phys. stat. sol. (b) **237**, No. 2, 500–512 (2003) / **DOI** 10.1002/pssb.200301654

Weak localization and the Mooij rule in disordered metals

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Received 29 May 2002, revised 19 November 2002, accepted 27 November 2002
Published online 24 April 2003

PACS 72.10.Di, 72.15.Cz, 72.15.Rn, 72.60.+g

Weak localization leads to the same correction to both the conductivity and the McMillan's electron–phonon coupling constant λ (and λ_{tr}, transport electron–phonon coupling constant). Consequently the temperature dependence of the thermal electrical resistivity is decreasing as the conductivity is decreasing due to weak localization, which results in the decrease of the temperature coefficient of resistivity (TCR) with increasing the residual resistivity. When λ and λ_{tr} are approaching zero, only the residual resistivity part remains and it gives rise to the negative TCR. Accordingly, the Mooij rule is a manifestation of weak localization correction to the conductivity and the electron–phonon interaction. This understanding provides a new means of probing the phonon-mechanism in exotic superconductors and an opportunity of fabricating new novel devices.

1 Introduction

Although weak localization has greatly deepened our understanding of the normal state of disordered metals [1–3], its effect on superconductivity and the electron–phonon interaction has not been understood well [2]. Recently, it has been shown that weak localization leads to the same correction to the conductivity and the phonon-mediated interaction in superconductivity [4, 5]. In fact, there are overwhelming numbers of experiments which support this idea [4]. For instance, tunneling [6–8], specific heat [9], X-ray photoemission spectroscopy (XPS) [10], correlation of T_c and the residual resistivity [11–13], universal correlation of T_c and the resistance ratio [14–16], and loss of the thermal electrical resistivity [17] with decreasing T_c clearly show a decrease of the electron–phonon interaction accompanying the decrease of T_c with disorder. It is then anticipated that the electron–phonon interaction in the normal state of metals will also be influenced strongly by weak localization. We expect that phonon-limited electrical resistance, attenuation of a sound wave, thermal resistance, and a shift in phonon frequencies may change due to weak localization [18].

Indeed, the Mooij rule [19] in strongly disordered metallic systems seems to be a manifestation of the effect of weak localization on the electron–phonon interaction and the conductivity. In early seventies, Mooij found a correlation between the residual resistivity and the temperature coefficient of resistivity (TCR). In particular, TCR is decreasing with increasing the residual resistivity. Then it becomes negative for resistivities above 150 $\mu\Omega$ cm. We stress that this behavior is consistent with the above superconducting properties: correlation of T_c and the residual resistivity [11–13], universal

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 0370-1972/03/23706-0500 $ 17.50+.50/0

correlation of T_c and the resistance ratio [14–16], and loss of the thermal electrical resistivity [17] with decreasing T_c.

There are already several theoretical works on this problem. Jonson and Girvin [20] performed numerical calculations for an Anderson model on a Cayley tree and found that the adiabatic phonon approximation breaks down in the high-resistivity regime producing the negative TCR. Imry [21] pointed out the importance of incipient Anderson localization (weak localization) in the resistivities of highly disordered metals. He argued that when the inelastic mean free path, ℓ_{ph}, is smaller than the coherence length, ξ, the conductivity increases with temperature like ℓ_{ph}^{-1} and thereby leads to the negative TCR. On the other hand, Kaveh and Mott [22] generalized the Mooij rule. Their results are as follows: the temperature dependence of the conductivity of a disordered metal as a function of temperature changes slope due to weak localization effects, and if interaction effects are included, the conductivity changes its slope three times. Belitz and coworkers [23, 24] introduced a theory with phonon-induced tunneling. There is also the extended Ziman theory [25], and Jayannavar and Kumar [26] suggested that the Mooij rule can arise from strong electron–phonon interaction taking into account qualitatively different roles of the diagonal and off-diagonal modulations. Zhao et al. [27] used the first-principles electronic structure calculation for 100- and 200-atom model for metallic glasses to compute the electronic transport properties. They noticed that the magnitudes and the shape of the conductivity function can give rise to a negative TCR.

In this paper, we propose an explanation of the Mooij rule based on the effect of weak localization on the electron–phonon interaction. We show that TCR decreases with increasing the residual resistivity, since weak localization decreases the electron–phonon interaction [4, 5]. The negative TCR is therefore due to weak localization correction to the Boltzmann conductivity. Note that when TCR is approaching zero there is no temperature-dependent resistivity left. (This latter point is similar to Kaveh and Mott's interpretation [22].) In Section 2, we briefly describe the Mooij rule. In Section 3, weak localization correction to the McMillan's electron–phonon coupling constant λ and λ_{tr} is calculated. A possible explanation of the Mooij rule is given in Section 4, and its implication is briefly discussed in Section 5. In particular, this study provides a means to probe the phonon-mechanism in exotic superconductors [5, 28]. Furthermore, since weak localization basically leads to the decoupling of electrons and phonons, this property can be employed to fabricate new novel devices [29].

2 The Mooij rule

Mooij [19] was the first to point out that the size and sign of the temperature coefficient of resistivity (TCR) in many disordered systems correlate with its residual resistivity ρ_0 as follows:

$$\begin{aligned} \mathrm{d}\rho/\mathrm{d}T > 0 \quad &\text{if} \quad \rho_0 < \rho_M\,, \\ \mathrm{d}\rho/\mathrm{d}T < 0 \quad &\text{if} \quad \rho_0 > \rho_M\,. \end{aligned} \tag{1}$$

Thus, TCR changes sign when ρ_0 reaches the Mooij resistivity $\rho_M \cong 150\,\mu\Omega\,\mathrm{cm}$. An approximate equation for $\rho(T)$ is given by [2]

$$\rho(T) = \rho_0 + (\rho_M - \rho_0)\,AT\,, \tag{2}$$

where A is a constant which depends on the material.

Figure 1 shows the temperature coefficient of resistance α versus resistivity for transition-metal alloys obtained by Mooij. It is clear that α (and TCR) is correlated with the residual resistivity. Note that above $150\,\mu\Omega\,\mathrm{cm}$ most α values are negative while no negative α is found for resistivities below $100\,\mu\Omega\,\mathrm{cm}$. Figure 2 shows the resistivity as a function of temperature for pure Ti and TiAl alloys containing 3, 6, 11, and 33% Al. TCR is decreasing as the residual resistivity is increasing. For TiAl alloy with 33% Al shows a negative TCR. The solid line denotes the temperature range which will be considered in our theoretical calculation. The resistivity saturation above 1000 K is not yet well understood [2]. We note that the positive TCR is basically a high temperature phenomenon, presumably related to the phonon-limited resistivity, whereas the negative TCR is rather a low temperature beha-

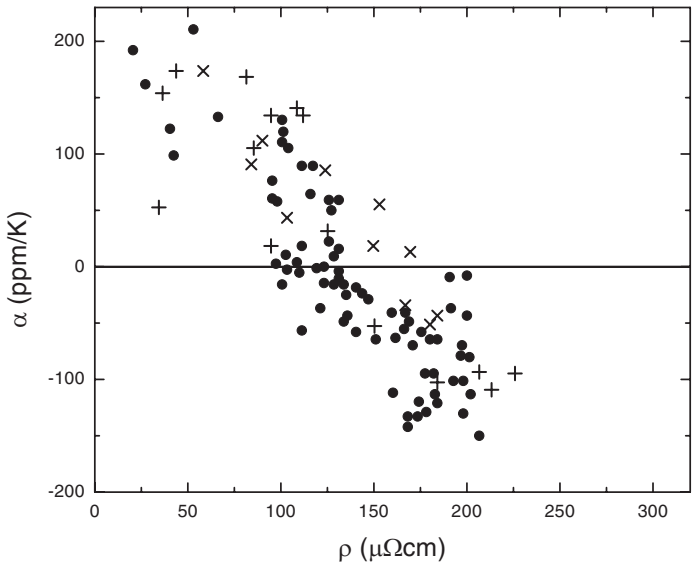

Fig. 1 Temperature coefficient of resistance α versus resistivity for bulk alloys (+), thin films (●), and amorphous (×) alloys. Data are from Mooij, Ref. [19].

vior, probably connected with the residual resistivity part. Since this behavior is generally found in strongly disordered metals and alloys, amorphous metals, and metallic glasses [2], it is called the Mooij rule. However, the physical origin of this rule has remained unexplained until now.

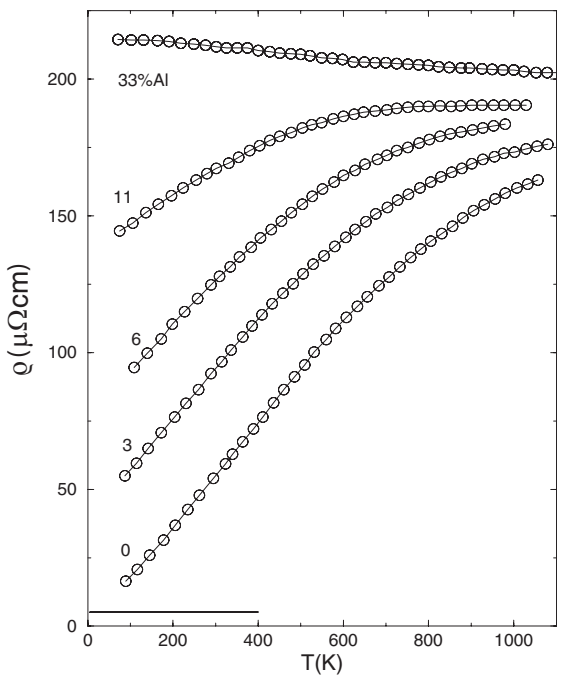

Fig. 2 Resistivity versus temperature for Ti and TiAl alloys containing 0, 3, 6, 11, and 33% Al. Data are from Mooij, Ref. [19]. The solid line represents the temperature range where our theoretical calculation will be compared with the experimental data.

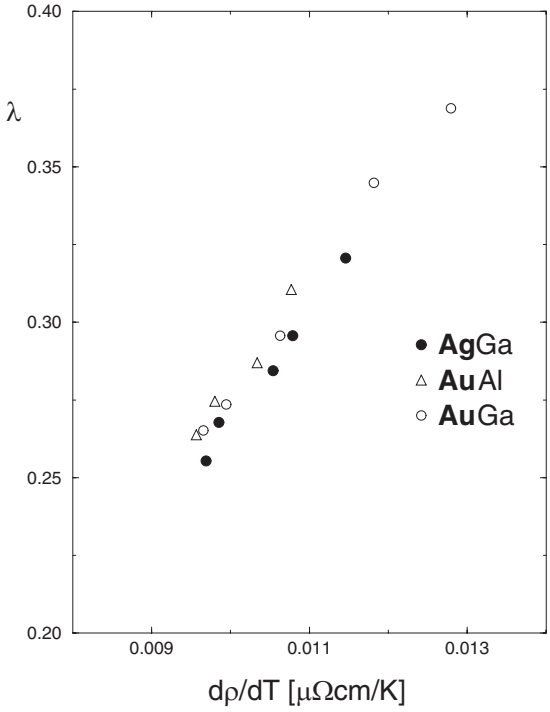

Fig. 3 McMillan's coupling constant λ versus $\mathrm{d}\rho/\mathrm{d}T \propto \lambda_{\mathrm{tr}}$ for Ag–Ga, Au–Al, and Au–Ga alloys. Data are from Rapp, Ref. [55] and Grimvall, Ref. [35].

3 Weak localization correction to the electron–phonon interaction

Since the electron–phonon interaction in metals gives rise to both the (high temperature) resistivity and superconductivity, these properties are closely related, which was noticed by many workers [30–34]. Gladstone et al. [30] pointed out that λ and the high temperature electrical resistivity are closely related each other. Hopfield [31, 32] noted that the electronic relaxation time due to electron–phonon interaction, as measured in optical experiments above the Debye temperature, should be approximately equal to $2\pi\lambda k_B T/\hbar$. He applied this idea to Nb, Mo, Al and Sn and found a good agreement with experiment. Grimvall [33] estimated λ for noble metals from Ziman's high temperature resistivity formula. Maksimov and Motulevich [34] followed the idea of Hopfield and estimated λ from optical measurements for Pb, Sn, In, Al, Zn, Nb, V, Nb_3Sn, and V_3Ga, which are in good agreement with the McMillan's coupling constant λ from superconductivity data.

In this section, we show that weak localization leads to the same correction to the conductivity, the McMillan's electron–phonon coupling constant λ and $\lambda_{\rm tr}$.

3.1 High temperature resistivity

At high temperatures, the phonon limited electrical resistivity is [35–38]

$$\rho_{\rm ph}(T) = \frac{4\pi m k_B T}{n e^2 \hbar} \int \frac{\alpha_{\rm tr}^2 F(\omega)}{\omega}\, {\rm d}\omega\,, \\ = \frac{2\pi m k_B T}{n e^2 \hbar}\, \lambda_{\rm tr}, \tag{3}$$

where $\alpha_{\rm tr}^2 F(\omega)$ is the transport electron–phonon coupling function which includes an average of a geometrical factor $1 - \cos\theta_{\boldsymbol{kk'}}$ in the Eliashberg coupling function $\alpha^2 F(\omega)$. $F(\omega)$ is the phonon density of states. On the other hand, in the strong-coupling theory of superconductivity [39, 40], the McMillan's electron–phonon coupling constant is defined by [40]

$$\lambda = 2 \int \frac{\alpha^2(\omega)\, F(\omega)}{\omega}\, {\rm d}\omega\,. \tag{4}$$

Assuming $\alpha_{\rm tr}^2 \cong \alpha^2$ [35, 41–43], we obtain

$$\rho_{\rm ph}(T) = \frac{2\pi m k_B T}{n e^2 \hbar}\, \lambda_{\rm tr} \tag{5}$$

$$\cong \frac{2\pi m k_B T}{n e^2 \hbar}\, \lambda\,. \tag{6}$$

Consequently the McMillan's coupling constant λ also determines the size and sign of TCR.

The existence of this relationship was well confirmed theoretically and experimentally. Table 1 shows the comparison of $\lambda_{\rm tr}$ and λ by Economou [41] for various materials. He obtained $\lambda_{\rm tr}$ from Eq.

Table 1 Comparison of $\lambda_{\rm tr}$ and the McMillan's electron-phonon coupling constant λ. Data are from Economou, Ref. [41] and Grimvall, Ref. [42].

metal	$\lambda_{\rm tr}$	λ	metal	$\lambda_{\rm tr}$	λ
Li	0.40	0.41 ± 0.15	Na	0.16	0.16 ± 0.04
K	0.14	0.13 ± 0.03	Rb	0.19	0.16 ± 0.04
Cs	0.26	0.16 ± 0.06	Mg	0.32	0.35 ± 0.04
Zn	0.67	0.42 ± 0.05	Cd	0.51	0.40 ± 0.05
Al	0.41	0.43 ± 0.05	Pb	1.79	1.55
In	0.85	0.805	Hg	2.3	1.6
Cu	0.13	0.14 ± 0.03	Ag	0.13	0.10 ± 0.04
Au	0.08	0.14 ± 0.05	Nb	1.11	0.9 ± 0.2

(5) and compared with λ, as obtained from T_c measurements, and/or tunneling experiments, and/or first principle calculations [42]. The overall agreement between $\lambda_{\rm tr}$ and λ is impressive. Grimvall estimated λ for noble metals [33] and noble metal alloys [44] from Eq. (6). Maksimov [43] also noted the direct relation between λ and the high temperature resistivity. Hayman and Carbotte [45] pointed out that information on the volume dependence of an electron–phonon coupling strength can be obtained from high temperature resistivity. Chakraborty et al. [46] used Eq. (5) to obtain the empirical values of $\lambda_{\rm tr}$ for Nb, Mo, Ta, and W. They found that $\lambda_{\rm tr}$ from resistance and the McMillan's coupling constant λ from superconductivity are very similar in magnitude for these materials. We can also mention experimental confirmations by Rapp and Crawfoord [47] for Nb–V alloys, Rapp and Fogelholm [48] for Al–Mg alloys, Flükiger and Ishikawa [49] for Zr–Nb–Mo alloys, Fogelholm and Rapp [50] for In-Sn alloys, Lutz et al. [51] for Nb_3Ge films, Mankovskii et al. [52] for thin Sn films, Rapp et al. [53] for Au–Ga alloys, and Sundqvist and Rapp [54] for aluminum under pressure. Figure 3 shows the McMillan's coupling constant λ versus $\mathrm{d}\rho/\mathrm{d}T \propto \lambda_{\rm tr}$ for Au–Ga, Au–Al, and Ag–Ga alloys [55], which examplifies the correlation implied by Eq. (6).

3.2 Weak localization correction to the McMillan's coupling constants λ and $\lambda_{\rm tr}$

Now we need to calculate the McMillan's electron–phonon coupling constant λ for highly disordered systems. We follow McMillan's approach to the strong-coupling theory [5, 40]. (For simplicity we consider an Einstein model with frequency ω_D). He showed that λ can be written as [40]

$$\lambda = 2\int \frac{\alpha^2(\omega)\, F(\omega)}{\omega}\,\mathrm{d}\omega \tag{7}$$

$$= N_0\,\frac{\langle I^2\rangle}{M\langle\omega^2\rangle}\,, \tag{8}$$

where M is the ionic mass and N_0 is the electron density of states at the Fermi level. $\langle I^2\rangle$ is the average over the Fermi surface of the square of the electronic matrix element and $\langle\omega^2\rangle = \omega_D^2$. In the presence of impurities, weak localization mainly leads to a correction to $\alpha^2(\omega)$ or $\langle I^2\rangle$. We disregards the changes of $F(\omega)$ and N_0, since experimental data do not show any significant changes of $F(\omega)$ [56, 57] and N_0 [58, 59].

There are two ways to obtain the McMillan's coupling constant λ in the presence of impurities. One method is to calculate λ directly from Eq. (8), using the electronic matrix element for disordered systems and the other is to carry out the canonical transformation of Fröhlich in the scattered state basis [4, 60]. We have found that both methods lead to the same λ.

In this paper, we use the latter method in a simple manner by observing that the Fröhlich interaction can be derived from the phonon Green's function [61]. We note that the equivalent electron–electron potential in the electron–phonon problem is given by the phonon Green's function $D(x-x')$ [61–63]

$$V(x-x') \to \frac{I_0^2}{M\omega_D^2}\, D(x-x')\,, \tag{9}$$

where $x = (\boldsymbol{r}, t)$ and I_0 is the electronic matrix element for the plane wave states. The Fröhlich interaction at finite temperatures is then given by [61]

$$\begin{aligned} V_{nn'}(\omega,\omega') &= \frac{I_0^2}{M\omega_D^2}\int\!\!\int \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}'\,\psi^*_{n'}(\boldsymbol{r})\,\psi^*_{\bar{n}'}(\boldsymbol{r}')\,D(\boldsymbol{r}-\boldsymbol{r}',\omega-\omega')\,\psi_{\bar{n}}(\boldsymbol{r}')\,\psi_n(\boldsymbol{r}) \\ &= \frac{I_0^2}{M\omega_D^2}\int |\psi_{n'}(\boldsymbol{r})|^2\,|\psi_n(\boldsymbol{r})|^2\,\mathrm{d}\boldsymbol{r}\,\frac{\omega_D^2}{\omega_D^2+(\omega-\omega')^2} \\ &= V_{nn'}\,\frac{\omega_D^2}{\omega_D^2+(\omega-\omega')^2}\,, \end{aligned} \tag{10}$$

where

$$D(\boldsymbol{r}-\boldsymbol{r}',\omega-\omega') = \sum_{\boldsymbol{q}} \frac{\omega_D^2}{(\omega-\omega')^2+\omega_D^2}\, \mathrm{e}^{i\boldsymbol{q}\cdot(\boldsymbol{r}-\boldsymbol{r}')} = \frac{\omega_D^2}{(\omega-\omega')^2+\omega_D^2}\, \delta(\boldsymbol{r}-\boldsymbol{r}')\,. \tag{11}$$

Here ω means the Matsubara frequency and ψ_n and $\psi_{\bar{n}}$ denote the scattered state and its time-reversed partner, respectively. Therefore, we get the strong-coupling gap equation [4]

$$\Delta(n,\omega) = T\sum_{\omega'}\sum_{n'} V_{nn'}(\omega,\omega')\, \frac{\Delta(n',\omega')}{\omega'^2+E_{n'}^2(\omega')} = T\sum_{\omega'} \frac{\omega_D^2}{(\omega-\omega')^2+\omega_D^2} \sum_{n'} V_{nn'}\, \frac{\Delta(n',\omega')}{\omega'^2+E_{n'}^2(\omega')}\,, \tag{12}$$

where

$$E_{n'}(\omega') = \sqrt{\epsilon_{n'}^2+\Delta_{n'}^2(\omega')}\,, \tag{13}$$

and the McMillan's electron–phonon coupling constant λ

$$\lambda = N_0\langle V_{nn'}(0,0)\rangle = N_0\, \frac{I_0^2}{M\omega_D^2} \left\langle \int |\psi_n(\boldsymbol{r})|^2\, |\psi_{n'}(\boldsymbol{r})|^2\, \mathrm{d}\boldsymbol{r} \right\rangle. \tag{14}$$

Here ϵ_n means the eigenenergy of the scattered state ψ_n. As expected, Eq. (14) is the same as Eq. (8). It is remarkable that the McMillan's electron–phonon coupling constant is determined by the density (or eigenstate) correlatin function, $\int |\psi_n(\boldsymbol{r})|^2\, |\psi_{n'}(\boldsymbol{r})|^2\, \mathrm{d}\boldsymbol{r}$ [60].

Note also that in the presence of impurities, the density correlation function has a free-particle form for $t<\tau$ (scattering time) and a diffusive form for $t>\tau$ [64]. As a result, for $t>\tau$ (or $r>\ell$), one finds [65–69]

$$R(t>\tau) = \int\limits_{t>\tau} |\psi_n(\boldsymbol{r})|^2\, |\psi_{n'}(\boldsymbol{r})|^2\, \mathrm{d}\boldsymbol{r} = \sum_{\boldsymbol{q}} |\langle\psi_n|\, \mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{r}}\, |\psi_{n'}\rangle|^2 = \sum_{\pi/L<\boldsymbol{q}<\pi/\ell} \frac{1}{2\pi\hbar N_0 D\boldsymbol{q}^2} \tag{15}$$

$$= \frac{3}{2(k_F\ell)^2}\left(1-\frac{\ell}{L}\right). \tag{16}$$

Here ℓ is the mean free path and L is the inelastic diffusion length. D means the diffusion constant and k_F is the Fermi wave vector. Whereas the contribution from the free-particle-like density correlation for $t<\tau$ is [4, 65]

$$R(t<\tau) = \int\limits_{t<\tau} |\psi_n(\boldsymbol{r})|^2\, |\psi_{n'}(\boldsymbol{r})|^2\, \mathrm{d}\boldsymbol{r} = \left[1-\frac{3}{(k_F\ell)^2}\left(1-\frac{\ell}{L}\right)\right]. \tag{17}$$

Since the phonon-mediated interaction is retarded for $t_{\mathrm{ret}}\sim 1/\omega_D$, only the free-particle-like density correlation contributes to λ. This is also true of λ_{tr}, simply because the conductivity is determined by the behavior of the wavefunction ψ for $t<\tau$ (or $r<\ell$) [70].

Consequently, we obtain weak localization correction to the McMillan's coupling constants λ and λ_{tr}

$$\lambda = N_0\, \frac{I_0^2}{M\omega_D^2}\left[1-\frac{3}{(k_F\ell)^2}\left(1-\frac{\ell}{L}\right)\right] \tag{18}$$

and

$$\lambda_{\mathrm{tr}} = 2\int \frac{\alpha_{\mathrm{tr}}^2(\omega)\, F(\omega)}{\omega}\, \mathrm{d}\omega \cong \frac{N_0 I_0^2}{M\omega_D^2}\left[1-\frac{3}{(k_F\ell)^2}\left(1-\frac{\ell}{L}\right)\right] = \frac{N_0 I_0^2}{M\omega_D^2}\left[1-\frac{3}{(k_F\ell)^2}\right]. \tag{19}$$

For $\lambda_{\rm tr}$ we have used $L = \infty$ at $T = 0$, since the zero temperature electron−phonon coupling constant is required [39]. Note that the weak localization correction term is essentially the same as that of the conductivity.

4 Explanation of the Mooij rule

As noted in the Section 2, the positive TCR is high temperature phenomenon whereas the negative TCR is low temperature phenomenon. Thus, the decrease of the positive TCR is mainly due to the decrease of the phonon-limited resistivity, which is a manifestation of weak localization correction to the electron−phonon interaction. On the other hand, the negative TCR originates from the residual resistivity, which is also a manifestation of weak localization correction to the conductivity. Accordingly, weak localization seems to be the physical origin of the Mooij rule in disordered metals. One should note that this observation agrees with the superconducting behavior of disordered system, when the electrons are weakly localized [14−17].

4.1 Decrease of TCR at high temperatures

Upon substituting Eq. (19) into Eq. (3), one finds the phonon-limited high temperature resistivity

$$\rho_{\rm ph}(T) = \frac{2\pi m k_B T}{n e^2 \hbar}\, \lambda_{\rm tr} \cong \frac{2\pi m k_B T}{n e^2 \hbar}\, \frac{N_0 I_0^2}{M\omega_D^2} \left[1 - \frac{3}{(k_F \ell)^2}\right]. \tag{20}$$

Note that as the disorder parameter $1/k_F\ell$ is increasing, both the magnitude of the phonon-limited resistivity and the TCR decrease. This behavior is due to the reduction of the McMillan's electron−phonon coupling constant when electrons are weakly localized. It is remarkable that the slope of the high temperature resistivity varies as $\sim 1/(k_F\ell)^2$, in accord with the behavior of the residual resistivity.

The phonon-limited resistivity $\rho_{\rm ph}$ versus temperature T is shown in Fig. 4a for six values of $k_F\ell$. Since conventional transport theory uses $(k_F\ell)^{-1}$ as an expansion parameter [2], $k_F\ell$ is a good measure of the degree of disorder. We used $k_F = 0.8\ \text{Å}^{-1}$, $n = k_F^3/3\pi^2$, and $N_0 I_0^2/(M\omega_D^2) = 0.5$. It is clear that TCR is decreasing significantly as the electrons are weakly localized.

4.2 Negative TCR at low temperatures

At low temperatures the conductivity and the residual resistivity are given by [2, 3]

$$\sigma = \sigma_B \left[1 - \frac{3}{(k_F\ell)^2}\left(1 - \frac{\ell}{L}\right)\right] \tag{21}$$

and

$$\rho_0 = \frac{1}{\sigma_B \left[1 - \dfrac{3}{(k_F\ell)^2}\left(1 - \dfrac{\ell}{L}\right)\right]}, \tag{22}$$

where $\sigma_B = ne^2\tau/m$. When $1/k_F\ell$ becomes comparable to ~ 1, the magnitude and slope of $\rho_{\rm ph}(T)$ are negligible. In that case, only the residual resistivity will play an important role. Therefore, the observed negative TCR may be understood from the residual part. With decreasing T, since the inelastic diffusion length L increases, the residual resistivity will also increase, leading to the negative TCR. We stress that both the phonon-limited resistivity and the residual resistivity have the same quadratic dependence on the disorder parameter $1/k_F\ell$.

Figure 4b shows the temperature dependence of the residual resistivity ρ_0 for $k_F\ell = 2.2, 2.4,\ 2.8,\ 3.4, 5,$ and 15. Since it is difficult to evaluate $k_F\ell$ up to a factor of two [71], we assumed

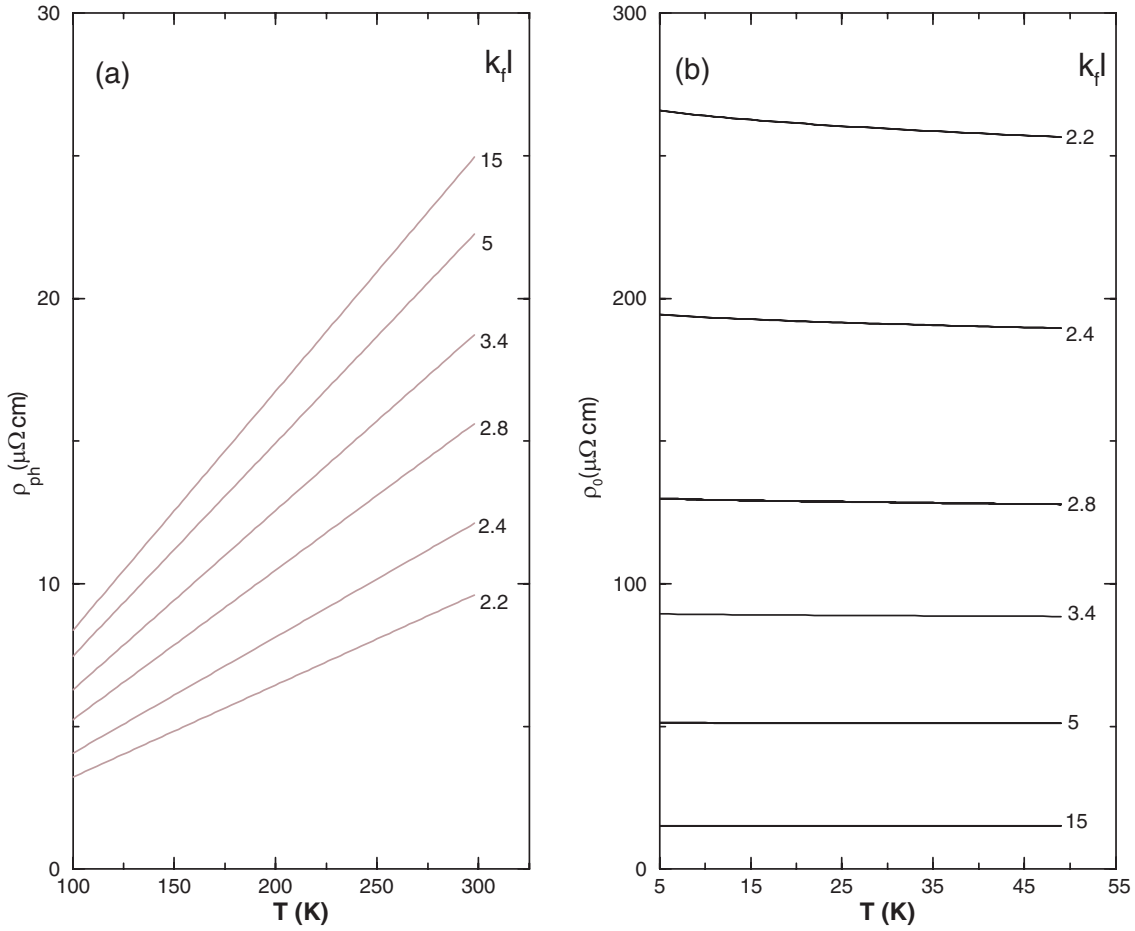

Fig. 4 (online colour at: www.interscience.wiley.com) a) Phonon-limited resistivity $\rho_{\rm ph}$ versus T for $k_F\ell = 15$, 5, 3.4, 2.8, 2.4, and 2.2. b) Residual resistivity ρ_0 versus T for the same six values of $k_F\ell$.

that $\rho_0 = 100\,\mu\Omega\,\mathrm{cm}$ corresponds to $k_F\ell = 3.2$. We used the same k_F as in Fig. 4a and $L = \sqrt{D\tau_i} = \sqrt{\ell} \times 350/\sqrt{T}\,(\text{Å})$. Here τ_i denotes the inelastic scattering time. When $k_F\ell$ is comparable to 1, the negative TCR emerges. Notice the scale difference between Figs. 4a and b.

4.3 Comparison with experiment

In sections 4.1 and 4.2, we have explained the physical origin of the Mooij rule. In this section, we compare our theoretical resistivity curve and the experimental data (Fig. 2) for extended temperature range, up to 400 K. Let us remind the approximate formula for $\rho(T)$ suggested by Lee and Ramakrishnan, i.e. [2],

$$\rho(T) = \rho_0 + (\rho_M - \rho_0)\,AT\,. \tag{23}$$

This form of equation can be obtained if we add the residual resistivity Eq. (22) and the phonon-limited resistivity Eq. (20), that is

$$\rho(T) = \rho_0 + \rho_{\rm ph}(T) = \frac{1}{\sigma_B\left[1 - \dfrac{3}{(k_F\ell)^2}\left(1 - \dfrac{\ell}{L}\right)\right]} + \frac{2\pi m k_B T}{ne^2\hbar}\,\frac{N_0 I_0^2}{M\omega_D^2}\left[1 - \frac{3}{(k_F\ell)^2}\right]. \tag{24}$$

It should be noticed that the addition of both resistivities does not mean the Matthiessen's rule. Here we included the interference effect between the electron−phonon and electron−impurity interactions:

$$\rho(T) = \rho_0 + \rho_{\rm ph}(T, c=0) + \Delta\rho_{\rm ph}^{\rm int}\,, \tag{25}$$

where c denotes an impurity concentration. Whereas Altshuler [72] and Reizer and Sergeev [73] investigated corrections to the impurity resistivity due to the interference, we have considered its correction to the phonon-limited resistivity. Since the interference correction to the impurity resistivity is $\sim 1\%$ of the residual resistivity [73, 74], we neglect its effect for simplicity.

In general, the phonon-limited resistivity at any temperature T is given by [35−38]

$$\rho_{\rm ph}(T) = \frac{4\pi m}{ne^2} \int \frac{(\beta\hbar\omega)\,\alpha_{\rm tr}^2(\omega)\,F(\omega)}{(\mathrm{e}^{\beta\hbar\omega}-1)\,(1-\mathrm{e}^{-\beta\hbar\omega})}\,\mathrm{d}\omega\,, \tag{26}$$

where $\beta = 1/k_B T$. For an Einstein phonon model with [75]

$$\alpha_{\rm tr}^2(\omega)\,F(\omega) = \frac{N_0 I_0^2}{2M\omega_D}\,\delta(\omega-\omega_D)\,, \tag{27}$$

it is rewritten as [76]

$$\rho_{\rm ph}(T) = \frac{2\pi m}{ne^2}\,\frac{N_0 I_0^2}{M\omega_D^2}\,\frac{(\beta\hbar\omega_D)\,\omega_D}{(\mathrm{e}^{\beta\hbar\omega_D}-1)\,(1-\mathrm{e}^{-\beta\hbar\omega_D})}\,. \tag{28}$$

It is necessary to emphasize that this result is exact for the phonon-limited resistivity in an Einstein model [74]. Including the weak localization correction to $\alpha^2(\omega) \cong \alpha_{\rm tr}^2(\omega)$,

$$\alpha_{\rm tr}^2(\omega)F(\omega) = \frac{N_0 I_0^2}{2M\omega_D}\left[1-\frac{3}{(k_F\ell)^2}\right]\delta(\omega-\omega_D)\,, \tag{29}$$

one finds

$$\rho_{\rm ph}(T) = \frac{2\pi m}{ne^2}\,\frac{N_0 I_0^2}{M\omega_D^2}\left[1-\frac{3}{(k_F\ell)^2}\right]\frac{(\beta\hbar\omega_D)\,\omega_D}{(\mathrm{e}^{\beta\hbar\omega_D}-1)\,(1-\mathrm{e}^{-\beta\hbar\omega_D})}\,. \tag{30}$$

Finally, we obtain the total resistivity at any temperature T

$$\begin{aligned}\rho(T) &= \rho_0 + \rho_{\rm ph}(T) \\ &= \frac{1}{\sigma_B\left[1-\dfrac{3}{(k_F\ell)^2}\left(1-\dfrac{\ell}{L}\right)\right]} + \frac{2\pi m}{ne^2}\,\frac{N_0 I_0^2}{M\omega_D^2}\left[1-\frac{3}{(k_F\ell)^2}\right]\frac{(\beta\hbar\omega_D)\,\omega_D}{(\mathrm{e}^{\beta\hbar\omega_D}-1)\,(1-\mathrm{e}^{-\beta\hbar\omega_D})}\,.\end{aligned} \tag{31}$$

(If we consider the Debye and realistic phonon models, there are minor changes. However, the overall behavior is the same. More details will be published elsewhere.)

Figure 5 shows the resistivity as a function of temperature for $k_F\ell = 2.3, 2.5, 2.8, 3.4, 5$, and 15. The solid lines represent the resistivity from an accurate expression Eq. (31), while the dashed lines are obtained from Eq. (24). We used the same parameters as those in Fig. 4 and $\hbar\omega_D = 250$ K. It is noteworthy that both equations give rise to almost the same curve as the system is more disordered. For low temperatures τ_i is determined by electron−electron scattering while for high temperatures it is determined by the electron−phonon scattering. Since we are interested in rather high temperatures, we assumed $\tau_i \sim T^{-1}$ corresponding to the electron−phonon scattering [2, 3] as in Fig. 4b, i.e., $L = \sqrt{D\tau_i} = \sqrt{\ell}\times 350/\sqrt{T}$ (Å). Considering the crudeness of our calculation, the overall behavior is in good agreement with experiment, Fig. 2 (up to temperature 400K). In Fig. 2 the resistivity saturation near 1000 K still remains unresolved [2, 34].

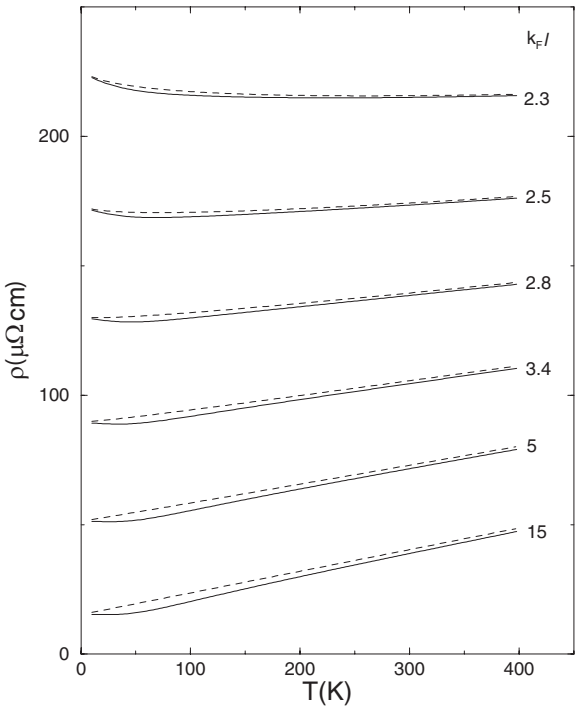

Fig. 5 Calculated resistivity versus temperature for $k_F\ell = 15$, 5, 3.4, 2.8, 2.5, and 2.3. The solid lines are $\rho(T)$ from an accurate formula, Eq. (31). The dashed lines represent the resistivity obtained from the approximate expression, Eq. (24).

5 Discussion

At low temperatures the interference of the Coulomb interaction and the impurity scattering leads to the interaction correction to the conductivity [2, 67]. This effect is described by [77]

$$\sigma = \sigma_B \left[1 - \frac{3}{(k_F\ell)^2} \left(1 - \frac{\ell}{L} \right) - \frac{C}{(k_F\ell)^2} \left(1 - \frac{\ell}{L_T} \right) \right], \tag{32}$$

where $L_T = (\hbar D/k_B T)^{1/2}$ and $C \sim 1$. The second correction term is the interaction term. The constant C, however, changes sign depending on the exchange and Hartree terms and since it is difficult to determine C [2, 3, 77], we did not include this term. But it may be important at much lower temperatures.

It is clear that weak localization effect on the electron–phonon interaction needs more theoretical and experimental studies. In particular, weak localization effect on the attenuation of a sound wave, shear modulus, thermal resistance, and a shift in phonon frequencies will be very interesting. Since superconductivity is also caused by the electron–phonon interaction, comparative study of the normal and superconducting properties of the metallic samples will be beneficial. There is already compelling evidence that this is the case. as shown by Testardi's universal correlation of T_c and the resistance ratio [11–17]. Recently, Elliot et al. [78] studied the conductance and superconducting transition temperature of Mo/Si multilayers as a function of the metal layer thickness, from 7 Å to 85 Å. They found the Mooij rule with a crossover resistivity of 125 μΩ cm and approximate correlation between the resistance ratio and T_c. Since their very thin films may be inhomogeneous macroscopically, some deviations are expected.

Observe that this study may provide a means of probing the phonon-mechanism in exotic superconductors, such as, heavy fermion superconductors, organic superconductors, fullerene superconductors, and high T_c cuprates. For superconductors caused by the electron–phonon interaction we expect the following behavior. As the electrons are weakly localized by impurities or radiation damage, the electron–phonon interaction is weakened. As a result, both T_c and TCR are decreasing at the same

rate. When λ is approaching zero, both T_c and TCR drops to zero almost simultaneously. When this happens we may say that the electron−phonon interaction is the origin of the pairing in the superconductors. This behavior was already confirmed in A15 superconductors [14−17] and Ternary superconductors [79]. Recently, it has been shown that MgB_2 also shows the same behavior, implying that MgB_2 is a BCS superconductor [5, 28]. In particular, Buzea and Yamashita [28] elaborated this approach in their review paper.

It is also noteworthy that this understanding leads to the fabrication of new novel devices. For example, Gershenson et al. [29] have suggested fabricating the hot-electron detectors of far-infrared radiation using ultra-thin disordered metal films, which have millisecond electron−phonon relaxation time at millikelvin temperatures. The long relaxation time is due to the decoupling of electrons and phonons caused by weak localization. We may also devise high-Q resonant-mass antennas and test masses for gravitational wave detectors [80, 81], since weak localization can make the displacement induced by gravitational waves to be free from the dissipation caused by the electron−phonon interaction. More details will be published elsewhere.

6 Conclusion

It is shown that weak localization decreases both the conductivity and the electron−phonon interaction at the same rate and thereby leads to the Mooij rule. As the residual resistivity is increasing due to weak localization, so the thermal electrical resistivity is decreasing, producing the decrease of TCR. When the electron−phonon interaction is near zero, only the residual resistivity is left and therefore the negative TCR obtains. We emphasize that weak localization induced correlation of normal and superconducting properties provides a means of probing the phonon-mechanism in exotic superconductors, such as, heavy fermion superconductors, organic and fullerene superconductors, and high T_c superconductors. Furthermore, the decoupling between phonons and electrons caused by weak localization can be employed to fabricate new novel devices.

Acknowledgements Y. J. K. is grateful to Faculty of Arts and Sciences at UPR-Mayaguez for release time. M. P. thanks the FOPI at the University of Puerto Rico-Humacao for release time.

References

[1] E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).
[2] P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).
[3] N. F. Mott and M. Kaveh, Adv. Phys. **34**, 329 (1985).
[4] Mi-Ae Park and Yong-Jihn Kim, Phys. Rev. B **61**, 14733 (2000).
[5] Mi-Ae Park, Kerim Savran, and Yong-Jihn Kim, Supercond. Sci. Technol. **14**, L31 (2001).
[6] K. E. Kihlstrom, D. Mael, and T. H. Geballe, Phys. Rev. B **29**, 150 (1984).
[7] D. A. Rudman and M. R. Beasley, Phys. Rev. B **30**, 2590 (1984).
[8] R. C. Dynes, A. E. White, J. M. Graybeal, and J. Garno, Phys. Rev. Lett. **57**, 2195 (1986).
[9] C. C. Tsuei, S. von Molnar, and J. M. Coey, Phys. Rev. Lett. **41**, 664 (1978).
[10] R. A. Pollak, C. C. Tsuei, and R. W. Johnson, Solid State Commun. **23**, 879 (1977).
[11] A. T. Fiory and A. F. Hebard, Phys. Rev. Lett. **52**, 2057 (1984).
[12] H. Wiesmann, M. Gurvitch, A. K. Ghosh, H. Lutz, K. W. Jones, A. N. Goland, and M. Strongin, J. Low. Temp. Phys. **30**, 513 (1978).
[13] T. P. Orlando, E. J. McNiff, Jr., S. Foner, and M. R. Beasley, Phys. Rev. B **19**, 4545 (1979).
[14] J. M. Poate, L. R. Testardi, A. R. Storm, and W. M. Augustyniak, Phys. Rev. Lett. **35**, 1291 (1975).
[15] L. R. Testardi, R. L. Meek, J. M. Poate, W. A. Royer, A. R. Storm, and J. H. Wernick, Phys. Rev. B **11**, 4304 (1975).
[16] J. M. Poate, R. C. Dynes, L. R. Testardi, and R. H. Hammond, Phys. Rev. Lett. **37**, 1308 (1976).
[17] L. R. Testardi, J. M. Poate, and H. J. Levinstein, Phys. Rev. B **15**, 2570 (1977).
[18] D. Pines, Elementary Excitations in Solids (Benjamin, New York, 1963), Chap. 5.
[19] J. H. Mooij, phys. stat. sol. (a) **17**, 521 (1973).
[20] M. Jonson and S. M. Girvin, Phys. Rev. Lett. **43**, 1447 (1979).

[21] Y. Imry, Phys. Rev. Lett. **44**, 469 (1980).
[22] M. Kaveh and N. F. Mott, J. Phys. C **15**, L707 (1982).
[23] D. Belitz and W. Götze, J. Phys. C Solid State Phys. **15**, 981 (1982).
[24] D. Belitz and W. Schirmacher, J. Non-cryst. Solids **61/62**, 1073 (1983).
[25] N. F. Mott, Conduction in Non-Crystalline Materials (Oxford University Press, Oxford, 1987), p.15.
[26] A. M. Jayannavar and N. Kumar, Phys. Rev. B **37**, 573 (1988).
[27] G.-L. Zhao, Y. He, and W. Y. Ching, Phys. Rev. B **42**, 10 887 (1990).
[28] C. Buzea and T. Yamashita, Supercond. Sci. Technol. **14**, R115 (2001).
[29] M. E. Gershenson, D. Gong, T. Sato, B. S. Karasik, and A. V. Sergeev, cond-mat/0104083 (2001).
[30] G. Gladstone, M. A. Jensen, and J. R. Schrieffer, in: Superconductivity, Vol. 2, edited by R. D. Parks (Marcel Dekker, New York, 1969), p. 665.
[31] J. J. Hopfield, Comments Solid State Phys. **3**, 48 (1970).
[32] J. J. Hopfield, in: Superconductivity in d- and f-Band Metals, edited by D. H. Douglass (AIP, New York, 1972), p. 358.
[33] G. Grimvall, Phys. Condens. Mater. **11**, 279 (1970).
[34] E. G. Maksimov and G. P. Motulevich, Zh. Eksp. Teor. Fiz. **61**, 414 (1971) [Sov. Phys. JETP **34**, 219 (1972)].
[35] G. Grimvall, The Electron–Phonon Interaction in Metals, (North-Holland, Amsterdam, 1981).
[36] B. Hayman and J. P. Carbotte, Can. J. Phys. **49**, 1952 (1971).
[37] B. Hayman and J. P. Carbotte, J. Phys. F **1**, 828 (1971).
[38] P. B. Allen, Phys. Rev. B **3**, 305 (1971).
[39] G. M. Eliashberg, Zh. Eksp. Teor. Fiz. **38**, 966 (1960) [Sov. Phys. JETP **11**, 696 (1960)].
[40] W. L. McMillan, Phys. Rev. **167**, 331 (1968).
[41] E. N. Economou, in: Metal Hydrides, edited by G. Bambakidis (Plenum, New York, 1981), p. 1.
[42] G. Grimvall, Phys. Scripta **14**, 63 (1976).
[43] E. G. Maksimov, Zh. Eksp. Teor. Fiz. **57**, 1660 (1969) [Sov. Phys. JETP **30**, 897 (1970)].
[44] G. Grimvall, Phys. Condens. Mater. **14**, 101 (1972).
[45] B. Hayman and J. P. Carbotte, Solid State Commun. **15**, 65 (1974).
[46] B. Chakraborty, W. E. Pickett, and P. B. Allen, Phys. Rev. B **14**, 3227 (1976).
[47] Ö. Rapp and C. Crawfoord, phys. stat. sol. (b) **64**, 139 (1974).
[48] Ö. Rapp and R. Fogelholm, J. Phys. F **5**, 1694 (1975).
[49] R. Flükiger and M. Ishikawa, in: Proc. 14th Internat. Conf. on Low Temperature Physics, Vol. 2, edited by M. Krusius and M. Vuorio (North-Holland and Elsevier, Helsinki, 1975) p. 63.
[50] R. Fogelholm and Ö. Rapp, ibid, p. 429.
[51] H. Lutz, H. Weismann, O. F. Kammerer, and M. Strongin, Phys. Rev. Lett. **36**, 1576 (1976).
[52] K. K. Mankovskii, V. V. Pilipenko, Y. F. Komnik, and I. M. Dmitrenko, Zh. Eksp. Teor. Fiz. **59**, 740 (1970) [Sov. Phys. JETP **32**, 404 (1971)].
[53] Ö. Rapp, A. C. Mota, and R. F. Hoyt, Solid State Commun. **25**, 855 (1978).
[54] B. Sundqvist and Ö. Rapp, J. Phys. F **9**, 1694 (1979).
[55] Ö. Rapp, (1980), unpublished data quoted in Ref. [34], p. 252.
[56] J. Bolz and F. Pobell, Z. Phys., B **20**, 95 (1975).
[57] G. Bergmann, Phys. Rep. **27**, 159 (1976).
[58] R. A. Pollak, C. C. Tsuei, and R. W. Johnson, Solid State Commun. **23**, 879 (1977).
[59] J. Kwo, T. P. Orlando, and M. R. Beasley, Phys. Rev. B **24**, 2506 (1981).
[60] Yong-Jihn Kim and A. W. Overhauser, Phys. Rev. B **47**, 8025 (1993).
[61] W. Jones and N. H. March, Theoretical Solid State Physics (Dover, New York, 1985), p. 897.
[62] A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, Methods of Quantum Field Theory in Statistical Physics (Dover, New York, 1975), p. 79.
[63] A. Fetter and J. D. Walecka, Quantum Theory of Many-Particle Systems (McGraw-Hill, New York, 1971), p. 401.
[64] P. W. Anderson, K. A. Muttalib, and T. V. Ramakrishnan, Phys. Rev. B **28**, 117 (1983).
[65] M. Kaveh, Philos. Mag. B **51**, 453 (1985).
[66] W. L. McMillan, Phys. Rev. B **24**, 2739 (1981).
[67] B. L. Altshuler and A. G. Aronov, Zh. Eksp. Teor. Fiz. **77**, 2028 (1979) [Sov. Phys. JETP **50**, 968 (1979)].
[68] E. Abrahams, P. W. Anderson, P. A. Lee, and T. V. Ramakrishnan, Phys. Rev. B **24**, 6783 (1981).
[69] Y. Imry, Y. Gefen, and D. J. Bergman, Phys. Rev. B **26**, 3436 (1982).
[70] M. Kaveh and N. F. Mott, J. Phys. C **14**, L177 (1981).

[71] H. Gutfreund, M. Weger, and O. Entin-Wohlman, Phys. Rev. B **31**, 606 (1985).
[72] B. L. Altshuler, Zh. Eksp. Teor. Fiz. **75**, 1330 (1978) [Sov. Phys. JETP **48**, 670 (1978)].
[73] M. Yu. Reizer and A. V. Sergeev, Zh. Eksp. Teor. Fiz. **92**, 2291 (1987) [Sov. Phys. JETP **65**, 1291 (1987)].
[74] P. M. Echternach, M. E. Gershenson, and H. M. Bozler, Phys. Rev. B **47**, 13659 (1993).
[75] S. V. Vonsovsky, Y. A. Izyumov, and E. Z. Kurmaev, Superconductivity of Transition Metals (Springer-Verlag, Berlin, 1982), Chap. 2.
[76] H.-L. Engquist, Phys. Rev. B **21**, 2067 (1980).
[77] N. F. Mott, Metal-Insulator Transitions (Taylor & Francis, London, 1990), p. 152.
[78] G. S. Elliot, A. D. Gromko, F. V. Veegaete, C. D. Johnson, and D. C. Johnson, Phys. Rev. B **58**, 8805 (1998).
[79] R. C. Dynes, J. M. Rowell, and P. H. Schmidt, in: Ternary Superconductors, edited by G. K. Shenoy, B. D. Dunlap, and F. Y. Fradin (North-Holland, Amsterdam, 1981), p. 169.
[80] National Research Council, Gravitational Physics: Exploring the Structure of Space and Time (National Academy Press, Washington DC, 1999).
[81] E. Coccia, G. Pizzella, and F. Ronga (Ed.), Gravitational Wave Experiments (World Scientific, Singapore, 1995).