Coupled Matrix Pencil Method for Frequency Extrapolation of Electromagnetic Solutions †

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Abstract

Matrix pencil method (MPM) is used to extrapolate the available electromagnetic solutions in frequency domain to estimate the high-frequency solutions. A new approach, namely, coupled MPM, is introduced to obtain the electromagnetic solutions at intermediate frequencies using the available low-frequency and high-frequency data.

I. Introduction

Accurate solvers of computational electromagnetics become prohibitively expensive as the frequency increases. High-frequency prediction techniques, which are not computationally expensive at any frequency, become inaccurate as the frequency decreases. The purpose of this paper is to bridge the gap at intermediate frequencies by performing frequency extrapolation.

Frequency-domain solutions of electromagnetic problems, when discretized by the method of moments (MoM) or a similar scheme, require more computational resources as the frequency increases. Fast solvers, such as the fast multipole method (FMM) and the multi-level fast multipole algorithm (MLFMA), also have frequency limits, albeit higher, even when they are implemented in parallel computing environments. In this paper, we use physical optics (PO) to obtain the solutions of electromagnetic problems at higher frequencies, however, the accuracies of the PO solutions degrade for relatively lower frequencies.

In this work, we propose using extrapolation to estimate the solution signal for the intermediate frequency band using the information in low-frequency and high-frequency bands. We construct the extrapolation problem based on the model-based parameter estimation [1]–[2]. We choose using weighted sum of complex exponentials to model the solution signal,

$$y\left[k\right] = \sum_{i=1}^{M} R_i z_i^k \qquad k = 0, \cdots, N-1, \tag{1}$$

where

$$z_i^k = e^{s_i F_s k}. \tag{2}$$

There are two popular approaches for the solution of the parameters in (1), namely, the polynomial method and the matrix pencil method [3]. In this work, due to its noise tolerance and computational efficiency, we use the matrix pencil method (MPM) to determine the parameters of (1).

II. Matrix Pencil Method (MPM)

In (1), the residuals $\{R_i\}$ and the complex exponentials $\{z_i\}$ are unknowns to be determined by the N known values of the solution signal y. Number of exponentials used in the model, M, is a parameter of choice, which has a direct influence on the accuracy of the constructed model. The method uses a mathematical tool called matrix pencil,

$$\left[Y_2\right] - \lambda \left[Y_1\right], \tag{3}$$

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where λ is a scalar parameter. When $[Y_1]$ and $[Y_2]$ are chosen as

$$[Y_1] = \begin{bmatrix} y[0] & y[1] & \cdots & y[L-1] \\ y[1] & y[2] & \cdots & y[L] \\ \vdots & \vdots & & \vdots \\ y[N-L-1] & y[N-L] & \cdots & y[N-2] \end{bmatrix}_{(N-L)\times L}, \tag{4}$$

$$[Y_2] = \begin{bmatrix} y[1] & y[2] & \cdots & y[L] \\ y[2] & y[3] & \cdots & y[L+1] \\ \vdots & \vdots & & \vdots \\ y[N-L] & y[N-L+1] & \cdots & y[N-1] \end{bmatrix}_{(N-L)\times L}, \tag{5}$$

the generalized eigenvalues of matrix pencil (3) will be equal to the unknown complex exponentials of (1) [4]. Once we choose M and determine the complex exponentials, we can easily determine the residuals from

$$\begin{bmatrix} y[0] \\ y[1] \\ \vdots \\ y[N-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ z_1 & z_2 & \cdots & z_M \\ \vdots & \vdots & & \vdots \\ z_1^{N-1} & z_2^{N-1} & \cdots & z_M^{N-1} \end{bmatrix} \cdot \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_M \end{bmatrix}, \tag{6}$$

which follows from (1).

III. Coupled Matrix Pencil Method (COMPM)

For electromagnetic modelling problems, the frequency-domain solution signal $y(f)$ can be obtained from two different sources, i.e., low-frequency data from accurate electromagnetic solvers and high-frequency data from high-frequency prediction techniques, such as the PO method. In order to incorporate all available information into the model, a new modelling scheme, namely, the coupled matrix pencil method (COMPM), is developed.

COMPM, with its foundations over MPM takes all the available scattering solution data and constructs a single model. If N is the index of the highest frequency solved by the high-frequency solver and the scattering solution signal is y, the available data can be written as,

$$y_L[k] = y[0 : N_1], \tag{7}$$

$$y_H[k] = y[N_2 : N-1], \tag{8}$$

where y_L is the available low-frequency data and y_H is the available high-frequency data. Eqs. (7)–(8) are treated as separate signals and MPM is used to find their complex exponential sets. These two sets of exponentials carry the signatures of the corresponding available data, which are indeed assumed to be the approximate signature of the whole data. Therefore, the two sets of complex exponentials are combined into a single set. This combination of complex exponentials requires the calculation of the residuals as

$$\begin{bmatrix} y_L \\ y_H \end{bmatrix} = \begin{bmatrix} Z_L & Z_H \end{bmatrix} \cdot R, \tag{9}$$

where $[Z_L]$ and $[Z_H]$ are the corresponding low- and high-frequency exponential matrices defined in (6). After constructing (1) with the coupled residuals and complex exponentials, we can evaluate the signal model for k values between $(N_1 + 1 : N_2 - 1)$ to perform extrapolation.

In COMPM, there are two M parameters, i.e., one for the low-frequency signal and another for the high-frequency signal. We determine the optimal choice of the M parameters by scanning all possible combinations of M values for both low-frequency and high-frequency cases.

IV. Numerical Results

In order to demonstrate the effectiveness of the proposed method, we will first consider the problem of scattering from a conducting sphere under plane-wave illumination, which also has an analytical solution. Hence the proposed method can be appraised by comparing the extrapolation result with the analytical solution. As a second example, we will consider the scattering from a conducting patch. In this case, there is no analytical solution; therefore all of the available information is obtained from numerical solvers.

A. *Backscattering from a Conducting Sphere*

For this example, the backscattering signal of a conducting sphere with a radius of 30 cm in the 1–64 GHz frequency range is analytically obtained. 1–5 GHz portion of this data is considered as the available low-frequency information. It is reasonable to assume that the low-frequency solution is not available beyond 5 GHz since the numerical solution of this scattering problem at 5 GHz would require about 100,000 unknowns. Even though we can solve larger problems with the MLFMA, a 100,000-unknown problem is commonly considered to be large by contemporary standards. With a frequency-sampling interval of 40 MHz, there are 101 samples of backscattering data available in the low-frequency region.

Initially, 1–5 GHz data is used to perform the MPM extrapolation in an effort to estimate the data in the 5–64 GHz region. Fig. 1(a) shows the performance of the MPM extrapolation in terms of the error between the extrapolated signal and the analytical solution. It can be seen that the magnitude of the error grows with the increasing frequency. Note that the error is below 10^{-7} (difference in the 7th digit or smaller) in the interpolation region.

Next, both the low-frequency data in the 1–5 GHz band and the high-frequency data in the 60–64 GHz band are assumed available, and a COMPM extrapolation is performed to estimate the data in the intermediate 5–60 GHz region. By using both low-frequency and high-frequency data, coupled residuals and complex exponentials are determined. Fig. 1(b) shows the extrapolation error of COMPM.

Comparing Fig. 1(b) to Fig. 1(a), COMPM extrapolation is clearly superior to MPM extrapolation. Coupling the residuals and the complex exponentials increases the error performance of the extrapolation in the extrapolation region. The slight increase of error in the interpolation region is acceptable since the data is already known in that region. Fig. 1(b) exhibits a maximum error below 10^{-3}, i.e., 0.1%, over a broad frequency band of 5–60 GHz.

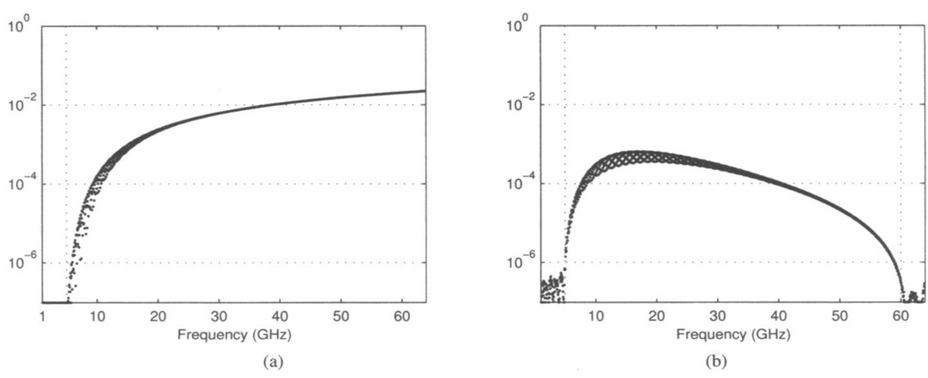

Fig. 1. (a) Error of the MPM extrapolation of the backscattering solution of the conducting sphere. (b) Error of the COMPM extrapolation, which incorporates the high-frequency solution of the conducting sphere in the model.

B. Backscattering from a Conducting Patch

The problem of scattering from a conducting square patch with edges of 60 cm residing on the x-y plane is considered. The patch is illuminated with a plane-wave propagating in the $-z$ direction and the backscattering solution is sought. Since the analytical solution of the patch problem is not available, the backscattering data is computed with an MLFMA solver in the 1–22 GHz range and with a PO solver in the 1–35 GHz range. 1–8 GHz portion of the MLFMA data is considered as the available low-frequency information. We pretend as if the data in the 8–22 GHz range is not available and instead save this portion of data as reference data to be used for comparisons with the extrapolation results.

Fig. 2(a) shows the MPM extrapolation error, which is defined as the difference between the extrapolated signal and the reference MLFMA data. Fig. 2(b) presents the COMPM eror, where the 30–35 GHz high-frequency PO data is also incorporated in the model. The error in Fig. 2(b) is below 10^{-2} for almost all frequencies. In this example, the data in the broad 8–30 GHz band is satisfactorily estimated with the COMPM by using only 7 GHz of computationally expensive low-frequency data and 5 GHz of computationally inexpensive high-frequency data.

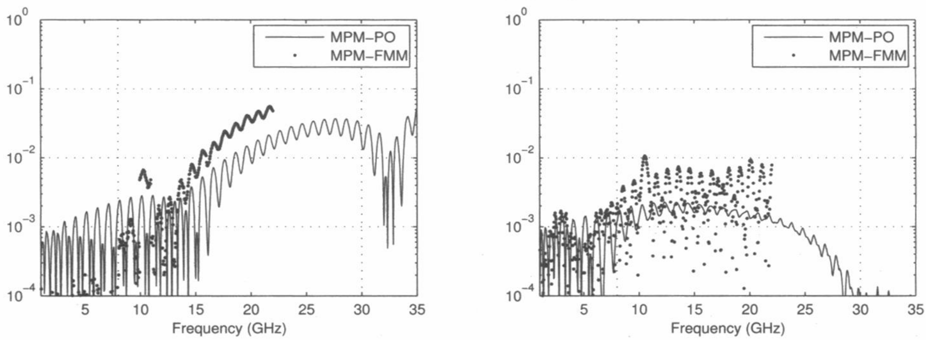

Fig. 2. (a) Error of the MPM extrapolation of the backscattering solution of the conducting patch. (b) Error of the COMPM extrapolation.

When compared to the analytical case, extrapolation of computational data manifests higher error levels. In the analytical case, both low-frequency and high-frequency information are obtained from the same analytical expression; however, in the computational case, those two solutions are obtained from different numerical solvers, under different approximations and assumptions. Furthermore, numerical noise that arises in the solution is also a cause of increase in the error levels. Nevertheless, COMPM extrapolation results are still accurate for practical applications.

V. Conclusion

In this paper, we present a method to extrapolate the solutions of electromagnetic scattering problems at intermediate frequencies using the available low-frequency and high-frequency data. The proposed COMPM extrapolation scheme displays a significant performance improvement over the MPM extrapolation.

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