PHYSICAL REVIEW B VOLUME 42, NUMBER 14 15 NOVEMBER 1990-I

Ballistic transport through a quantum point contact: Elastic scattering by impurities

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(Received 5 March 1990; revised manuscript received 15 May 1990)

The effects of elastic scattering due to impurities in a quasi-one-dimensional constriction are investigated with an exact calculation of the conductance. It is found that the quantization of conductance is distorted owing to scattering by a single impurity which exists in an infinite constriction. The extent of deviation from quantized values depends on the strength, position, and lateral range of the scattering potential. The resonance structure due to interference of current-carrying waves is still apparent for a constriction of finite length containing an impurity. However, both the magnitude and position of these oscillations in the resonance structure are affected as a result of elastic scattering. A resonant tunneling effect is found due to a state bound to the attractive impurity potential.

I. INTRODUCTION

Using high-mobility two-dimensional (2D) electron gas (EG) and split-gate structures, van Wees *et al.*[1] and Wharam *et al.*[2] fabricated quantum point contacts (QPC) with length scales smaller than the electron mean free path and comparable with the Fermi wavelength λ_F. They observed that the two-terminal conductance of the QPC is quantized in units of $2e^2/h$ as a function of the width of the constriction w. Recently, assuming that the transport is ballistic, several groups[3-5] developed theories to explain the quantization of conductance. Furthermore, they predicted resonances superimposed on the quantized plateaus. The deviations from exact quantization and the lack of the resonance structure in the experimental results[1,2] have become important issues, and were attributed to various effects. For example, Glazman *et al.*[6] showed that the current-carrying states evolve adiabatically without reflection and without intersubband scattering in certain hornlike connections to the reservoirs. The authors,[7] on the other hand, showed that sharply quantized conductance $G(w)$ devoid of resonance structure can occur even if the adiabaticity requirements are not satisfied for certain QPC geometries. The elastic scattering by impurities in a ballistic channel can also affect the above-mentioned quantization of conductance and the resonance structure. Earlier works[4,8,9] have indicated such a possibility, but a clear understanding of impurity effects on experimentally relevant systems is not fully developed yet. Therefore, scattering by an impurity in a QPC has several interesting features which deserve further study.

In this paper we investigate the effects of elastic scattering by an impurity in a ballistic channel. Using a Green's-function technique, we obtain the expression for conductance for an infinite quasi-1D constriction with a single impurity represented by a model potential. The form of the model potential is realistic and enables us to obtain exact solutions for scattering events. Moreover, it is appropriate to carry out a systematic analysis on the effects of the position and lateral extent of the impurity. The formalism developed for an infinite constriction is further extended to treat a finite-length QPC with a single impurity. Our results are in overall agreement with the results of the earlier studies,[4,8,9] which were obtained by using completely different approaches. Present study investigates several aspects of scattering by an impurity in a QPC (which were not treated earlier) by using more realistic scattering potentials and boundary conditions. In Sec. II we describe the method of calculation and introduce the model potential. A critical comparison of our method with the earlier ones is also presented in this section. In Sec. III we present the results obtained by using this formalism for the infinite and finite constrictions and discuss the similarities and differences with those of the earlier ones. Important aspects of our study are stressed by way of conclusions in Sec. IV.

II. METHOD

We first consider an infinite constriction, for which z is the propagation direction and y is the transverse direction as described by the inset in Fig. 1. We also assume that the confinement in the x direction is complete. The eigenstates for such a uniform quasi-1D constriction (electron wave guide) in the presence of a scattering potential $v_I(y,z)$ can be written as

$$\psi_j(y,z)=e^{i\gamma_j z}\phi_j(y)+\int dy'\int dz'\, g(z-z';y,y') \times v_I(y',z')\psi_j(y',z') \ . \qquad (1)$$

The first term on the right-hand side represents the incident wave, which is the unperturbed solution for the jth subband with the wave function $\phi_j(y)$, the eigenenergy ε_j, and the corresponding wave vector $\gamma_j^2=2m^*(E-\varepsilon_j)/\hbar^2$ along the z direction. Details for the unperturbed solutions for the current-carrying states and the variation of conductance calculated thereof for the uniform and tapered quasi-1D constrictions can be found in Refs. 5 and 7. The above expression in Eq. (1) is the well-known Lippmann-Schwinger equation adapted to quasi-1D systems with the retarded Green's function g. The exact solution of Eq. (1) can be written using the t operator as

$$\psi_j(y,z)=e^{i\gamma_j z}\phi_j(y)+\sum_n \phi_n(y)\int dk\, e^{ikz}\tilde{\mathbf{G}}_n(k)\tilde{\mathbf{T}}_{nj}(k)\ , \qquad (2)$$

where $\tilde{\mathbf{T}}_{nj}(k)$ is given by

$$\tilde{\mathbf{T}}_{nj}(k)=\tilde{\mathbf{V}}_{nj}(k-\gamma_j)+\sum_m \int dk'\tilde{\mathbf{V}}_{nm}(k-k')\tilde{\mathbf{G}}_m(k')\tilde{\mathbf{T}}_{mj}(k')\ . \qquad (3)$$

Note that the Fourier transforms of g, v_I, and t are matrices $\tilde{\mathbf{G}}$ (diagonal), $\tilde{\mathbf{V}}$, and $\tilde{\mathbf{T}}$, respectively. An element of such matrices are calculated from the integral described by the following expression:

$$\tilde{\mathbf{F}}_{nm}(k)=\int dy\, \phi_n(y)\phi_m(y)\int dz\, e^{-ikz}f(y,z)\ . \qquad (4)$$

By solving Eq. (3) for $\tilde{\mathbf{T}}$, one obtains the solution for the scattering problem for a right-going incident wave in the jth subband. The solution $\bar{\psi}_j(y,z)$ is found similarly for a left-going incident wave.

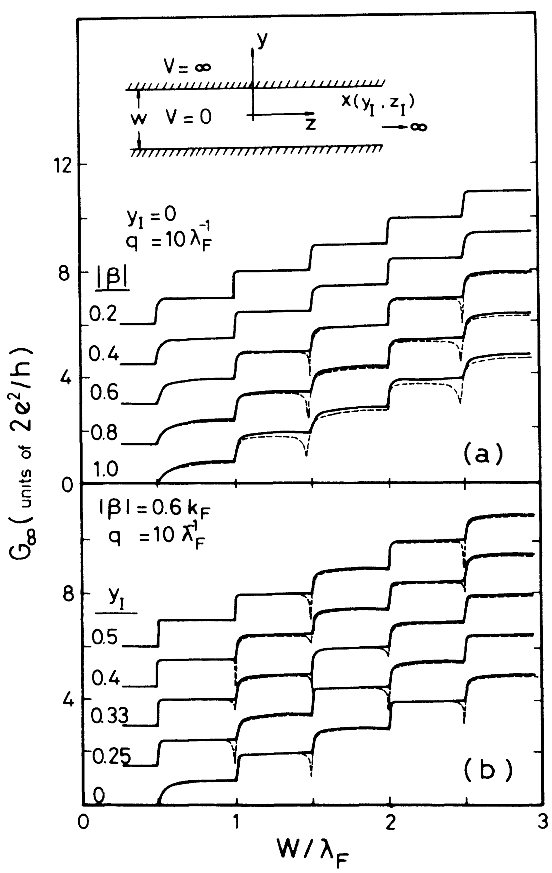

FIG. 1. The conductance G_∞ vs the width w of an infinite constriction containing an impurity. (a) $y_I=0$ and $q=10\lambda_F^{-1}$, the strength $|\beta|$ varying (in units of k_F). (b) $|\beta|=0.6k_F$ and $q=10\lambda_F^{-1}$, the position y_I varying (in units of λ_F). Solid (dashed) curves correspond to repulsive (attractive) impurities, and are vertically offset by an amount $1.5\times(2e^2/h)$ for clarity. The geometry of the channel is described by the inset.

It is important to note that in the present study we calculate the conductance of the constriction by using a two-terminal geometry. That is, two reservoirs are connected to the ends of the constriction (or the 2D EG for the finite constriction) so that the voltage difference between the reservoirs is just the difference of the electrochemical potential deep in the reservoirs (which is taken to be infinitesimal). The conductance $G_\infty(w)$ of an infinite constriction is then calculated from the expectation value of the momentum operator,

$$G_\infty(w)=\frac{2e^2}{h}\sum_j^{\rm occ}\frac{1}{\hbar\gamma_j}\langle\psi_j|\hat{p}_z|\psi_j\rangle\ . \qquad (5)$$

The solution of Eq. (3) for a general potential $v_I(y,z)$ is complicated and may require extensive computations. In order to obtain an analytical solution which leads to a clear picture of the effects of elastic scattering, we use the following model potential for a scatterer located at (y_I,z_I):

$$v_I(y,z)=\frac{\hbar^2\beta}{m^*}\exp(-q|y-y_I|)\delta(z-z_I)\ , \qquad (6)$$

which is a Dirac δ function in the z direction, and has the exponentially decaying form in the y direction with a decay length of q^{-1}. The strength of this potential is set by the magnitude of β, which may be both attractive ($\beta<0$) and repulsive ($\beta>0$). For this form of the potential, Eq. (3) is exactly solvable and the $\tilde{\mathbf{T}}$ matrix is given by

$$\tilde{\mathbf{T}}_{mj}(k)=\frac{1}{\sqrt{2\pi}}e^{-i(k-\gamma_j)z_I}\tilde{\mathbf{\Omega}}_{mj}\ , \qquad (7)$$

where $\tilde{\mathbf{\Omega}}=\tilde{\mathbf{u}}(\tilde{\mathbf{\Gamma}}+i\tilde{\mathbf{u}})^{-1}\tilde{\mathbf{\Gamma}}$ with $\tilde{\mathbf{\Gamma}}_{ij}=\delta_{ij}\gamma_j$ and

$$\tilde{\mathbf{u}}_{ij}=\beta\int dy\, \phi_i(y)\phi_j(y)\exp(-q|y-y_I|)\ . \qquad (8)$$

The conductance for an infinite constriction containing an elastic scatterer as described in Eq. (6) is expressed in terms of these matrices as

$$G_\infty=\frac{2e^2}{h}\sum_{\substack{j\\(\varepsilon_j<E_F)}}\left[1+2\frac{\mathrm{Im}[(\tilde{\mathbf{\Omega}})_{jj}]}{\gamma_j}+\frac{\mathrm{Re}[(\tilde{\mathbf{\Omega}}^\dagger\tilde{\mathbf{\Gamma}}^{-1}\tilde{\mathbf{\Omega}})_{jj}]}{\gamma_j}\right]. \qquad (9)$$

It should be noted that the effect of the evanescent waves with $\varepsilon_j>E_F$ is included in the above formalism of G_∞. This is provided through the intersubband coupling in $\tilde{\mathbf{u}}$ and yields novel effects described in Sec. III. These effects do not exist in strictly 1D systems.

To calculate the conductance for a QPC of finite length d, we furthermore assume that the impurity potential $v_I(y,z)$ is zero outside the QPC region $0\le z\le d$. Thus, the solution of the Schrödinger equation in the 2D EG ($z\le0$ and $z\ge d$) is a linear combination of plane waves, each plane wave being a solution of the 2D EG reservoirs. This assumption simplifies the solution since elastic scattering takes place only in the constriction, and thus the use of Green's function in the 2D EG is not necessary. Note that the model potential in Eq. (6) satisfies this condition if the impurity is located in the constriction (i.e., $0 \leq z_I \leq d$). The solution in the constriction is expressed in terms of ψ_j and $\bar{\psi}_j$ as

$$\Psi(y,z) = \sum_j [\tilde{\mathbf{A}}_j \psi_j(y,z) + \tilde{\mathbf{B}}_j \bar{\psi}_j(y,z)] . \quad (10)$$

The boundary conditions at $z=0$ and $z=d$ are used to find the coefficients $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$. Next we express $\Psi(y,z)$ in terms of a linear combination of exponentials either for $0<z<z_I$ or $z_I<z<d$ as

$$\Psi(y,z) = \sum_j \phi_j(y)(e^{i\gamma_j z}\tilde{\Theta}_j + e^{-i\gamma_j z}\tilde{\Delta}_j) . \quad (11)$$

Finally, the conductance $G_d(w)$ is expressed in terms of these vectors of coefficients $\tilde{\Theta}$ and $\tilde{\Delta}$ as described elsewhere:[5]

$$G_d = \frac{e^2}{\pi h} \int_{-k_F}^{k_F} d\kappa \frac{1}{k_z(\kappa)} [(\tilde{\Theta}^\dagger \mathrm{Re}\{\tilde{\Gamma}\}\tilde{\Theta} - \tilde{\Delta}^\dagger \mathrm{Re}\{\tilde{\Gamma}\}\tilde{\Delta}) + 2\,\mathrm{Im}(\tilde{\Theta}^\dagger \mathrm{Im}\{\tilde{\Gamma}\}\tilde{\Delta})] . \quad (12)$$

In Eq. (12) the coefficients $\tilde{\Theta}$ and $\tilde{\Delta}$ depend on the parameters of the impurity, namely $\tilde{\Omega}$ and z_I, as well as the parameters of the constriction. In the numerical studies presented in Sec. III we used an infinite-well confinement in the transverse direction. Nevertheless, Eqs. (1) and (3) have general validity, and Eqs. (9) and (12) are valid for the impurity potential given by Eq. (6).

At this point it is in order to compare our model with the earlier ones.[4,8,9] Haanapel and van der Marel[4] used the tight-binding method to analyze the effects of an impurity in or near the constriction for short QPC's. They argued that the presence of the impurity in or near the constriction prevents the quantization of conductance. However, since the potential of the impurity was taken as a 2D Dirac δ function, their study was not able to reveal the scattering effects in detail. Recently Chu and Sorbello[8] calculated the conductance of an infinite constriction in the presence of an impurity by using a scattering theoretical formulation. They provided an exact analytic solution for the conductance in terms of phase shifts pointing out interesting features of impurity scattering. However, the applicability of their analysis made by the isotropic (s-like) scatterer in an infinite wave guide is limited for an experimentally relevant system. Masek and co-workers[9] employed the Anderson model to analyze the conductance of a disordered quasi-1D conductor. Their results may be significant for ensemble-averaged effects of impurities. Although their results are in agreement with those obtained by other methods, the microscopic aspects of scattering due to a single impurity cannot be extracted from that study. The present model provides exact and partly analytical solution for the conductance. As seen, the form of the potential and thus the formalism is versatile and enables us to study various parameters such as the position, lateral extent, and strength of the impurity. The weakness of the model potential used in this study is that it is highly anisotropic. Consequently, a direct quantitative comparison with the experimental systems may not be straightforward.

Finally, we comment on the effects of the self-consistent potential and inelastic scattering. Earlier, Landauer[10] argued that self-consistent charge due to nonequilibrium electrons is accumulated near the impurity, which yields corrections to the conductance. This is closely related to the question of which Landauer formula, $G \sim T$ or $G \sim T/R$ (T and R being transmission and reflection probabilities, respectively), has to be applied. An extensive discussion of this issue is beyond the scope of our work, however. Relevant references, which present comprehensive reviews of several efforts and debates, are given in Ref. 10. It becomes clear now that a different Landauer formula applies to different measurement geometry. For the system we are considering, the voltage difference is measured between the reservoirs. As stated above, this is a two-terminal geometry. That is, expressing in terms of relevant Landauer formula,[10] the conductance is given by $G \sim T$ [for multichannel case[11] $\mathrm{Tr}(\tilde{\mathbf{t}}\tilde{\mathbf{t}}^\dagger)$, $\tilde{\mathbf{t}}$ being the transmission matrix]. In the present approach the finite temperature effects are also neglected. Despite this, we think that progress towards a better understanding of elastic scattering in a ballistic channel is made by the present work. Moreover, our findings have close bearings to the resonant tunneling, especially in scanning tunneling microscopy.

III. RESULTS AND DISCUSSION

A. Infinite constriction

The variation of $G_\infty(w)$ for an infinite constriction having a single impurity is shown in Figs. 1 and 2. As seen, the ideal quantization is distorted in the presence of the scatterer. If the potential of the impurity is weak [e.g., $|\beta| \lesssim 0.5k_F$ for $q=10\lambda_F^{-1}$ in Fig. 1(a)], $G_\infty(w)$ still reflects a staircase structure with smoothed steps and with plateaus very close to the quantized values, $2e^2N_c/h$ (N_c being the number of subbands below E_F). Another observation is that for weak scatterers the sign of the potential does not have a pronounced effect on the conductance. This result is in compliance with the first-order Born approximation, since the lowest-order correction to the conductance is proportional to β^2 in the perturbative treatment of the impurity. Therefore, both repulsive ($\beta>0$) and attractive ($\beta<0$) impurities have the same effects on the transport. In order for the Born approximation to be valid, and thus for only a single scattering event to take place, the velocity or equivalently the wave vector of electrons has to be large. In the quasi-1D system under investigation the related wave vector is the propagation constant γ_j and is equal to zero whenever a new subband dips the Fermi level, i.e., $w=N_c\lambda_F/2$. Thus, the Born approximation fails for w values just above $N_c\lambda_F/2$ and it is necessary to include the multiple scattering events.

For relatively stronger impurities [$0.5k_F \lesssim |\beta| \lesssim k_F$ for $q=10\lambda_F^{-1}$ in Fig. 1(a)] not only the steps are smoothed, but also the plateaus exhibit deviations from the quantized values $2e^2N_c/h$. The most remarkable effect observed in this range of β is the difference between the at-

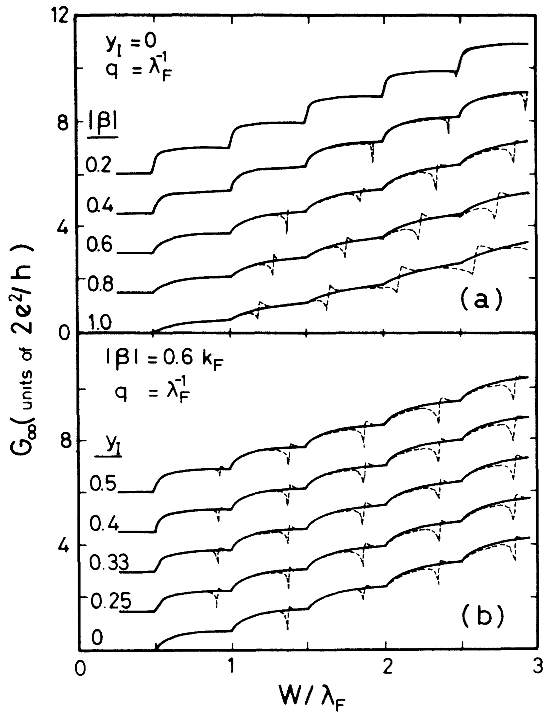

FIG. 2. The conductance G_∞ vs the width w of an infinite constriction containing an impurity. (a) $y_I=0$ and $q=\lambda_F^{-1}$, the strength $|\beta|$ varying (in units of k_F). (b) $|\beta|=0.6k_F$ and $q=\lambda_F^{-1}$, the position y_I varying (in units of λ_F). Solid (dashed) curves correspond to repulsive (attractive) impurities, and are vertically offset by an amount $1.5\times(2e^2/h)$ for clarity.

tractive ($\beta<0$) and repulsive ($\beta>0$) scatters. Also for strong scatterers the Born approximation begins to fail for the whole range of w. As seen, for attractive impurities the dips in the G versus w curves appear below the steps. The conductances at these minima are approximately equal to $2e^2(N_c-1)/h$ for laterally confined impurities (i.e., large q), and there are sharp rises to the next quantized value above these dips. One important point we notice is that dips do not occur below all of the steps. To analyze this, we calculated $G_\infty(w)$ for different transverse positions (y_I) of the impurity as shown in Fig. 1(b), and consider $\tilde{u}_{jj}$ given by Eq. (8). For a laterally confined impurity (i.e., large q), $\tilde{u}_{jj}$ is approximately proportional to $|\phi_j(y_I)|^2$. To a first approximation, the effect of the impurity is large on the N_cth plateau when $|\phi_{N_c}(y_I)|^2$ is a maximum, but is small when it is negligible. For example, for $y_I=0$ the deviations from the quantized values will be large on the odd-numbered plateaus and small on the even-numbered ones. On the other hand, the size, width, and existence of the dips below the N_cth step for the attractive impurities[12] are determined by magnitudes of $\tilde{u}_{iN_c}$ for $i<N_c$. Analyzing these dips in detail we find that they originate from the enhancement of backscattering due to the intersubband scattering.[13] For the strictly 1D problem total backscattering is not allowed since the boundary condition at $z=z_I$ for the derivative of the wave function cannot be satisfied. For the quasi-1D case there are subbands which may be coupled in the presence of the impurity. Therefore, the total backscattering can occur in a subband by inclusion of the evanescent states in a quasi-1D system. Since the first-order Born approximation employs the equivalent 1D problem for each subband, the dips cannot be obtained perturbatively. The backscattering effect is visible in Fig. 2(a). For an impurity positioned at the center of the channel, even- and odd-numbered subbands are completely decoupled. Therefore, there is no dip below the second step, and the dip below the third step is due to enhanced backscattering in the first subband caused by the evanescent third-subband state. In the presence of a large number of impurities,[9] all the subbands are mixed and it is possible to observe dips below all of the steps.

For laterally spread impurity potentials with small q the deviations from the quantized steps [see Fig. 2(a)] are enhanced compared to those with large q. For example, the dips do not have conductance $2e^2(N_c-1)$. This is due to the large integrated strength $\sim\beta/q$. Note that $\tilde{u}$ given in Eq. (8) is determined by this integrated strength and not solely by the strength β. Another observation is that for attractive impurity potentials the dips are shifted to values of w which are smaller than $N_c\lambda_F/2$ and appear together with peaks. Since the impurity potential influences a wide range of the constriction, the wave function evaluated at y_I cannot give an idea about the effect of the scatterers. Although the deviations from the quantized plateaus vary with y_I [Fig. 2(b)], this effect is not as drastic as it was for large q.

Comparing these results with those obtained by Chu and Sorbello[8] and Masek *et al.*,[9] it is concluded that the present model potential is more appropriate to analyze the transport in a ballistic channel with a single impurity. Although the dips were also found by those authors,[8,9] all of the steps were alike in the results given by Chu and Sorbello since the position of the scatterer is chosen to yield coupling of all of the subbands. In addition to that, their approach does not allow one to vary the strength and the integrated strength independently. Therefore the results presented in Fig. 2 are unique to the present study. Another important advantage of the present approach is that it enables the control of the intersubband coupling. For large values of q the scatterer looks like a δ function, which enhances the intersubband interactions. In contrast, the potential becomes flat in the lateral direction and the intersubband interaction vanishes for small values of q. In this case the dips disappear.

B. Finite constriction

Having discussed elastic scattering due to a single impurity in an infinite constriction, we next consider the situation in a QPC of finite length d. Using the formalism described in Sec. II, we calculated the conductance $G_d(w)$. The results are summarized in Fig. 3. As for the impurity-free constriction,[5] the main effect of finite length is to smooth out the sharp changes in $G_\infty(w)$ (or its first derivative) due to inclusion of evanescent states. This effect is of major importance for short constrictions ($d\lesssim\lambda_F$). For longer constrictions the effect of evanescent states decreases, but a new feature due to interference of left- and right-going waves arises, namely the resonance structure. Since the effects of only elastic scattering by a single impurity are taken into account, neither a phase breaking due to an inelastic event nor a phase averaging due to a large number of scatterers[9] can take place. In other words, the system we are investigating here is the quasiballistic regime, which still contains well-defined interference effects leading to the resonance structure in Fig. 3. The dramatic effect of the impurity is revealed by comparing conductances of finite (neglecting the contribution due to tunneling) and infinite constrictions. For an impurity-free channel the conductance of the finite constriction is smaller than that of an infinite constriction (i.e., smaller than the ideal quantized steps) for all w. In contrast, for a constriction with a single impurity $G_d(w)$ may be larger than $G_\infty(w)$. This is a result of the combined scattering from the impurity and the ends of the constriction ($z=0$ and $z=d$). That is, scattering from the ends may depress the effect of scattering by the impurity.

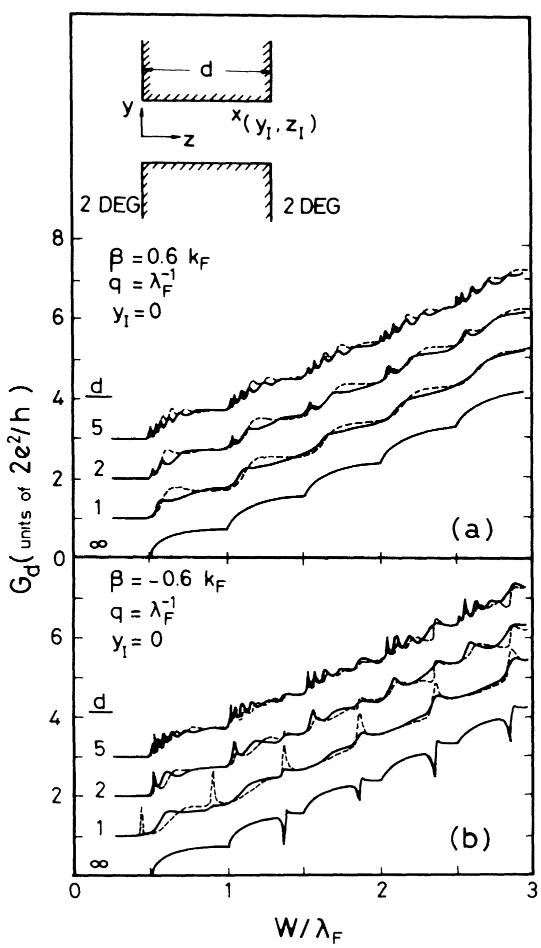

FIG. 3. The conductance G_d vs the width w of a finite length constriction for $q=\lambda_F^{-1}$, $y_I=0$, and (a) $\beta=0.6k_F$, (b) $\beta=-0.6k_F$. The length of the constriction d is varying (in units of λ_F). Solid (dashed) curves denoting $z_I=0.2\lambda_F$ ($z_I=0.5\lambda_F$), and are vertically offset by an amount $(2e^2/h)$ for clarity. The geometry of the channel is described by the inset.

Clearly the main features of $G_d(w)$ shown in Fig. 3, in particular the heights and positions of the resonances and antiresonances, are strongly dependent on the position of the impurity along the z direction. That is, moving the impurity along the channel will give rise to oscillations in the conductance. The magnitude and period of the oscillations are related to the length and width of the constriction, as well as the properties of the impurity. A similar effect is observed by moving defects in a metallic nanoconstriction.[14] In Fig. 4 the resistance of a typical QPC is shown when an impurity is present in the constriction. Clearly, for large q the deviation from the quantized values is approximately constant for a given N_c and decreases with increasing N_c. This result closely resembles the experimental observation of Wharam *et al.*[15] These examples show that it is possible to observe the effects of elastic scattering in the channel. However, additional experimental studies are still needed to fully exploit this conclusion.

Finally, we wish to point out a novel feature of attractive impurities. For short constrictions with an attractive impurity placed near their center ($z_I=d/2$) the conductance curve $G_d(w)$ has sharp peaks just below the steps [Fig. 3(b)]. The widths of these peaks decrease with increasing d, and for very long constrictions the peaks cannot even be resolved. Moving the impurity away from the center of the constriction (by changing either the position of the impurity z_I or the length of the constriction d) has the same effect. Similarly increasing the strength or integrated strength of the impurity causes the peaks to shift the lower w values. A detailed analysis of these results shows that these peaks are associated with resonant tunneling through quasi-0D states bound to the impurity. The properties of this resonant-tunneling effect are analogous to those obtainable from the double-barrier resonance tunneling structures. Hence, similar to formation of quasi-0D states due to geometrical effects[7] (local widening of the constriction) in an impurity-free ballistic channel, it is possible to obtain bound states in a constriction in the presence of an attractive impurity potential. A final remark about these resonances is that the peaks

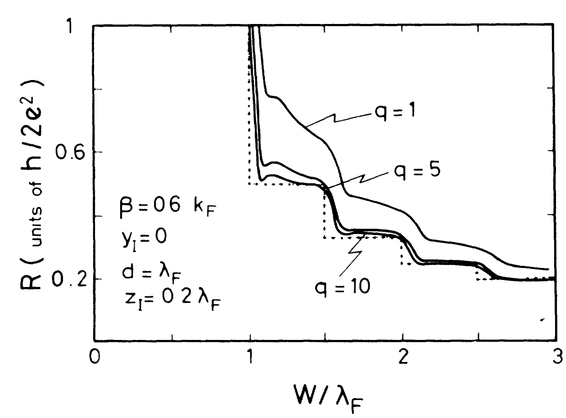

FIG. 4. The resistance of a QPC of length $d=\lambda_F$ containing an impurity at $(y_I;z_I)=(0;0.2)\lambda_F$ with $\beta=0.6k_F$. q is varying (in units of λ_F^{-1}). The dotted lines indicate the value corresponding to the exact quantization.

appear exactly at the same positions with the peaks above the dips in $G_\infty(w)$. This is due to the presence of two orthogonal solutions, one being a quasi-0D state and the other the current-carrying state with unity transmission. Although $G_\infty(w)$ is calculated by including only the current-carrying states, $G_d(w)$ has contributions from both of the above states. Therefore the effects of both quasi-0D state and current-carrying states are visible in Fig. 3. The resonant-tunneling effect is usually depressed for laterally confined impurities (i.e., large q) since the resonance peaks and steps are very close to each other, yielding the overlap of corresponding features in $G_d(w)$. An important remark is about the difference of the evanescent states leading to the dips and peaks. Although the dips are formed as a result of the enhanced backscattering stimulated by the intersubband scattering, the peaks are related to bound (or resonance) states localized around the impurity. By decreasing q it is possible to turn off the intersubband scattering and thus to discard the dips. On the other hand, the resonance states become real bound states in the absence of subband mixing. Thus, while the dips are specific to quasi-1D systems, the peaks are due to resonant tunneling and are achievable for all dimensions.

IV. CONCLUSION

We investigate the effects of elastic scattering by an impurity in a ballistic channel. Using a model potential we obtained exact expressions for the conductance both for finite and infinite constrictions. We summarize the important findings of this study as follows. (i) In agreement with the earlier studies,[4,8,9] we found that the presence of an impurity in the ballistic channel distorts the quantization of conductance. The deviation from quantized values increases with increasing strength or increasing integrated strength of the impurity potential. (ii) For attractive impurity potentials the dips in the conductance curve form as a result of complete reflection. The position and strength of the impurity determines the structure of these dips. (iii) For finite constrictions the effect of the impurity may be depressed by combined scattering from the ends of the constriction. The resonance structure due to interference of current-carrying states is still visible in the conductance curve. (iv) A resonant-tunneling event takes place for attractive impurity potentials. This is a result of formation of quasi-0D states bound to the impurity.

ACKNOWLEDGMENTS

This work is partially supported by a joint project agreement between IBM Zurich Research Laboratory and Bilkent University. We acknowledge stimulating discussions with Dr. M. Büttiker, Dr. P. Gueret, Dr B. J. van Wees, and Dr. L. P. Kouwenhoven, and we wish to thank to Dr. C. S. Chu and Dr. R. S. Sorbello for providing us with their results prior to publication.

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