PO-MLFMA HYBRID TECHNIQUE FOR THE SOLUTION OF ELECTROMAGNETIC SCATTERING PROBLEMS INVOLVING COMPLEX TARGETS

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Abstract

The multilevel fast multipole algorithm (MLFMA) is a powerful tool for efficient and accurate solutions of electromagnetic scattering problems involving large and complicated structures. On the other hand, it is still desirable to increase the efficiency of the solutions further by combining the MLFMA implementations with the high-frequency techniques such as the physical optics (PO). In this paper, we present our efforts in order to reduce the computational cost of the MLFMA solutions by introducing PO currents appropriately on the scatterer. Since PO is valid only on smooth and large surfaces that are illuminated strongly by the incident fields, accurate solutions require careful choices of the PO and MLFMA regions. Our hybrid technique is useful especially when multiple solutions are required for different frequencies, illuminations, and scenarios, so that the direct solutions with MLFMA become expensive. For these problems, we easily accelerate the MLFMA solutions by systematically introducing the PO currents and reducing the matrix dimensions without sacrificing the accuracy.

1 Introduction

For the solution of scattering problems involving large and complicated targets, surface integral equations provide accurate results when they are discretized appropriately by using small elements (such as triangles) compared to wavelength. Simultaneous discretizations of the integral equations and the objects lead to dense matrix equations that can be solved iteratively, where the matrix-vector multiplications are accelerated by the multilevel fast multipole algorithm (MLFMA). For an $N \times N$ matrix equation, MLFMA performs the matrix-vector multiplications in $O(N \log N)$ time using $O(N \log N)$ memory. Due its low complexity, MLFMA provides the solution of electromagnetic scattering problems involving large numbers of unknowns on relatively inexpensive computing platforms [4]. On the other hand, most of the real-life problems require multiple solutions for different illuminations, frequencies, and scenarios. For these problems, it is desirable to accelerate the solutions by using fast but less accurate techniques, such as the physical optics (PO). In the literature, there are many studies on developing hybrid techniques based on combining the method of moments (MOM) and PO technique to utilize both the accuracy of MOM and the efficiency of PO for the solution of scattering and radiation problems [2],[4],[7],[8]. In general, these hybrid techniques are successfully used to improve the accuracy of the PO solutions by introducing MOM in some limited regions, where it is critical to account for the electromagnetic interactions for accurate simulations.

In this paper, we present a robust hybrid technique, which involves the combination of MLFMA and PO for the solution of scattering problems involving complicated structures. Similar to the other hybrid techniques in the literature, we employ integral equations for specific regions (MoM regions), such as the locations near the edges, cavities, and surfaces around the shadow boundaries, while the surface currents in other (smooth) regions are approximated by PO. The surface currents in the MOM region are solved by MLFMA with a low complexity. To achieve a desired level of accuracy with the minimum number of unknowns and processing time, we systematically introduce the PO currents on the scatterer and reduce the dimensions of the matrix equations solved by MLFMA. Effectiveness of this hybrid technique is demonstrated on a scattering problem involving a disc target with smooth edges.

2 Formulation

We consider scattering problems involving three-dimensional conducting surfaces with arbitrary shapes. For a numerical solution, surface of object is discretized by using small planar triangles, on which the Rao-Wilton-Glisson (RWG) functions [9] are defined to expand the unknown surface current density, i.e.,

$$\boldsymbol{J}(\boldsymbol{r}) = \sum_{n=1}^{N} a_n \boldsymbol{b}_n(\boldsymbol{r}), \tag{1}$$

where a_n represents the unknown coefficient of the nth basis function $\boldsymbol{b}_n(\boldsymbol{r})$. When a scattering problem is formulated directly by using an integral-equation formulation, simultaneous discretizations of the geometry and the integral equation lead to $N \times N$ dense matrix equation, i.e.,

$$\sum_{n=1}^{N} Z_{mn} a_n = v_m \qquad (m = 1, 2, ..., N). \tag{2}$$

Using the combined-field integral equation (CFIE) [6], which is obtained by the combination of the electric-field integral equation (EFIE) and the magnetic-field integral equation (MFIE), matrix elements in (2) can be written as

$$Z_{mn} = \alpha_m Z_{mn}^{E} + (1 - \alpha_m) Z_{mn}^{M}, \tag{3}$$

where α_m represents a combination parameter between 0 and 1 for each $m = 1, 2, ..., N$ [3]. In (3), contributions of EFIE and MFIE are derived as

$$\begin{aligned} Z_{mn}^{E} &= ik \int_{S_m} d\boldsymbol{r}\ \boldsymbol{t}_m(\boldsymbol{r}) \cdot \int_{S_n} d\boldsymbol{r}'\ \boldsymbol{b}_n(\boldsymbol{r}') g(\boldsymbol{r}, \boldsymbol{r}') \\ &\quad - \frac{i}{k} \int_{S_m} d\boldsymbol{r}\ \boldsymbol{t}_m(\boldsymbol{r}) \cdot \int_{S_n} d\boldsymbol{r}'\ \boldsymbol{b}_n(\boldsymbol{r}') \cdot \left[\nabla \nabla' g(\boldsymbol{r}, \boldsymbol{r}') \right] \end{aligned} \tag{4}$$

and

$$\begin{aligned} Z_{mn}^{M} &= -\int_{S_m} d\boldsymbol{r}\ \boldsymbol{t}_m(\boldsymbol{r}) \cdot \boldsymbol{b}_n(\boldsymbol{r}) \\ &\quad + \int_{S_m} d\boldsymbol{r}\ \boldsymbol{t}_m(\boldsymbol{r}) \cdot \hat{\boldsymbol{n}} \times \int_{S_n} d\boldsymbol{r}'\ \boldsymbol{b}_n(\boldsymbol{r}') \times \nabla' g(\boldsymbol{r}, \boldsymbol{r}'), \end{aligned} \tag{5}$$

respectively, where k is the wavenumber, $\boldsymbol{t}_m(\boldsymbol{r})$ represents mth testing function, $\hat{\boldsymbol{n}}$ is outward normal vector on the surface, and

$$g(\boldsymbol{r}, \boldsymbol{r}') = \frac{\exp(ik\,|\boldsymbol{r} - \boldsymbol{r}'|)}{4\pi\,|\boldsymbol{r} - \boldsymbol{r}'|} \tag{6}$$

denotes the free-space Green's function in phasor notation using $\exp(iwt)$ convention. In (4) and (5), S_n and S_m represent the spatial supports of the nth basis function and mth testing function, respectively. Using a Galerkin scheme, we choose the testing functions also as the RWG functions. Finally, in (2), elements of the excitation vector are derived as

$$v_m = -\int_{S_m} d\boldsymbol{r}\ \boldsymbol{t}_m(\boldsymbol{r}) \cdot \left[\frac{\alpha_m}{\eta} \boldsymbol{E}^{inc}(\boldsymbol{r}) + (1 - \alpha_m) \hat{\boldsymbol{n}} \times \boldsymbol{H}^{inc}(\boldsymbol{r}) \right] \tag{7}$$

where η is the wave impedance, while $\boldsymbol{E}^{inc}$ and $\boldsymbol{H}^{inc}$ are the incident electric and magnetic fields, respectively, due to the external sources.

The matrix equation in (2) can be solved iteratively, where the matrix-vector multiplications are accelerated by MLFMA [10]. For more efficient solutions, however, we propose a hybrid technique based on approximating the currents on smooth surfaces by using PO. In this technique, PO and MOM regions are determined on the object by considering the trade-off between the efficiency and accuracy. To achieve a desired level of accuracy with the minimum number of unknowns and processing time, we apply MLFMA only on such regions where the PO currents cannot provide accurate results. These regions usually correspond to the locations near the edges, cavities, and surfaces around the shadow boundaries. For smooth surfaces that are illuminated strongly by the incident fields, we expand the PO currents in a series of basis functions, i.e.,

$$\boldsymbol{J}^{PO}(\boldsymbol{r}) = \frac{1}{2} \hat{\boldsymbol{n}} \times \boldsymbol{H}^{inc}(\boldsymbol{r}) = \sum_{n=1}^{N_{PO}} a_n \boldsymbol{b}_n(\boldsymbol{r}), \tag{8}$$

where coefficients of the basis functions (a_n for $n = 1, 2, ..., N_{PO}$) can be found by testing the equation as

$$\begin{aligned} \int_{S_m} d\boldsymbol{r}\ \boldsymbol{t}_m(\boldsymbol{r}) \cdot \frac{1}{2} \hat{\boldsymbol{n}} \times \boldsymbol{H}^{inc}(\boldsymbol{r}) &= \int_{S_m} d\boldsymbol{r}\ \boldsymbol{t}_m(\boldsymbol{r}) \cdot \sum_{n=1}^{N_{PO}} a_n \boldsymbol{b}_n(\boldsymbol{r}) \\ &= \sum_{n=1}^{N_{PO}} a_n \int_{S_m} d\boldsymbol{r}\ \boldsymbol{t}_m(\boldsymbol{r}) \cdot \boldsymbol{b}_n(\boldsymbol{r}) \qquad (m = 1, 2, ..., N_{PO}). \end{aligned} \tag{9}$$

This way, we obtain a sparse matrix equation in the form of

$$\sum_{n=1}^{N_{PO}} I_{mn} a_n = w_m \qquad (m = 1, 2, ..., N_{PO}), \tag{10}$$

where

$$w_m = \frac{1}{2} \int_{S_m} d\boldsymbol{r}\ \boldsymbol{t}_m(\boldsymbol{r}) \cdot \hat{\boldsymbol{n}} \times \boldsymbol{H}^{inc}(\boldsymbol{r}) \tag{11}$$

and

$$I_{mn} = \int_{S_m} d\boldsymbol{r}\ \boldsymbol{t}_m(\boldsymbol{r}) \cdot \boldsymbol{b}_n(\boldsymbol{r}) \tag{12}$$

is the inner product of mth testing and nth basis functions, which is nonzero only when the functions overlap in space. Using RWG basis and testing functions, I_{mn} is extremely sparse and the matrix equation in (10) can be solved easily in a few iterations using a Krylov subspace iterative algorithm [1].

After the coefficients in (10) are determined, the PO currents are radiated to the MOM region by performing a matrix-vector multiplication, i.e.,

$$y_m = \sum_{n=1}^{N_{PO}} Z_{mn} a_n \qquad \left(m = (N_{PO}+1),(N_{PO}+2),\ldots,N\right). \quad (13)$$

In (13), y_m for $m = (N_{PO}+1),(N_{PO}+2),\ldots,N$ corresponds to the testing of the radiated field due to the PO currents on the mth testing function located in the MOM region. The multiplication in (13) can be performed efficiently by employing MLFMA with reduced complexity. Then, the coefficients of the basis functions in the MOM region can be calculated by solving the matrix equation

$$\sum_{n=N_{PO}+1}^{N} Z_{mn} a_n = v'_m \qquad \left(m = (N_{PO}+1),(N_{PO}+2),\ldots,N\right), \quad (14)$$

where

$$v'_m = v_m - y_m \qquad \left(m = (N_{PO}+1),(N_{PO}+2),\ldots,N\right) \quad (15)$$

involves the testing of the incident fields due to both external sources and the PO currents.

The matrix equation in (14) can also be solved iteratively by employing MLFMA. Using the hybrid technique, dimensions of the matrix equation is reduced from $N \times N$ to $(N - N_{PO}) \times (N - N_{PO})$. The extra cost is only due to the solution of the extremely-sparse matrix equation in (10) to expand the PO currents in a series of basis functions and the matrix-vector multiplication in (13) to radiate the PO currents to the MOM region. Both of these operations require negligible time compared to the iterative solution of (14). As a result, by choosing the PO region appropriately, matrix dimensions can be reduced significantly [from (2) to (14)] to accelerate the solutions without sacrificing the accuracy.

3 Results

To demonstrate the accuracy and the efficiency of the proposed hybrid technique, Figure 1 presents a scattering problem involving a disc with smooth edges. The target is illuminated by a plane wave propagating at 45° from the z axis with the electric field polarized in the y direction. The problem is solved at 10 GHz and discretization of the geometry with λ/10 mesh size leads to about 260,000 unknowns.

Figure 2 depicts the real part of the surface current density induced on the target shown in Figure 1. Comparing Figure 2(a) and 2(b), we observe that the currents obtained by PO differ significantly compared to the currents obtained by MLFMA especially around the shadow boundary. The inaccuracy of PO is also illustrated in Figure 2(c), where we plot the difference between the real parts of the currents obtained by PO and MLFMA.

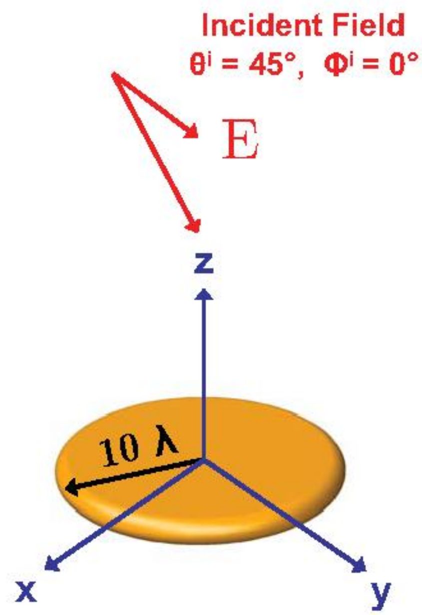

Figure 1. A scattering problem involving a disc with smooth edges.

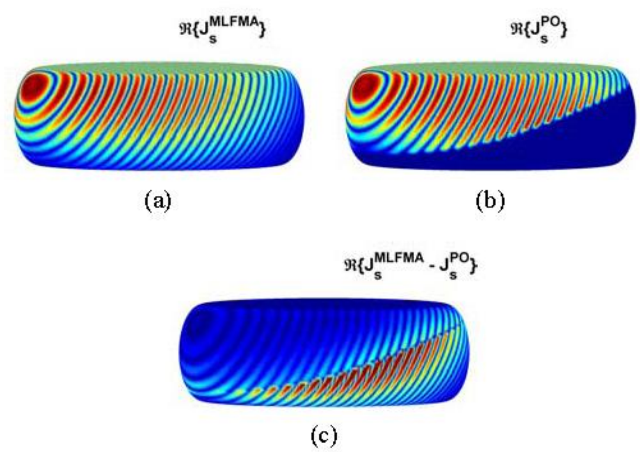

Figure 2. Real part of the surface current density induced on the target depicted in Figure 1. (a) MLFMA. (b) PO. (c) MLFMA-PO.

To further investigate the accuracy of the PO solution, Figure 3 presents the bistatic radar cross section (RCS) values in decibels (dBms) as a function of bistatic angle from 225° to 270° on the z-x plane, where 225° corresponds to the forward-scattering direction. We observe that the PO solution is inaccurate compared to the reference solution by MLFMA.

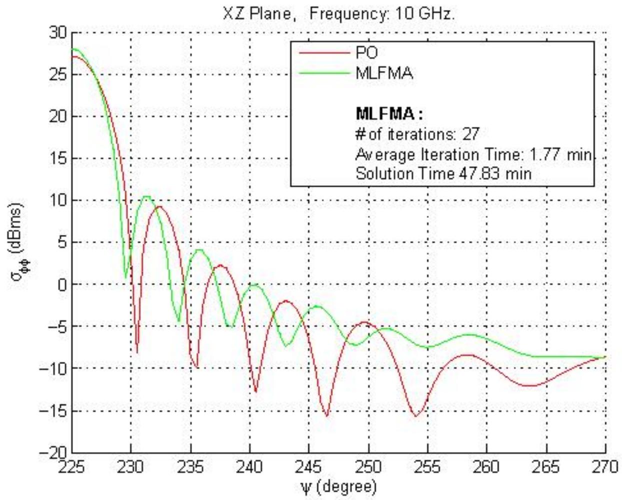

Figure 3. Bistatic RCS of the target in Figure 1 calculated by MLFMA and PO.

The MLFMA solution of the scattering problem in Figure 1 is performed in about 48 minutes on an AMD Opteron processor. In order to reduce the processing time without loosing the accuracy, we employ the hybrid MLFMA-PO technique by systematically introducing PO currents on the object. This is achieved by using PO currents on the lit and shadow regions while keeping the MLFMA on the shadow boundary (MOM region). By adjusting the area of the MOM region, we examine the trade-off between the efficiency and accuracy. In Figure 4, we present the error of the solutions by plotting the difference between the currents obtained by the hybrid technique and the reference MLFMA. It can be observed that the currents obtained by the hybrid technique becomes more and more accurate as the MOM region is enlarged. This is also confirmed by the RCS plots in Figure 5. Using MLFMA in a narrow region discretized with 19,500 unknowns, we obtain the solution in only about 4.2 minutes, while the results are close to the reference solution as depicted in Figure 5(a). Then, by increasing the area of the MOM region, accuracy of the results can be further improved as depicted in Figure 5(b), Figure 5(c), and Figure 5(d), while the solution time increases. In general, the choice of the PO and MLFMA regions depends on the desired level of accuracy and the efficiency requirements.

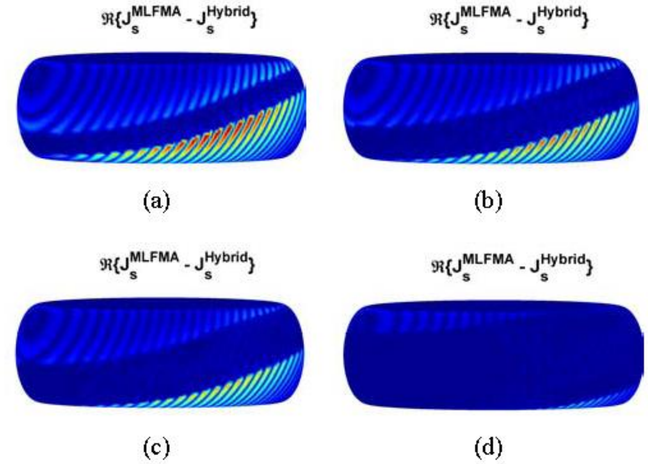

Figure 4. Error in the real part of the induced currents obtained by the hybrid technique compared to reference MLFMA solution. (a) S_{MoM} : 0.5λ around edges with 19,500 unknowns. (b) S_{MoM} : 0.8λ around edges with 31,050 unknowns. (c) S_{MoM} : 1λ around edges with 39,750 unknowns. (d) S_{MoM} : 2λ around edges with 82,500 unknowns.

Conclusion

We present a robust hybrid technique for the solution of scattering problems involving three-dimensional complicated targets. Our strategy is based on introducing PO currents systematically on the target to reduce the dimensions of the matrix-equation solved by MLFMA. We consider the trade-off between the accuracy and efficiency of the results by adjusting the PO and MOM regions carefully. This way, we are able to accelerate the solutions without sacrificing the accuracy.

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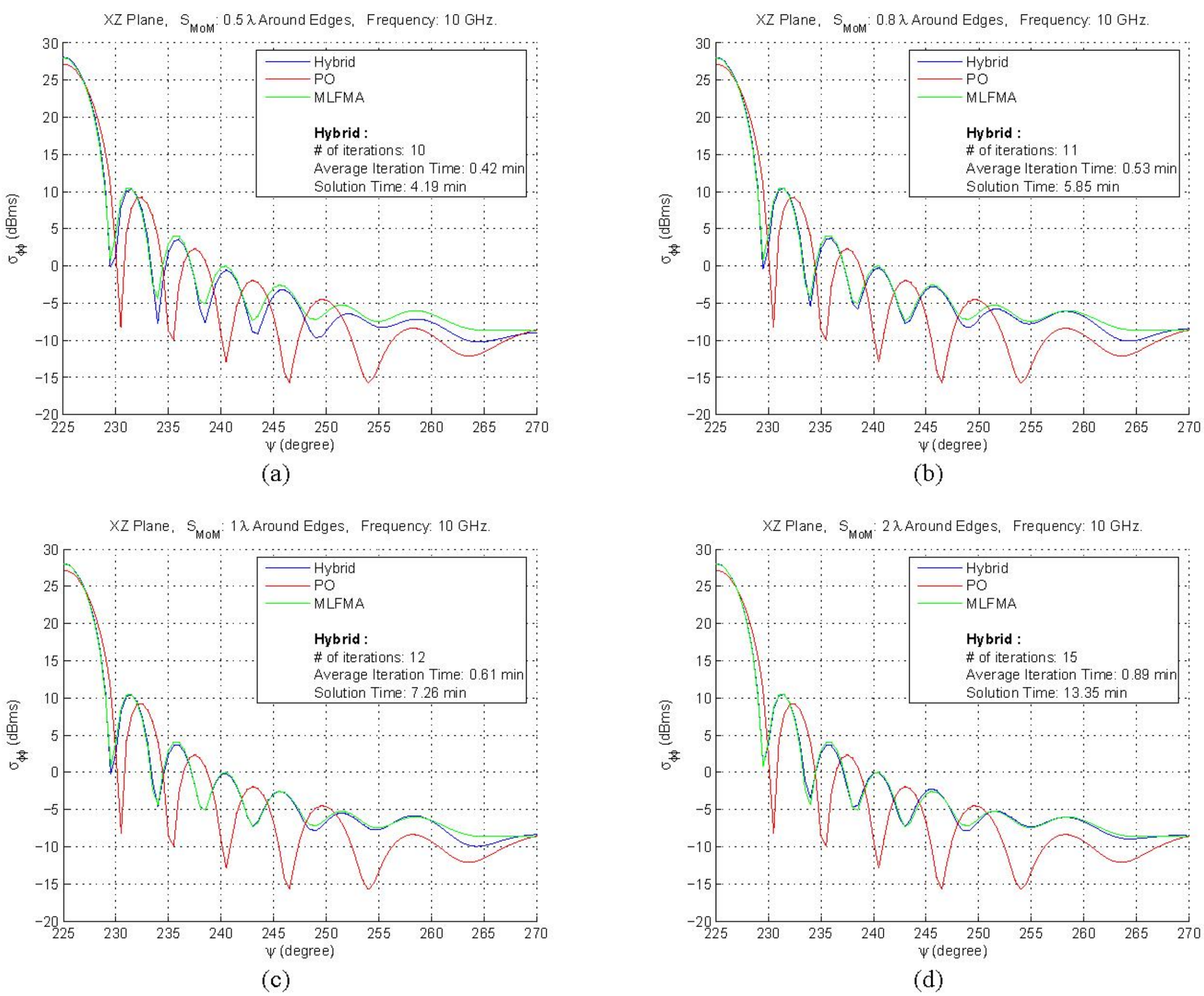

Figure 5. Bistatic RCS of the target in Figure 1 calculated by MLFMA, PO, and the hybrid technique. (a) S_{MoM} : 0.5λ around edges with 19,500 unknowns. (b) S_{MoM} : 0.8λ around edges with 31,050 unknowns. (c) S_{MoM} : 1λ around edges with 39,750 unknowns. (d) S_{MoM} : 2λ around edges with 82,500 unknowns.