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AN ECONOMIC APPROACH TO ACHIEVEMENT AND IMPROVEMENT INDEXES

(Accepted 1 June 2001)

ABSTRACT. This study proposes a useful alternative to the "aggregate deprivation index" which is used to measure the well-beings of individuals in different countries or geographic locations. Furthermore, we also propose an improvement index which alleviates well known difficulties associated with overtime comparisons of "aggregate deprivation index". While deriving our indexes, we pursued an economic approach to index numbers theory and relied on the assumptions of optimizing behavior. The proposed achievement index has its roots in the theory of quantity indexes whose axiomatic properties are well established. The roots of our improvement index on the other hand, is well grounded in the productivity growth literature. The study also provides a numerical example.

1. INTRODUCTION

The consensus on the deficiency of per capita income as a measure of standard of living has led to a search for better means of measuring the quality of life. While research over the last ten to fifteen years has considerably improved our understanding of alternative measures of quality of life, it has also generated a certain amount of controversy. The issues discussed have centered on two areas. One, how to define the standard of living (Sen, 1985, 1987) and its "constituents" and "determinants" (Dasgupta and Weale, 1992) and two, how to aggregate the different indicators to obtain a commonly acceptable single index of quality of life and then measure its improvement. On either front, the debate is far from being settled.

While there is a common agreement on the definition of standard of living within the conceptual framework whose foundations are laid out by Sen (1985, 1987), the disagreements prevail on the optimal bundle of indicators which would measure the quality of life and its improvement. Dasgupta and Weale (1992), for

Social Indicators Research **56:** 91–118, 2001.

example, argue that the indexes used by the World Bank and United Nations Development Program (UNDP) are more concentrated on the "socioeconomic sphere of life", failing to pay attention to "political and civil spheres". Along the same lines, Bunge (1981) argues that any bundle of social indicators which leaves indicators of "self reliance" (independence) and "fairness" (equity) out will be far from being complete.

Once a set of indicators of well being (e.g., life expectancy at birth, literacy rate, infant mortality (survival) rate) are chosen, the problem is confined to translating these to indexes that would signify the success of a country in provision of quality of life (achievement indexes) and how this improves over time (improvement indexes). The construction of such indexes is subject to scrutiny as well. One important question is with respect to which benchmark achievement should be measured. Should this benchmark be a biological benchmark as in the case of a biologically maximum longevity for the indicator "life expectancy at birth", or should the achievement be measured with respect to a country which is taken as a baseline. One other question is, whether a non-linear relationship between the achievement index and values of indicators is a more preferred property over a linear relationship (Kakwani, 1993). Aggregation over individual indexes is one other issue, which remains unresolved. Disagreements on the weights that should be assigned to individual components of the index still prevail. Most important of all, (as also pointed out by Ivanova et al., 1999; Anand and Ravalion, 1993; McGillivray, 1991) the existing indexes, such as Human Development Index (HDI) of UNDP fail to measure performance comparisons across time, since by construction they are designed to measure performance at a point of time rather than being a measure of over-time comparisons.

In a recent study of the axiomatic properties of the Human Development Index, Ivanova et al. (1999), in a concluding paragraph suggest the following direction of research in this area:

> Clearly, the process of measuring human development is only in its infancy. Further refinements in its construction as well as additional theoretical support as a quantitative measure are needed. . . . Additional research is certainly needed to arrive at an improved index as a measure of one of the most critical aspects of a nation's competitiveness, namely its human capital.

Motivated by this statement, the objective of this study is to propose a useful alternative to the HDI with desirable axiomatic properties. Furthermore, we will propose an improvement index, which alleviates well known difficulties associated with comparisons of HDI overtime.

Our approach is known as the economic approach to index number theory and relies on the assumptions of optimising behavior. While deriving our achievement index, we will heavily rely on the theory of quantity indexes whose axiomatic properties are well established. The roots of our improvement index on the other hand are well grounded in the productivity growth literature. All our measures will depend upon computation of distance functions which are complete characterizations of production technology (Färe and Primont, 1995).

The proposed indexes in this study improve upon the empirical literature on social indicators in two aspects. First, unlike the previous studies which typically produce a synthetic indicator which aggregates over its constituents using artificially assigned weights, our approach implicitly recognise the underlying production process which transforms inputs into private and social goods. The success of country "*i*" in the provision of social goods with respect to another country "*j*" is evaluated by measuring the distance of both countries with respect to a common benchmark, i.e., a best practice technology in the provision of social and private goods. The best practice technology is of course constructed over the observations on inputs, social goods and private goods. Thus, while providing an economic content to social indicators, we exploit the aggregator characteristics of distance function that aggregate over the components with optimally chosen weights determined by the data. Furthermore, distance functions yield index numbers consistent with the axiomatic properties laid out by Fisher (1922). Second, our approach that is based on axiomatic production theory allows us to construct an improvement index, which measures the success of a particular country in expanding its social goods from one year to another. In deriving the improvement index, since we allow the best practise technology to change over time, we can capture the improvement in the performance better than alternative indexes

which have less tolerance for improvement for the best achievers (Ivanova et al., 1999).

The paper unfolds as follows. The following section will introduce the methodology. Section 3 is allocated to a numerical example which compares the indexes that are proposed in this study with the "aggregate deprivation index" used in the construction of HDI. Finally section 4 concludes.

2. METHODOLOGY

The very need for alternative measures of human well-being (rather than per capita GNP itself) arises due to the commonly agreed upon fact that per capita GNP (or GDP) does not translate into human well being. This actually means that economies, using their scarce resources (x), are producing private goods (y) and social goods (s) and that producing more of (y) might be even at the expense of (s) production.[1] To describe the theoretical underpinnings, suppose we observe a sample of K countries each of which use inputs $x = (x_1, \ldots, x_N) \in R_+^N$, to produce a vector of private goods $y = (y_1, \ldots, y_M) \in R_+^M$, and a vector of social goods $s = (s_1, \ldots, s_J) \in R_+^J$. Using the notation at hand, for a particular country k, the technology can be described as all feasible vectors (x, y, s) i.e., $T^k = \{(x^k, y^k, s^k)$: x^k can produce $(y^k, s^k)\}$. If knowledge is freely transferable between countries, one can also assume a common technology[2] i.e., $T^k = T$ for $k = 1, \ldots, K$.

The technology T may be alternatively modeled by output sets $P(x)$, $x \in R_+^N$, each consisting of all vectors (y, s) that can be produced by the input vector x. The output set is assumed to satisfy the standard set of properties which include[3]

P.1 $P(0) = \{0,0\}$.
P.2 $P(x)$ is compact for each $x \in R_+^N$.
P.3 $P(x) \supseteq P(x')$, $x \geq x'$.
P.4 $(y, s) \in P(x)$ and $y' \leq y$ and $s' \leq s$ imply $(y', s') \in P(x)$.

The first property states that one can not produce positive output without any inputs. The second property points out to the fact that scarce inputs can only produce finite output. The third and the

fourth properties impose free disposability of inputs and outputs respectively.[4]

Among alternative approaches, distance functions prove to be a particularly useful tool not only to represent a technology with distinctive characteristics, but also as being a perfect aggregator and a performance measure. Hence, for example, for country k, which is endowed with resource vector x^k and producing private goods y^k and social goods s^k, a sub-vector distance function is defined by[5]

$$D_s^k(x^k, y^k, s^k) = \inf\{\theta^k : (x^k, y^k, s^k/\theta^k) \in P(x)\}.$$

This function expands the social goods vector (i.e., $\theta^k \leq 1$), so that the expanded social goods vector, the input vector and the private goods vector fall on the frontier which is common for all the countries. In other words, this distance function measures the success of a country in expanding its social goods with respect to a frontier common to all countries. Since the common frontier technology $P(x)$ is not observed it has to be constructed over the observations on inputs and outputs of K countries, i.e., $\{(x^k, y^k, s^k)$: $k = 1, \ldots, K\}$. For this purpose we formulate an Activity Analysis or DEA problem[6] that satisfies the properties discussed above.

The DEA or piecewise linear output set (see Färe et al., 1994), is

$$\begin{aligned} P(x) = \{(y, s) : \quad & \sum_{k=1}^{K} z_k y_{km} \geq y_m, \quad m = 1, \ldots, M, \\ & \sum_{k=1}^{K} z_k s_{kj} \geq s_j, \quad j = 1, \ldots, J, \\ & \sum_{k=1}^{K} z_k x_{kn} \leq x_n, \quad n = 1, \ldots, N, \\ & z_k \geq 0, \quad k = 1, \ldots, K\}, \end{aligned}$$

where z_k are the intensity variables, which serve to form the technology from convex combinations of the data.

The sub-vector distance function together with the common frontier technology $P(x)$ help us to construct an achievement index which relies on the construction of a quantity index of social goods.

Intuitively, the quantity index of social goods shows the relative success of an observation, say "*i*", in expanding its social goods[7] (with respect to a common frontier) while using the same level of inputs and producing the same level of private goods as another observation say "*j*". As is the standard convention in the index number literature, "*i*" and "*j*" can refer to observations of a given country – for example in different time periods – or they may refer to different countries in a single time period.

More specifically the quantity index of social goods

$$Q_s(x^0, y^0, s^i, s^j) = \frac{D_s(x^0, s^i, y^0)}{D_s(x^0, s^j, y^0)}$$

compares social goods s^i and s^j given a vector of inputs x^0 and a vector of private goods y^0.

This quantity index, which is essentially a Malmquist quantity index (see Färe and Primont, 1995) satisfies a number of desirable properties due to Fisher (1922). These are:

(1) Homogeneity: $Q_s(x^0, y^0, \lambda s^i, s^j) = \lambda Q_s(x^0, y^0, s^i, s^j)$
(2) Time-reversal: $Q_s(x^0, y^0, s^i, s^j)Q_s(x^0, y^0, s^j, s^i) = 1$
(3) Transitivity: $Q_s(x^0, y^0, s^i, s^j)Q_s(x^0, y^0, s^j, s^t) = Q_s(x^0, y^0, s^i, s^t)$
(4) Dimensionality: $Q_s(x^0, y^0, \lambda s^i, \lambda s^j) = Q_s(x^0, y^0, s^i, s^j)$

As for the improvement index, we will measure the success of a particular country in expanding its social goods from year t to year $t + 1$ measured with respect to a common (world) benchmark technology constructed for the period t. Our improvement index

$$IMP^{t,t+1} = \frac{D_s^{k,t}(x^{k,t}, y^{k,t}, s^{k,t+1})}{D_s^{k,t}(x^{k,t}, y^{k,t}, s^{k,t})}$$

is the ratio of two distance functions where

$$D_s^{k,t}(x^{k,t}, y^{k,t}, s^{k,t+1}) = \inf\{\theta^{k,t+1} : (x^{k,t}, y^{k,t}, s^{k,t+1}/\theta^{k,t+1}) \in P^t(x^t)\}$$

and

$$D_s^{k,t}(x^{k,t}, y^{k,t}, s^{k,t}) = \inf\{\theta^{k,t} : (x^{k,t}, y^{k,t}, s^{k,t}/\theta^{k,t}) \in P^t(x^t)\}.$$

The first distance function shows the success of an observation, say k, in expanding its social goods in year $t + 1$ (with respect to a common frontier which represent the technology at t) while using the same level of inputs and producing the same level of private goods as in year t (i.e., $x^{k,t}$ and $y^{k,t}$). Similarly, the second distance function measures the success of the same observation in expanding its social goods in t period with respect to a common frontier representing the technology at t. Note that, since the distances are measured with respect to the same benchmark (while holding resources and private goods at their year t levels), the ratio provides the improvement[8] in social good provision for observation k.

3. A NUMERICAL EXAMPLE

In constructing an example for the achievement and the improvement indexes proposed in this study we consider a sample of 55 countries for the years 1977, 1980, 1982, 1987 and 1990. The data set includes the OECD countries, developing and newly industrialized countries.[9] We proxy the vector of social goods with infant survival rate,[10] life expectancy at birth (total years), primary school enrolment rate (% gross) and secondary school enrolment rate (% gross). Our proxy for private goods is real gross domestic product. The resource constraint is represented with two aggregate inputs, capital stock and labor. The source for variables which represent social goods, is World Bank Social Indicators Database. Other variables, real gross domestic product, capital stock and employment are retrieved from the Penn World Tables.

In computing the distance functions, we chose the data envelopment analysis (DEA) methodology so as to take advantage of the fact that the distance functions are reciprocals of Farrell efficiency measures.

In this particular application, we chose Australia as our reference country. Thus we are assuming that $j = 0$ which then refers to the associated quantities for Australia. We let $k = 1, \ldots, K$ index the countries in our sample. Thus for a particular year, for each country $k' = 1, \ldots, K$ we may compute

$$(D_y(x^0, s^{k'}, y^0))^{-1} = \max \theta$$

$$
\begin{aligned}
&st\\
&\sum_{k=1}^{K} z_k s_j^k \geq \theta s_j^{k'}, \quad j = 1, \ldots, J,\\
&\sum_{k=1}^{K} z_k y_m^k \geq y_m^0, \quad m = 1, \ldots, M,\\
&\sum_{k=1}^{K} z_k x_n^k \leq x_n^0, \quad n = 1, \ldots, N,\\
&z_k \geq 0, \quad k = 1, \ldots, K
\end{aligned}
$$

which is the numerator for $Q_s(x^0, y^0, s^i, s^j)$. The denominator is computed by replacing $s^{k'}$ on the right hand side of the social goods constraint with the observed social goods for Australia, i.e., s^0. This problem constructs the best practice frontier from the observed data, and computes the scaling factor on social goods required for each observation to attain best practice. Note that, this scaling factor is an aggregate performance measure where weights (z's) are determined optimally using observations on inputs, social goods and private goods over the countries for a particular year.[11]

The achievement index constructed using the methodology above is presented in Table I for the years 1977, 1980, 1982, 1987 and 1990. Note that, figures greater than 1 (and less than 1) represent a better achievement (and an inferior achievement) with respect to Australia (respectively). However, since our index is transitive it allows for bilateral comparisons among all country pairs. To facilitate an easier exposition, for each year, we normalized all the indexes by the value of the best performer,[12] so as to assign a value of 100 for the best achiever. These are provided in Table II. A quick glance over the table shows that, although ranking of individual countries[13] differ considerably from one year to another, Australia, Austria, Belgium, Canada, Denmark, Finland, Ireland, Netherlands, Norway and Portugal have always maintained their position within the best twenty performers. As for the worst performers, our achievement index consistently places Bolivia, Guatemala, India, Israel, Luxembourg, Mauritius, Sierra Leone, Thailand and Zambia among the last 20.

To provide a comparison, in Table III, we also report the scores obtained from the conventional "aggregate deprivation index"[14] (Mazumdar, 1999; Ivanova et al., 1999) used to construct the HDI. A comparison of quantity index for a particular year, with that of aggregate deprivation index, reveals that variation in the aggregate deprivation index is larger.[15] This is as theoretically expected since the quantity index is homogenous of degree one in social goods and the aggregate deprivation index has a larger range.[16] Nevertheless, although by construction the aggregate deprivation index and the quantity index proposed in this study are quite different from each other, for the year 1977, they are in agreement in ranking Finland, Japan, Ireland, Norway, Netherlands, USA, Canada, Austria, Denmark, France and Iceland among the best fifteen and Dominican Republic, Kenya, Zambia, Thailand, Honduras, Zimbabwe, Guatemala, Bolivia, Nigeria, Mauritius, India, Malawi, and Sierra Leone among the worst fifteen.

In an appendix table (Table AI) we report all the Spearman rank correlations between the indexes derived in this study. While the Spearman rank correlation between the aggregate deprivation index and the quantity index is rather high (0.86) in 1977, the same is not true for other years. This of course is due to the differences in methodologies employed to construct these indexes. While our index accounts for the differences in resource use and the provision of private goods across countries, the aggregate deprivation index does not. One other difference worth noting is that, while quantity index produces quite different rankings of countries in subsequent time periods (as evidenced by low Spearman rank correlations) aggregate deprivation index produces more or less the same ranking. Since for a single variable case both the indexes share similar axiomatic properties like scale invariance and translation invariance with respect to the rank,[17] both indexes would produce similar ranking of countries across time if the variable under consideration changes slowly over time. For a multi-variable case while the aggregate deprivation index maintains this characteristic, the quantity index does not, since the weights attached to each component were kept constant through time in the deprivation index while the optimally chosen weights in the quantity index change over time. Thus, one can argue that quantity index satisfies a desirable property (respon-

TABLE I

Achievement indexes – distance function approach

	1977	1980	1982	1987	1990
Argetina	0.892	0.9473	0.9406	1.0257	0.987
Australia	1	1	1	1	1
Austria	0.8182	1.0515	1.0011	1.1342	1.0331
Belgium	0.9659	1.0322	1.0043	1.1693	1.0251
Bolivia	0.7152	0.7732	0.7863	0.8745	0.8793
Canada	1.0114	0.9947	1.0065	1.1342	1.0032
Chile	0.9402	0.9732	0.9353	0.9734	0.9276
Colombia	1.0045	0.9991	0.9176	0.9658	0.9489
Denmark	0.9432	1.19	1.1262	1.2191	1.0879
Dominican Republic	0.7634	1.05	1.0177	0.9306	0.909
Ecuador	0.8518	1.0491	1.0576	1.1426	1.0817
Finland	1.0568	1.1321	1.0949	1.2066	1.1596
France	0.9432	0.992	0.9707	1.0475	1.0074
Germany	0.9886	1.0219	0.9925	1.0866	0.9935
Greece	0.9205	0.922	0.9131	1.0527	0.9295
Guatemala	0.7233	0.7886	0.8009	0.872	0.8567
Honduras	0.7313	0.8777	0.9193	1.058	0.9749
Hong Kong, China	0.9241	0.9554	0.9486	0.9981	0.9471
Iceland	0.9432	0.9731	0.9342	1.0583	0.9923
India	0.6931	0.7619	0.7801	0.903	0.9006
Ireland	1.0455	1.0185	1.0022	1.1206	1.0012
Israel	0.7888	0.8482	0.8599	0.9508	0.8987
Italy	0.8295	0.892	0.8777	0.9249	0.9573
Jamaica	0.7875	0.9205	0.9371	0.9601	0.9406
Japan	1.0568	1.056	1.0033	1.098	0.9664
Kenya	0.7634	1.0286	0.9929	0.9335	0.8821
Korea, Rep.	0.8598	0.9821	0.9486	1.0527	0.9749
Luxembourg	0.7875	0.8508	0.8591	0.9228	0.9011
Madagascar	0.8839	1.1625	1.1197	1.0019	0.9554
Malawi	0.6549	0.7156	0.7279	0.804	0.7848
Mauritius	0.8759	0.8332	0.9007	1.0751	1.0139
Mexico	0.9081	1.075	1.07	1.0951	1.0576
Morocco	0.7082	0.7753	0.7896	0.8664	0.8496
Netherlands	1.0455	1.0503	1.0518	1.3323	1.1905
New Zealand	0.9205	0.9911	0.9681	1.0086	0.9805
Nigeria	0.7122	0.9714	1.0887	0.8506	0.8487
Norway	1.0227	1.0662	1.0475	1.0776	1.0261
Panama	0.9643	0.9509	0.9344	1.0209	0.9861
Paraguay	0.8197	0.9429	0.9415	0.9829	0.9786
Peru	0.9	1.0152	1.0417	1.0998	1.1003
Philippines	0.8679	1	0.9663	1.0456	1.0306
Portugal	0.9402	1.1	1.0887	1.2386	1.1458

TABLE I

Continued

	1977	1980	1982	1987	1990
Sierra Leone	0.6429	0.697	0.7053	0.7639	0.7362
Spain	0.8864	0.9867	0.9787	1.1704	1.0371
Sri Lanka	0.7631	0.9196	0.9309	0.9981	0.9842
Sweden	0.9091	0.9981	0.9418	1.0346	0.9266
Switzerland	0.8104	0.8757	0.8771	1.0187	0.9378
Syrian Arab Republic	0.7875	0.8929	0.9131	1.0551	1.0028
Thailand	0.7511	0.8839	0.8688	0.9221	0.9192
Turkey	0.8438	0.8571	0.9131	1.0361	0.9192
United Kingdom	0.9432	0.9481	0.9159	1.0038	0.9675
United States	1.0227	1.0356	1.0205	1.1093	0.9499
Yugoslavia, FR	0.8977	0.9425	0.84	0.9168	0.887
Zambia	0.7634	0.8027	0.8147	0.9743	0.9164
Zimbabwe	0.7273	0.7918	1.1064	1.2576	1.0743

siveness to changes in its components), by being quite sensitive to weights attached to each component and hence more responsive to even small changes in some variables if importance of these variables change over time.

Since neither of the indexes allow for the analysis of improvement overtime, we refrain from year to year comparisons. However, the analysis of distributions pertaining achievement indexes reveals further information. To derive the distributions of the achievement indexes we employed nonparametric kernel density estimation, for which the preliminaries of the technique are provided in Appendix B. These distributions[18] pertaining to each year, are plotted in Figures 1(a)–1(e) for the quantity index and in Figures 2(a)–2(e) for the aggregate deprivation index. For both the indexes, the distributions for the year 1977 point to a bi-modal structure dividing the countries into low and high achievers. However, over the years, this bi-modal distribution is transformed into a uni-modal one, implying convergence in quality of life as the countries at the lower tails move towards the center. Note that this transformation is faster as measured by the quantity index (from 1977 to 1980), than that of aggregate deprivation index.

TABLE II

Achievement indexes (best = 100)

	1977	1980	1982	1987	1990
Argetina	84.41 (26)	79.61 (33)	83.52 (30)	76.99 (28)	82.91 (22)
Australia	94.63 (9)	84.03 (18)	88.79 (19)	75.06 (34)	84.00 (19)
Austria	77.42 (36)	88.36 (8)	88.89 (18)	85.13 (9)	86.78 (10)
Belgium	91.40 (11)	86.74 (13)	89.18 (15)	87.77 (7)	86.11 (13)
Bolivia	67.68 (50)	64.97 (52)	69.82 (52)	65.64 (50)	73.86 (50)
Canada	95.70 (7)	83.59 (22)	89.37 (14)	85.13 (9)	84.27 (16)
Chile	88.97 (17)	81.78 (27)	83.05 (32)	73.06 (39)	77.92 (39)
Colombia	95.05 (8)	83.96 (20)	81.48 (37)	72.49 (40)	79.71 (34)
Denmark	89.25 (13)	100.00 (1)	100.00 (1)	91.50 (4)	91.38 (5)
Dominican Republic	72.24 (42)	88.24 (10)	90.37 (13)	69.85 (44)	76.35 (44)
Ecuador	80.60 (32)	88.16 (11)	93.91 (8)	85.76 (8)	90.86 (6)
Finland	100.00 (1)	95.13 (3)	97.22 (4)	90.57 (5)	97.40 (2)
France	89.25 (13)	83.36 (23)	86.19 (23)	78.62 (24)	84.62 (15)
Germany	93.55 (10)	85.87 (15)	88.13 (21)	81.56 (16)	83.45 (20)
Greece	87.10 (20)	77.48 (36)	81.08 (39)	79.01 (22)	78.08 (38)
Guatemala	68.44 (49)	66.27 (50)	71.12 (50)	65.45 (51)	71.96 (51)
Honduras	69.20 (47)	73.76 (42)	81.63 (36)	79.41 (20)	81.89 (27)
Hong Kong, China	87.44 (19)	80.29 (30)	84.23 (26)	74.92 (35)	79.55 (35)
Iceland	89.25 (13)	81.77 (28)	82.95 (34)	79.43 (19)	83.35 (21)
India	65.58 (53)	64.03 (53)	69.27 (53)	67.78 (49)	75.65 (46)
Ireland	98.93 (3)	85.59 (16)	88.99 (17)	84.11 (11)	84.10 (18)
Israel	74.64 (38)	71.28 (46)	76.35 (46)	71.37 (42)	75.49 (47)
Italy	78.49 (34)	74.96 (40)	77.93 (43)	69.42 (45)	80.41 (31)
Jamaica	74.52 (39)	77.35 (37)	83.21 (31)	72.06 (41)	79.01 (36)
Japan	100.00 (1)	88.74 (7)	89.09 (16)	82.41 (14)	81.18 (30)
Kenya	72.24 (42)	86.44 (14)	88.16 (20)	70.07 (43)	74.09 (49)
Korea, Rep.	81.36 (31)	82.53 (26)	84.23 (26)	79.01 (22)	81.89 (27)
Luxembourg	74.52 (39)	71.50 (45)	76.28 (47)	69.26 (46)	75.69 (45)
Madagascar	83.64 (28)	97.69 (2)	99.42 (2)	75.20 (33)	80.25 (32)
Malawi	61.97 (54)	60.13 (54)	64.63 (54)	60.35 (54)	65.92 (54)
Mauritius	82.88 (29)	70.02 (47)	79.98 (42)	80.70 (18)	85.17 (14)
Mexico	85.93 (23)	90.34 (5)	95.01 (7)	82.20 (15)	88.84 (8)
Morocco	67.01 (52)	65.15 (51)	70.11 (51)	65.03 (52)	71.36 (52)
Netherlands	98.93 (3)	88.26 (9)	93.39 (9)	100.00 (1)	100.00 (1)
New Zealand	87.10 (20)	83.29 (24)	85.96 (24)	75.70 (31)	82.36 (25)
Nigeria	67.39 (51)	81.63 (29)	96.67 (5)	63.84 (53)	71.29 (53)
Norway	96.77 (5)	89.60 (6)	93.01 (10)	80.88 (17)	86.19 (12)
Panama	91.25 (12)	79.91 (31)	82.97 (33)	76.63 (29)	82.83 (23)
Paraguay	77.56 (35)	79.24 (34)	83.60 (29)	73.77 (37)	82.20 (26)
Peru	85.16 (24)	85.31 (17)	92.50 (11)	82.55 (13)	92.42 (4)
Philippines	82.13 (30)	84.03 (18)	85.80 (25)	78.48 (25)	86.57 (11)
Portugal	88.87 (17)	92.44 (4)	96.67 (5)	92.97 (3)	96.25 (3)

TABLE II

Continued

	1977	1980	1982	1987	1990
Sierra Leone	60.83 (55)	58.57 (55)	62.63 (55)	57.34 (55)	61.84 (55)
Spain	83.88 (27)	82.92 (25)	86.90 (22)	87.85 (6)	87.11 (9)
Sri Lanka	72.21 (45)	77.28 (38)	82.66 (35)	74.92 (35)	82.67 (24)
Sweden	86.02 (22)	83.87 (21)	83.63 (28)	77.66 (27)	77.83 (40)
Switzerland	76.68 (37)	73.59 (43)	77.88 (44)	76.46 (30)	78.77 (37)
Syrian Arab Republic	74.52 (39)	75.03 (39)	81.08 (39)	79.19 (21)	84.23 (17)
Thailand	71.07 (46)	74.28 (41)	77.14 (45)	69.21 (47)	77.21 (41)
Turkey	79.84 (33)	72.03 (44)	81.08 (39)	77.77 (26)	77.21 (41)
United Kingdom	89.25 (13)	79.67 (32)	81.33 (38)	75.34 (32)	81.27 (29)
United States	96.77 (5)	87.03 (12)	90.61 (12)	83.26 (12)	79.79 (33)
Yugoslavia, FR	84.95 (25)	79.20 (35)	74.59 (48)	68.81 (48)	74.51 (48)
Zambia	72.24 (42)	67.45 (48)	72.34 (49)	73.13 (38)	76.98 (43)
Zimbabwe	68.82 (48)	66.54 (49)	98.24 (3)	94.39 (2)	90.24 (7)

To shed light on the dynamics of this transformation, we employed a series of statistical tests for the comparison of distributions across the years. In particular, for both the indicators of quality of life, we employ the nonparametric tests proposed by Fan and Ullah (1999) and Li (1996), to compare the two unknown distributions that belong to sequential years i.e., we test H_0: $f(x_{1977}) = g(x_{1980})$ for all x against the alternative H_1: $f(x_{1977}) \neq g(x_{1980})$ for all x.[19] In Table IV, we report the test statistics and the critical values for all the tests performed over sequential years. For the quantity index, the null hypotheses that there is no difference between the distribution for the year 1977 and the distribution for the year 1980 (i.e., Figure 1(a) and Figure 1(b)) is rejected at all significance levels. However, we fail to reject the null hypotheses for the distributions belonging to sequential year pairs 1980, 1982 and 1982, 1987. As for the last pair 1987, 1990, we reject the null hypotheses that there is no difference between the distributions. Taken together, these hypotheses tests reveal that, if there has been a convergence in quality of life as measured by the quantity index, this has occurred between 1977 and 1980 followed by a stagnant period which lasted until 1987. As for the aggregate deprivation index the hypotheses tests rejected equality of distributions for all sequential pairs except

TABLE III

Aggregate deprivation indexes

	1977	1980	1982	1987	1990
Argetina	76.82 (29)	72.07 (31)	73.83 (31)	73.22 (27)	76.42 (28)
Australia	91.62 (3)	83.93 (14)	85.74 (12)	80.54 (20)	84.67 (15)
Austria	83.16 (20)	83.41 (18)	84.11 (17)	82.88 (9)	86.90 (9)
Belgium	88.05 (16)	85.34 (9)	85.51 (13)	83.80 (8)	86.38 (11)
Bolivia	39.98 (48)	39.12 (49)	41.41 (51)	43.40 (48)	49.54 (47)
Canada	89.66 (8)	84.07 (13)	88.26 (3)	84.57 (5)	87.60 (5)
Chile	75.08 (31)	72.33 (30)	74.16 (30)	73.84 (26)	77.20 (27)
Colombia	73.05 (32)	66.58 (35)	65.81 (39)	63.70 (40)	67.96 (39)
Denmark	88.46 (13)	87.26 (2)	88.23 (4)	83.81 (6)	86.35 (12)
Dominican Republic	57.64 (41)	63.17 (39)	66.17 (38)	62.00 (41)	70.59 (38)
Ecuador	63.43 (36)	65.90 (36)	69.73 (33)	68.87 (35)	71.64 (37)
Finland	90.28 (7)	85.65 (7)	87.33 (7)	84.86 (4)	88.60 (4)
France	91.24 (5)	86.97 (4)	87.53 (6)	84.91 (3)	88.68 (3)
Germany	87.83 (17)	83.77 (15)	84.58 (16)	82.48 (11)	87.21 (7)
Greece	87.13 (18)	82.48 (19)	83.79 (18)	82.01 (14)	83.62 (20)
Guatemala	36.40 (52)	37.43 (53)	37.09 (53)	40.51 (50)	46.92 (49)
Honduras	47.49 (44)	52.43 (43)	56.61 (43)	59.39 (42)	58.17 (45)
Hong Kong, China	82.90 (21)	80.23 (20)	82.05 (21)	79.90 (22)	82.92 (23)
Iceland	89.32 (9)	85.15 (10)	86.34 (9)	82.86 (10)	87.31 (6)
India	37.43 (49)	38.02 (51)	41.96 (50)	44.07 (47)	52.98 (46)
Ireland	90.44 (6)	83.55 (16)	84.68 (15)	82.01 (13)	85.79 (13)
Israel	81.50 (23)	77.37 (24)	80.19 (22)	77.40 (23)	80.29 (25)
Italy	85.11 (19)	79.36 (21)	79.86 (23)	76.56 (24)	83.39 (22)
Jamaica	76.27 (30)	76.29 (27)	77.26 (27)	71.80 (30)	75.62 (30)
Japan	91.71 (2)	87.24 (3)	88.80 (2)	83.80 (7)	86.98 (8)
Kenya	45.38 (45)	51.45 (44)	52.47 (46)	46.28 (46)	48.77 (48)
Korea, Rep.	77.21 (28)	78.33 (23)	79.42 (24)	75.89 (25)	81.19 (24)
Luxembourg	81.14 (24)	76.95 (26)	77.90 (26)	72.88 (29)	76.19 (29)
Madagascar	40.67 (47)	46.78 (46)	48.38 (48)	38.85 (51)	43.11 (51)
Malawi	11.94 (54)	10.82 (54)	11.35 (54)	11.02 (54)	18.73 (54)
Mauritius	71.28 (33)	64.42 (37)	68.25 (35)	69.43 (34)	72.85 (33)
Mexico	68.10 (35)	70.83 (32)	74.30 (29)	70.22 (33)	72.81 (34)
Morocco	36.97 (50)	41.33 (47)	45.81 (49)	42.06 (49)	45.02 (50)
Netherlands	91.80 (1)	86.53 (6)	86.99 (8)	88.16 (1)	91.36 (1)
New Zealand	88.61 (11)	85.55 (8)	85.96 (10)	80.68 (18)	84.58 (16)
Nigeria	32.53 (53)	40.21 (48)	50.69 (47)	33.79 (53)	40.17 (52)
Norway	91.32 (4)	86.75 (5)	87.56 (5)	87.56 (5)	81.11 (17)
Panama	82.12 (22)	74.15 (29)	74.38 (28)	71.49 (31)	74.81 (31)
Paraguay	62.01 (39)	61.00 (40)	62.37 (41)	59.27 (43)	64.28 (41)
Peru	62.58 (38)	63.28 (38)	66.39 (37)	64.93 (39)	72.49 (36)
Philippines	69.95 (34)	68.39 (34)	69.01 (34)	66.85 (36)	72.59 (35)
Portugal	80.30 (27)	75.33 (28)	78.52 (25)	82.04 (12)	84.53 (17)

TABLE III

Continued

	1977	1980	1982	1987	1990
Sierra Leone	2.5 (55)	2.37 (55)	2.95 (55)	3.70 (55)	2.15 (55)
Spain	88.45 (14)	87.33 (1)	89.48 (1)	86.90 (2)	89.88 (2)
Sri Lanka	63.16 (37)	69.99 (33)	72.60 (32)	72.90 (28)	77.75 (26)
Sweden	88.32 (15)	84.51 (11)	85.33 (14)	81.47 (16)	84.45 (18)
Switzerland	80.62 (26)	78.95 (22)	82.53 (20)	80.68 (19)	84.81 (14)
Syrian Arab Republic	60.69 (40)	59.79 (41)	63.52 (40)	65.85 (37)	67.89 (40)
Thailand	54.12 (42)	57.46 (42)	58.98 (42)	56.35 (44)	61.18 (43)
Turkey	52.75 (43)	48.44 (45)	52.55 (45)	56.14 (45)	60.91 (44)
United Kingdom	88.55 (12)	83.50 (17)	83.55 (19)	80.23 (21)	83.55 (21)
United States	89.11 (10)	84.12 (12)	85.86 (11)	81.90 (15)	84.16 (19)
Yugoslavia, FR	80.94 (25)	77.35 (25)	67.03 (36)	71.39 (32)	74.54 (32)
Zambia	41.77 (46)	37.70 (52)	38.95 (52)	35.28 (52)	39.30 (53)
Zimbabwe	36.49 (51)	38.16 (50)	56.26 (44)	65.32 (38)	62.33 (42)

the years 1982 and 1987, implying that convergence is an ongoing phenomena.

Now we turn our attention to the computation of the improvement index proposed in this study. For the numerator of $IMP^{t,t+1}$, for each k', we solve the following linear programming problem:

$$
\begin{aligned}
&(D_s^{k't}(x^{k'}, s^{k'}, y^{k'}))^{-1} = \max \theta^{k',t+1} \\
&st \\
&\sum_{k=1}^{K} z_k s_{kj}^t \geq \theta^{k',t+1} s_{k'j}^{t+1}, \quad j = 1, \ldots, J, \\
&\sum_{k=1}^{K} z_k y_{km}^t \geq y_{k'm}^t, \quad m = 1, \ldots, M, \\
&\sum_{k=1}^{K} z_k x_{kn}^t \leq x_{k'n}^t, \quad n = 1, \ldots, N, \\
&z_k \geq 0, \quad k = 1, \ldots, K.
\end{aligned}
$$

The denominator can be computed in a similar fashion by replacing $\theta^{k',t+1}$ with $\theta^{k',t}$ and $s_{k'j}^{t+1}$ on the right side of the first inequality with $s_{k'j}^t$.

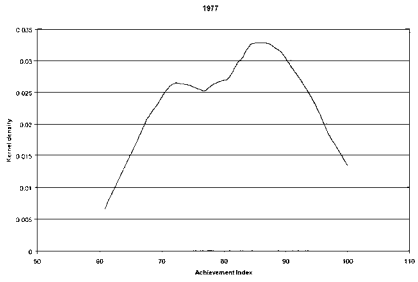

Fig. 1(a) Achievement index 1977

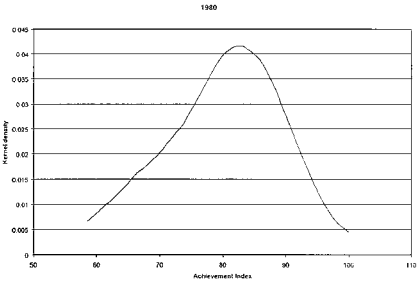

Fig. 1(b) Achievement index 1980

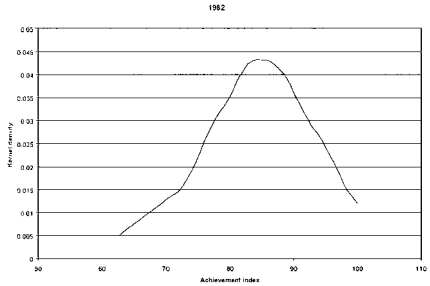

Fig. 1(c) Achievement index 1982

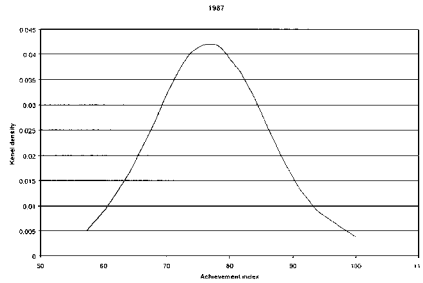

Fig. 1(d) Achievement index 1987

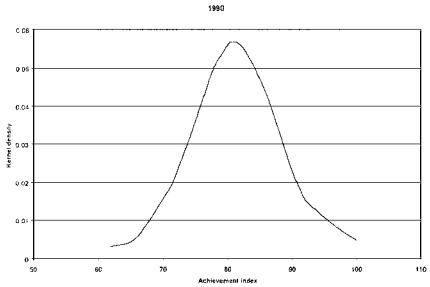

Fig. 1(e) Achievement index 1990

Figure 1. Achievement indexes – distance function approach.

In Table V below we provide the improvement indexes for the sub-periods as well as for the entire period between 1977–1990. The improvement between 1977 and 1990 is computed by the sequential multiplication of the improvements during the sub-periods. An analysis of the figures in Table V reveals that, although improvement indexes exhibit a large variation both between the countries and also from one sub-period to another one, the most significant improvement has been during 1977–1980 period[20] (last row of

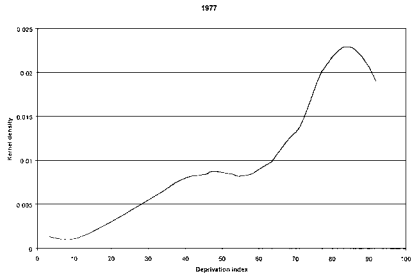

Fig. 2 (a) deprivation index 1977

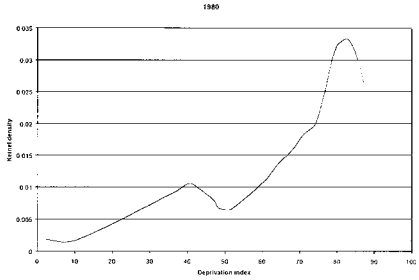

Fig. 2(b) deprivation index 1980

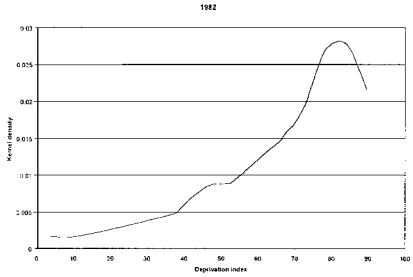

Fig. 2 (c) deprivation index 1982

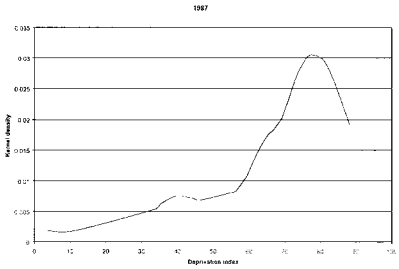

Fig. 2(d) deprivation index 1987

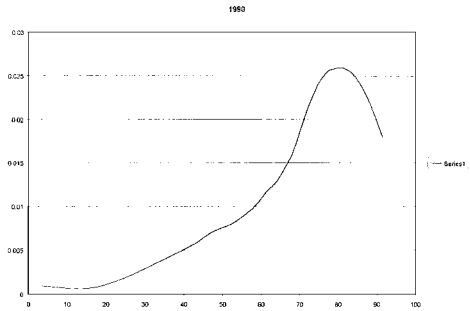

Fig. 2 (e) deprivation index 1990

Figure 2. Achievement indexes – aggregate deprivation indexes.

Table V). Evaluated with respect to the entire time span between 1977 and 1990 (last column in Table V), we observe that 11 countries have shown decline in the quality of life. These are: Colombia, Australia, Chile, Panama, Madagascar, Turkey, Argentina, Nigeria, Hong Kong, New Zealand and United Kingdom. As for the countries which showed improvement, the most striking one is that of Sierra Leone. Note that in spite of the fact that this country

TABLE IV

Hypotheses test on the closeness of distributions

Null hypotheses	Test statistics	10% significance level (critical value = 1.281)	5% significance level (critical value = 1.645)
Kernel distributions: quantity index			
$f(x_{1977}) = g(x_{1980})$	7.216	Reject	Reject
$f(x_{1980}) = g(x_{1982})$	0.181	Do not reject	Do not reject
$f(x_{1982}) = g(x_{1987})$	0.216	Do not reject	Do not reject
$f(x_{1987}) = g(x_{1990})$	8.621	Reject	Reject
Kernel distributions: aggregate deprivation index			
$f(x_{1977}) = g(x_{1980})$	7.793	Reject	Reject
$f(x_{1980}) = g(x_{1982})$	2.580	Reject	Reject
$f(x_{1982}) = g(x_{1987})$	0.776	Do not reject	Do not reject
$f(x_{1987}) = g(x_{1990})$	4.164	Reject	Reject

ranked last with regards to achievement in all the sub-periods, she is the one with the highest improvement score. Other countries which fall into the category of best 15 with respect to improvement are: Paraguay, Denmark, Netherlands, Switzerland, Luxembourg, Zimbabwe, Finland, Belgium, Iceland, Norway, Mauritius, Honduras and India.

In an appendix table (Table AII), we also report the improvement indexes computed as taking the difference between two aggregate deprivation indexes pertaining to two different time periods. We refrain from interpreting and comparing those with the improvement index proposed in this study, since such comparisons would be misleading.[21] Nevertheless, a couple of points are persuasive enough to argue that the index proposed in this study produces estimates of changes in well-being superior to those of computed by taking the difference between two aggregate deprivation indexes. For example, since aggregate deprivation index is scale invariant, a given percent improvement in all variables for all countries from one year to another will produce no improvement for any of the observations, while the proposed index will show an across the board improvement by the same percentage since $IMP^{t,t+1}$ is homogeneous of degree one in $s^{k,t+1}$. One other implausible result

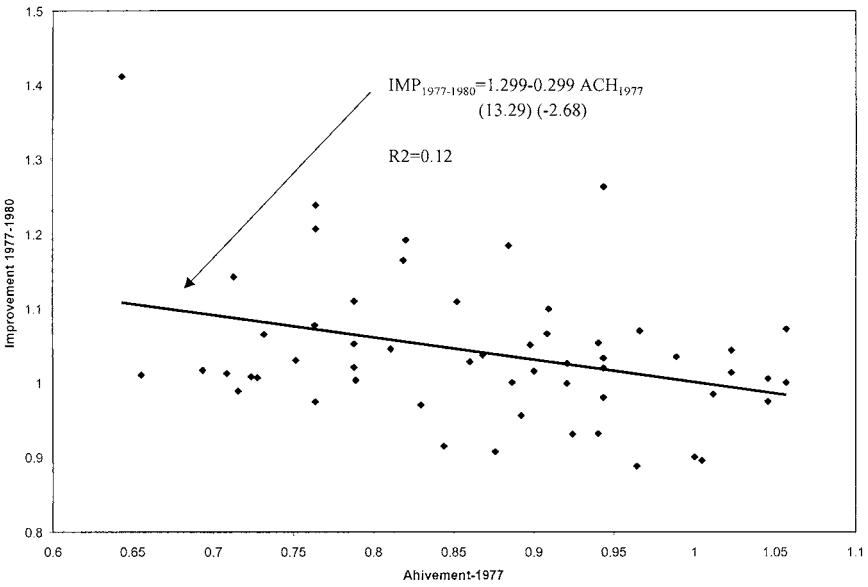

Figure 3. Relation between improvement and achievement 1977–1980.

produced by year by year comparisons of two aggregate achievement indexes is when only the country with lowest achievement succeeds in increasing her social goods proportionately. In this case, while neither the lowest nor the highest achiever will show any change in performance from one year to another as measured by the conventional index, all other countries will seem like deteriorating in performance. The improvement index proposed in this study however, will appropriately record a proportionate improvement for only the relevant country (i.e., the lowest achiever). These peculiar results (which could be extended) stem from trying to use an index constructed to measure performance at a point in time for over time comparisons. The improvement index proposed in this study however, is constructed specially for overtime comparisons and will not have such shortcomings.

In a final analysis, we also investigated if the initial achievement is an important determinant of improvement. Figures 3 and 4 show the separate scatter diagrams and the predicted regression equations, which establish the relation between achievement in 1977 and improvement during periods 1977–1980 and 1977–1990. The negative and significant coefficient of the achievement

TABLE V

Improvement indexes – distance function approach

	1977–1980	1980–1982	1982–1987	1987–1990	1977–1990
Argetina	0.9559 (48)	1 (36)	1.017 (31)	0.9852 (45)	0.95776 (49)
Australia	0.9 (53)	1.0071 (30)	0.9326 (51)	1.0238 (15)	0.86542 (54)
Austria	1.1642 (7)	1.0022 (33)	1.0797 (8)	1.0349 (10)	1.30372 (4)
Belgium	1.0694 (14)	1.0242 (20)	1.1096 (5)	0.9961 (38)	1.21058 (10)
Bolivia	0.9887 (42)	1.0153 (25)	1.0313 (26)	1.0099 (24)	1.0455 (30)
Canada	0.9843 (43)	1.0651 (7)	1.074 (11)	1.005 (31)	1.13159 (16)
Chile	0.9316 (49)	0.9679 (53)	0.9706 (48)	1.002 (32)	0.87649 (53)
Colombia	0.8952 (54)	0.9249 (55)	0.9816 (44)	1.0059 (30)	0.81753 (55)
Denmark	1.2627 (2)	0.9962 (40)	1.0316 (25)	1.0139 (21)	1.31569 (3)
Dominican Republic	1.2379 (3)	0.9762 (48)	0.8528 (53)	1.0824 (4)	1.11547 (19)
Ecuador	1.1085 (10)	1.0153 (25)	1.0075 (36)	0.9692 (49)	1.09898 (24)
Finland	1.072 (13)	1.0181 (23)	1.0502 (20)	1.0919 (1)	1.25153 (9)
France	1.0193 (29)	0.9856 (45)	1.0305 (27)	1.0083 (26)	1.04386 (32)
Germany	1.0345 (23)	1.0222 (21)	1.0435 (22)	1.024 (13)	1.12995 (17)
Greece	0.999 (41)	1.001 (35)	1.051 (18)	0.9774 (46)	1.02725 (37)
Guatemala	1.0079 (35)	1.0052 (32)	1.0163 (32)	1.0083 (26)	1.0382 (36)
Honduras	1.0648 (16)	1.0549 (8)	1.0733 (12)	0.9434 (53)	1.13736 (14)
Hong Kong, China	0.9304 (50)	1 (36)	0.9813 (45)	1.0667 (6)	0.97392 (47)
Iceland	1.0325 (24)	1.0172 (24)	1.0797 (8)	1.0652 (7)	1.2079 (11)
India	1.0164 (30)	1.0287 (18)	1.0591 (16)	1.0239 (14)	1.13383 (15)
Ireland	0.975 (45)	1.0357 (13)	1.0657 (13)	1.0152 (20)	1.09251 (25)
Israel	1.0312 (38)	1.0012 (34)	1.0028 (37)	1.0012 (33)	1.00832 (40)
Italy	0.9699 (47)	0.991 (43)	0.9954 (40)	1.0511 (8)	1.00564 (44)
Jamaica	1.052 (18)	1.0252 (19)	0.9589 (49)	1.0157 (18)	1.05042 (29)
Japan	1 (39)	1 (36)	1.043 (23)	1 (36)	1.043 (33)
Kenya	1.2063 (4)	0.9722 (51)	0.8875 (52)	0.9674 (50)	1.0069 (42)
Korea, Rep.	1.028 (26)	0.9727 (50)	1.0274 (28)	1.0166 (17)	1.04439 (31)
Luxembourg	1.1094 (9)	1.0423 (10)	1.0151 (33)	1.087 (3)	1.27591 (7)
Madagascar	1.1836 (6)	0.97 (52)	0.8345 (54)	0.9763 (47)	0.93538 (51)
Malawi	1.0102 (34)	1.0126 (28)	0.9989 (39)	0.9965 (37)	1.01823 (39)
Mauritius	0.9072 (52)	1.0393 (11)	1.1132 (4)	1.0885 (2)	1.14247 (13)
Mexico	1.0655 (15)	1.0025 (32)	0.9544 (50)	0.9887 (42)	1.00794 (41)
Morocco	1.0121 (33)	1.0308 (16)	0.9926 (42)	1.0065 (29)	1.04228 (34)
Netherlands	1.0054 (37)	1.0541 (9)	1.2072 (1)	1.0153 (19)	1.29896 (5)
New Zealand	1.0259 (27)	0.9838 (46)	0.9716 (47)	0.9953 (39)	0.97601 (46)
Nigeria	1.1419 (8)	1.1287 (3)	0.7324 (55)	1.0259 (11)	0.96841 (48)
Norway	1.0433 (21)	1.0341 (14)	0.9804 (46)	1.0819 (5)	1.14436 (12)
Panama	0.8875 (55)	0.9897 (44)	1.019 (29)	0.9888 (41)	0.88502 (52)
Paraguay	1.1913 (5)	1.1022 (4)	1.0116 (34)	1.0232 (16)	1.3591 (2)
Peru	1.0152 (31)	1.0334 (15)	0.9847 (43)	1.0242 (12)	1.05806 (27)
Philippines	1.037 (22)	0.9732 (49)	1.0092 (35)	1.0454 (9)	1.06473 (26)
Portugal	1.053 (17)	0.9968 (39)	1.0611 (14)	0.947 (52)	1.05473 (28)

TABLE V

Continued

	1977–1980	1980–1982	1982–1987	1987–1990	1977–1990
Sierra Leone	1.4108 (1)	1.1972 (2)	1.1882 (2)	0.989 (40)	1.98481 (1)
Spain	1 (39)	1.0128 (27)	1.0902 (7)	1.0068 (28)	1.11166 (20)
Sri Lanka	1.0771 (12)	1.0194 (22)	1 (38)	1.0095 (25)	1.10843 (21)
Sweden	1.0987 (11)	0.9932 (42)	1.047 (21)	0.9869 (43)	1.12755 (18)
Switzerland	1.0448 (20)	1.0798 (5)	1.125 (3)	1.0111 (23)	1.28329 (6)
Syrian Arab Republic	1.0204 (28)	1.03 (17)	1.0777 (10)	0.973 (48)	1.10209 (23)
Thailand	1.0295 (25)	0.9941 (41)	0.9954 (40)	1.0012 (33)	1.01994 (38)
Turkey	0.9143 (51)	1.0729 (6)	1.0583 (17)	0.9136 (54)	0.94845 (50)
United Kingdom	0.98 (44)	0.9767 (47)	1.0507 (19)	0.9867 (44)	0.99232 (45)
United States	1.0133 (32)	1.0373 (12)	1.0359 (24)	1.0129 (22)	1.10288 (22)
Yugoslavia, FR	1.0506 (19)	0.9392 (54)	1.0184 (30)	1.001 (35)	1.00588 (43)
Zambia	0.9741 (46)	1.0099 (29)	1.0995 (6)	0.9629 (51)	1.0415 (35)
Zimbabwe	1.0066 (36)	1.3612 (1)	1.0601 (15)	0.8745 (55)	1.27024 (8)
Geomean	1.03619	1.02084	1.01848	1.0069	1.08476

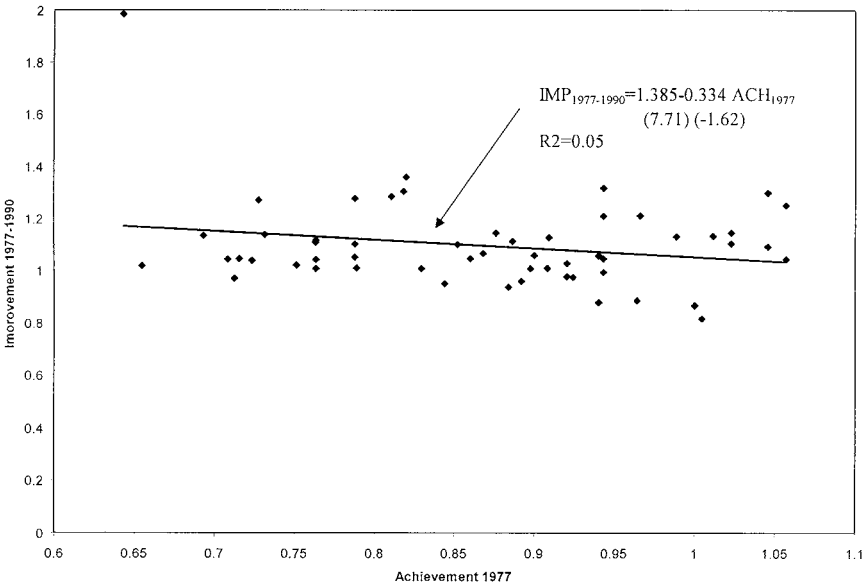

Figure 4. Relation between improvement and achievement 1977–1990.

variable in explaining the improvement during 1977–1980, and the insignificant coefficient of the achievement variable in explaining improvement during the 1977–1990 period, is an indication that convergence in quality of life took place during 1977–1980 which is followed by a rather stagnant period. This in fact, provides supporting evidence for our analysis of the differences between distributions of the achievement indexes.

4. CONCLUSIONS

In this study we provide a useful alternative to the aggregate deprivation index, – an index utilized to measure the well-beings of individuals in different countries or geographic locations. We also propose an improvement index, which alleviates well known difficulties associated with overtime comparisons of "aggregate deprivation index". While deriving our indexes, we pursued a microeconomic approach to index numbers theory and relied on the assumptions of maximizing behavior. The proposed achievement index has its roots in the theory of quantity indexes whose axiomatic properties are well established. Furthermore, the desirable property of the index proposed is that, it aggregates over the constituent indexes without having to impose artificial weights. The roots of our improvement index on the other hand, is well grounded in the productivity growth literature.

The study also provides a numerical example, where more conventional methods of measuring welfare are compared with the proposed index in this study. The analysis of results reveal that, in addition to multilateral comparisons, the distributions of the indexes provide additional insight. The analysis of distribution functions of achievement indexes over the years, in conjunction with the results obtained from improvement indexes, showed that, for this particular sample of countries, there has been convergence in well-being during 1977–1980.

APPENDIX A

TABLE AI

Spearman rank correlations*

	Q77	Q80	Q82	Q87	Q90	DEP77	DEP80	DEP82	DEP87	DEP90
Q77	1.000									
Q80	0.710	1.000								
Q82	0.537	0.859	1.000							
Q87	0.630	0.621	0.702	1.000						
Q90	0.580	0.612	0.887	1.000						
DEP77	**0.863**	0.553	0.379	0.555	0.510	1.000				
DEP80	0.806	**0.587**	0.429	0.580	0.520	0.966	1.000			
DEP82	0.799	0.585	**0.472**	0.636	0.596	0.961	0.986	1.000		
DEP87	0.771	0.554	0.445	**0.692**	0.609	0.926	0.953	0.965	1.000	
DEP90	0.773	0.546	0.424	0.650	**0.600**	0.939	0.957	0.969	0.987	1.000

*Q indicates quantity index, DEP indicates aggregate deprivation index.

TABLE AII

Improvement indexes – aggregate deprivation index approach

	1977–1980	1980–1982	1982–1987	1987–1990	1977–1990
Argetina	−0.048 (44)	0.018 (23)	−0.006 (16)	0.032 (34)	−0.004 (30)
Australia	−0.077 (54)	0.0018 (22)	−0.052 (49)	0.041 (20)	−0.070 (54)
Austria	0.0022 (16)	0.007 (42)	−0.012 (18)	0.040 (22)	0.037 (20)
Belgium	−0.027 (28)	0.002 (51)	−0.017 (20)	0.026 (46)	−0.017 (35)
Bolivia	−0.009 (19)	0.023 (18)	0.020 (9)	0.061 (8)	0.096 (8)
Canada	−0.056 (49)	0.042 (4)	−0.037 (40)	0.030 (38)	−0.021 (39)
Chile	−0.028 (29)	0.018 (20)	−0.003 (14)	0.034 (30)	0.021 (25)
Colombia	−0.065 (51)	−0.008 (54)	−0.021 (24)	0.043 (18)	−0.051 (52)
Denmark	−0.012 (24)	0.010 (36)	−0.044 (46)	0.025 (47)	−0.021 (40)
Dominican Republic	0.055 (5)	0.030 (15)	−0.042 (45)	0.086 (2)	0.130 (4)
Ecuador	0.025 (10)	0.038 (9)	−0.009 (17)	0.028 (44)	0.082 (9)
Finland	−0.046 (42)	0.017 (25)	−0.025 (27)	0.037 (28)	−0.017 (36)
France	−0.043 (38)	0.006 (45)	−0.026 (29)	0.038 (27)	−0.026 (42)
Germany	−0.041 (33)	0.008 (41)	−0.021 (23)	0.047 (16)	−0.006 (32)
Greece	−0.046 (43)	0.013 (30)	−0.018 (21)	0.016 (52)	−0.035 (43)
Guatemala	0.010 (13)	0.003 (53)	0.034 (5)	0.064 (6)	0.105 (6)
Honduras	0.049 (6)	0.042 (5)	0.028 (6)	−0.012 (53)	0.107 (5)
Hong Kong, China	−0.027 (27)	0.018 (21)	−0.022 (25)	0.030 (39)	0.000 (28)

TABLE AII

Continued

	1977–1980	1980–1982	1982–1987	1987–1990	1977–1990
Iceland	−0.042 (36)	0.012 (32)	−0.035 (37)	0.045 (17)	−0.020 (38)
India	0.006 (15)	0.039 (7)	0.021 (8)	0.089 (1)	0.156 (2)
Ireland	−0.069 (53)	0.011 (33)	−0.027 (31)	0.038 (26)	−0.047 (47)
Israel	−0.041 (35)	0.028 (16)	−0.028 (32)	0.029 (43)	−0.012 (34)
Italy	−0.058 (50)	0.005 (47)	−0.033 (35)	0.068 (5)	−0.017 (37)
Jamaica	0.000 (18)	0.010 (37)	−0.055 (51)	0.038 (25)	−0.006 (33)
Japan	−0.045 (40)	0.016 (27)	−0.050 (47)	0.032 (36)	−0.047 (48)
Kenya	0.061 (4)	0.010 (35)	−0.062 (52)	0.025 (49)	0.034 (21)
Korea, Rep.	0.011 (12)	0.011 (34)	−0.035 (38)	0.053 (11)	0.040 (19)
Luxembourg	−0.042 (37)	0.009 (38)	−0.050 (48)	0.033 (33)	−0.050 (50)
Madagascar	0.061 (3)	0.016 (26)	−0.095 (54)	0.043 (19)	0.024 (23)
Malawi	−0.011 (22)	0.005 (46)	−0.003 (15)	0.077 (3)	0.068 (15)
Mauritius	−0.069 (52)	0.038 (8)	0.012 (10)	0.034 (29)	0.016 (26)
Mexico	0.027 (9)	0.035 (12)	−0.041 (44)	0.026 (45)	0.047 (16)
Morocco	0.044 (7)	0.045 (3)	−0.037 (41)	0.030 (42)	0.081 (11)
Netherlands	−0.053 (48)	0.005 (48)	0.012 (11)	0.032 (35)	−0.004 (31)
New Zealand	−0.031 (30)	0.004 (49)	−0.053 (50)	0.039 (24)	−0.040 (45)
Nigeria	0.077 (1)	0.105 (2)	−0.169 (55)	0.064 (7)	0.076 (12)
Norway	−0.046 (41)	0.008 (40)	−0.065 (53)	0.057 (10)	−0.045 (46)
Panama	−0.080 (55)	0.002 (50)	−0.029 (33)	0.033 (32)	−0.073 (55)
Paraguay	−0.1913 (5)	0.014 (29)	−0.031 (34)	0.050 (12)	0.023 (24)
Peru	0.007 (14)	0.031 (14)	−0.015 (19)	0.076 (4)	0.099 (7)
Philippines	−0.016 (25)	0.0062 (43)	−0.022 (26)	0.057 (9)	0.026 (22)
Portugal	−0.0 50 (45)	0.032 (13)	0.035 (4)	0.025 (48)	0.042 (17)
Sierra Leone	0.001 (17)	0.006 (44)	0.007 (12)	−0.016 (54)	−0.001 (29)
Spain	−0.011 (23)	0.022 (19)	−0.026 (28)	0.030 (40)	0.014 (27)
Sri Lanka	0.068 (2)	0.026 (17)	0.003 (13)	0.049 (13)	0.146 (3)
Sweden	−0.038 (32)	0.008 (39)	−0.039 (42)	0.030 (41)	−0.039 (44)
Switzerland	−0.017 (26)	0.036 (11)	−0.019 (22)	0.041 (21)	0.042 (18)
Syrian Arab Republic	−0.009 (20)	0.037 (10)	0.023 (7)	0.020 (51)	0.072 (13)
Thailand	0.033 (8)	0.015 (28)	−0.026 (30)	0.048 (14)	0.071 (14)
Turkey	−0.043 (39)	0.041 (6)	0.036 (3)	0.048 (15)	0.082 (10)
United Kingdom	−0.051 (47)	0.001 (52)	−0.033 (36)	0.033 (31)	−0.050 (51)
United States	−0.050 (46)	0.017 (24)	−0.040 (43)	0.023 (50)	−0.049 (49)
Yugoslavia, FR (Serbia/Mon)	−0.036 (31)	−0.103 (55)	0.044 (2)	0.031 (37)	−0.064 (53)
Zambia	−0.041 (34)	0.012 (31)	−0.037 (39)	0.040 (23)	−0.025 (41)
Zimbabwe	0.017 (11)	0.181 (1)	0.091 (1)	−0.030 (55)	0.258 (1)

APPENDIX B

Kernel Density Estimator and the Test for Closeness of Distributions

Let $X_1, \ldots, X_n$ be independent observations with probability density function f and let $Y_1, \ldots, Y_n$ be independent observations with probability function g. The density functions f and g can be consistently estimated by kernel estimators:

$$f(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{X_i - x}{h}\right)$$

$$g(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{Y_i - x}{h}\right)$$

where h is an optimally chosen smoothing parameter and K is a density function satisfying $\int_{-\infty}^{+\infty} K(\varphi)d\varphi = 1$ where $\varphi = \frac{X_i - x}{h}$ and $\varphi = \frac{Y_i - x}{h}$. In our application K is chosen as Epaninchinov kernel function.

To test the closeness (equality) of the two density functions $f(x)$ and $g(x)$ the test statistic

$$T = \frac{nh^{1/2} I}{\hat{\sigma}^2} \sim N(0, 1)$$

relies on integrated square difference

$$I = \int [f(x) - g(x)]^2 dx = \frac{1}{nh^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left\{K\left(\frac{X_i - X_j}{h}\right) + K\left(\frac{Y_i - Y_j}{h}\right) + 2K\left(\frac{X_i - Y_j}{h}\right)\right\}.$$

To avoid small sample bias, bootstrap approximation to the distribution of T is used. For details see Fan and Ulah (1999) and Li (1996).

NOTES

[1] Assume for example there exists a production possibilities frontier with P on the horizontal and S on the vertical axis.

[2] This assumption could be relaxed if one wants bilateral comparisons with a country which is chosen as a baseline.

[3] For the comprehensive discussion of representation of technology and its properties see Färe et al. (1994).
[4] The model is flexible enough to accommodate joint production of desirable and undesirable outputs (such as emissions of pollutants). In this case the technology is assumed to satisfy weak disposability of undesirable outputs, which states that it may not be possible to freely dispose of an undesirable output without sacrificing some of the desirable output. For examples of such models see Ball et al. (2001), Färe et al. (2000) and Zaim and Taskin (2000).
[5] For the comprehensive discussion of distance functions as a representation of technology and their properties, see Färe and Primont (1995).
[6] DEA stands for Data Envelopment Analysis, a term coined by Charnes, Cooper and Rhodes (1978).
[7] When undesirable outputs (b) are also considered, the output set would include an additional constraint $\sum z_k b_{ki} = 1$ (where equality implies weak disposability of undesirable outputs) and while evaluating the performance, undesirable outputs would be contracted while simultaneously expanding desirable outputs (social goods).
[8] While figures greater than 1 will indicate an improvement in social good provision for observation k, figures less than 1 will indicate deterioration.
[9] The availability of data was one constraining factor in our choice of countries.
[10] This variable is defined as 1000-infant mortality rate per 1000 births.
[11] This property alleviates the alleged difficulties associated with conventional "aggregate deprivation index". Ivanova et al. (1999) criticizes HDI index by stating that "it is a synthetic indicator and thus the methodology used in aggregating the three components is artificial and not based upon empirical observation". With the index proposed in this study, the aggregation over individual components is based on empirical observation (i.e., data determined).
[12] The transitivity characteristic allows for such a normalization.
[13] Numbers in parentheses indicate the ranking of each country.
[14] The index for a particular indicator is defined as:

$$A_{ij} = \frac{X_{ij} - X_{i\,\min}}{X_{i\,\max} - X_{i\,\min}}$$

where X_{ij} is the value of i'th indicator for the j'th country, $X_{i\,\min}$ and $X_{i\,\max}$ are the minimum and maximum values for the particular indicator respectively. Hence, the aggregate achievement index for the j'th country at a particular time is defined as:

$$A_j = \frac{1}{n} \sum_{i=1}^{n} A_{ij}.$$

[15] For 1977 while quantity index varies between 100 and 60.83, aggregate deprivation index varies between 91.80 and 32.53.

[16] This implies that, everything else the same, while a 10% difference in social goods vector between two countries will cause 10% difference in the quantity index of two countries, the difference will be greater than 10% as measured by deprivation index.

[17] Scale invariance states that the index remains unchanged if all variables proportionately change (i.e., double) for all countries. Translation invariance with respect to the rank implies that if all variables are improved by the same amount the relative positions of the countries with respect to each other will not be affected.

[18] Intuitively one may view these distributions as smoothed histograms.

[19] We gratefully acknowledge R. Robert Russell and Subodh Kumar for providing us with the algorithm required to perform these tests.

[20] Note that, during this period, averaged over countries, social good provision has increased by 3.62%.

[21] As Ivonova et al. (1999) states, "The growth rate of HDI (aggregate deprivation index) is a meaningless figure, which explains its lack of acceptability. The rationale for this lies in the way the index is constructed. The best achievements have less tolerance range for improvement and the growth rate at the top does not have the same meaning as the growth rate for bottom countries" (pp. 172–173).

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