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ABSTRACT

Creation of a dynamic 3-D display based on holography, in which a 3-D scene is encoded in terms of optical diffraction, transformed into the fringe patterns of the hologram that is further converted into a signal for a spatial light modulator (SLM) and displayed in real time, is an extremely challenging enterprise. There are various approaches targeted to solve associated problems.

Keywords: Holography, spatial light modulator, 3-D display, diffraction optics

1. INTRODUCTION

Capturing three-dimensional visual information of a real-life scene and creating an exact (except the scale) optical duplicate of it at a remote site instantaneously, or at a later time, are ultimate goals in visual communications. Medical imaging, computer aided design, automated robotic systems, molecular biology, air traffic control, telepresence, education, game industry, cultural heritage as well as mass TV industry exhibit increasing demand for implementation of 3DTV technologies. Although a lot of 3-D display systems have become available recently, realization of a 3-D dynamic display continues to be an object of intensive research and development¹.

To achieve image realism, a 3-D display should provide high spatial resolution and depth perception where the main depth cues are binocular disparity, motion parallax, ocular accommodation, occlusion etc. Currently existing 3-D imaging systems are usually based on stereoscopic and auto-stereoscopic displays^{2,3}. Stereoscopic displays exploit the ability of the human brain to combine different perspectives of the object as an integrated 3-D visual scene. Auto-stereoscopic displays form a wider class of display systems; re-imaging, volumetric, parallax and holographic systems, and present a 3-D image to a viewer without the need for viewing aids. Many authors, however, pay special credit to holographic displays as a separate class of “true” 3-D displays due to the inherent property of holography to reconstruct a wavefront that is identical to the wavefront emanated from the object. Thus, at least theoretically, the holographic display is able to reproduce the original 3-D scene with all of the depth cues and high resolution that would allow a truly realistic 3-D image. Paradoxically, the correct approach for realization of the 3-D display has been formulated long ago, in the dawn of the laser and holographic epoch, by N. A. Valuys – one of the most distinguished scientists in the field of stereoscopy. He writes in his book “Stereoscopy” (published by “Nauka” – Moscow in 1962, p.203) that “*the only way to solve the problem is to accomplish an integrated reconstruction of spatial images in which each point emanates light from that point of space where it is localized on the reconstructed object*”, in other words only by holography.

Introduction of the fourth, time-coordinate to the 3-D display further aggravates the task. Creation of a dynamic 3-D display based on holography, in which a 3-D scene is encoded in terms of optical diffraction, transformed into the fringe patterns of the hologram that is further converted into a signal for a spatial light modulator (SLM) and displayed in real time, is an extremely challenging enterprise. The aim of this review paper is to present trends in development of the dynamic holographic display (known also as holovideo or electroholographic display⁴) considering both signal processing issues and the underlying optics in the process of generation of a 3-D image from a 3-D description of a scene.

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2. GENERATION OF DIFFRACTION STRUCTURES

The computational complexity associated with representation of complicated 3-D scenes and the computation of related high-resolution computer-generated holograms (CGH) or the simulation of related light propagation and interference is huge, especially when it should be done in real time. A simple estimation, performed in Ref.5, shows that a 1.8-GHz Pentium-4 processor⁵ needs approximately 10 seconds to compute a low-resolution CGH consisting of 800×600 samples from a 3D object with a simple structure represented as 1,000 points. Special hardware may be needed for real time computation of higher resolution CGH associated with a high resolution 3D object.

Like all other displays, the holographic display must also stimulate in the most efficient way the human visual system (HVS). At a normal viewing distance of 600 mm from the display, the HVS is characterized with a lateral spatial resolution of about 0.175 mm and angular resolution of an image of 5 milliradians⁶. An optically produced hologram provides spatial resolution that exceeds by two orders of magnitude the limit imposed by the HVS and a continuously varying parallax. In view of this information redundancy and taking in account parallelism and the time-response of the HVS, different approaches for reduction of information content have been proposed. A few examples are horizontal parallax only (HPO) display architectures, decreased viewing zone and the image size, non-uniform sampling of the hologram, and various encoding and compression methods.

Historically, one of the first attempts to generate a CGH for 3D object presentation is to consider the object as a collection of parallel planar projections or slices. Two approaches have been developed - Born approximation for transparent objects and "ping-pong" algorithm for solid opaque objects⁷⁻⁹. Another approach is to acquire multiple projection images from different view points using a computer graphics rendering process or a real scene capture by an array of cameras. A technique of the holographic stereogram in which the object is observed from different points of view has been proposed and developed^{7,10-13}. An attempt to construct a large CGH by "stitching" together a number of pre-computed small sub-holograms is described in Ref.14 and 15. The drawback of all methods that rely on the assumption of planar object and hologram surfaces is that they require very high-resolution displays for large viewing angles. In addition, it is impossible to observe the 3-D image from the opposite side of the hologram. Leseberg and Frere¹⁶ consider 3D objects as composed of finite line segments and further from tilted and shifted planar segments. Tommasi and Bianco¹⁷ built an off-axis hologram that consists of tilted and shifted surfaces in the spatial-frequency domain by using translation and rotation transformations of angular spectra. Mishina *et al.* describe aliasing associated with the pixilated structure of the CGH and develop a technique for viewing angle enlargement¹⁸. A 360° holography is proposed in Refs.19,20, that requires solution of the diffraction task on non-planar surfaces. In Ref.20 a CGH is generated on a spherical surface without application of the FFT which makes the computation rather slow. In Ref.21 a fast calculation method based on a convolution theorem with the FFT algorithm is proposed for the case of concentric cylindrical object and observation surfaces. As is stated in Ref.22, the method of multiple projection images can facilitate the outdoor recording because it can be performed using incoherent illumination and hence is not so vulnerable to external vibrations. Recording of CGHs of real 3-D objects at incoherent white illumination is proposed in Ref.23. A further improvement of the algorithm is made in Ref.24 where the authors suggest to generate a hologram of real 3D objects by fusing the angular projections captured by a digital camera; this makes it possible to move the viewpoint in both horizontal and vertical directions. In Ref.25 a method is proposed for synthesis of Fresnel holograms from 3-D Fourier spectra without need of a lens for reconstruction. A strong feature of the described methods is the possibility to extend them to color presentation^{22,26}.

A natural design of a CGH for 3-D object presentation²⁷ is to consider the object as consisting of independent point sources. The interference of the waves coming from these point sources with the reference wave forms the resulting complex amplitude distribution on the hologram plane. The continuous parallax and focusing on different transverse planes through the volume (accommodation effect) ensure depth information. Lighting effect, surface reflection properties and occlusion can be also incorporated. Both wireframe and smooth-shaded CGHs have been computed by interference modeling. The method is enriched by the idea of incremental computing²⁸⁻³³ which makes use of stored pre-computed general hologram elements. Accordingly, the object is considered as consisting of analytically defined "holoprimitives", as has been proposed in Ref.29. The resultant CGH is built from pre-computed fringes, each of them corresponding to a given holoprimitive, and can be easily updated. The incremental updating evolves from the bipolar intensity approach that takes advantage of digital representation of holograms and computes only the useful part of the image excluding the object self-interference and the reference-bias terms. The result is the elimination of unwanted noise terms and an increase of the dynamic range. Real-valued linear summation of pre-computed elemental fringes

stored in a “look-up” table enables CGH computation at interactive rates^{33,34}. The main problem of the incremental updating is the trade-off between the computational speed and quality of the 3-D image. An additional drawback is the huge size of the look-up table. A method for accelerating the full-parallax 3-D CGH calculation based on recurrent formulas for precise computation of distances between the object points and sampling points on the CGH is developed in Ref.35. The interference-based approach to the dynamic holographic 3-D display directly simulates propagation of light and produces fringe patterns that closely resemble the fringes recorded in optical holography. The result is high-quality images with all depth-cues. The drawbacks are the computational burden imposed by the large number of object points and limited options to develop holographic bandwidth compression techniques³⁶. Nevertheless, most of the recently reported 3-D dynamic holographic displays rely on various modifications to achieve CGH computation.

In search of effective real-time data updating, a diffraction-specific (DS) approach, which keeps the idea to use pre-computed look-up tables, has been developed^{6,37}. The DS method relies only on two computing steps – direct encoding of data and decoding. Two holographic encoding techniques - “hogel-vector” and “fringelet” – have been designed to directly compute the encoded formats by spatial and spectral subsampling. The DS method makes simulation of interference unnecessary, and does not include the noise terms. It generates the required fringes by backwards imitation of diffraction. A pre-computed set of basis fringes with specific diffractive functions is stored in a diffraction table which can be also used to map location, size, and orientation of higher-order image elements. The speed of the DS approach results from the direct basis-fringe summation in the decoding step. Hogel-vector encoding compresses the bandwidth, however, at the expense of expansion of the point spread function and speckle-like image. As all algorithms for data compression, the DS approach offers a trade-off between image quality and computational speed.

Obviously, signal processing plays a central role in translation of 3D scene information into signals which drive the display. From the point of view of future development, the 3-D holographic display must face two main problems. The first one is the forward problem of calculating the complex wave field emanating from the object. Calculation of diffraction structures is well formulated in classical texts. Various approximation techniques, each with its own benefits and limitations, are also well developed and frequently applied to diffraction problems. However, fast and elegant solutions to fundamental problems associated with holographic 3D display are still missing; e.g. solution of the forward problem for non-planar objects is expected to lead to a number of challenges. The second problem that must be addressed is the inverse problem of how to drive the physical devices to recreate this complex field that will result in the desired 3-D perception. The reconstruction of the complex field is highly technologically dependent. Real-time TV operation requires fast implementations of these solutions, and this is another severe problem. Physical display devices that will eventually generate the desired 3D optical fields have their own properties and constraints. Computation of the appropriate driving signals for these devices is another challenging problem.

3. PHYSICAL REALIZATIONS OF THE 3-D HOLOGRAPHIC DISPLAY

Real-time high-resolution computer-controlled SLMs with large light throughputs and good diffraction efficiencies are necessary for generation of dynamic CGHs needed for the implementation of true holographic displays. In principle, amplitude and phase can be modulated. Only recently low cost devices with the desired modulating properties have become available. Until now three types of SLMs have been used in the holographic 3D displays: acousto-optic modulators (AOMs), liquid crystal displays (LCDs), and digital micromirror devices (DMDs).

The first holographic display system, capable of displaying 3D images in real-time, is the Mark I Holographic Video System created in 1989 by the MIT Media Laboratory Spatial Imaging Group. It is a HPO system that uses as a SLM an acoustooptic Bragg cell with a vertical channel arrangement to maintain the required temporal bandwidth³⁸⁻⁴¹. Both interference-based and DS approaches have been used for calculation of the diffraction pattern in the MIT display. The created image occupies a volume of $150 \times 75 \times 160 \text{ mm}^3$, and the viewing zone in the horizontal plane is 30 degrees. The necessity to convert the CGH into an analog signal before applying it to the AOM, and the need of moving optical parts are among the main drawbacks of this system. To avoid optical scanning without sacrificing the display temporal bandwidth, a multi-channel AOM that consists of six TeO_2 acoustooptic Bragg cells is proposed and tested⁴². The cells are addressed simultaneously, and this permits to keep the temporal bandwidth of the whole SLM equal to that of a single cell at six-times increase of the length of the resulting AOM. The developed AOM has been implemented in the pulsed-laser holographic video system of the Korea Institute of Science and Technology, and can display $36,864 \times 128$ points at refresh rate 60 frames per second using a personal computer.

Later, several holographic displays on the basis of the constantly evolving LCD technology have been described in the scientific literature^{5,43-45}. One of the first projects for monochrome display is under the guidance of the Telecommunications Advancement Organization of Japan (TAO)². In this design, increase of resolution in the horizontal plane is achieved by optical tiling of several LCDs. The volume of the produced image is $150 \times 50 \times 50 \text{ mm}^3$. The viewing zone is very narrow reaching only 65 mm width at around one meter distance. As in the MIT display, the vertical parallax is eliminated. Last achievements in LCD technology make possible other display solutions. Ref.5 describes a monochrome holographic display with a LCD that can realize full parallax. The authors use a reflective LCD with a resolution of 800×600 pixels, a pixel pitch of $12 \mu\text{m}$, an active area of $9.6 \text{ mm} \times 7.2 \text{ mm}$, and a maximum refresh rate of 360 Hz. The system can produce a CGH of 800×600 samples from a 3D object consisting of approximately 400 points in approximately 0.15 seconds, i.e. the refresh rate is 7 Hz. For the purpose, the authors developed a four module system that consists of a USB controller, a special-purpose chip for holography with a multiple pipeline architecture, an LCD controller, and a reflective LCD panel mounted on a printed circuit board. A reconstructed 3-D animated image can be seen when a reference light illuminates the LCD panel. However, the viewing zone of the display is only 3 degrees for the wavelength of 500 – 700 nm. For expanding of the viewing zone multiple units can be added to the optical system.

Several technical solutions for a color dynamic holographic display have been reported recently. Similarly to an ordinary 2D display, a 3D color holographic display that is based on three optical setups for three primary-color images (RGB) has been proposed in Ref.43. A predecessor of this system is a one LCD panel color holographic display with time-division multiplexing⁴⁴. The R, G, and B reference beams illuminating in turn the panel are synchronized by an electronic control shuttering system. The three-panel system is large and requires calibration. The time-division multiplexing system is small but also relies on synchronization of reference light and the hologram. In Ref.45 a simpler method for color holographic display, that employs one LCD display panel with no need of an electronic shutter, is proposed. The R, G, and B reference beams illuminate the hologram simultaneously all the time. The experimental setup consists of the reflective LCD panel with pixel pitch $10.4 \mu\text{m} \times 10.4 \mu\text{m}$ and $14.6 \text{ mm} \times 10.9 \text{ mm}$ in size, RGB high-brightness type LEDs and pinhole filters as a reference light system, a beam splitter, a collimator lens, a field lens, and a personal computer. The proposed devices still not satisfy technical requirements for a real color dynamic display at desired image quality and a frame refresh rate, but are promising for further developments.

The Texas Instruments Digital Micromirror Device (DMD)⁴⁶ has several key advantages which make it an almost ideal SLM for holography: a light modulator efficiency of the order of 65%, high contrast ratio (typically 1000:1), modulation rate of the mirrors of about 180 Hz, and ability to operate in a wide spectral range. One disadvantage of the DMD as a phase modulator is the multiple Fraunhofer diffraction orders that occur at coherent light illumination due to the DMD grating pattern. In the case of a standard DMD with $19.3 \mu\text{m}$ mirror diagonal periodicity and 12 degrees tilt angle, angular distance between diffractive peaks is as little as 2 - 3 degrees causing a large number of diffractive orders in optical systems with sufficiently large viewing angles. However, DMDs with ± 9.2 degree mirror tilt have been developed to create a switchable blaze grating with 88% of the diffracted energy in a single diffractive order. This property is used in a 3-D dynamic holographic display described in Refs. 47,48. The CGH is calculated by the interference-based approach. Using an optical system composed of a 15 mW He-Ne laser, spatial filter, collimating lens, DMD, converging lens and an image reconstructor for real image viewing (a suspension of micro-scatterers created from a thick Agarose gel), the authors succeed to create a monochrome real-time image.

Recently, to apply digital holography to industry, a joint venture company, Holographic Imaging LLC (Royal Oak, MI, USA; www.holographicimaging.com), has been formed by Qinetiq (www.qinetiq.com) and the Ford Motor Co. (Dearborn, MI, USA; www.ford.com). Coming to the conclusion that currently available commercial SLMs scarcely satisfy the demands of holographic display systems, Qinetiq proposes the so called Active Tiling modulator system that exploits the high frame rate of electrically addressable spatial light modulators (EASLMs) and the high resolution of optically addressable spatial light modulators (OASLMs). In other words, the system benefits from the wide temporal bandwidth of the EASLM and the wide spatial bandwidth of the OASLM. The EASLMs can be realized on the basis of DMDs, liquid crystal on silicon and a ferroelectric liquid crystal on silicon. Both interference-based and diffraction specific approaches have been applied for CGH creation. The CGH is divided into segments that are displayed sequentially on the EASLM. Each segment is projected onto the OASLM and stored. Special care should be taken to ensure a uniform light intensity across the OASLM during the projection to achieve even switching as data are delivered to its various regions. The OASLM is addressed with incoherent light in order to minimize speckle. The process is

repeated until a complete diffractive structure has been written. The written complex, high-resolution CGH is reconstructed under coherent illumination. The complete image can be updated at rate of over 40 Hz.

The most appropriate for realization of the OASLM are the controllable reversible materials. This has resulted in tremendous efforts in the development of holographic materials ranging from photorefractive crystals⁴⁹ to different types of photopolymers, azobenzene polyester films and composite materials⁵⁰. The application of photorefractive crystals for dynamic displays is limited because of their low sensitivity and small aperture that could be realized⁴⁹. The main advantages of the organic media are simple (dry), one step processing, high sensitivity, proper mechanical characteristics (plasticity) – possibility to integrate them easily in different compact devices, high signal to noise ratio and spatial resolution. The most promising are the so called composite nano-sized particle containing polymer-dispersed liquid crystals (PDLCS)^{51,52}.

4. CONCLUDING REMARKS

The topic of development of the 3-D display with its five stages – scene capture, scene representation, compression and coding of the scene, transmission, and display, is inherently multidisciplinary and requires a diversity of research directions. Advances in electronic technologies, optics, computer science, signal processing, and telecommunications, seem to create an exciting synergy towards achieving fully digital 3-D TV. Recently, a new European consortium⁵³ is formed as a Network of Excellence to integrate the research works of 19 institutions in the field of 3DTV. The consortium is funded by EC under the FP6 thematic area Information Society Technologies within the strategic objective Cross-media Content for Leisure and Entertainment. The technical focus of the consortium is 3DTV with all its aspects except audio. A rich variety of different display techniques, including stereoscopy and holographic displays are among the main focus of the consortium.

The research involved will be dealing with application of specific signal processing techniques which operate on the abstract representation of captured motion 3-D scenery, and convert that to those specific signals needed to drive the designated display. Furthermore, considering the propagating optical waves as a multidimensional signal to be generated by a holographic display device, the efforts will be also concentrated on the effects of various approximations at different stages to the subsequent optical quality of the generated true 3D scene by a given type of holographic display. The fundamental problems outline above entails development of some primary signal processing techniques. Of primary importance are the issues associated with discretization and quantization in diffraction problems, sampling, interpolation, digital capturing of holographic signals and generation of purely computational diffraction masks. Development of space-frequency techniques such as plane-wave decompositions, atomic decompositions, wavelets, fractional Fourier transform and existing optimization techniques deserve special attention as tools for 3D displays^{54,55}.

Practical application of the developed prototypes of the dynamic holographic display based on different dynamic SLMs – AOM, DMD or LCD – is limited mainly because of the low quality of the reconstructed images, connected to the low spatial resolution of the utilized SLMs. One of the ways to overcome this drawback is to use optically addressable spatial light modulators (OASLMs) for creation of the large format and high resolution dynamic synthetic holograms, by consecutive recording of initially calculated diffractive structures, displayed onto smaller resolution, but faster SLMs (AOM, DMD or LCD). The most promising light sensitive materials for OASLMs are the reversible nano-sized particles containing composite polymer dispersed liquid crystals which are the object of intensive research.

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