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Operational approach to quantum limits on polarization measurement

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Abstract

We discuss the polarization measurement of a fully polarized coherent field in the weak intensity quantum limit within the framework of operational approach which was recently discussed by Noh, Fougères and Mandel. We mainly focus on the fluctuations in the measurement of the parameters of polarization and on the uncertainty relations between them. A dependence of these parameters and their quantum fluctuations on the parameters of the initial coherent field is found.

The existence of a well-behaved Hermitian quantum mechanical phase operator is a controversy lasting from the early days of quantum mechanics [1–4]. Most of the literature in this area is mainly concentrated on the mathematical description of the optical quantum phase for the free electromagnetic field. Although these descriptions are equally satisfying for the classical limiting case, they give mutually conflicting results in the cases of extremely low intensities or in strongly non-classical (e.g. squeezed light) distributions [5]. An operational approach to the measurement of the relative quantum phase between two optical fields has been developed recently by Noh, Fougères and Mandel (NFM) [6,7]. In this formulation, the relative phase is described in terms of the photon counting operators which are defined directly by the measurement.

In this Letter, we investigate the quantum measurement of the polarization of a coherent optical field using an operational approach. The advantage of this approach is that the quantum fluctuations of the polarization state can be formulated in terms of the quantum statistical fluctuations of the number of detected photons, even in the weak field limit. We first outline a classical polarization measurement scheme and then investigate the quantum mechanical case.

Classically, the state of polarization of a monochromatic optical field, $E_i = \epsilon_i \cos(\omega t + \delta_i)$, where $i = 1, 2$ are the indices of two pre-selected orthogonal polarization eigenmodes, can be fully described by the four Stokes parameters s_m ($m = 0, 1, 2, 3$) [8–10]. Here we only consider the Stokes parametrization of a fully polarized field. For this case

$$\begin{aligned} s_0 &= \frac{1}{2}(\langle E_1^2 \rangle + \langle E_2^2 \rangle), & s_1 &= \frac{1}{2}(\langle E_1^2 \rangle - \langle E_2^2 \rangle), \\ s_2 &= (\langle E_1^2 \rangle \langle E_2^2 \rangle)^{1/2} \cos \phi, \\ s_3 &= (\langle E_1^2 \rangle \langle E_2^2 \rangle)^{1/2} \sin \phi, \end{aligned} \quad (1)$$

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where $\phi = \delta_2 - \delta_1$ is the optical phase and $\langle E_i^2 \rangle$ is the intensity of the i th component with the brackets indicating the average over the time interval of the measurement. For an optical field this measurement interval is much greater than the period but it is also smaller than the longitudinal coherence time [7,11]. Note that the Stokes parameters are not all independent, but are related by the equality $s_0^2 = s_1^2 + s_2^2 + s_3^2$.

In the classical case, the s_m can be measured using a somewhat similar optical setup to that of NFM, as shown in Fig. 1. The beam whose polarization is to be measured is firstly split into two at the non-polarizing beam splitter BS₁. One of the output beams from BS₁ is sent to a polarizing beam splitter PBS₁, which defines two linearly polarized orthogonal polarization eigenmodes $i = 1, 2$. Then I_1 and I_2 are measured as the output field intensities. The other output beam from BS₁ is further split at BS₂ with the purpose of measuring $\sin \phi$ and $\cos \phi$ simultaneously. One of the output beams from BS₂ is analyzed with PBS₂, which is oriented at a 45° angle with respect to the axes $i = 1, 2$ defined by PBS₁. The intensities measured after PBS₂ are I_3 and I_4 . The other output beam from BS₂ goes through a quarter-wave ($\frac{1}{4}\lambda$) plate whose fast and slow axes are aligned along the $i = 1, 2$ directions. This beam is then analyzed at PBS₃ whose axes are aligned with PBS₂. The intensities measured after PBS₃ are I_5 and I_6 .

For 50% non-polarizing beam splitters, the measured intensities I_i are related to the input field by

$$\begin{aligned} I_1 &= \frac{1}{2} \langle E_1^2 \rangle, & I_2 &= \frac{1}{2} \langle E_2^2 \rangle, \\ I_3 &= \frac{1}{4} (\langle E_1^2 \rangle + \langle E_2^2 \rangle) + 2\sqrt{\langle E_1^2 \rangle \langle E_2^2 \rangle} \cos \phi, \\ I_4 &= \frac{1}{4} (\langle E_1^2 \rangle + \langle E_2^2 \rangle) - 2\sqrt{\langle E_1^2 \rangle \langle E_2^2 \rangle} \cos \phi, \\ I_5 &= \frac{1}{4} (\langle E_1^2 \rangle + \langle E_2^2 \rangle) + 2\sqrt{\langle E_1^2 \rangle \langle E_2^2 \rangle} \sin \phi, \\ I_6 &= \frac{1}{4} (\langle E_1^2 \rangle + \langle E_2^2 \rangle) - 2\sqrt{\langle E_1^2 \rangle \langle E_2^2 \rangle} \sin \phi. \end{aligned} \quad (2)$$

From these relations the classical Stokes parameters can be written as

$$\begin{aligned} s_0 &= I_1 + I_2, & s_1 &= I_1 - I_2, \\ s_2 &= I_3 - I_4, & s_3 &= I_5 - I_6. \end{aligned} \quad (3)$$

The s_m defined by Eqs. (2) and (3) depend on the initial choice of the orthogonal axes 1, 2. The polar-

ization state, hence the orientation of the polarization ellipse, for any input field will be measured relative to these axes. We are interested in the fluctuations associated with the measurement of the polarization state. Such fluctuations are very important when the measured field is very weak or in some quantum state. In relation with the measurement of these fluctuations we first define the classical angular functions,

$$\begin{aligned} \sin \theta &= 2 \frac{(I_1 I_2)^{1/2}}{I_1 + I_2}, & \cos \theta &= \frac{I_1 - I_2}{I_1 + I_2}, \\ \cos \phi &= \frac{I_3 - I_4}{[(I_3 - I_4)^2 + (I_5 - I_6)^2]^{1/2}}, \\ \sin \phi &= \frac{I_5 - I_6}{[(I_3 - I_4)^2 + (I_5 - I_6)^2]^{1/2}}. \end{aligned} \quad (4)$$

Using Eq. (4), Eq. (3) can be described in a polar decomposition. If the axes 1, 2 are the principle axes of the ellipse of polarization, then $\cos \theta$ is related to the ellipticity angle, and $\phi = \delta_2 - \delta_1$ becomes the optical phase between the major and minor axis components [8]. Although θ and ϕ can be mixed by rotating the frame in which they are defined, they are closely connected with the geometry of the ellipse.

The setup in Fig. 1 allows us to make measurements on θ and ϕ related to the polarization state without employing any interference. In the laboratory, it is technically difficult to eliminate the influence of the random path length fluctuations on the measurement of the optical phase. It should be emphasized that in our proposed setup, each component goes through the same optical path in both the $\cos \phi$ and the $\sin \phi$ measurements. There is also no restriction on the arm lengths BS₂–PBS₃ and BS₂–PBS₂ imposed by the temporal coherence length. Therefore, the setup in Fig. 1 is immune to random phase fluctuations resulting from mechanical vibrations of various optical components.

Having defined our measurement, we next look at the quantum description of the polarization state. Quantum mechanically, the unused ports in the beam splitters are considered as input for vacuum fields. Therefore, we include in Fig. 1 the vacuum fields $v^{(1)}, v^{(2)}$ for BS₁ and BS₂ and $w^{(1)}, w^{(2)}, w^{(3)}$ for PBS₁, PBS₂ and PBS₃, respectively.

The annihilation operators associated with the optical fields at the output arms of PBS₁ are related to those at the input through

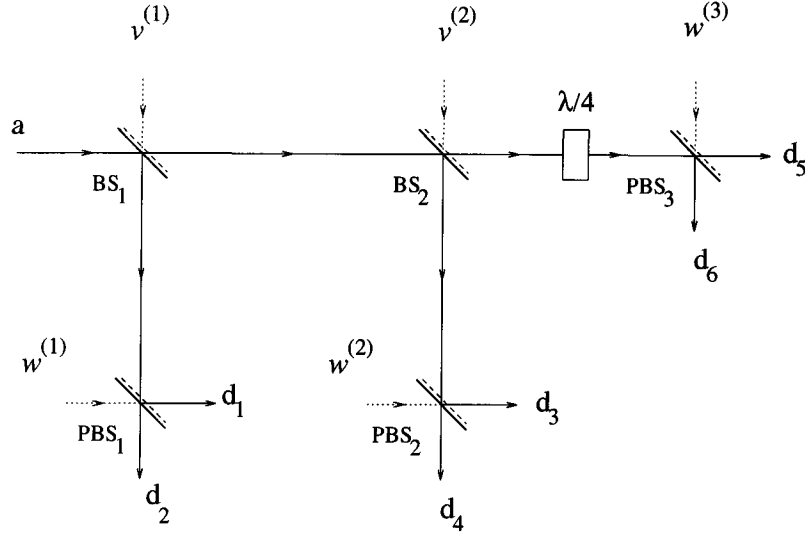


Fig. 1. Experimental setup for the measurement of all Stokes parameters. Note that this setup can be used in the measurement of a partially polarized field as well. However, here we only concentrate on the fully polarized case.

$$\hat{d}_1 = r\hat{a}_1 + t\hat{v}_1^{(1)}, \quad \hat{d}_2 = r\hat{a}_2 + t\hat{v}_2^{(1)}, \quad (5)$$

where $r = i/\sqrt{2}$ and $t = 1/\sqrt{2}$ are the field reflection and transmission coefficients, and $\hat{v}_j^{(1)}$ ($j = 1, 2$) describes the polarized vacuum field in the j direction entering through the vacuum port of BS₁. The vacuum fields $\hat{w}^{(k)}$ ($k = 1, 2, 3$) do not couple with the measured field operators since they are orthogonal to them. Therefore, we do not include any of these orthogonal vacuum field components, as they do not contribute to the fluctuations of the intensity in our operational formalism. We next define the operators

$$\hat{S}_\theta = 2 \frac{(\hat{n}_1 \hat{n}_2)^{1/2}}{\hat{n}_1 + \hat{n}_2}, \quad \hat{C}_\theta = \frac{\hat{n}_1 - \hat{n}_2}{\hat{n}_1 + \hat{n}_2}, \quad (6)$$

where $\hat{n}_j = \hat{d}_j^\dagger \hat{d}_j$, $j = 1, 2$, are the photon number operators for the fields measured at the detectors 1, 2. Assuming ideal detectors with unit quantum efficiency, the measured value of \hat{n}_j represents the photocurrents at the detectors. Since $[\hat{v}_1^{(1)}, \hat{v}_2^{(1)\dagger}] = 0$ and $[\hat{a}_1, \hat{a}_2^\dagger] = 0$ due to independence, we have $[\hat{d}_1, \hat{d}_2^\dagger] = 0$. This relation together with the use of the semiclassical analogy justifies the definition of the operators in Eq. (6). As a consequence of $[\hat{d}_1, \hat{d}_2^\dagger] = 0$, we also have $[\hat{S}_\theta, \hat{C}_\theta] = 0$ and $\hat{S}_\theta^2 + \hat{C}_\theta^2 = 1$.

On the other hand, at the output ports of PBS₂ and PBS₃, the field operators $\hat{d}_3, \hat{d}_4, \hat{d}_5, \hat{d}_6$ are

$$\begin{aligned} \hat{d}_3 &= \frac{1}{\sqrt{2}} [(tr\hat{a}_1 + r^2\hat{v}_1^{(1)} + t\hat{v}_1^{(2)}) \\ &\quad + (tr\hat{a}_2 + r^2\hat{v}_2^{(1)} + t\hat{v}_2^{(2)})], \\ \hat{d}_4 &= \frac{1}{\sqrt{2}} [-(tr\hat{a}_1 + r^2\hat{v}_1^{(1)} + t\hat{v}_1^{(2)}) \\ &\quad + (tr\hat{a}_2 + r^2\hat{v}_2^{(1)} + t\hat{v}_2^{(2)})], \\ \hat{d}_5 &= \frac{1}{\sqrt{2}} [i(t^2\hat{a}_1 + tr\hat{v}_1^{(1)} + r\hat{v}_1^{(2)}) \\ &\quad + (t^2\hat{a}_2 + tr\hat{v}_2^{(1)} + r\hat{v}_2^{(2)})], \\ \hat{d}_6 &= \frac{1}{\sqrt{2}} [-i(t^2\hat{a}_1 + tr\hat{v}_1^{(1)} + r\hat{v}_1^{(2)}) \\ &\quad + (t^2\hat{a}_2 + tr\hat{v}_2^{(1)} + r\hat{v}_2^{(2)})]. \end{aligned} \quad (7)$$

Here $\hat{v}_j^{(2)}$ ($j = 1, 2$) describes the polarized vacuum field in the j direction through the vacuum port of BS₂. Then, by the same analogy with Eq. (4) we have the following relations for the optical phase operators,

$$\begin{aligned} \hat{C}_\phi &= \frac{\hat{n}_3 - \hat{n}_4}{[(\hat{n}_3 - \hat{n}_4)^2 + (\hat{n}_5 - \hat{n}_6)^2]^{1/2}}, \\ \hat{S}_\phi &= \frac{\hat{n}_5 - \hat{n}_6}{[(\hat{n}_3 - \hat{n}_4)^2 + (\hat{n}_5 - \hat{n}_6)^2]^{1/2}} \end{aligned} \quad (8)$$

with $\hat{n}_j = \hat{d}_j^\dagger \hat{d}_j$ ($j = 3, 4, 5, 6$). Here $\hat{n}_3 - \hat{n}_4$ and $\hat{n}_5 - \hat{n}_6$ are

$$\begin{aligned} \hat{n}_3 - \hat{n}_4 &= [(r^* t^* \hat{a}_2^\dagger + r^* r^* \hat{v}_2^{(1)} + t^* \hat{v}_2^{(2)\dagger}) \\ &\quad \times (rt\hat{a}_1 + r^2\hat{v}_1^{(1)} + t\hat{v}_1^{(2)}) + \text{h.c.}], \\ \hat{n}_5 - \hat{n}_6 &= i[(t^* \hat{a}_2^\dagger + t^* r^* \hat{v}_2^{(1)} + r^* \hat{v}_2^{(2)\dagger}) \\ &\quad \times (t^2\hat{a}_1 + tr\hat{v}_1^{(1)} + r\hat{v}_1^{(2)}) - \text{h.c.}], \end{aligned} \quad (9)$$

where h.c. denotes Hermitian conjugate. Using Eqs. (7) it can be shown that neither \hat{n}_5 nor \hat{n}_6 commute with \hat{n}_4 or \hat{n}_3 . Here $[\hat{C}_\phi, \hat{S}_\phi] = 0$ follows explicitly from the existence of the vacuum fields in the combination $\hat{n}_3 - \hat{n}_4$ and $\hat{n}_5 - \hat{n}_6$. On the other hand, for the commutation $[\hat{S}_\theta, \hat{C}_\theta] = 0$ of the previous case, the vacuum field $\hat{v}^{(1)}$ does not play any role. This is because at PBS₁ the orthogonal field operators \hat{d}_1 and \hat{d}_2 do not have the same components of the vacuum field $\hat{v}^{(1)}$.

We first analyze the fluctuation

$$D_\theta = \sqrt{(\Delta S_\theta)^2 + (\Delta C_\theta)^2}$$

in the measurement of \hat{S}_θ and \hat{C}_θ where the variances are $(\Delta S_\theta)^2 = \langle (\hat{S}_\theta - \langle \hat{S}_\theta \rangle)^2 \rangle$ and $(\Delta C_\theta)^2 = \langle (\hat{C}_\theta - \langle \hat{C}_\theta \rangle)^2 \rangle$, respectively. The brackets indicate quantum averages calculated with respect to the initial state $|\psi_{IN}\rangle$. In this semiclassical operational approach, the measured field is treated classically, and the joint probability distribution of the detectors is treated quantum mechanically [11,12]. At PBS₁ the measured value of a general operator $f(\hat{n}_1, \hat{n}_2)$ is given by [7]

$$\langle \hat{f}(\hat{n}_1, \hat{n}_2) \rangle = \mathcal{N} \sum_{n_1, n_2} f(n_1, n_2) \langle \mathcal{P}(\hat{n}_1, \hat{n}_2) \rangle, \quad (10)$$

where \mathcal{N} is the normalization, and $\langle \mathcal{P}(\hat{n}_1, \hat{n}_2) \rangle = \langle \psi_{IN} | \mathcal{P}(\hat{n}_1, \hat{n}_2) | \psi_{IN} \rangle$, where $|\psi_{IN}\rangle$ is the initial photon state as observed in PBS₁. Here $\mathcal{P}(\hat{n}_1, \hat{n}_2)$ is given by the normal ordered product

$$\mathcal{P}(\hat{n}_1, \hat{n}_2) = \prod_{j=1}^2 : (\hat{d}_j^\dagger \hat{d}_j)^{n_j} \exp(-\hat{d}_j^\dagger \hat{d}_j) / n_j! :, \quad (11)$$

which is the joint probability distribution of detecting n_j photons at detectors 1, 2. To calculate the fluctuation Δf in the measurement of the operator \hat{f} , we find $\langle \hat{f} \rangle$ and $\langle \hat{f}^2 \rangle$ using Eq. (10). An important point is that

there is an ill-defined contribution of the unobserved data corresponding to $n_1 = n_2 = 0$ in the calculation of D_θ . The relative contributions of these unobserved events in the total average is not small if the initial average field intensities are weak (i.e. $\bar{n}_1, \bar{n}_2 < 1$). Therefore, we discard these data points by subtracting the probability $\langle \mathcal{P}(0, 0) \rangle$ from the normalization and also excluding the corresponding terms in the summation (indicated by the prime). Hence the normalization, in the operational sense, is given by $\mathcal{N}^{-1} = 1 - \langle \mathcal{P}(0, 0) \rangle$ [7].

According to the measurement scheme in Fig. 1 the orientation of the first polarizing beam splitter PBS₁ is arbitrary. Thus, fluctuations should not only be examined with respect to the independent average number of particles \bar{n}_1 and \bar{n}_2 but also with respect to their ratio $\eta = \bar{n}_2 / \bar{n}_1$. Let us assume that the initial beam is given in a product coherent state with average number of particles $|\alpha|^2$ and $|\beta|^2$ with respect to some fixed reference axes 1^0 and 2^0 . Then $|\langle n_1 n_2 | \psi_{IN} \rangle|^2 = P(n_1, |\alpha|^2) \otimes P(n_2, |\beta|^2)$ where P describes the number probability distribution of a coherent state with mean photon numbers $|\alpha|^2$ and $|\beta|^2$. If $\alpha = |\alpha| e^{i\delta_\alpha}$ and $\beta = |\beta| e^{i\delta_\beta}$ one also has a coherent distribution in 1, 2 frame with average number of particles,

$$\begin{aligned} \bar{n}_1 &= \frac{1}{2} |\alpha \cos \gamma - \beta \sin \gamma|^2, \\ \bar{n}_2 &= \frac{1}{2} |\beta \cos \gamma + \alpha \sin \gamma|^2, \end{aligned} \quad (12)$$

with γ describing the arbitrary angle between the frames $1^0, 2^0$ and 1, 2, and the overall $\frac{1}{2}$ is due to the BS₁. In Fig. 2 D_θ is shown as a function of \bar{n}_1 for $\eta = 0.1, 1, 10$. The fluctuation increases rapidly to its maximum value from the classically expected value of zero as the average number of detected photons decreases. The largest fluctuation is associated with $\eta = 1$. Even if $|\alpha| \neq |\beta|$ η can still be very close to one if the axes of PBS₁ are oriented relative to $1^0, 2^0$ by certain amount which is determined by Eq. (12).

We now study $D_\phi = [(\Delta \hat{S}_\phi)^2 + (\Delta \hat{C}_\phi)^2]^{1/2}$ for \hat{C}_ϕ and \hat{S}_ϕ operators. Here the operator $\hat{f}(\hat{n}_3, \hat{n}_4, \hat{n}_5, \hat{n}_6)$ in Eq. (10) is given by the corresponding \hat{C}_ϕ and \hat{S}_ϕ operators defined by Eq. (8), and $\mathcal{P}(\hat{n}_1, \hat{n}_2)$ is replaced by $\mathcal{P}(\hat{n}_3, \hat{n}_4, \hat{n}_5, \hat{n}_6)$. The calculated D_ϕ is shown in Fig. 3. Figs. 2 and 3 indicate that for $\eta = 1$, D_θ takes its largest and D_ϕ its smallest values compared to other possible values of η for small values of \bar{n}_1 . As \bar{n}_1 gets large enough, the roles of D_θ and D_ϕ

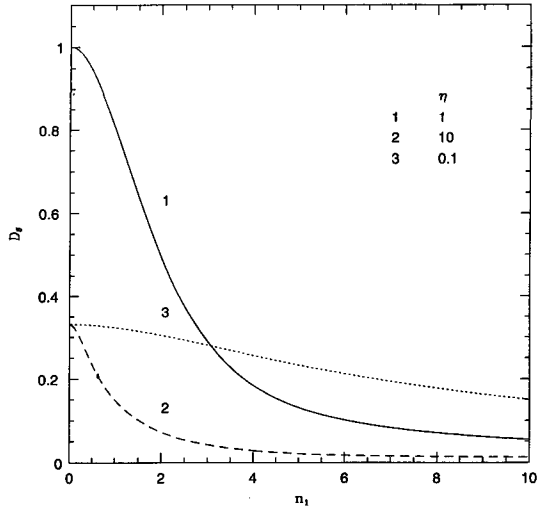


Fig. 2. Fluctuation in θ as a function of the average number of particles measured at the detector d_1 .

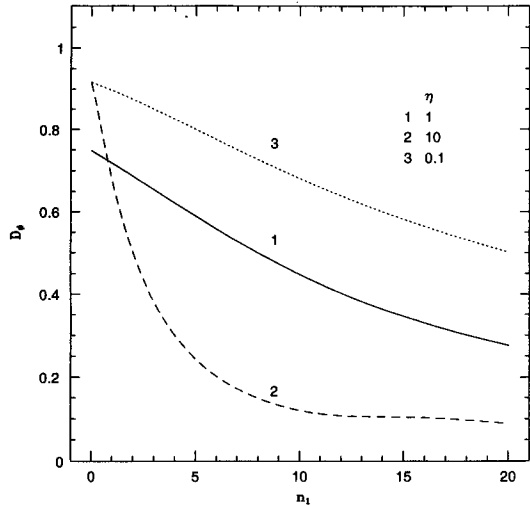


Fig. 3. Fluctuation in ϕ as a function of the average number of particles measured at the detector d_2 .

interchange. We will now briefly study this behaviour in connection with an uncertainty principle.

The analysis of fluctuations in the polarization measurement of weak fields yields interesting results. Figs. 2 and 3 indicate that the quantum statistics of the initial field has an important influence on the measured parameters of polarization. Although the choice of the axes on which one measures the polarization is arbitrary,

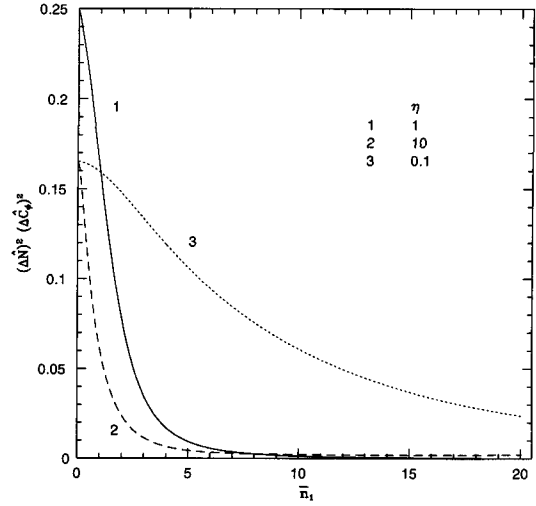


Fig. 4. Uncertainty product described by the l.h.s. of Eq. (13) for $\eta = 0.1, 1, 10$.

fluctuations strongly depend on the relative intensity of the field components measured along the axes. Theoretically, the optical phase ϕ is the canonically conjugated operator to the number difference operator $\hat{N} = \hat{n}_2 - \hat{n}_1$ [1,2]. Then from Eq. (6) we have $\hat{N} = -\hat{C}_\theta (\hat{n}_1 + \hat{n}_2)$. Hence we expect the fluctuations in θ - and ϕ -dependent operators to be regulated by the uncertainty relations

$$\begin{aligned}
 (\Delta \hat{N})^2 (\Delta \hat{C}_\phi)^2 &\geq \frac{1}{4} \langle \hat{S}_\phi \rangle^2, \\
 (\Delta \hat{N})^2 (\Delta \hat{S}_\phi)^2 &\geq \frac{1}{4} \langle \hat{C}_\phi \rangle^2.
 \end{aligned}
 \tag{13}$$

In Fig. 4 the first one of Eqs. (13) is plotted as a function of \bar{n}_1 for $\eta = 0.1, 1, 10$. We examined it and observed that the inequality is respected for all values of \bar{n}_1 even though one would expect it to be questionable for $\bar{n}_1 < 1$ due to the subtracted statistics in the averages. For small values of \bar{n}_1 , $\Delta \hat{N}$ is small due to the number coherent state but does not go to zero like in the pure coherent state. This is also a result of the subtracted, unobservable data. Also in Fig. 4, an interesting feature of Eqs. (13) with respect to η deserves some attention. Curves corresponding to two different η 's cross each other at \bar{n}_1 values fixed by the ratio of those η 's. For instance, in the range $\bar{n}_1 < 1$ $\eta = 1$ yields the largest uncertainty among all possible η values. At $\bar{n}_1 \approx 1$ a crossover is observed between the curves corresponding to $\eta = 0.1$ and $\eta = 1$. The

$\eta = 1$ and $\eta = 10$ curves cross each other at $\bar{n}_1 \sim 10$ and then $\eta = 1$ yields the smallest uncertainty of all three curves for $\bar{n}_1 > 10$. If, instead of Fig. 4, we had plotted $(\Delta \hat{C}_\theta)^2 (\Delta \hat{C}_\phi)^2$ against \bar{n}_1 we would have seen the same crossing pattern at approximately the same \bar{n}_1 values. Naturally because of the form of Eqs. (4), (6) and (8) the point where the crossover is observed scales with respect to η^2 . Thus, although $\eta = 1$ corresponds to the largest uncertainty in the weak field limit, it is the fastest decaying one with increasing average number of particles.

In conclusion, we believe that the relative intensity dependence of the uncertainty relations (13) described in Fig. 4 is of experimental importance for the measurement of the polarization of weak fields. These theoretically observed features can be easily examined in an experiment described in Fig. 1. There have been a number of studies on the measurement of the probability distribution of the moments of the phase operator [13]. It can be shown that in polarization measurement the widths of the corresponding probability distributions P_θ and P_ϕ of θ and ϕ dependent operators are correlated in the reciprocal sense due to the uncertainty relations (13). A more detailed analysis of this manifestation of the uncertainty in weak polarization measurements will be given in another publication.

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² Small shifts of the crossover points from their fixed value determined by η 's is due to the truncation errors in the calculation of otherwise infinite summation in Eq. (10).