Mesoscopic Fano Effect in Aharonov-Bohm Rings with an Embedded Double Dot

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Abstract. We investigate theoretically in a tight-binding model the transport properties of the Aharonov-Bohm interferometer (ABI) with one dot embedded in each of its arms. For weak interdot coupling the model Hamiltonian describes the system considered in the experiments of Holleitner *et al*. [*Phys. Rev. Lett*. **87**, 256802 (2001)]. The electronic transmittance of the interferometer is computed within the Landauer-Büttiker formalism while the coexistence of resonant and coherent transport is explicitly emphasized by using the Feschbach formula. The latter produces effective Hamiltonians whose spectral properties describe the tunneling processes through each dot. We reproduce numerically the stability charging diagrams reported in the experiments of Holleitner *et al*. When the magnetic flux is fixed and one dot is set to resonance the interferometer transmittance shows Fano lineshapes as a function of the gate voltage applied to the other dot. Our model includes the effect of the magnetic field on the dot levels and explains the change of the asymmetric tail as the magnetic flux is varied. The transmittance assigned to the Fano dips located in the almost crossing point of the charging diagrams shows Aharonov-Bohm oscillations.

Keywords: Quantum dots, Aharonov-Bohm interferometer, Fano effect
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GENERAL FRAMEWORK

The Aharonov-Bohm interferometers are hybrid systems composed of one or several quantum dots embedded in the arms of a mesoscopic ring. The interferometer Hamiltonian H^I acts on $\mathscr{H}^I = \mathscr{H}^R \oplus \mathscr{H}^C$, where $\mathscr{H}^C$ and $\mathscr{H}^R$ are the Hilbert spaces of the quantum dot cluster and the truncated ring. The latter is coupled to several semi-infinite noninteracting leads labelled $\alpha, \beta ....$ H^I is conveniently written as:

$$H^I = H^C + H^R + H^{CR} + H^{RC} \quad (1)$$

$$H^{CR} + H^{RC} = \tau \sum_m (e^{-i\varphi_m} |m\rangle\langle 0m| + h.c). \quad (2)$$

The off-diagonal parts H^{CR},H^{RC} connect the two subsytems, τ being the ring-dot coupling constant. m is the site of the cluster that is the closest one to the site $0m$ of the truncated ring. H^C contains a sum of single-dot terms H^{D_k} and their couplings:

$$H^{D_k} = -eV_k \sum_{i \in QD_k} |i\rangle\langle i| + t_D \sum_{<i,i'>} e^{2\pi i \varphi_{ii'}} |i\rangle\langle i'| \quad (3)$$

The on-site term V_k simulates the gate potential applied to the dot k $< i, i' >$ denotes the nearest neighbor summation and t_D is the hopping integral on dots. The magnetic flux ϕ is described through Peierls phases and will be expressed in units of quantum flux Φ_0. The conductance matrix G can be computed from the Landauer-Büttiker formula provided one knows an effective Green function G^I_{eff} of the sample in the presence of the leads. If the leads are weakly coupled the transport through the sample is easily studied by looking at the complex poles of G^I_{eff} [1, 2]. This situation is different in the experiments with ABI because the weak-coupling is set between the ring and the dot cluster while the electrons from leads reach freely the interferometer. Moreover, the complexity of the system yields complicated contributions to transport which have to be discerned at the level of the effective Green function. The remedy is to use the Feschbach formula (see [2] for details) to express the effective resolvent in the following form:

$$G^I_{\text{eff}} = \begin{pmatrix} G^C & -G^C H^{CR} G^R \\ -G^R H^{RC} G^C & G^R + G^R H^{RC} G^C H^{CR} G^R \end{pmatrix} \quad (4)$$

The new effective Green functions G^C and G^R describe *individually* the dot and the truncated ring:

$$G^R(z) := (H^R - \Sigma^L(z) - z)^{-1} \quad (5)$$

$$G^C(z) := (H^D - \Sigma^C(z) - z)^{-1}. \quad (6)$$

$\Sigma^L(z)$ is the lead's self-energy and the cluser self-energy $\Sigma^C(z) = H^{CR}(QH^I_{\text{eff}}Q - z)^{-1}H^{RC}$ where Q projects on the Hilbert space of the truncated ring. The conductance across the interferometer is given by

$$g_{\alpha\beta}(E_F) = 4\tau^4 \sin^2 k \left| \sum_{m,n} e^{i(\varphi_m - \varphi_n)} G^R_{\alpha,0m} G^C_{mn} G^R_{0n,\beta} \right|^2. \quad (7)$$

This formula captures all the resonant processes inside the interferometer. Our method involves only Green functions is an alternative to the scattering theoretical approach [3].

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FANO EFFECT IN DOUBLE-DOT INTERFEROMETER

In this section we concentrate on Eq. (7) in which a double-dot interferometer is characterized. Thus $H^C = H^{\mathrm{QD}_1} + H^{\mathrm{QD}_2} + H^{\mathrm{tun}}$, the last term describing the inter-dot tunnel coupling. Clearly, the condition for quantum interference is that both dots transmit. This means that the electron tunnels simultaneuosly through two levels of the *isolated* double dot system $E_i(V_1,V_2)$ and $E_j(V_1,V_2)$. Here V_1,V_2 are the gate potentials applied on each dot. Following Holleitner *et al*. [4] we plot in Fig. 1 the calculated charging diagram of an interferometer with 4×5 sites noninteracting quantum dots in the weak coupling regime. For each fixed value of V_2, we varied V_1 in the interval shown in the figures and we selected only conductances g_{12} that are larger than 0.65, which means that what we obtain is roughly a map for the peak positions in the plane (V_1,V_2). Each horizontal (vertical) trace represents the trajectory of a conductance peak associated with a resonant tunneling process through QD_2 (QD_1).

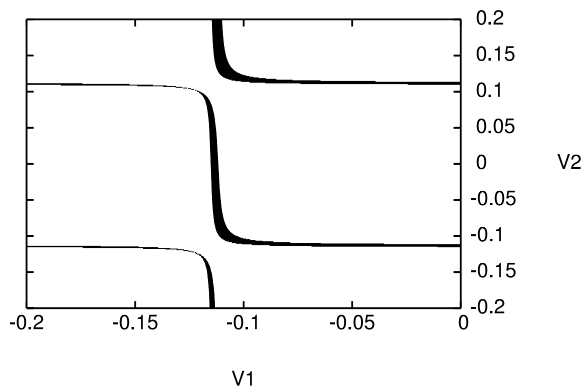

FIGURE 1. Charging diagram of the double dot interferometer with ring-dot coupling constant $\tau = 0.3$ and $\phi = 3\Phi_0$.

The idea is then to isolate the resonant contribution of a pair of eigenvalues in the effective resolvent. To this end one has first to use again the Feschbach formula in order to single out an effective resolvent acting in the two-dimensional spectral subspace of the two chosen eigenvalues. As a consequence, G^C is approximated by a 2×2 matrix $\tilde{G}^D_{\mathrm{eff}}$. Secondly, a Dyson equation for $\tilde{G}^C_{\mathrm{eff}}$ is written down, with respect to its off-diagonal part. The unperturbed resolvent involved in the Dyson expansion is the sum of two Breit-Wigner-like terms associated with the resonances located near E_i and E_j. By plugging the Dyson expansion for $\tilde{G}^D_{\mathrm{eff}}$ in (7) one recovers all the electronic paths within the interferometer. More technical details were given in [2]. In Fig. 2 we show a detail from the charging diagram in Fig. 1, taken in the neighborhood of an almost crossing point. We observe

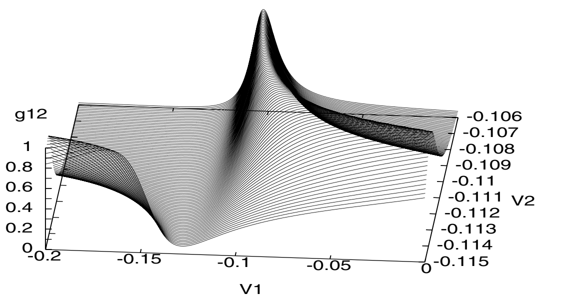

FIGURE 2. Fano effect in the double-dot interferometer.

an asymmetric large tail of the peaks, showing clearly that in this regime the interferometer acts as a Fano system. This happens because one dot (QD_2) is always set to a resonance thus the corresponding arm of the ring is 'free', providing the continuum component for the interference. As V_2 is slightly modified the orientation of the Fano tail changes. This is the so-called electrostatic control of the Fano interference [5]. Moreover, the transmittance assigned to the Fano dips shows Aharonov-Bohm oscillations, in full agreement with the observations of Holleitner *et al*. These results were thoroughly discussed in [2].

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