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An exact tool allocation approach for CNC machines

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Abstract. An exact approach is developed to determine the optimum machining conditions and tool allocation decisions simultaneously to minimize the total production cost on a CNC turning machine. There are multiple machining operations and we consider a set of alternative cutting tool types for each operation. The existing tool management approaches at the system level fail to relate the tooling issues to the machining conditions, and ignore the tool availability and tool wear restrictions. Consequently, we not only improve the overall solution by exploiting the interactions between these two decision making problems, but also prevent any unfeasibility that might occur for the tool allocation problem due to tool contention among the operations for a limited number of tool types by considering the machining operation, tool availability and tool life limitations. The computational results indicated that the average computation time to find an optimum solution was 1.11 s, whereas the maximum time was 11.45 s, for a set of randomly generated problems.

1. Introduction

There is an increasing requirement for manufacturing industries to achieve effective, diverse, small lot production, so as to meet diversified user needs. Numerical control (NC) is a form of programmable automation, which is designed to accommodate variations in product configurations. Its principal applications are in low and medium volume situations, primarily in a batch production mode. The results of a US Census Bureau survey of nearly 10 000 manufacturing firms in 1990 offered insight into use of 17 manufacturing technologies, such as CAD/ CAE, robots. NC machine tools, with 41.5% of the respondents indicating its use, was the most widely used manufacturing technology. Machinery production statistics released by the Japanese Ministry of International Trade and Industry showed that the number of NC machine tools produced in Japan was equal to 61 695 in 1990, which made more than 75% of total machine tool production shares (Asai and Takashima 1994). Furthermore, one of the major components of a flexible manufacturing system (FMS) is computer numerical control (CNC) machine tools. A FMS is usually defined as a group of CNC machine tools interconnected by a material handling system and controlled by a computer system.

In view of the high investment and operating costs of CNC machines and hence of FMSs, attention should be paid to their effective utilization. Gray *et al.* (1993) and Veeramani *et al.* (1992) give extensive surveys on the tool management issues of automated manufacturing systems, and emphasize that the lack of tooling considerations has resulted in the poor performance of these systems. Kouvelis (1991) identified cutting tool utilization as an important parameter for the overall system performance. In this study, the cost of tooling has been reported to be 25–30% of the fixed and variable costs of production. Gray *et al.* (1993) also present an integrated conceptual framework for resource planning to examine how tool management issues can be classified into tool-level, machine-level, and system-level concerns. Tool management decisions arise in production planning and scheduling, and involve machine grouping, part type selection and loading, and tool allocation at the system level. The key tool management issues at the single machine level are loading and placing a set of tools in the machine's magazine, determining the part input sequence to meet certain magazine constraints and establishing tool replacement strategies. Tool management issues at the tool level include tool selection activities, such as the number and type of cutting tools, and tool cutting speeds and feed rates for each manufacturing operation.

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For solving the tool allocation problem at the system level, most of the published studies use 0–1 binary variables, i.e. a particular tool j is assigned to operation i, to represent tool requirements. Stecke (1983) formulates the FMS loading problem as a nonlinear mixed-integer programming (MIP) problem and solves it through linearization techniques. Sarin and Chen (1987) give an integer programming (IP) formulation under the assumption that the total machining costs depend upon the tool–machine combination. Ram *et al.* (1990) develop a new formulation for the same problem using discrete generalized networks to propose an efficient algorithm for solving the resulting mathematical model. Both the machining costs and tool lives are considered as fixed system parameters regardless of the machining conditions. Leung *et al.* (1993) propose a linear integer model to solve part assignment and tool allocation simultaneously to minimize the sum of machine process, in-process tool use and material handling costs. Maheshwari and Khator (1995) extend the IP loading model of Leung *et al.* to evaluate several operational control strategies by utilizing a simulation model. All of these studies assume constant processing times and tool lives as *a priori* information by ignoring their interaction with the machining conditions selection and the tool availability restrictions. Therefore, they cannot consider the actual tool wear and the corresponding tool life limitations, hence the resulting tool replacement needs and their impact on the total cost. Furthermore, depending on the batch size, the number of tools required to produce a certain operation might be greater than one. Finally, most of the studies determine the tool requirements for each operation independently, and fail to consider the contention among the operations for a limited number of tools. The operational characteristics of the system components, such as machining conditions, tool availability and tool life, should be taken into account for the reliable modelling of CNCs, or the absence of such crucial issues could lead to unfeasible or inferior results.

At the machine level, most of the studies emphasize the minimization of tool switches due to a change in a part mix (Tang and Denardo 1988, Kouvelis 1991, Crama *et al.* 1994). Unfortunately, these studies also assume constant processing times and tool lives, even though the tool wear, consequently the tool replacement frequency, is directly related with the machining conditions selection. Further, in the multiple operation case, non-machining time components, such as the tool replacements, can have a significant impact on the total cost of production because of the relatively short tool lives of many turning tools as stated by Gray *et al.* (1993). In the same study, they reported that tools are changed ten times more often due to tool wear than to part mix.

The machining conditions optimization for a single operation is a well known problem, where the decision variables are the cutting speed and feed rate. Several models and solution methodologies have been developed in the literature (Gopalakrishnan and Al-Khayyal 1991, Tan and Creese 1995). However, these models only consider the contribution of machining time and tooling cost to the total cost of operation, and they usually ignore the contribution of non-machining time components to the operating cost, which could be very significant for the multiple operation case. Furthermore, the existing studies exclude the tooling issues such as the tool availability and the tool life capacity limitations. As a result, their results can lead to infeasibility due to tool contention among the operations for a limited number of tool types.

The remainder of this paper is organized as follows. In the next section, we define the scope of the study with the underlying assumptions and state a mathematical formulation of the problem. In section 3, we present the proposed solution procedure, which is applied in an example problem in section 4. The computational results are discussed in section 5. Finally, some concluding remarks are provided in the last section.

The notation used throughout the paper is as follows:

- $\alpha_j, \beta_j, \gamma_j$: speed, feed, depth of cut exponents for tool j
- B : batch size
- C_j : Taylor's tool life constant for tool j
- C_m, b, c, e : specific coefficient and exponents of the machine power constraint
- C_o : operating cost of the CNC machine ($/min)
- C_s, g, h, l : specific coefficient and exponents of the surface roughness constraint
- C_{t_j} : cost of the tool j ($/ per tool)
- D_i : diameter of the generated surface for the operation i (in)
- d_i : depth of cut for operation i (in)
- f_{ij} : feed rate for operation i using tool j (ipr)
- H : maximum available machine power for all operations (hp)
- I : set of all operations
- J : set of the available tools
- J_i : set of the candidate tools that can be used for the operation i
- L_i : length of the generated surface for the operation i (in)
- M : a very large positive number

n_{ij} : number of tool type j required for completion of operation i
N_j : number of available tools on hand for tool type j
p_{ij} : number of times that an operation i can be performed by a tool type j
S_i : maximum allowable surface roughness for the operation i (μin)
$t_{m_{ij}}$: machining time of operation i using tool j (min)
t_{l_j} : tool magazine loading time for a single tool j (min)
t_{r_j} : tool replacing time for tool j (min)
T_{ij} : tool life of tool j in operation i (min)
U_{ij} : usage rate of tool j in operation i
v_{ij} : cutting speed for operation i using tool j (fpm)
x_{ij} : 0–1 binary decision variable which is equal to 1 if tool j is assigned to operation i.

2. Problem definition

We develop a new mathematical model and propose an efficient solution procedure to determine concurrently the optimal machining conditions of cutting speed and feed rate, the optimal operation–tool assignment, and the optimal allocation of tools, for single-pass operations of a batch of parts processed on a single CNC turning machine. In a previous study by Avci and Akturk (1996), we address the tooling issues related to tool sharing and loading of duplicate tools at a single CNC machine level. A new algorithm is proposed to solve the tool magazine arrangement and operations sequencing problems subject to tool allocation, precedence and tool magazine capacity restrictions for the given machining conditions for each manufacturing operation. In this study, we emphasize the tool management issues at the tool level such as the optimum machining conditions and tool selection–allocation decisions in connection with the tool life, machining operations and tool availability constraints to minimize the total production cost.

The following assumptions are made to define the scope of this study. Each machining operation has a set of alternative tool types. For each type of cutting tool there is only a limited number of tools available. For the machining operations, the cutting speed and the feed rate will be taken as the decision variables, and the depth of cut is assumed to be given as an input. Initial tool loading and subsequent tool replacements are only allowed while the machine is off-line and only a single tool can be changed at a time. This implies that tool changing times are additive. Since the tool changing events during an operation might adversely affect the surface finish requirements, each machining operation is assumed to be completed by a single tool type, even though alternative tools are considered for each operation. The batch size of each part is known, although there might be a significant interaction between the lot sizing and tool allocation decisions as discussed in Akturk and Onen (1997). In the existing decision-making hierarchy, we determine the optimum machining conditions and the corresponding tool allocations. Once calculated, processing and set-up time data are passed up to the system planning level, in which decisions such as batch sizes and schedules are determined from the timing data along with system level objective functions.

Advances in cutting tool materials and designs will increase the cutting speeds at which machining is carried out, consequently reduce the machining time, but the initial tooling cost might be higher. Therefore we consider a set of alternative cutting tool types for each machining operation, such as HSS, carbides, coated tools, since no one cutting tool type is best for all purposes. Furthermore, the total production cost should be expressed in terms of both machining time and non-machining time components, and the tooling cost. Machining time, $t_{m_{ij}}$, is the time required to complete a turning operation. Tool life is generally defined as the machining time in minutes taken to produce a given wear land for a set of machining conditions. The relationship between the tool life, T_{ij}, and machining time can be expressed as a function of the machining conditions by using an extended form of the Taylor's tool life equation. For the turning operation, a new expression is defined for the machining time to tool life ratio, which is called the usage rate of tool j in operation i, and denoted by U_{ij}. A similar expression can be defined for other machining operations.

$$U_{ij} = \frac{t_{m_{ij}}}{T_{ij}} = \frac{(\pi D_i L_i)/(12 v_{ij} f_{ij})}{C_j/(v_{ij}^{\alpha_j} f_{ij}^{\beta_j} d_i^{\gamma_j})} = \frac{\pi\ D_i\ L_i\ d_i^{\gamma_j}}{12\ C_j\ v_{ij}^{(1-\alpha_j)}\ f_{ij}^{(1-\beta_j)}}$$

Consequently, $p_{ij} = \lfloor 1/U_{ij} \rfloor$ and $n_{ij} = \lceil B/p_{ij} \rceil$. For practical purposes, p_{ij} must be found in order to instruct either the CNC program or the operator to change tools after a predetermined number of pieces have been machined.

All time consuming events except the actual cutting operation are called the non-machining time components. Even though there might be many distinct non-machining time components such as tool tuning, workpiece loading/unloading, etc., we only consider the ones that can be expressed as a function of both the machining conditions and alternative operation–tool pairs, such as tool replacing times, t_{r_j}, and loading times, t_{l_j}.

A general mathematical formulation of the problem is stated below, where the total cost of manufacturing for a particular batch is expressed as the sum of operating cost due to machining time and non-machining time components, the tooling cost, and tool waste cost, respectively. Depending on the batch size and machining conditions, the number of tools required to produce a certain operation might be greater than one, i.e. $BU_{ij} > 1$. If the last copy of tool type j is not fully utilized for machining operation i then it can be used for machining other parts, although the remaining tool life of the previous copies may not be enough to produce a single operation due to tool life constraint. Therefore, the cost of unused remaining tool life prior to the tool replacement due to tool wear is denoted as tool waste cost. There are four sets of decision variables. The first set of decision variables, x_{ij}, represents the tool allocation decisions. The second set of decision variables, n_{ij}, depicts the number of tools of a given type allocated to an operation. The third and fourth sets, v_{ij} and f_{ij}, respectively, represent the machining conditions selection decisions.

$$\text{Minimize } C_{tm} = BC_o\left(\sum_{i\in I}\sum_{j\in J} x_{ij} t_{m_{ij}}\right) + C_o\left(\sum_{i\in I}\sum_{j\in J} x_{ij}\left((n_{ij}-1)t_{r_j} + t_{l_j}\right)\right) + \sum_{i\in I}\sum_{j\in J} x_{ij} n_{ij} C_{t_j} + \sum_{i\in I}\sum_{j\in J} C_{t_j}\lfloor B/p_{ij}\rfloor(1 - p_{ij}U_{ij}) .$$

Subject to:

(Tool Assignment Constraints)

$$\sum_{j\in J_i} x_{ij} = 1 \text{ for every } i \in I$$

$$n_{ij} \le Mx_{ij} \text{ for every } i \in I, j \in J_i$$

$$x_{ij} \ge U_{ij} \text{ for every } i \in I, j \in J_i$$

(Tool Availability Constraint)

$$\sum_{i\in I} x_{ij}\ n_{ij} \le N_j, \text{ for every } j \in J$$

(Tool Life Constraint)

$$x_{ij}\ U_{ij}\ p_{ij} \le 1, \text{ for every } i \in I, j \in J$$

(Machine Power Constraint)

$$x_{ij} C_m v_{ij}^b f_{ij}^c d_i^e \le H, \text{ for every } i \in I, j \in J_i$$

(Surface Roughness Constraint)

$$x_{ij} C_s v_{ij}^g f_{ij}^h d_i^l \le S_i, \text{ for every } i \in I, j \in J_i$$

(Non-negativity and Integrality Constraints)

$$v_{ij},\ f_{ij} > 0\ ,\ x_{ij} = \{0, 1\}$$

and n_{ij}, p_{ij} positive integers for every $i \in I, j \in J$.

In this nonlinear MIP formulation, there exist three types of constraints, namely, operational, tool related and machining operation constraints. The first three sets of constraints represent the operational constraints which ensure that each operation is assigned to a single tool type from its candidate tools set. The tool availability and tool life constraints are the tool related constraints which guarantee that the solution will not exceed the available quantity on hand and the available tool life capacity for any tool type. The last two sets of constraints are the machining operation constraints. The machining resistance is in general given by the power function of cutting speed and feed rate, and it must not exceed the motor power of the machine tool employed. The surface roughness represents the quality requirement for the operation and should be less than a certain amount to ensure good product accuracy.

The proposed formulation can be very helpful in defining the influence of the machining conditions on the total production cost. If we increase either v_{ij} or f_{ij}, or both, then we can reduce the machining time but this will increase the machine horsepower and the number of tool requirements, and equivalently non-machining and tooling costs. On the other hand, a heavy feed rate is conducive to the formation of a built-up edge and a rough surface finish, whereas high cutting speed improves the surface finish since it decreases the built-up edge formation on the face of a cutting tool. Therefore, a new approach is proposed to determine concurrently the optimal machining conditions, the optimal operation–tool assignments and the optimal allocation of tools that minimize the total production cost of a batch of parts processed on a CNC machine.

3. Solution procedure

The constraints and the decision variables for machining conditions and tool allocation interact with each other. In order to solve these two interrelated problems simultaneously, we propose a new solution procedure by relaxing the set of tool availability constraints, which can be called coupling constraints. In this resource directed decomposition procedure, we first find the optimum machining conditions for all possible operation–tool pairs and select the tool that gives the minimum cost measure by using the single machining operation problem (SMOP). This will provide a lower bound for the tool allocation and machining conditions optimization problem. If the required number of tools for any tool type exceeds the number of tools available on hand then we generate different tool requirement levels for every operation–tool pair. Consequently, the nonlinear MIP formulation with several sets of constraints given in the previous section is polynomially transformed to a much simpler IP formulation as outlined below.

3.1. *Single machining operation problem*

In SMOP, the objective function includes the tooling cost and operating cost due to the machining time, and it is possible to impose the machining operation constraints on that problem together with a tool life constraint. In the tool life constraint, p_{ij} is a positive integer corresponding to a desired level of tool requirement, n_{ij}. The following mathematical formulation of geometric programming (GP) can be written for the SMOP for every possible operation and tool pair:

$$\text{Minimize } M_{ij} = C_1 v_{ij}^{-1} f_{ij}^{-1} + C_2 v_{ij}^{(\alpha_j - 1)} f_{ij}^{(\beta_j - 1)}$$

Subject to:

(Tool Life Constraint)

$$C'_t v_{ij}^{(\alpha_j-1)} f_{ij}^{(\beta_j-1)} \leq 1$$

(Machine Power Constraint)

$$C'_m v_{ij} f_{ij}^c \leq 1$$

(Surface Roughness Constraint)

$$C'_s v_{ij}^g f_{ij}^h \leq 1$$

$$v_{ij}, f_{ij} > 0$$

where

$$C_1 = \frac{\pi D_i L_i C_o}{12}, \quad C_2 = \frac{\pi D_i L_i d_i^{\gamma_j} C_{t_j}}{12 C_j}$$

$$C'_t = \frac{\pi D_i L_i d_i^{\gamma_j} p_{ij}}{12 C_j}, \quad C'_m = \frac{C_m d_i^e}{H} \quad \text{and} \quad C'_s = \frac{C_s d_i^l}{S_i}$$

The associated GP–Dual problem for the above formulation is given below. The objective function for the dual problem is still a nonlinear one, but the constraints of the dual formulation are well-defined linear equations.

$$\text{Maximize } Q^* = \left(\frac{C_1}{Y_1}\right)^{Y_1} \left(\frac{C_2}{Y_2}\right)^{Y_2} \left(C'_t\right)^{Y_3} \left(C'_m\right)^{Y_4} \left(C'_s\right)^{Y_5}$$

Subject to:

$$Y_1 + Y_2 = 1$$

$$-Y_1 + (\alpha_j - 1) Y_2 + (\alpha_j - 1) Y_3 + bY_4 + gY_5 = 0$$

$$-Y_1 + (\beta_j - 1) Y_2 + (\beta_j - 1) Y_3 + cY_4 + hY_5 = 0$$

$$Y_1, Y_2, Y_3, Y_4, Y_5 \geq 0$$

The dual problem is solved by using the complementary slackness conditions in conjunction with the primal and dual constraints. Each of the constraints of the primal problem can be either loose or tight at optimality and the corresponding solution should be feasible in both the dual and primal problems. Since we have three constraints in the primal problem, there are eight different cases for the dual, but only six of them are feasible as implied by Theorem 1. Thus, the machining conditions should always be set to a point on the boundary of the feasible region as shown in figure 1.

Theorem 1: *In the constrained SMOP, at least one of the surface roughness or machine power constraints must be tight at the optimal solution.*

Proof: There are only two possibilities where both constraints can be loose at optimality. (1) Only the tool life constraint is tight. Then the dual variables Y_4 and Y_5, which correspond to the machine power and surface roughness constraints, respectively, are both equal to zero due to the complementary slackness conditions. Therefore, they can be eliminated from the set of linear equations in the dual problem. We also know that the inequality of, $\alpha_j > \beta_j, \gamma_j > 1$, always holds for the extended Taylor's tool life expression, T_{ij}, as shown by Gorczyca (1987). Since $\alpha_j \neq \beta_j$, the solution for this case is $Y_1 = 0, Y_2 = 1$ and $Y_3 = -1$. Therefore, this case is unfeasible since $Y_3 < 0$. As a

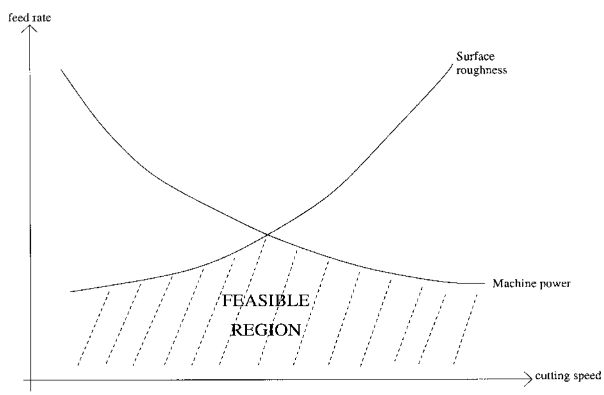

Figure 1. Feasible region.

result, the tool life constraint *cannot* be tight just itself. (2) All the constraints are loose, i.e. $Y_3 = Y_4 = Y_5 = 0$. This system is unfeasible since α_j and β_j cannot be equal to each other, which makes the system of equalities inconsistent. Therefore, the occurrence of such a case in constrained SMOP is also impossible. The remaining cases include one of the mentioned constraints.

The exact solution for the extended version of SMOP can be found by solving each of the aforementioned six cases for the worst case. Lets look at one of the remaining six cases to show how we derived closed form expressions for primal and dual variables. If both the tool life and surface roughness constraints are tight then Y_3 and Y_5 should be non-negative because of the dual feasibility constraints. Furthermore the machine power constraint is loose, so the corresponding dual variable Y_4 is equal to zero due to the complementary slackness conditions. Therefore, the following system can be written by using the complementary slackness conditions:

$$C'_t v_{ij}^{(\alpha_j - 1)} f_{ij}^{(\beta_j - 1)} = 1$$

$$C'_s v_{ij}^{g} f_{ij}^{h} = 1$$

By taking the logarithmic transform, the above system turns to a system of linear equations with two equations and two unknowns, which is solved for v_{ij} and f_{ij}, as follows:

$$v_{ij} = \exp\left(\frac{h \ln(1/C'_t) - (\beta_j - 1)\ln(1/C'_s)}{h(\alpha_j - 1) - g(\beta_j - 1)}\right)$$

$$f_{ij} = \exp\left(\frac{(\alpha_j - 1)\ln(1/C'_s) - g \ln(1/C'_t)}{h(\alpha_j - 1) - g(\beta_j - 1)}\right)$$

where $h(\alpha_j - 1) - g(\beta_j - 1) \neq 0$, since $g < 0$, $\alpha_j, \beta_j > 1$ and $h > 0$. After finding v_{ij}, f_{ij} and corresponding M_{ij}, dual variables Y_1 and Y_2 can be calculated as they give the weight of each term in the primal objective function:

$$Y_1 = \frac{C_1 v_{ij}^{-1} f_{ij}^{-1}}{M_{ij}} \quad \text{and} \quad Y_2 = 1 - Y_1$$

If the solution is dual feasible in terms of Y_1 and Y_2, i.e. $0 \leq Y_1, Y_2 \leq 1$, then the following system is solved for Y_3 and Y_5:

$$(\alpha_j - 1)Y_3 + gY_5 = Y_1 - (\alpha_j - 1)Y_2$$

$$(\beta_j - 1)Y_3 + hY_5 = Y_1 - (\beta_j - 1)Y_2$$

The overall solution for this case is dual feasible if $Y_3, Y_5 \geq 0$. Therefore, we can find the exact solution very quickly since the explicit analytic expressions of the solution in each case are derived due to the proposed decomposition procedure. As a result, the proposed approach finds the optimum machining conditions after solving J_i problems for each operation $i \in I$ and has a polynomial time complexity of $O(IJ_i)$.

3.2. *Algorithm*

The following algorithm is proposed to reduce the initial candidate tool set to a single tool for every operation, by considering the tool availability constraints, and to determine the optimum tool allocation and machining conditions for every operation. The steps of the proposed algorithm can be summarized as follows. In step 1, we solve SMOP for all possible operation–tool pairs. In step 2, we propose a new cost measure to extend the results of SMOP to handle the multiple operations and find the global minimum of the proposed cost measure for every possible operation–tool pair. The best tool allocation is determined in step 3, which also provides a lower bound for this problem. In step 4, we check the tool availability constraint, if it is violated for any tool type then the possible tool requirement levels and their costs are calculated in step 5. An optimum solution is found in step 6. A numerical example is given in the next section.

Step 1. For every possible operation (i, j), such that $j \in J_i$, solve SMOP using the procedure defined above, and p_{ij} values are initially equal to $\lceil B/N_j \rceil$ to ensure the feasibility in terms of the tool availability constraint. Then, update p_{ij} according to the optimum v_{ij}, f_{ij} and U_{ij}, and calculate the corresponding n_{ij}.

Step 2. In the multiple operation case, a lower cost measure can be obtained while increasing the cost of SMOP, M_{ij}, due to a possible decrease in tool waste and tool replacement costs. Therefore, for every operation (i, j), the minimum cost measure must be searched among the possible p_{ij} and n_{ij} pairs. The following cost measure is proposed to rank a set of alternative tools for a particular operation in terms of their desirability for this operation.

$$\overline{C}_{ij} = BM_{ij} + C_o\left[(n_{ij} - 1)t_{r_j} + t_{l_j}\right]$$
$$+ C_{t_j}\lfloor B/p_{ij} \rfloor (1 - p_{ij} U_{ij})$$

where the first term projects the cost of SMOP over the batch, while the second and third terms account for operating costs due to the non-machining time components and the tool waste cost, respectively. Therefore the initial n_{ij} value is decreased to the next alternative n'_{ij} setting, which corresponds to a different p'_{ij} and U'_{ij} pair, and the cost measure is evaluated for the new parameters. The proposed cost measure is a convex function of the integer n_{ij} values, provided that $p_{ij}U_{ij} \le p'_{ij}U'_{ij}$ for $n'_{ij} < n_{ij}$. The convexity of the proposed cost measure is proven in theorem 2 given in the Appendix. This theorem implies that if an increase in the cost measure is found then we stop and the previous solution corresponds to the global minimum.

Step 3. Create a primal tools set, J_p, such that $J_p = \{j \mid \arg\min_{j \in J_i} \bar{C}_{ij}$ for every $i \in I\}$. For every $j \in J_p$, define the corresponding set of operation assignments, I_j, such that $I_j = \{i \mid j \in J_i$ and $\arg\min_{i \in I} \bar{C}_{ij}$ for every $j \in J_p\}$.

Lower bound is equal to:

$$\mathrm{LB} = \sum_{j \in J_p} \sum_{i \in I_j} \bar{C}_{ij}.$$

Step 4. For every $j \in J_p$, calculate the total tool requirement, $R_j = \sum_{i \in I_j} n_{ij}$. If $R_j \le N_j$ for every $j \in J_p$ then solution is optimum, STOP.

Step 5. Since the tool availability constraint is violated, a reduction in their tool requirements is needed, and in this case, the alternative tools should also be considered because a possible increase in the cost of SMOP due to a reduction of tool usage might justify the use of them. Therefore, solve SMOP for the requirement level, $k \in \{1, 2, \ldots, n_{ij}\}$, of every operation (i, j) to find p^k_{ij}, U^k_{ij}, and the corresponding M^k_{ij}. Evaluate the following cost measure for every operation–tool pair (i, j) at the tool requirement level k.

$$\bar{C}^k_{ij} = BM^k_{ij} + C_o\left[(k-1)t_{r_j} + t_{l_j}\right] + C_{t_j}\lfloor B/p^k_{ij} \rfloor (1 - p^k_{ij}U^k_{ij})$$

Step 6. Solve the following IP to find the best allocation for every operation that satisfies the tool availability constraints:

$$\text{Minimize} \quad \sum_{i \in I} \sum_{j \in J_i} \sum_{k=1}^{n_{ij}} \bar{C}^k_{ij} x^k_{ij}$$

Subject to:

$$\sum_{j \in J_i} \sum_{k=1}^{n_{ij}} x^k_{ij} = 1 \;\; \forall i \in I$$

$$\sum_{i \in I} \sum_{k=1}^{n_{ij}} k x^k_{ij} \le N_j \;\; \forall j \in J$$

where x^k_{ij} is a 0–1 binary decision variable which is equal to 1 if the machining of volume i is assigned to tool j at the tool requirement level of k tools. In this formulation, the first constraint ensures that a single allocation will be selected for each operation. The second constraint guarantees that total number of tool allocations will not exceed the tool availability constraints.

4. A numerical example

In this section, an example part is studied which has twelve pre-specified machinable volumes as shown in figure 2 with the geometrical data and the required surface qualities given in table 1. Each machinable volume, V_i, can be machined by a set of candidate tools denoted by an operation–tool pair (i, j). There are six different cutting tool types available. Their technological parameters and the other input data are presented in tables 2 and 3, respectively.

The possible operation–tool assignments are given by the following 0–1 matrix Y:

$$Y = \begin{bmatrix} 0&0&1&0&0&0&1&0&0&0&1&1 \\ 0&0&1&0&0&0&1&0&0&0&1&1 \\ 1&1&1&1&1&1&1&1&1&1&0&0 \\ 1&1&0&1&1&1&0&1&1&1&0&0 \\ 1&1&1&1&1&1&1&1&1&1&0&0 \\ 0&0&0&0&0&0&1&0&0&0&1&1 \end{bmatrix}^{\mathrm{T}}$$

In the first two steps of the algorithm, the best machining conditions for all possible operation–tool pairs are determined for different n_{ij} values. In table 4, this procedure is illustrated for the Volume-11 and Tool-6 pair, i.e. operation (11, 6), as an example. At the end of step 1, $n_{11,6}$ was equal to 3. In the multiple operation case, the optimal solution of the SMOP may

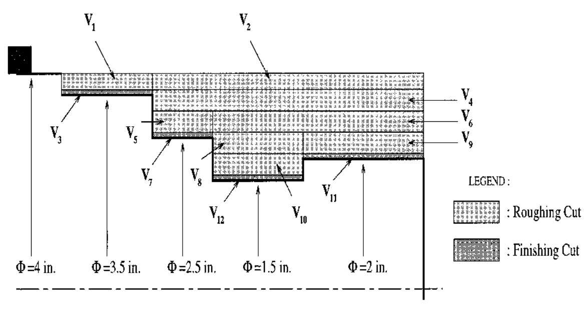

Figure 2. Machinable volume presentation.

not correspond to the minimum of proposed cost measure as illustrated in table 4 for the operation (11, 6). We found a better solution by decreasing the number of tool requirements, which slightly increased the cost of SMOP but decreased the overall cost measure for the multiple operation case. Furthermore, we can easily conjecture that the proposed cost measure, $\overline{C}_{ij}$, is more effective than the SMOP approaches, which do not consider the non-machining time components and the tool waste cost.

In step 3, the following sets are formed by using the best machining operation conditions for every possible pair: $I_3 = \{1, 2, 4, 5, 6, 8, 9, 10\}$, $I_5 = \{3\}$, $I_6 = \{7, 11, 12\}$ and $J_p = \{3, 5, 6\}$. Therefore, a lower bound on the minimum cost value is equal to 119.84. In step 4, we check the tool availability constraint for every $j \in J_p$ as follows:

$$R_3 = n_{1,3} + n_{2,3} + n_{4,3} + n_{5,3} + n_{6,3} + n_{8,3} + n_{9,3} + n_{10,3}$$

$$= 3 + 6 + 6 + 2 + 4 + 2 + 3 + 2 = 28 > N_3 = 20$$

$$R_5 = n_{3,5} = 2 < N_5 = 4$$

$$R_6 = n_{7,6} + n_{11,6} + n_{12,6} = 1 + 2 + 1 = 4 > N_6 = 2$$

Since the tool availability constraints are violated for tools 3 and 6, we calculate the tool requirement levels and their cost values in step 5. The optimum tool allocations with the corresponding machining conditions found in step 6 are given in table 5, where the total production cost is equal to 122.06. The final tool allocation is also represented by the following sets: $I_3 = \{2, 3, 4, 6, 7, 9\}$, $I_4 = \{8\}$, $I_5 = \{1, 5, 10\}$, $I_6 = \{11, 12\}$ and $J = \{3, 4, 5, 6\}$. When we analyse the optimum solution for the allocation of Tool-6, this solution suggests to use Tool-1 for the manufacturing of Volume-7 instead of Tool-6, a reduction of a single Tool-6 in the processing of the Volume-11, and it leaves the SMOP solution for the Volume-12 without any reduction in the usage of Tool-6. As a summary, the initial solution of SMOP was inferior to the proposed cost measure for the multiple operation case as indicated in table 4, and it was also infeasible due to tool availability constraint resulting from the tool contention among the operations for a limited number of tools.

5. Computational results

The SMOP algorithm presented earlier and the matrix generator for the problem formulation were coded in C language and compiled with the Gnu C compiler. An optimal solution was found by using the CPLEX MIP solver on a SPARC Station 10 under SunOS 5.4. In this section, the efficiency of the proposed exact approach for the tool allocation and machining conditions optimization problem is tested in terms of the computation time to find an optimal solution.

Table 1. Machinable volume data.

$V\#$	D_i	L_i	d_i	S_i	$V\#$	D_i	L_i	d_i	S_i
V_1	4	3	0.2	300	V_7	2.6	2	0.05	50
V_2	4	9	0.2	400	V_8	2.6	3	0.25	400
V_3	3.6	3	0.05	75	V_9	2.6	4	0.25	300
V_4	3.6	9	0.25	400	V_{10}	2.1	3	0.25	300
V_5	3.1	2	0.25	300	V_{11}	2.1	4	0.05	40
V_6	3.1	7	0.25	400	V_{12}	1.6	3	0.05	30

Table 3. Tooling information.

	T_1	T_2	T_3	T_4	T_5	T_6
t_{rj}	0.75	0.75	0.75	0.75	1	0.75
t_{lj}	1	1	1	1	1.5	0.75
N_j	2	3	20	10	4	2
C_{tj}	0.50	0.70	0.70	0.70	0.75	0.75

Table 2. Technological exponents and coefficients of the available tools.

$T\#$	α	β	γ	C_j	b	c	e	C_m	g	h	l	C_s
T_1	4.0	1.40	1.16	40960000	0.91	0.78	0.75	2.394	− 1.52	1.004	0.25	204620000
T_2	4.3	1.60	1.20	37015056	0.96	0.70	0.71	1.637	− 1.60	1.005	0.30	259500000
T_3	3.7	1.28	1.05	11001020	0.80	0.75	0.70	2.415	− 1.63	1.052	0.30	205740000
T_4	4.1	1.26	1.05	48724925	0.80	0.77	0.69	2.545	− 1.69	1.005	0.40	204500000
T_5	3.7	1.30	1.05	13767340	0.83	0.75	0.73	2.321	− 1.63	1.015	0.30	203500000
T_6	4.2	1.65	1.20	56158018	0.90	0.78	0.65	1.706	− 1.54	1.104	0.32	211825000

Table 4. Finding the minimum cost measure for operation (11,6).

n_{ij}	p_{ij}	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	M_{ij}	$\bar{C}_{ij}$
3	12	659.02	0.01655	0.2015	2.5721	0.0784	0.1595	6.00
2	15	633.60	0.01567	0.2214	3.3217	0.0667	0.1607	5.57
1	30	535.20	0.01238	0.3318	9.9528	0.0333	0.1909	6.10

Table 5. Optimum tool allocation and machining conditions.

V#	T#	p_{ij}	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	M_{ij}	n_{ij}	$\bar{C}_{ij}$
1	5	16	286.08	0.02548	0.4308	7.1731	0.0601	0.2604	2	9.09
2	3	5	256.73	0.03189	1.1506	5.9650	0.1929	0.7103	6	23.83
3	3	15	475.57	0.02507	0.2370	3.5554	0.0667	0.1652	2	5.83
4	3	6	236.50	0.02635	1.3604	8.1623	0.1667	0.7969	5	25.91
5	5	30	270.56	0.02181	0.2749	8.2552	0.0333	0.1616	1	5.60
6	3	8	242.92	0.02747	0.8510	7.0095	0.1214	0.5105	4	17.00
7	3	30	498.20	0.01833	0.1490	4.4712	0.0333	0.0979	1	3.44
8	4	15	214.75	0.03025	0.3142	4.7125	0.0667	0.2038	2	6.99
9	3	15	259.98	0.02321	0.4509	6.7640	0.0667	0.2721	2	9.04
10	5	30	270.56	0.02181	0.2793	8.5375	0.0327	0.1642	1	5.69
11	6	30	535.20	0.01238	0.3318	9.9528	0.0333	0.1909	1	6.10
12	6	30	639.16	0.01222	0.1608	4.8244	0.0333	0.1054	1	3.54

Table 6. Experimental factors.

Factors	Definition	Low	High
A	Number of operations	50	100
B	Number of tool types	6	10
C	Assignment matrix	Random	Clustered
D	Tool availability	80%	60%
E	Tooling cost variability	UN $\sim$[1.2,1.6]	UN $\sim$[0.6,2.2]

There are five experimental factors that can affect the efficiency of the proposed algorithm, which are listed in table 6. Both the number of operations and the cutting tool types are most likely to affect the computation times since they directly affect the total number of possible operation–tool pairs. The third factor determines the assignment matrix, i.e. random or clustered. At the random level, each cutting tool type can be assigned to a candidate tool set of each operation with an equal probability. But in the clustered case, 80% of the operations are taken to be roughing operations whereas the remaining 20% are taken to be finishing operations. The fourth factor directly specifies the tightness of the tool availability constraints. The number of available tools on hand for tool type j, N_j, is taken as 80% or 60% of the required number of tools for each tool type at low and high levels, respectively. As a result, the tool availability constraint was always violated in step 4 so we had to solve the IP formulation given in step 6. Finally, the fifth factor gives the tooling cost variability. Since there are five factors and two levels, our experiment is 2^5 full-factorial design, which corresponds to 32 treatment combinations. The number of replications of each combination is taken as five, that gives 160 different randomly generated runs.

Other variables in the system were treated as fixed parameters and generated as follows:

- System related parameters, $B = 30$ parts, $C_o = \$0.5/\text{min}$, and $H = 5$ hp.
- Operation related parameters, D_i and L_i were selected randomly from the interval UN$\sim$[1.5, 3] and UN$\sim$[4, 8], respectively, where UN stands for the uniform distribution.
- The values of S_i and d_i were related with the assignment matrix. For random assignment matrix, $S_i =$ UN$\sim$[30, 500] and $d_i =$ UN$\sim$[0.025, 0.3]. In the clustered case, there were two types of operations, namely roughing and finishing. For roughing operations, $S_i =$ UN$\sim$[300, 500] and $d_i =$ UN$\sim$[0.2, 0.3]. For finishing operations, $S_i =$ UN$\sim$[30, 70] and $d_i =$ UN$\sim$[0.025, 0.075].
- Tool related technological exponents were already given in table 2. t_{r_j} and t_{l_j} were selected

randomly from the interval UN~[0.75, 1.0] and UN~[1.0, 1.5], respectively.

Table 7 summarizes the CPU times (in seconds) to find the optimum solution for each run, along with the minimum, average and maximum CPU times (based on five random replications) for each factor combination. In this table, low and high levels for each factor are represented by 0 and 1, respectively. For all 160 problems reported in this table, the maximum CPU time was 11.45 s, whereas the average time was 1.11 s. The maximum CPU time was found for the factor combination of (1 0 1 1 0). In other words, the number of operations and the restriction on the tool availability constraints were at their high levels, and the initial tooling cost variability and the number of tool types were at their low levels. On the other hand, the minimum CPU time of 0.06 s found for a clustered assignment matrix with a high initial tooling cost variability and other factors were at their low levels, i.e. (0 0 1 0 1). As mentioned above, the levels of the fourth factor were selected in a way that the tool availability constraint was always binding for at least one of the tool types. Therefore, we had to solve an IP formulation in each run. In order to give an idea about the size of the IP formulation, the range of the number of 0–1 variables were between 1000 and 5000 for all runs.

Table 7. Results of the computational experiments.

Factors					CPU Times (seconds)		
A	B	C	D	E	Minimum	Average	Maximum
0	0	0	0	0	0.23	0.59	1.17
1	0	0	0	0	0.64	1.36	2.61
0	1	0	0	0	0.30	1.13	2.69
1	1	0	0	0	0.37	1.54	5.57
0	0	1	0	0	0.06	0.09	0.15
1	0	1	0	0	0.11	0.14	0.17
0	1	1	0	0	0.09	0.21	0.42
1	1	1	0	0	0.22	0.43	0.98
0	0	0	1	0	0.29	0.61	0.87
1	0	0	1	0	0.91	1.53	2.24
0	1	0	1	0	0.42	1.47	3.72
1	1	0	1	0	0.36	1.20	3.44
0	0	1	1	0	0.07	0.10	0.18
1	0	1	1	0	0.15	2.48	11.45
0	1	1	1	0	0.10	0.36	0.92
1	1	1	1	0	0.25	0.95	3.16
0	0	0	0	1	0.12	0.73	2.20
1	0	0	0	1	0.70	1.55	3.67
0	1	0	0	1	0.13	2.49	3.78
1	1	0	0	1	0.51	2.68	10.38
0	0	1	0	1	0.06	0.08	0.09
1	0	1	0	1	0.12	0.16	0.23
0	1	1	0	1	0.08	0.38	1.01
1	1	1	0	1	0.16	0.24	0.33
0	0	0	1	1	0.56	2.97	9.99
1	0	0	1	1	0.36	1.59	2.81
0	1	0	1	1	0.33	3.25	5.42
1	1	0	1	1	0.78	3.04	10.90
0	0	1	1	1	0.09	0.11	0.13
1	0	1	1	1	0.18	0.47	0.94
0	1	1	1	1	0.36	1.02	2.99
1	1	1	1	1	0.30	0.45	0.75
overall					0.06	1.11	11.45

Finally, a two-way analysis of variance (ANOVA) test was applied on two performance measures of the optimum value of the total production cost and the computation time to test the equality of observed responses from the different treatments of the chosen factors. As expected, factors A, B, C and D were found to be significant at the 0.5% significance level, whereas factor E is only significant at the 25% level, on the total production cost. For a combination of factors, the interactions AB and AC, which directly affect the number of possible operation–tool pairs and the assignment matrix, were found to be significant at the 0.5% significance level. For the computation time criterion, factor C was the only significant one at the 0.5% significance level. When factor C was at the high level, i.e. clustered case, the overall problem was decomposed into two separate problems for roughing and finishing operations, which reduced the number of possibilities. For the remaining factors, factor D was significant at the 10% significance level and the others were not statistically significant on the computation time to find the optimum solution, which also indicated the robustness of the proposed algorithm to changing conditions of the experimental factors.

Another important question is the sensitivity of machining conditions and tool allocation–selection decisions with respect to the technological coefficients of the usual machining operation constraints. In the literature, the manufacturing optimization problems are solved for a given set of fixed technological coefficients as indicated earlier in an example problem in table 2. However, these coefficients are different for each change in work material, tool material, tool form and shape, size and shape of cut, machine tools used, and cutting fluid. Their values have been determined empirically for many specific conditions and are given in reference books and handbooks. Therefore, we performed another 2^9 full-factorial design for the factor combination of (1 1 1 0 0) giving 2560 different randomly generated runs for the representative ranges of 9 technological coefficients as summarized in table 8. ANOVA tests were applied on three performance measures of lower bound, optimum value and computation time. Our results indicated that all of the factors were significant on all three measures as shown in table

Table 8. Evaluation of technological coefficients.

Constraints	Factors	Low	High
Tool life	α	UN~[2.8, 3.0]	UN~[3.2, 3.4]
	β	UN~[1.25, 1.30]	UN~[1.35, 1.40]
	C_j	UN~[10000000, 20000000]	UN~[30000000, 40000000]
Horsepower	b	UN~[0.81, 0.87]	UN~[0.91, 0.97]
	c	UN~[0.70, 0.73]	UN~[0.77, 0.80]
	C_m	UN~[1.5, 1.8]	UN~[2.3, 2.6]
Surface finish	g	UN~[1.50, 1.55]	UN~[1.65, 1.70]
	h	UN~[1.00, 1.02]	UN~[1.08, 1.10]
	C_s	UN~[200000000, 210000000]	UN~[220000000, 230000000]

Table 9. F values and significance levels (p) for ANOVA results.

	Lower bound		Optimum		Comp. time	
Factors	F	p	F	p	F	p
α	86.3	0.000	89.6	0.000	94.7	0.013
β	4.5	0.034	4.7	0.030	5.3	0.022
C_j	21.4	0.000	23.6	0.000	65.6	0.000
b	68092.0	0.000	69585.6	0.000	35.5	0.000
c	17521.1	0.000	17957.2	0.000	20.5	0.000
C_m	37218.2	0.000	37933.6	0.000	37.1	0.000
g	690.5	0.000	715.6	0.000	41.1	0.000
h	98.6	0.000	101.1	0.000	8.1	0.004
C_s	8.9	0.003	9.4	0.002	3.0	0.086

9. Consequently, the optimum solution and the corresponding computation time are dependent on the operational and tooling parameters.

6. Conclusions

In this paper, an exact approach is presented for solving the tool allocation and machining conditions selection problems simultaneously to find the minimum production cost, where alternative tools can be used for each operation. For this purpose, the classical SMOP formulation is extended by adding a new tool life constraint, which enabled us to include tooling issues like tool wear and tool availability. Furthermore, a new cost measure is proposed to exploit the interaction between the number of tools required with the machining, tool replacing and loading times, and tool waste cost in conjunction with the optimum machining conditions for alternative operation–tool pairs. Consequently, the proposed algorithm can prevent any unfeasibility that may occur for the tool allocation problem at the system level due to tool contention and tool life restrictions through a feedback mechanism. As indicated in the example problem, a decision made at a higher-level without considering its impact on the lower-levels can lead to unfeasible or inferior results when we consider both constraints and parameters of the lower-level problems. As a final point, an effective tool management is a major requirement for the implementation of an FMS, hence the CNC machine tools as stated by several authors. In the automated environments, sophisticated computerized decision making tools are needed for effective operation and control of the system. In this respect, this study can be considered as a part of the fully automated process planning system.

Appendix

Theorem 2: *The following cost measure is a convex function of the integer n_{ij} values:*

$$\bar{C}_{ij} = BM_{ij} + C_o\left[(n_{ij} - 1)t_{r_j} + t_{l_j}\right]$$

$$+C_{t_j}\lfloor B/p_{ij}\rfloor(1 - p_{ij}U_{ij})$$

provided that $p_{ij}U_{ij} \leq p'_{ij}U'_{ij}$ for $n'_{ij} < n_{ij}$.

Proof: To prove this theorem, the following properties of the convex functions will be devised: (i) a linear function is convex and (ii) the sum of convex functions is also convex. The proposed cost measure has three components, namely, SMOP, operating cost due to non-machining events, and tool waste cost. The SMOP component is a convex function since its Hessian matrix is positive definite over the possible values of v_{ij} and f_{ij}, hence the integer n_{ij} values (Bazaraa *et al.* 1993). The non-machining time component is a linear function of the integer n_{ij} values, so it is a convex function due to the first property. The third component of the measure is the tool waste cost. Let's consider two consecutive integer tool requirements such that $n'_{ij} < n_{ij}$ and $n_{ij} - n'_{ij} \geq 1$. We can write the following statement in general:

$$\lfloor B/p_{ij} \rfloor = \begin{cases} n_{ij} & \text{if } B/p_{ij} \in Z^+ \\ n_{ij} - 1 & \text{otherwise} \end{cases}$$

Now, consider the worst case for these two consecutive tool requirements, such that $\lfloor B/p'_{ij} \rfloor = n'_{ij}$ and $\lfloor B/p_{ij} \rfloor = n_{ij} - 1$. That is, $n_{ij} - n'_{ij} \geq 1 \Rightarrow \lfloor B/p_{ij} \rfloor \geq \lfloor B/p'_{ij} \rfloor$. Therefore the tool waste cost component is a non-decreasing function, i.e. a convex function, if the following condition is satisfied $p_{ij} U_{ij} \leq p'_{ij} U'_{ij}$ for $n'_{ij} < n_{ij}$. Consequently, the proposed cost measure is also a convex function over the integer values of n_{ij} due to the second property.

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