COMPARISON OF THE CAF-DF AND SAGE ALGORITHMS IN MULTIPATH CHANNEL PARAMETER ESTIMATION

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ABSTRACT

In this paper, performance of the recently proposed Cross Ambiguity Function – Direction Finding (CAF-DF) technique is compared with the Space Alternating Generalized Expectation Maximization (SAGE) technique. The CAF-DF, iteratively estimates direction of arrival (DOA), time-delay, Doppler shift and amplitude corresponding to each impinging signal onto an antenna array by utilizing the cross ambiguity function. On synthetic signals, based on Monte Carlo trials, performances of the algoritms are tested in terms of root Mean Squared Error (rMSE) at different Signal-to-Noise Ratios (SNR). Cramer-Rao lower bound is included for statistical comparisons. Simulation results indicate the superior performance of the CAF-DF technique over SAGE technique for low and medium SNR values.

1. INTRODUCTION

The next generation mobile radio communication systems are faced with the ever increasing demand for higher communication rates. In order to meet this demand, the communication systems should obtain an accurate model for the communication channel. For fuller utilization of multipath communication channels, communication systems utilize antenna arrays and related signal processing techniques to produce estimates for communication channel parameters such as DOA of each path, their time-delays, Doppler shifts and amplitudes. Various array signal processing techniques are proposed for reliable and accurate estimation for these channel parameters [1]. The proposed algorithms can be categorized as spectral estimation, parametric subspace-based estimation and deterministic parametric estimation techniques. The well known MUltiple SIgnal Classification (MUSIC) algorithm is in the category of spectral estimation techniques [2]. The ESPRIT technique is a good example for the second category [3]. The Expectation Maximization (EM) algorithm is in the category of deterministic parametric estimation techniques [4]. The SAGE algorithm, which is an extention of the EM, jointly estimates DOA, time-delay and Doppler shift of impinging signals [5,6]. The recently proposed CAF-DF technique, which is in the same category as the SAGE technique, also provides estimates for DOA, time-delay and Doppler shift of impinging signals [7]. In this paper, a comparative performance study of the CAF-DF and SAGE techniques in a multipath environment is presented. For further comparison, performance curves for the MUSIC algorithm and the Cramer-Rao lower bounds are also reported.

2. SIGNAL AND CHANNEL MODEL

Communication systems typically employ transmission of training sequences to obtain reliable estimates for the channel parameters of interest. Transmitted signal, $b(t)$, for a training sequence of length q, can be written as:

$$b(t) = \sum_{k=1}^{q} p(t-(k-1)T) \quad , \tag{1}$$

where $p(t)$ is the pulse waveform of duration T_p which is less than the pulse repetition interval T. As shown in Fig. 1, delayed, Doppler shifted and attenuated copies of the transmitted signal from a transmitter (TX) impinge on an M element receiver antenna array from d different paths.

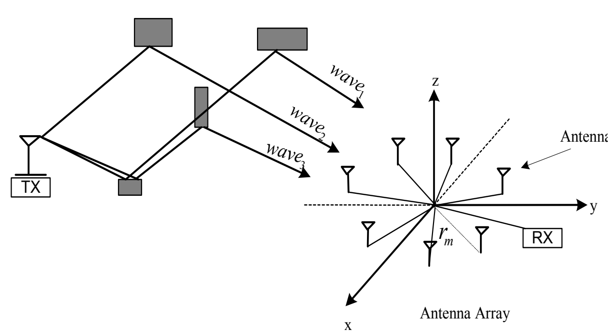

Fig. 1. An illustration of receiver antenna array intercepting multipath signals from a transmitter.

In such an environment, baseband signal on the m^{th} antenna can be modeled as:

$$y_m(t) = \sum_{i=1}^{d} x_{m,i}(t) \quad , \tag{2}$$

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where $x_{m,i}(t)$ are the individual multipath signals that are modeled as:

$$\begin{aligned} x_{m,i}(t) &= s_{m,i}(t) + n_{m,i}(t) \\ &= \zeta_{m,i} b(t-\tau_{o,i}) e^{j2\pi v_i t} e^{-j2\pi v_c \xi_{m,i}(\theta,\phi)} + n_{m,i}(t) \end{aligned} \quad . \tag{3}$$

Furthermore, the receiver array output can be given as:

$$\boldsymbol{y}(t) = \sum_{i=1}^{d} \boldsymbol{a}(\theta_i,\phi_i) b(t-\tau_{o,i}) e^{j2\pi v_i t} + \boldsymbol{n}_i(t) \quad . \tag{4}$$

In these equations, d is the number of signal paths, m is the antenna index, $\zeta_{m,i}$ includes the phase shift and attenuation of the i^{th} signal path on the m^{th} antenna, $\tau_{o,i}$ is the time-delay of the i^{th} signal path with respect to the origin of the array, v_i is the Doppler shift of the i^{th} signal path, v_c is the carrier frequency, θ and ϕ represents the elevation and azimuth angles respectively, $\xi_{m,i}(\theta,\phi)$ is the relative phase difference with respect to the array origin on m^{th} antenna, $\boldsymbol{a}(\theta_i,\phi_i)$ is the steering vector and $n_{m,i}(t)$ is the circularly symmetric complex Gaussian noise. In this work it is assumed that the bandwidth of the $b(t)$ is much smaller than the carrier frequency, justifying the use of narrowband approximations.

3. CAF-DF ALGORITHM

The recently proposed CAF-DF technique makes use of the cross-ambiguity function (CAF), which is commonly used for the delay and Doppler estimation in radar applications [8]. In order to obtain initial estimates for the delay and Doppler shifts of individual multipath components, the CAF of each antenna output and the known transmitted training signal is calculated as:

$$\chi_{y_m,b}(\tau,v) = \int_{-\infty}^{\infty} y_m\left(t+\frac{\tau}{2}\right) b^*\left(t-\frac{\tau}{2}\right) e^{-j2\pi v t} dt \quad . \tag{5}$$

The $\left|\chi_{y_m,b}(\tau,v)\right|$ can be used to detect the presence of incoming signals and their respective delays and Doppler shifts. Since, antennas are typically spaced at a fraction of the wavelength corresponding to the carrier frequency of communication, their separation is very small relative to the TX-RX distance. Therefore, time-delay and Doppler shift of individual multipath components are almost the same on each antenna. To obtain more reliable initial delay Doppler estimates in the absence of DOA information, an incoherent integration of the absolute values of each CAF surfaces is computed as:

$$\chi_{incoh} = \left|\chi_{y_1,b}\right| + \left|\chi_{y_2,b}\right| + \cdots\cdots + \left|\chi_{y_M,b}\right| \quad . \tag{6}$$

Compared to individual CAF surfaces at each antenna output, detection of peaks on the $\chi_{incoh}(\tau,v)$ yields significantly improved results. Let (τ_p,v_p) be the coordinates of the highest peak on the $\chi_{incoh}(\tau,v)$ surface. To obtain an estimate for the DOA of this path, with delay and Doppler shift is initially estimated as (τ_p,v_p), the following complex valued vector is formed:

$$\boldsymbol{P} = \begin{bmatrix} \chi_{y_1,b}(\tau_p,v_p) \\ \chi_{y_2,b}(\tau_p,v_p) \\ \vdots \\ \chi_{y_M,b}(\tau_p,v_p) \end{bmatrix} = \begin{bmatrix} \left|\chi_{y_1,b}(\tau_p,v_p)\right| e^{j\psi_1} \\ \left|\chi_{y_2,b}(\tau_p,v_p)\right| e^{j\psi 2} \\ \vdots \\ \left|\chi_{y_M,b}(\tau_p,v_p)\right| e^{j\psi_M} \end{bmatrix} \quad . \tag{7}$$

In this vector, $\psi_{\boldsymbol{m}}$ is the phase of $\chi_{y_m,b}(\tau,v)$ at the detected peak location (τ_p,v_p). The DOA of the signal, whose time-delay and Doppler shift is initially estimated as (τ_p,v_p), can be estimated as:

$$(\hat{\theta},\hat{\phi}) = \underset{\theta,\phi}{\arg\max}\left(1-\frac{\left|P^H S(\theta,\phi)\right|}{\|P\|}\right)^{-1} \quad , \tag{8}$$

where $\boldsymbol{S}(\theta,\phi)$ includes the phase shift of each antenna with respect to the array origin:

$$\boldsymbol{S}(\theta,\phi) = \frac{1}{\sqrt{M}}\left[e^{j\xi_{1,i}(\theta,\phi)}, e^{j\xi_{2,i}(\theta,\phi)}, \cdots, e^{j\xi_{M,i}(\theta,\phi)}\right]^T \quad . \tag{9}$$

To perform the required maximization, a grid search can be conducted on θ-ϕ plane where the grid spacing is chosen in accordance with the resolution limitations in azimuth and elevation directions. Once $(\hat{\theta},\hat{\phi})$ be obtained as the estimated DOA corresponding to the detected signal path on the $\chi_{incoh}(\tau,v)$ surface, the initial delay and Doppler estimates of this path can be updated. For this purpose, by using the obtained DOA estimate, CAFs on each antenna output are coherently integrated as:

$$\chi_{coh}(\tau,v) = \chi_{y1,b}(\tau,v) e^{j2\pi v_c \xi_{1,i}(\hat{\theta},\hat{\phi})} + \cdots + \chi_{yM,b}(\tau,v) e^{j2\pi v_c \xi_{M,i}(\hat{\theta},\hat{\phi})} \quad . \tag{10}$$

The initial delay and Doppler shift estimates can be updated with the coordinates of the peak on the $\chi_{coh}(\tau,v)$ in the vicinity of the initial estimates (τ_p,v_p).

Using the estimated multipath parameters, received signal on the m^{th} antenna can be written as,

$$s_{m,i}(t) = \zeta_{m,i} b(t-\hat{\tau}_{o,i}) e^{j2\pi \hat{v}_i t} e^{-j2\pi v_c \xi_{m,i}(\hat{\theta},\hat{\phi})} \qquad m=1,2,\ldots,M \quad . \tag{11}$$

Depending on the accuracy on the calibration of the antennas, $\zeta_{m,i}$ may be very close or significantly different for each antenna. In the case of low quality calibration, the individual $\zeta_{m,i}$ can be estimated as the minimizer of the following cost function:

$$J_m(\zeta_{m,i}) = \int_0^T \left|y_m(t) - s_{m,i}(t)\right|^2 dt \quad . \tag{12}$$

Optimal estimate for $\zeta_{m,i}$ can be easily obtained as:

$$\hat{\zeta}_{m,i} = \frac{\int_0^T s^*(t-\hat{\tau}_{0,i}) e^{-j2\pi \hat{v}_i t} e^{j2\pi v_c \xi_{m,i}(\hat{\theta},\hat{\phi})} y_m(t) dt}{\int_0^T s^*(t-\hat{\tau}_{0,i}) s(t-\hat{\tau}_{0,i}) dt} \quad . \tag{13}$$

A copy of the multipath component can be generated by using (11) with the estimated path parameters. The generated copy of the multipath component is eliminated from each antenna output. Then CAF-DF technique starts a new iteration to detect and estimate path parameters of another multipath component. This iterative detection, estimation and elimination phases are concluded once there is no significant peak left above the noise floor of the incoherent detection surface.

4. SAGE ALGORITHM

The SAGE algorithm is an alternative version of the classical EM algorithm [4]. The use of the SAGE in multipath channel parameter estimation is presented firstly in [6]. Each iteration of the algorithm contain EM iteration phase where some of the parameters are fixed at the previous iteration values, while other parameters are re-estimated. Instead of simultaneous parameter estimation, parameters are estimated sequentially. In order to reduce the complexity of the algorithm, suboptimal but faster one dimensional optimization procedures along each parameter are used.

SAGE algorithm can be divided into two parts namely; expectation and maximization phases. First phase starts with forming what is known as "the complete information" as :

$$\hat{x}_{m,i}(t) = s_{m,i}(t) + y_m(t) - \sum_{i'=1}^{d} s_{m,i'}(t) \quad . \tag{14}$$

Once the complete information is formed, the maximization phase takes place to yield a new set of parameter estimates for each multipath component:

$$\hat{\tau}_i'' = \arg\max_{\tau} \left\{ \left| g(\tau,(\hat{\theta}_i',\hat{\phi}_i'),\hat{\nu}_i';\hat{x}_i(t)) \right| \right\} \quad , \tag{15}$$

$$\left(\hat{\theta}_i'',\hat{\phi}_i''\right) = \arg\max_{\theta,\phi} \left\{ \left| g(\hat{\tau}_i'',(\theta,\phi),\hat{\nu}_i';\hat{x}_i(t)) \right| \right\} \quad , \tag{16}$$

$$\hat{\nu}_i'' = \arg\max_{\nu} \left\{ \left| g(\hat{\tau}_i'',(\hat{\theta}_i'',\hat{\phi}_i''),\nu;\hat{x}_i(t)) \right| \right\} \quad , \tag{17}$$

$$\hat{\zeta}_i'' = \frac{1}{\left(b(t)\times b(t)^*\right)\left\| a(\hat{\theta}_i'',\hat{\phi}_i'')\right\|^2} g(\hat{\tau}_i'',(\hat{\theta}_i'',\hat{\phi}_i''),\hat{\nu}_i'';\hat{x}_i(t)) \quad . \tag{18}$$

In these equations $g(\tau,\phi,\nu;x_i)$ is defined as

$$g(\tau,(\theta,\phi),\nu;x_i(t)) \triangleq \sum_{m=1}^{M} \int_{-\infty}^{\infty} b^*(t-\tau)\, x_{m,i}(t)\, e^{-j2\pi\nu t} e^{j\xi_m(\theta,\phi)} dt \quad . \tag{19}$$

There are various methods to initialize the algorithm. One can use MUSIC algorithm to provide initial time-delay values and then for the remaining signal parameters initialization iterations of the SAGE can be used [6]. In this paper, a different initialization procedure is preferred [6]. Since, initially, phase of the complex amplitudes $\zeta_{m,i}$ are not known, time-delays and DOAs are estimated incoherently. For this purpose, in the initialization part, maximization procedures used for time-delay and DOA estimations given in (15) and (16) are changed with the equations below.

$$\hat{\tau}_i'' = \arg\max_{\tau} \left\{ \sum_{m=1}^{M} \left| \int_{-\infty}^{\infty} b^*(t-\tau)\, \hat{x}_{m,i}(t)\, dt \right|^2 \right\} \quad . \tag{20}$$

$$\left(\hat{\theta}_i'',\hat{\phi}_i''\right) = \arg\max_{\theta,\phi} \left\{ \sum_{m=1}^{M} \left| \int_{-\infty}^{\infty} b^*(t-\tau)\, e^{j\xi_{m,i}(\theta,\phi)} \hat{x}_{m,i}(t)\, dt \right|^2 \right\} \quad . \tag{21}$$

As seen from the equations above, signal estimates of the waves with initialized parameters are subtracted from observed data $y(t)$. Parameter update procedure is continued until there is no improvement in the sense of rMSE between consecutive iterations.

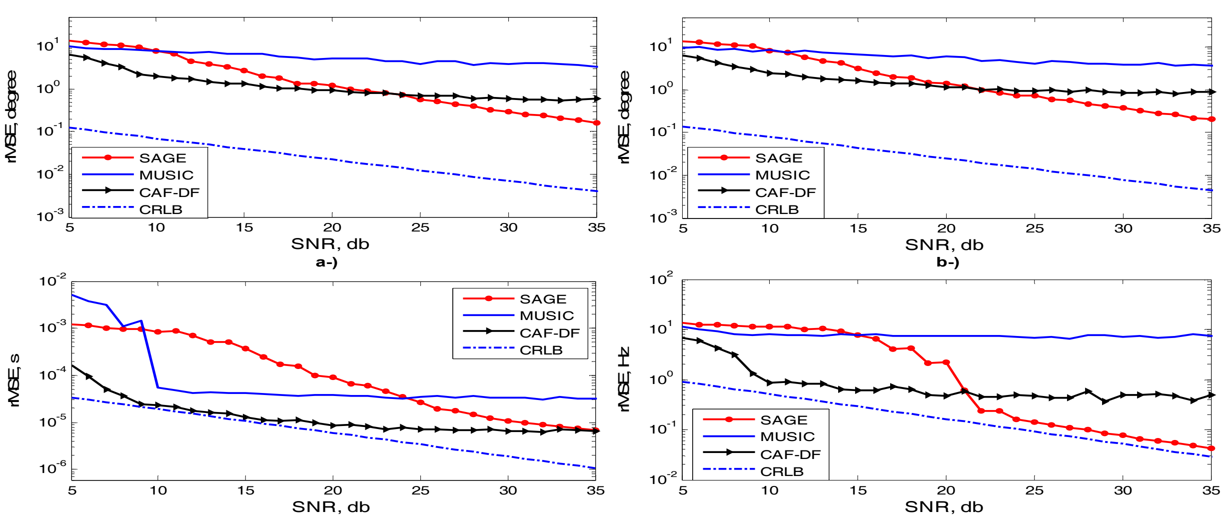

Fig. 2. The rMSE of estimates vs SNR in a)azimuth, b)elevation, c)delay and d)Doppler shift of CAF-DF, SAGE and MUSIC techniques for the 1st multipath component. Dashed line represents the unbiased Cramer-Rao lower bound.

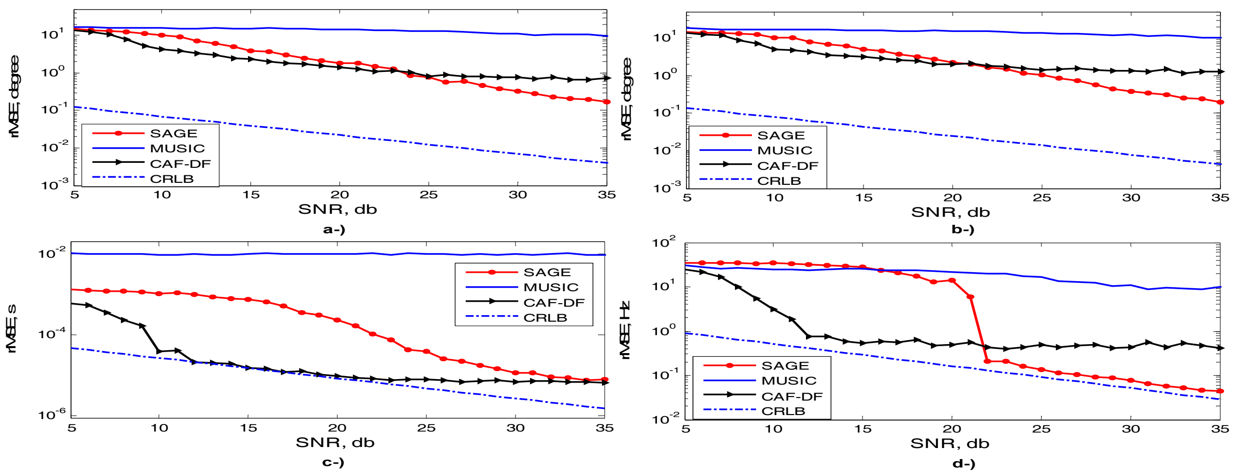

Fig. 3. The rMSE of estimates vs SNR in a)azimuth, b)elevation, c)delay and d)Doppler shift of CAF-DF, SAGE and MUSIC techniques for the 2nd multipath component. Dashed line represents the unbiased Cramer-Rao lower bound.

5. COMPARISON OF THE CAF-DF, SAGE AND MUSIC ALGORITHMS

In this part, performances of the CAF-DF, SAGE and MUSIC algorithms are compared on synthetic signals using Monte Carlo simulations. A circular receiver array with six antennas is modeled. In order to prevent spatial aliasing, distance between each antenna is selected smaller than the half of the signal wavelength. Transmitted training signal consists of 6 Barker-13 coded pulses with 7.8 ms duration. The pulse repetition interval is 18 ms resulting a total signal duration of 108 ms. Using (2), two multipath signal components having path parameters in a vectorized form as: $\tau = [0.6\,;0.6]\,ms$, $\nu = [8\,;22]\,Hz$, $\theta = [190\,;188]^{\,o}$, $\phi = [32\,;34]^{\,o}$ are created. $\zeta_{m,i}$ of each signal is chosen such that its magnitude is 1 and its phase is uniformly distributed in $[0{,}2\pi]$. Time-delay, Doppler shift and DOA estimates of each algorithm are estimated based on 300 Monte Carlo trials conducted at various SNR. Obtained rMSE of the first and second multipath channel parameters are presented in Fig. 2 and Fig. 3, respectively. Obtained results show that both the CAF-DF and the SAGE techniques provide significantly better parameter estimates than the MUSIC technique. Furthermore, the CAF-DF technique provides more reliable estimates than the SAGE technique for a wide range of SNR values extending from medium to low SNR values. Finally, the SAGE technique outperforms the CAF-DF at high SNR values.

6. CONCLUSIONS

Recently proposed CAF-DF technique for the estimation of multipath channel parameters is compared with the commonly used the SAGE and the MUSIC techniques. Obtained results show that CAF-DF provides better parameter estimates in terms of rMSE than the MUSIC technique. More importantly, the CAF-DF technique is found to be superior to the SAGE technique over a wide range of SNR values extending from medium to low. Since this range of SNR values creates the most challenging situations in practice, it can be concluded that the CAF-DF technique serves well in a wide range of applications. Our current research effort is focused on developing a hybrid algorithm where the SAGE technique is initialized with the estimated multipath channel parameters provided by the CAF-DF technique. The hybrid technique is expected to be superior to both the CAF-DF and the SAGE techniques at all SNR values.

11. REFERENCES

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