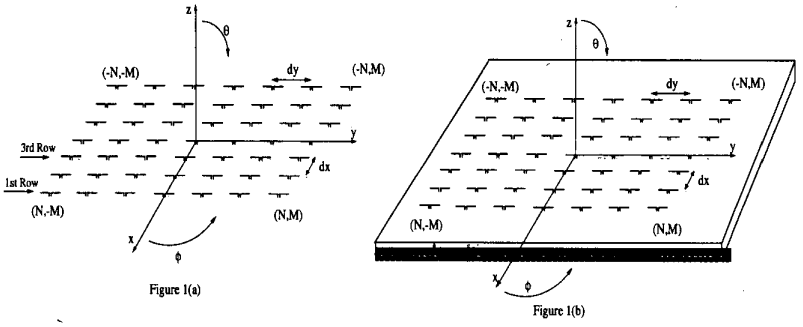

Figure 1: (a) Geometry of a periodic array of $(2N+1) \times (2M+1)$ dipoles in air, (b) geometry of a periodic array of $(2N+1) \times (2M+1)$ printed dipoles.

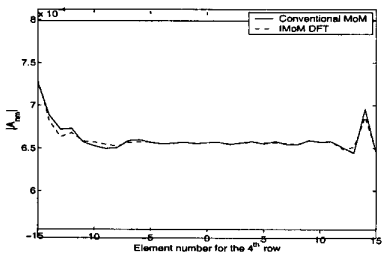

Figure 2 (a)

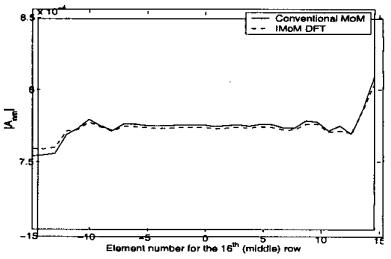

Figure 2 (b)

Figure 2: Current distribution on a 31×31 free-standing dipole array; scan angle: ($\theta = 20°$, $\phi = 80°$), $d_x = 0.2\lambda_0$ and $d_y = 0.6\lambda_0$, length of dip.= $0.4\lambda_0$, width of dip.= $10^{-3}\lambda_0$, (a) 4^{th} row, (b) middle row.

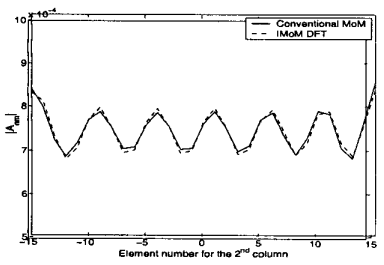

Figure 3 (a)

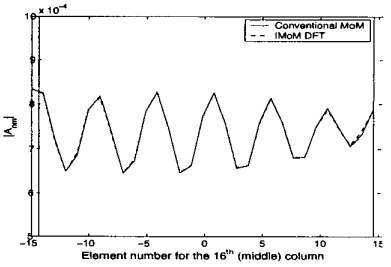

Figure 3 (b)

Figure 3: Same array with the same parameters; (a) 2^{nd} column, (b) middle column.