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Accelerated Born–Infeld metrics in Kerr–Schild geometry

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Abstract

We consider Einstein Born–Infeld theory with a null fluid in Kerr–Schild geometry. We find accelerated charge solutions of this theory. Our solutions reduce to the Plebanski solution when the acceleration vanishes and to the´ Bonnor–Vaidya solution as the Born–Infeld parameter b goes to infinity. We also give the explicit form of the energy flux formula due to the acceleration of the charged sources.

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1. Introduction

Accelerated charge metrics in Einstein–Maxwell theory have been studied in two equivalent ways. One way uses the Robinson–Trautman metrics [1–4] and the other way is the Bonnor–Vaidya approach [5] using the Kerr–Schild ansatz [6, 7]. In both cases one can generalize the metrics of non-rotating charged static spherically symmetric bodies by introducing acceleration. Radiation of energy due to the acceleration is a known fact both in classical electromagnetism [8, 9] and in Einstein–Maxwell theory [5].

Recently, we have given accelerated solutions of the D-dimensional Einstein–Maxwell field equations with a null fluid [10]. The energy flux due to acceleration in these solutions are all finite and have the same sign for all $D \geqslant 4$. It is highly interesting to study the same problem in nonlinear electrodynamics.

The nonlinear electrodynamics of Born–Infeld [11] shares two separate important properties with Maxwell theory. The first is that its excitations propagate without the shocks commontogenericnonlinearmodels[12], andthesecondiselectromagneticdualityinvariance [13] (see also the references therein). For this reason we consider the Einstein Born–Infeld theory in this work. We assume that the spacetime metric is of the Kerr–Schild form [6, 7] with an appropriate vector potential and a fluid velocity vector. We derive a complete set

of conditions for the Einstein Born–Infeld theory with a null fluid. We assume vanishing pressure and cosmological constant. Under such assumptions we give the complete solution. This generalizes the Plebanski solution [14]. We also obtain the energy flux formula which´

turns out to be the same as that obtained in Einstein–Maxwell theory. For the sake of completeness we start with some necessary tools that will be needed in the following sections. For conventions and details we refer the reader to [10].

Let $z^\mu(\tau)$ describe a smooth curve *C* in four-dimensionalMinkowski manifold **M** defined by z : I ⊂**R** → **M**. Here τ is the arclength parameter of the curve, and *I* is an interval on the real line. For the null case, τ is a parameter of the curve. The distance between an arbitrary point *P* (not on the curve) with coordinates x^μ in **M** and a point *Q* on the curve *C* with coordinates z^μ is given by

$$\Omega = \eta_{\mu\nu}(x^\mu - z^\mu(\tau))(x^\nu - z^\nu(\tau)).$$

Let $\tau = \tau_0$ define the point on the curve *C* so that $\Omega = 0$ and $x^0 > z^0(\tau_0)$ (the retarded time).

Then we find the following:

$$\lambda_\mu \equiv \partial_\mu \tau_0 = \frac{x_\mu - z_\mu(\tau_0)}{R}, \tag{1}$$

$$R \equiv \dot{z}^\mu(\tau_0)(x_\mu - z_\mu(\tau_0)). \tag{2}$$

From here on, a dot denotes differentiation with respect to τ_0. We then have

$$\lambda_{\mu,\nu} = \frac{1}{R}[\eta_{\mu\nu} - \dot{z}_\mu\lambda_\nu - \dot{z}_\nu\lambda_\mu - (A-\epsilon)\lambda_\mu\lambda_\nu],$$
$$R_{,\mu} = (A-\epsilon)\lambda_\mu + \dot{z}_\mu, \tag{3}$$

where

$$A \equiv \ddot{z}^\mu(x_\mu - z_\mu), \qquad \dot{z}^\mu \dot{z}_\mu = \epsilon = 0, \pm 1.$$

For timelike curves we take $\epsilon = -1$. We introduce some scalars a_k (k = 0,1,2,...)

$$a_k = \lambda_\mu \frac{d^k \ddot{z}^\mu}{d\tau_0^k}, \qquad k = 0, 1, 2, \ldots . \tag{4}$$

In what follows, we shall take $a_0 \equiv a = \frac{A}{R}$. For all *k* we have the following property (see [10] for further details):

$$\lambda^\mu a_{k,\mu} = 0. \tag{5}$$

For the flux expressions that will be needed in section 3, we take

$$\lambda_\mu = \epsilon \dot{z}_\mu + \epsilon \frac{n_\mu}{R}, \qquad \epsilon \neq 0, \tag{6}$$

where n_μ is a spacelike vector orthogonal to $\dot{z}^\mu$ (see [10] for more details). For the remaining part of this work, we always assume and take $\epsilon \neq 0$.

2. Accelerated Born–Infeld metrics

We now consider the Einstein Born–Infeld field equations with a null fluid distribution in four dimensions. The Einstein equations

$$G_{\mu\nu} = \kappa T_{\mu\nu} = \kappa T^{\mathrm{BI}}_{\mu\nu} + \kappa T^{f}_{\mu\nu} + \Lambda g_{\mu\nu}$$

with the fluid and Maxwell equations are given by [15, 16]

$$G_{\mu\nu} = \kappa \left\{ \frac{b^2}{\Gamma}\left[F_{\mu\alpha}F_\nu{}^\alpha - (b^2 + F^2/2)g_{\mu\nu}\right] + b^2 g_{\mu\nu} + (p+\rho)u_\mu u_\nu + p g_{\mu\nu} \right\} + \Lambda g_{\mu\nu}, \tag{7}$$

$$\Gamma \equiv b^2\sqrt{1 + F^2/2b^2}$$

$$\mathcal{F}_{\mu\nu} \equiv b^2 \frac{F_{\mu\nu}}{\Gamma},$$

$$F^2 \equiv F^{\mu\nu}F_{\mu\nu}.$$

$$p_{,\nu} = (J^{\mu}F_{\mu\sigma}u^{\sigma})u_{\nu} - (p + \rho)u^{\mu}{}_{;\mu}u_{\nu} - (p + \rho)u^{\mu}u_{\nu;\mu} - \rho_{,\mu}u^{\mu}u_{\nu} - J^{\mu}F_{\mu\nu}, \quad (8)$$

$$F_{\mu\nu;\nu} = J_{\mu}, \quad (9)$$

where b is the Born–Infeld parameter and

, (10)

(11)

(12)

When b → ∞, Born–Infeld theory goes to the Maxwell theory. We assume that the metric of the four-dimensional spacetime is the Kerr–Schild metric. Furthermore, we take the null vector λ_μ in the metric as the same null vector defined in (1). With these assumptions the Ricci tensor takes a special form.

Proposition 1. *Let* $g_{\mu\nu} = \eta_{\mu\nu} - 2V\lambda_\mu\lambda_\nu$ *and* λ_μ *be the null vector defined in (1) and let* V *be a differentiable function, then the Ricci tensor and the Ricci scalar are, respectively, given by*

$$R_{\alpha\beta} = \zeta_\beta\lambda_\alpha + \zeta_\alpha\lambda_\beta + r\delta_{\alpha\beta} + q\lambda_\beta\lambda_\alpha, \quad (13)$$

$$R_s = -2\lambda^\alpha K_{,\alpha} - 4\theta K - \frac{4V}{R^2}, \quad (14)$$

where

$$r = -\frac{2V}{R^2} - \frac{2K}{R}, \quad (15)$$

$$q = \eta^{\alpha\beta}V_{,\alpha\beta} + \epsilon r + 2a(K + \theta V) - \frac{4}{R}(\dot{z}^\mu V_{,\mu} + AK - \epsilon K), \quad (16)$$

$$\zeta_\alpha = -K_{,\alpha} + \frac{2V}{R^2}\dot{z}_\alpha, \quad (17)$$

$K \equiv \lambda^\alpha V_{,\alpha}$ *and* $\theta \equiv \lambda^\alpha{}_{,\alpha} = \frac{2}{R}$.

and

Let us assume that the electromagnetic vector potential A_μ is given by $A_\mu = H\lambda_\mu$ where H is a differentiable function. Let p and ρ be, respectively, the pressure and the energy density of a null fluid distribution with the velocity vector field $u_\mu = \lambda_\mu$. Then the difference tensor $\mathcal{T}_{\mu\nu} = G_{\mu\nu} - \kappa\left(T^{\mathrm{BI}}_{\mu\nu} + T^{f}_{\mu\nu}\right) - \Lambda g_{\mu\nu}$ is given by the following proposition.

Proposition 2. *Let* $g_{\mu\nu} = \eta_{\mu\nu} - 2V\lambda_\mu\lambda_\nu, A_\mu = H\lambda_\mu$, *where* λ_μ *is given in (1), and* V *and H be differentiable functions. Let p and* ρ *be the pressure and energy density of a null fluid with velocity vector field* λ_μ*. Then the difference tensor becomes*

$$T^\alpha{}_\beta = \lambda^\alpha W_\beta + \lambda_\beta W^\alpha + P\delta^\alpha{}_\beta + Q\lambda^\alpha \lambda_\beta \tag{18}$$

where

$$\mathcal{P} = \frac{2K}{R} + \lambda^\alpha K_{,\alpha} - \kappa b^2(1-\Gamma_0) - (\kappa p + \Lambda), \tag{19}$$

$$\mathcal{Q} = \eta^{\alpha\beta} V_{,\alpha\beta} - \frac{4}{R}(\dot{z}^\alpha V_{,\alpha}) - \frac{2K}{R}(A-\epsilon) + \frac{4AV}{R^2} - \frac{2\epsilon V}{R^2} - \kappa(p+\rho) - \frac{\kappa}{\Gamma_0}(\eta^{\alpha\beta} H_{,\alpha} H_{,\beta}), \tag{20}$$

$$W_\alpha = \frac{2V}{R^2}\dot{z}_\alpha - K_{,\alpha} + \frac{\kappa}{\Gamma_0}(\lambda^\mu H_{,\mu})H_{,\alpha} \tag{21}$$

and

$$\Gamma_0 \equiv \sqrt{1 - \frac{(\lambda^\mu H_{,\mu})^2}{b^2}}.$$

We shall now assume that the functions V and *H* depend on *R* and on some *R*-independent functions c_i,(i = 1,2,...) such that

$$c_{i,\alpha}\lambda^\alpha = 0, \tag{22}$$

for all *i*. It is clear that due to the property(5) of a_k, all these functions (c_i) are functions of the scalars*a* anda_k (k = 1,2,...), andτ_0. Infactwe shouldwrite this as $c_i = c_i(\tau_0; a, a_1, a_2, \ldots)$ where all the acceleration scalars $a, a_1, a_2, \ldots$ implicitly depend on the arclength parameter τ_0. If one uses the Serret–Frenet frame in four dimensions, one sees that all these scalars $a, a_k, k = 1,2,\ldots$, are functions of the curvaturescalars $\kappa_1, \kappa_2, \kappa_3$ of the curve *C* (see [10] for furtherdetails). We remarkthatthe scalars$a, a_1, a_2, \ldots$maynotnecessarilybeall functionally independent. We only want to emphasize that a c_i of the form $c_i = c_i(\tau_0; a, a_1, a_2, \ldots)$ identically satisfies (22). We now have the following proposition.

Proposition 3. *Let* V *and H depend on R and functions* c_i (i = 1,2,...), *that satisfy (22), then the Einstein equations given in proposition 2 reduce to the following set of equations:*

$$\kappa p + \Lambda = V'' + \frac{2}{R}V' - \kappa b^2[1 - \Gamma_0], \tag{23}$$

$$\kappa \frac{(H')^2}{\Gamma_0} = V'' - \frac{2V}{R^2}, \tag{24}$$

$$\kappa(p+\rho) = \sum_{i=1} \left[V_{,c_i}(c_{i,\alpha}{}^{,\alpha}) - \frac{4}{R} V_{,c_i}(c_{i,\alpha}\dot{z}^\alpha) - \frac{\kappa}{\Gamma_0}(H_{,c_i})^2 (c_{i,\alpha} c_i{}^{,\alpha}) \right] - \frac{2A}{R}\left(V' - \frac{2V}{R}\right) \tag{25}$$

$$\sum_{i=1} w_i c_{i,\alpha} = \left[\sum_{i=1} (w_i c_{i,\beta} \dot{z}^\beta)\right] \lambda_\alpha, \tag{26}$$

where

$$w_i = V'_{,c_i} - \frac{\kappa H'}{\Gamma_0} H_{,c_i}, \tag{27}$$

$$\Gamma_0 = \sqrt{1 - \frac{(H')^2}{b^2}}, \tag{28}$$

and the prime denotes partial differentiation with respect to R. Equation (9) defines the electromagnetic current vector J_μ,

$$\mathcal{J}^\nu = \frac{\partial}{\partial x^\mu}\left(\frac{F^{\mu\nu}}{\Gamma_0}\right), \tag{29}$$

$$F^{\mu\nu} = H'(\dot{z}^\mu \lambda^\nu - \dot{z}^\nu \lambda^\mu) + \sum_{i=1} [H_{,c_i}(c_i{}^{,\mu}\lambda^\nu - c_i{}^{,\nu}\lambda^\mu)]. \tag{30}$$

The above equations can be described as follows. Equations (23) and (25) determine, respectively, the pressure and mass density of the null fluid distribution with null velocity λ_μ. Equation (24) gives a relation between the electromagnetic and gravitational potentials *H* and V. Since this relationis quite simple, when oneof them is given,one can easily solve the other. Equation (26) implies that there are some functions c_i (i = 1,2,...) where this equation is satisfied. The functions c_i (i = 1,2,...) arise as integration constants (with respect to the variable *R*) while determining the *R* dependence of the functions V and *H*. Assuming the existence of such c_i, the above equations give the most general form of the Einstein Born– Infeld field equations with a null fluid distribution under the assumptions of proposition 2. Assuming now that the null fluid has no pressure and the cosmological constant vanishes, we have the following special case. (From now on, we set $\kappa = 8\pi$ so that one finds the correct Einstein limit as one takes $b \to \infty$ [5, 10].)

Proposition 4. *Let* $p = \Lambda = 0$. *Then*

$$V = \frac{m}{R} - 4\pi e^2 \frac{F(R)}{R}, \tag{31}$$

$$H = c - \epsilon e \int^R \frac{\mathrm{d}R}{\sqrt{R^4 + e^2/b^2}}, \tag{32}$$

where

$$m = M(\tau) + 8\pi\epsilon e c, \tag{33}$$

$$F(R) = \int^R \frac{\mathrm{d}R}{R^2 + \sqrt{R^4 + e^2/b^2}}, \tag{34}$$

$$\rho = -\frac{\dot{M}}{4\pi R^2} - \frac{(c_{,\alpha}c^{,\alpha})}{R^2}\sqrt{R^4 + e^2/b^2} + \epsilon\frac{e}{R}(c_{,\alpha}{}^{,\alpha}) - \epsilon\frac{4e}{R^2}(c_{,\alpha}\dot{z}^{\alpha}) + 6\epsilon\frac{aec}{R^2} + \frac{3Ma}{4\pi R^2}$$
$$- \frac{3ae^2}{R^2}F(R) + \frac{ae^2}{R}\frac{\mathrm{d}F}{\mathrm{d}R} - \frac{2\epsilon}{R^2}\dot{e}c + \frac{e\dot{e}}{R^2}\int^R \frac{\mathrm{d}R}{\sqrt{R^4 + e^2/b^2}}. \quad (35)$$

Here e is assumed to be a function of τ *only but the functions m and c which are related through the arbitrary function* M(τ) *(depends on* τ *only) do depend on the scalars a and* a_k $(k \geqslant 1)$*. The current vector (30) reduces to the following form:*

$$\mathcal{J}^{\mu} = \left\{\epsilon\frac{2ac_{,a}}{R^4}\left[\frac{e^2}{b^2R^2\gamma_0} + R^2\gamma_0\right] + 2\epsilon\frac{ea}{R^2} - \epsilon\frac{\dot{e}}{R^2} + \frac{\gamma_0}{R^2}c_{,a,a}(\ddot{z}_{\alpha}\ddot{z}^{\alpha} + \epsilon a^2)\right\}\lambda^{\mu}$$
$$+ \frac{2}{R^6}\frac{e^2}{b^2\gamma_0}c_{,a}(\ddot{z}^{\mu} - a\dot{z}^{\mu}) \quad (36)$$

for the simple choice c = c(τ,a). *Here* $\gamma_0 \equiv \sqrt{1 + \frac{e^2}{b^2R^4}}$.

Note that equation (23) with zero pressure and (24) determine the *R* dependence of the potentials V and *H* completely. Using proposition 3 we have chosen the integration constants (*R* independent functions) as the functions c_i (i = 1,2,3) so that c_1 = m , c_2 = e and c_3 = c, and

c = c(τ,a,a_k), e = e(τ), m = M(τ) + 8(πe)c

where a_k are defined in (4).

Remark 1. There are two limiting cases. In the first limit one obtains the Bonnor–Vaidya solutions when b → ∞. In the Bonnor–Vaidya solutions the parameters *m* and *c* (which are related through (33)) depend upon τ and *a* only. In our solution, these parameters depend not only on τ and a but also on all other scalars a_k $(k \geqslant 1)$. The scalars a_k are related to the scalar curvatures of the curve C (see [10] for further details). The second limit is the static case where the curve C is a straight line or $a_k = 0$ for all $k = 0,1,\ldots$. Our solution then reduces to the Plebanski solution [14].´

Remark 2. In the case of classical electromagnetism the Lienard–Wiechert potentials lead´ to the accelerated charge solutions [8–10]. In this case, due to the nonlinearity, we do not have such a solution. The current vector in (36) is asymptotically zero for the special choice $c = -ea$ and $e =$ constant. This means that $J = O(1/R^6)$ as $R \to \infty$. Hence the solution we found here is asymptotically pure source free Born–Infeld theory. With this special choice the currentvector is identically zero in the Maxwell case [10]. Note also that the current vector vanishes identically when $e =$ constant, $c = c(\tau)$ and $a = 0$.

Remark 3. It is easy to prove that the Born–Infeld field equations

$$\partial_\mu F_{\mu\nu} = 0$$

in flat spacetime do not admit solutions with the ansatz

$$A_\mu = H(R,\tau,a,a_k)\lambda_\mu.$$

Furthermore, the ansatz $A_\mu = H(R,\tau)\dot{z}_\mu$ is also not admissible.

Remark 4. Note that $\rho = 0$ only when the curve C is a straight line in **M** (static case). This means that there are no accelerated vacuum Einstein–Born–Infeld solutions in Kerr–Schild form.

3. Radiation due to acceleration

In this section we give the energy flux due to the acceleration of charged sources in the case of the solution given in proposition 4. The solutions described by the functions c,e and M give the energy density ρ in (35). Remember that at this point $c = c(\tau,a,a_k)$ are arbitrary. Asymptotically(as b goesto infinity) oursolutionapproachesthe Einstein–Maxwellsolutions. With the special choice $e =$ constant , $c = -ea$ our solution is asymptotically (as R goes to infinity) gauge equivalent to the flat space Lienard–Wiechert solution and reduces to the (as´ b goes to infinity) Bonnor–Vaidya solution [5]. Hence we shall use this choice in our flux expressions. The flux of null fluid energy is then given by

$$N_f = -\int_{S^2} T_f{}^{\alpha}{}_{\beta}\dot{z}_\alpha n^\beta R \,\mathrm{d}\Omega \tag{37}$$

and since $T_f{}^{\alpha}{}_{\beta} = \rho\lambda^\alpha\lambda_\beta$ for the special case $p = \Lambda = 0$ that we are examining, one finds that

$$N_f = \int_{S^2} \rho R^2 \,\mathrm{d}\Omega \tag{38}$$

where ρ is given in (35). The flux N_{BI} of Born–Infeld energy is similarly given by

$$N_{\rm BI} = -\int_{S^2} T_{\rm BI}{}^{\alpha}{}_{\beta}\dot{z}_{\alpha}n^{\beta} R\,{\rm d}\Omega \tag{39}$$

and for the solution we are examining, one finds that (as R → ∞)

$$N_{\rm BI} = \epsilon e^2 \int_{S^2} {\rm d}\Omega\,[a^2 + \epsilon(\dddot{z}^{\alpha}\ddot{z}_{\alpha})]. \tag{40}$$

Here we took e = constant and c = −ea. The total energy flux is given by

$$N = N_{\rm BI} + N_f = \epsilon \int_{S^2} \left[\frac{-\epsilon}{4\pi}\dot{M} + \frac{3\epsilon}{4\pi} aM + 2e^2 a_1 - 8e^2 a^2\right] {\rm d}\Omega \tag{41}$$

for large enough *R*. For a charge with acceleration $|\ddot{z}_{\alpha}| = \kappa_1$, we have (see [10])

$$N = -\dot{M} - \epsilon\frac{8\pi}{3}e^2(\kappa_1)^2, \tag{42}$$

whereκ_1 is the first curvaturescalar of *C*. This is exactlythe result of Bonnorand Vaidyain [5]. Hence with the choice of c = −ea, the linear classical electromagnetism and the Born–Infeld theories give the same energy flux for the accelerated charges. This, however, should not be surprising considering the fact that the Born–Infeld theory was originally introduced to solve the classical self-energy problem of an electron described by the Maxwell theory in the shortdistance limit [11]. For other choices of c = c(τ,a,a_k), one obtains different expressions for the energy flux.

4. Conclusion

We have found exact solutions of the Einstein Born–Infeld field equations with a null fluid source. Physically, these solutions describe electromagnetic and gravitational fields of a charged particle moving on an arbitrary curve *C*. The metric and the electromagnetic vector potential arbitrarily depend on a scalar c(τ_0,a,a_k) which can be related to the curvatures of the curve *C*. When the Born–Infeld parameter *b* goes to infinity, our solution reduces to the Bonnor–Vaidya solution of the Einstein–Maxwell field equations [5, 10]. On the other hand, when the curve *C* is a straight line in **M**, our solution matches with the Plebanski solution´ [14]. We have also studied the flux of Born–Infeld energy due to the acceleration of charged particles. We observed that the energy flux formula depends on the choice of the scalar *c* in terms of the functions a,a_k (or the curvature scalars of the curve *C*).

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