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Note on "The Backroom Effect in Retail Operations"

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Eroglu et al. (2013) study a retailer with limited shelf capacity and a backroom. They study a continuous review (r, q) ordering policy with a known order quantity, q. Assuming that backorders can be satisfied from the backroom inventory (if available), they find the expression for the optimal reorder level, r. Our work builds on Eroglu et al. (2013). We correct an erroneous derivation of the expected overflow term, as well as derive an exact expression for the expected cost function, and hence optimal reorder level, instead of the approximate one used by Eroglu et al. (2013).

Key words: retail operations; backroom; (r, q) policy
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1. Introduction

We consider a retailer with expected annual demand of D units. Every time the inventory drops to level $r \geq 0$, a replenishment order of size q, which arrives exactly after L time units, is placed. The demand during lead time, LTD, is a random variable with a continuous and differentiable cumulative distribution function, $F(\cdot)$. The corresponding density function is $f(\cdot)$. Consistent with Hadley and Whitin (1963), we assume that there is never more than a single outstanding order, although our results will hold for arbitrary number of outstanding orders under backordering regime.

Inventory can be stored at two locations: a shelf and a backroom. Upon arrival of a replenishment order, initially the shelf is filled to its capacity, c, and items that do not fit on the shelf are stored in the backroom. Therefore, if the total number of items in the system upon arrival of the replenishment order is $u \geq c$, $u - c$ items are stored in the backroom. We refer to the total number of items in the backroom immediately after the allocation of the replenishment order as *overflow*. Customer demands are first satisfied from the shelf. After depletion of the shelf inventory, backroom inventory is used for demand satisfaction.

The fixed cost of placing an order is a. Per unit purchase cost is v. The unit holding cost per year is h. The penalty cost is b per unit backordered. Backordering cost does not depend on the length of the time for which the backorder exists. The retailer incurs a cost of k for each item that does not fit on the shelf at the time of replenishment. Our objective is to find the reorder level that minimizes the expected annual cost.

2. Analysis

Given a realization of the lead time demand x, the net inventory just before the arrival of the replenishment order is $r - x$. The total net inventory after q units arrive becomes $r - x + q$. Note that if the initial net inventory is negative, that is, if we have backorders, some of the incoming inventories will be used for clearing the backorders. Thus, the overflow becomes $(r - x + q - c)^+$, where $(z)^+$ equals z for $z \geq 0$ and 0 otherwise. The expected overflow, w, is 0 if $q \leq c - r$ and $w = \int_0^{r+q-c} F(x)dx$ otherwise. In fact, the error in Eroglu et al. (2013) is in the expression of w. They make a mistake by implicitly treating the model as lost sales even though they assume that unsatisfied customer demands are backordered. The expected annual cost can be written as follows

$$TC(r|q,c) = vD + a\frac{D}{q} + h\left(\frac{q}{2} + r - E[LTD]\right) + \left(h + b\frac{D}{q}\right)\int_r^{\infty}(x-r)f(x)dx + kw\frac{D}{q}, \tag{1}$$

where the first term in the annual purchasing cost, the second term is the ordering cost, the third term is the holding cost, the fourth term is the backordering cost and the final term is the overflow cost. $TC(r|q, c)$ is convex in r and the expressions for the optimal reorder level, r^*, are as in Table 1.

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Table 1 Optimal Reorder Level

$q \le c$		$q > c$	
r^*	Condition	r^*	Condition
$F^{-1}\left(\frac{bD}{hq+bD}\right)$	$F(c - q) \ge \frac{bD}{hq+bD}$	0	$F(q - c) \ge \frac{b}{k}$
$F^{-1}\left(\frac{bD-kDF(r^*+q-c)}{hq+bD}\right)$	$F(c - q) < \frac{bD}{hq+bD}$	$F^{-1}\left(\frac{bD-kDF(r^*+q-c)}{hq+bD}\right)$	$F(q - c) < \frac{b}{k}$

3. Numerical Analysis

The error in Eroglu et al. (2013) influences the case with $q > c$ only. In this section, we investigate the cost benefit of using r^* in Table 1 instead of the value suggested by Eroglu et al. (2013) for this case. We fix the values of D, h and v to 10, 1, and 1, respectively. Consistent with Eroglu et al. (2013) the lead time demand is assumed to have a Gamma distribution with three different (α,β) = (shape,scale) combinations: (1,2), (2,2), and (4,2). The rest of the parameters take values from the following sets: $q \in \{8,10,12\}$, $c \in \{1,2,4,8\}$, $a \in \{1,5,10\}$, $b \in \{1,1.5,2\}$, and $k \in \{1,5,10\}$. Given that r^E is the reorder level suggested by Eroglu et al. (2013), we calculate the percentage cost differences by $\Delta = \frac{TC(r^E|q,c)-TC(r^*|q,c)}{TC(r^*|q,c)}100\%$. In addition, excluding the components vD and $a\frac{D}{q}$, which do not depend on the reorder level, we calculate the *relevant* percentage cost difference by $\Delta^{Rel} = \frac{TC(r^E|q,c)-TC(r^*|q,c)}{TC(r^*|q,c)-vD-a\frac{D}{q}}100\%$. The results are summarized in the first part of Table 2.

We observe that Δ and Δ^{Rel} increase as the average demand during the lead time decreases and k, q, and c decrease. Based on these observations, we construct a (partial) worst-case bound for Δ^{Rel} by studying the parameter values, where Eroglu et al. (2013) set r^E to 0; more specifically a worst-case bound for the problems, where $q > c$ and $\frac{hq}{bD} \ge 1$. By setting b to $\frac{hq}{D}$, c to 0 and k to 0, we obtain a worst-case bound that depends only on the values of q and the distribution of the lead time demand. The results are in the second part of Table 2. Note that Δ^{Rel} can be quite significant especially for the lead time demand distributions with low coefficient of variations (CV).

Table 2 Summary Statistics and Worst-Case Bounds

Statistics		Δ		Δ^{Rel}
Average		2.82		7.58
SD		4.04		10.58
Maximum		25.01		58.55
(α,β)	$CV \diagdown q$	8	10	12
(1,2)	1	11.39	9.61	8.31
(2,2)	0.71	31.07	26.70	23.40
(4,2)	0.5	69.45	60.86	54.16

4. Conclusions

The model studied in this note needs to be extended to mimic the real-world retailer operations more closely. A cost accounting scheme that takes the fixed cost of operating the backroom and the additional handling cost of moving the items from the backroom to the shelf into account needs to be adapted. A more realistic setup should study the lost sales assumption and the periodic nature of the shelf replenishment process from the backroom.

References

Eroglu, C., B. D. Williams, M. A. Waller. 2013. The backroom effect in retail operations. *Prod. Oper. Manag.* **22**(4): 915–923.

Hadley, G., T.M. Whitin. 1963. *Analysis of Inventory Systems*. Prentice-Hall, Inc., Englewood Cliffs, NJ.