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Ground-State Properties and Collective Excitations in a 2D Bose-Einstein Condensate with Gravity-Like Interatomic Attraction

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Abstract We study the ground-state properties of a Bose-Einstein condensate (BEC) with the short-range repulsion and gravitylike $1/r$ interatomic attraction in two-dimensions (2D). Using the variational approach, we obtain the ground-state energy and show that the condensate is stable for all interaction strenghts in 2D. We also determine the collective excitations at zero temperature using the time-dependent variational method. We analyze the properties of the Thomas-Fermi-gravity (TF-G) and gravity (G) regimes.

Keywords Cold atom Bose and Fermi systems · Excitations in quantum systems

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1 Introduction

Investigations of Bose-Einstein condensates (BEC) have mostly concentrated on systems with the two-body short-ranged interaction which is characterized by s-wave scattering length. Recently, O'Dell *et al.* [1] have proposed a configuration for the occurrence of $1/r$ interaction which is a totally new regime for cold gases, having a long-range attractive interaction. The analysis of this configuration is also important since it suggests a new way to examine the stellar $1/r$ interaction in the laboratory. Apart from this possibility, it is interesting that such a system results in stable condensates even in the absence of external trap potential. Recent experiments [2] started to probe the properties of such systems.

The gravity-like interaction of atoms in a condensate is mostly a result of dipole-dipole interactions [3]. Adjusting the configuration of intense off-resonant laser

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beams, one can obtain $1/r$ interaction [1] where the interaction potential has the form $U(\mathbf{r}) = -u/r$ in which u is related to material parameters and laser intensity. In this system, kinetic energy and gravity-like attractive interaction forms a stable configuration without a trap potential[1, 3]. Ghosh [4] has studied the collective excitation frequencies of this system in 3D within the time-dependent variational method. He has shown that variational analysis agrees very well with the results of Giovanazzi *et al*. [3] in which the sum-rule approach was used.

The recent progress on the cooling and trapping of neutral atomic gases with electromagnetic field has opened the way to the study of 2D Bose gases [5]. The 2D atomic BECs have many interesting properties as revealed by experiments. In this paper we study a 2D condensate with the attractive $1/r$ interaction. We calculate the ground-state properties using a variational approach and show that the condensate is stable without the external potential. We also consider the dynamics of the condensate within the time-dependent variational method and calculate the monopole and quadrupole mode frequencies.

2 Ground-State Properties

We will use the mean field theory together with the variational method. For a dilute gas of bosonic atoms, we can write the equation of motion for the system

$$i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{ext}(\mathbf{r}) + V_H(\mathbf{r})\right]\psi(\mathbf{r},t) \tag{1}$$

where $V_{ext} = m\omega_0^2 r^2/2$ is the external harmonic potential and V_H is the Hartree potential composed of hard sphere and gravity interactions in the form

$$V_H(\mathbf{r}) = g \mid \Psi(\mathbf{r}) \mid^2 - u \int d^2\mathbf{r}' \frac{\mid \Psi(\mathbf{r}') \mid^2}{\mid \mathbf{r}' - \mathbf{r} \mid} \tag{2}$$

where $g = 2\sqrt{2\pi}\hbar^2 a/ma_z$ is the 2D contact interaction strength with a is the s-wave scattering length, and a_z is the harmonic oscillator length. One can obtain the energy functional from (1)

$$E = \int d^2r \left[\frac{\hbar^2}{2m}|\nabla\Psi(\mathbf{r})|^2 + V_{ext}|\Psi(\mathbf{r})|^2 + \frac{g}{2}|\Psi(\mathbf{r})|^4 - \frac{u}{2}\int d^2r' \frac{|\Psi(\mathbf{r}')|^2|\Psi(\mathbf{r})|^2}{|\mathbf{r}' - \mathbf{r}|}\right]. \tag{3}$$

We use the variational wavefunction in the form

$$\Psi(\mathbf{r},\lambda) = \frac{(N/\pi)^{1/2}}{(\lambda l_0)} \exp(-r^2/2\lambda^2 l_0^2) \tag{4}$$

where $l_0 = \sqrt{\hbar/m\omega_0}$ and this form satisfies the normalization condition with the total number of particles N. Using this function in the energy functional, energy per particle can be obtained as

$$\frac{E(\lambda)}{N\hbar\omega_0} = \frac{1}{2}(\lambda^{-2} + \lambda^2 + \tilde{s}\lambda^{-2} - 2\tilde{u}\lambda^{-1}) \tag{5}$$

Table 1 Comparison of four asymptotic regions

	G	TF-G	TF-0	I
defn:	$\tilde{u} \gg 1$	$\tilde{s} \ll \tilde{u}^{4/3}$	$\tilde{s} \gg 1$	$\tilde{u} \ll 1$
	$\tilde{s} \ll 1$		$\tilde{s} \gg \tilde{u}^{4/3}$	$\tilde{s} \ll 1$
λ:	$1/\tilde{u}$	$\tilde{s}/\tilde{u}$	$\tilde{s}^{1/4}$	1
$E_{rel}/\hbar\omega_0$:	$\frac{1}{2}N\tilde{u}^2 \propto N^3$	$\frac{1}{2}N\tilde{u}^2\tilde{s}^{-1} \propto N^2$	$\frac{1}{2}N\tilde{s}^{1/2} \propto N^{3/2}$	$\frac{1}{2}N \propto N$
ρ_{max}:	$\frac{N^3u^2}{16l_0^4\hbar^2\omega_0^2}$	$\frac{Na_z^2u^2}{16a^2l_0^4\hbar^2\omega_0^2}$	$\frac{N^{1/2}a_z^{1/2}}{(2\pi^5)^{1/4}a^{1/2}l_0^2}$	$\frac{N}{\pi l_0^2}$

where we choose the dimensionless interaction parameters as

$$\tilde{s} = \frac{Na}{\sqrt{2\pi}a_z} \quad \text{and} \quad \tilde{u} = \frac{\pi u N}{4l_0\hbar\omega_0}. \tag{6}$$

Minimizing the energy with respect to variational parameter λ yields

$$1 - (1+\tilde{s})\lambda^{-4} + \tilde{u}\lambda^{-3} = 0. \tag{7}$$

Table 1 gives the comparison of the four asymptotic regions on some experimental quantities such as condensate radius, release energy and peak density. Ideal noninteracting region (I) and ordinary Thomas-Fermi region (TF-O) are familiar from the ordinary condensates but two new regions are realized for atomic BECs. The regions G and TG-G are related to the balance of the gravity-like potential with the kinetic energy or the contact interaction, respectively. These regions are not sensitive to the external potential, so that it can adiabatically be turned off. The gravity-like attraction does not induce the collapse of the condensate unlike the contact interaction. In contrast to the 3D system [1], there is no turning point in which the minimum disappears in 2D solutions. This means that there is no instability in the case of 2D gravity-like interactions, even for the negative scattering lengths. From Fig. 1 one can conclude that the condensate is stable without the external trap, i.e., it is self-bound.

3 Time-Dependent Variational Analysis

We use the time-dependent variational approach [6] to obtain the dynamics of the condensate. The Lagrangian density can be written as

$$\mathcal{L} = \frac{i\hbar}{2}\left(\psi\frac{\partial\psi^*}{\partial t} - \psi^*\frac{\partial\psi}{\partial t}\right) - \frac{\hbar^2}{2m}|\nabla\psi|^2 + \frac{1}{2}V_H(\mathbf{r})|\psi|^2, \tag{8}$$

in which the external potential is set equal to zero. The oscillation frequencies obtained by a Gaussian ansatz in 3D are shown to be in good agreement with the exact calculations [3]. Thus, we choose the trial function

$$\psi(x, y, t) = \sqrt{\frac{N}{\alpha_1\beta_1\pi}}\exp\left(-\frac{1}{2}[\alpha(t)x^2 + \beta(t)y^2]\right) \tag{9}$$

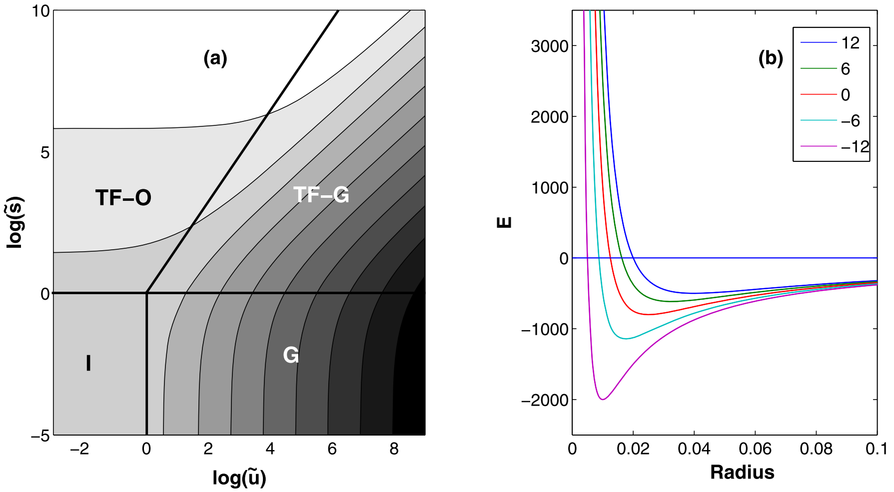

Fig. 1 (Color online) **a** Contour plot of the logarithm of the condensate radius as a function of $\log\tilde{u}$ and $\log\tilde{s}$, *darker shade* corresponds to smaller radius. Four asymptotic regions can be seen from the plot. **b** Energy of the condensate for different values of variational parameter $\tilde{s}\tilde{u}$ as a function of radius for large u. The energy is scaled with $N\hbar\omega_0$

where the wave function is normalized to N and x and y are variables in units of $l_0 = \sqrt{\hbar/m\omega_g}$ and $\omega_g = mu^2N^2/\hbar^3$ is the gravitational frequency. $\alpha(t) = 1/\alpha_1^2 + i\alpha_2$ and $\beta(t) = 1/\beta_1^2 + i\beta_2$ are the dimensionless time-dependent variational parameters. α_1 and β_1 are condensate widths along the x and y directions, respectively. The complex parts of the variational parameters are necessary for an accurate description of the condensate dynamics [7]. Substituting this wavefunction into the Lagrangian density and integrating over 2D spatial coordinates, we obtain the following Lagrangian

$$L = \frac{SN^2u^2}{g}\left[\frac{1}{2}(\alpha_1^2\dot{\alpha_2} + \beta_1^2\dot{\beta_2}) - \frac{1}{2}\left(\frac{1}{\alpha_1^2} + \alpha_1^2\alpha_2^2\right) - \frac{1}{2}\left(\frac{1}{\beta_1^2} + \beta_1^2\beta_2^2\right)\right.$$
$$\left. - \frac{1}{\pi}\frac{S}{\alpha_1\beta_1} + \sqrt{\frac{\pi}{2}}\frac{{}_2F_1[\frac{1}{2}, \frac{1}{2}; 1; (1-\frac{\beta_1^2}{\alpha_1^2})]}{\alpha_1}\right] \quad (10)$$

where $S = gmN/2\hbar^2$ is a dimensionless scattering parameter and ${}_2F_1[\frac{1}{2}, \frac{1}{2}; 1; (1 - \beta_1^2/\alpha_1^2)]$ is the hypergeometric function. The energy as a function of the variational parameter in an isotropic system is obtained as

$$E = \frac{SN^2u^2}{g}\left[\frac{1}{\alpha^2} + \frac{1}{\pi}\frac{S}{\alpha^2} - \sqrt{\frac{\pi}{2}}\frac{1}{\alpha}\right]. \quad (11)$$

From minimizing the energy functional with respect to the variational parameter, equilibrium point is obtained as $w = (2/\pi)^{1/2}(2 + 2S/\pi)$. The chemical potential $\mu = E/N$ and sound velocity $c_s^2 = \mu/m$ can be calculated from (11). Using the Euler-

Lagrange equation, time evolution of the widths is

$$\ddot{\alpha}_1 = \frac{1}{\alpha_1^3} + \sqrt{\frac{\pi}{2}}\left(\frac{\tilde{S}}{\alpha_1^2\beta_1} + F_{\alpha_1}\right), \tag{12}$$

$$\ddot{\beta}_1 = \frac{1}{\beta_1^3} + \sqrt{\frac{\pi}{2}}\left(\frac{\tilde{S}}{\alpha_1\beta_1^2} + F_{\beta_1}\right) \tag{13}$$

where $\tilde{S} = \sqrt{2/\pi^3}S$ and $F_{\alpha_1}, F_{\beta_1}$ are derivatives of ${}_2F_1[1/2, 1/2; 1; (1-\beta_1^2/\alpha_1^2)]/\alpha_1$ with respect to α_1 and β_1, respectively. We are looking for low-energy excitations which correspond to small oscillations around the equilibrium point. Thus we need to expand around the equilibrium width by letting $\alpha_1 = w + \delta\alpha_1$ and $\beta_1 = w + \delta\beta_1$ for the isotropic system. Time evolution of the widths is given by

$$\delta\ddot{\alpha}_1 = \left[-\frac{3}{w^4} + \sqrt{\frac{\pi}{2}}\left(-\frac{2\tilde{S}}{w^4} + \frac{5}{8w^3}\right)\right]\delta\alpha_1 + \sqrt{\frac{\pi}{2}}\left[-\frac{S}{w^4} + \frac{3}{8w^3}\right]\delta\beta_1, \tag{14}$$

$$\delta\ddot{\beta}_1 = \sqrt{\frac{\pi}{2}}\left[-\frac{\tilde{S}}{w^4} + \frac{3}{8w^3}\right]\delta\alpha_1 + \left[-\frac{3}{w^4} + \sqrt{\frac{\pi}{2}}\left(-\frac{2\tilde{S}}{w^4} + \frac{5}{8w^3}\right)\right]\delta\beta_1. \tag{15}$$

Substituting $e^{i\omega t}$ type solution and solving the system, the following excitation frequencies are obtained

$$\omega_+^2 = \frac{3}{w^4} + \sqrt{\frac{\pi}{2}}\left(\frac{3\tilde{S}}{w^4} - \frac{1}{w^3}\right), \tag{16}$$

$$\omega_-^2 = \frac{3}{w^4} + \sqrt{\frac{\pi}{2}}\left(\frac{\tilde{S}}{w^4} - \frac{1}{4w^3}\right). \tag{17}$$

The excitation spectrum is plotted in Fig. 2 as a function of the scattering parameter. We observe that, in contrast to 3D, the 2D system can bear the negative scattering parameter. There is no collapse in the system in 2D. For large values of scattering parameter, pseudo-potential term dominates the gravitational energy and the monopole mode becomes above the quadrupole mode. Thus, we conclude that the 2D systems are not sensitive for negative scattering parameters. At $S = 9.42$, there is an intersection of the two modes which can be seen from the inset of Fig. 2.

TF-G regime When the gravity-like potential is balanced by contact interaction, i.e., for a large s-wave scattering length, the kinetic and harmonic potential energy can be neglected. The ground-state energy is $E_0 = -0.62(N^2u^2/g)$ showing that energy per particle varies as N. In this regime the monopole and the quadrupole frequencies are obtained as $\omega_M = 2.1867\omega_g/S^{3/2}$ and $\omega_Q = 1.5462\omega_g/S^{3/2}$, respectively. Their ratio is $\omega_Q/\omega_M = 0.7$.

G regime In this regime we neglect the trap potential and contact interaction. This is the analog of the nonrelativistic boson star. The ground-state energy per particle varies as N similar to the TF-G regime. The quadrupole mode frequency is $\omega_Q = 0.6269\omega_g$ and monopole mode frequency is $\omega_M = 0.3927\omega_g$. Their ratio is $\omega_Q/\omega_M = 1.58$ in this regime.

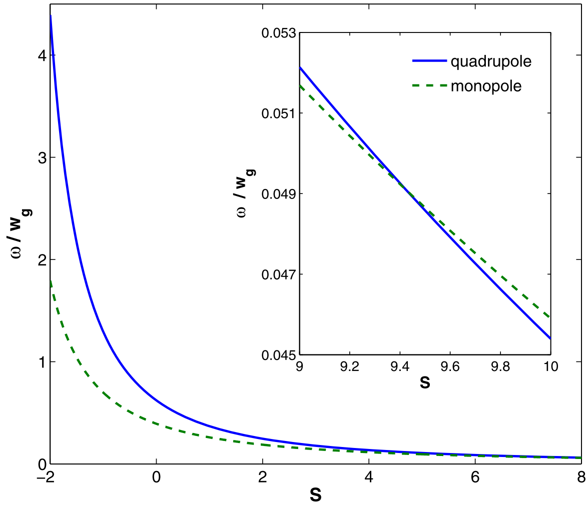

Fig. 2 (Color online) The monopole and quadrupole mode frequencies as a function of S. The inset shows the intersect of two modes

4 Conclusions

In this paper we show that the laser-induced attractive $1/r$ interaction gives rise to a stable condensate in 2D as in 3D without a trap. In contrast to the 3D case there is no collapse for any value of the scattering parameter. We calculated quantities such as release energy, peak velocity and condensate radius for I, TF-O, TF-G, and G regions. We also study the dynamics of the system and calculated the monopole and quadrupole frequencies. For TF-G and G regimes, we calculate the ground-state energy, monopole and quadrupole modes. These modes depend on scattering length a in the TF-G regime unlike the ordinary TF regime. We show that the monopole mode exists for negative values of S unlike the situation in 3D [4].

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