Physica B 293 (2001) 283–288

PHYSICA B

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Bose–Einstein condensation of noninteracting charged Bose gas in the presence of external potentials

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Received 21 June 1999; received in revised form 7 March 2000; accepted 6 July 2000

Abstract

We investigate thermodynamic properties of noninteracting charged bosons in the presence of externally applied electric and magnetic fields. Using the semiclassical density of states, we obtain the condensate fraction, chemical potential, total energy, and specific heat of a system of finite number of charged Bose particles. We conclude that Bose–Einstein condensation of the charged Bose gas occurs in the crossed electric and magnetic fields.

PACS: 03.65.Sq; 03.75.Fi; 05.30.Jp; 67.40.Kh; 64.60. − i

Keywords: Bose–Einstein condensation; External potentials; Density of states

1. Introduction

The recent observations of Bose–Einstein condensation (BEC) in trapped atomic gases [1–6] have renewed interest in bosonic systems [7–9]. The condensate clouds obtained in the experiments consist of a finite number of atoms (ranging from several thousands to several millions), and are confined in externally applied trapping potentials. The ground state properties of the condensed gases, including the finite size effects on the temperature dependence of the condensate fraction, are of primary interest. BEC is characterized by a macroscopic occupation of the ground state for $T < T_c$, where T_c depends on the system parameters.

From a theoretical standpoint BEC has been extensively studied as a possible explanation of the superfluid [10] transition in ^{4}He and a plausible theory of the conventional (low-temperature) superconductors [11]. Although condensation of a charged Bose gas (CBG) is not a correct picture of superconductivity in metals, the CBG has recently been studied to understand the high-temperature superconductivity in cuprates [12,13].

Many years ago Schafroth [11] pointed out that the CBG does not condense at any finite temperature in the presence of a homogeneous magnetic field. Later, the charged Bose system in a magnetic field was studied by various groups [14–18]. Recently, Rojas [19] has discussed the possibility of obtaining BEC for CBG under a constant magnetic field. Standen and Toms [20] have shown that three-dimensional CBG does not have phase transition for any value of the magnetic field. CBG

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PII: S0921-4526(00)00561-5

in the presence of a harmonic trapping potential and a constant magnetic field was studied within path-integral formalism [21].

The aim of this paper is to study the effects of the externally applied electric and magnetic fields on BEC of the CBG. The long-range interactions between the charged bosons are neglected, with the assumption that screening effects somehow render them short ranged. We use a model density of states which takes the finite sample size into account to calculate the thermodynamic quantities [22]. The importance of constructing an accurate density of states has been recognized in various works [23–29]. We obtain quantities such as condensate fraction, chemical potential, total energy, and specific heat of the system using the semiclassical density of states. We first concentrate on the CBG in the homogeneous electric field only. We then investigate the possibility for achieving BEC in three-dimensional noninteracting CBG under the crossed electric and magnetic fields.

2. Theory

We consider N particles of a charged Bose gas in an external field $\mathscr{F}$ which is described by a monotonic potential $V(x)$ and trapped by two infinite barriers at $x = 0$ and L. Using the semiclassical (WKB) approximation, the quantization condition for the energy ε_n is given by [30,22]

$$\sqrt{2m}\int_0^{x_n}\sqrt{\varepsilon_n - V(x)}\,\mathrm{d}x = \hbar\pi(n + \phi/4 + \varphi/2), \quad (1)$$

where $n = 0, 1, 2, \ldots$ and the classical turning point x_n and the phase factors ϕ and φ are given by $V(x_n) = \varepsilon_n$, $\phi = 1$ and $\varphi = 1$ for $\varepsilon_n < V(L)$, while $x_n = L$, $\phi = 2$ and $\varphi = 1$ for $\varepsilon_n \geqslant V(L)$. For large values of L, ε_n becomes a quasi-continuous function of n and the semiclassical approximation is identical to the exact results (see Ref. [30]). The density of states (DOS) can be calculated from the trace formula

$$\rho(E) = \mathrm{Tr}\,\delta(E - \hat{H}), \quad (2)$$

where $\hat{H}$ is the Hamiltonian of the system. The total number of particles is implicitly related to the chemical potential μ by

$$N = N_0 + \int \rho(E)n(E)\,\mathrm{d}E, \quad (3)$$

where N_0 is the number of the particles in the ground state and $n(E) = (\exp[(E-\mu)/k_\mathrm{B}T] - 1)^{-1}$. The critical temperature T_c can be determined from Eq. (3) by taking $N_0 = 0$ and $\mu = 0$ at $T = T_\mathrm{c}$. For $T < T_\mathrm{c}$, the condensate fraction N_0/N can be determined from Eq. (3) and total energy of the system is given by

$$E_T(T) = \int \frac{E\rho(E)\,\mathrm{d}E}{\exp(E/k_\mathrm{B}T) - 1}. \quad (4)$$

For $T > T_\mathrm{c}$, after finding μ from Eq. (3) ($N_0 = 0$), total energy is calculated from

$$E_T(T) = \int E\rho(E)n(E)\,\mathrm{d}E. \quad (5)$$

The specific heat of the system $C_V(T) = \partial E_T(T)/\partial T$ can be shown to be

$$C_V(T) = \frac{1}{k_\mathrm{B}T}\int E\rho(E)n(E)^2 \times\left[\mu'(T) + \frac{E-\mu}{T}\right]\exp\left(\frac{E-\mu}{k_\mathrm{B}T}\right)\mathrm{d}E, \quad (6)$$

where $\mu'(T) = \partial\mu(T)/\partial T$. The discontinuity in the specific heat at T_c is given by [23,24]

$$\Delta C_V(T_\mathrm{c}) = C_V(T_\mathrm{c}^-) - C_V(T_\mathrm{c}^+) = \frac{1}{k_\mathrm{B}T_\mathrm{c}^2}\frac{[\int E\rho(E)n(E)^2\exp(E/k_\mathrm{B}T_\mathrm{c})\,\mathrm{d}E]^2}{\int\rho(E)n(E)^2\exp(E/k_\mathrm{B}T_\mathrm{c})\,\mathrm{d}E}. \quad (7)$$

In the above formulation of the thermodynamic properties, the density of states plays an important role. The effects of external potentials are embodied in the DOS, and the resulting thermodynamic properties depend crucially on the choice and construction of the DOS. The importance of the DOS in the BEC of harmonically confined systems has been emphasized by Kirsten and Toms [26].

3. BEC in an applied electric field

We consider first a three-dimensional CBG in a constant electric field $\mathscr{E}$ along the x direction. In this case, trapping potential becomes $V(r) = V_0 x/L$, where $V_0 = e\mathscr{E}L$ and $\mathscr{E}L$ is the total voltage drop across the sample. The semiclassical density of states, using the method of Kubisa and Zawadzki [22], can be obtained from Eqs. (1) and (2)

$$\rho(E) = \begin{cases} \alpha E^{3/2} & \text{if } E < V_0, \\ \alpha[E^{3/2} - (E - V_0)^{3/2}] & \text{if } E \geqslant V_0, \end{cases} \tag{8}$$

where $\alpha = \frac{1}{3}\pi^2 E_0 V_0$, $E_0 = \hbar^2/2mL^2$ and we normalize all energies with E_0. Note that our expression for $\rho(E)$ differs from that of Bagnato et al. [23,24] in that we include the finite sample size effects. For vanishing electric field, $V_0 \to 0$, one gets the well-known result $\rho(E) \sim E^{1/2}$ for homogeneous systems. The main effect of the applied electric field is to shift the DOS from low to high energies due to acceleration of particles. In the sequel, we shall examine the results of this effect on the thermodynamic quantities.

The critical temperature is determined by solving the following integral equation:

$$N = \frac{(k_B T_c)^{5/2}}{3\pi^2 V_0}[g_{5/2}(0) - g_{5/2}(-V_0/k_B T_c)], \tag{9}$$

where

$$g_\nu(z) = \int_0^\infty \frac{x^{\nu-1}\,dx}{\exp(x-z) - 1}, \tag{10}$$

is the much-studied Bose function [31]. The temperature dependence of the condensate fraction and the total energy are given by

$$N_0/N = 1 - \frac{(k_B T)^{5/2}}{3\pi^2 N V_0}[g_{5/2}(0) - g_{5/2}(-V_0/k_B T)] \tag{11}$$

and

$$E_T(T) = \frac{(k_B T)^{7/2}}{3\pi^2 V_0}\left[g_{7/2}(0) - g_{7/2}(-V_0/k_B T) - \frac{V_0}{k_B T} g_{5/2}(-V_0/k_B T)\right], \tag{12}$$

respectively. For $T > T_c$, μ is determined from solution of the following equation:

$$N = \frac{(k_B T)^{5/2}}{3\pi^2 V_0}[g_{5/2}(\mu/k_B T) - g_{5/2}((\mu - V_0)/k_B T)]. \tag{13}$$

Finally, E_T can be found from

$$E_T(T) = \frac{(k_B T)^{7/2}}{3\pi^2 V_0}\left[g_{7/2}(\mu/k_B T) - g_{7/2}((\mu - V_0)/k_B T) - \frac{V_0}{k_B T} g_{5/2}((\mu - V_0)/k_B T)\right]. \tag{14}$$

Fig. 1 displays the temperature dependence of the condensate fraction N_0/N for various field strengths or external potential values V_0. Our results fall between the two extreme cases. In the case of a homogeneous system, the temperature dependence of the condensate fraction is given by $N_0/N = 1 - (T/T_c)^{3/2}$. On the other extreme is the bosons trapped by a linear potential as discussed by Bagnato et al. [23,24]. The corresponding depletion of the condensate is given by $N_0/N = 1 - (T/T_c)^{5/2}$. For small values of V_0, as the discussion on the DOS shows, we recover the homogeneous system result. As V_0 increases, our results

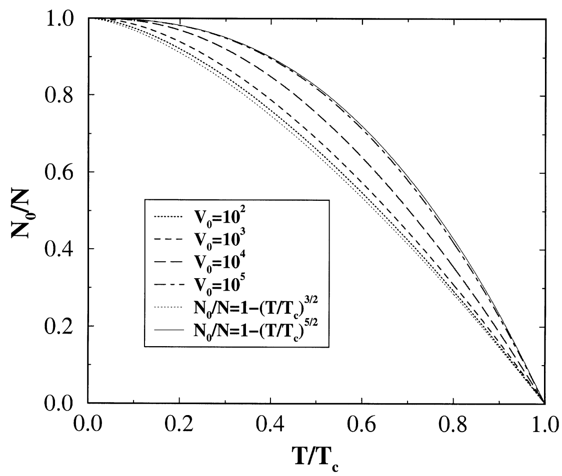

Fig. 1. The condensate fraction N_0/N versus normalized temperature T/T_c for $N = 10^5$ and for various values of the trapping potential (electric field) V_0.

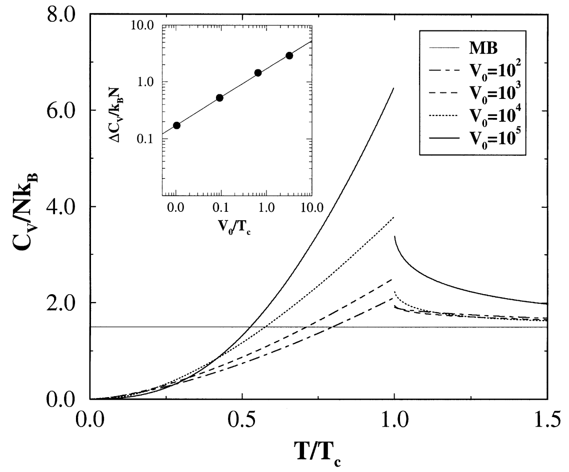

Fig. 2. The temperature dependence of the specific heat $C_V(T)$ for $N = 10^5$ and for various values of the trapping potential V_0. Inset: (●) symbols show variation of discontinuity in the specific heat $\Delta C_V/k_B N$ at T_c with V_0/T_c, (–) is the best fit.

approach the latter case, indicating that the confinement effects become important. The specific heat $C_V(T)$ as a function of temperature is shown in Fig. 2. We note that a discontinuity in C_V at $T = T_c$ develops as the external (trapping) potential is increased. Based on the numerical results shown in the inset of Fig. 2, we estimate the discontinuity in the specific heat as $\Delta C_V/Nk_B \sim (V_0/T_c)^{1/2}$. As V_0 increases our results approach that of Bagnato et al. [23,24] and as $V_0 \to 0$, we recover the homogeneous system result with no discontinuity. The net effect of the external electric field in our model is to provide a confining potential to produce BEC in a linear potential.

Strictly speaking, the continuum model of a density of states should be applicable only in the thermodynamic limit, viz. $N \to \infty$ and $V \to \infty$ (V is the volume of the system) while keeping the average density $\bar{\rho} = N/V$ fixed. Thus, our results for finite N are more meaningful for cases of large N. For illustration purposes we have used $N = 10^5$ in Figs. 1 and 2.

4. BEC in crossed electric and magnetic fields

We next consider the CBG in crossed electric and magnetic fields. Taking a constant electric field along x-axis and a magnetic field along z-axis, one can find the semiclassical density of states [22] for $E < \varepsilon_n + V_0$,

$$\rho(E) = \beta \sum_n (E - \varepsilon_n)^{1/2} \tag{15}$$

and for $E \geqslant \varepsilon_n + V_0$,

$$\rho(E) = \beta \sum_n [(E - \varepsilon_n)^{1/2} - (E - \varepsilon_n - V_0)^{1/2}], \tag{16}$$

where $\beta = \hbar\omega_c/\pi E_0 V_0$, $\varepsilon_n = \hbar\omega_c(n + \frac{1}{2}) + \gamma^2$, $n = 0, 1, 2, \ldots$ and $\gamma = V_0/\hbar\omega_c$, $\omega_c = eB/mc$. For vanishing electric field, $V_0 \to 0$, one gets the well-known Landau level singularities $\rho(E) \sim (E - \varepsilon_n)^{-1/2}$, with $\varepsilon_n = \hbar\omega_c(n + \frac{1}{2})$. The sharp divergences at $\gamma = 0$ (zero electric field) become finite peaks when the electric field is turned on. At higher values of γ the density of states exhibits a smooth dependence on the energy.

The critical temperature in the present case is obtained from

$$N = \frac{(k_B T_c)^{3/2}}{\pi\gamma} \sum_n [g_{3/2}(-\varepsilon_n/k_B T_c) - g_{3/2}(-(\varepsilon_n + V_0)/k_B T_c)]. \tag{17}$$

Note that N and subsequent thermodynamic quantities not only depend on the ratio of electric and magnetic fields, γ, but also on the value of V_0. The condensate fraction and total energy are given by

$$N_0/N = 1 - \frac{(k_B T)^{3/2}}{\pi\gamma N} \sum_n [g_{3/2}(-\varepsilon_n/k_B T) - g_{3/2}(-(\varepsilon_c + V_0)/k_B T)] \tag{18}$$

and

$$E_T(T) = \frac{(k_B T)^{5/2}}{\pi\gamma} \sum_n \left[g_{5/2}(0) + \frac{\varepsilon_n}{k_B T} g_{3/2}(-\varepsilon_n/k_B T) - g_{5/2}(-(\varepsilon_n + V_0)/k_B T) - \frac{\varepsilon_n + V_0}{k_B T} g_{3/2}(-(\varepsilon_n + V_0)/k_B T) \right], \tag{19}$$

respectively. For $T > T_c$, μ is determined from

$$N = \frac{(k_B T)^{3/2}}{\pi\gamma} \sum_n [g_{3/2}((\mu - \varepsilon_n)/k_B T) - g_{3/2}((\mu - \varepsilon_n - V_0)/k_B T)], \tag{20}$$

and the total energy is given by

$$E_T(T) = \frac{(k_B T)^{5/2}}{\pi\gamma} \sum_n \Bigg[g_{5/2}(\mu/k_B T) + \frac{\varepsilon_n}{k_B T} g_{3/2}((\mu - \varepsilon_n)/k_B T) - g_{5/2}((\mu - \varepsilon_n - V_0)/k_B T) - \frac{\varepsilon_n + V_0}{k_B T} g_{3/2}((\mu - \varepsilon_n - V_0)/k_B T) \Bigg]. \tag{21}$$

We now present our results for the case of externally applied crossed electric and magnetic fields. The expressions to be evaluated are slightly more demanding because of the infinite sums in the above equations. Since the system can readily undergo a BEC in a linear potential, i.e. electric field, we set out to investigate the effects of the external magnetic field.

The condensate fraction for various combinations of the crossed electric and magnetic field strengths is shown in Fig. 3. Here the presence of a magnetic field and hence the peaked nature of the DOS gives rise to a nonmonotone dependence in terms of various combinations of the parameters V_0 and E_c. Finally, the specific heat and the discontinuity at T_c are displayed in Fig. 4. In the presence of the magnetic field, the specific heat still shows a discontinuity at the critical temperature. Our results may be interpreted as indicating the occurance of a BEC in a confining potential when the applied magnetic field is not too strong. Previously, Brosens et al. [21] have predicted the possibility of BEC in a parabolic confining potential and magnetic field. As shown in Fig. 4, if we decrease the amplitude of trapping potential V_0, while keeping the magnetic field constant, the discontinuity in the specific heat decreases. This is in line with the disappearance of BEC in a magnetic field for homogeneous systems.

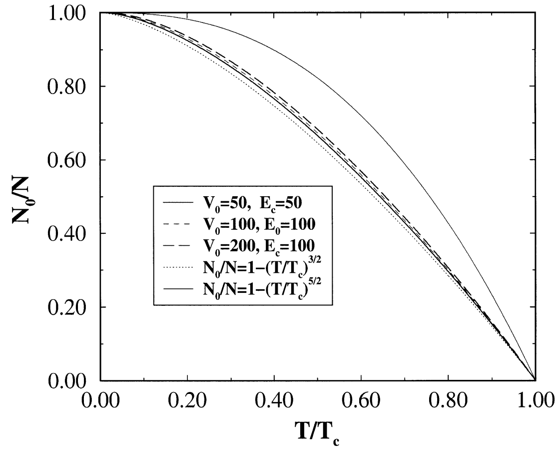

Fig. 3. The condensate fraction N_0/N versus normalized temperature T/T_0 for $N = 10^5$ and for various values of the electric and magnetic fields.

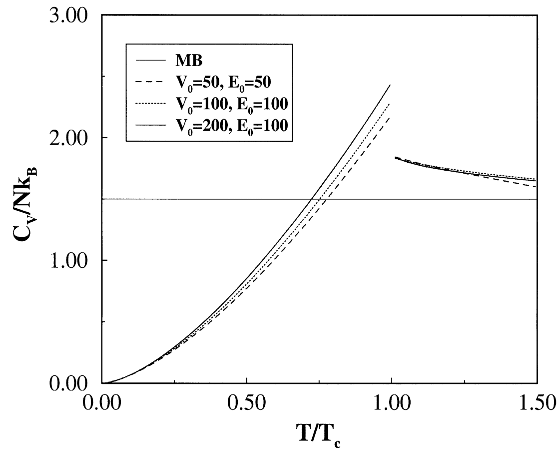

Fig. 4. The temperature dependence of the specific heat $C_V(T)$ for $N = 10^5$ and for various values of the electric and magnetic fields.

5. Conclusion

In this work, we have considered a system of noninteracting charged bosons and have studied the BEC phenomenon in the presence of externally applied electric and magnetic fields. The external fields make the system inhomogeneous and alter the BEC characteristics compared to the homogeneous case. We employ a recently introduced semiclassical density of states [22] to calculate the temperature dependence of the condensate fraction and the specific heat. We find that the noninteracting system of charged bosons undergo BEC when external electric and magnetic fields are applied. The density of states which includes finite sample size dimension effects gives rise to interesting dependencies. The discontinuity in the specific heat is obtained as a function of the external potentials. It would be interesting to look for experimental verifications of our predictions. Our results may also provide a starting point for more involved theories that take the interaction effects into account.

Acknowledgements

This work was supported by the Scientific and Technical Research Council of Turkey (TUBITAK) under Grant No. TBAG-1662. It is a pleasure to acknowledge useful discussions with Professors G. Host and C. Yalabık.

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