variation of the moment of inertia in the uncertain system is 1%. For easy comparison, suitable parameters are selected such that similar asymptotic properties are obtained for different-order HOSM controllers. Simulation results are shown in Fig. 2.

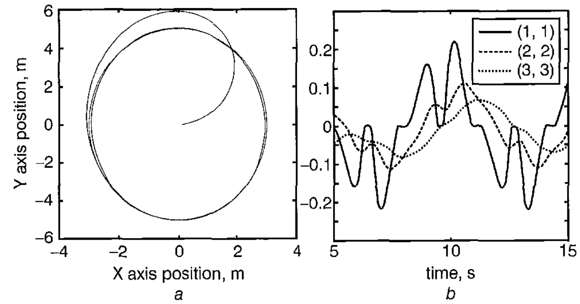

Fig. 2 *Trajectory tracking and switching function*
a Trajectory tracking of uncertain system
b Switching function S_1: (1, 1), (2, 2) and (3, 3) orders

Simulation studies show that the controller is able to achieve trajectory tracking control successfully. Another point is that the higher the sliding order, the less the chattering problem. Hence chattering can be greatly reduced via the high order sliding mode control approach. Also, the output tracking controller has a certain robustness with respect to the initial error and parametric variation.

Conclusions: HOSM controllers of different order are designed for the output tracking of a typical mobile robot with uncertain parameters. Simulations results show that the approach tracks the desired trajectory successfully and reduces chattering greatly. It also has robustness with respect to the initial state error and parametric variation.

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References

1 TARN, T.J., BEJCZY, A.K., YUN, X.P., and LI, Z.F.: 'Effect of motor dynamics on nonlinear feedback robot arm control', *IEEE Trans. Robot. Autom.*, 1991, **7**, (1), pp. 114–121
2 ELDEEB, Y., and ELMARAGHY, W.H.: 'Robust adaptive control of a robotic manipulator including motor dynamics', *J. Robot. Syst.*, 1998, **15**, (11), pp. 661–669
3 LEVANT, A.: 'Sliding order and sliding accuracy in sliding mode control', *Int. J. Control*, 1993, **58**, (6), pp. 1247–1263
4 FRIDMAN, L., and LEVANT, A.: 'Higher order sliding modes as a natural phenomenon in control theory' in GAROFALO, F., and GLIELMO, L. (Eds.): 'Robust control via variable structure and Lyapunov techniques' (Springer-Verlag, London, 1996)
5 BARTOLINI, G., FERRARA, A., and GIACOMINI, L.: 'A robust control design for a class of uncertain nonlinear systems featuring a second-order sliding mode', *Int. J. Control*, 1999, **72**, (4), pp. 321–331

Approximations in compensator design: A duality

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In classical controller design, poles far to the left of dominant poles are sometimes ignored. Similarly, in some proportional-integral compensation techniques, the controller zero is placed close to the origin and design proceeds after cancelling this zero with a pole at the origin. A rigorous basis for these methods is provided, it being shown that there is a duality between the two.

Introduction: Most classical control textbooks include heuristic methods on low-order approximations to loop-gain transfer functions and employ such approximations in design [1–3].

One method relies on identifying dominant *versus* insignificant poles. The general rule is that poles having real parts at least five times as large as poles which are nearest the $j\omega$ axis are considered insignificant [1], provided there are no zeros nearby [4]. Such poles can be deleted from a transfer function, taking care to keep the low-frequency gain unchanged, and design can be carried out on the approximate model obtained. Although the dominant pole-based approximation is normally used on a closed-loop transfer function, occasionally such approximations are also used on the open-loop or loop-gain transfer functions ([1], p.416).

A second approximation that is used is part of a specific design method of proportional-integral (PI) controllers for plants of type equal to or greater than 1. PI controllers are usually employed to improve the steady-state (low-frequency) performance of a control system. If any transient performance specification further exists, then the controller zero is placed much closer to the origin than any other stable plant pole and the transient requirement is satisfied as if the controller is a proportional one, i.e. the multiple of the PI controller and the plant transfer functions is approximated by the original plant transfer function under constant gain ([1], p.695). Since PID controllers can be designed by consecutive PI and PD design stages, the method described becomes quite convenient, simplifying the second stage.

The connection between the two approximation methods is obscure. The two methods even seem contradictory since in the second method a pole in the dominant region is cancelled from the loop-gain transfer function.

Our purpose in this Letter is to justify these approximation methods from the viewpoint of closed-loop stability. We also show that the two approximation procedures are dual to each other under the transformation $s \to s^{-1}$ in the complex plane.

Main results: Throughout this Section, the set of stable proper real rational functions of s (real-rational H_∞ functions) will be denoted by $\mathbf{S}$ [5].

The following Lemma is a result by Smith and Sondergeld stated in [6] for the scalar case and for stable controllers. Although the extension is straightforward, a proof is still supplied in order to make the choice of a critical constant ϵ_0 clear.

Lemma 1: If a strictly proper transfer matrix $G(s)$ is stabilised by a controller transfer matrix $G_c(s)$, then for any $q \geq 1$, there exists a small enough $\epsilon_0 > 0$, possibly depending on q, such that $(\epsilon s+1)^{-q}G(s)$ is also stabilised by $G_c(s)$ for all $\epsilon \in (0, \epsilon_0]$.

Proof: Let $G = D^{-1}N$ be a left coprime representation of G over $\mathbf{S}$, where N is strictly proper as is G. Let $N_c D_c^{-1}$ be a right coprime representation of G_c over $\mathbf{S}$. Since G is stabilised by G_c, $U := DD_c + NN_c$ is unimodular over $\mathbf{S}$. Let

$$\delta := 1/\|U(s)\|_\infty$$

By Lemma 19 of [5], any matrix V over $\mathbf{S}$ that satisfies $\| V(s) - U(s) \|_\infty < 1/ \| U(s) \|_\infty$ is unimodular. To show the existence of ϵ such that $V(s) := D(s)D_c(s) + (\epsilon s+1)^{-q}N(s)N_c(s)$ is unimodular, it is hence enough to show that there exists $\epsilon > 0$ for which

$$\|[(\epsilon s+1)^{-q} - 1]N(s)N_c(s)\|_\infty < \delta \tag{1}$$

By strict properness of $H := NN_c$, there exists ω_0 for which

$$\sup_{\omega \geq \omega_0} \bar{\sigma}(H(j\omega)) < \frac{\delta}{2},$$

where $\bar{\sigma}(H(j\omega))$ is the largest singular value of $H(j\omega)$. It follows that, for any finite ϵ,

$$\sup_{\omega \geq \omega_0} \bar{\sigma}([(\epsilon j\omega + 1)^{-q} - 1]H(j\omega)) < \delta \tag{2}$$

Conversely, in the interval $[0, \omega_0]$, we have that $\max_{\omega \in [0,\omega_0]}|(\epsilon j\omega + 1)^{-q} - 1| < k\omega_0^q\epsilon$ for some k that depends on q. Let $\mu := \max_{\omega \in [0,\omega_0]}\bar{\sigma}(H(j\omega))$ so that for every choice of ϵ less than $\delta/\mu k\omega_0^q$, it holds that

$$\sup_{\omega \in [0,\omega_0]} \bar{\sigma}([(\epsilon j\omega + 1)^{-q} - 1]H(j\omega)) < \delta \tag{3}$$

By (2) and (3), the norm equality (1) follows. ∎

This result justifies (and generalises to the multivariable case) methods in which a stabilising controller is determined by neglecting an insignificant pole of multiplicity q in a loop-gain transfer function

$$\frac{G(s)}{(\epsilon s+1)^q} \tag{4}$$

and performing the design on the lower order approximation $G(s)$. The discarded term is such that the low-frequency gain $G(0)$ of $G(s)$ and of (4) are the same. Note that a pole of (4) is *insignificant* if it is less than $-\epsilon_0^{-1}$. It follows that ϵ_0 determined in the proof gives a rigorous definition of an insignificant pole. The bound in the preceding proof for ϵ_0, however, is generally very conservative.

A transfer matrix $G(s)$ is of type greater or equal to 1 if $\lim_{s\to\infty} sG(s)$ is finite. The following result concerns such transfer matrices. We supply a proof based on Lemma 1 and the transformation $s \to s^{-1}$ in the complex plane. We omit a direct proof which makes the choice of ϵ_0 more explicit [7].

Theorem 1: Let a transfer matrix $G(s)$ be of type greater than or equal to 1. If $G(s)$ is stabilised by a controller transfer matrix $G_c(s)$, then for any $q \geq 1$, there exists a small enough $\epsilon_0 > 0$, possibly depending on q, such that

$$\frac{(s+\epsilon)^q}{s^q} G(s)$$

is also stabilised by G_c for all $\epsilon \in (0, \epsilon_0]$.

Proof: This proof uses the simple 'Fact: Let $U(s)$ be left (right) unimodular over **S** (see [5]). Then, $\hat{U}(s) := U(s^{-1})$ is also left (right) unimodular over **S**.' We first observe that by the hypothesis on the type of G, $G(s^{-1})^{-1}$ is strictly proper. Let $G(s^{-1}) = \hat{D}(s)^{-1}\hat{N}(s)$ be a left coprime representation over **S**. Note that $\hat{D}(s)$ is strictly proper and $\hat{N}(s)$ is biproper. Let $D(s) := \hat{D}(s^{-1})$ and $N(s) := \hat{N}(s^{-1})$. Then, $G(s) = D(s)^{-1}N(s)$, where $(D(s), N(s))$ is left coprime over **S** by the Fact. Let $G_c(s) = N_c(s)D_c(s)^{-1}$ be a right coprime representation. Since, G_c stabilises G, we have that $DD_c + NN_c =: U$ is unimodular over **S**. Substituting s^{-1} for s, we obtain $\hat{D}(s)\hat{D}_c(s) + \hat{N}(s)\hat{N}_c(s) = \hat{U}(s)$, where $\hat{D}_c(s) := D_c(s^{-1})$, $\hat{N}_c(s) := N_c(s^{-1})$, and $\hat{U}(s) := U(s^{-1})$. Note that $\hat{N}_c(s)\hat{D}_c(s)^{-1} = G_c(s^{-1})$ is proper. Also by the Fact, $\hat{U}(s)$ is unimodular over **S**. By Lemma 1, given any $q \geq 1$, there exists $\epsilon_0 > 0$ such that $(1+\epsilon s)^{-q}\hat{D}(s)\hat{D}_c(s) + \hat{N}(s)\hat{N}_c(s) = \hat{V}(s)$ is unimodular for every $\epsilon \leq \epsilon_0$. Substituting s^{-1} for s, we obtain

$$\frac{s^q}{(s+\epsilon)^q} D(s)D_c(s) + N(s)N_c(s) = V(s),$$

where $V(s) = \hat{V}(s^{-1})$ is unimodular by the Fact. ∎

The result of Theorem 1 justifies methods of design where a loop-gain transfer matrix

$$\frac{(s+\epsilon)^q}{s^q} G(s) \tag{5}$$

is approximated by the type ≥ 1 function $G(s)$ in designing a stabilising controller. The term that is discarded is such that the high-frequency gain of $G(s)$ and that of (5) are the same, i.e. a zero is always cancelled with exactly one pole at the origin. The cancelled zeros must be in the vicinity of the origin.

The proof given of Theorem 1 clarifies the relation between the two design methods: An insignificant pole term $\epsilon s + 1$ under the transformation $s \to s^{-1}$ gives a PI controller

$$\frac{s+\epsilon}{s}$$

with its zero close to the origin. The approximation procedures involved in the two methods are hence dual procedures and have the same logical foundation.

Conclusions: In Lemma 1 and Theorem 1, we have provided rigorous support and established a duality for two classical design methods from the viewpoint of closed-loop stability. The virtue of such design methods lie not only in closed-loop stability but also in their use in satisfying performance specs in an easy manner. Hence, further rigorous support from the viewpoint of closed-loop performance is needed.

The crucial constants ϵ_0 of Lemma 1 and its counterpart for Theorem 1 provide a rigorous distinction between dominant and insignificant poles on one hand, and between cancellable and uncancellable PI controller zeros on the other. A closer look into the construction of ϵ_0 will yield a new approximation-based design method. This is currently under investigation.

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References

1 KUO, B.C.: 'Automatic control systems' (Prentice Hall, NJ, USA, 1995), 7th edn.

2 ROHRS, C.E., MELSA, J.L., and SCHULTZ, D.G.: 'Linear control systems' (McGraw Hill, NJ, USA, 1993)

3 DORF, R.C., and BISHOP, R.H.: 'Modern control systems' (Prentice Hall, NJ, USA, 2001), 9th edn.

4 OGATA, K.: 'Modern control engineering' (Prentice Hall, NJ, USA, 1997), 3rd edn.

5 VIDYASAGAR, M.: 'Control system synthesis: a factorization approach' (The MIT Press, MA, USA, 1985)

6 SMITH, M.C., and SONDERGELD, K.P.: 'On the order of stable compensators', *Automatica*, 1986, **22**, (1), pp. 127–129

7 Özgüler, A.B. and Gündeş, A.N.: 'Common controllers for plants with fixed zeros', *Preprint*, Electrical and Electronics Engineering Department, Bilkent University, Bilkent, Ankara, TR-06533, Turkey, 2001

Proof for effective and efficient web caching

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Perfect-LFU, the optimal replacement policy for Web caches, achieves high performance (cache hit-rate) exploiting Zipf's law, but is very expensive to implement. It is proved that an alternative policy, called Window-LFU, achieves equivalent performance at a significantly lower cost.

Introduction: Web caches take advantage of Zipf's law, which governs user requests, in general. The optimal replacement policy is perfect-LFU (P-LFU) [1], which replaces Web objects based on their popularity. P-LFU is expensive to implement, because correct calculation of popularity requires statistics on all objects accessed in the whole past.

One can make replacement decisions based on Web object popularity in the recent past, called the recent time window. This policy, called window-LFU (W-LFU), clearly has a significantly lower implementation cost. In this Letter, we prove that W-LFU achieves high hit rates, equivalent to P-LFU, for small window sizes. Our only assumptions are Zipf's law and statistical independence among user requests.

Model and notation: We consider an enterprise network (or LAN) connected to the Internet through a gatewith with a cache, as shown in Fig. 1. User (client) requests are either served by the cache, if the requested objects are cache-resident, or forwarded to the Internet.