Spin Squeezing and Entanglement

Barış Öztop, Alexander A. Klyachko, Alexander S. Shumovsky
Faculty of Science, Bilkent University, Bilkent,
Ankara, 06800 Turkey
boztop@fen.bilkent.edu.tr

Abstract: We reformulate definition of spin squeezing and spin coherence in terms of the total variance in spin operators. We propose a new measure of spin squeezing. We show equivalence of spin squeezing and spin entanglement.

OCIS codes: (270.6570) Squeezed states, (000.6800) Theoretical physics

Spin-*s* squeezing is usually defined in the following way [1]. In a state ψ, the direction of mean-spin vector $\vec{S}_\psi = \sum_j \vec{e}_j \langle S_j \rangle_\psi$ ($j = x, y, z$, $\vec{e}_j \cdot \vec{e}_k = \delta_{jk}$) is considered as a new quantization axis. The state ψ is said to be spin squeezed iff one of the components of spin operator orthogonal to $\vec{S}_\psi$ has uncertainty less than *s/2*. We show that this definition can be reformulated in terms of the total variance (uncertainty) as follows

$$s < V_\psi \leq s(s+1), \quad V_\psi = \sum_j \langle S_j^2 \rangle_\psi - \langle S_j \rangle_\psi^2 . \tag{1}$$

The state ψ is a spin-coherent one iff $V_\psi = s$. This statement agrees with the conventional definition of spin coherence [2].

This definition (1) allows us to introduce a natural measure of spin squeezing in terms of deviation of the total variance from its minimal value

$$\Delta_\psi = V_\psi - V_{coh} = V_\psi - s . \tag{2}$$

The maximal squeezing corresponds to the states with $\Delta_\psi = s^2$. In other words, these are the states with maximal total uncertainty $V_{max} = s(s+1)$. Thus, it is possible to define the measure of spin squeezing in the interval [0,1] by simple normalization of (2):

$$\mu^2(\psi) = \frac{\Delta_\psi}{s^2} = \frac{V_\psi - V_{coh}}{V_{\max} - V_{coh}} . \tag{3}$$

Let us now note that the square root of the measure (3) coincides with the measure of entanglement of pure states that has been introduced in [3]. For bipartite systems, this measure is equivalent to the general representation of concurrence of the work [4]. At the same time, our measure is valid beyond bipartite systems as well [5].

As a matter of fact, this perfect coincidence of measures of spin-squeezing and entanglement agrees with the main idea of the approach [6], in which complete entanglement is associated with states of maximal total variance, while all other entangled states can be obtained from the complete entangled one by means of SLOCC (stochastic local operations assisted by classical communications) [7]. Let us stress that the condition of spin squeezing (1) is valid for a single spin-*s* object as well as for a multi-spin system. It can be easily seen that in the case of $s = 1/2$ all states are coherent so that neither squeezing nor entanglement is possible for this system. It was shown in [8,9] that a single spin-*1* can manifest entanglement. This entanglement can be measured by means of $\mu(\psi)$ [5]. Thus, the definition of spin-squeezing (1) can be considered as a definition of spin-entanglement for an arbitrary system *with* $s \geq 1$.

The known possibility to consider a spin-*s* system with $s \geq 1$ as a collective system of *2s* spins *1/2* (qubits) [10] allows us to interpret the single-spin entanglement as that related to the "intrinsic" degrees of freedom of a single "particle" [6]. At the same time, it should be stressed that the definition (1) specifies the spin-*s* system as a whole and treats single-spin entanglement (squeezing) regardless of the internal structure of the object. This is a novelty with respect to the works [11] that treated entanglement of qubit systems in terms of multi-"mode" squeezed states.

In particular, it follows from our definition (1) that entanglement in an arbitrary spin-*s* system can be detected by only three measurements of spin components S_x, S_y, S_z. This generalizes results of Ref. [12] that have been obtained for qubit-systems.

Physically, the single-spin entanglement may manifest itself by means of decay of entangled state into entangled states of well-defined qubits that can be spatially separated. A simple example of interest is provided by a biphoton [13] (a system of two photons created at once and propagating in the same direction). In this case, we can associate spin-*1* operators with the Stokes operators, measuring three polarizations of a single biphoton. The spin-squeezed (entangled) state in this case coincides with the unpolarized spin-projection state $|0\rangle$. By means of a beam splitter, this state can be decomposed into the polarization-entangled state of two photons

$$\frac{1}{\sqrt{2}}\left(\left|HV\right\rangle+\left|VH\right\rangle\right),$$

where H and V denote the horizontal and vertical photon polarizations, respectively.

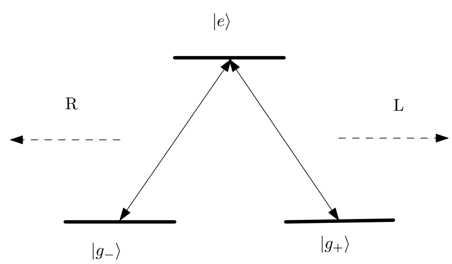

Fig. 1: Λ-type transition in *Cs* atom.

Another interesting example is given by a single three-level atom in any allowed configuration that can also be associated with a single spin-*1* system. For example, consider the case of the Λ-type configuration, corresponding to the transitions between sublevels ($6S_{1/2}, F = 3, m = 0$) and ($6P_{1/2}, F = 3, m = \pm 1$) in *Cs* atom [14] denoted in Fig. 1 as the states $|e\rangle$ and $|g_\pm\rangle$, respectively. These states can be associated with the spin-1 states as $|e\rangle = |0\rangle$, $|g_\pm\rangle = |\pm 1\rangle$. It follows from definition (1) that the excited state $|e\rangle$ is completely squeezed (the measure (3) $\mu(e) = 1$) and hence completely entangled. Its decay gives rise to the state

$$\frac{1}{\sqrt{2}}\left(\left|g_+\right\rangle\otimes\left|1_L\right\rangle+\left|g_-\right\rangle\otimes\left|1_R\right\rangle\right), \qquad (4)$$

where $|1_P\rangle$ denotes the single-photon state with either left (L) or right (R) circular polarization. This state (4) can be interpreted as a completely entangled state of two qubits. The atomic qubit is provided by the degenerated ground state $|g_\pm\rangle$, while the photon qubit is given by the two polarization states. This state is usually considered in connection with the atom-photon entanglement (e.g., see Ref. [15]). This result can be easily obtained with the V and Ξ configurations as well.

Thus, we have shown that the spin squeezing can be defined in terms of total variance in the same way as spin entanglement, so that these two notions should be considered as synonyms. The notion of spin entanglement is regarded to a whole system independent of its structure in terms of elementary spins *1/2* (qubits). Thus, entanglement of such a system can be detected by means of only three measurements for any system with $s \geq 1$. The above examples can be easily generalized in the case of high-spin states.

One of the authors (B.Ö.) would like to acknowledge the Scientific and Technical Research Council of Turkey (TÜBİTAK) for financial support.

References

[1] M. Kitagawa and M. Ueda, Phys. Rev. A **47**, 5138 (1993).

[2] J.M. Radcliffe, J. Phys. A: Math. General **4**, 313 (1971); F.T. Arecchi, E. Courtens, R. Gilmore, and H. Thomas, Phys. Rev. A **6**, 2211 (1972).

[3] A.A. Klyachko, B. Öztop and A.S. Shumovsky, Appl. Phys. Lett. **88**, 124102 (2006).

[4] P. Rungta, V. Bužek, C.M. Caves, M. Hillery, and G.J. Milburn, Phys. Rev. A, **64**, 042315 (2001).

[5] A.A. Klyachko, B. Öztop and A.S. Shumovsky, Phys. Rev. A (in print), E-print quant-ph/0606119.

[6] A.A. Klyachko and A.S. Shumovsky, J. Phys: Conf. Series **36**, 87 (2006).

[7] W. Dür, G. Vidal, and J.I. Cirac, Phys. Rev. A **62**, 062314 (2000); F. Verstraete, J. Dehaene, and B. De Moor, Phys. Rev. A **68**, 012103 (2003).

[8] H. Barnum, E. Knill, G. Ortiz, R. Somma, and L. Viola, Phys. Rev. Lett. **92**, 107902 (2004).

[9] M.A. Can, A.A. Klyachko, A.S. Shumovsky, J. Opt. B: Quant. Semiclass. Opt. **7**, L1 (2005).

[10] L.D. Landau and E.M. Lifshitz, *Quantum Mechanics* (Butterworth-Heinemann, Oxford, 1981).

[11] A. Sørensen, L.M. Duan, J.I. Cirac, and P. Zoller, Nature (London) **409**, 63 (2001); L.K. Thomsen, S. Mancini, and H.M. Wiseman, Phys. Rev. A **65**, 061801 (2002); D.W. Berry and B.C. Sanders, J. Phys. A **38**, L205 (2005).

[12] J.M.G. Sancho and S.F. Huelga, Phys. Rev. A **61**, 042303 (2000).

[13] M.V. Chekhova, L.A. Krivitsky, S.P. Kulik, and G.A. Maslennikov, Phys. Rev. A **70**, 053801 (2004).

[14] W. Lange and H.J. Kimble, Phys. Rev. A **61**, 063817 (2000).

[15] B.B. Blinov, D.L. Moehring, L.-M. Duan, and C. Monroe, Nature **428**, 153 (2004).