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Mathematical Knowledge and Skills Expected by Higher Education in Engineering and the Social Sciences: Implications for High School Mathematics Curriculum

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One important function of school mathematics curriculum is to prepare high school students with the knowledge and skills needed for university education. Identifying them empirically will help making sound decisions about the contents of high school mathematics curriculum. It will also help students to make informed choices in course selection at high school. In this study, we surveyed university faculty members who teach first year university students about the mathematical knowledge and skills that they would like to see in incoming high school graduates. Data were collected from 122 faculty members from social science (history, law, psychology) and engineering departments (electrical/electronics and computer engineering). Participants were asked to indicate which high school mathematics topics and skills they thought were important to be successful at university education in their field. Results were compared across social science and engineering departments. Implications were drawn for curriculum specialists, students, and mathematics educators.

Keywords: High school, mathematical knowledge, mathematical topics, mathematical skills, mathematics curriculum

INTRODUCTION

High schools served a multitude of goals within societies in the history of education in the 20th century. General goals of high schools included developing citizenship, supporting personal intellectual growth, preparing students for jobs and occupations and preparing students for higher education (deMarrais & LeCompte,1995; Ornstein & Levine, 1984).

Depending on the perceived functions of high schools, what to teach in high schools in general and mathematics in particular have been a topic of debate both in Turkey and abroad (Alacaci, 2004; Cockcroft, 1982; Jones, 1970; MoNE, 1973, 2012a). Relative

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State of the literature

- One function of high schools has been to equip the students with the mathematical knowledge and skills needed for university education in professions.
- There is evidence that existing curriculum options do not exactly serve to prepare students for university education in engineering and social sciences.
- Identifying the mathematical knowledge and skills needed by students empirically is an useful method in setting the scope of curriculum in the design process and in evaluating existing curricula.

Contribution of this paper to the literature

- Through a structured survey, this study explored the mathematical knowledge and skills needed by high school students for university education, as perceived the teaching staff in engineering and social sciences.
- Findings showed that social science and engineering fields have unique needs in terms of required mathematical knowledge, but they also have some overlap.
- There is however a variance between the contents of existing mathematics curriculum options in Turkey and the knowledge needed for university education, especially in the field of social sciences education.

importance of high schools' functions have changed over time depending on the socio-cultural context and economic life of the country. For example, when high schools served only a select few like the case in 1950s in Turkey, they were mainly expected to support students' intellectual growth and preparation for higher education. As a small portion of high ability students had the opportunity to pursue secondary education, its curriculum was quite rigorous and academic. However, in 2014 when high schools serve more than two thirds of the youth cohort in Turkey (MoNE, 2013a), its curriculum and programs must reflect the diversity of abilities and interests of a larger group of students.

Some students join economic life right after graduation from a general high school and seek to get a job. Others choose to attend vocational high schools and take a vocational job or occupation following their program at high school. Yet many other students continue to higher education to obtain professional degrees in fields such as engineering and social sciences.

Compulsory education was extended to grade 12 in 2012 in Turkey following the 4+4+4 law (MoNE, 2012b). High schools now serve as the capstone of "general education." Hence one function of high schools is to ensure that the graduates possess the minimum knowledge and skills to function within Turkish society before they leave the formal educational system (MoNE, 1973, 2012a). Accordingly, all graduates should possess the basic literacy skills, understand how the society is governed, attain the basic scientific knowledge and skills for health and self-care and possess the mathematical skills needed for personal finance and daily life. Yet, one important function of high school mathematics is to help students' transition into higher education and prepare them for the mathematical needs or demands of education in professions such as business, law, engineering, medicine or history (MoNE, 1973, 2012a).

High schools prepare students' transition into higher education by providing the background mathematical knowledge and skills commensurate to their planned fields of study in many countries. For example, in the United Kingdom, explicit mathematical pathways for students intending to pursue university education are provided within the course modules of General Certificate of Secondary Education (Lee, Browne, Dudzic, & Stripp, 2010). Similarly, in Frech lycées, students can select customized mathematics courses for three streams; science and engineering, economics and social sciences and humanities and literature (Özdemir Erdoğan, 2014). In the US, students can take courses up to calculus level if they wish to go to a technical field in college. Some of these students can even take advanced placement university credits for these courses if they take and pass qualifying examinations (Adelman, 2006).

In Turkey, as part of the curricular update following the 4+4+4 law, all students follow the same core mathematics curriculum at grades 9 and 10 regardless of the type of high school they attend (MoNE, 2012b). The purpose of the core curriculum is to develop quantitative skills needed by an average citizen. The structure of current high school mathematics curriculum is depicted in Figure 1.

After grade 10, general high schools offer two options, two-hours a week basic mathematics curriculum or an advanced 6 hours a week mathematics curriculum (MoNE, 2013b). Advanced mathematics curriculum is geared towards those students who intend to pursue a program in higher education in technical fields such as engineering and basic sciences. This option covers advanced algebra, trigonometry and calculus. Basic program on the other hand covers such topics as data analysis and probability, rates, proportions and scaling, number rules and number sequences. It appears that the basic mathematics option is intended for those who do not wish to go to university al all or those who intend to study social science or non-technical fields in higher education. It is important to note that the differentiation between the two pathways

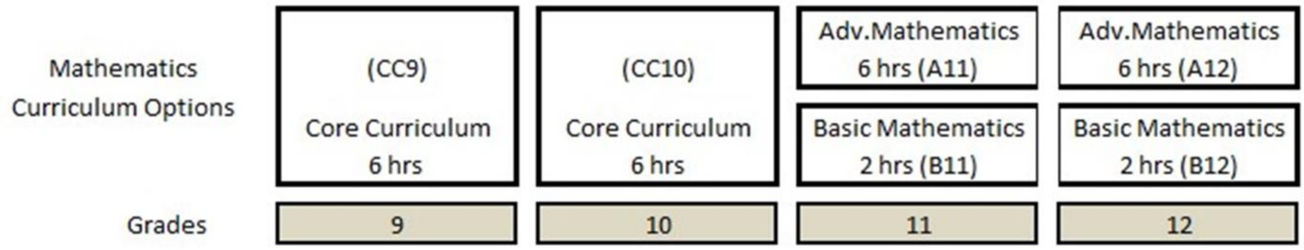

Figure 1. The structure of high school mathematics curriculum (MoNE, 2013a)

is not solely based on a matter of degree as seen in the weekly instructional times, but also of content (MoNE, 2013b).

There is anecdotal evidence about mathematics used in the practice of technical and non-technical professions. The kind of mathematics used in the professions may give ideas for the required mathematical training and the background needed to be successful in these trainings. For example, lawyers typically use a step-by-step method of logical analysis for legal cases which resembles structuring a mathematical proof (Çelikel, 1996; TBA, 2003). It requires advanced logical thinking and analytical reasoning skills. Lawyers who specialize in real estate, taxation, estates, contracts and bankruptcy will need a good understanding of financial mathematics. So, students who want to go to law school may need taking statistics courses and mathematics basic algebra level at high school (Öztürk, 2010).

Engineers model and deal with dynamic physical systems such as stresses a damn must withstand or an efficient operating weight of an airplane. Accordingly they are trained in university for an operational knowledge of advanced mathematical principles. These students must learn calculus, trigonometry, geometry and analytic geometry and probability and statistics (Gençoğlu & Cebeci, 1999). Although there are a few reports focusing on the required mathematical background for engineering based on student survey data (e.g., Güner, 2008); no empirical study was located from the perspective of academicians who teach these students at university.

Purpose of Study

This study is an empirical attempt to investigate high school mathematical knowledge and skills needed for university education in engineering and social sciences as perceived by the academic staff teaching in these fields. If decisions regarding curriculum and textbook content are informed by this type of empirical data, rather than guess-work, students can see higher relevancy in high school mathematics courses and be more motivated and successful. It can also inform curriculum planners to make sound decisions about the contents of curricular materials and evaluate the effectiveness of existing curriculum for students with university plans. In this study, we sought to answer the following research questions:

1. What high school mathematics topics and skills are considered important by university teaching staff to prepare students for higher education programs in engineering and social science fields?
2. How do mathematical topics and skills that are rated important compare between engineering and social science fields?

METHODOLOGY

The data for this study was collected in Fall 2012. The official high school mathematics curriculum in use at the time was put in place by Turkish National Board of Education in 2005, and geometry and analytic geometry curricula were enacted in 2010. National mathematics curriculum was later changed in 2013, however the change was mainly in the structuring and organization rather than the scope of the mathematical topics. In 2013, mathematics, geometry and analytic geometry curriculum were integrated into one curriculum under "mathematics," and two options were provided to students, basic and advanced as depicted in Figure 1 above (for the current mathematics curriculum, see MoNE, 2013b).

To construct the data collection instrument, an inclusive list of mathematical topics from national curriculum in use at the time of the study was prepared. The list included 43 mathematical topics from grades 9-12 from the official mathematics, geometry and analytic geometry curricula (MoNE, 2005, 2010a, 2010b). The list was supplemented by 6 more topics from International Baccalaureate Diploma Program (IBDP) that were not covered in the official Turkish curriculum. The topics were added to address the possibility that university faculty members might think there were topics students needed to learn but were not taught in the Turkish national curriculum. IBDP curriculum and textbooks are used in an increasing number of private schools in Turkey (IBO, 2015). Altogether, the list consisted of 49 mathematical topics. In a separate section, participants were asked to indicate the importance of the following mathematical skills as well; mathematical problem solving, mathematical modelling, mathematical reasoning, mathematical communication, mathematical representations, mathematical connections, analytical thinking and critical thinking. These skills were also included as target competencies in official Turkish mathematics curriculum (MoNE, 2005, 2010a, 2010b). To make sure all participants attribute the same meaning to these terms, a brief description was given following each.

The survey explained that the topics and skills are chosen mainly from the official Turkish mathematics curriculum. Participants were asked to rate the mathematical topics and skills in a Likert-type scale from 1 to 5 for how important they thought it was (1: not important at all, 5: very important) for incoming students to attain at high school to be successful in university education in their field (e.g., law, history, or computer engineering, etc.). They were also given space to add any other topic and skill that were not included in the list, but they consider important. The list of topics and skills covered in the survey is given in the Appendix.

Participants were chosen from two leading universities in Ankara, Turkey, one public and one private. These universities were considered to be representative of other higher education institutions in the contents of undergraduate programs. Academic staff who participated in the study were chosen from 5 departments; law, history, psychology, computer and electrical/electronics engineering. Data from departments of law, history, psychology departments were combined under "social science fields" and the data from computer engineering and electrical/electronics engineering were combined under "engineering fields." The data were collected electronically. All teaching staff in these departments were initially invited to participate by an e-mail message explaining the purpose of the study. To those who volunteered, an online link to a form was sent. They could fill in the form electronically. Among the volunteering participants, there were 72 faculty members from engineering (computer engineering 42, electrical/electronics engineering 30) and 50 from social science departments (psychology 17, law 25 and history 8), totalling 122. Participants were from all ranks; instructors and assistant, associate and full professors.

To answer the first research question, for the mathematical topics and skills, mean ratings were computed across social science and engineering departments. Arithmetic means of 3.5 or above were considered "important" by the researchers. This was because values between 3.5 and 4.0 are closer to (and hence can be rounded to) 4 which stood for a rating of "important." . For the second research question, ratings received by mathematical topics from teaching staff of social science and engineering departments were compared by using Mann-Whitney U test. A parametric test could not be used as a ceiling effect was detected due to skewness of the response data.

RESULTS

Table 1 presents the mean values and standard deviations for the perceived importance of the 49 mathematical topics. For easy display of ratings, summary information is given in the last two columns. Mean values at or above 3.5 is shown by a plus sign.

For social science departments, nine out of 49 topics were rated important by the staff; logic, mathematical proof and proof methods, rates and proportions, basic probability, statistical measures of central tendency, data representation, hypothesis testing and correlation and regression. It is important to note that statistical topics carried a relatively big weight among these topics along with logic, mathematical proof and proof methods. Also, rates and proportions, a central concept in pre-algebra and geometry was rated important for social sciences.

For the engineering departments, all but six topics in the list were rated important. The topics that were not rated as important were tessellations, geometry of 3d objects, triangle similarity, proofs in geometry, conic sections and interest computations. It is possible that triangle similarity was thought to be too elementary by the engineering staff. Geometric proof was not rated important, however general methods of mathematical proof and logic were rated important elsewhere. Engineering departments considered it would be helpful to have background in topics from a wide range of mathematics including algebra, elementary functions, trigonometry, calculus, basic analytic geometry, vectors, matrices, statistics and probability. Average ratings for mathematical topics were generally higher for engineering departments than social sciences. Only "correlation and regression," an important tool for social sciences and "interest computation," an important concept of financial mathematics received higher ratings for social science departments.

Table 2 shows the mean ratings and standard deviations for perceived importance of mathematical skills. It is remarkable that all of the 6 mathematical skills given in the survey were rated important by both social and engineering fields. Mathematical problem solving, mathematical reasoning, critical thinking, analytical reasoning and ability to see mathematical connections among mathematical topics received high endorsement by the faculty members in both social science and engineering departments with average mean ratings of above 4.0.

Table 1. Mean Ratings (and Standard Deviations) of Importance for Mathematical Topics

		Mean (SD)		Importance	
Grade Level	Topics	Social Sci.	Engineering	Social Sci.	Engineering
9th	logic	4.22 (1.08)	4.57 (0.73)	+	+
	mathematical proof	3.98 (0.94)	4.49 (0.71)	+	+
	sets	3.20 (1.11)	4.33 (0.79)	-	+
	relations	3.28 (1.07)	4.31 (0.66)	-	+
	functions	3.02 (1.13)	4.74 (0.48)	-	+
	modular arithmetic	2.44 (1.15)	4.43 (0.80)	-	+
	exponential/root exps	2.64 (1.26)	4.36 (0.86)	-	+
	divisibility	2.92 (1.32)	4.19 (0.78)	-	+
	rates/proportions	4.06 (1.00)	4.36 (0.92)	+	+
	vectors	2.46 (1.31)	4.44 (0.67)	-	+
	line/circle in plane	2.42 (1.31)	3.93 (0.91)	-	+
	distance in plane	2.5 (1.30)	4.11 (0.85)	-	+
	point/line/angle	2.28 (1.09)	3.56 (1.11)	-	+
	triangle/polygons	2.36 (1.19)	3.58 (1.08)	-	+
	3D objects	2.24 (1.29)	3.39 (1.04)	-	-
	tesselations	2.12 (1.10)	2.74 (0.84)	-	-
10th	polynomials	2.56 (1.15)	4.32 (0.75)	-	+
	quadratics	3.08 (1.26)	4.43 (0.73)	-	+
	trigonometry/ratios	2.28 (1.13)	4.28 (0.86)	-	+
	trigonometry	2.22 (1.09)	4.15 (0.91)	-	+
	similarity/triangle	2.60 (1.26)	3.42 (1.07)	-	-
	transformation	2.30 (1.18)	3.58 (1.11)	-	+
	proofs/geometry	2.48 (1.28)	3.46 (1.06)	-	-
11th	complex numbers	2.34 (1.15)	3.82 (1.08)	-	+
	exponential eqn.	2.38 (1.18)	4.22 (0.72)	-	+
	logarithmic eqn.	2.38 (1.14)	4.42 (0.62)	-	+
	proof methods	3.88 (1.12)	4.38 (0.83)	+	+
	sequences	2.92 (1.16)	4.11 (0.78)	-	+
	matrices	2.42 (1.25)	4.51 (0.65)	-	+
	linear eqn.s	2.90 (1.23)	4.49 (0.65)	-	+
	counting	3.14 (1.21)	4.38 (0.80)	-	+
	pascal/binomial	2.40 (1.20)	4.04 (0.86)	-	+
	conic sections	2.18 (1.17)	3.35 (1.02)	-	-
	circular region	2.46 (1.33)	3.68 (1.07)	-	+
	probability	4.04 (0.93)	4.54 (0.79)	+	+
	stat/presentation	3.94 (1.25)	4.38 (0.91)	+	+
	stat/tend-disp	3.68 (1.45)	4.10 (1.08)	+	+

12th	limits/cont.	2.46 (1.15)	4.10 (0.97)	-	+
	graphs of functions	3.02 (1.29)	4.49 (0.71)	-	+
	derivatives	2.60 (1.25)	4.43 (0.75)	-	+
	integration	2.30 (1.27)	4.33 (0.82)	-	+
	vectors in 3D	2.34 (1.22)	4.25 (0.87)	-	+
	plane in space	2.32 (1.13)	4.01 (0.83)	-	+
IB	finite random var.	2.92 (1.28)	3.71 (0.99)	-	+
	Statistical distr.s	3.42 (1.42)	3.75 (1.16)	-	+
	Bayes theorem	3.02 (1.02)	3.82 (1.12)	-	+
	hypothesis testing	3.64 (1.31)	3.69 (1.18)	+	+
	correlation/regression	3.64 (1.34)	3.61 (1.22)	+	+
	interest	3.36 (1.31)	2.90 (1.19)	-	-

Table 2. Ratings of Importance for Mathematical Skills

Skills	Social Science	Engineering	Social Sci.	Engineering
Mathematical problem solving	4.12 (0.83)	4.80 (0.40)	+	+
Mathematical modelling	3.81 (0.92)	4.52 (0.58)	+	+
Mathematical reasoning	4.31 (0.80)	4.72 (0.48)	+	+
Mathematical communication	3.83 (1.12)	4.86 (0.46)	+	+
Mathematical connections	4.10 (1.04)	4.52 (0.56)	+	+
Mathematical representations	3.56 (1.24)	4.37 (0.68)	+	+
Analytical reasoning	4.60 (0.68)	4.83 (0.45)	+	+
Critical thinking	4.77 (0.52)	4.70 (0.60)	+	+

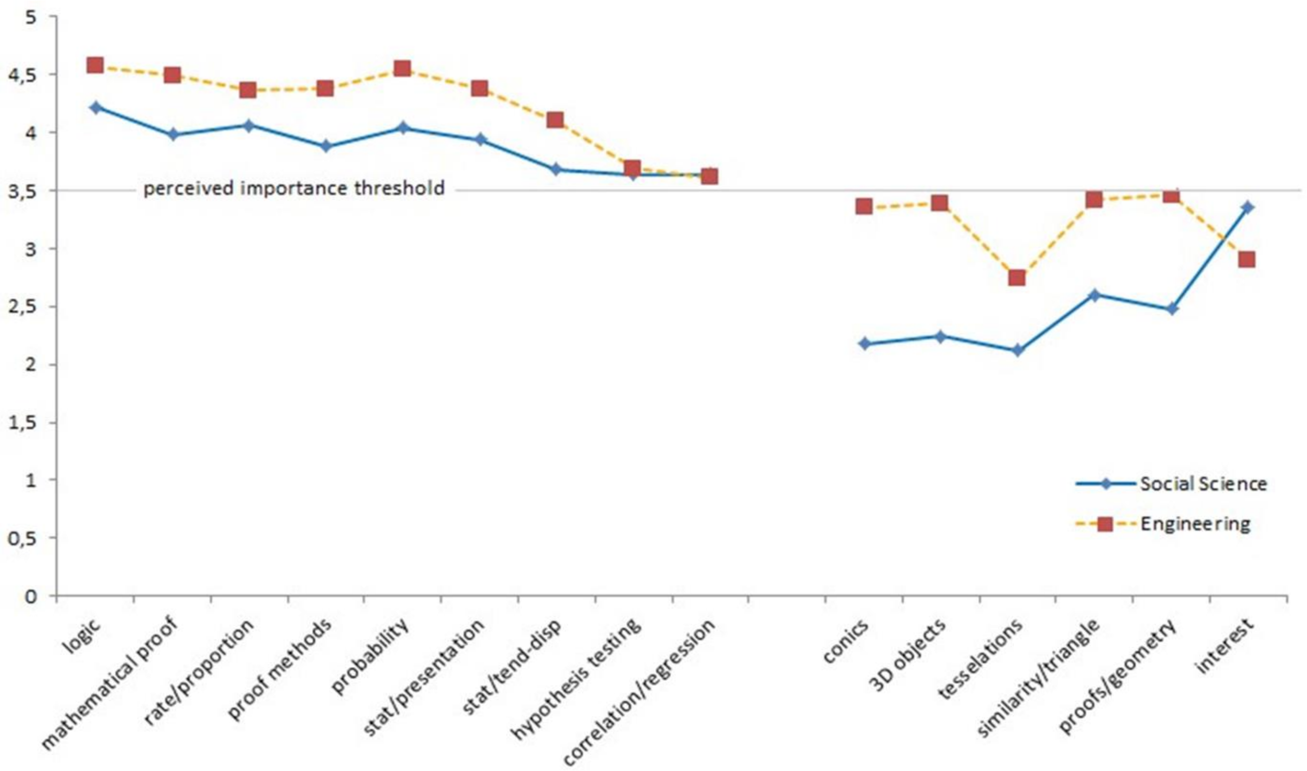

Figure 2. Perceived levels of importance of mathematical topics in the same rating categories across academic fields

To answer the second research question, individual ratings for the mathematical topics were compared across social science and engineering disciplines using Mann-Whitney U Test. In this comparison, for practical purposes mathematical topics were classified into three categories; namely i. the topics which were rated important for both social science and engineering departments (with two + signs in the last two columns of Table 1), ii. the topics that were rated important by only one of social science and engineering departments (with only one + sign in the last two columns of table 1), and iii. the topics that were not rated as important by either of social science and engineering departments (with two - signs in the last two columns of Table 1). The ratings received by the topics in the first and the third categories are displayed in Figure 2. For the practical purposes of this research, the topics in the first and the third categories did not need to be compared, because they were rated as important (with mean ratings at or above 3.5 or they were rated as not important by both of the two types of fields (with mean ratings below 3.5).

There were 34 topics in the second category which are displayed in Figure 3. For these topics, a measure was needed to decide whether the differences in ratings were large enough to attribute to the perceived importance, rather than chance. For example, a topic with mean ratings of 2.45 and 4.65 was more likely to carry a real difference than a topic with mean ratings of 3.45 and 3.60, even though they were both in the second category. For the second topic in the example above, the difference was more likely to be due to chance. In other words, a measure was needed to assess the significance of differences in perceived importance. Mann-Whitney U test was used to compare the ratings received by the mathematical topics from teaching staff in social science and engineering departments. Table 3 presents results of comparison for the 34 mathematical topics in this category.

Table 3 shows that the differences in the ratings of the mathematical topics were statistically significant for all of the topics with one exception, statistical distributions.

All of the eight skills listed in Table 2 received "important" ratings by social science and engineering departments. Because average ratings were in the same category, no statistical test was run for comparison by the two types of academic fields. Even though they were rated important by the staff in both fields, mathematical skills received somewhat higher ratings in general by engineering staff (the only exception was "critical thinking" (please see Table 2).

Participants of the study were asked to suggest any other mathematical topics or skills which they considered important to attain at high school and were not given in the lists of the survey. Three suggestions came from electrical/electronics engineering staff, and sixteen from computer engineering staff. There were no suggestions from social science departments. The topics and skills suggested by engineering faculty members are given in Table 4.

Eight suggestions for inclusion in high school mathematics curriculum were related to knowledge of algorithms (f=4) and programming languages (f=4). Three suggestions were made about discrete mathematics. As to the skills, there were 15 suggestions. Finding multiple approaches to problems (f=5) and being able to lay out solutions clearly or "solution development" (f=4) received the highest frequencies. These can be interpreted that teaching staff wished students to develop higher order thinking skills at high school. This resonates well with the relatively high ratings mathematical skills received by the staff in both types of disciplines.

DISCUSSIONS AND IMPLICATIONS

Mathematical Needs of High School Students Intending to Pursue Social Science and Engineering Fields at University

As presented above, the topics that were rated important for social science fields included 9 topics; logic, mathematical proof and proof methods, rates and proportions, basic probability, statistical measures of central tendency, data representation, hypothesis testing and correlation and regression. It is notable that social science programs rated logic, mathematical proof and proof methods an important background for their discipline. All three of these competencies indeed represent tools of systematic thinking and analytical reasoning in social science fields (e.g., McGovern, Furumoto, Halpern, Kimble, & McKeachie, 1991; Öztürk, 2010).

In fact, when a lawyer structures his or her argument in a legal case in defense of her client, she has to consider what is given, identify the missing information, search for the missing by doing research. Then she structures and presents the information in a logical and persuasive manner leading to a favorable conclusion. Similarly, when a historian is engaged in field research, he takes an account of the existing information, search for objective historical evidence to construct a new argument that is logically defensible for his colleagues. All of these activities require possession of the ability to logically breakdown what is available, identify and find out what is missing to reach a conclusion, a process resembling the steps of a mathematical proof. Additionally, facility with a strong understanding of the use of rates and proportions, a central concept of quantitative reasoning is indeed necessary for any student of social sciences, in fact a

Table 3. Results of Mann-Whitney U Test Comparisons between Social Science and Engineering Fields for Mathematical Topics

Grade Level	Topics	Mann-Whitney U Score	z-score	Level of significance (2-tailed)
9	sets	778.00	-5.57	< .001
	relations	822.00	-5.45	< .001
	functions	303.50	-8.33	< .001
	modular arithmetic	324.50	-7.97	< .001
	exponential/root exps	507.00	-7.01	< .001
	divisibility	823.50	-5.30	< .001
	vectors	383.50	-7.67	< .001
	line/circle in plane	694.00	-5.96	< .001
	distance in plane	614.00	-6.40	< .001
	point/line/angle	797.00	-5.37	< .001
	triangle/polygons	852.50	-5.06	< .001
10	polynomials	391.50	-7.62	< .001
	quadratics	668.50	-6.21	< .001
	trigonometry/ratios	341.50	-7.82	< .001
	trigonometry	373.00	-7.63	< .001
	transformations	820.50	-5.24	< .001
11	complex numbers	661.50	-6.10	< .001
	exponential eqn.	388.00	-7.62	< .001
	logarithmic eqn.	273.50	-8.24	< .001
	sequences	778.00	-5.56	< .001
	matrices	294.00	-8.16	< .001
	linear eqn.	499.50	-7.10	< .001
	counting	722.00	-5.96	< .001
	pascal/binomial	520.50	-6.89	< .001
	circular region	898.50	-4.84	< .001
12	limits & cont.	530.00	-6.81	< .001
	graphs of functions	617.00	-6.49	< .001
	derivatives	418.00	-7.48	< .001
	integration	391.50	-7.57	< .001
	vectors in 3D	422.50	-7.39	< .001
	plane in space	473.50	-7.15	< .001
IB	finite random var.	1166.50	-3.42	0.001
	Statistical distr.s	1590.50	-1.13	0.260[1]
	Bayes theorem	1063.00	-3.98	0.001

[1] asymptotic significance level does not indicate statistical significance at $p<0.01$.

competency probably needed by a high school graduate who may not even pursue higher education. In a related note, it is possible that there may be an overlap between the mathematical knowledge and background needed for social science education at university and the mathematics needed in daily life by all high school graduates, university bound or otherwise.

All of the remaining five topics rated important for social science departments are related to statistics and probability. Missing the certainty of physical deterministic methods, social sciences typically search for "truth" by using probabilistic tools of reasoning. For example, when an event is observed one way 95 out of 100 random times, it is considered satisfactory evidence to attribute the observation to a "factor," rather than pure chance. For example in psychology, when children who are exposed to high levels of lead in environment display lower intelligence compared to their peers 95 out of 100 times, this is considered satisfactory "evidence" of the negative effect of lead on children's cognitive development, even though how this happens at the cellular level in brain or body is not exactly known (Lalonde & Gardner, 1993).

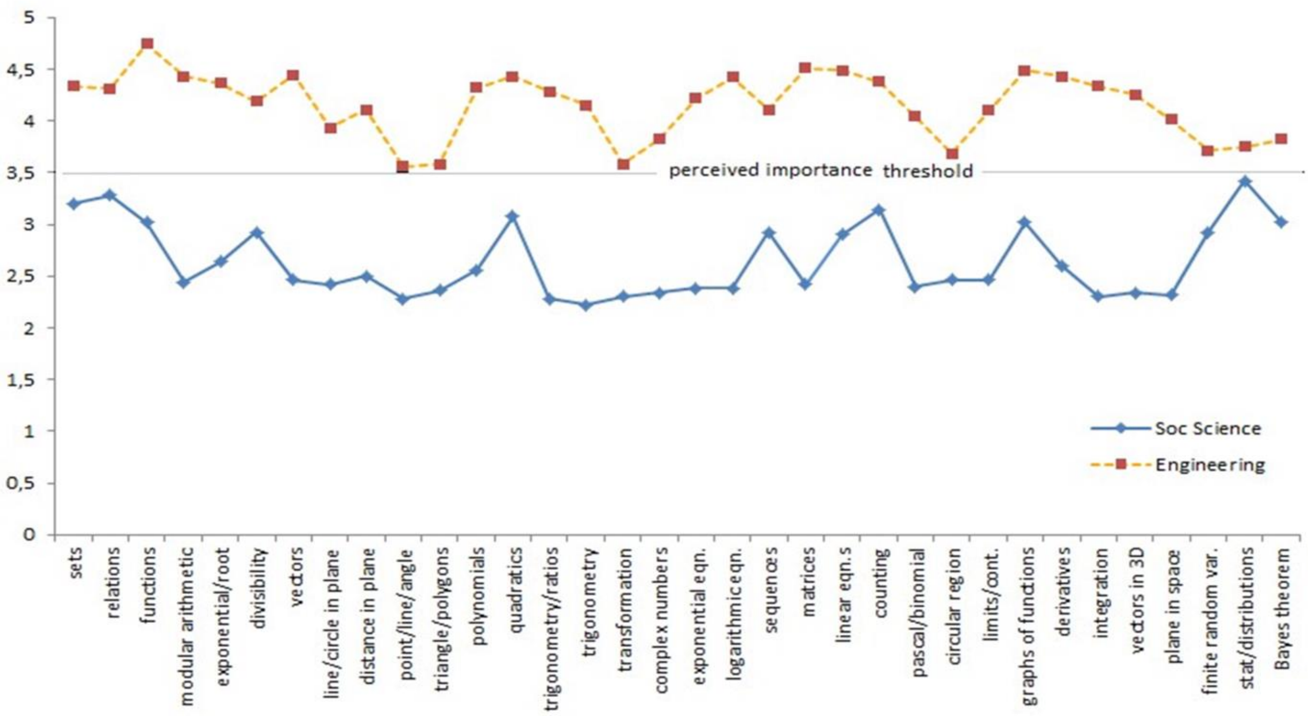

Figure 3. Perceived importance levels of mathematical topics for which social science and engineering departments differed in rating categories

Table 4. Topics and Skills Suggested by Academic Staff

Topics	f	Skills	f
Algorithms	4	Finding multiple approaches	5
Computer languages	4	Solution development	4
Discrete mathematics	3	Effective study skills	3
Graph theory	1	Ability to analyze	2
Logic and deduction	1	Presentation skills	1
Numerical methods	1	Total	15
Logic design	1		
Taylor series	1		
Complexity analysis	1		
Data structures	1		
History of mathematics	1		
Total	19		

Table 5. Importance of Mathematical Topics by the Type of Field and the Places of the Topics in the 2013 National Curriculum (MoNE, 2013a)

	Rating Status by The Type of Field		Place in the National Curriculum[1]
Topics	Social Sci.	Engineering	
logic	+	+	A11
mathematical proof	+	+	A11
sets	-	+	CC9
relations	-	+	--
functions	-	+	CC9, CC10
modular arithmetic	-	+	A11
exponential/root exps	-	+	CC9

divisibility	-	+	B11, A11
rates/proportions	+	+	B11
vectors	-	+	CC9, A12
line/circle in plane	-	+	CC10, A12
distance in plane	-	+	CC10
point/line/angle	-	+	CC10, A12
triangle/polygons	-	+	CC9, CC10
3D objects	-	-	CC10, A12
tesselations	-	-	--
polynomials	-	+	CC10
quadratics	-	+	CC10
trigonometry/ratios	-	+	CC9
trigonometry	-	+	A11
similarity/triangle	-	-	CC9, B12
transformation	-	+	A11
proofs/geometry	-	-	--
complex numbers	-	+	CC9
exponential eqn.	-	+	CC9, A11
logarithmic eqn.	-	+	A11
proof methods	+	+	A11
sequences	-	+	B11
matrices	-	+	--
linear eqn.s	-	+	CC9
counting	-	+	A12
pascal/binomial	-	+	CC10
conic sectionss	-	-	A12
circular region	-	+	CC10
probability	+	+	CC9, CC10
stat/presentation	+	+	CC9
stat/tend-disp	+	+	CC9, B11
limits/cont.	-	+	A12
graphs of functions	-	+	B12
derivatives	-	+	A12
integration	-	+	A12
vectors in 3D	-	+	--
plane in space	-	+	A12
finite random var.	-	+	--
statistical distr.s	-	+	--
Bayes theorem	-	+	--
hypothesis testing	+	+	--
correlation/regression	+	+	--
interest	-	-	B11

[1] CC9: Core curriculum of grade 9, CC10: Core curriculum of grade 10, B11: Basic mathematics option at grade 11, B12: Basic mathematics option at grade 12, A11: Advanced mathematics option at grade 11, A12: Advanced mathematics option at grade 12.

Accordingly, it is no surprise that university social science staff think familiarity with the tools of logical, probabilistic and statistical reasoning at high school facilitates a student's professional education, and perhaps the life of any educated citizen (Ottaviani, 1991; Öztürk, 2010).

Based on current curriculum options, after the compulsory core curriculum at grades 9 or 10, students who intend to pursue a social science program at university are most likely to follow the "basic mathematics" option in grades 11 and 12. To analyze the place of topics rated high for the social science departments, the grade levels at which the topics were placed in the (current) high school mathematics curriculum are shown in Table 5. The table shows that logic, mathematical proof and proof methods are placed in advanced mathematics programs at grade 11, and not in the type of option that are most likely to be followed by the students intending to go to social science departments. The two IB topics; "hypothesis testing" and "correlation and regression" are not covered in the current national high school mathematics programs (except that there is passing reference to correlation limited to the visual/graphical context of scatter plots.) On the other hand, four topics (probability, data presentation, measures of central tendency and dispersion are covered in the core curriculum. Consequently, it can be concluded that national high school mathematics curriculum at present falls short of providing high school students the necessary mathematical background for prospective social science students at university.

For engineering departments, a wide range of 43 mathematical topics at high school were rated important to prepare for engineering education. These topics ranged from proof and logic, discrete mathematics, basic number theory, modular arithmetic, analytic and synthetic geometry, algebra, trigonometry, polynomials, and linear algebra, analysis to statistics and probability. The robust mathematical background for high school graduates who intend to go to engineering departments at university is probably not a surprise. Engineers make heavy use of mathematics in their profession, and learn high level of mathematics to become an engineer (Crowther, Thompson, & Cullingford, 1997; Güner, 2008). For example, computer engineers who are engaged in software production have to be good in algorithms. For algorithms, there is heavy reliance on discreet mathematics such as Boolean algebra, sets, combinatorics, graph theory, computational number theory, probability and linear algebra. Further, programs that involve numerical analysis presume knowledge of calculus and differential equations. Electrical and electronics engineers produce and deal with electrical, electromagnetic and electronic devices. Devices with electronic circuits have ubiquitous use in the modern world. For designing electronic circuits, one needs to use Boolean algebra, algebra of polynomials, logarithms and trigonometry. Calculus, the mathematics of change is often used to model electrical currents. Analytic geometry or the geometry of circle, lines, points and curves are used in modeling and designing new products. Accordingly, prospective engineering students who develop an aptitude in a wide range of mathematical topics as shown in this study will certainly be at an advantage at university (Crowther et al., 2007; Ismaila et al., 2012).

When compared to the existing national mathematics curriculum, most of the topics rated important for engineering departments were placed in the core curriculum, or the basic and advanced mathematics options at grades 11 and 12 (please see Table 5). Of the 43 topics, 19 topics were placed in the core curriculum. These were sets, functions, exponential and root expressions, vectors, point, line, angle and circle geometry, distance in plane, triangles, quadrilaterals and polygons, trigonometric ratios, introduction to complex numbers, linear, quadratic and polynomial equations, Pascal and binomial expansions, analytics of circle, basic probability, data presentation and measures of central tendency. Four topics were placed in basic mathematics options of grades 11 and 12 and these were rates and proportions, number sequences, graphs of functions and interest computations. These four topics are probably covered in the contexts of other topics of advanced mathematics option. For example, function graphs can be learned while discussing elementary functions such as logarithmic and trigonometric functions in the advanced option. Thirteen topics belonged to the advanced mathematics option. These mathematical topics were logic, mathematical proof and proof methods, divisibility rules, geometric transformations, plane in space, trigonometric identities and functions, logarithmic functions, counting methods, and such calculus topics as limits and continuity, derivatives, integration. The remaining eight topics were not covered in the current national mathematics curriculum. These were the concept of relations, matrices, vectors in 3d, finite random variables, statistical distributions, Bayes theorem, hypothesis testing and correlation and regression (relations is briefly mentioned in the definition of functions, but not treated as a separate topic). We believe that the concept of relations is a topic not worth teaching by itself as it is implied and used within the definition of "functions." Most of the remaining topics were from IB statistics topics. Overall, it can be concluded that high school mathematics curriculum serves relatively well to the needs of engineering intending high school students, more so than the students who intend to go to social science departments at university.

Readers will remember that a number of topics rated important for social science fields (3 out of 9) and also engineering fields (19 out of 43 topics) are covered in the core curriculum. One way to interpret this finding is related to the possible overlap between mathematical background needed by an average high school graduate who may not intend to go university and the mathematical background expected from social science and engineering intending students. In other words, mathematical knowledge needed by an average citizen is also possibly good for university-bound students.

Comparison of Engineering and Social Science Fields

There were 34 topics that were rated important by engineering departments but not the social science departments (please see Figure 3). Comparisons using Mann-Whitney U tests revealed that the differences in the ratings between the two academic fields were statistically significant for all of these topics. The fact that for the 34 topics, the differences were statistically significant points a clear distinction between the presumed background knowledge for social science and engineering departments, a finding that makes sense and would be expected.

Only six topics were not rated important by both the engineering departments and social science departments (please see Figure 2). These were conic sections, geometry of 3d objects, tessellations, triangle similarity, and proofs in geometry and interest computations. Perhaps proofs in geometry was considered part of mathematical proof in general and hence not worthy on its own. Readers will remember that mathematical proof was rated high by the staff of both types of academic fields. Tessellations were possibly considered a topic of mathematical enrichment and recreation rather than a topic of instruction. It is possible that triangle similarity was seen as a too elementary topic for high school, and a knowledge needed by an average citizen rather than specifically by a social scientist or an engineer.

Implications for Teaching

One message that is loud and clear from the findings of this study is related to how much value is attributed to mathematical process skills such as problem solving, mathematical reasoning and communication, ability to use mathematical connections and representations as well as analytical thinking and critical reasoning. There was also evidence of this in the additional suggestions made by the teaching staff (please see Table 4). Mathematical skills were rated high by both types of academic disciplines (please see Tables 2), a finding that has strong support from professionals of the field as well (Gençoğlu & Cebeci, 1999; Murray, 1997; Öztürk, 2010).

For instructional purposes, skills are related to how we teach rather than what we teach. Regardless of students' future track, if mathematical topics are taught in a manner in which understanding and reasoning are valued rather than memorization and imitation, we believe students are more likely to develop these higher order skills. If mathematics is taught by highlighting the connections between mathematics and its applications and the connections between mathematical concepts, we believe that students are more likely to feel at ease for using them in their future careers and in daily lives. If students are expected to explain their thinking clearly both verbally and in writing in mathematics classes, we believe they are more likely to develop clear solutions on paper to the problems in their discipline in their university education, an ability highly valued by engineering staff.

Implications Curriculum Design and Testing

The findings of this study lend support to the idea of differentiated pathways in mathematics curriculum for high school students. At present, the national mathematics curriculum for general high schools reflects fulfilling a double role, i. to teach the basic mathematical knowledge and skills needed by an average citizen, a role that can be accomplished reasonably well by the core curriculum of grades 9 and 10 supplemented by the basic mathematics option in grades 11 and 12; and ii. to teach further mathematics to prepare students for technical and engineering fields in higher education. The findings of the present study showed that the second goal can also be accomplished to a certain extend with the advanced mathematics option at present.

However, what is clearly missing in the current configuration is the needs of students who intend to go to social science fields in higher education. They certainly do not need to take advanced mathematics option or a lesser version of it. A special and customized track designed to cover probabilistic and statistical reasoning, hypothesis testing, correlation, regression, data analysis and presentation techniques supplemented by other "basic math" topics such as rates and proportions, financial and consumer mathematics would best fit their needs for higher education, similar to options in other countries (e.g., Lee et al., 2010; Özdemir Erdoğan, 2014).

Some of these topics may be appropriately placed in the existing core curriculum or in an "advanced math for social science," an option that does not exist at present. For example, correlation and regression is a topic that is not directly covered in the existing national curriculum options. Given the centrality and the perceived importance of these topics for both engineering and social science fields, curriculum planners may consider including it in the core mathematics curriculum at grades 9 and 10. Hypothesis testing is another topic rated high for both types of fields. This topic may be placed in the basic and advanced options of grades 11 and 12 with different depths of treatment. Hypothesis testing would probably not be suitable for the core curriculum, as it requires knowledge of probability and probability distributions.

Another implication of the findings of this study concerns university entrance examinations. As stress to enter universities gradually eases up in Turkey in the coming years, customized examinations can be structured for social science and engineering departments covering relevant topics. We know that "what you test is what you get" is a well-known phenomenon in education. Seeing the relevance of high school mathematics curriculum to their future careers as assessed in the university entrance examination will help motivate students to do their best in learning the curriculum. However, although not directly warranted by this study, we believe the university entrance exam should find ways to reward the use of higher order mathematical skills discussed above. This will certainly require including items in alternative (e.g., open ended) formats, beyond the sole use of multiple choice questions in these exams.

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Appendix: Topics of Mathematics Curriculum Included in the Survey

Abbreviation	Corresponding topic from the official curriculum
9th Grade*	
logic	Logic, truth tables, propositions, etc.
proof	Generic methods of mathematical proof (induction, proof by contradiction, etc.)
sets	Sets and operations with sets
relations	Relations (relations between sets)
functions	Concept of function (domain and range sets of functions, operations on functions)
modular arithmetic	Modular arithmetic (the numbers that are not in 10 base)
exponential/root exps	Exponential numbers and root numbers
divisibility	Divisibility of integers
rates/proportions	Rate/proportion
vectors	Vectors in analytic plane, operations and vectors
line/circle in plane	Line and circle properties in the analytic plane
distance in plane	Distance and applications in analytic plane
point/line/angle	Synthetic geometry: point, line, angle, ray, plane, space
triangle/polygons	Synthetic geometry: angles and areas of triangles and polygons
3D objects	Cylinder, cone, sphere, prism, pyramid and their properties
tessellations	Tessellations on the plane (e.g., Escher's drawings)
10th Grade Level	
polynomials	Polynomials (operations on polynomials and factorization)
quadratics	Quadratic equations and functions
trigonometry/ratios	Trigonometric ratios (sine, cosine, etc.)
trigonometry	Trigonometric functions
similarity/triangle	Similarity theorems for triangles
transformations	Transformations on the plane (translation, revolution, reflection)
proofs/geometry	The proof of theorems in geometry
11th Grade Level	
complex numbers	Complex numbers
exponential eqn.	Exponential equations and functions
logarithmic eqn.	Logarithmic equations and functions, natural logarithm
proof methods	Proof by induction and other proof methods
sequences	Sequences (arithmetic and geometric sequences)
matrices	Matrices, matrices operations and determinants
linear eqn.	Linear equation systems and applications
counting methods	Counting methods (permutation and combination)
pascal/binomial expn	Pascal triangle and binomial expansion
conic sections	Analytical investigation of conics (parabola, hyperbola and ellipse)
circular region	Circular region and area of circular region, the angles of a circle, etc.)
probability	Basic probability concepts (experiment, output, sample, conditional probability)
stat/ data presentation	Statistics - Data presentation (graphs such as column, line, box, scatter,)
stat/tend-disp	Statistics - Central tendency and dispersion

12^{th} Grade Level	
limits & cont.	Limits and continuity
graphs of functions	Drawing and interpreting functions graphs
derivatives	Derivatives and their application
integration	Integration (Indefinite/definite integrals, application of integrals)
vectors in 3D	Vectors in space (three dimensional), operations and vectors
plane	Plane in space and analytic properties
Topics from IBDP	
finite random var.	Finite random variables
Statistical distr.s	Statistical distributions (binomial, Poisson, chi -square, etc.)
Bayes theorem	Bayes theorem
hypothesis tests	Significance and hypothesis testing
correlation/regression	Correlation and regression
interest	Interest, depreciation and cost

* These topics were not identified by grade level in the survey. They are grouped by grade level in this table for ease of reference to the national mathematics curriculum.

Mathematical Skills Included In the Survey

Skill	Descriptions
Problem solving	Mathematical problem solving: ability to apply mathematical concepts and rules effectively in order to solve non-routine problems
Modelling	Mathematical modelling: ability to construct mathematical models satisfying and explaining matters in science, social science, engineering, economics, etc. through mathematical language and concepts
Reasoning	Mathematical reasoning: ability to understand the logic behind mathematical rules, generalizations and solutions and ability to go beyond memorization of mathematical formulas
Communication	Mathematical communication: ability to explain one's mathematical reasoning by mathematical terminology and symbols so that other people can understand it
Connections	Mathematical connections: ability to establish connections among mathematical concepts, mathematics and other science fields, mathematics and real life
Representations	Mathematical representations: ability to demonstrate a mathematical concept in different ways as through algebra, graph, table, diagram etc. ability to make a link between relations and transitions
Critical thinking	Critical thinking skills: ability to think systematically to evaluate the validity of arguments in speeches, news, or research
Analytical reasoning	Analytical reasoning skills: ability to see parts and relations among parts in order to manipulate the functioning of a whole