Robust entanglement in atomic systems via Λ-type processes

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It is shown that the system of two three-level atoms in the Λ configuration in a cavity can evolve into a long-lived maximum entangled state if the Stokes photons vanish from the cavity by means of either leakage or damping. The difference in the evolution picture corresponding to the general model and effective model with two-photon process in a two-level system is discussed.

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I. INTRODUCTION

During the last decade, the problem of engineered entanglement in atomic systems has attracted a great deal of interest (see Refs. [1–8], and references therein). In particular, the atomic entangled states were successfully realized through the use of cavity QED [1] and the technique of ion traps [3]. At present, one of the most important problems under consideration is how to make a long-lived and easy-monitored atomic entangled state with existing experimental techniques.

An interesting scheme has been proposed recently [9]. In this scheme, the two identical atoms are placed into a cavity tuned to resonance with one of the dipole-allowed transitions. Initially, both atoms are prepared in the ground state, while the cavity field consists of a single photon. It is easy to show that the atom-field interaction leads in this case to a maximum atomic entangled state such that the single excitation is shared between the two atoms with equal probability. It was proposed in Ref. [9] to consider the absence of photon leakage from a nonideal cavity as a signal that the atomic entangled state has been created. The scheme can also be generalized to the case of any even number of atoms $2n$, sharing n excitations. In this case, the atomic entangled states are represented by the so-called SU(2) phase states [10].

Another interesting proposal is to use a strong coherent drive to provide the multipartite entanglement in a system of two-level atoms in a high-Q cavity [11]. This approach can be used to produce the atomic entanglement as well as that of atoms and cavity modes and even of different cavity modes.

In the schemes of Refs. [9,11], the lifetime of the entanglement is defined by the specific time scale of the dipole-allowed radiative processes in atoms. Unfortunately, this lifetime is usually quite short [12].

Generally speaking, the lifetime of atomic entanglement is specified by the interaction of atoms with environment. For example, in the model of Ref. [10], the environment is represented by the vacuum field that causes emission of a photon getting out of the cavity.

The interaction with environment can also be used to create a long-lived entanglement in atomic systems. For example, the initial nonentangled system may evolve to an entangled state connected with the atomic states that cannot be depopulated by radiation decay. In this case, the lifetime of the entangled state is specified by considerably long nonradiative processes. A possible realization is provided by the use of a three-level Λ-type process instead of the two-level scheme of Refs. [9,11]. The process is illustrated by Fig. 1. Here, the levels 1 and 2 as well as the levels 2 and 3 are connected by the electric dipole transitions. In turn, the dipole transition between the levels 3 and 1 is forbidden because of the parity conservation [13]. The absorption of pumping photon by the transition $1\leftrightarrow 2$ with further jump of the electron to the level 3 can be interpreted as a kind of Raman process in atomic system with emission of Stokes photon (see Ref. [14], and references therein). It is clear that the atom excited to the level 3 can change the state either by absorption of the Stokes photon resonant with respect to the transition $3\leftrightarrow 2$ or trough a nonradiative decay.

Now we assume that the two identical Λ-type atoms are placed into a cavity of high quality with respect to the pumping photons resonant to the transition $1\leftrightarrow 2$ and also that the Stokes photons created by the transition $2\rightarrow 3$ either leave the cavity freely or are absorbed by the cavity walls. Then, the atom-field interaction may lead to creation of the maximum entangled atomic state

$$\frac{1}{\sqrt{2}}(|3,1\rangle+|1,3\rangle), \tag{1}$$

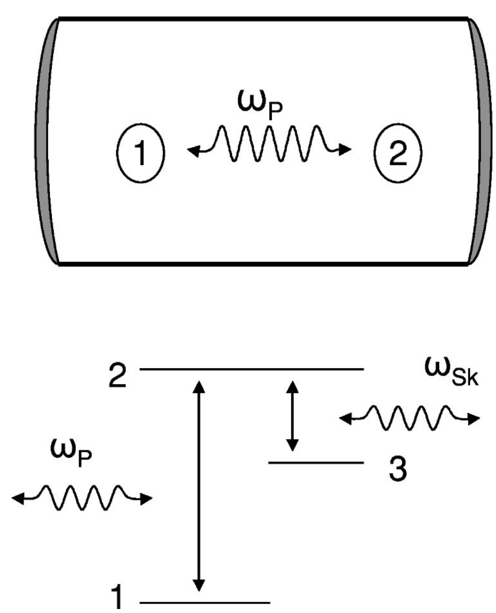

FIG. 1. Scheme of the process and configuration of atomic levels and transitions.

whose lifetime is determined by the slow processes of nonradiative decay.

The above scheme has been proposed in Ref. [10] and briefly discussed in Ref. [15]. The main objective of the present paper is to consider in detail the evolution towards the long-lived atomic entangled state (1).

The paper is organized as follows. In Sec. II, we discuss the model Hamiltonians that can be used to describe the process under consideration. Viz, we discuss the model of the one-photon three-level interaction and an effective model of the two-photon process in a two-level system. Then, in Sec. III, we examine the irreversible dynamics, leading to state (1) in a cavity with leakage of Stokes photons. We show that both models describe the exponential evolution to state (1). At the same time, the effective model, corresponding to a rough time scale, is unable to take into account the possible oscillations of population between the states 1 and 2. Let us stress that the monitoring of Stokes photons outside the cavity can be used to detect the atomic entangled state (1) in this case.

Another way of creation of state (1) through the use of a cavity with very low quality with respect to the Stokes photons is discussed in Sec. IV. Finally, in Sec. V, we discuss the possible realization of entanglement in the system of Λ-type atoms.

II. THE MODELS OF THE Λ-TYPE PROCESS

Assume that a system of N identical three-level atoms with Λ-type transitions shown in Fig. 1 interacts with the cavity mode close to resonance with $1\leftrightarrow 2$ transition and with the Stokes radiation that can leave the cavity freely. Then, following Refs. [13,14], we can choose the model Hamiltonian in the following form:

$$H=H_0+H_{int},$$

$$H_0=\omega_P a_P^+ a_P+\sum_k \omega_{Sk}a_{Sk}^\dagger a_{Sk}+\sum_f [\omega_{21}R_{22}(f)+\omega_{31}R_{33}(f)], \tag{2}$$

$$H_{int}=\sum_f \lambda_P R_{21}(f)a_P+\sum_{f,k}\lambda_{Sk}R_{23}(f)a_{Sk}+\text{H.c.}. \tag{3}$$

Here, a_P denotes the photon annihilation operator of the cavity mode with frequency ω_P, a_{Sk} is the annihilation operator of Stokes photon with frequency ω_{Sk}, and ω_{21}, ω_{31} are the energies of the corresponding atomic levels with respect to the ground level 1. The operator

$$R_{ij}(f)=|i_f\rangle\langle j_f|$$

describes the transition from level j to level i and index f marks the number of atom. In Eq. (3), λ_P and λ_{Sk} are the coupling constants, specifying the dipole transitions $2\leftrightarrow 1$ and $3\leftrightarrow 2$, respectively. Summation over k in Eq. (3) implies that the Stokes photons do not feel presence of the cavity walls. This summation involves the modes, corresponding to the natural line breadth near

$$\omega_S\equiv\omega_{23}=\omega_{21}-\omega_{31}. \tag{4}$$

Apart from the total electron occupation number, the model, Eqs. (2) and (3), has two integrals of motion

$$N_P=a_P^+a_P+\sum_f \{R_{22}(f)+R_{33}(f)\},$$

$$N_S=\sum_k a_{Sk}^+a_{Sk}-\sum_f R_{33}(f). \tag{5}$$

Consider the system of only two atoms. Assume that both atoms are prepared initially in the ground state 1, the cavity contains a single photon of frequency ω_P, and the Stokes field is in the vacuum state. Then, because of the integrals of motion (5), the evolution of the system occurs in a single-excitation domain of the Hilbert space spanned by the vectors

$$|\psi_1\rangle=|1,1\rangle\otimes|1_P\rangle\otimes|0_S\rangle,$$

$$|\psi_2^{(\pm)}\rangle=\frac{1}{\sqrt{2}}(|1,2\rangle\pm|2,1\rangle)\otimes|0_P\rangle\otimes|0_S\rangle, \tag{6}$$

$$|\psi_{3k}^{(\pm)}\rangle=\frac{1}{\sqrt{2}}(|1,3\rangle\pm|3,1\rangle)\otimes|0_P\rangle\otimes|1_{Sk}\rangle.$$

By construction, the four states (6) labeled by the superscripts $\pm$ manifest the maximum entanglement. It is easily seen that the action of operator (3) cannot transform the states

$$\{|\psi_1\rangle,|\psi_2^{(+)}\rangle,|\psi_{3k}^{(+)}\rangle\} \tag{7}$$

into the states

$$\{|\psi_2^{(-)}\rangle,|\psi_{3k}^{(-)}\rangle\} \tag{8}$$

and vice versa. Thus, the evolution of the system from the initial nonexcited state $|\psi_1\rangle$ takes place in the subspace spanned by only three vectors (7). Thus, states (8) can be discarded.

Instead of the one-photon three-level model described by the Hamiltonian, Eqs. (2) and (3), an effective model of two-photon process can also be used under a certain condition [16,17]. Viz., if the cavity is tuned consistent with two-photon energy conservation, i.e.,

$$E_3-E_1=\omega_1-\omega_2,$$

we are left only with one detuning parameter

$$\Delta=E_1-E_2-\omega_1=E_2-E_3-\omega_S.$$

Here, E_i denotes the energy of corresponding atomic level. Then, it was shown in Ref. [16] that under the condition

$$\Delta\gg E_3-E_1,$$

the dynamics of the system is governed by the effective Hamiltonian of the form

$$H^{eff}=\omega_P a_P^\dagger a_P+\omega_S a_S^\dagger a_S+\sum_f \omega_{31}R_{33}(f)$$
$$+\sum_f \lambda[R_{31}(f)a_S^\dagger a_P+a_P^\dagger a_s R_{13}(f)]. \qquad (9)$$

This Hamiltonian (9) describes an effective two-level two-photon system with simultaneous absorption of pumping photon and creation of Stokes photon and vice versa. Here λ is an effective coupling constant.

III. DYNAMICS DESCRIBED BY THE HAMILTONIAN [EQS. (2) AND (3)]

Under the assumption that there are only two three-level Λ-type atoms in the cavity and that the system is initially prepared in the state $|\psi_1\rangle$ in Eq. (5), in view of the results of previous section, we should choose the time-dependent wave function as follows:

$$|\Psi(t)\rangle=C_1|\psi_1\rangle+C_2|\psi_2\rangle+\sum_k C_{3k}|\psi_{3k}\rangle, \qquad (10)$$

$$C_1(0)=1, \quad C_2(0)=0, \quad \forall k \quad C_k(0)=0, \qquad (11)$$

using the reduced basis (7). Here, we use the notations $|\psi_2\rangle\equiv|\psi_2^{(+)}\rangle$ and $|\psi_{3k}\rangle\equiv|\psi_{3k}^{(+)}\rangle$, for simplicity. The time-dependent Schrödinger equation with the Hamiltonian, Eqs. (2) and (3), then leads to the following set of equations for the coefficients in Eq. (11):

$$i\dot{C}_1=\omega_P C_1+\lambda_P\sqrt{2}C_2,$$
$$i\dot{C}_2=\omega_{21}C_2+\lambda_P\sqrt{2}C_1+\sum_k \lambda_{Sk}C_{3k}, \qquad (12)$$
$$i\dot{C}_{3k}=(\omega_{31}+\omega_{Sk})C_{3k}+\lambda_{Sk}C_2.$$

To find solutions of Eq. (12), let us represent the last equation in Eq. (12) in the form

$$C_{3k}(t)=-i\lambda_{Sk}\int_0^t C_2(\tau)e^{i(\omega_{31}+\omega_{Sk})(\tau-t)}d\tau. \qquad (13)$$

Then, we should take the time derivative on both sides of the first equation in Eq. (12) and substitute the second equation together with integral representation (13). We get

$$i\ddot{C}_1=(\omega_P+\omega_{21})\dot{C}_1+i(\omega_{21}\omega_P-2\lambda_P^2)C_1$$
$$-\sum_k \lambda_{Sk}^2\int_0^t(i\dot{C}_1-\omega_P C_1)e^{i(\omega_{31}+\omega_{Sk})(\tau-t)}d\tau.$$

Carrying out the integration by parts, we get the following integro-differential equation with respect to only one unknown variable $C_1(t)$:

$$i\ddot{C}_1=(\omega_P+\omega_{21})\dot{C}_1+i\left(\omega_{21}\omega_P-2\lambda_P^2-\sum_k \lambda_{Sk}^2\right)C_1$$
$$+i\sum_k \lambda_{Sk}^2 e^{-i(\omega_{31}+\omega_{Sk})t}-\sum_k \lambda_{Sk}^2(\omega_{31}+\omega_{Sk}-\omega_P)$$
$$\times\int_0^t C_1(\tau)e^{i(\omega_{31}+\omega_{Sk})(\tau-t)}d\tau. \qquad (14)$$

In contrast to the conventional Wigner-Weisskopf theory (e.g., see Ref. [18]), Eq. (14) contains the second-order derivatives. This integro-differential equation (14) can be analyzed through the use of Laplace transformation as in the Wigner-Weisskopf theory [18,19]. We get

$$\int_0^\infty C_1(t)e^{-st}dt=\mathcal{L}(C_1),$$

$$\int_0^\infty \dot{C}_1(t)e^{-st}dt=s\mathcal{L}-1,$$

$$\int_0^\infty \ddot{C}_1(t)e^{-st}dt=s^2\mathcal{L}-s-\dot{C}_1(0)=s^2\mathcal{L}-s+i\omega_P.$$

Then, Eq. (14) is reduced to the following algebraic equation with respect to $\mathcal{L}$:

$$\mathcal{L}\left[is^2-s(\omega_P+\omega_{21})-i\left(\omega_{21}\omega_P-2\lambda_P^2-\sum_k \lambda_{Sk}^2\right)\right]$$
$$=-(\omega_P+\omega_{21})+i\sum_k \lambda_{Sk}^2\frac{s+i(\omega_{31}+\omega_{Sk})}{s^2+(\omega_{31}+\omega_{Sk})^2}$$
$$-\int_0^\infty e^{-st}\left\{\sum_k \lambda_{Sk}^2(\omega_{31}+\omega_{Sk}-\omega_P)\right.$$
$$\left.\times\int_0^t C_1(\tau)e^{i(\omega_{31}+\omega_{Sk})(\tau-t)}d\tau\right\}dt. \qquad (15)$$

The last term in the right-hand side of this expression can be represented as follows:

$$\int_0^\infty e^{-st}\left\{\int_0^t e^{i(\omega_{31}+\omega_{Sk})(\tau-t)}C_1(\tau)d\tau\right\}dt$$
$$=\int_0^\infty C_1(\tau)e^{i(\omega_{31}+\omega_{Sk})\tau}d\tau\int_\tau^\infty e^{-[s+i(\omega_{31}+\omega_{Sk})]t}dt$$
$$=\frac{\mathcal{L}}{s+i(\omega_{31}+\omega_{Sk})}.$$

Thus, Eq. (15) takes the form

$$\mathcal{L}=\left[i\sum_k \lambda_{Sk}^2\frac{s+i(\omega_{31}+\omega_{Sk})}{s^2+(\omega_{31}+\omega_{Sk})^2}-(\omega_P+\omega_{21})\right]\times\left[is^2-s(\omega_P+\omega_{21})+i\left(\omega_P\omega_{21}-2\lambda_P^2-\sum_k \lambda_{Sk}^2\right)+\sum_k \frac{\lambda_{Sk}^2(\omega_{31}+\omega_{Sk}-\omega_P)}{s+i(\omega_{31}+\omega_{Sk})}\right]^{-1}. \tag{16}$$

Then, the exact form of the time behavior of the coefficient $C_1(t)$ in Eq. (10) is governed by the inverse Laplace transformation:

$$C_1(t)=\frac{1}{2\pi i}\int_{\epsilon-i\infty}^{\epsilon+i\infty}e^{st}\mathcal{L}(s)ds, \tag{17}$$

where ϵ is a infinitesimal real positive number and s is considered to be a complex parameter. As soon as the explicit time behavior of $C_1(t)$ is known, the other coefficients in Eq. (10) can be defined through the use of Eqs. (12) and (13). In particular, it follows from Eqs. (6), (10), and (11) that the probability to have the atomic entangled state (1) has the form

$$\sum_k |C_{3k}|^2=1-|C_1(t)|^2-|C_2(t)|^2=1-|C_1(t)|^2-\frac{|i\dot{C}_1(t)-\omega_P C_1(t)|^2}{2\lambda_P^2}. \tag{18}$$

Thus, Eqs. (16) and (17) completely determine the probability of having the robust entangled state (1). It can be shown that Eq. (16) describes the reversible, Poincaré-type behavior (e.g., see Ref. [19]). The irreversible evolution can be obtained under the further assumption that the atomic transition $2\leftrightarrow 3$ interacts with continuum of Stokes modes rather than with a discrete spectrum

$$\sum_k \cdots \rightarrow \int_{-\infty}^{\infty}\cdots\rho(\omega)d\omega, \quad \omega=ck.$$

Here, measure $\rho(\omega)d\omega$ defines the density of states of Stokes photons with different frequencies.

Let us stress that, unlike the conventional Wigner-Weisskopf theory, Eqs. (16) and (17) describe a superposition of exponential decay and harmonic oscillations. The latter are caused by the interaction between the $1\leftrightarrow 2$ transitions and cavity field.

Further analysis shows that the coefficients C_1 and C_2 have the form

$$C_1(t)\approx\left[-\frac{2\lambda^2}{(\Gamma-i\Delta)^2}e^{(-\Gamma+i\Delta)t}+\left(1+\frac{2\lambda^2}{(\Gamma-i\Delta)^2}\right)e^{-[2\lambda^2/(\Gamma-i\Delta)]t}\right]e^{-i\omega_P t},$$

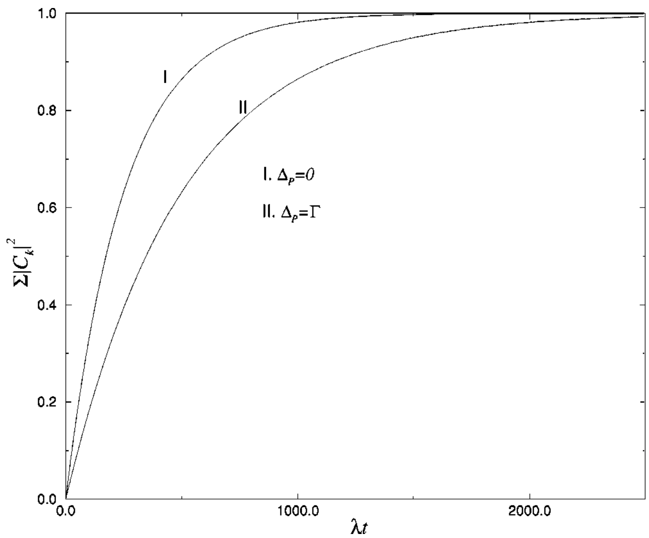

FIG. 2. Time evolution of probability (18) to have the robust entanglement at $\lambda_P=0.001\Gamma$ for (I) $\Delta_P=0$ and (II) $\Delta_P=\Gamma$.

$$C_2(t)\approx-\frac{\sqrt{2}\lambda}{i\Gamma+\Delta}[e^{-\Gamma t}-e^{-[2\lambda^2/(\Gamma-i\Delta)+i\Delta]t}]e^{-i\omega_{21}t} \tag{19}$$

to the second order in $\lambda/(\Gamma-i\Delta_P)$. Here,

$$\Delta_P=\omega_P-\omega_{12}$$

is the detuning factor for the pumping mode and

$$\Gamma=\rho(\omega_S)\lambda_{Sk}(k=\omega_S/c), \quad \omega_S=\omega_{21}-\omega_{31}.$$

Equation (19) proves to be a good approximation because $\rho(\omega_S)\gg 1$ and $\Gamma\gg\lambda_P,\lambda_{Sk}$.

It is seen that Eq. (19) describes the damped oscillations of the coefficient $C_1(t)$ in Eq. (10). According to Eq. (11), $C_2(t)$ manifests similar behavior. Thus, the probability (18) to get the robust entangled state tends to 1 as $t\rightarrow\infty$ (see Fig. 2). The decay time γ^{-1} is defined by the coupling constant and detuning parameter for the pumping mode and by the width of the Stokes line Γ. The contribution of oscillations into the evolution described by Eq. (19) is un-noticeable at small detuning $\Delta_P\leqslant\Gamma$ and becomes apparent at $\Delta_P\gg\Gamma$ (see Fig. 2).

Similar result can also be obtained in terms of the effective Hamiltonian (9) [15]. It should be stressed that the assumptions made in the process of derivation of Eq. (9) lead to an effective roughening of the time scale. In fact, the effective removal of the level 2 leads to the negligence of the Rabi oscillations between the levels 1 and 2. Therefore, the effective model (9) gives only rough picture of purely exponential evolution of probability (18).

While the atomic system evolves to the maximum entangled state (1), the Stokes photon leaves the cavity. Thus, the observation of Stokes photon outside the cavity can be considered as a signal that the robust entangled state has been prepared.

IV. CAVITY WITH ABSORPTION OF STOKES PHOTONS

The atomic entangled state (1) can also be realized when the Stokes mode is strongly damped in the cavity. For simplicity, we again assume no damping for the pumping mode. At the same time, the Stokes photons are supposed to be absorbed by the cavity walls. This situation corresponds to a number of experiments with single-atom Rydberg maser [20,21]. In this case, the effect of damping can be calculated through the use of the so-called dressed-atom approximation [22].

The model Hamiltonian, describing the process under consideration, can be chosen as follows:

$$H = H_0 + H_{int},$$

$$H_0 = \omega_P a_P^\dagger a_P + \omega_S a_S^\dagger a_S + \sum_f [\omega_{21} R_{22}(f) + \omega_{31} R_{33}(f)], \tag{20}$$

$$H_{int} = \sum_f [\lambda_P R_{21}(f) a_P + \lambda_S R_{23}(f) a_S] + \text{H.c.}.$$

This corresponds to the single-Stokes-mode approximation in Eqs. (2) and (3). The eigenstates of Hamiltonian (20) have the form

$$|\psi_0\rangle = \frac{\lambda_S}{\epsilon}|\psi_1\rangle - \frac{\lambda_P\sqrt{2}}{\epsilon}|\psi_3\rangle,$$

$$|\psi_\pm\rangle = \pm\frac{\lambda_P}{\epsilon}|\psi_1\rangle + \frac{1}{\sqrt{2}}|\psi_2\rangle \pm \frac{\lambda_S}{\epsilon\sqrt{2}}|\psi_3\rangle, \tag{21}$$

where $|\psi_1\rangle$ coincides with the first state in Eq. (6), $|\psi_2\rangle = |\psi_2^{(+)}\rangle$, and

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}(|3,1\rangle + |1,3\rangle) \otimes |0_P\rangle \otimes |1_S\rangle.$$

In Eq. (21),

$$\epsilon = \sqrt{2\lambda_P^2 + \lambda_S^2}.$$

Under the assumption of exact resonance

$$\omega_P = \omega_{21} = \omega_{31} + \omega_S$$

that we use hereafter for simplicity, the corresponding eigenvalues are

$$H|\psi_0\rangle = \omega_P|\psi_0\rangle, \quad H|\psi_\pm\rangle = (\omega_P \pm \epsilon)|\psi_\pm\rangle.$$

Besides this, there is one more eigenstate

$$|\psi_4\rangle = \frac{1}{\sqrt{2}}(|3,1\rangle + |1,3\rangle) \otimes |0_P\rangle \otimes |0_S\rangle, \tag{22}$$

such that

$$H|\psi_4\rangle = \omega_{31}|\psi_4\rangle.$$

It is clear that this eigenstate corresponds to the maximum atomic entanglement (1). Physically, this state is achieved when the Stokes photon is absorbed by the cavity walls.

To take into account the cavity damping of Stokes photons, consider the interaction with a "phonon reservoir" responsible for the absorption of photons by cavity walls [18]. Then, Hamiltonian (20) should be supplemented with the term

$$H_{loss} = \sum_q \eta_q (b_q^\dagger a_S + a_S^\dagger b_q) + \sum_q \Omega_q b_q^\dagger b_q, \tag{23}$$

where $b_q, b_q^\dagger$ are the Bose operators of "phonons" in the cavity walls.

The density matrix of the system can be chosen as follows:

$$\rho(t) = \sum_{j,\ell} \rho_{j\ell}(t)|\psi_j\rangle\langle\psi_\ell|, \quad j,\ell = 0,\pm,4, \tag{24}$$

where $|\psi_j\rangle$ are eigenstates (21) and (22) and $\rho_{j\ell}(t)$ is the time-dependent c number.

With the total Hamiltonian

$$H_{tot} = H + H_{loss}$$

in hand, we can now write the Master Equation, eliminating the cavity degrees of freedom (e.g., see Ref. [23]),

$$\dot{\rho} = -i[H,\rho] + \kappa\{2a_S\rho a_S^\dagger - a_S^\dagger a_S\rho - \rho a_S^\dagger a_S\}, \tag{25}$$

so that the contribution of Eq. (23) is taken into account effectively through the Liouville term. Here $1/\kappa$ is the lifetime of a Stokes photon in the cavity and $Q = \omega_{31}/\kappa$ is the quality factor with respect to the Stokes photons. Let us choose the same initial condition as in the preceding section, so that

$$\rho(0) = |\psi_1\rangle\langle\psi_1|, \tag{26}$$

where the initial state $|\psi_1\rangle$ is expressed in terms of eigenstates (21) as follows:

$$|\psi_1\rangle = \frac{\lambda_S}{\epsilon}|\psi_0\rangle + \frac{\lambda_P}{\epsilon}(|\psi_+\rangle - |\psi_-\rangle).$$

Equation (25) can now be solved numerically at different values of parameter κ, specifying the absorption of Stokes photons. The results are shown in Fig. 3. It is seen that the system evolves to the robust atomic entangled state (1). The stairslike structure is again caused by competition between the transitions $1 \leftrightarrow 2$ and $2 \leftrightarrow 3$. Although such a behavior is an inherent property of the model under consideration, the stairs become more visible with increase of κ (see Fig. 3).

A similar result can also be obtained within the framework of effective model with Hamiltonian (9) and the damping described by Eq. (23). In this case, the density matrix consists of only six elements because the state $|\psi_0\rangle$ in Eq. (21) should be discarded and the states $|\psi_\pm\rangle$ are changed by the states

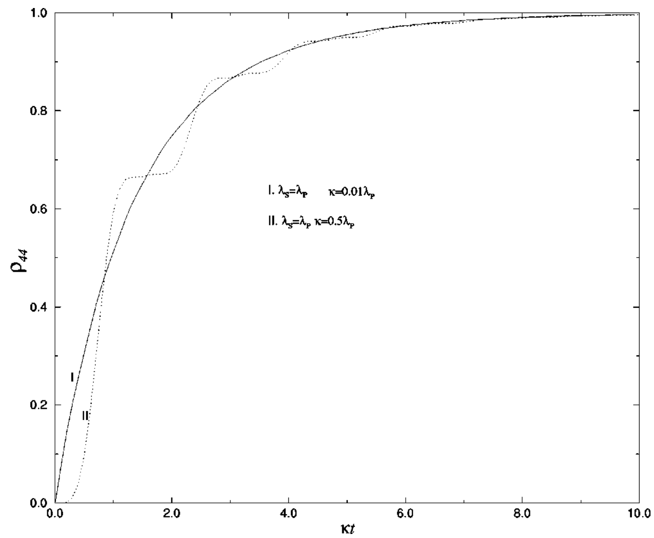

FIG. 3. Time evolution of $\rho_{44}(t)$ at $\lambda_P = \lambda_S$ and $\kappa = 0.01\lambda_P$ (I) and $\kappa = 0.5\lambda_P$ (II).

$$|\phi_\pm\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle \pm |\psi_3\rangle),$$

with the eigenvalues

$$\varepsilon_\pm = \omega_P \pm \lambda\sqrt{2}.$$

It should be stressed that the effective model does not show the stairslike behavior of $\rho_{44}(t)$.

V. SUMMARY AND DISCUSSION

In this paper, we have studied the quantum dynamics of a system of two three-level atoms in the Λ configuration interacting with two modes of quantized electromagnetic field in a cavity under the assumption that the Stokes-mode photons either leave the cavity freely or are damped rapidly. It is shown that in both cases, the system evolves from the state when both atoms are in the ground state and cavity contains a pumping photon into the robust entangled state (1). The lifetime of this final state is defined completely by the nonradiative processes and is therefore relatively long.

In the case of cavity transparent for the Stokes photons, the creation of Stokes photon signalizes the rise of atomic entanglement. Such a photon can be monitored outside the cavity.

Let us stress that the general models with Hamiltonians (2), (3), and (20), which take into account all the three atomic levels, admit a certain peculiarities in the evolution towards the robust entangled state caused by the competition of transitions $1 \leftrightarrow 2$ and $2 \leftrightarrow 3$. The effective model with adiabatically eliminated highest excited level is incapable of description of these peculiarities, while predicts correct asymptotic behavior. Moreover, the general model admits also a number of intermediate maximum entangled states [$|\psi_2\rangle$ and $|\psi_{3k}\rangle$ in Eq. (6)] that do not exist in the effective model. Unfortunately, the lifetime of these entangled states are defined by the dipole radiative processes and are therefore too short.

One of the most important conditions of experimental realization of the robust entanglement discussed in this paper is that the transitions $1 \leftrightarrow 2$ and $2 \leftrightarrow 3$, used for absorption of pumping photons and generation of Stokes photons, should have quite different frequencies. The considerable difference of frequencies ω_{21} and ω_{23} makes it possible to design a multimode cavity with high quality with respect to ω_{12}, permitting either leakage or strong absorption of Stokes photons. An important example is provided by the $3S \leftrightarrow 4P$ and $4P \leftrightarrow 4S$ transitions in sodium atom and similar transitions in other alkaline atoms (see Ref. [24]). These atoms are widely used in quantum optics, in particular, in investigation of Bose-Einstein condensation [25]. Λ-type structures obeying the condition $\omega_{12} \gg \omega_{23}$ can also be found in other atoms and molecules [24]. The multimode cavities are also well known [26]. In particular, the cavities with necessary properties may be assembled using distributed Bragg reflectors (DBR) and double DBR structures to single out two different wavelengths [27].

The initial state of the system can be prepared in the same way as in Ref. [21]. The atoms can propagate through the cavity, using either the same opening or two different openings. The velocity of atoms should be chosen in a proper way so that the time they spend in the cavity will be $\tau \gg \lambda_P^{-1}, \lambda_S^{-1}$. All measurements aimed at the detection of atomic entanglement can be performed outside the cavity. Thus, the discussed realization of robust entanglement seems to be feasible with the present experimental technique.

Although our results were obtained for a system of two atoms, they can be generalized with ease to the case of big atomic clusters, using the method of Ref. [10]. In fact, it is possible to show that a certain robust entanglement can be obtained in a system with any even number $2N$ of three-level Λ-type atoms initially prepared in the ground state and interacting with N pumping photons.

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