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Integrated Routing and Scheduling of Hazmat Trucks with Stops En Route

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We consider an integrated routing and scheduling problem in hazardous materials transportation where accident rates, population exposure, and link durations on the network vary with time of day. We minimize risk (accident probability multiplied by exposure) subject to a constraint on the total duration of the trip. We allow for stopping at the nodes of the network. We consider four versions of this problem with increasingly more realistic constraints on driving and waiting periods, and propose pseudopolynomial dynamic programming algorithms for each version. We use a realistic example network to experiment with our algorithms and provide examples of the solutions they generate. The computational effort required for the algorithms is reasonable, making them good candidates for implementation in a decision-support system. Many of the routes generated by our models do not exhibit the circuitous behavior common in risk-minimizing routes. The en route stops allow us to take full advantage of the time-varying nature of accident probabilities and exposure and result in the generation of routes that are associated with much lower levels of risk than those where no waiting is allowed.

Key words: hazardous materials; routing and scheduling; dynamic programming

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1. Introduction

Transportation of hazardous materials (hazmats) is a problem that interests shippers, government agencies, insurance companies, and the public at large, primarily due to the possibility of accidents and the undesirable consequences associated with them. Although the number of hazmat accidents constitutes a very small percentage of all traffic accidents, the few accidents that result in a significant consequence (such as a spill or a fire) attract considerable attention in the national media. Public perception of risks associated with hazmats is influenced and amplified by the involuntary and the potentially catastrophic nature of hazmat accidents. There are a number of ways to reduce the risks associated with hazmat transport, such as improved driver training, frequent vehicle maintenance, and the building of sturdier and safer tanks. It is also possible to reduce risks through route planning, which allows operations research (OR) to make a contribution in this area. This paper develops methods that can reduce hazmat transport risks by combining routing and scheduling decisions for shipments.

Hazmat route planning has been a relatively popular area of research in OR. Surveys of the area can be found in List et al. (1991) and Erkut and Verter (1995). Most of the past research has focused on

selecting minimum risk paths for hazmat shipments. While there is no consensus on how the hazmat transport risk should be modeled, almost all authors have used either accident probabilities or an estimate of the consequences, or both. Accident probabilities are derived from historical truck accident frequencies, and they are usually in the order of 0.1 per million kilometers (km) of highway travel. Consequences are usually estimated using a count of the population to be impacted by an accident. Some authors multiply probabilities and consequences to model risk, while others pose the routing problem as a multicriteria problem where probabilities and consequences are treated as separate objectives, usually along with route length. Using a simplifying assumption on accident probabilities, all these route-selection problems are converted to shortest path problems, which facilitates their solution.

Most past research in this area uses static estimates for accident probabilities and consequences. This choice negates a need to solve a scheduling problem, because the route attributes do not depend on the timing of the trip. However, there is some empirical evidence that suggests accident probabilities are higher at night than during the day. Furthermore, it is clear that there are cyclic population movements (for example, from home to work) that are likely to impact

consequences. If we model probabilities and consequences as functions of time, then the scheduling problem becomes as important as the routing problem. In fact, the two are intertwined and must be tackled simultaneously. This observation was made first by Nozick, List, and Turnquist (1997), who proposed an algorithm to find efficient routing/scheduling combinations for hazmat trips. This paper improves on the applicability of integrated routing/scheduling decisions by generating schedules that allow stops along the way.

2. Motivating Example

We use a very simple example to motivate our problem. Consider a shipment on a route that consists of 10 links of equal length and equal traffic density. Suppose each link is 100 km in length and the hazmat vehicle travels at a constant speed of 100 kilometers per hour (km/hr). The time-dependent arc attributes are given in Table 1. For the purposes of this example, suppose that the consequence is measured in the number of vehicles on the road that would be impacted by a hazmat accident. That number is highly variable, with peaks during rush hours and a low during the early morning hours. In contrast, the accident probability is higher at night than during the day.

Assume that the accident probability and the consequence on a link are constant for each hour. If the trip

starts at time zero (midnight) and there is no waiting at the nodes, then the total accident probability and the average consequence for the path are 91×10^{-6} and 28.9 vehicles, respectively.

If stopping en route is allowed, then it is possible to reduce the consequence significantly. For example, starting the trip at midnight and stopping at Node 7 for 3 hours (to avoid the morning rush hour) results in an average consequence of 21.9—a 24% reduction over an uninterrupted trip. This stop also reduces the accident probability for the trip by a small amount. Extending the stop at Node 7 to 14 hours (or restarting the trip at 8:00 P.M. and ending it at 6:00 A.M.) reduces the average consequence by 47% while increasing the accident probability by 15%. Whether this is desirable or not depends on the way a user might model risk.

This example demonstrates that stopping en route may be desirable in some cases. Furthermore, stopping en route would be required if the trip duration exceeds a certain upper bound on driving time. Multiple stops may be necessary to complete a long trip.

3. Literature Review

Our problem can be characterized as a constrained shortest path problem with time-varying parameters where stopping en route is allowed. We minimize risk subject to a constraint on path duration where both arc attributes (risk and duration) are time dependent. Our model can be considered as a biobjective routing problem, because we can generate efficient solutions by varying the upper bound on path duration. Papers relevant to ours can be found in different areas of the OR literature: multiobjective routing problems and related constrained shortest path problems, time-varying shortest path problems, and hazmat-routing problems. In this section we review the most relevant papers in these areas.

Current and Marsh (1993) provide a review on multiple-objective shortest path problems. Warburton (1987) presents approximation algorithms for multi-objective shortest path problems, where the algorithm complexity is polynomial in terms of the approximation parameter.

Joksch (1966), Aneja and Nair (1978), Handler and Zang (1980), and Ribeiro and Minoux (1985) are among the studies that deal with shortest path problems with constraints. All use time-invariant arc attributes. Joksch (1966) first presents a linear programming (LP) formulation of the problem and then a dynamic programming algorithm to solve the problem. Aneja and Nair (1978) also present an LP formulation and offer a labeling algorithm. Handler and Zang (1980) present a dual algorithm for the constrained shortest path problem. Ribeiro and Minoux

Table 1 Exposure (in Vehicles) and Accident Probability (Per km) as a Function of Time of Day

From	To	Consequence (vehicles/accident)	Probability ($\times 10^{-9}$) per km
0:00	1:00	13	105
1:00	2:00	7	105
2:00	3:00	4	105
3:00	4:00	3	105
4:00	5:00	4	105
5:00	6:00	12	105
6:00	7:00	47	85
7:00	8:00	95	65
8:00	9:00	61	65
9:00	10:00	43	65
10:00	11:00	45	65
11:00	12:00	46	65
12:00	13:00	42	65
13:00	14:00	47	65
14:00	15:00	51	65
15:00	16:00	67	65
16:00	17:00	102	65
17:00	18:00	94	65
18:00	19:00	55	65
19:00	20:00	47	85
20:00	21:00	36	105
21:00	22:00	31	105
22:00	23:00	24	105
23:00	0:00	20	105

Note. We converted all per km values in Tables 1, 2, and 3 to per mile figures for the tests on the U.S. road network.

(1985) deal with the shortest path problem with a double-sided inequality constraint. The authors suggest a pseudopolynomial algorithm for solving the general parametric shortest path problem.

A basic version of the shortest path problem with resource constraints is considered by Desrosiers et al. (1995). Gamache et al. (1999) deal with an airline crew-scheduling problem where a subproblem turns out to be a constrained shortest path problem with reset variables. Our problem is different from these because of the time-dependent nature of arc attributes.

Cooke and Halsey (1966) and Halpern (1977) provide pioneering studies in shortest path problems with time-varying arc attributes. They deal with single-attribute objective functions. Halpern (1977) also considers limited waiting possibilities at nodes.

Perhaps the papers that are the most relevant to ours are Orda and Rom (1991), Cai, Kloks, and Wong (1997), and Nozick, List, and Turnquist (1997). We describe these papers at some length and discuss the similarities and differences between these papers and ours.

Orda and Rom (1991) consider shortest paths in time-dependent networks. Their objective is to minimize a single attribute (the length of the path) where the arc lengths vary with time. They consider three versions of the problem: (i) unlimited waiting is allowed at each node; (ii) no waiting is allowed; and (iii) waiting is only allowed at the origin. They show that the first and the third versions are relatively easy to solve (a polynomial algorithm can be devised). However, if waiting at nodes is not permitted, then the problem becomes NP-hard. They also point out that the exclusion of waiting in the problem may result in strange routing behavior, such as looping. (It is easy to design an example where a hazmat truck might loop in a rural area with low exposure to avoid driving through an urban area during rush hour if the departure time is fixed and no waiting en route is allowed.) Our model is more complicated than the one considered by Orda and Rom (1991), because we deal with two attributes and we impose constraints on waiting periods.

Cai, Kloks, and Wong (1997) also solve a time-varying shortest path problem. Their approach is similar to ours in the sense that they minimize the (time-dependent) length of the path subject to an upper bound on the total path duration. They present three variants of the model: (i) arbitrary waiting times at the nodes are possible; (ii) no waiting is allowed; and (iii) there are upper bounds on the waiting times at the nodes. The authors present pseudopolynomial algorithms for these problems. The simplest of the four problems we consider is similar to one of the problems described by Cai, Kloks, and Wong (1997),

and it can be solved by their algorithm. However, the rest of our problems are more complicated than theirs because of the constraints imposed on waiting and driving times, and our study can be considered as an extension of this paper.

There are a significant number of papers in hazmat routing. However, only one is closely related to our study, which we discuss next. For background on the hazmat-routing literature we refer the reader to the references provided in the first section. Nozick, List, and Turnquist (1997) consider an integrated routing and scheduling problem for hazmat transportation where the accident rates and the population exposed on the road network vary with the time of day. They propose a multiobjective routing algorithm based on time-varying parameters, where the objectives are the minimization of the route length, the total on-the-road population exposed, and the total accident probability.

This is a heuristic algorithm based on the exact multiobjective routing algorithm of Cox (1984) that works with time-invariant parameters. The authors find nondominated routes for a given departure time and repeat this for a set of possible departure times to construct a set of nondominated route/schedule combinations. They show that the time-invariant analysis computes the route exposures and probabilities incorrectly and classifies some routes incorrectly as dominated or efficient. The case study used provides a convincing argument that taking time-of-day variations into account provides a richer and more accurate decision-making environment than using average attribute values.

While we appreciate the richness gained by using time-variant attributes for integrated routing and scheduling, we believe that the methodology outlined by Nozick, List, and Turnquist (1997) does not take full advantage of the time-varying nature of the data. Although the authors consider different departure times from the origin, they do not allow for stopping en route. This may prevent the generation of certain desirable routes and identify dominated routes as efficient.

Suppose a vehicle approaches a major city before rush hour. If stopping is allowed, then the driver could take a break during the rush hour and drive through the city (or on the ring road around the city) afterward. However, if stopping is not allowed, then the driver would either drive through the city during rush hour and increase exposure, or take a detour and increase accident probability as well as route length. While allowing for stopping at intermediate nodes is likely to produce better solutions, the resulting problem is more complicated, because there is a new variable associated with every node representing the duration of the wait.

Although we add a new dimension to the integrated routing and scheduling problem by allowing for stops, our objective function is somewhat simpler than the one considered by Nozick, List, and Turnquist (1997). We minimize one arc attribute (which can be exposure, probability, or risk), subject to a constraint on the total time allowed for completion of the route, whereas Nozick, List, and Turnquist (1997) consider three objectives: probability, exposure, and length. Yet limiting the problem to a single objective allows us to find optimal solutions, while Nozick, List, and Turnquist (1997) resort to a heuristic to generate an efficient frontier.

4. Models and Algorithms

Consider a directed graph where nodes represent population centers and highway intersection points, and arcs represent highway segments. We consider a specific hazmat shipment on this network with a given origin and a destination node. All arc attributes—namely duration, accident probability, and population exposure—are time dependent. The arc attributes that apply to the tracing of an arc are determined by the entry time for the arc.

We consider two objectives: risk and duration. Although we use the expected consequence definition of risk, our algorithms would work equally well if one wanted to minimize the total accident probability or the total population exposure. Specifically, we compute the risk associated with an arc by multiplying the accident probability for that arc with the number of individuals who would be adversely impacted by an accident on that arc. While all arc attributes are time dependent, we assume that they are fixed once the vehicle starts traversing an arc, at values that are determined by the start time for the arc. For implementation purposes we assume that time is discretized into small units (such as five-minute periods).

We allow for the hazmat truck to stop and wait at each node. The intent of introducing a delay is to reduce the transport risk in arcs that will be traversed in the future. Stops between nodes are not allowed. (A rest stop along an arc can be modeled as a node in the network to allow for stopping.) Because waiting at nodes is permitted, our problem is a path-selection problem together with the determination of the departure times from each node on the selected path.

We wish to minimize trip risk and duration. Note that there is a trivial solution to the risk-minimization problem: Find a minimum risk route on a time-invariant network by using the minimum possible risk for each arc (this can be determined easily given the time-dependent risk function), and schedule the trip on this route by injecting sufficient waiting times at each node so that each arc is traversed when its

risk is the lowest. However, this route/schedule combination is unlikely to be practical, because the truck would probably travel for only a small portion of each day, so a trip may take many days.

Likewise, the minimum time route can be found easily using a shortest path algorithm. However, this route may go through major urban centers and may be associated with high risk. Rerouting the truck around certain areas or delaying the entrance times for certain arcs may increase the total path duration but may decrease the total risk. Hence, it is unlikely that the optimal solution to either single-objective problem will present a reasonable solution. We solve the bicriteria problem by minimizing risk subject to an upper bound on duration. Setting parameters on the upper bound allows us to generate a set of efficient solutions, which can be presented to a decision maker to help with the tradeoff between trip duration and risk.

It is clear that waiting at nodes may reduce transport risks. If the trip is sufficiently long, then waits are not only beneficial from a risk-minimization perspective, but they also are mandated by the authorities. For example, a trip that takes 20 hours cannot be completed without stopping (unless multiple drivers are used). Hence, we consider different scenarios with restrictions on waiting and driving times. In the simplest scenario, we disregard the needs of the driver and the regulations and set waiting times to minimize risk. In the most complex and the most realistic scenario, we consider typical transport department regulations. For example, according to U.S. Department of Transportation (DoT) regulations in effect during 1939–2004, the driver must be off duty for a minimum of 8 hours after driving for 10 hours or being “on duty” (includes driving and rest stops) for 15 hours (DoT 2004). In the following subsections we present our four scenarios and provide formal models and algorithms to the corresponding problems.

4.1. Unrestricted Waiting and Driving Times

First we consider the case where there are no restrictions on the waiting and driving times. The truck is allowed to wait for an arbitrary period of time at each node and to be on the road for an arbitrary period of time between stops. Even though this simplest case is not a very realistic setting for hazmat transport, we analyze it because it provides insight for more complicated formulations, and the results may be useful for benchmarking purposes.

4.1.1. Mixed-Integer Programming Formulation.

We need the following notation for this and the following sections:

$G(V, E)$: A directed graph where V is the set of nodes and E is the set of arcs,

N : Number of nodes,

m : Number of arcs,

$P(i)$: Set of predecessor nodes of node i ,
 $S(i)$: Set of successor nodes of node i ,
 $d_{ij}(t)$: Duration of arc (i, j) when the entry time is t ,
 $r_{ij}(t)$: Risk experienced on arc (i, j) when the entry time is t ,
 T : Upper bound on the total duration of the path,
 $X_{ijt} = 1$, if arc (i, j) is entered at time t ; 0, otherwise.
 Arbitrarily selecting Node 1 as the origin and Node N as the destination, we formulate the following linear binary programming problem.

MP-I:

$$\min \sum_{t=1}^T \sum_{(i,j) \in E} r_{ij}(t) \cdot X_{ijt}$$

$$\text{s.t. } \sum_{t=1}^T \sum_{i \in S(1)} X_{i1t} = 1 \tag{1}$$

$$\sum_{t=1}^T \sum_{j \in S(i)} X_{ijt} - \sum_{t=1}^T \sum_{j \in P(i)} X_{jit} = 0$$

$$i = 2, \dots, N - 1 \tag{2}$$

$$\sum_{t=1}^T \sum_{i \in P(N)} X_{iNt} = 1 \tag{3}$$

$$\sum_{t=1}^T \sum_{j \in P(i)} X_{jit}(t + d_{ji}(t)) \leq \sum_{t=1}^T \sum_{j \in S(i)} X_{ijt} \cdot t$$

$$i = 2, \dots, N - 1 \tag{4}$$

$$\sum_{t=1}^T \sum_{i \in P(N)} x_{iNt}(t + d_{iN}(t)) \leq T \tag{5}$$

$$X_{ijt} = \{0, 1\} \quad \forall i, j, t. \tag{6}$$

In MP-I, constraints (1)–(3) are flow-conservation constraints. Constraint (4) ensures that the departure time from a node is greater than or equal to the departure time from the preceding node plus the duration of the arc in between. Constraint (5) satisfies the ending condition of the path at time T . This formulation has Tm binary variables and $2N - 1$ constraints. Note that even for moderate-size problems, the number of variables is very large, making the solution using an off-the-shelf solver impractical. This observation applies to the formulations of the subsequent models as well, and we will resort to dynamic programming for solutions of all four problems.

4.1.2. DP Formulation. Using the following additional notation, this problem can also be formulated as a dynamic programming (DP) problem. This formulation is due to Cai, Kloks, and Wong (1997).

Let

$f_1(y, t)$: total risk of the minimum-risk path from the origin to node y of duration t or less, when there are no restrictions on waiting and driving times,
 u_d : departure time from a node.

DP-I:

$$f_1(1, 0) = 0, \quad f_1(y, t) = \infty \quad \forall y = 2, \dots, N; t = 1, \dots, T$$

$$f_1(y, t) = \min \{f_1(y, t - 1),$$

$$\min_{\{x: (x,y) \in E\}} \min_{\{u_d: u_d + d_{xy}(u_d) = t\}} \{f_1(x, u_d) + r_{xy}(u_d)\}$$

$$y = 1, 2, \dots, N \text{ and } t = 2, \dots, T.$$

PROPOSITION 1. DP-I finds the optimal minimum-risk path when there exist no restrictions on waiting times and driving times.

PROOF. The proof of this proposition follows from Lemma 1 and Corollary 1 of Cai, Kloks, and Wong (1997). Hence, it is omitted here. \square

Cai, Kloks, and Wong (1997) suggest a solution algorithm for this model and show that this formulation can be solved in $O(T(N + m))$ time. The optimal path risk is given by $f_1(N, T)$, and the optimal path can be found by backtracking.

4.2. Restricted Waiting and Unrestricted Driving Times

In the second problem we consider, there are no restrictions on driving times, but the waiting time at a node must be either zero or between an upper and a lower bound. The lower bound is there to make sure the driver can get a minimum amount of rest, and the upper bound prevents excessively long stops. Reasonable lower and upper bounds might be one hour and eight hours, respectively.

4.2.1. Mixed-Integer Programming Formulation.

We need the following additional notation for the formulation of this case:

L_i : lower bound on waiting at node i , if the waiting time is positive,

U_i : upper bound on waiting at node i , if the waiting time is positive,

a_i : arrival time at node i (if $a_i = 0$, then node i is not visited),

p_i : departure time from node i (if $p_i = 0$, then node i is not visited),

$w_i = 1$, if waiting occurs at node i ; 0, otherwise.

MP-II:

$$\min \sum_{t=1}^T \sum_{(i,j) \in E} r_{ij}(t) \cdot X_{ijt}$$

$$\text{s.t. } (1), (2), (3), (5), (6)$$

$$\sum_{t=1}^T \sum_{j \in S(i)} X_{ijt} \leq 1 \quad i = 1, 2, \dots, N - 1 \tag{7}$$

$$a_i = \sum_{t=1}^T \sum_{j \in P(i)} X_{jit} \cdot (t + d_{ji}(t)) \quad \forall i \tag{8}$$

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$$p_i = \sum_{t=1}^T \sum_{j \in S(i)} X_{ijt} \cdot t \quad \forall i \quad (9)$$

$$L_i \cdot w_i \leq p_i - a_i \leq U_i \cdot w_i \quad \forall i \quad (10)$$

$$w_i = \{0, 1\} \quad \forall i. \quad (11)$$

Constraint (7) assures construction of elementary paths. Constraints (8) and (9) directly follow from constraint (4) in MP-I. Constraint (10) imposes the upper and lower bounds on the waiting time or forces it to be equal to zero. Note that constraints (8)–(10) can be merged into a single constraint set, negating the need to define a_i and p_i . However, we choose to provide this detail for ease of exposition.

4.2.2. DP Formulation. We introduce the following additional notation for the DP formulation:

$f_{II}(y, t)$: total risk of the minimum risk path from origin to node y with a path duration exactly of t and with waiting time zero at node y , subject to the constraint that the waiting time at any node x on the path is either 0 or between L_x and U_x . If the path is infeasible for the current value of t then $f_{II} = \infty$.

$f_{II}^*(y)$: total risk of the minimum risk path from the origin to node y ,

u_a : arrival time at a node.

DP-II:

$$f_{II}^*(N) = \min_{t \leq T} f_{II}(N, t) \quad \text{with} \\ f_{II}(1, t) = 0, \quad t = 0, \dots, T \quad (12)$$

$$f_{II}(y, t) = \min_{\{x: (x, y) \in E\}} \min_{\{(u_a, u_d) \in F(x, y, t)\}} \{f_{II}(x, u_a) + r_{xy}(u_d)\}, \quad (13)$$

where

$$F(x, y, t) = \{(u_a, u_d): u_d + d_{xy}(u_d) = t \quad \text{and} \\ (u_d = u_a \text{ or } L_x \leq u_d - u_a \leq U_x)\}, \\ y = 2, \dots, N \text{ and } t = 1, \dots, T.$$

PROPOSITION 2. *DP-II finds the optimal minimum-risk path when there exist simple restrictions on waiting times and no restrictions on driving times.*

PROOF. By the definition of $f_{II}(y, t)$, the t value ($t = 1, \dots, T$) that results in the minimal $f_{II}(N, t)$ gives the total risk of the minimum-risk path from the origin to the destination. The proof of the conjecture that (12) and (13) can be used to calculate $f_{II}(N, t)$ follows from Lemma 6 in Cai, Kloks, and Wong (1997) and is omitted here. \square

Our algorithm (Erkut and Alp 2005) for DP-II is similar to our algorithm for DP-I, and it has computational complexity of $O(T(m + N \log T))$.

4.3. Restricted Waiting and Driving Times

In this version of the problem, we impose an upper bound on the driving times between stops in addition to the constraints of §4.2. The upper bound on the

uninterrupted driving time (for example, eight hours) prevents unreasonably long stretches without a break.

4.3.1. Nonlinear Mixed-Integer Programming Formulation. We need the following additional notation:

D : maximum uninterrupted driving time permissible,

v_i : uninterrupted driving time on arrival at node i .

MP-III:

$$\min \sum_{t=1}^T \sum_{(i, j) \in E} r_{ij}(t) \cdot X_{ijt}$$

s.t. (1)–(3), (5)–(11)

$$v_i = \sum_{j \in P(i)} \left\{ v_j \cdot \left(\sum_{t=1}^T X_{jit} \cdot (1 - w_i) \right) + \sum_{t=1}^T X_{jit} \cdot d_{ji}(t) \right\} \quad \forall i \quad (14)$$

$$0 \leq v_i \leq D \quad \forall i. \quad (15)$$

Constraint (14) calculates the uninterrupted driving time according to the value of w_i , and constraint (15) imposes the bounds of the consecutive driving times. Note that constraint (14) is nonlinear, further complicating the model.

4.3.2. DP Formulation. We need the following additional notation for the DP formulation:

$f_{III}(y, t, v)$: total risk of the minimum risk path from the origin to node y with a path duration of t , last uninterrupted driving time of v , and with waiting time at node y of zero, subject to the constraints that the waiting time at any vertex x on the path is either 0 or between L_x and U_x , and the uninterrupted driving time is no more than D . If the path is infeasible with the current values of t and v , then f_{III} is set to ∞ ;

$f_{III}^*(y)$: total risk of the minimum risk path from origin to node y ,

u_r : uninterrupted driving time on arrival at a node.

DP-III:

$$f_{III}^*(N) = \min_{\substack{t \leq T \\ v \leq D}} f_{III}(N, t, v) \quad \text{with}$$

$$f_{III}(1, t, 0) = 0 \quad \forall t,$$

$$f_{III}(1, t, v) = \infty \quad \forall v > 0; t = 1, \dots, T$$

$$f_{III}(y, t, v)$$

$$= \min_{\{x: (x, y) \in E\}} \min_{\{(u_a, u_d, u_r) \in F(x, y, t, v)\}} \{f_{III}(x, u_a, u_r) + r_{xy}(u_d)\}, \quad (16)$$

where

$$F(x, y, t, v) = \{(u_a, u_d, u_r): u_d + d_{xy}(u_d) = t; u_a = u_d \text{ and} \\ u_r = v - d_{xy}(u_d) \text{ or } L_x \leq u_d - u_a \leq U_x, \\ d_{xy}(u_d) = v, \text{ and } 0 \leq u_r \leq D\} \\ 2 \leq y \leq N, 1 \leq t \leq T, 1 \leq v \leq D. \quad (17)$$

PROPOSITION 3. *DP-III finds the optimal minimum-risk path in the presence of simple restrictions on waiting and driving times.*

PROOF. See Erkut and Alp (2005). \square

In DP-III, uninterrupted driving times are tracked as an additional resource on the minimum-risk path selected. Note that DP-III uses forward recursion and requires the computation of $f_{III}(x, u_a, u_r)$ for all $u_a < t$, $1 \leq u_r \leq D$, and $x \in V$ prior to computing $f_{III}(y, t, v)$.

We treat the break and no-break cases separately. If there is no break at node x , then we consider all departure times u_d on node x that satisfy $u_d + d_{xy}(u_d) = t$. Moreover, the arrival time at node x , u_a , must be equal to u_d because break time ($u_d - u_a$) is zero. For the same reason, the accumulated resource on uninterrupted driving time on reaching node x , u_r , is not reset and therefore must satisfy $u_r = v - d_{xy}(u_d)$ to maintain feasibility.

On the other hand, if the driver takes a break at node x , then we consider all departure times u_d on node x that satisfy $u_d + d_{xy}(u_d) = t$ and $d_{xy}(u_d) = v$. Moreover, in this case, any arrival time u_a satisfying $u_d - L_x \leq u_a \leq u_d - U_x$ will yield a feasible break time. For each of these u_a values, the uninterrupted driving time on arrival at node x , u_r , may take on any value between 1 and D , as this resource is reset when a break is taken. Set $F(x, y, t, v)$ in (17) stores all such feasible (u_a, u_d, u_r) vectors for each predecessor node x so that all feasible paths reaching node y at time t with an uninterrupted driving time v can be evaluated and compared.

In the appendix we offer an algorithm for DP-III that utilizes binary heap data structures effectively.

PROPOSITION 4. *Model DP-III can be implemented in $O(TD(m + N) + TN \log T)$ time.*

PROOF. See appendix. \square

We make two observations that result in an efficient implementation of the algorithm.

(1) For given (y, t) , the total risk function $f_{III}(y, t, v)$ has a finite value only for a small subset of all possible v values. Limiting the function evaluation to the subset of v values that result in finite function evaluations in the previous iteration of the forward DP algorithm reduces the computational effort considerably.

(2) We observe that if $f_{III}(y, t, v) \leq f_{III}(y, t, v')$ for $v < v'$, then $f_{III}(y, t, v')$ is dominated by $f_{III}(y, t, v)$ and can be eliminated. (Lower uninterrupted driving times are preferred to higher ones.) Not storing the dominated solutions during the implementation of the algorithm saves considerable memory and speeds up the algorithm.

4.4. Complex Restrictions on Waiting and Driving Times

Finally we present the most realistic version of the problem that imposes the U.S. DoT regulations on

the trip schedule. In our computational experiment we used the regulations that were in effect in 2004: Drivers must be off duty for a minimum of 8 hours following 15 hours of duty or 10 hours of uninterrupted driving. We note that these regulations have been changed recently. Now the drivers must be off duty for a minimum of 10 hours following 14 hours of duty or 11 hours of uninterrupted driving. The new regulations are under appeal and they may be revised again in 2005 (DoT 2004). We note that such minor changes in the regulations would not affect the structure of our formulations or our algorithms.

We do not formulate this case as a mathematical programming model because it would be considerably more complicated and less tractable than the nonlinear model of §4.3.1. However, we present a tractable dynamic programming formulation for this problem after presenting the following definitions. We use the term “short break” to refer to waiting at a certain node mainly for the purpose of delaying the entrance to the arcs ahead for risk-minimization purposes. The driver is considered to be on duty during this break. For example, these breaks could be one or two hours long—similar to a lunch or dinner break. In contrast, the waiting at a node for the purpose of a long rest is called a “long break.” The lower bound on this type of break is eight hours. A reasonable upper bound might be 10 or 12 hours. The driver is considered to be off duty during this break. The term “on duty period” refers to the total duration of driving and short break times between two long breaks. The on-duty period cannot last more than 15 hours.

4.4.1. Dynamic Programming Formulation. The following additional notation is needed for this case:

W : maximum length of the on duty period,

$f_{IV}(y, t, v, w)$: risk value of the minimum risk path from the origin to node y , with a path duration of exactly t , uninterrupted driving time of v , duration of the current on duty period of w , and with waiting time at node y of zero, subject to the constraints that the length of a long break taken at any vertex x on the path is between L_x and U_x , the length of a short break taken at any vertex x on the path is at least l_x , the length of uninterrupted driving time is no more than D , and the length of on duty period does not exceed W . If the path is infeasible with the current values of t , v , and w , then f_{IV} is set to ∞ ,

$f_{IV}^*(y)$: total risk of the minimum risk path from the origin to node y ,

u_w : length of the on duty period on arrival at a node,

u_{sb} : duration of a short break taken at a node,

u_{lb} : duration of a long break taken at a node,

l_i : lower bound on the short break given at node i .

DP-IV:

$$f_{IV}^*(N) = \min_{\substack{t \leq T \\ v \leq D \\ w \leq W}} f_{IV}(N, t, v, w) \quad \text{with}$$

$$f_{IV}(1, t, 0, 0) = 0, \quad \forall t,$$

$$f_{IV}(1, t, v, w) = \infty \quad \forall v > 0; w > 0$$

$$f_{IV}(y, t, v, w)$$

$$= \min_{\{x: (x, y) \in E\}} \min_{\{(u_a, u_d, u_r, u_w, u_{sb}, u_{lb}) \in F(x, y, t, v, w)\}} \{f_{IV}(x, u_a, u_r, u_w) + r_{xy}(u_d)\} \quad (18)$$

where

$$F(x, y, t, v, w)$$

$$= \{(u_a, u_d, u_r, u_w, u_{sb}, u_{lb}):$$

$$(u_{lb} = 0; u_{sb} = 0; u_d: u_d + d_{xy}(u_d) = t,$$

$$u_a = u_d; u_r = v - d_{xy}(u_d); u_w = w - d_{xy}(u_d)) \quad (19)$$

or

$$(u_{lb} = 0; l_x \leq u_{sb} \leq W; u_d: u_d + d_{xy}(u_d) = t \text{ and}$$

$$d_{xy}(u_d) = v, u_a = u_d - u_{sb}; 0 \leq u_r \leq D;$$

$$u_w = w - u_{sb} - d_{xy}(u_d)) \quad (20)$$

or

$$(L_x \leq u_{lb} \leq U_x; u_{sb} = 0; u_d: u_d + d_{xy}(u_d) = t \text{ and}$$

$$d_{xy}(u_d) = v \text{ and } d_{xy}(u_d) = w; u_a = u_d - u_{lb};$$

$$0 \leq u_r \leq D; 0 \leq u_w \leq W)\} \quad (21)$$

$$2 \leq y \leq N, 1 \leq t \leq T, 1 \leq v \leq D, 1 \leq w \leq W.$$

PROPOSITION 5. *DP-IV finds the optimal minimum-risk path in the presence of complex restrictions on waiting and driving times.*

PROOF. The proof of this proposition is omitted, as it is similar to the proof of Proposition 3. \square

In DP-IV, uninterrupted driving and working times are tracked as resources on the nodes of the minimum-risk path selected. In contrast to DP-III, short and long breaks must be defined as decision variables because different types of breaks have different implications. Prior to computing $f_{IV}(y, t, v, w)$ for a given state combination (y, t, v, w) , subpaths reaching node y from each predecessor node x must be evaluated and compared for all feasible values of arrival time u_a , departure time u_d , uninterrupted driving time u_r , uninterrupted working time u_w , short break time u_{sb} , and long break time u_{lb} on node x . Set $F(x, y, t, v, w)$ in (19) stores all such feasible $(u_a, u_d, u_r, u_w, u_{sb}, u_{lb})$ vectors.

If there is a short break given at node x , then the accumulated resource on uninterrupted driving time is reset on reaching node x , but the one on the uninterrupted working time is not. In such a case, a feasible

subpath reaching node y at a state (y, t, v, w) can be constructed from subpaths reaching node x with all $(u_a, u_d, u_r, u_w, u_{sb}, u_{lb})$ vectors satisfying (21).

Similarly, if there is a long break given at node x , the accumulated resources on uninterrupted driving and working times are reset. In such a case, a feasible subpath reaching node y at a state (y, t, v, w) can be constructed from subpaths reaching node x with all $(u_a, u_d, u_r, u_w, u_{sb}, u_{lb})$ vectors satisfying (21). Finally, if there is no break at node x , then none of the accumulated resources are reset. In this case, a feasible subpath reaching node y at a state (y, t, v, w) can be constructed from subpaths reaching node x with all $(u_a, u_d, u_r, u_w, u_{sb}, u_{lb})$ vectors satisfying (20). In the appendix we offer a sketch of the algorithm for DP-IV that utilizes binary heap data structures. The full algorithm is available in Erkut and Alp (2005).

PROPOSITION 6. *Model DP-IV can be implemented in $O(TDW(m + N) + TN \log T)$ time.*

PROOF. See Erkut and Alp (2005). \square

In implementing this algorithm, we take advantage of the two observations made in §4.3.2. Dominance rules for DP-IV are similar to those for DP-III: $f_{IV}(y, t, v', w)$ is a dominated solution if $f_{IV}(y, t, v, w) \leq f_{IV}(y, t, v', w)$ for $v < v'$, and $f_{IV}(y, t, v, w')$ is also a dominated solution if $f_{IV}(y, t, v, w) \leq f_{IV}(y, t, v, w')$ for $w < w'$.

5. Computational Experience

The goal of our computational experiment is threefold: to demonstrate the viability of the proposed algorithms, to produce some realistic numerical examples, and to compare the solutions produced for the different versions of the problem. We use the northeastern U.S. interstate highway network from Nozick, List, and Turnquist (1997) and consider a hypothetical shipment between Wilmington, Delaware, and Portland, Maine. This network has 138 nodes and 368 arcs. Each arc has three attributes: arc length, time-dependent travel duration, and time-dependent travel risk. Risk is defined as the time-dependent accident probability multiplied by the time-dependent exposure. Exposure is defined as exposure to other drivers on the road. Hence, the hypothetical shipment is assumed to be one that may create a small fire or explosion with consequences limited to the road.

We use five-minute time intervals. The distance between the origin and the destination in our network is too short to demonstrate some of the differences between our models—it is possible to go from the origin to the destination in one working day. Hence, we multiply the lengths of all arcs by a factor of two. Although this reduces the realism in the results somewhat, it provides us with a better comparison of our models. The parameters used for the time-dependent arc attributes are summarized in Tables 2 and 3.

Table 2 Accident Release Rates

Highway category	Release accidents per million vehicle km	
	Day (7 A.M.–6 P.M.)	Night (6 P.M.–7 A.M.)
Urban freeway	0.065	0.104
Rural highway	0.028	0.044

5.1. The Minimum-Risk Path and the Shortest Path

Figure 1 displays the minimum-risk path found using average attribute values for each link and ignoring the time dependency. Note that the minimum-risk path is rather circuitous; it avoids the more direct route that goes through the eastern part of the network, where population exposures are higher. The minimum-risk path is 61% longer than the shortest path, which is very undesirable from a shipper’s perspective. Later we demonstrate that it is possible to find low-risk paths that are much shorter if waiting is allowed en route.

Figure 2 displays the shortest path between Wilmington and Portland. Using the methodology developed by Nozick, List, and Turnquist (1997), we find that a departure time of 10:00 P.M. results in a minimum risk value of 397.6×10^{-6} on the shortest path. The duration of this trip with no waiting is 15.75 hours. Figure 3 displays the solution of DP-I for an upper bound on the trip duration of 30 hours. The resulting risk is 226.0×10^{-6} —a 43% reduction over



Figure 1 The Risk-Minimizing Path with Average Attribute Values

Notes. Path length = 1,462 miles; path duration = 1,468 minutes; path risk = 523.3×10^{-6} .

the no-wait solution. Although the solution to DP-I results in a large decrease in the path risk, this is not a desirable solution, because it contains too many stops—some very short (five minutes). Figure 4 displays the solution to DP-II, where a minimum wait of 30 minutes (a reasonable coffee break) has been imposed on the solution. The risk associated with this solution is virtually identical to the risk associated with the solution of DP-I. The resulting schedule is reasonably realistic, with only three stops along the way. In fact, this schedule satisfies the restrictions of our Models III and IV.

5.2. Experience with the Four Models

5.2.1. Unrestricted Waiting and Driving Times.

We solve DP-I for a maximum trip duration of three days (4,320 minutes consisting of $T = 864$ five-minute

Table 3 Exposure Values (Per km)

Hour of day	Urban highway	Rural highway
0	12.04	2.51
1	6.52	1.46
2	3.51	1.04
3	2.51	1.04
4	3.51	1.46
5	11.04	3.76
6	45.15	10.75
7	138.45	14.30
8	62.70	12.73
9	40.13	13.05
10	42.64	14.30
11	43.64	15.03
12	39.13	15.03
13	44.65	15.45
14	49.66	17.54
15	70.73	19.62
16	252.32	21.40
17	133.94	18.79
18	54.68	13.99
19	44.65	10.75
20	33.11	8.45
21	28.59	7.52
22	22.07	6.26
23	18.56	4.49



Figure 2 The Shortest Path (Path Length = 892 Miles)

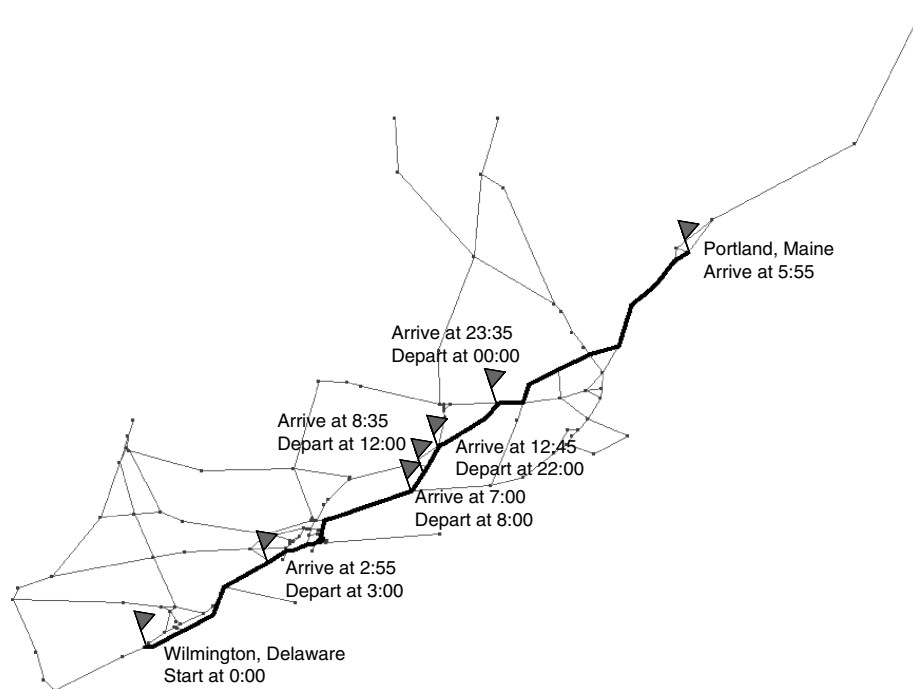


Figure 3 Solution of DP-I on Shortest Path Between Wilmington and Portland for Maximum Trip Duration of 30 Hours

Notes. The stops are indicated by flags. The solution corresponds to the following drive-wait sequence: (2:55, 0:05), (4:00, 1:00), (0:30, 3:30), (0:45, 9:15), (1:35, 0:25), (5:25, —), with a midnight departure. The total driving time is 15:10, and the total waiting time is 14:15. The risk associated with this schedule is only 57% of the risk associated with the minimum-risk, no-wait schedule on this path.

intervals). The objective is to find minimum-risk paths subject to the constraint that the path duration is no more than t , for all $t \leq 4,320$. Then, we extract the efficient paths from this solution pool. We find a total of 171 efficient solutions, with durations varying from 945–4,320 minutes. These solutions occur on 19 different paths with path lengths varying from 894 miles to 1,284 miles.

Figure 5 displays the efficient frontier with respect to risk and duration. Note that the risk reduction is very dramatic as the trip duration increases from 945 minutes to 980 minutes, gradual between 1,000 and 1,800 minutes, and very little after 1,800 minutes. Figure 6 provides four of the efficient solutions with total durations of 980, 1,470, 1,680, and 3,225 minutes. As more time is allowed for the trip, the algorithm takes advantage of the slack to avoid high-risk travel periods, and reduces risk.

In contrast, Figure 7 provides an efficient solution for the no-wait model, with trip duration of 1,690 minutes. Note that one of the four efficient solutions displayed in Figure 6 has comparable trip duration, yet it is 37% shorter and 26% less risky—further evidence that a schedule incorporating waiting can be superior to a no-wait schedule on both criteria: length and risk.

5.2.2. Restricted Waiting Times. The only difference between DP-I and DP-II is the bounds imposed

on the waiting times. In the interest of space, we display no solutions to DP-II in addition to the one displayed in Figure 4. While the models are very similar, the imposition of the bounds increases the computational effort significantly. The average computational time for a given departure time goes from three seconds to eight seconds on a 750-MHz Sun Blade 1000 computer as we go from DP-I to DP-II.

5.2.3. Restricted Driving Times. We solve DP-III for maximum trip duration of 4,320 minutes. We set the maximum uninterrupted driving time as 10 hours. We find a total of 116 efficient solutions with durations varying from 950 minutes to 4,320 minutes. These solutions occur on 11 different paths, with path lengths varying from 894 miles to 1,208 miles. Figure 8 displays the efficient frontier with respect to risk and duration, and Figure 9 provides four of the efficient solutions generated for T values of 1,780, 2,185, 2,770, and 3,070 minutes. The average computational effort for DP-III is 3.3 seconds for a given departure time.

5.2.4. Complex Restrictions on Waiting and Driving Times. We solve DP-IV for a maximum trip duration of 4,320 minutes. We set the maximum uninterrupted driving and working times to 10 and 15 hours, respectively. We find a total of 92 efficient solutions with durations ranging from 1,430 minutes to 4,320 minutes. These solutions occur on 10 different paths, with path lengths varying from 894 miles

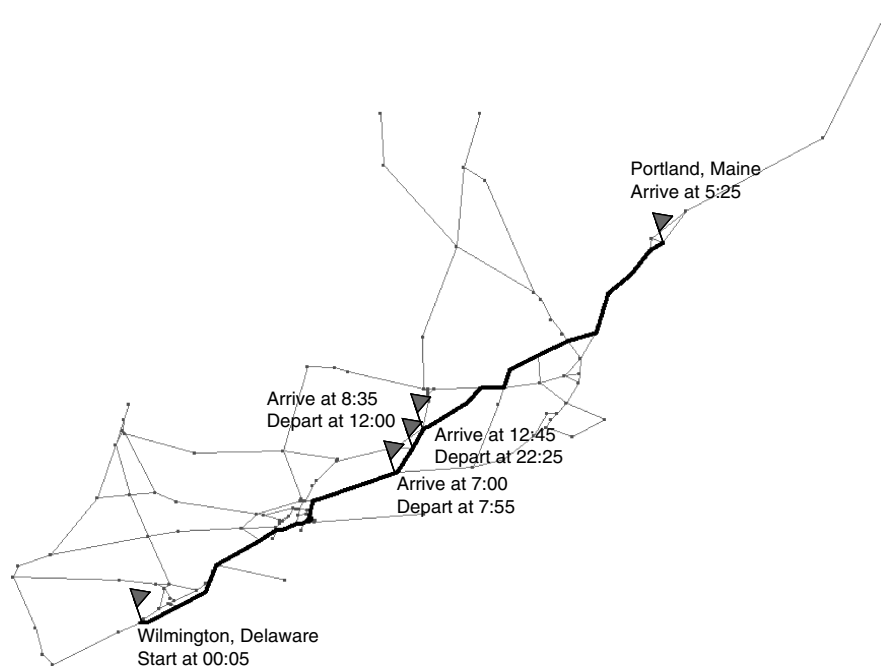


Figure 4 Solution of DP-II on Shortest Path Between Wilmington and Portland for Maximum Trip Duration of 30 Hours

Notes. The stops are indicated by flags. The solution corresponds to the following drive-wait sequence: (6:55, 0:55), (0:40, 3:25), (0:45, 9:35), (7:00, —). The total driving time is 15:20, and the total waiting time is 13:55. The risk associated with this schedule is virtually identical to the risk associated with the solution of DP-I displayed in Figure 3.

to 1,208 miles. Figure 10 displays the efficient frontier with respect to risk and duration, and Figure 11 provides four of the efficient solutions generated: for T values of 1,430, 1,555, 2,845, and 4,165 minutes. The average computational effort for DP-IV is 12.1 minutes per departure time.

We note that all efficient solutions displayed in Figures 6, 7, 9, and 11 have departure times of 12:00 A.M. to facilitate comparison. However, the efficient solution sets contain many other departure times.

6. Concluding Remarks

In this paper we consider the hazmat routing and scheduling problems simultaneously and extend existing methodology by allowing for stops along the route. We consider time-varying link attributes (accident, exposure, and duration) and solve a risk-minimization problem subject to a constraint on the path duration. We study four different versions of the problem with increased restrictions on driving and stopping times, which increases the realism, as

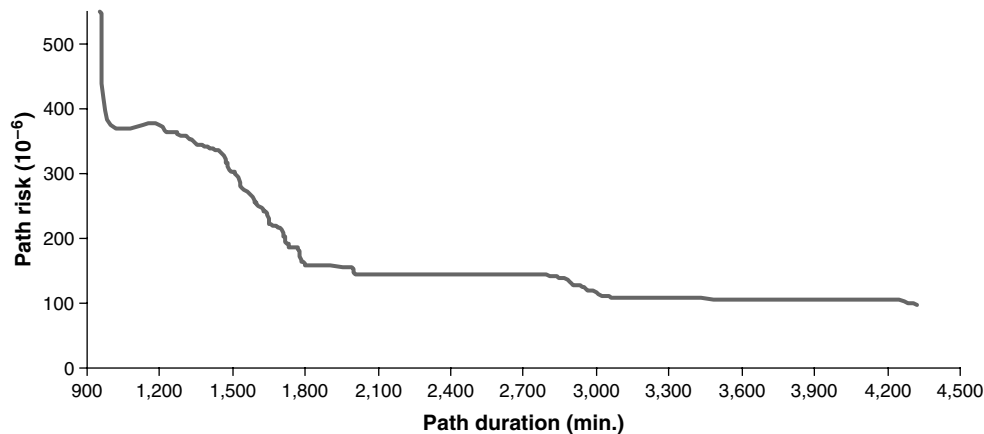
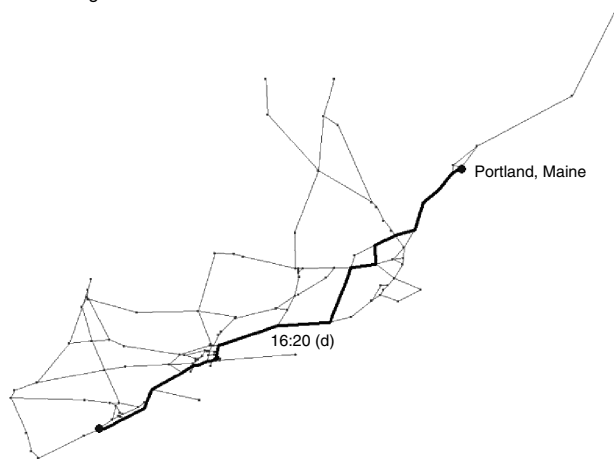


Figure 5 Efficient Frontier for DP-I with Respect to Two of Three Objectives: Path Duration and Path Risk

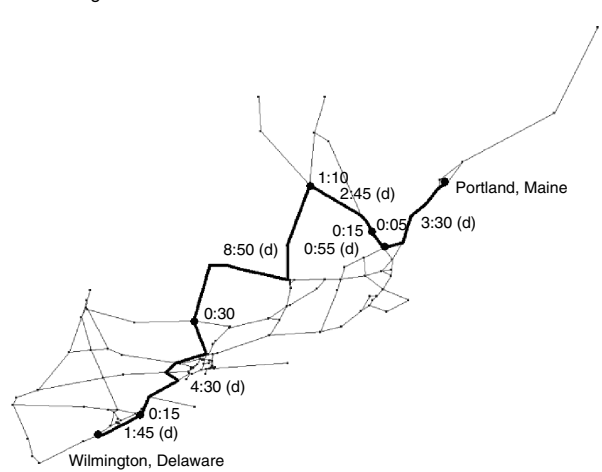
Notes. The apparent increase in the path risk between path durations of 900 and 1,400 miles is due to paths that are efficient with respect to the third objective: path length.

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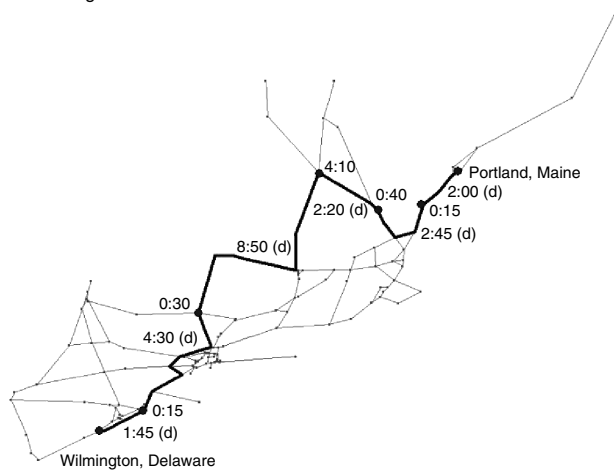
(a) Duration = 980 minutes, risk = 382.3×10^{-6} , length = 930 miles, waiting time = 0 minutes



(b) Duration = 1,470 minutes, risk = 317.5×10^{-6} , length = 1,284 miles, waiting time = 135 minutes



(c) Duration = 1,680 minutes, risk = 218.4×10^{-6} , length = 1,284 miles, waiting time = 350 minutes



(d) Duration = 3,235 minutes, risk = 109.3×10^{-6} , length = 984 miles, waiting time = 2,200 minutes

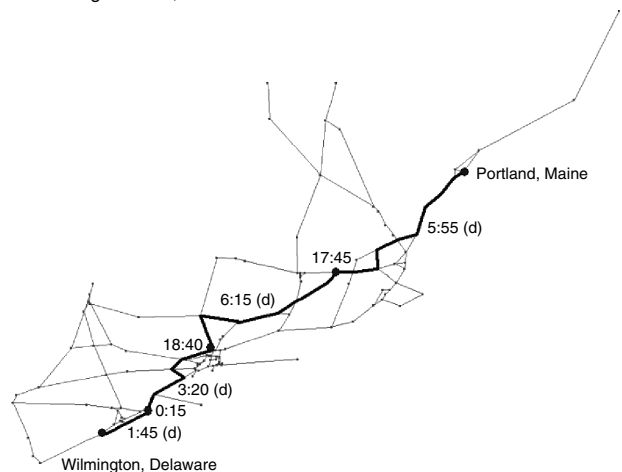


Figure 6 Four Efficient Solutions to DP-I for Increasing Values of Trip Duration (and Decreasing Risk)

Notes. Driving times are indicated with “(d).” Stop times are indicated by circles along the path, and waiting times are indicated next to the circles.

well as the complexity, of the problem. We develop pseudopolynomial implicit enumeration algorithms to generate a subset of the efficient frontier by varying the path duration. Our computational experience indicates that our dynamic programming algorithms can solve all four problems considered with reasonable computational effort for a realistic network. However, developing heuristic algorithms may make sense in case of significantly larger networks.

Nozick, List, and Turnquist (1997) study the same problem with no stops and report that some efficient routes are as much as 66% longer than the shortest path. We believe that their algorithm is forced into such solutions, as waiting is not allowed en route. To avoid entering a high-exposure area of the network during rush hour, the hazmat truck is sent on circuitous alternate routes. In contrast, in our model the truck simply stops and waits for the rush hour to

pass. Consequently, most of our routes are relatively close in length to the shortest path.

We assume that the probability of an accident is zero when the vehicle is not moving. This may not be true. For example, another vehicle may strike a parked hazmat truck and cause a release of the contents. Consideration of nonzero accident probabilities during stops would reduce the incentive for the vehicle to stop. However, we believe that accident probabilities, as well as consequences during a stop, would be considerably lower than their counterparts while the vehicle is moving. Hence, it would still be possible to reduce the overall risk by stopping en route. The data structures we use can easily accommodate a discrete set of nonzero accident probabilities for stops—for example, a smaller probability for short stops and a larger probability for longer stops. Furthermore, our algorithms can readily accommodate node-dependent

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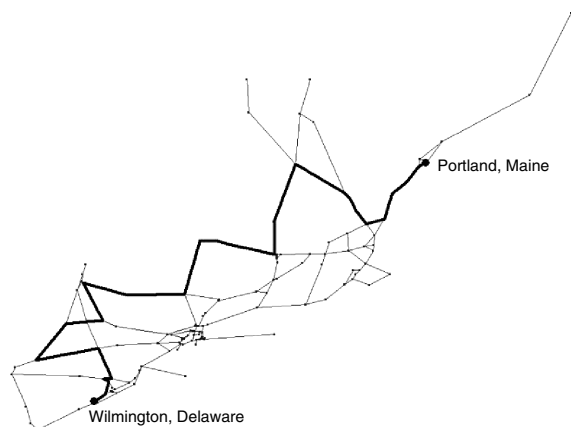


Figure 7 Efficient Solution to the No-Wait Model for Trip Duration of 1,690 Minutes

Notes. Though efficient among no-wait solutions, this solution is dominated by solutions to DP-I. The total risk on this path is 263.2×10^{-6} .

accident probabilities for stops. However, the incorporation of accident probabilities that are proportional to the duration of the stop would result in slightly increased complexity for the algorithms.

We observed that many solutions to DP-I have very short stops (5–10 minutes). This is merely because we use discrete link data. If a truck arrives at a Node 5 minutes before 8:00 A.M., it can stop for five minutes and enjoy a steep drop in the exposure for the start of its trip on the next link. If continuous data are used, such unreasonable waits are much less likely to occur. If it is not possible to get continuous data, one could fit a curve to the existing discrete data. Another way to eliminate such unreasonably short stops is to use a higher lower bound on the duration of a stop (i.e., 15 minutes instead of 5 minutes). This would reduce the computational effort.

In our computational implementation we assumed that it was possible to stop at every node. If stopping is allowed only at a subset of the nodes (for example only at full-service truck stops), then the number of efficient solutions, as well as the computational effort,

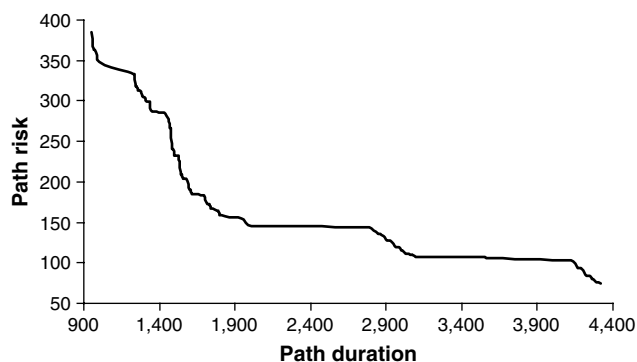


Figure 8 Efficient Frontier with Respect to Two of the Three Objectives: Path Duration and Path Risk for DP-III

would go down. Hence, the computational times we report can be considered worst-case times for the given network size.

All of our models rely on the driver traveling consistently at a certain speed. If the driver goes faster or slower than anticipated, he or she will not cross the links at the planned time intervals and may incur risks that are quite different from those computed. Although we can expect a professional hazmat truck driver to follow a trip plan fairly closely, late or early arrivals at network nodes do not invalidate the models. Given the location of the truck along the route, the problem can be resolved in real time to provide the driver with a trip plan update. In fact, this can be very useful in cases where estimated accident or exposure figures deviate significantly from the expected—for example, in the case of inclement weather or heavy traffic due to a sports event. Provided it is possible to link different weather and road conditions to accident probabilities, the problem can be resolved every time there is a change in a link attribute.

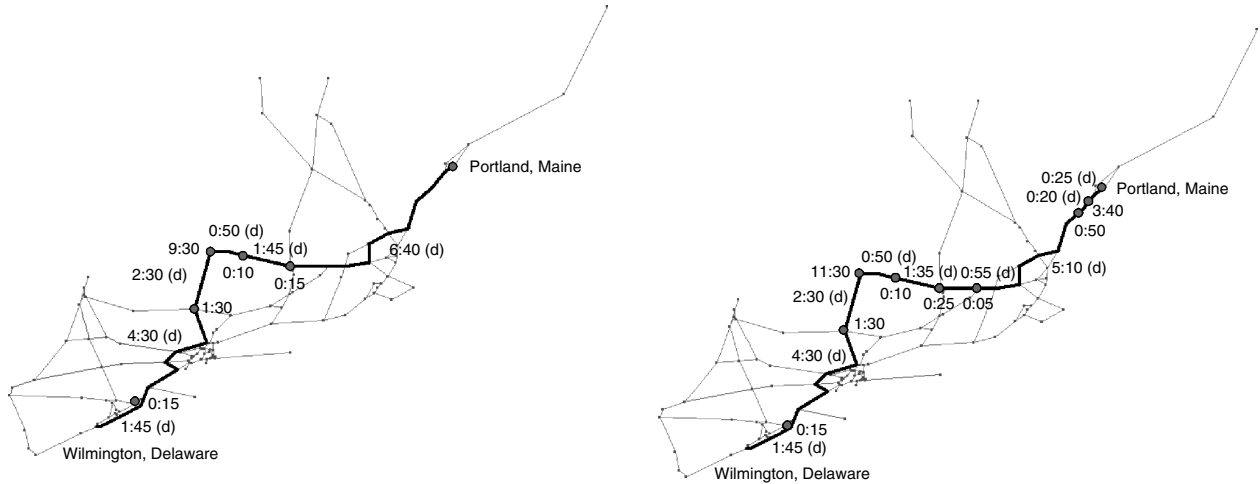
We finish the paper by discussing some enhancements of varying complexity. While we imposed a limit on path duration, it is just as easy to impose a limit on the path length. This may be more relevant for a shipper that is interested, for example, in a risk-minimizing path as long as it is no more than five percent longer than the shortest path. Likewise, although we considered on-road population to estimate exposure, it is possible to use off-road population as well. Time-dependent population data for all geographical areas may be difficult to obtain. However, it may be possible to model the most obvious population shifts (such as increased population in a downtown during the day).

Other possible enhancements deal with accident probabilities. We assumed that the accident probability is only a function of the time of day. However, it is arguable that the accident probability of a given truck driver increases as he or she becomes fatigued (say after 6 hours of uninterrupted driving). It is possible to incorporate into our DP algorithms accident probabilities that depend on the driving history since the last long rest. Likewise, it is possible to model the accident probability as a function of traffic density on the road, where the probability might increase with density up to a certain point and then decrease again as the road becomes so congested that travel speeds go down dramatically. We believe such improvements in the modeling of accident probability make the models more realistic; yet the problems are no more complicated to solve than those we considered.

Acknowledgments

This research has been supported in part by a grant from the Natural Sciences and Engineering Council of Canada

(a) Duration = 1,780 minutes, risk = 164.3×10^{-6} , length = 1,030 miles (b) Duration = 2,185 minutes, risk = 145.1×10^{-6} , length = 1,030 miles



(c) Duration = 2,770 minutes, risk = 143.4×10^{-6} , length = 1,030 miles (d) Duration = 3,070 minutes, risk = 109.4×10^{-6} , length = 1,030 miles

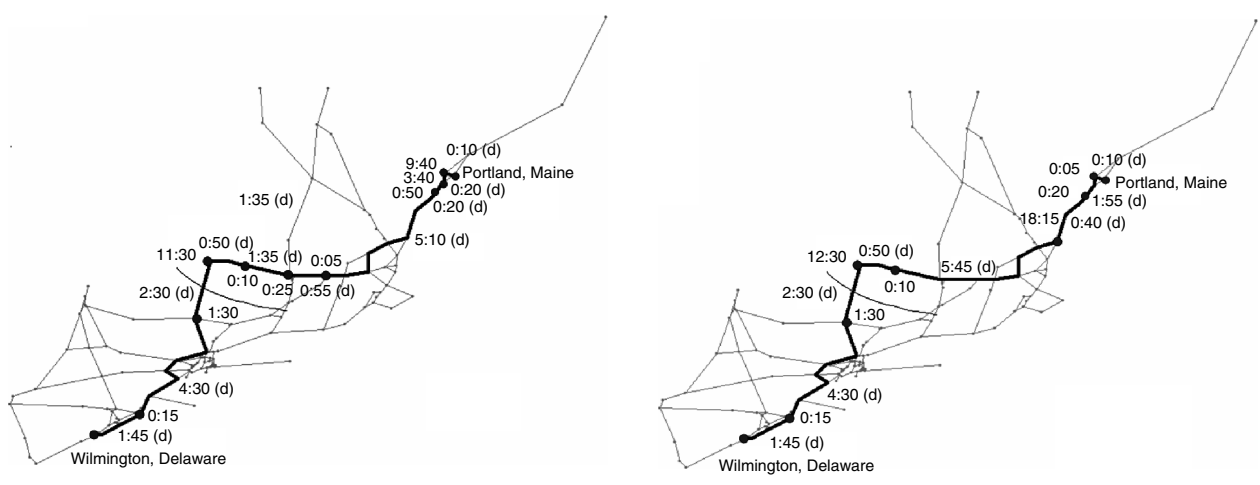


Figure 9 Four Efficient Solutions to DP-III for Increasing Values of Trip Duration (and Decreasing Risk)

Notes. Driving times are indicated with “(d).” Stops are indicated by circles along the path, and waiting times are indicated next to the circles.

(RGPIN 25481). The authors thank Dr. Linda Nozick for providing the data used in the computational experiment. Part of this research was conducted in the University of Alberta School of Business.

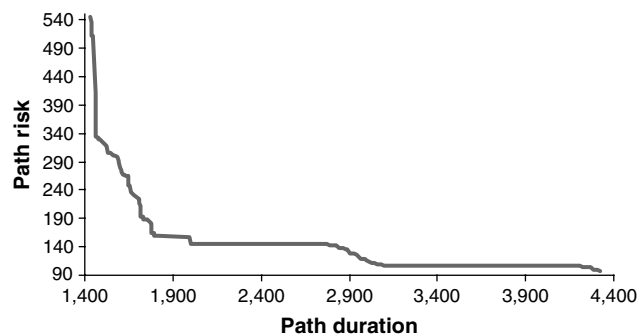


Figure 10 Efficient Frontier with Respect to Two of the Three Objectives: Path Duration and Path Risk for DP-IV

Appendix

ALGORITHM TO SOLVE DP-III.

The following algorithm can be used to solve DP-III efficiently by utilizing a binary heap data structure to keep the necessary information of feasible minimum risk paths reaching each node at every state combination.

For each arc $(x, y) \in E$, $1 \leq t \leq T$ and $1 \leq v \leq D$, let

$$\gamma_{xy}(t, v) = \min_{\{(u_a, u_d, u_r) \in F(x, y, t, v)\}} \{f_{III}(x, u_a, u_r) + r_{xy}(u_d)\}$$

with the convention that $\gamma_{xy}(t, v) = \infty$ whenever $F(x, y, t, v)$ (as defined in (16)) is empty. Then

$$f_{III}(y, t, v) = \min_{\{(x: (x, y) \in E)\}} \gamma_{xy}(t, v) \text{ for } 1 \leq t \leq T, \text{ and } 1 \leq v \leq D.$$

Let,

Heap_y = a binary heap maintained for each node y,

M(y, t) = minimum risk of departing from node y at time t with a feasible break time at node y.

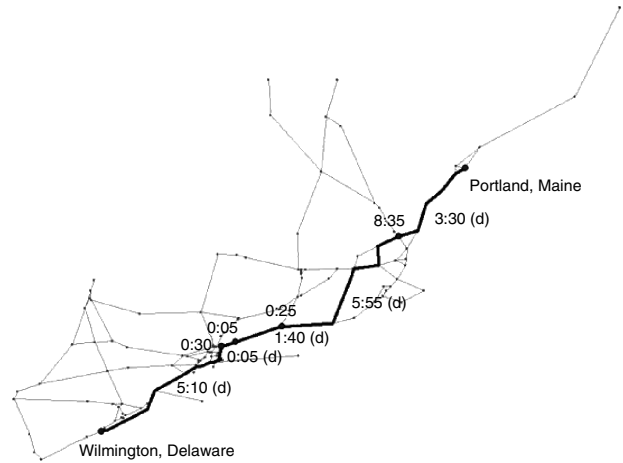
Each element of Heap_y consists of two pieces of information, tHeap_y and vHeap_y, in addition to its key (the sorting

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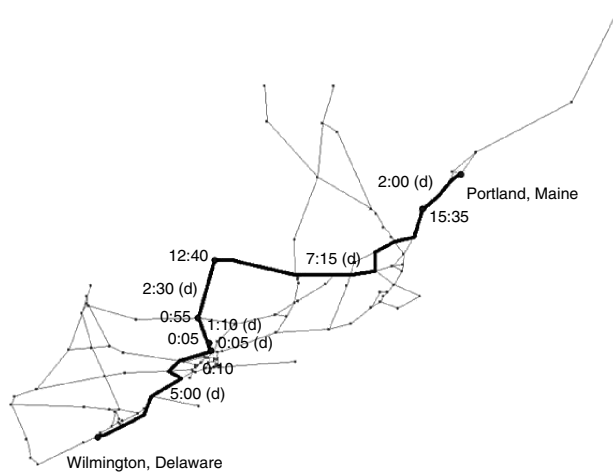
(a) Duration = 1,430 minutes, risk = 544.4×10^{-6} , length = 894 miles



(b) Duration = 1,555 minutes, risk = 301.8×10^{-6} , length = 930 miles



(c) Duration = 2,845 minutes, risk = 140.4×10^{-6} , length = 1,030 miles



(d) Duration = 4,165 minutes, risk = 104.8×10^{-6} , length = 1,208 miles

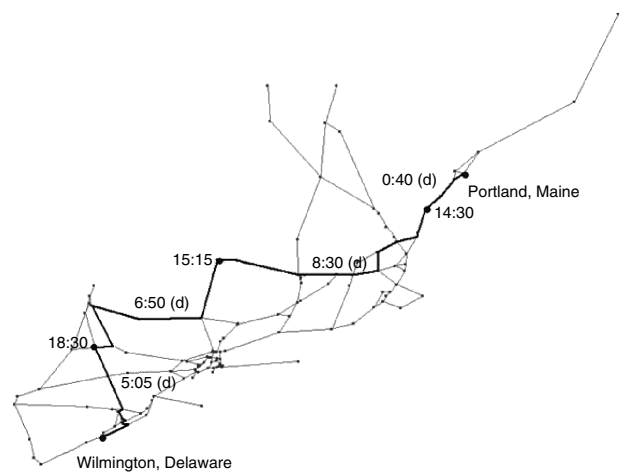


Figure 11 Four Efficient Solutions to DP-IV for Increasing Values of Trip Duration (and Decreasing Risk)

Notes. Driving times are indicated with “(d).” Stops are indicated by circles along the path, and waiting times are indicated next to the circles.

criterion). Key of each element at time t corresponds to the risk value of a minimum-risk path that starts at origin and arrives at node y at time t Heap_y so that

$$L_y \leq t - t\text{Heap}_y \leq U_y,$$

with an uninterrupted driving time of $v\text{Heap}_y$ where

$$v\text{Heap}_y = \arg \min_{1 \leq v \leq D} \{f_{\text{III}}(y, t\text{Heap}_y, v)\}.$$

$M(y, t)$ corresponds to the root element of this heap at any time.

The algorithm now can be stated as follows.

- Let $M(1, t) = 0$ for all t .
- Let $M(y, t) = \infty$ for all $y \neq 1, t$.
- Let $f_{\text{III}}(1, t, v) = 0$ for all t, v .
- Let $f_{\text{III}}(y, t, v) = \infty$ for all $y \neq 1, t, v$.
- Sort all values of $u_d + d_{xy}(u_d)$ for all $u_d = 1, \dots, T$ and for all arcs $(x, y) \in E$.
- For $t = 1, \dots, T$
- For $v = 1, \dots, D$

For all arcs (x, y) and all u_d such that

$$u_d + d_{xy}(u_d) = t. \tag{L1}$$

If $v = d_{xy}(u_d)$, then

$$\gamma_{xy}(t, v) = \min\{\infty, M(x, u_d) + r_{xy}(u_d)\}.$$

If $v > d_{xy}(u_d)$, then

$$\gamma_{xy}(t, v) = \min\{\infty, f_{\text{III}}(x, u_d, v - d_{xy}(u_d)) + r_{xy}(u_d)\}.$$

Next

For every vertex y let

$$f_{\text{III}}(y, t, v) = \min_{(x, y) \in E} \gamma_{xy}(t, v) \tag{L2}$$

$$\Phi(y, t) = \min_{1 \leq v \leq D} f_{\text{III}}(y, t, v) \tag{L3}$$

Next

For every vertex $y \neq 1$ (L4)

 If $t \geq L_y$

 Insert $\Phi(y, t - L_y)$ into Heap_y .

 If $t > U_y$, then delete the element with

$t\text{Heap}_y = t - U_y - 1$ from Heap_y .

$u_a = t\text{Heap}_y$ of the root element of Heap_y .

$u_r = v\text{Heap}_y$ of the root element of Heap_y .

$M(y, t) = f_{\text{III}}(y, u_a, u_r)$.

 End if

Next

Next

For every vertex y , $f_{\text{III}}^*(y) = \min_{0 \leq t \leq T, 0 \leq v \leq D} f_{\text{III}}(y, t, v)$ (L5)

For a given (x, y, t) , u_d can easily be found if we have a sorted list of values $u_d + d_{xy}(u_d)$. In loop (L1) $v = d_{xy}(u_d)$ indicates that stopping has occurred at node x ; therefore, $M(x, u_d)$ is used to update $\gamma_{xy}(t, v)$. On the other hand, $v > d_{xy}(u_d)$ indicates that no stopping has occurred at node x ; therefore, $f_{\text{III}}(x, u_d, v - d_{xy}(u_d))$ is used to update $\gamma_{xy}(t, v)$.

PROOF OF PROPOSITION 4. Initialization takes $O(TDN)$ time. The $u_d + d_{xy}(u_d)$ values can be sorted by bucket sorting in $O(Tm)$ time. Loop (L1) can be implemented in $O(m)$ time because the u_d values satisfying (17) can be found in $O(1)$ time from the output of the bucket sort. Loops (L2) and (L3) can be implemented without any additional effort inside loop (L1). Loop (L4) can be completed in $O(N \log T)$ time because the insertion and deletion operator on the binary heap takes $O(\log T)$ time, retrieving the root element takes $O(1)$ time, and the size of the heap is at most T . Finally, (L5) can be completed in $O(TDN)$ time. Therefore, DP-III can be solved by using this algorithm in $O(TmD + TND + TN \log T)$ time. \square

Sketch of Algorithm for DP-IV

Because the algorithm for DP-IV is similar to the algorithm for DP-III, we only summarize the main differences here. We treat long-break, short-break, and no-break situations separately. For long breaks, a binary heap structure similar to that of DP-III is maintained with an additional piece of information on uninterrupted working times. For the short breaks, we define an additional function for state variables y , t , and w to calculate the minimum risk of departing from node y at time t with an uninterrupted working time of w at the departure time and a feasible short break at node y . This function is calculated in the algorithm iteratively while maintaining the binary heap. No break case is handled as in DP-III. Erkut and Alp (2005) provide the full algorithm and further detail.

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