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A Hamiltonian-based solution to the mixed sensitivity optimization problem for stable pseudorational plants

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Abstract

This paper considers the mixed sensitivity optimization problem for a class of infinite-dimensional stable plants. This problem is reducible to a two- or one-block H^∞ control problem with structured weighting functions. We first show that these weighting functions violate the genericity assumptions of existing Hamiltonian-based solutions such as the well-known Zhou–Khargonekar formula. Then, we derive a new closed form formula for the computation of the optimal performance level, when the underlying plant structure is specified by a pseudorational transfer function.

Keywords: Pseudorational transfer function; Infinite-dimensional systems; Mixed sensitivity optimization; H^∞ control; Skew-Toeplitz approach

1. Introduction

Since mid-1980s various methods have been developed for the H^∞ control of infinite-dimensional systems. In particular, for the one-block problem of finding

$$\rho_{\rm opt} := \inf_{Q\in H^\infty} \|W - mQ\|_\infty$$

with m being an pure time delay, and W given as a strictly proper rational function, a closed form expression has been obtained by Zhou and Khargonekar [15]. The formula has been extended to more general cases in [3,8,10,14]: Let $H_{\rho,W}$ be the Hamiltonian matrix associated with W and ρ:

$$H_{\rho,W} := \begin{bmatrix} A & BB^{\rm T}/\rho \\ -C^{\rm T}C/\rho & -A^{\rm T} \end{bmatrix}, \tag{1}$$

where (A, B, C) is a minimal realization of W. Suppose that m is analytic on the set of eigenvalues of $H_{\rho,W}$. Then the optimal sensitivity $\rho_{\rm opt}$ is the maximal ρ such that $\det(\tilde{m}(H_{\rho,W})|_{22}) = 0$ where $M|_{22}$ denotes

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the $(2,2)$-block of matrix M partitioned accordingly to (1). Recently, in [7], it was shown that when a plant is pseudorational [12], $\tilde{m}(H_{\rho,W})$ is easily obtainable without numerical computations of poles and zeros of the transfer function; see Lemma 2.

In this paper, we consider the mixed sensitivity optimization problem

$$\gamma_{\text{opt}} := \inf_{C \text{ stabilizing } P} \left\| \begin{bmatrix} W_s(1+PC)^{-1} \\ W_t PC(1+PC)^{-1} \end{bmatrix} \right\|_\infty, \quad (2)$$

where W_s and W_t are rational weights, and P is a stable pseudorational plant. This problem is known to be a typical two-block problem, for which a Hamiltonian-based formula is obtained [4]. However, this result is not directly applicable, since a "generic" assumption of the formula is almost always violated [6]. In view of this, we derive a Hamiltonian-based formula for the optimal mixed sensitivity computation, by reducing this structured two-block problem to a one-block problem. This result can be viewed as an extension of the Zhou–Khargonekar formula to a specifically structured one-block problem.

The paper is organized as follows: in the next section we review some preliminary results on pseudorational systems. In Section 3, we briefly summarize the observations made in [7] and state drawbacks in more precise terms. In Section 4, we derive a Hamiltonian-based solution for the structured one-block problem. A numerical example is given in Section 5, and concluding remarks are made in the last section.

Notation and Convention

As usual, H^p and H^p_- denote the Hardy p-spaces on the open right- and left-half complex planes, respectively. Let $\tilde{q}(s) := q(-s)$. For an inner function m, let $H(m)$ be the orthogonal complement of mH^2 in Hilbert space H^2. It is known [5] that

$$H(m) = \{x \in H^2 : \tilde{m}x \in H^2_-\}. \quad (3)$$

For a given distribution (in the sense of Schwartz [9]) α, $\operatorname{supp}\alpha$ denotes its *support* [9], and

$$\ell(\alpha) := \inf\{t : t \in \operatorname{supp}\alpha\},$$
$$r(\alpha) := \sup\{t : t \in \operatorname{supp}\alpha\}.$$

Let $\mathscr{E}'(\mathbb{R}_-)$ denote the space of distributions having compact support in $(-\infty, 0]$. $\mathscr{D}'_+(\mathbb{R})$ is the space of distributions having support bounded on the left. Clearly $\mathscr{E}'(\mathbb{R}_-) \subset \mathscr{D}'_+(\mathbb{R})$. If a distribution α is Laplace transformable, its Laplace transform is denoted by $\hat{\alpha}(s)$.

2. Preliminaries on pseudorational systems

In this section we review certain basic facts on pseudorational systems. This class of systems has been introduced in the late 1980's, and plays a crucial role in realization, modeling, and control of infinite-dimensional systems, especially delay-differential systems [12,13]:

Definition 1. Let f be a distribution having support in $[0,\infty)$. It is said to be *pseudorational* if there exist $q, p \in \mathscr{E}'(\mathbb{R}_-)$ such that

(1) q^{-1} exists over $\mathscr{D}'_+(\mathbb{R})$,
(2) $\operatorname{ord} q^{-1} = -\operatorname{ord} q$,
(3) f can be written as $f = q^{-1} * p$,

where q^{-1} is taken with respect to convolution and $\operatorname{ord} q$ denotes the *order* of a distribution q [9].

If f is pseudorational, its associated transfer function $\hat{f}$ is also said to be pseudorational. From the Paley–Wiener–Schwartz theorem [9], in the Laplace domain, every pseudorational transfer function is a ratio of entire functions of exponential type—the simplest extension of rational functions. For a stable pseudorational plant P, even if P is not necessarily inner,

$$\rho_{\text{opt}} := \inf_{Q \in H^\infty} \|W - PQ\|_\infty \quad (4)$$

can be computed by the following:

Lemma 2 (*Kashima and Yamamoto [7]*). *Suppose that P can be factorized as $\hat{p}_1\hat{p}_2/\hat{q}$ with $q, p_1, p_2 \in \mathscr{E}'(\mathbb{R}_-)$ such that $\hat{q}^{-1}, \hat{p}_1^{-1}, \mathrm{e}^{r(p_2)s}\tilde{\hat{p}}_2^{-1} \in H^\infty$, that is, $\hat{p}_1$ and $\hat{p}_2$ denote the stable and anti-stable parts of the numerator, respectively. Assume also that $1/\hat{p}_2$ is analytic on the set of eigenvalues of $H_{\rho,W}$. Define $L := -\ell(q) + \ell(p_1) - r(p_2)$ and*

$$m_v(s) = \mathrm{e}^{-Ls}\frac{\hat{p}_2(s)}{\tilde{\hat{p}}_2(s)}. \quad (5)$$

Then ρ_{opt} *in* (4) *is the maximal* ρ *that satisfies* $\det(m_v\tilde{}(H_{\rho,W})|_{22})=0$.

3. Mixed sensitivity optimization problem

3.1. Two-block problem

In this section, we show that the weighting functions have a specific structure when we reduce the mixed sensitivity optimization problem to the standard two-block problem. Throughout the paper the plant P is assumed to be stable. By the Youla parameterization, all stabilizing controllers are given in the form $C = Q(1-PQ)^{-1}$, $Q \in H^\infty$. Hence we obtain

$$\gamma_{\mathrm{opt}} = \inf_{Q\in H^\infty} \left\| \begin{bmatrix} W_s(1-PQ) \\ W_t PQ \end{bmatrix} \right\|_\infty . \tag{6}$$

First, introduce the following spectral factorization G

$$G\tilde{}(W_s\tilde{}W_s + W_t\tilde{}W_t)G = 1, \tag{7}$$

where both G and G^{-1} have no unstable poles. Then it follows that

$$L_1 := \begin{bmatrix} (W_sG)\tilde{} & (-W_tG)\tilde{} \\ W_tG & W_sG \end{bmatrix}, \quad L_2 := \begin{bmatrix} m_d & 0 \\ 0 & 1 \end{bmatrix}$$

are unitary, where m_d is a finite Blaschke product that makes

$$W := m_d W_s (W_s G)\tilde{} \tag{8}$$

stable [2]. Multiplying (6) by L_2L_1 from the left, we obtain

$$\gamma_{\mathrm{opt}} = \inf_{Q\in H^\infty} \left\| \begin{bmatrix} W - m_d PQ \\ V \end{bmatrix} \right\|_\infty , \tag{9}$$

where

$$V := W_s W_t G. \tag{10}$$

Note that both W and V are rational and stable. The problem in the form (9) has been considered in [4], and a solution based on a Hamiltonian related to a realization of $\gamma^2 - WW\tilde{} - VV\tilde{}$ is derived. It is however assumed in [4] that *V and W have no common poles*. For arbitrary rational functions V and W, this assumption is satisfied generically. However, in the mixed sensitivity problem, the functions W and V need to be in the form (8) and (10). As seen in Appendix, this means that unless W_s and W_t are chosen in a specific way, *W and V will have common poles*, i.e., the assumption above is almost always violated.

On the other hand, by (7), (8) and (10), we have

$$\begin{aligned} \gamma^2 - WW\tilde{} - VV\tilde{} &= \gamma^2 - W_sW_s\tilde{}G(W_sW_s\tilde{} + W_tW_t\tilde{})G\tilde{} \\ &= \gamma^2 - W_sW_s\tilde{}. \end{aligned} \tag{11}$$

Thus (11) may help us to avoid the "genericity" assumption. However, in the argument in [4], it is difficult to introduce such structures on V and W, since no relationship between these weights was assumed. In view of this, we reduce the specifically structured two-block problem to a one-block problem to make use of such structures explicitly.

3.2. Reduction to one-block problem

Again, applying the standard techniques, see e.g. [2], we now reduce the two-block H^∞ problem (9) to a one-block problem. First, suppose that $\gamma > \|V\|_\infty$ satisfies $\gamma = \gamma_{\mathrm{opt}}$. Then there exists $Q \in H^\infty$ such that

$$|W - m_d PQ|^2 + |V|^2 = \gamma^2 \quad \text{a.e.}$$

on the imaginary axis. Here, since $\gamma > \|V\|_\infty$, there exists a unique spectral factor F_γ:

$$F_\gamma\tilde{}(\gamma^2 - V\tilde{}V)F_\gamma = 1 \quad \text{a.e.} \tag{12}$$

where both F_γ and $F_\gamma^{-1} \in H^\infty$. Therefore, by defining $W_\gamma := F_\gamma W$, we obtain

$$|W_\gamma - m_d PQ|^2 = 1 \quad \text{a.e.}$$

on the imaginary axis. Furthermore it is shown [11] that γ_{opt} is given by the maximal γ such that 1 is a singular value of the *compression* operator W_c of W_γ to $H(m)$ defined by

$$W_c : H(m) \to H(m) : x \mapsto \pi^m[W_\gamma x],$$

where $m := m_d m_v$ and $\pi^m[\cdot]$ denotes the orthogonal projection from H^2 onto $H(m)$.

Lemma 2 characterizes the singular values of the corresponding compression operator [14], that is, 1 is a singular value of W_c if and only if $m\tilde{}(H_{1,W_\gamma})|_{22}$ is not of full rank, when m is analytic on the set of eigenvalues of H_{1,W_γ}. However, W_γ and m_d have a specific structure, and we must be careful in applying Lemma 2. To see this, let us consider the eigenvalues of H_{1,W_γ}. Notice that the eigenvalues of the Hamiltonian matrix $H_{\rho,W}$ coincide with the zeros of $\rho^2 - \tilde{W}W$. Eqs. (11) and (12) yield

$$1 - \tilde{W_\gamma}W_\gamma = (\gamma^2 - \tilde{V}V - \tilde{W}W)\tilde{F_\gamma}F_\gamma$$
$$= (\gamma^2 - \tilde{W_s}W_s)\tilde{F_\gamma}F_\gamma.$$

Therefore, the eigenvalues of H_{1,W_γ} arise from those of H_{γ,W_s} or zeros of F_γ and $\tilde{F_\gamma}$. Unfortunately, the zeros of F_γ coincide with poles of m_d; see Appendix and [6,11] for details. In other words, there always exists a nonsingular matrix T such that

$$H_{1,W_\gamma} = T^{-1}\,\text{blockdiag}(H_{\gamma,W_s}, A_d, -A_d)T, \tag{13}$$

with $(sI - A_d)^{-1} \in H(m_d)$. This means that the poles of m_d are eigenvalues of H_{1,W_γ}, i.e., the assumption of the lemma is also almost always violated. In practice, we can circumvent this problem by slightly altering V and obtain upper and lower bounds for the optimal value [6].

In what follows, we derive a Hamiltonian-based formula for the optimal mixed sensitivity computation, i.e., the problem of finding the maximal γ such that 1 is a singular value of W_c.

4. Main result

Consider the singular value equation

$$y = W_c x, \quad x = W_c^* y,$$

where W_c^* is the adjoint operator of W_c. Let $(A_\gamma, B_\gamma, C_\gamma)$ be a minimal realization of W_γ. Following exactly the same argument in [14, Proposition 2.6], we can show that these equations are characterized by finite dimensional vectors as follows:

$$y = W_\gamma x - m(s)C_\gamma(sI - A_\gamma)^{-1}\xi,$$
$$x = \tilde{W_\gamma}y + B_\gamma(sI + A_\gamma^{\mathrm{T}})^{-1}\zeta,$$

where $\xi, \zeta \in \mathbb{R}^{n+p}$ and n and p are the degrees of W_s and m_d, respectively. Combining these equations together, and following the same argument as given in [14], we obtain the following Hamiltonian-based characterization:

Lemma 3. *Under the definitions above*, 1 *is a singular value of W_c if and only if there exists a nonzero vector $[\xi^{\mathrm{T}}\ \ \zeta^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{2(n+p)}$ such that*

$$\Phi(s) := (sI - H_{1,W_\gamma})^{-1}\begin{bmatrix} m(s)\xi \\ \zeta \end{bmatrix} \in H(m). \tag{14}$$

By invoking the Dunford integral, we can reduce this lemma to a rank condition [14]. Partition T accordingly to (13),

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \\ T_{31} & T_{32} \end{bmatrix} := T, \tag{15}$$

where $T_{11}, T_{12} \in \mathbb{R}^{2n\times(n+p)}$ and other four matrices are in $\mathbb{R}^{p\times(n+p)}$. We are now ready to give a formula for the optimal mixed sensitivity for stable plants.

Theorem 4. *Define the matrices $H_{1,W_\gamma}, H_{\gamma,W_s}$ and T_{ij} $(i = 1, 2, 3,\ j = 1, 2)$ by* (1), (13) *and* (15). *Suppose that m is analytic on the set of the eigenvalues of H_{γ,W_s}. Then the optimal mixed sensitivity γ_{opt} in* (6) *is the maximal γ such that*

$$\begin{bmatrix} T_{11} & m_{\tilde{v}}(H_{\gamma,W_s})T_{12} \\ T_{21} & 0 \\ 0 & T_{32} \end{bmatrix} \tag{16}$$

is not of full rank.

Proof. From Lemma 3, it suffices to show that there exists a nonzero vector $[\xi^{\mathrm{T}}\ \ \zeta^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{2(n+p)}$ satisfying (14) if and only if the matrix in (16) is not of full rank.

Since T is nonsingular, $\Phi(s)$ belongs to $H(m)$ if and only if so does $T\Phi(s)$, or equivalently,

$$(sI - H_{\gamma,W_s})^{-1}[T_{11}\ \ T_{12}]\begin{bmatrix} m(s)\xi \\ \zeta \end{bmatrix} \in H(m), \tag{17}$$

$$(sI - A_d)^{-1}[T_{21}\ \ T_{22}]\begin{bmatrix} m(s)\xi \\ \zeta \end{bmatrix} \in H(m) \tag{18}$$

and,

$$(sI + A_d)^{-1}[T_{31}\ \ T_{32}]\begin{bmatrix} m(s)\xi \\ \zeta \end{bmatrix} \in H(m). \tag{19}$$

First consider (17). Let Δ be a closed rectifiable contour that encircles all eigenvalues of H_{γ,W_s}, but none of the singularities of $\tilde{m}$. Since $\tilde{m}$ is analytic at eigenvalues of H_{γ,W_s}, this is possible. Consider now the integral

$$\frac{1}{2\pi j}\int_{\Delta}(sI - H_{\gamma,W_s})^{-1}[T_{11}\ \ T_{12}]\begin{bmatrix}\xi\\ \tilde{m}(s)\zeta\end{bmatrix}\mathrm{d}s.$$

Notice that, by spectral integral theory [1], this integral equals

$$T_{11}\ \ T_{12}\begin{bmatrix}\xi\\ 0\end{bmatrix} - \tilde{m}(H_{\gamma,W_s})[T_{11}\ \ T_{12}]\begin{bmatrix}0\\ \zeta\end{bmatrix}.$$

Since (17) holds if and only if this integral is equal to 0, [14], we obtain

$$T_{11}\xi = \tilde{m}(H_{\gamma,W_s})T_{12}\zeta. \tag{20}$$

We now consider condition (18). Recall that we have $(sI - A_d)^{-1}T_{22}\zeta \in H(m_d) \subset H(m)$. Hence in view of (3), (18) is equivalent to $(sI - A_d)^{-1}T_{21}\xi \in H^2_-$. Since A_d has no unstable eigenvalues, this implies

$$T_{21}\xi = 0. \tag{21}$$

For (19), we have $m(s)(sI + A_d)^{-1} \in m_v H(m_d) \subset H(m)$ and all eigenvalues of $-A_d$ are unstable. Therefore we must have

$$T_{32}\zeta = 0. \tag{22}$$

Combining (20)–(22) together yields

$$\begin{bmatrix}T_{11} & \tilde{m_v}(H_{\gamma,W_s})T_{12}\\ T_{21} & 0\\ 0 & T_{32}\end{bmatrix}\begin{bmatrix}\xi\\ \zeta\end{bmatrix} = 0. \tag{23}$$

There exists a nonzero $[\xi^{\mathrm{T}}\ \ \zeta^{\mathrm{T}}]^{\mathrm{T}}$ satisfying (23) if and only if the matrix in (16) is not of full rank. This completes the proof. □

Remark 5. When $W_t = 0$, this problem becomes the sensitivity optimization, and $[T_{11}\ \ T_{12}] = I$ and $p = 0$. In this case, we can verify that the rank condition in Theorem 4 is equivalent to $\tilde{m_v}(H_{\gamma,W_s})|_{22}$ is not of full rank, which is the generalized Zhou–Khargonekar formula for the one-block case as expected.

5. Example

Suppose that the weighting functions are given by $W_s(s) = 1/(s+1)$ and $W_t(s) = (s+0.5)/(s+1)$, and a stable pseudorational plant $P(s) = (\mathrm{e}^s - 2)/(2\mathrm{e}^{2s} - 1) \in H^\infty$. Then the function m_v in (5) is given by

$$m_v := \mathrm{e}^{-s}\frac{2\mathrm{e}^{-s} - 1}{2 - \mathrm{e}^{-s}}.$$

Then, by (7), (8) and (10), m_d, V and W are given by

$$m_d(s) = \frac{s+\lambda}{s-\lambda}, \quad V(s) = \frac{1}{(s+1)(s-\lambda)},$$
$$W(s) = \frac{s+0.5}{(s+1)(s-\lambda)},$$

where $\lambda = -\sqrt{5}/2$. We see that V and W have common poles. Function W_γ is given by

$$W_\gamma = \frac{1}{\gamma(s^2 + bs + a)},$$

where

$$a = \frac{\sqrt{5 - \gamma^{-2}}}{2} \quad \text{and} \quad b = \sqrt{\frac{9}{4} + 2a - \gamma^{-2}}.$$

The eigenvalues of H_{1,W_γ} are $s = \pm\lambda,\ \pm\sqrt{1 - \gamma^{-2}}$, including the pole of m_d.

In [6], by changing the weighting function W_γ slightly, it has been shown that $0.852 < \gamma_{\mathrm{opt}} < 0.857$. Fig. 1 shows the smallest singular values of the matrix in (16) versus γ. According to Theorem 4, the optimal mixed sensitivity γ_{opt}, the maximal γ such that this minimal singular value equals to zero, is approximately 0.8567 and this satisfies the estimate above.

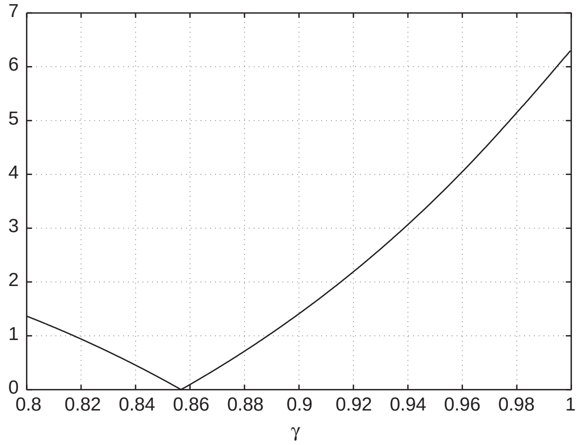

Fig. 1. $\sigma_{\min}$ versus γ.

6. Conclusions

We have derived a new closed form Hamiltonian-based formula to the optimal mixed sensitivity optimization problem for stable pseudorational plants with rational weights. This result can be viewed as an extension of the Zhou–Khargonekar formula to a specifically structured one-block problem.

Appendix. Constraint on the derived weighting functions

Here we see the structure of weighting functions, when we reduce the mixed sensitivity optimization problem to the two-block problem (9) or the problem of finding the singular values of the compression operator W_c. Consider rational weighting functions $W_s = n_s/d_s,\ W_t = n_t/d_t$ where pairs of polynomials (d_s, n_s) and (d_t, n_t) are coprime. For simplicity, we assume that d_s and d_t have no common zeros. Let us take a stable polynomial d_G such that

$$d_G^{\sim} d_G = n_s n_s^{\sim} d_t d_t^{\sim} + n_t n_t^{\sim} d_s d_s^{\sim}.$$

Then we have $G = d_s d_t / d_G$ and $m_d = d_G^{\sim}/d_G$. Hence weighting functions in the two-block problem (9) are given by $W = n_s n_s^{\sim} d_t^{\sim}/d_s d_G$ and $V = n_s n_t / d_G$, and have common poles. Now let us define a stable polynomial d_F such that

$$d_F^{\sim} d_F = \gamma^2 d_G d_G^{\sim} - n_s n_s^{\sim} n_t n_t^{\sim}.$$

The spectral factor F_γ in (12) is given by $F_\gamma = d_G/d_F$, and its zeros are poles of m_d.

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